

Topics about the Two Higgs Doublet Model

Pedro Ferreira

ISEL and CFTC, UL



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OUTLINE

- 1 – The Two-Higgs Doublet potential
- 2 – Symmetries of the 2HDM
- 3 – The vacuum structure of the 2HDM
- 4 – One-loop contributions to inert minima
- 5 – LHC constraints on 2HDM parameters
- 6 – Current limitations of the 2HDM

- LHC discovered a new particle with mass ~ 125 GeV.
- Up to now, all is compatible with the Standard Model (SM) Higgs particle.

BORING!

Two-Higgs Dublet model, 2HDM (Lee, 1973) : one of the easiest extensions of the SM, with a richer scalar sector. **Can help explain the matter-antimatter asymmetry of the universe, provide dark matter candidates, ...**

1 – The Two-Higgs Doublet potential

Most general $SU(2) \times U(1)$ scalar potential:

$$\begin{aligned} V = & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - [m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{H.c.}] \\ & + \frac{1}{2} \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1) \\ & \times (\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) + [\frac{1}{2} \lambda_5 (\Phi_1^\dagger \Phi_2)^2 \\ & + \lambda_6 (\Phi_1^\dagger \Phi_1) (\Phi_1^\dagger \Phi_2) + \lambda_7 (\Phi_2^\dagger \Phi_2) (\Phi_1^\dagger \Phi_2) + \text{H.c.}] \end{aligned}$$

m_{12}^2 , λ_5 , λ_6 and λ_7 complex - seemingly **14** independent real parameters

Most frequently studied model: **softly broken theory with a Z_2 symmetry**,

MODEL I: Only Φ_2 couples to fermions.

MODEL II: Φ_2 couples to up-quarks, Φ_1 to down quarks and leptons.

...

Scalar sector of the 2HDM is richer => more stuff to discover

Two doublets => 4 neutral scalars (h, H, A) + 1 charged scalar (H^\pm).

$\left. \begin{matrix} \mathbf{h} \\ \mathbf{H} \end{matrix} \right\}$ CP-even scalars

$\mathbf{h}, \mathbf{H} \rightarrow \gamma \gamma$
 $\mathbf{h}, \mathbf{H} \rightarrow \mathbf{ZZ}, \mathbf{WW}$ (real or off-shell)
 $\mathbf{h}, \mathbf{H} \rightarrow \mathbf{ff}$
 $\mathbf{H} \rightarrow \mathbf{hh}$ (if $m_{\mathbf{H}} > 2m_{\mathbf{h}}$)
...

\mathbf{A} - CP-odd scalar
(pseudoscalar)

$\mathbf{A} \rightarrow \gamma \gamma$
 ~~$\mathbf{A} \rightarrow \mathbf{ZZ}, \mathbf{WW}$~~
 $\mathbf{A} \rightarrow \mathbf{ff}$
...

Certain versions of the model provide a simple and natural candidate for Dark Matter – *INERT MODEL*, based on an unbroken discrete symmetry.

Deshpande, Ma (1978); Ma (2006); Barbieri, Hall, Rychkov (2006); Honorez, Nezri, Oliver, Tytgat (2007)

Doublet field components:

$$\Phi_a = \begin{pmatrix} \phi_a^+ \\ (v_a + \rho_a + i\eta_a) / \sqrt{2} \end{pmatrix}, \quad a = 1, 2$$

Definition of β angle:

$$\tan \beta \equiv \frac{v_2}{v_1}$$

**Definition of α angle
(h, H: CP-even scalars):**

$$\begin{aligned} h &= \rho_1 \sin \alpha - \rho_2 \cos \alpha, \\ H &= -\rho_1 \cos \alpha - \rho_2 \sin \alpha \end{aligned}$$

(without loss of generality: $\pi/2 \leq \alpha \leq +\pi/2$)

Couplings of scalars to fermions and gauge bosons depend on α , β .

For gauge bosons, for instance:

$$(g_{hZZ})^{2HDM} = \sin(\beta - \alpha) (g_{hZZ})^{SM}$$

Models without tree-level FCNC

Each type of fermion only couples to ONE of the doublets. Four possibilities, with the convention that the up-quarks always couple to Φ_2 :

	Type I	Type II	Lepton-specific	Flipped
ξ_h^u	$\cos \alpha / \sin \beta$	$\cos \alpha / \sin \beta$	$\cos \alpha / \sin \beta$	$\cos \alpha / \sin \beta$
ξ_h^d	$\cos \alpha / \sin \beta$	$-\sin \alpha / \cos \beta$	$\cos \alpha / \sin \beta$	$-\sin \alpha / \cos \beta$
ξ_h^ℓ	$\cos \alpha / \sin \beta$	$-\sin \alpha / \cos \beta$	$-\sin \alpha / \cos \beta$	$\cos \alpha / \sin \beta$
ξ_H^u	$\sin \alpha / \sin \beta$	$\sin \alpha / \sin \beta$	$\sin \alpha / \sin \beta$	$\sin \alpha / \sin \beta$
ξ_H^d	$\sin \alpha / \sin \beta$	$\cos \alpha / \cos \beta$	$\sin \alpha / \sin \beta$	$\cos \alpha / \cos \beta$
ξ_H^ℓ	$\sin \alpha / \sin \beta$	$\cos \alpha / \cos \beta$	$\cos \alpha / \cos \beta$	$\sin \alpha / \sin \beta$
ξ_A^u	$\cot \beta$	$\cot \beta$	$\cot \beta$	$\cot \beta$
ξ_A^d	$-\cot \beta$	$\tan \beta$	$-\cot \beta$	$\tan \beta$
ξ_A^ℓ	$-\cot \beta$	$\tan \beta$	$\tan \beta$	$-\cot \beta$

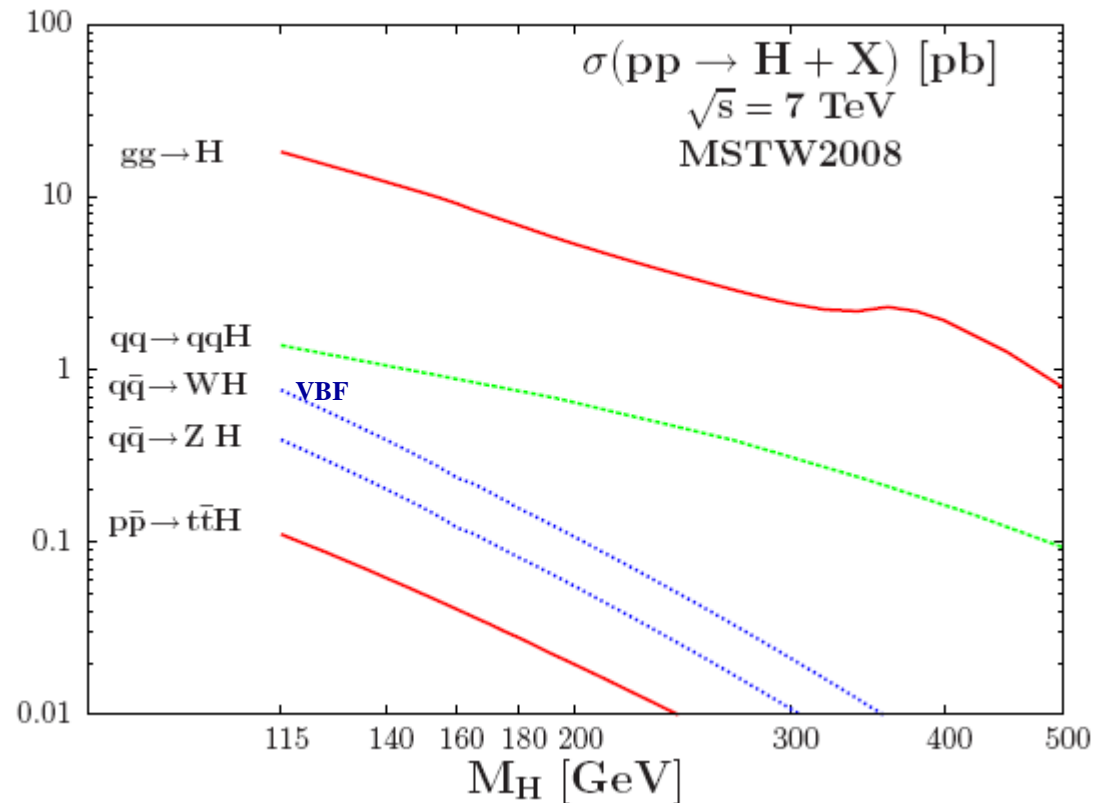
$$= \left\{ \frac{1}{v} \bar{u} (m_u \xi_A^L + m_d \xi_A^R) u \right\} + \frac{1}{v} \bar{\nu}_L \nu_R + \text{h.c.} \Big\}$$

What we compare to data:

$$\mu_f = \frac{\sigma^{2HDM}(pp \rightarrow h) BR^{2HDM}(h \rightarrow f)}{\sigma^{SM}(pp \rightarrow h) BR^{SM}(h \rightarrow f)}$$

Plenty of different
production processes
possible at the LHC:

J. Baglio and A. Djouadi, JHEP 03
(2011) 055



2 – Symmetries of the 2HDM

Higgs Family
Symmetries:

$$\mathbf{Z}_2: \Phi_1 \rightarrow \Phi_1, \quad \Phi_2 \rightarrow -\Phi_2$$

$$\mathbf{U}(1): \Phi_1 \rightarrow \Phi_1, \quad \Phi_2 \rightarrow e^{i\theta} \Phi_2 \quad \theta \neq \{0, \pi\}$$

$$\mathbf{U}(2): \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} \rightarrow U \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} \quad \forall U \in U(2)$$

Generalized CP
Transformations:

$$\mathbf{CP1}: \Phi_1 \rightarrow \Phi_1^*, \quad \Phi_2 \rightarrow \Phi_2^*$$

$$\mathbf{CP2}: \Phi_1 \rightarrow \Phi_2^*, \quad \Phi_2 \rightarrow -\Phi_1^*$$

$$\mathbf{CP3}: \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} \rightarrow \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \Phi_1^* \\ \Phi_2^* \end{pmatrix} \quad 0 < \theta < \pi/2$$

Symmetries of the potential of 2HDM

symmetry	m_{11}^2	m_{22}^2	m_{12}^2	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	
Z_2			0					real	0	0	7
U(1)			0					0	0	0	6
U(2)	m_{11}^2		0	λ_1		$\lambda_1 - \lambda_3$		0	0	0	3
CP1			real					real	real	λ_6	10
CP2	m_{11}^2		0	λ_1				real	0	0	5
CP3	m_{11}^2		0	λ_1			$\lambda_1 - \lambda_3 - \lambda_4$ (real)		0	0	4

$$\begin{aligned}
 V = V = & m_{11}^2(\varphi_1^\dagger\varphi_1) + m_{11}^2(\varphi_2^\dagger\varphi_2) \\
 & + \frac{1}{2}\lambda_1(\varphi_1^\dagger\varphi_1)^2 + \frac{1}{2}\lambda_1(\varphi_2^\dagger\varphi_2)^2 + \lambda_3(\varphi_1^\dagger\varphi_1)(\varphi_2^\dagger\varphi_2) \\
 & + (\lambda_1 - \lambda_3)(\varphi_1^\dagger\varphi_2)(\varphi_2^\dagger\varphi_1)
 \end{aligned}$$

Symmetries of the LAGRANGIAN of 2HDM

symmetry	m_{11}^2	m_{22}^2	m_{12}^2	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	
Z_2			0					real	0	0	7
U(1)			0					0	0	0	6
U(2)	m_{11}^2		0		λ_1		$\lambda_1 - \lambda_3$	0	0	0	3
CP1			real					real	real	λ_6	10
CP2	m_{11}^2		0		λ_1			real	0	0	5
CP3	m_{11}^2		0		λ_1		$\lambda_1 - \lambda_3 - \lambda_4$ (real)		0	0	4

MASSLESS FERMIONS: U(2), CP2, CP3 (if you're not careful!)

Three generations of massive fermions: CP1, Z_2 , U(1) and CP3 (but bad CKM!)

Plus absence of tree-level FCNC: Z_2 , U(1)

Remainder *USELESS*...? Not necessarily so...

OTHER POSSIBILITIES:

- Approximate symmetries broken by hypercharge, like *custodial symmetry*.
(A. Pilaftsis, Phys.Lett. B706 (2012) 465)
- Use alignment *ansatz* for the fermion sector, instead of symmetry.
(A.Pich, P.Tuzon, Phys.Rev. D80 (2009) 091702)
- Make these GLOBAL symmetries local, thus dropping the need of a soft breaking term but introducing new gauge bosons (S. Choi, S. Jung, P. Ko, JHEP 1310 (2013) 225)
- . . .

BGL Models

Before symmetry breaking, *before* rotating quarks into mass basis:

$$\mathcal{L}_{\text{Yukawa}} = - \left(\bar{p}_L, \bar{n}_L \right) \left[(\Phi_1 \Gamma_1 + \Phi_2 \Gamma_2) n_R + \left(\tilde{\Phi}_1 \Delta_1 + \tilde{\Phi}_2 \Delta_2 \right) p_R \right] + \text{H.c.},$$

Define:

Down-type quark mass matrices



$$M_n = \frac{v_1 \Gamma_1 + v_2 \Gamma_2}{\sqrt{2}}, \quad N_n = \frac{v_2 \Gamma_1 - v_1 \Gamma_2}{\sqrt{2}},$$



Down-type FCNC interactions

Up-type quark mass matrices



$$M_p = \frac{v_1 \Delta_1 + v_2 \Delta_2}{\sqrt{2}}, \quad N_p = \frac{v_2 \Delta_1 - v_1 \Delta_2}{\sqrt{2}}$$



Up-type quark mass matrices

BGL Models

Impose following symmetry on the Lagrangian:

$$Q_{L1} \rightarrow e^{i\sigma} Q_{L1}, \quad n_{R1} \rightarrow e^{2i\sigma} n_{R1}, \quad \phi_2 \rightarrow e^{-i\sigma} \phi_2$$

This symmetry distinguishes between flavours in the quark sector – *different flavours will have different interactions*

U(1) type symmetry in the scalar sector

$$\Gamma_1 \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & \times & \times \\ 0 & \times & \times \end{pmatrix}, \quad \Gamma_2 \sim \begin{pmatrix} \times & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \Delta_1 \sim \begin{pmatrix} 0 & 0 & 0 \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix}, \quad \Delta_2 \sim \begin{pmatrix} \times & \times & \times \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Rotating to the quark mass basis where $\mathbf{M}_n, \mathbf{M}_p$ are diagonal:

- The down-sector matrix \mathbf{N}_n is diagonal – NO FCNC.
- The up-sector matrix \mathbf{N}_p is non-diagonal - THERE ARE FCNC IN THIS SECTOR.

BGL Models

Flavour changing neutral interactions are extremely constrained by experimental data – if a model has tree-level FCNC mediated by neutral scalars, the respective couplings need to be extremely small – usually involves a lot of fine-tuning.

But, in BGL models, the symmetries imposed force all FCNC interactions to be governed by the CKM matrix!

Thus FCNC interactions are **NATURALLY** small, without fine-tuning!

There are *at least 6* different BGL-type models, depending on the type of symmetries one chooses for the quark sector – FCNC on the up or down sector, in each of the flavours.

The number of models increases (**36**) if one considers also FCNC in the leptons.

Example: FCNC in the up sector

Down-quark neutral scalar interactions:

$$\begin{aligned}\mathcal{L}_{\text{Yukawa}} = & \bar{d}_1 \frac{m_1}{v} \frac{h \cos \alpha + H \sin \alpha + i A \gamma_5 \cos \beta}{\sin \beta} d_1 \\ & + \sum_{j=2}^3 \bar{d}_j \frac{m_j}{v} \frac{-h \sin \alpha + H \cos \alpha - i A \gamma_5 \sin \beta}{\cos \beta} d_j + \dots\end{aligned}$$

Up-quark neutral scalar interactions:

$$\begin{aligned}& \dots + h \left[-\frac{\sin \alpha}{\cos \beta} \sum_{\psi=u,c,t} \bar{\psi} \frac{m_\psi}{v} \psi + \frac{\cos(\alpha - \beta)}{\sin \beta \cos \beta} \sum_{\psi,\chi=u,c,t} V_{\psi 1} V_{\chi 1}^* \bar{\psi} \left(\frac{m_\chi}{v} P_R + \frac{m_\psi}{v} P_L \right) \chi \right] \\ & + H \left[\frac{\cos \alpha}{\cos \beta} \sum_{\psi=u,c,t} \bar{\psi} \frac{m_\psi}{v} \psi + \frac{\sin(\alpha - \beta)}{\sin \beta \cos \beta} \sum_{\psi,\chi=u,c,t} V_{\psi 1} V_{\chi 1}^* \bar{\psi} \left(\frac{m_\chi}{v} P_R + \frac{m_\psi}{v} P_L \right) \chi \right] \\ & + A \left[\tan \beta \sum_{\psi=u,c,t} \bar{\psi} \frac{m_\psi}{v} i \gamma_5 \psi + \frac{i}{\sin \beta \cos \beta} \sum_{\psi,\chi=u,c,t} V_{\psi 1} V_{\chi 1}^* \bar{\psi} \left(-\frac{m_\chi}{v} P_R + \frac{m_\psi}{v} P_L \right) \chi \right].\end{aligned}$$

3 – The vacuum structure of the 2HDM

Vacuum structure more rich => different types of stationary points/minima possible!

The **NORMAL** minimum, $\langle \Phi_1 \rangle_N = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}$, $\langle \Phi_2 \rangle_N = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}$

The **CHARGE BREAKING (CB)** minimum, with

$$\langle \Phi_1 \rangle_{CB} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ c_1 \end{pmatrix} , \quad \langle \Phi_2 \rangle_{CB} = \frac{1}{\sqrt{2}} \begin{pmatrix} c_2 \\ c_3 \end{pmatrix}$$

c_2 has electric charge => breaks $U(1)_{em}$

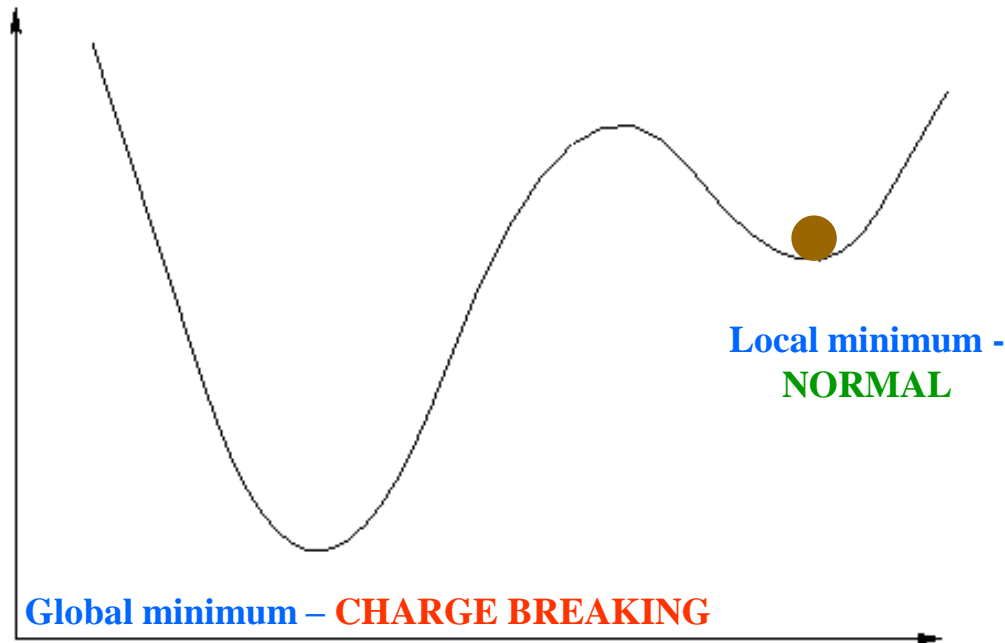
The **CP BREAKING** minimum, with

$$\langle \Phi_1 \rangle_{CP} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \bar{v}_1 \end{pmatrix} , \quad \langle \Phi_2 \rangle_{CP} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \bar{v}_2 e^{i\theta} \end{pmatrix}$$

$\theta \neq 0, \pi$ breaks CP

Would there be any problem if the potential had two of these minima simultaneously?

Answer: there might be, if the CB minimum, for instance, were “deeper” than the normal one (metastable).



$$\cancel{m_\gamma = 0}$$

$$m_\gamma \neq 0 !$$

THEOREM: if a Normal Minimum exists, the Global minimum of the theory is Normal - the photon is guaranteed to be massless.

(not so in SUSY, for instance)

$$V_{CB} - V_N = \left(\frac{m_{H^\pm}^2}{4v^2} \right)_N [(v_1 c_3 - v_2 c_1)^2 + v_1^2 c_2^2]$$

Barroso,
Ferreira,
Santos

So: if $m_{H^\pm}^2 > 0$, then $V_{CB} - V_N > 0 \Rightarrow V_{CB} > V_N$!

Plus, it is possible to prove that in this case **CB is a saddle point.**

$$V_{CP} - V_N = \left(\frac{m_A^2}{4v^2} \right)_N [(\bar{v}_2 v_1 \cos \theta - \bar{v}_1 v_2)^2 + \bar{v}_2^2 v_1^2 \sin^2 \theta]$$

Likewise: if $m_A^2 > 0$, then $V_{CP} - V_N > 0 \Rightarrow V_{CP} > V_N$!

Also, it is possible to prove that in this case **CP is a saddle point.**

If **N** is a minimum, it is the deepest one, and stable against **CB** or **CP**!

But there is another possibility –

Ivanov

AT MOST TWO NORMAL MINIMA COEXISTING...

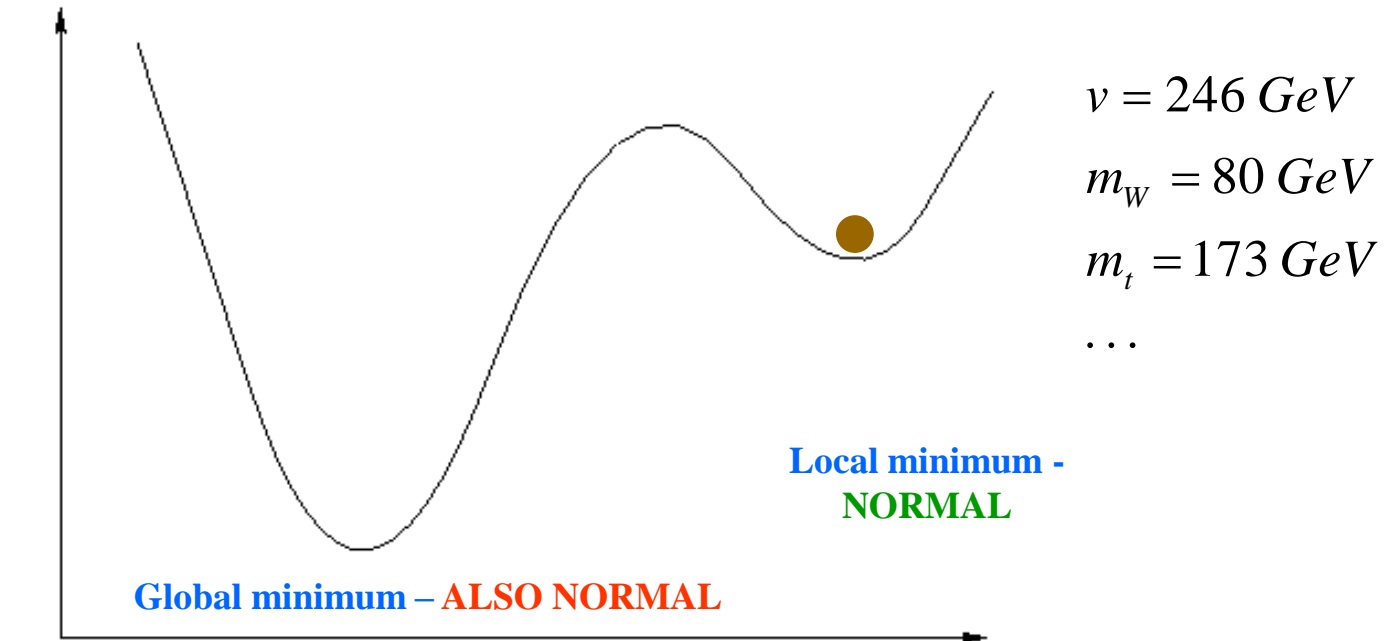
$$\langle \Phi_1 \rangle_N = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle \Phi_2 \rangle_N = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix} \quad \langle \Phi_1 \rangle_{N'} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v'_1 \end{pmatrix}, \quad \langle \Phi_2 \rangle_{N'} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v'_2 \end{pmatrix}$$

The relationship between the depths of the potential at both minima is given by

$$V_{N'} - V_N = \frac{1}{4} \left[\left(\frac{m_{H^\pm}^2}{v^2} \right)_N - \left(\frac{m_{H^\pm}^2}{v^2} \right)_{N'} \right] (v_1 v'_2 - v_2 v'_1)^2$$

**Now there isn't an obvious minimum,
each of them can be the deepest!**

So, though our vacuum cannot tunnel to a deeper CB or CP minimum, there is another scary prospect...



$v \neq 246 \text{ GeV}$

$m_W \neq 80 \text{ GeV}$

$m_t \neq 173 \text{ GeV}$

...

PANIC VACUUM!!

Cannot occur for SUSY, models with exact $U(1)$ or Z_2 symmetries
(exception – INERT MODEL!)

Can occur if there is soft symmetry breaking!
(or for potentials with only CP symmetry, or
no symmetry at all)

**Under what conditions can the 2HDM scalar potential
have two normal minima?**

**Necessary
condition:**

$$m_{11}^2 + k^2 m_{22}^2 < 0$$

$$\sqrt[3]{x^2} + \sqrt[3]{y^2} \leq 1,$$

**Interior of
an astroid**

with

$$x = \frac{4 k m_{12}^2}{m_{11}^2 + k^2 m_{22}^2} \frac{\sqrt{\lambda_1 \lambda_2}}{\lambda_{345} - \sqrt{\lambda_1 \lambda_2}}$$

$$y = \frac{m_{11}^2 - k^2 m_{22}^2}{m_{11}^2 + k^2 m_{22}^2} \frac{\sqrt{\lambda_1 \lambda_2} + \lambda_{345}}{\sqrt{\lambda_1 \lambda_2} - \lambda_{345}}$$

and

$$\lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5$$

$$k = \sqrt[4]{\frac{\lambda_1}{\lambda_2}}$$

I. P. Ivanov, Phys. Rev. D **75**, 035001 (2007) [Erratum-
ibid. D **76**, 039902 (2007)] [hep-ph/0609018]; *ibid*, **77**,
015017 (2008).

And out of those two minima, how can you know whether you are in a panic vacuum?

- **Extremely simple “panic” conditions may be obtained: valid even without checking whether there **ARE** two minima!**

***Our vacuum is the global minimum of the 2HDM potential
if, and only if,
 $D > 0$***

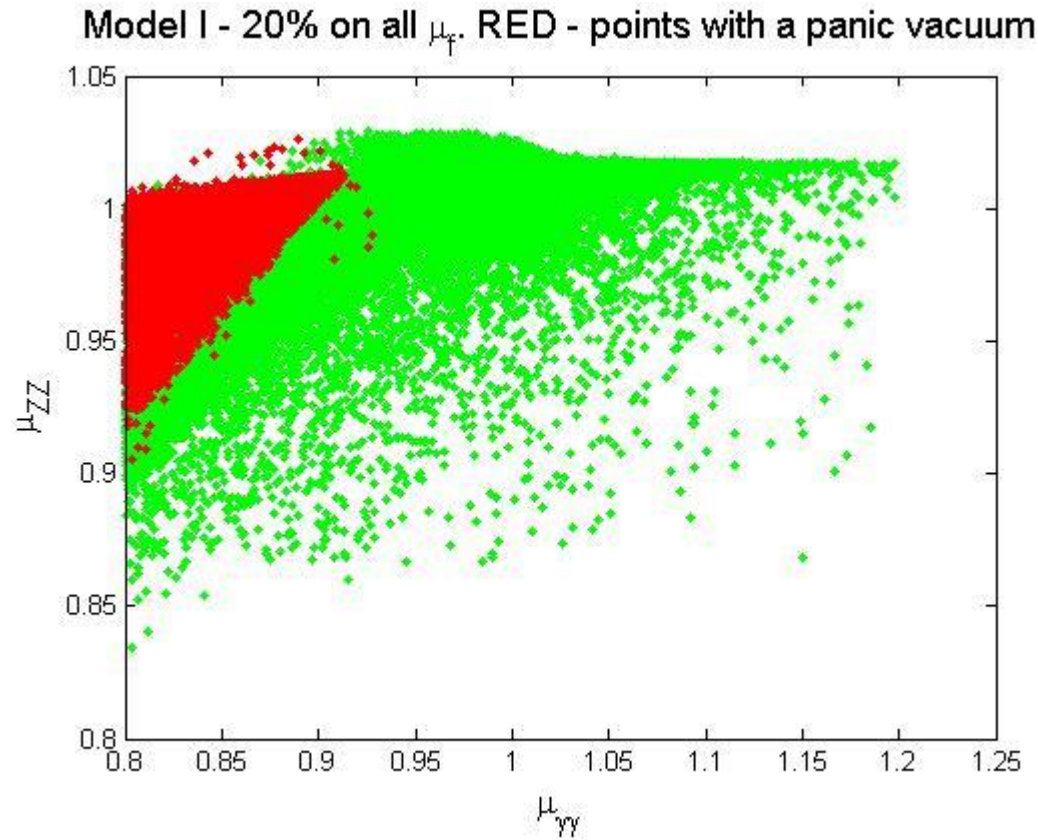
$$D = m_{12}^2 \left(\sqrt{\lambda_2} m_{11}^2 - \sqrt{\lambda_1} m_{22}^2 \right) \left(\sqrt[4]{\lambda_2} \tan \beta - \sqrt[4]{\lambda_1} \right)$$

Notice that this discriminant which specifies the existence of a second normal minimum

IS ONLY BUILT WITH QUANTITIES OBTAINED IN “OUR” MINIMUM.

(different discriminant D can be built for models without m_{12} or which do not satisfy the Z_2 symmetry)

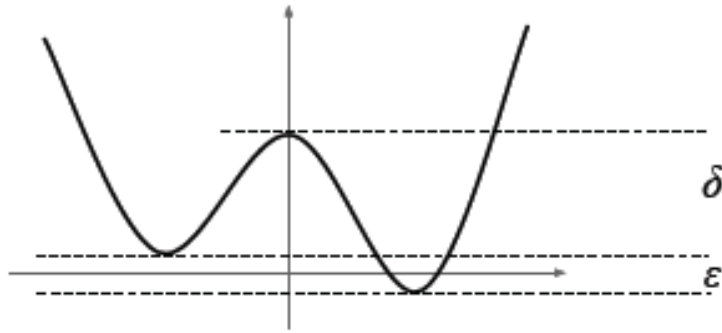
DOES THIS MATTER?



ATTENTION: the **RED** region isn't excluded, there are
 "green points" in there, "between/under" the red ones!

$0.8 < \{\mu_{ZZ}, \mu_{WW}, \mu_{bb}, \mu_{\tau\tau}, \mu_{\gamma\gamma}\} < 1.2$

Cosmological vacuum lifetime estimates



Should we worry about a deeper minimum?

What if the tunnelling time is bigger than the age of the universe?

- Very tricky calculation, full of assumptions.
- Usual criterion: if $\delta/\epsilon > \sim 1$, the tunnelling time is big and the vacuum is safe.
- Calculations show vast majority of panic vacua **NOT SAFE**.

S. R. Coleman, "The Fate of the False Vacuum. 1. Semiclassical Theory," Phys. Rev. D **15**, 2929 (1977) [Erratum-ibid. D **16**, 1248 (1977)].

V. A. Rubakov, "Classical theory of gauge fields," Princeton, USA: Univ. Pr. (2002) 444 p.

4 – One-loop contributions to inert minima

- All vacua analysis so far was performed at tree-level.
- No reason why one-loop contributions should *not* have considerable effects on the vacuum structure (just think of SUSY contributions to m_h mass).
- One-loop effective potential quite a complicated entity with limitations (gauge dependent away from minima...).
- Start by studying the simple case of the **Inert Model**...

Z_2 -symmetric model

$$V = m_{11}^2 |\Phi_1|^2 + m_{22}^2 |\Phi_2|^2 + \frac{1}{2} \lambda_1 |\Phi_1|^4 + \frac{1}{2} \lambda_2 |\Phi_2|^4 + \lambda_3 |\Phi_1|^2 |\Phi_2|^2 + \lambda_4 |\Phi_1^\dagger \Phi_2|^2 + \frac{1}{2} \lambda_5 \left[\left(\Phi_1^\dagger \Phi_2 \right)^2 + h.c. \right]$$

- **SEVEN** real independent parameters.
- The symmetry must be extended to the whole lagrangian, otherwise the model would not be renormalizable.

Coupling to fermions

MODEL I: Only Φ_2 couples to fermions.

Inert: Only Φ_1 couples to fermions.

MODEL II: Φ_2 couples to up-quarks, Φ_1 to down quarks and leptons.

• • •

Inert vacua – preserve Z_2 symmetry

- The **INERT** minimum,
$$\langle \Phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix} \quad \text{and} \quad \langle \Phi_2 \rangle = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$
- Since only Φ_1 has Yukawa couplings, fermions are massive – “**OUR**” minimum.
- The **INERT-LIKE** minimum,
$$\langle \Phi_1 \rangle = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \text{and} \quad \langle \Phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}$$
- Since only Φ_1 has Yukawa couplings, fermions are massless.

WHY BOTHER?

In the inert minimum, the second doublet originates perfect Dark Matter candidates!

Inert neutral scalars do not couple to fermions or have triple vertices with gauge bosons.

Tree-level vacuum solutions

INERT:

$$v_1^2 = -\frac{2 m_{11}^2}{\lambda_1}$$

- Needs $m_{11}^2 < 0$.

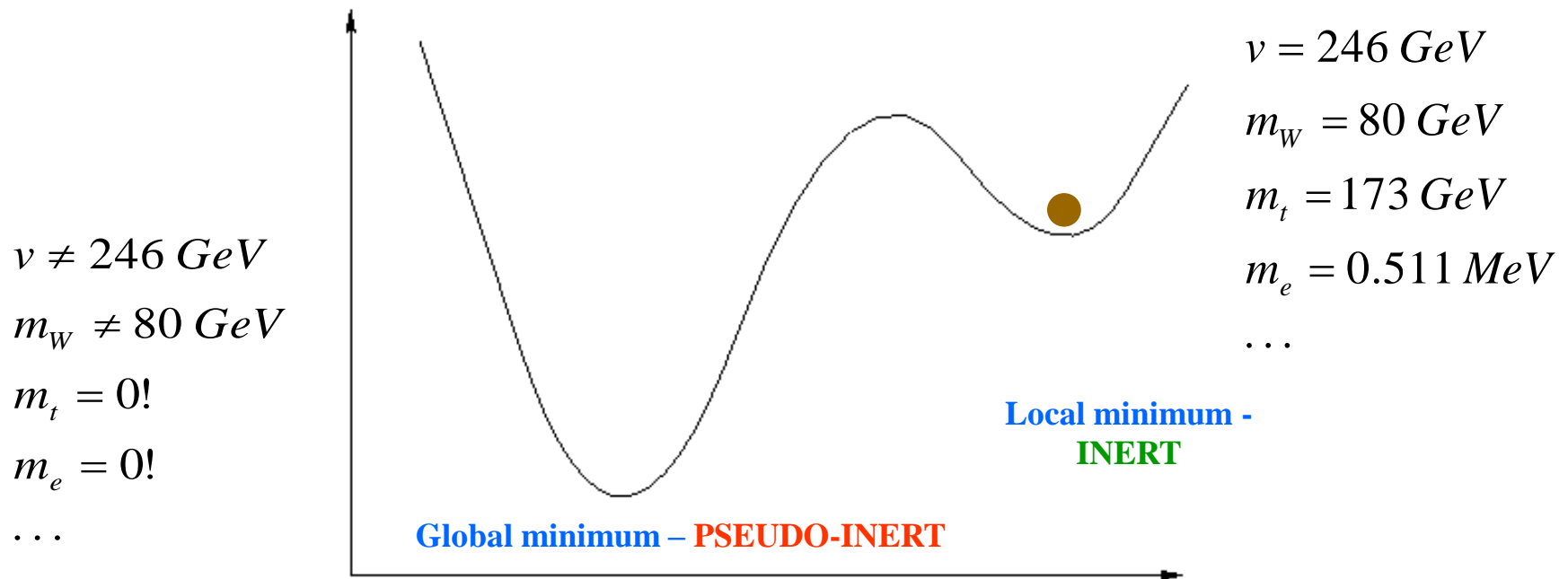
INERT-LIKE:

$$v_2^2 = -\frac{2 m_{22}^2}{\lambda_2}$$

- Needs $m_{22}^2 < 0$.

E. Ma, Phys. Rev. D73 077301 (2006); R. Barbieri, L.J. Hall and V.S. Rychkov Phys. Rev. D74:015007 (2006);
L. Lopez Honorez, E. Nezri, J. F. Oliver and M. H. G. Tytgat, JCAP 0702, 028(2007);
L. Lopez Honorez and C. E. Yaguna, JHEP 1009, 046(2010); L. Lopez Honorez and C. E. Yaguna, JCAP 1101, 002 (2011)

These minima can coexist in the potential, which raises a troubling possibility...



Tree-Level Conclusions:

Inert and inert-like minima can coexist in the potential if $m_{11}^2 < 0$ and $m_{22}^2 < 0$.

$$\begin{aligned}
 V_I - V_{IL} &= \frac{1}{2} \left(\frac{m_{22}^4}{\lambda_2} - \frac{m_{11}^4}{\lambda_1} \right) \\
 &= \frac{1}{4} \left[\left(\frac{m_{H^\pm}^2}{v_2^2} \right)_{IL} - \left(\frac{m_{H^\pm}^2}{v_1^2} \right)_I \right] v_1^2 v_2^2
 \end{aligned}$$

Tree-level to one-loop...

$$V = V_0 + V_1,$$

$$V_1 = \frac{1}{64\pi^2} \sum_{\alpha} n_{\alpha} m_{\alpha}^4(\varphi_i) \left[\log \left(\frac{m_{\alpha}^2(\varphi_i)}{\mu^2} \right) - \frac{3}{2} \right]$$

$$\frac{\partial V}{\partial \varphi_i} = \frac{\partial V_0}{\partial \varphi_i} + \frac{1}{32\pi^2} \sum_{\alpha} n_{\alpha} m_{\alpha}^2 \frac{\partial m_{\alpha}^2}{\partial \varphi_i} \left[\log \left(\frac{m_{\alpha}^2}{\mu^2} \right) - 1 \right]$$

$$\begin{aligned} \frac{1}{v_1} \frac{\partial V}{\partial v_1} = & m_{11}^2 + \frac{1}{2} \lambda_1 v_1^2 + \\ & \frac{1}{32\pi^2} \left\{ \lambda_1 m_{G_0}^2 \left[\log \left(\frac{m_{G_0}^2}{\mu^2} \right) - 1 \right] + 3 \lambda_1 m_{h_0}^2 \left[\log \left(\frac{m_{h_0}^2}{\mu^2} \right) - 1 \right] + \right. \\ & \lambda_{345} m_{H_0}^2 \left[\log \left(\frac{m_{H_0}^2}{\mu^2} \right) - 1 \right] + \bar{\lambda}_{345} m_{A_0}^2 \left[\log \left(\frac{m_{A_0}^2}{\mu^2} \right) - 1 \right] + \\ & 2 \lambda_3 m_{H_0^{\pm}}^2 \left[\log \left(\frac{m_{H_0^{\pm}}^2}{\mu^2} \right) - 1 \right] + 2 \lambda_1 m_{G_0^{\pm}}^2 \left[\log \left(\frac{m_{G_0^{\pm}}^2}{\mu^2} \right) - 1 \right] - \\ & 6 \lambda_t^2 m_{t_0}^2 \left[\log \left(\frac{m_{t_0}^2}{\mu^2} \right) - 1 \right] - 6 \lambda_b^2 m_{b_0}^2 \left[\log \left(\frac{m_{b_0}^2}{\mu^2} \right) - 1 \right] - 2 \lambda_{\tau}^2 m_{\tau_0}^2 \left[\log \left(\frac{m_{\tau_0}^2}{\mu^2} \right) - 1 \right] + \\ & \left. 3 \frac{g^2 + g'^2}{2} m_{Z_0}^2 \left[\log \left(\frac{m_{Z_0}^2}{\mu^2} \right) - 1 \right] + 3 g^2 m_{W_0}^2 \left[\log \left(\frac{m_{W_0}^2}{\mu^2} \right) - 1 \right] \right\} = 0. \end{aligned}$$

$$m_h^2 = m_{h_0}^2 + \frac{1}{32\pi^2} m_{h_1}^2, \quad m_{h_1}^2 = \text{Re}(m_{h_1,S}^2 + m_{h_1,G}^2 + m_{h_1,F}^2)$$

$$\begin{aligned} m_{h_1,S}^2 = & \lambda_1 A(m_{G_0}) + 2\lambda_1 A(m_{G_0^\pm}) + 3\lambda_1 A(m_{h_0}) + \lambda_{345} A(m_{H_0}) + \bar{\lambda}_{345} A(m_{A_0}) + 2\lambda_3 A(m_{H_0^\pm}) \\ & + \lambda_1^2 v_1^2 B(m_{G_0}, m_{G_0}, p^2) + 2\lambda_1^2 v_1^2 B(m_{G_0^\pm}, m_{G_0^\pm}, p^2) + 9\lambda_1^2 v_1^2 B(m_{h_0}, m_{h_0}, p^2) \\ & + \lambda_{345}^2 v_1^2 B(m_{H_0}, m_{H_0}, p^2) + \bar{\lambda}_{345}^2 v_1^2 B(m_{A_0}, m_{A_0}, p^2) + 2\lambda_3^2 v_1^2 B(m_{H_0^\pm}, m_{H_0^\pm}, p^2), \end{aligned}$$

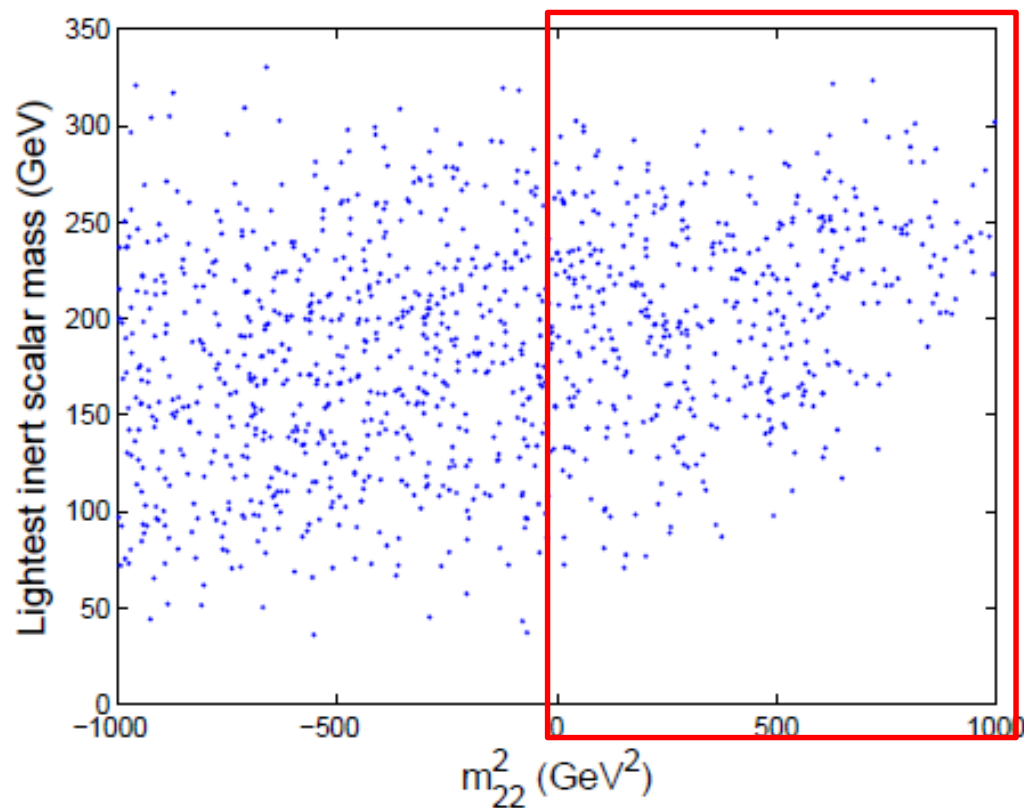
$$\begin{aligned} m_{h_1,G}^2 = & \frac{g^4}{2c_W^2} B_{SV}(m_{G_0}, m_{Z_0}, p^2) + g^2 B_{SV}(m_{G_0^\pm}, m_{W_0}, p^2) \\ & + \frac{g^2}{c_W^2} m_{Z_0}^2 B_{VV}(m_{Z_0}, m_{Z_0}, p^2) + 2g^2 m_{W_0}^2 B_{VV}(m_{W_0}, m_{W_0}, p^2) \\ & + \frac{3g^2}{2c_W^2} A(m_{Z_0}) + 3g^2 A(m_{W_0}), \end{aligned}$$

$$\begin{aligned} m_{h_1,F}^2 = & -6\lambda_t^2 \left[(4m_{t_0}^2 - p^2) B(m_{t_0}, m_{t_0}, p^2) + 2A(m_{t_0}) \right] \\ & -6\lambda_b^2 \left[(4m_{b_0}^2 - p^2) B(m_{b_0}, m_{b_0}, p^2) + 2A(m_{b_0}) \right] \\ & -2\lambda_\tau^2 \left[(4m_{\tau_0}^2 - p^2) B(m_{\tau_0}, m_{\tau_0}, p^2) + 2A(m_{\tau_0}) \right]. \end{aligned}$$

Tree-level results:

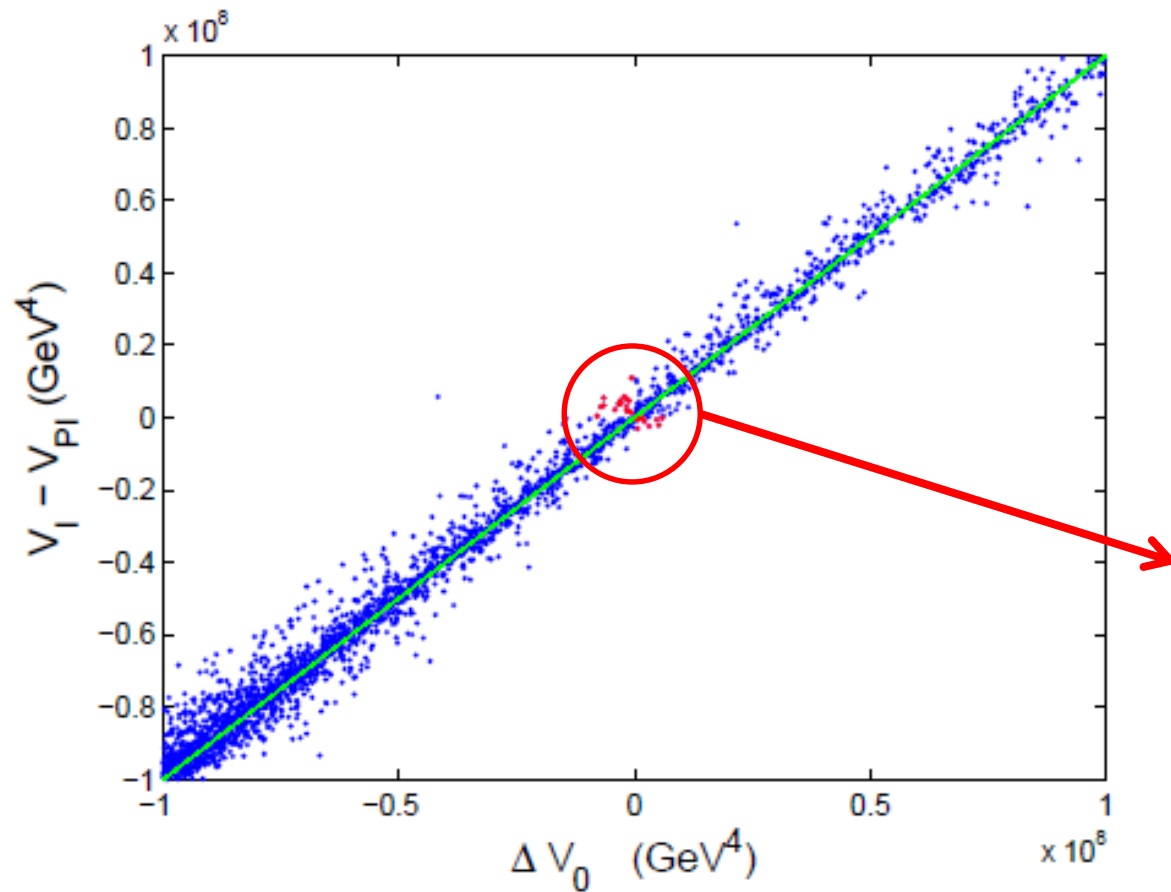
Inert and inert-like minima can coexist in the potential if $m_{11}^2 < 0$ and $m_{22}^2 < 0$.

One-loop results:



**IMPOSSIBLE TO HAVE SIMULTANEOUS
MINIMA IN THIS REGION AT TREE-LEVEL!**

**One-loop
Potential
values**



**Inversion of
minima**

**Tree-level
Potential
values**

$$\Delta V_0 = \frac{1}{4} \left[\left(\frac{m_{H^\pm}^2}{v_2^2} \right)_{IL} - \left(\frac{m_{H^\pm}^2}{v_1^2} \right)_I \right] v_1^2 v_2^2$$

5 – LHC constraints on 2HDM parameters

- LHC has found a CP-even scalar with mass ~ 125 GeV.
- This particle behaves a lot like the SM Higgs boson.
- Its interactions are – so far – so similar to the SM Higgs that the 2HDM parameter space is already severely constrained.
- 2HDM being pushed to the *decoupling limit* (all extra masses heavy) or the *alignment limit* (some couplings such that not all masses need to be heavy).

Theory constraints

Potential has to be
bounded from below:

$$\lambda_1 \geq 0, \quad \lambda_2 \geq 0,$$
$$\lambda_3 \geq -\sqrt{\lambda_1 \lambda_2}, \quad \lambda_3 + \lambda_4 - |\lambda_5| \geq -\sqrt{\lambda_1 \lambda_2}$$

Theory must respect
unitarity:

$$a_{\pm} = \frac{3}{2} (\lambda_1 + \lambda_2) \pm \sqrt{\frac{9}{4} (\lambda_1 - \lambda_2)^2 + (2\lambda_3 + \lambda_4)^2},$$

$$b_{\pm} = \frac{1}{2} (\lambda_1 + \lambda_2) \pm \frac{1}{2} \sqrt{(\lambda_1 - \lambda_2)^2 + 4\lambda_4^2},$$

$$c_{\pm} = \frac{1}{2} (\lambda_1 + \lambda_2) \pm \frac{1}{2} \sqrt{(\lambda_1 - \lambda_2)^2 + 4\lambda_5^2},$$

$$e_1 = \lambda_3 + 2\lambda_4 - 3\lambda_5$$

$$e_2 = \lambda_3 - \lambda_5,$$

$$f_+ = \lambda_3 + 2\lambda_4 + 3\lambda_5,$$

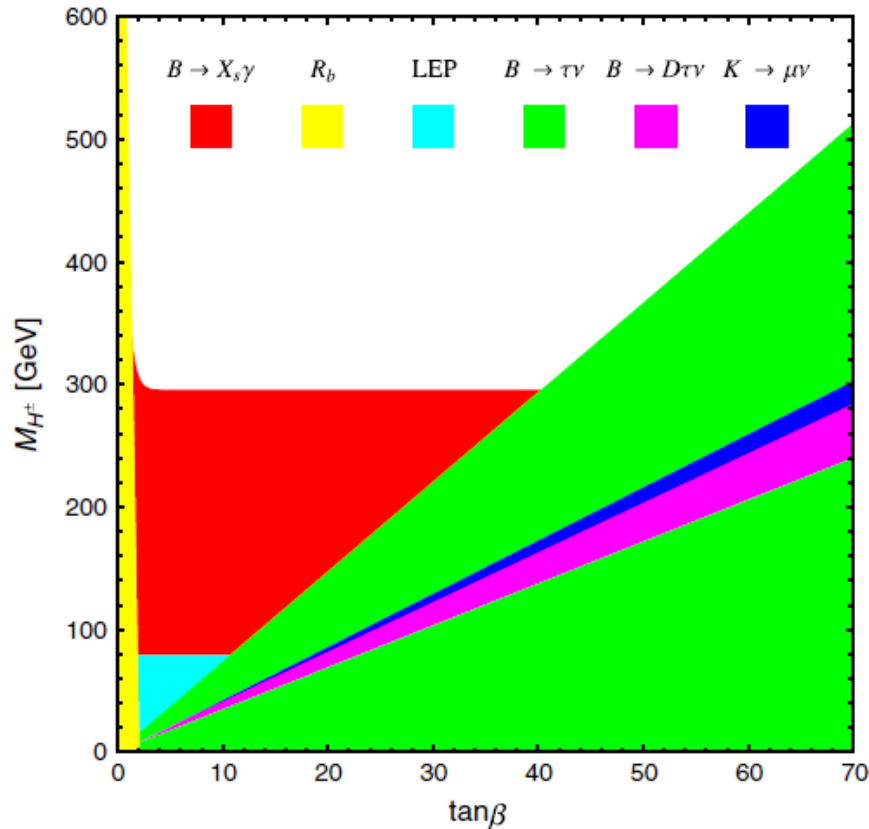
$$f_- = \lambda_3 + \lambda_5,$$

$$f_1 = \lambda_3 + \lambda_4,$$

$$p_1 = \lambda_3 - \lambda_4.$$

$$|a_{\pm}|, |b_{\pm}|, |c_{\pm}|, |f_{\pm}|, |e_{1,2}|, |f_1|, |p_1| < 8\pi$$

Constraints from b-physics and others



Most important/reliable constraints from **$b \rightarrow s\gamma$** and **$\Gamma(Z \rightarrow b\bar{b})$** observables.

Significant theory uncertainties!

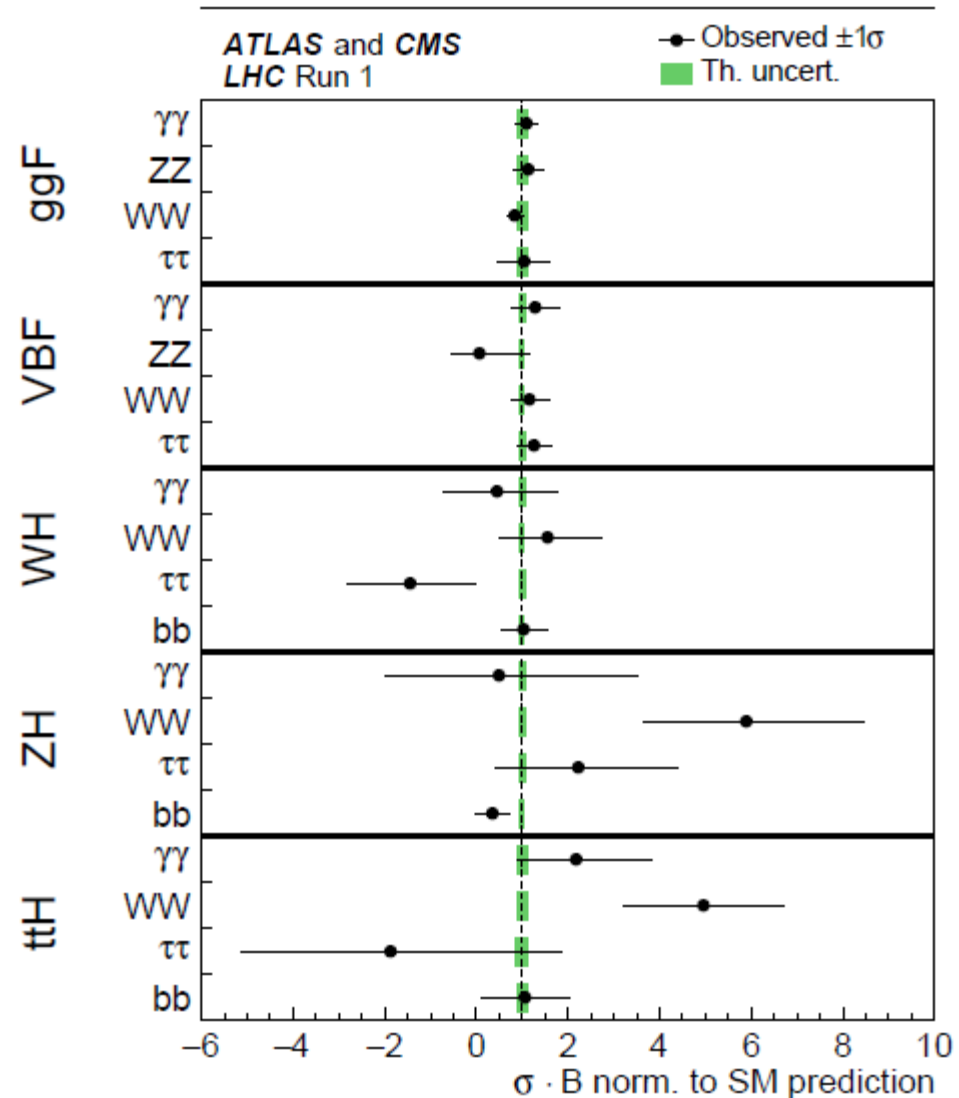
To good approximation, then:

Model I, Lepton specific: $\tan\beta \geq 1$

Model II, Flipped: $\tan\beta \geq 1$ and $m_{H^\pm} \geq 400 \text{ GeV}$

LHC results for h

$$\mu_X = \frac{\sigma(pp \rightarrow h)}{\sigma^{SM}(pp \rightarrow h)} \times \frac{BR(h \rightarrow X)}{BR^{SM}(h \rightarrow X)}$$



Simplified 2HDM analysis

Higgs production dominated by top contribution to gluon-gluon fusion:

$$\sigma(pp \rightarrow h) \simeq (\xi_h^u)^2 \sigma^{SM}(pp \rightarrow h)$$

Higgs decays dominated by width of decay to bottom quarks:

$$BR(h \rightarrow ZZ) \simeq \frac{\Gamma(h \rightarrow ZZ)}{\Gamma(h \rightarrow b\bar{b})} = \frac{(\xi_h^V)^2 \Gamma^{SM}(h \rightarrow ZZ)}{(\xi_h^d)^2 \Gamma^{SM}(h \rightarrow b\bar{b})} \simeq \left(\frac{\xi_h^V}{\xi_h^d} \right)^2 BR^{SM}(h \rightarrow ZZ)$$

To good approximation, then:

$$\mu_{ZZ} \simeq \left(\frac{\xi_h^u}{\xi_h^d} \xi_h^V \right)^2$$

**Different for each model,
since couplings to down-
quarks vary.**

$$\mu_{ZZ} \simeq \left(\frac{\xi_h^u}{\xi_h^d} \xi_h^V \right)^2$$

type-I and lepton-specific $\xi_h^u = \xi_h^d$

$$\mu_{ZZ} \simeq \sin^2(\beta - \alpha)$$

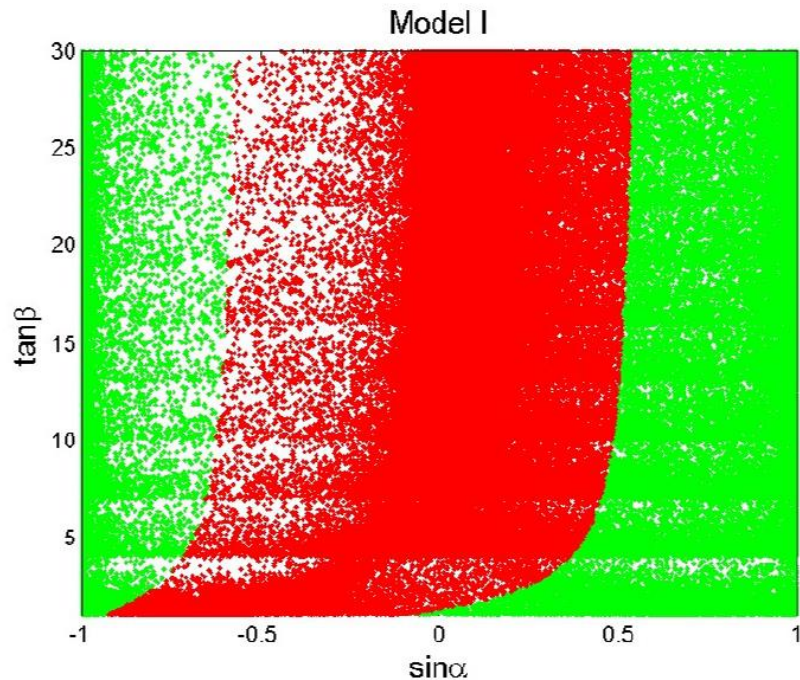
type-II and Flipped

$$\mu_{ZZ} \simeq \frac{\sin^2(\beta - \alpha)}{\tan^2 \alpha \tan^2 \beta}$$

Higgs production dominated by top contribution to gluon-gluon fusion: In both cases, a Higgs boson decaying SM-like to Z (and W) bosons implies

$$\sin(\beta - \alpha) \approx 1$$

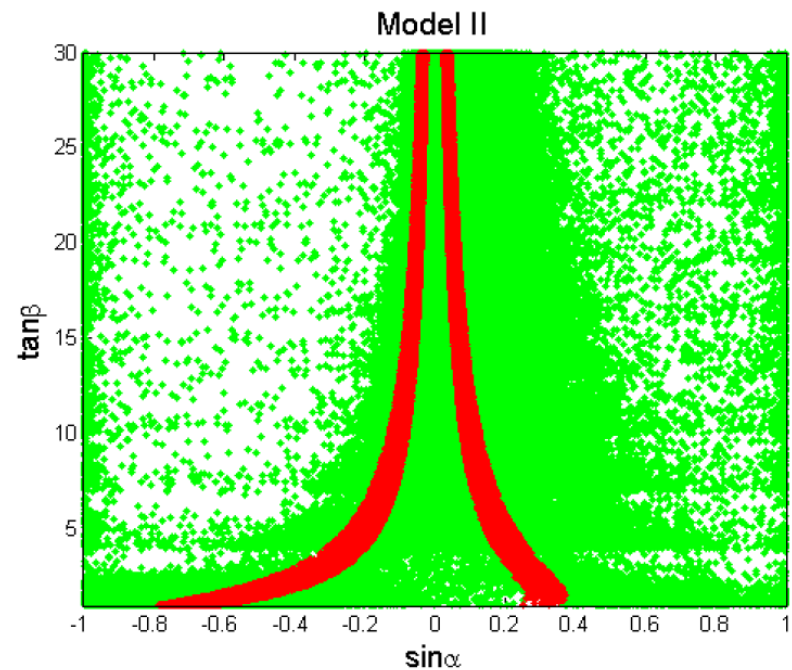
Run-I parameter space restrictions



Model I (Lepton Specific)

$$k_t = \frac{\cos \alpha}{\sin \beta} > 0$$

$$k_b = \frac{\cos \alpha}{\sin \beta} > 0$$



Model II (Flipped)

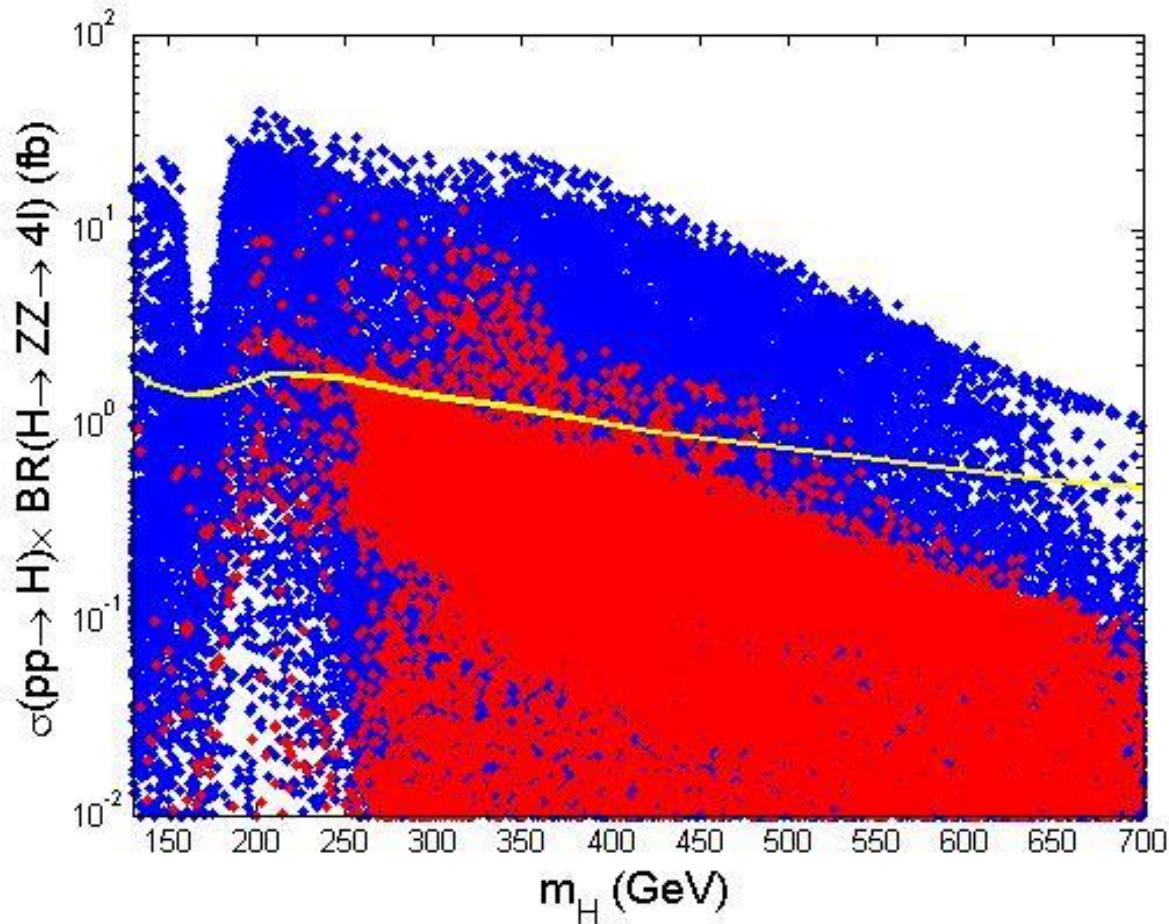
$$k_t = \frac{\cos \alpha}{\sin \beta} > 0$$

$$k_b = \frac{\sin \alpha}{\cos \beta} > 0 \text{ or } < 0$$

Wrong-Sign Limit

The Importance of Being Earnest ~~h~~ h

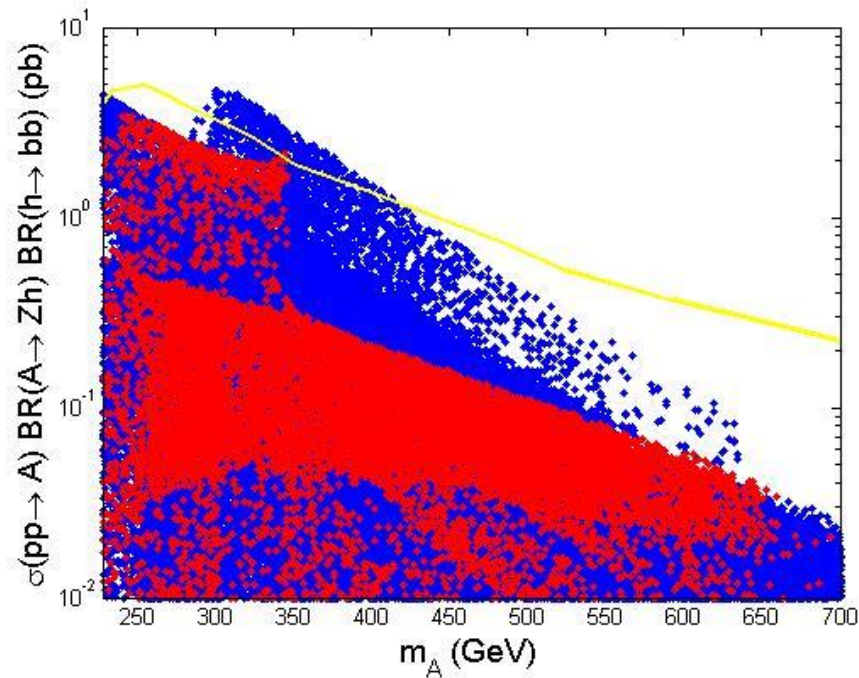
Run II has limits on high mass resonances in the 4 lepton channel...



(yellow line upper bound on non-observation from CMS PAS HIG-16-033)

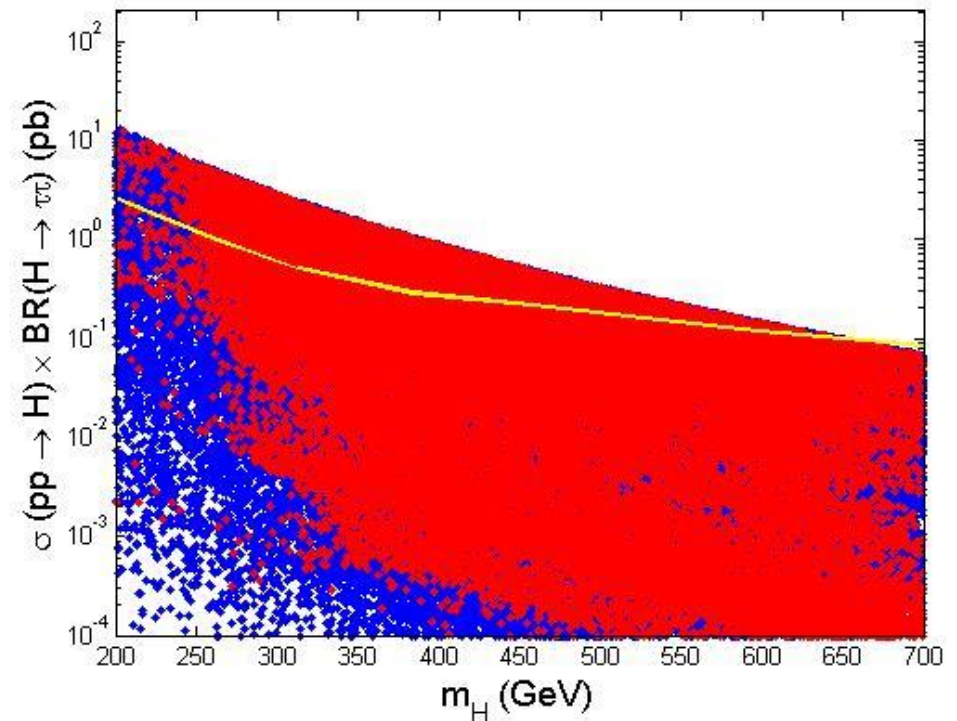
(red points are what remains after demanding “h” rates are within 30% of SM values)

(ATLAS limit)



Demanding “h” behaviour being SM-like complies with latest high-mass exclusions...

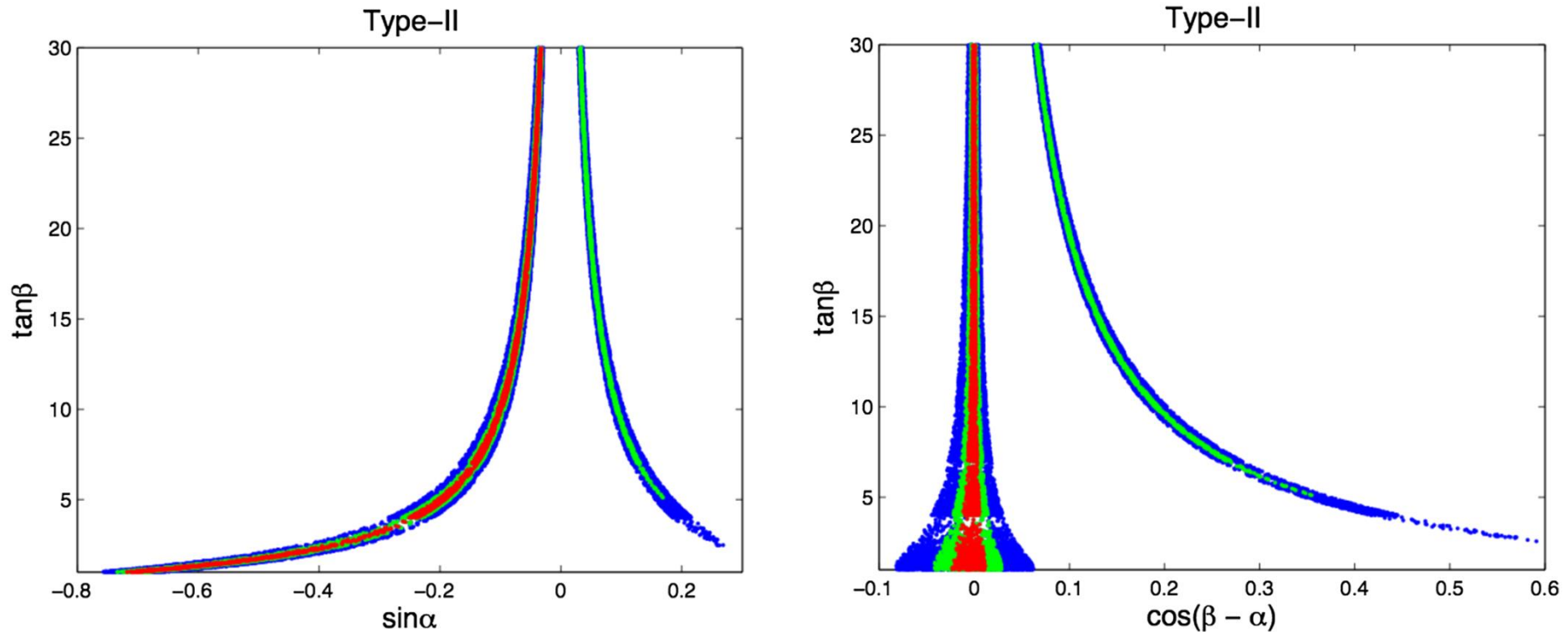
(ATLAS limit)



... Though not for *ALL* observables

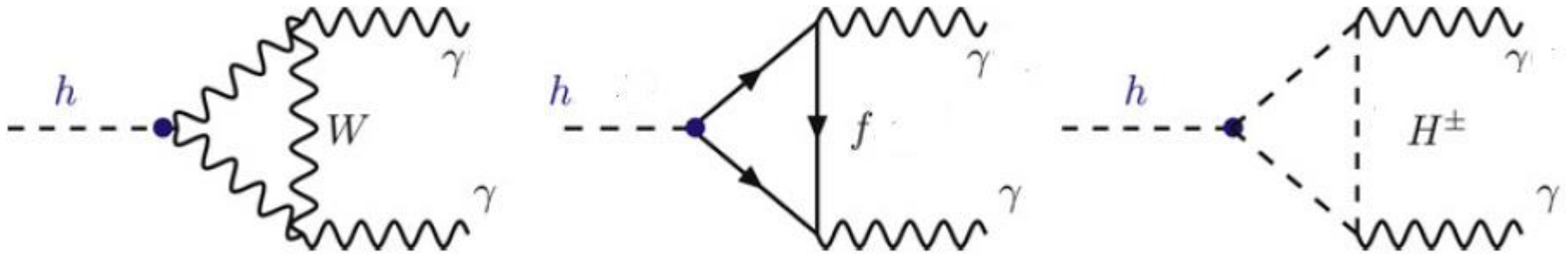
The wrong-sign region

Only possible (in the quark sector) in model II and Flipped models.



All ratios μ within 20% (Blue), 10% (Green) or 5% (Red) of their expected SM values (1).

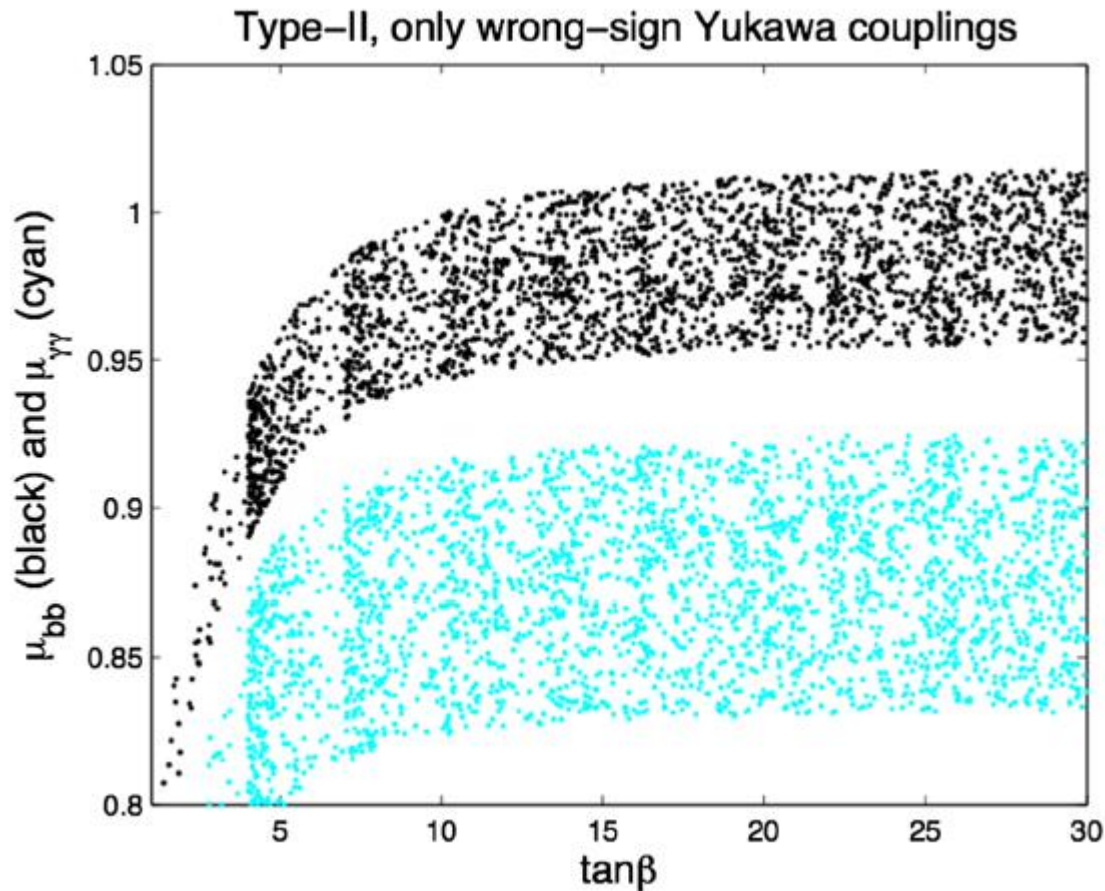
The wrong-sign region



In the SM there is a *destructive* interference between the W diagrams and the fermion ones.

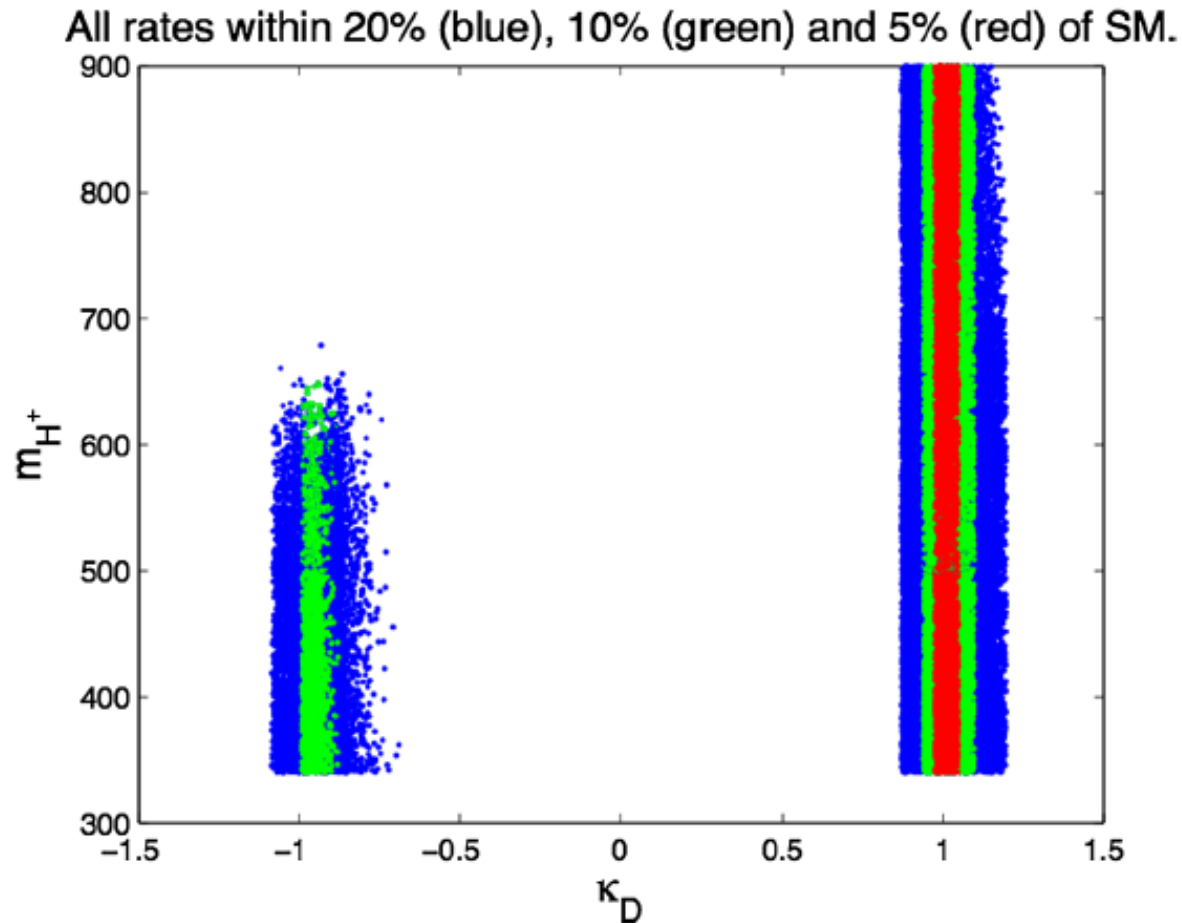
In the wrong-sign regime, that interference becomes *constructive* for the bottom quark contribution and there is an enhancement of the diphoton (and digluon) width.

The wrong-sign region



5% cut on
 μ_{bb} (black)
incompatible with
a 5% cut on
 $\mu_{\gamma\gamma}$ (blue).

The wrong-sign region

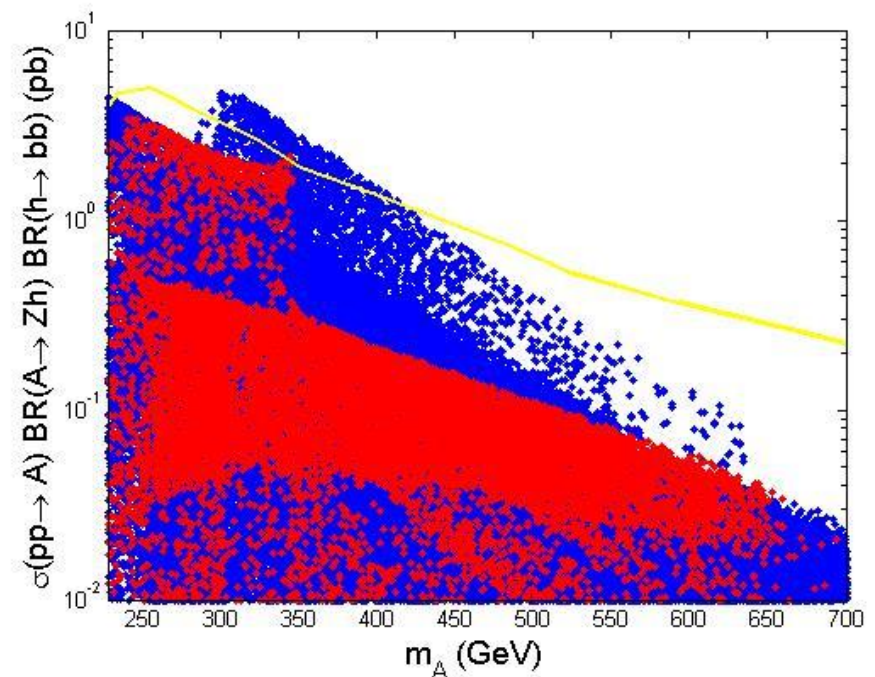
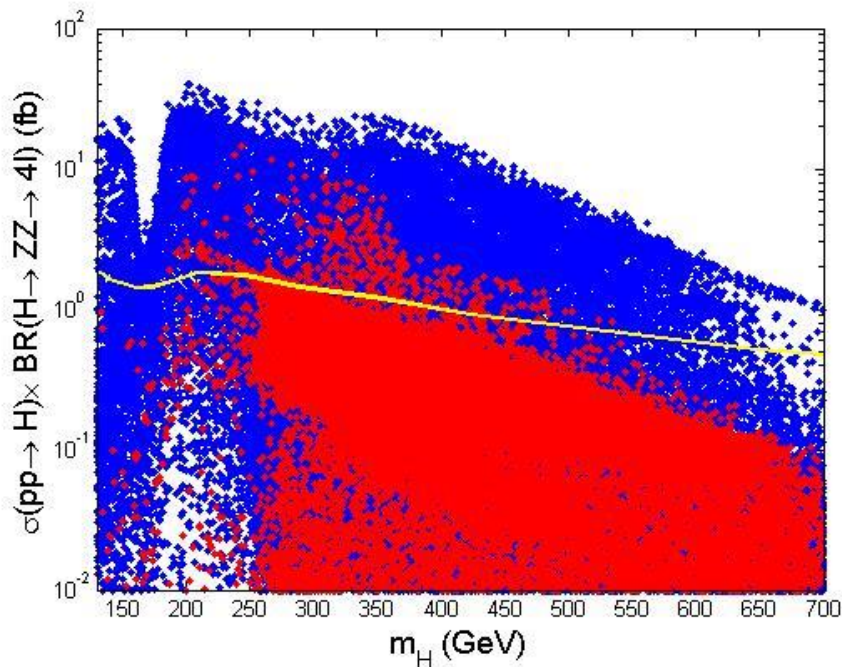


The incompatibility is due to a irreducible contribution to the diphoton width from the charged scalar – in the wrong sign regime, this becomes a “**non-decoupling**” contribution...

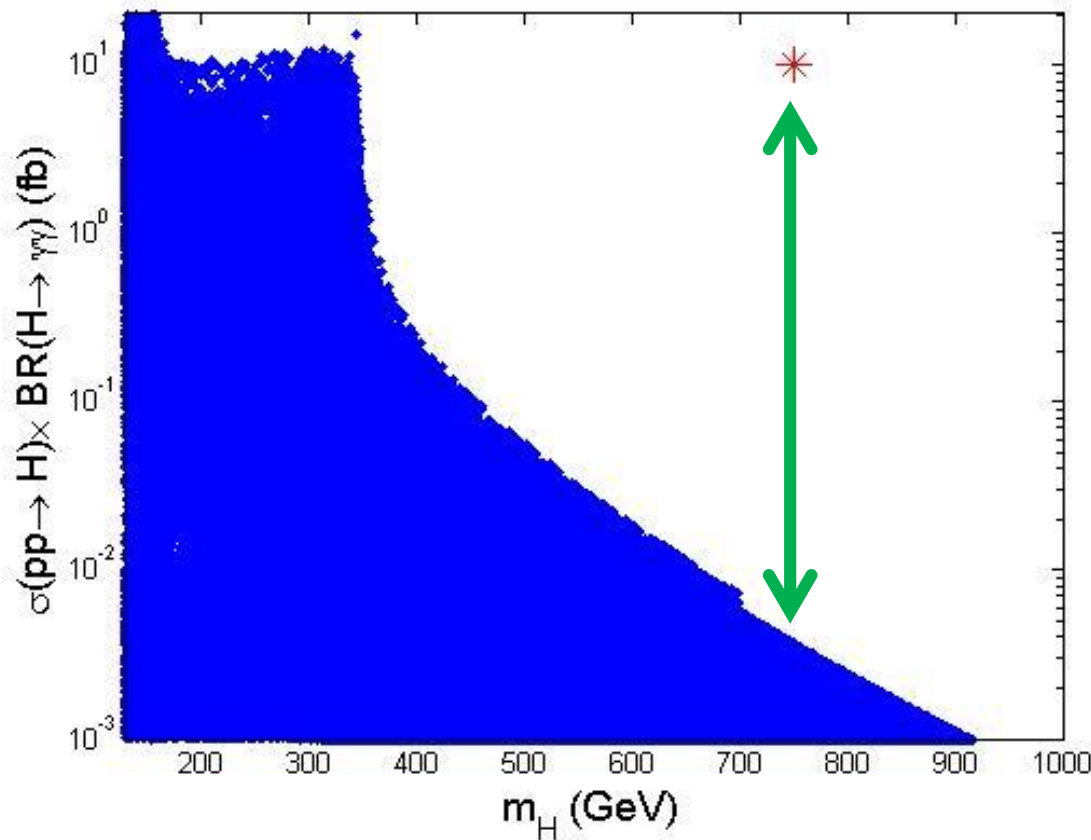
- In the Lepton-specific model, there is a wrong-sign regime in the lepton Yukawa couplings, which may be used to explain the muon $g - 2$.
(E.J. Chun, Z. Kang, T. Takeuchi, Y.S Tsai, JHEP 1511 (2015) 099)
- The muon $g - 2$ may also be interpreted, in the 2HDM, in terms of an *aligned* model.
(T. Han, S.K. Kang, J. Sayre, JHEP 1602 (2016) 097)
- So the 2HDM is incredibly *versatile*, despite all the constraints we put upon him!

6 – Current limitations of the 2HDM

The 2HDM is already so constrained that significant deviations from SM expected behaviour might *exclude it*.



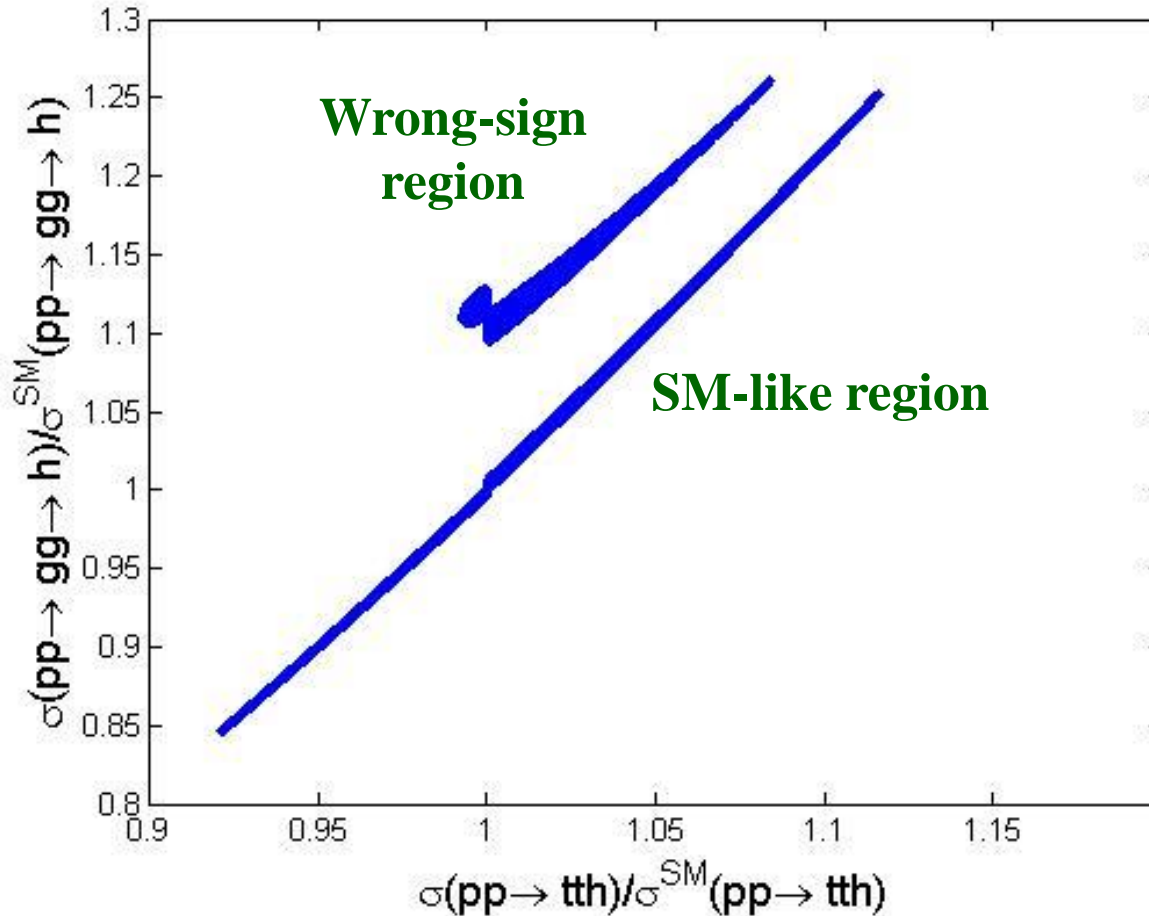
The 750 GeV “anomaly”



BLUE:
theoretically acceptable
points +
B physics +
requiring that “h” is SM-
like,

$$0.7 \leq \frac{\sigma(pp \rightarrow H)BR(H \rightarrow ZZ)}{\sigma^{SM}(pp \rightarrow H)BR^{SM}(H \rightarrow ZZ)} \leq 1.3$$

The tth “anomaly”



ATLAS:

$$\mu = 2.1^{+1.0}_{-0.9}$$

$\sigma_{t\bar{t}h}$ *versus* $\sigma_{gg h}$ in model II