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Remarks on the Standard Model predictions for $R(D)$ and $R(D^*)$

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Semileptonic $b \to c$ transitions, and in particular the ratios $R(D^{(*)}) = \frac{\Gamma(B\to D^{(*)}\tau\nu)}{\Gamma(B\to D^{(*)}\ell\nu)}$, can be used to test the universality of the weak interactions. In light of the recent discrepancies between the experimental measurements of these observables by BaBar, Belle and LHCb and the Standard Model predicted values, we study the robustness of the latter. Our analysis reveals that $R(D)$ might be enhanced by lepton mass effects associated to the mostly unknown scalar form factor. In constrast, the Standard Model prediction for $R(D^*)$ is found to be more robust, since possible pollutions from B^* contributions turn out to be negligibly small, which indicates that $R(D^*)$ is a promising observable for searches of new physics.

I. INTRODUCTION

Exclusive semileptonic $b \to c$ decays provide an ideal place to test the quark flavor mixing structure [1, 2] of the Standard Model (SM) and to look for the existence of new charged currents. Among the required theoretical ingredients, accurate calculations of the relevant hadronic matrix elements are necessary to achieve these goals. Observables like

$$R(D^{(*)}) = \frac{\Gamma(B \to D^{(*)}\tau\nu)}{\Gamma(B \to D^{(*)}\ell\nu)}\,, \tag{1}$$

with $\ell = e$ or μ, are particularly interesting to test the universality of weak interactions, since most hadronic uncertainties cancel in these ratios.

The most precise SM predictions for these ratios are shown in Table I; the quoted (hadronic) uncertainties stem, respectively, purely from lattice calculations [3] and from an estimate of higher order corrections to the ratio of A_0/A_1 form factors in HQET [4]. The experimental situation regarding $R(D^{(*)})$ has improved lately with new results from Belle [5, 6] and LHCb [7] collaborations, to be added to previous results from BaBar [8]. The current world averages reported by the HFAG [9] exceed the SM predictions by 1.9 and 3.3 σ, respectively.

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Source	$R(D)$	$R(D^*)$
HFAG Exp. Av. [9]	$0.397 \pm 0.040 \pm 0.028$	$0.316 \pm 0.016 \pm 0.010$
SM Prediction	0.300 ± 0.008 [3]	0.252 ± 0.003 [4]

TABLE I: Summary of experimental results and SM predictions for $R(D^{(*)})$.

These hints of a possible violation of lepton universality have prompted many theoretical proposals, which include the exchange of charged scalars [10–16], leptoquarks (or, equivalently, R-parity violating supersymmetry) [14, 16–31], vector resonances [32] or a W' boson [16, 31, 33–36]. Possible effects due to the presence of light sterile neutrinos have also been explored in [37, 38]. We also note that the pQCD approach with lattice QCD input [39] has shown drastically reduced discrepancies from the experimental results.

In this letter we check the robustness of the SM prediction for the $R(D^{(*)})$ ratios. While the vector form factor (VFF) predictions for $B \to D\ell\nu$ decays have been tested with some detail in measurements of the branching ratios and q^2-distributions for light lepton channels, this is not the case for the scalar form factor (SFF) which is visible only in decays with τ leptons. Small departures of the SFF from lattice calculations can make compatible the SM prediction with current measurements for $R(D)$. Inversely, we can argue that the present experimental result for the $B \to D\tau\nu$ rate can determine the mostly unknown SFF, and test our knowledge of nonperturbative QCD, instead.

In contrast, the compatibility of the SM with the observed value of $R(D^*)$ would require unreasonably large departures from current form factor calculations. Here we will show that if one defines $R(D^*)$ from a narrow window of the $D\pi$ invariant mass in $B \to D\pi\ell\nu_\ell$, the additional B^* pole contribution that pollutes this decay gives a negligible small contribution. Although the individual branching ratios are sensitive to the size of the chosen narrow window, the ratio $R(D^*)$ turns out to be rather insensitive. Thus, the robustness of the SM prediction for $R(D^*)$ indicates that this observable is more promising for new physics searches in view of current discrepancies.

II. SEMILEPTONIC $B \to P$ TRANSITIONS AND $R(D)$

The simplest semileptonic charged $B \to P$ transitions ($P = D, \pi$) are denoted by $B(p_B) \to P(p_P)\ell^-(p)\bar{\nu}_\ell(p')$. The square of the momentum transfer $q = p_B - p_P = p + p'$ that characterizes the hadronic current varies within the interval $m_\ell^2 \leq q^2 \leq (m_B - m_P)^2$. Up to terms of $O(q^2/m_W^2)$, the tree-level decay amplitude is written as

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} V_{cb} \langle P|\bar{q}\gamma_\mu b|B\rangle \cdot \bar{u}(p)\gamma^\mu(1-\gamma_5)\bar{v}(p') \ . \tag{2}$$

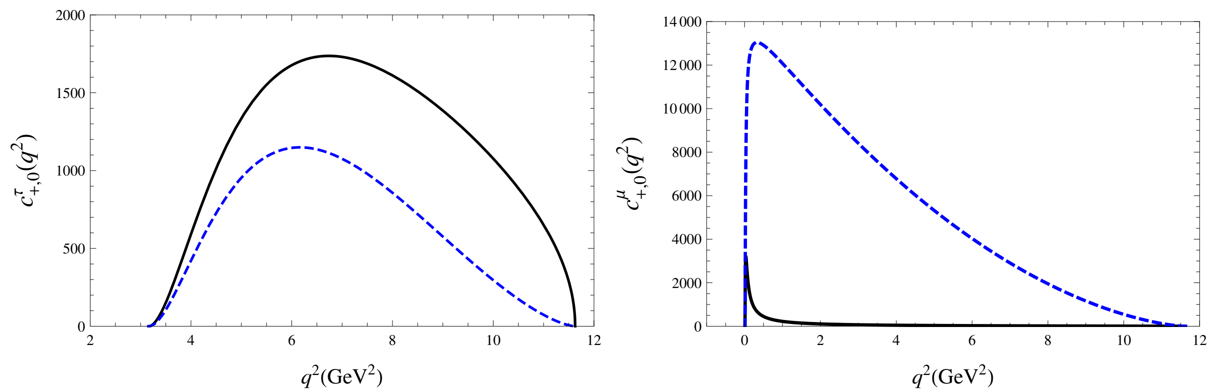

FIG. 1: The $c_0^\ell(q^2)$ ($c_+^\ell(q^2)$) coefficient of $B \to D\ell^-\bar{\nu}_\ell$ decays are plotted as a function of q^2 with a solid (dashed) line. The left (right) panel corresponds to $\ell = \tau$ ($\ell = \mu$).

Lorentz covariance fixes the hadronic matrix element to have the form[1]

$$\langle P|\bar{q}\gamma_\mu b|B\rangle = f_+(q^2)\left[(p_B + p_P)_\mu - \frac{\Delta_{BP}}{q^2}q_\mu\right] + f_0(q^2)\frac{\Delta_{BP}}{q^2}q_\mu \tag{3}$$

where we have defined $\Delta_{BP} \equiv m_B^2 - m_P^2$. The VFF and SFF are $f_+(q^2)$ and $f_0(q^2)$, respectively. They are related at $q^2 = 0$ as $f_+(0) = f_0(0)$.

The differential decay rate is given by

$$\frac{d\Gamma(B \to P\ell\nu_\ell)}{dq^2} = \frac{G_F^2|V_{cb}|^2}{192\pi^3 m_B^3}\eta_{\rm em}\left[c_+^\ell(q^2)\big|f_+(q^2)\big|^2 + c_0^\ell(q^2)\big|f_0(q^2)\big|^2\right] , \tag{4}$$

where $\eta_{\rm em}$ denotes the electroweak corrections [40].

The coefficients $c^\ell_{+,0}$ that multiply the squared form factors in the above expression are shown in Figure 1 for $\ell = \tau$ and $\ell = \mu$ [3, 41]. These plots clearly show that the effects of the SFF are sizable for the $B \to D\tau\nu$ transition, but negligibly small for $B \to D\ell\nu$ decays; also, the effects of the SFF are less important in the $B \to \pi\tau\nu$ transition.

We focus now on the $P = D$ case. The vector and scalar form factors calculated in Ref. [40] using lattice QCD are shown Figure 2. The shaded bands represent the quoted errors in Ref. [40]. From the behavior of the form factors obtained from lattice calculations and the condition $f_0(0) = f_+(0)$, the following scaling relation

$$f_0(q^2) = \left[1 + \alpha\, q^2 + \beta\, q^4\right] f_+(q^2)\,. \tag{5}$$

reproduces the scalar form factor within the kinematical range of $B \to D$ transitions. In particular, the linear approximation $\alpha = 0.020(1)$ GeV^{-2}, $\beta = 0$ [3, 41], the solid line in Figure 2, reproduces very well the central values for the SFF obtained in lattice calculations [40].

By taking a different choice for the (α, β) parameters one can get a SM prediction closer to the measured value of the $R(D)$ observable. Since the lattice results are expected to be more reliable

[1] The replacement $f_+(q^2) \to f_+(q^2)/(1 - q^2/m_W^2)$ accounts for the finite mass of the W gauge-boson. Its effects in $B \to D$ transition are relevant only for precision studies aiming at an accuracy at the few per mille level.

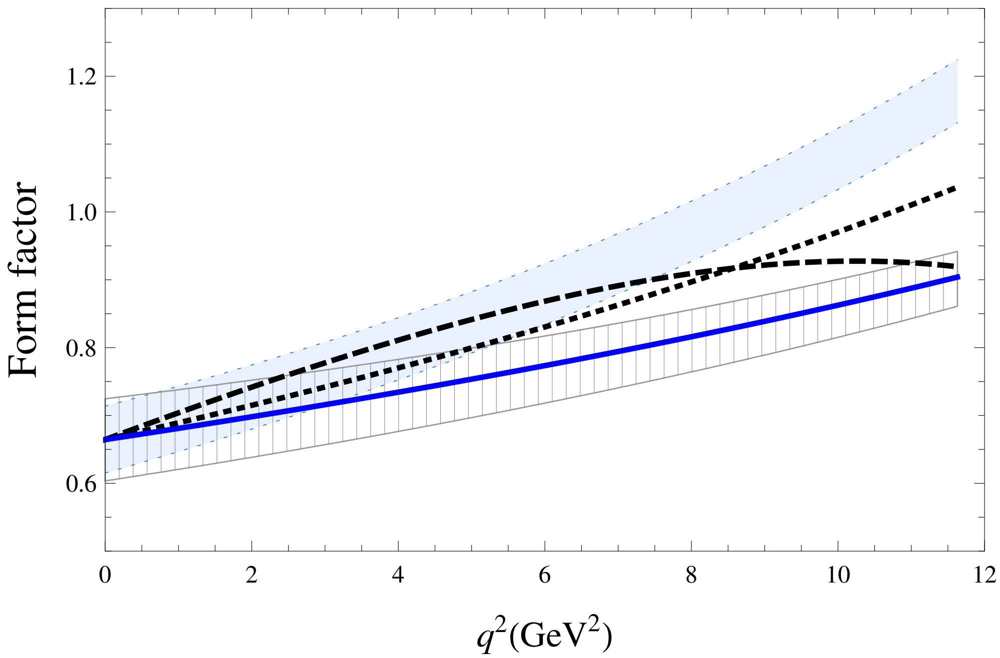

FIG. 2: The bands represent the Lattice predictions [40] for the vector (light shaded) and scalar (vertical stripes) form factors. The solid, dashed and dotted lines correspond to the scalar form factor obtained from the parametrization given in Eq. (5) with (α (GeV^{-2}), β (GeV^{-4}))=$(-0.020, 0.0)$, $(+0.016, -0.003)$ and $(-0.008, -0.0002)$, respectively.

at large q^2 values, one may choose (α, β) so that the quadratic relation (5) and the lattice results for $f_0(q^2)$ coincide at $q^2_{\rm max}$, as ilustrated by the dashed line in Figure 2. Another possible choice is to allow the scalar form factor to depart from its lattice QCD value at maximun q^2, as shown by the dotted line in Figure 2.

These two possible departures from the linear scaling [3, 41] between scalar and vector form factors lead to similar values of $R(D)$, as shown in Table II, in better agreement with the experimental measurement. We note that assuming an error bar for the dashed line as wide as the one for the lattice calculation of the SFF (vertical stripes band in Figure 2) would lead to an overlap among them. In this case, the resulting $R(D)$ value would be very close to the experimental measurement.

One may re-interpret this by stating that the current experimental value of $R(D)$ indicates that the scalar form factor departs from current lattice calculations by at least 10% for certain q^2 values. This conclusion is justified by the absence of an independent and more precise test of the scalar form factor besides the one provided by measurements of $R(D)$. Measurements of the q^2-distributions in $B \to D\tau\nu_\tau$ decays will then be helpful as tests of lattice calculations. The robustness of the SM calculation of $R(D)$ depends crucially on a better knowledge of the SFF.

Finally, let us comment that the effect of similar changes in the SFF are very small in the case of the ratio $R(\pi) = B(B \to \pi\tau\nu_\nu)/B(B \to \pi\mu\nu_\mu)$. This follows from the smaller difference between the c_0^τ and c_+^τ coefficients in $B \to \pi\tau\nu$ transitions.

	$\alpha(\mathrm{GeV}^{-2}),\ \beta(\mathrm{GeV}^{-4})$	$R(D)$
-	$-0.020,\ 0.000$	0.300
-	$-0.008,\ -0.0002$	0.339
-	$+0.016,\ -0.003$	0.335
SM Prediction [42]		0.300 ± 0.008
Measured Value [42]		$0.397 \pm 0.040 \pm 0.028$

TABLE II: Predictions for the $R(D)$ ratio using the parametrization given in Eq. (5).

III. $B \to D\pi\ell^-\bar{\nu}_\ell$ DECAYS AND THE DEFINITION OF $R(D^*)$

In contrast to the strong dependence of the $B \to D\tau\nu$ rate on the SFF, the $B \to D^*\tau\nu$ decay does not depend strongly on any unknown form factors, and thus the SM prediction for $R(D^*)$ is rather robust. Therefore, a discrepancy between the $R(D^*)$ experimental measurement and its predicted value in the SM would indicate a strong hint in favor of new physics. In the following we proceed to substantiate this claim. In particular, we will explore a possible deviation in $R(D^*)$ through an $R(D\pi)$ contribution, which we find to be negligibly small.

Theoretical calculations assume the D^* meson in $B \to D^*\ell\nu_\ell$ (denoted as $B_{\ell 3}(D^*)$) decays to be an asymptotic state. This allows to assume that $\langle D^*|j_\mu|B\rangle$ is the hadronic matrix element of the S-matrix in the factorization approximation. Previous studies that take into account the effects of decays of τ leptons and/or D^* mesons in some kinematical distributions were reported in [43, 44]. Experimentally, the observable process is $B \to D\pi(D\gamma)\ell\nu$, and the observables associated to $B_{\ell 3}(D^*)$ are obtained by chosing $D\pi$ $(D\gamma)$ events within a narrow window of their invariant mass distribution around the D^* mass (hereafter we focus our discussion only on the $D\pi$ final states). The possible effects of higher D^{**} resonances decaying into $D\pi$ are taken into account in simulations. Here we consider the possible effects of a B^* pole contribution in addition to the D^* pole and assess its effects in the extraction of the $R(D^*)$ observable.

We can define the ratio

$$R(D\pi) \equiv \frac{\Gamma(B \to D\pi\tau\nu_\tau)}{\Gamma(B \to D\pi\ell\nu_\ell)} \tag{6}$$

from the $B(p_B) \to D(p_1)\pi(p_2)\ell(p_3)\nu(p_4)$ $(B_{\ell 4}(D\pi))$ decays. The Feynman diagrams contributing to this decay are shown in Figure 3.

In the narrow D^* width approximation, the contribution in Figure 3(b) yields $B(B \to D\pi\ell\nu_\ell) = B(B \to D^*\ell\nu_\ell) \cdot B(D^* \to D\pi)$, thus the definition of $R(D\pi)$ and $R(D^*)$ are completely equivalent. This is not the case in the presence of the other contributions, which will lead to $R(D\pi) = R(D^*) \times (1 + \delta_{D\pi})$, where $\delta_{D\pi}$ is a pollution that remains owing to non-D^* contributions. We expect these additional contributions to be very small for a narrow window around the D^* mass, and we turn to evaluate them numerically. While the D^* pole gives rise to pure p-wave contributions of the $D\pi$ system, the B^* pole can contribute to other configurations as well.

In our calculation we assume that the pole contributions are the dominant ones. Thus, the

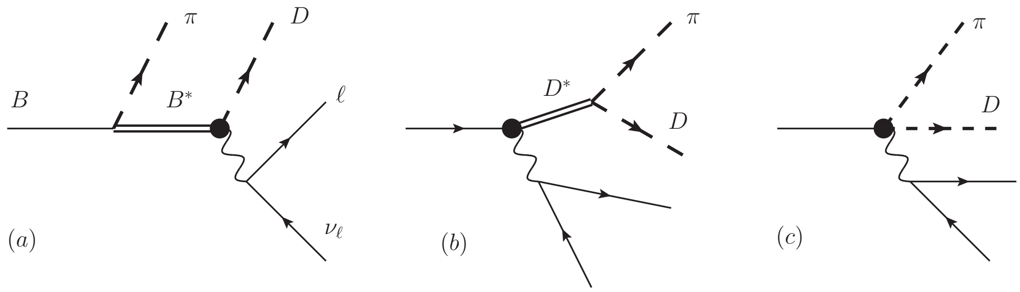

FIG. 3: Contributions to the four-body semileptonic decay $B \to D\pi\ell^-\bar{\nu}_\ell$. Double-lines are used for the intermediate vector resonances. The solid dot indicates the hadronic weak vertex.

decay amplitude becomes:

$$\begin{aligned}\mathcal{M} &= \mathcal{M}_a + \mathcal{M}_b \\ &= \frac{G_F V_{cb}}{\sqrt{2}} H_\alpha L^\alpha \ ,\end{aligned} \tag{7}$$

where L^α is the leptonic weak current and the hadronic matrix element is denoted as:

$$H_\alpha = \langle D(p_1)\pi(p_2)|j_\alpha|B(p_B)\rangle \ , \tag{8}$$

where j_α is the SM weak current for the $b \to c$ transition. Explicitly, the hadronic matrix elements corresponding to Figures 3 (a) and (b) are:

$$H^a_\alpha = g_{BB^*\pi}\langle D(p_1)|j_\alpha|\widetilde{B^*}_\beta(p_B - p_2)\rangle(p_B + p_2)_\nu \left[\frac{N^{\nu\beta}_{B^*}(p_B - p_2)}{(p_B - p_2)^2 - m^2_{B^*}}\right] \tag{9}$$

$$H^b_\alpha = g_{DD^*\pi}\langle\widetilde{D^*}_\beta(p_1 + p_2)|j_\alpha|B(p_B)\rangle(p_1 - p_2)_\nu \left[\frac{N^{\nu\beta}_{D^*}(p_1 + p_2)}{(p_1 + p_2)^2 - m^2_{D^*} + i m_{D^*}\Gamma_{D^*}}\right] , \tag{10}$$

where $g_{VV^*\pi}$ denote the strong coupling constants and $N^{\nu\beta}_{V^*}(q) = T_{\nu\beta}(q) + L_{\nu\beta}(q)(m^{*2}_V - q^2)/m^2_{V^*}$, with the transverse and longitudinal projectors defined by $T_{\nu\beta}(q) = g_{\nu\beta} - q_\nu q_\beta/q^2$ and $L_{\nu\beta}(q) = q_\nu q_\beta/q^2$. In the above expressions, the tildes denote the *off-shell* vector meson intermediate states with Lorentz index β replacing their polarization four-vectors. The D^* propagator has been provided with a finite width because it can be produced on-shell. Owing to similar $B^* - B$ and $D^* - D$ squared mass differences [42], the real parts in the denominators of the propagators may have similar sizes, thus the heavyness of the B^* meson in principle does not provide a kinematical suppression.

For the purposes of numerical evaluations, we use the results of Ref. [4] for the hadronic matrix element of the $B \to D^*$ transition. The $B^* \to D$ matrix element has a similar Lorentz structure as the $B \to D^*$ transition, although with different form factors; we can use the Heavy quark symmetry to relate it to the one of $B \to D$ decay, although for the purposes of the present work we use the form factors given in Ref. [45]. We will use $g_{BB^*\pi} = 20.0 \pm 1.2$ [46] which is consistent with other recent determinations [47, 48]; also, we use the experimental value $g_{DD^*\pi} = 8.39 \pm 0.08$ [42].

TABLE III: Sensitivity of the branching ratio $B(B^0 \to D^+\pi^0\ell^-\nu)$ and $R_\ell(D\pi)$ (the subscript ℓ in the definition of $R(D\pi)$ refers to the specific light ℓ channel used as normalization) to different regions of integration over s_{12}. In the calculation of the branching fractions we have used the $|V_{cb}|$ and form factor parameters of Ref. [50] and the average B^0 lifetime of Ref. [42].

Channel\Δ value	$0.5\Gamma_{D^*}$	Γ_{D^*}	$1.5\Gamma_{D^*}$	$2\Gamma_{D^*}$	1 MeV
$B(B^0 \to D^+\pi^0\tau^-\bar{\nu}_\tau)$	0.00354	0.00499	0.00563	0.00598	0.00689
$B(B^0 \to D^+\pi^0\mu^-\bar{\nu}_\mu)$	0.01399	0.01972	0.02225	0.02360	0.02725
$B(B^0 \to D^+\pi^0 e^-\bar{\nu}_e)$	0.01405	0.01981	0.02235	0.02372	0.02736
$R_\mu(D\pi)$	0.2532	0.2532	0.2532	0.2534	0.2532
$R_e(D\pi)$	0.2520	0.2520	0.2520	0.2520	0.2520

The four-body $B_{\ell 4}(D\pi)$ decay can be described in terms of five independent kinematical variables. We chose the special set defined in Ref. [49], with $s_{12} = (p_1+p_2)^2$ and $s_{34} = (p_3+p_4)^2$ as two relevant variables. For our example under consideration, the allowed phase space is determined by $(m_D+m_\pi)^2 \leq s_{12} \leq (m_B-m_\ell)^2$ and $m_\ell^2 \leq s_{34} \leq (m_B-\sqrt{s_{12}})^2$. Leaving s_{12} as the last integration variable, Eq. (6) can be written as:

$$R_{D\pi} = \frac{\int_{s_{12}^-}^{s_{12}^+} ds_{12} \dfrac{d\Gamma(B\to D\pi\tau\nu)}{ds_{12}}}{\int_{s_{12}^-}^{s_{12}^+} ds_{12} \dfrac{d\Gamma(B\to D\pi\ell\nu)}{ds_{12}}} \tag{11}$$

where the limits of integration $s_{12}^{\pm} = (m_D^* \pm \Delta)^2$. This allows to study the dependence of the decay rates upon the size of the small window around the D^* mass.

As it was already mentioned, in the narrow width approximation obtained by setting $\Delta = 0$, we recover the result $R(D\pi) = R(D^*)$ since the B^* pole gives a vanishing contribution. In Table III we show the result of our calculations of the branching rations and of $R(D\pi)$ using Eq. (11), for different values of Δ. As it can be noticed, the branching fractions are sensitive to the cuts employed to define the D^* mass window, although the ratio $R(D\pi)$ is insensitive to the value of Δ to the quoted accuracy. The relative size of the B^* pole contribution with respect to the D^*pole contribution is very small for the different intervals chosen for s_{12}. Using the muon or the electron channels to normalize the τ decay rate, makes a difference of only about 0.5% in $R(D\pi)$.

IV. SUMMARY AND CONCLUSIONS

The $R(D^{(*)})$ ratios are useful observables to study possible violations of the charged current universality. The SM prediction for $R(D)$ is sensitive to the scalar form factor (SFF) in $B \to D$ semileptonic decays, and the current prediction relies on lattice calculations which have not been provided with independent tests. We have confirmed that increasing the SFF by up to 10% with respect to lattice results affects mainly the tau decay channels and can render the SM prediction in agreement with current measurements.

The situation is different for $R(D^*)$, since it requires strong variations of the SFF to produce a sizable change in the $B \to D^*\tau\nu_\tau$ rate. Since D^* mesons are unstable states that are detected from

$D\pi$ events very close to threshold in $B \to D\pi\ell\nu$ decays, we have studied the possible contamination of the D^* signal by other allowed contributions. Considering the B^* pole as the dominant additional contribution, we evaluate its impact in the extraction of $R(D^*)$ and find that it gives a negligible contribution when choosing a narrow window in the $D\pi$ invariant mass distribution.

In conclusion, the SM prediction for $R(D^*)$ looks more robust than the one for $R(D)$ because it is less sensitive to hadronic form factors that are enhanced by lepton mass effects. Extracting the $R(D^*)$ ratio from observable S-matrix elements like $B \to D\pi\ell\nu_\ell$ may include additional B^* contributions that pollute the D^* signal; fortunately they turn out to be negligibly small.

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