Stochastic Evolution of Halo Spin

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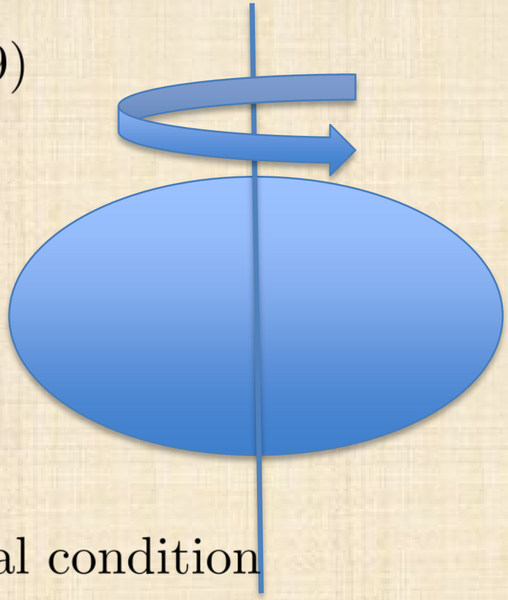
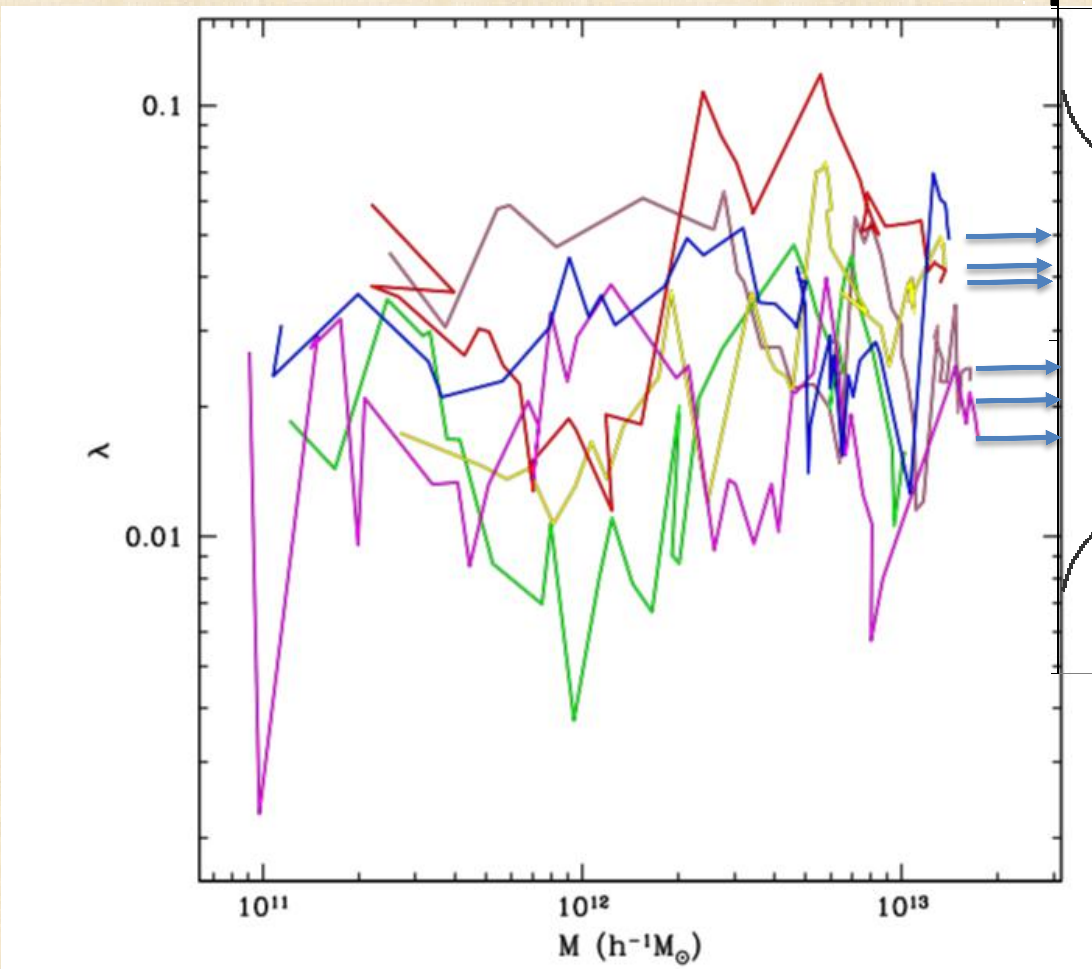
Halo Spin (λ)

- Original definition: $\lambda^o \equiv \frac{\sqrt{E}|\mathbf{J}|}{GM^{5/2}}$ (Peebles 1969)
  - E: halo total energy
  - $\mathbf{J}$: halo angular momentum
  - M: halo mass



- Modified version: $\lambda \equiv \frac{|\mathbf{J}|}{\sqrt{2}MRV}$ (Bullock+01)
  - Equivalent to the original under the virial condition
  - R (virial radius) & V (circular velocity) are determined by M and cosmology
- Unsolved Issues related to λ
  - **Why the log normal distribution of halo spin?**
  - **Environmental dependence exists?**
  - **The role of accretion and merger in shaping the spin distribution.**

Stochastic (random) Motion of Halo Spin



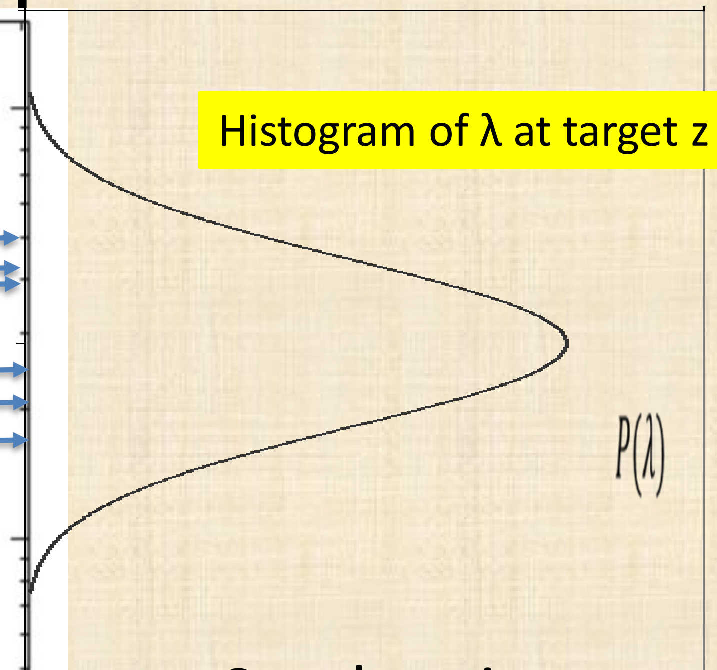
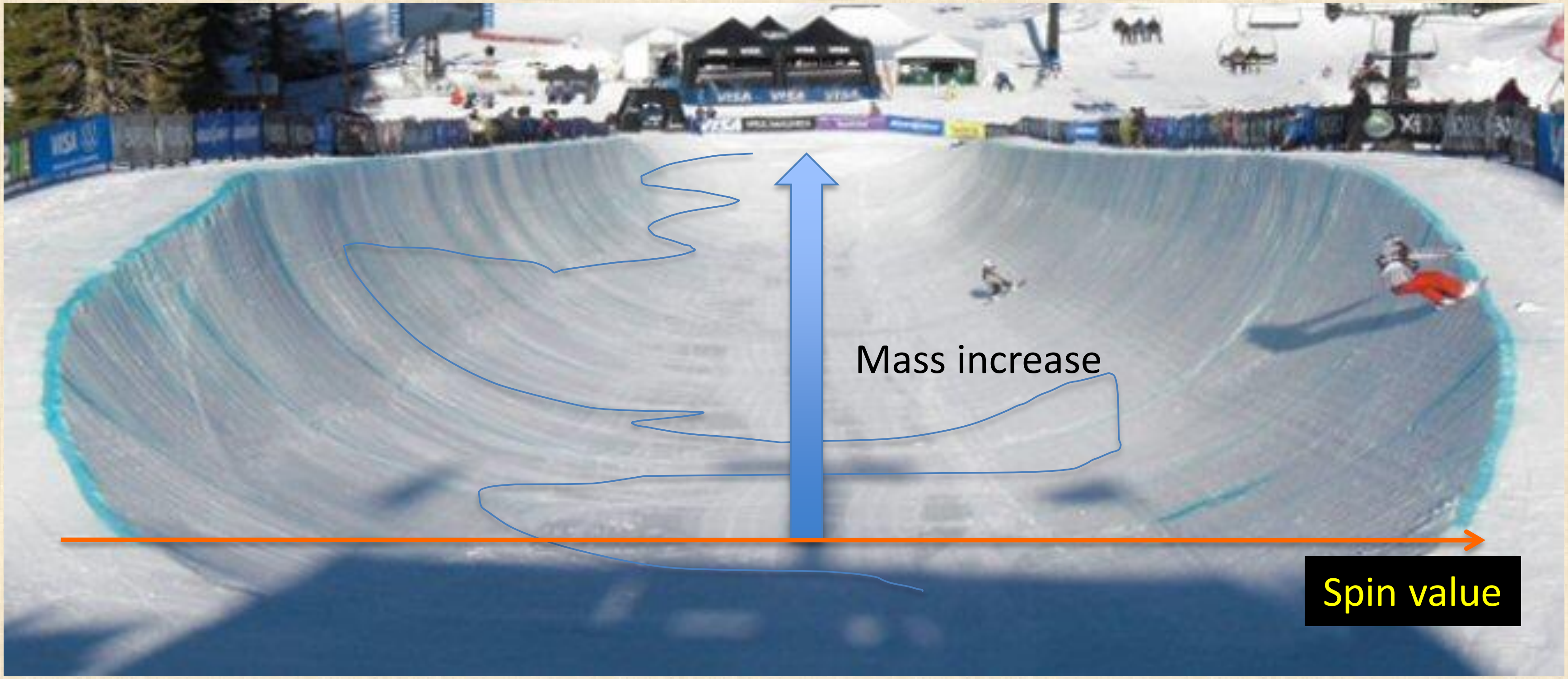


Figure 1. Several examples of spin evolution of simulated halos. Each color represents the spin trajectory of a single main-merging tree. For the x axis, we use halo mass rather than time or redshift. Halos at $z = 0$ are chosen with mass $10^{13}\,h^{-1}\,M_{\odot} \leqslant M < 2 \times 0^{13}\,h^{-1}\,M_{\odot}$.

- Stochastic
  - *Def.: Randomly determined; having a random probability distribution or pattern that may be analyzed statistically but may not be predicted precisely.*
  - Brownian motion,Monte Carlo simulation, and etc.

Contained Random Motion



Stochastic Equation of the Spin Evolution

- Stochastic Evolution of the modified spin

$$\frac{d\log_{10}\lambda}{d\log_{10}M} = \frac{d\log_{10}|\mathbf{J}|}{d\log_{10}M} - \frac{5}{3} + \alpha(z) \quad (1)$$

$$= D - \frac{5}{3} + \alpha(z) \quad (2)$$

where $D \equiv \frac{\Delta \log |\mathbf{J}|}{\Delta \log M}$ and $\alpha(z)$ depends on cosmology.

- In this equation, Halo mass plays a role as an independent variable (time).
- Distribution of D is a function of z, M, ΔM, ρ_{10}.
- $\alpha(z)$ a function of cosmological model (Ω_m, Ω_Λ, H, etc).
  - It is very small and negligible except $\Delta M \to 0$.
  - If $\Delta M \to 0$ which means there is no matter infall or accretion, the cosmology determines how the halo spin evolves.

Simulations

- Four PMTree2048p737.28s simulations in WMAP 5-year cosmology
  - Boxsize: $L_{box} = 737.28h^{-1}\mathrm{M}_{\odot}$
  - $n_p = 2048^3$, $z_i = 120$, $N_{step} = 3000$.
  - a different set of random numbers for each simulation $\rightarrow$ different initial generations
  - save FoF halo data at 44 redshifts.
  - measure halo characteristics: spin, mass, etc.
- Generate the halo merger trees
  - Using particle index to find progenitor halos & major descendent line
  - Finding the changes of spin and mass during the accretion and merger

Sub-Sampling of merger/accretion events

- make subsamples of spin changes in 6-dimensional parameter spaces, M, λ, $\Delta \log M$, $\Delta\rho_{10}$, and z.

$$\Delta\rho_{10} = \frac{\rho_{10}}{<\rho_{10}>} - 1 \quad (1)$$

where ρ_{10} is a density measured with a spline kernel from 10 neighbors.

  - $\Delta\rho_{10} < 0.7$: underdense (void) region
  - $0.7 < \Delta\rho_{10} < 2$: mean field
  - $2 < \Delta\rho_{10} < 10$: group region
  - $10 < \Delta\rho_{10} < 100$: cluster region
  - $\Delta\rho_{10} > 100$: highly clustered region

- for each subsample, we measured the distribution of $D \equiv \dfrac{\Delta \log \lambda}{\Delta \log M}$.

Parameter Ranges[a] Applied to the Measurement of $P(D)$

Id	Halo Mass (M_{12})	λ	$\Delta \log_{10} M$	$\Delta\rho_{10}$	z
1	0.1	0.004	0.3	0.7	5
2	0.3	0.006	0.1	2	4
3	0.5	0.0085	0.07	10	3
4	0.8	0.01	0.05	100	2
5	1	0.015	0.03	∞	1.5
6	2	0.02	0.02	…	1
7	4	0.025	0.01	…	0.6
8	5	0.03	0.0065	…	0.4
9	7	0.035	0.004	…	0.2
10	10	0.038	0	…	0.0
11	20	0.04	0.004	…	…
12	50	0.043	0.0065	…	…
13	500	0.045	0.008	…	…
14	5000	0.047	0.01	…	…
15	∞	0.05	0.015	…	…
16	…	0.053	0.02	…	…
17	…	0.056	0.03	…	…
18	…	0.06	0.05	…	…
19	…	0.063	0.08	…	…
20	…	0.067	0.1	…	…
21	…	0.07	0.15	…	…
22	…	0.072	0.2	…	…
23	…	0.075	0.3	…	…
24	…	0.078	0.7	…	…
25	…	0.08	∞	…	…
26	…	0.09	…	…	…
27	…	0.1	…	…	…
28	…	0.11	…	…	…
29	…	0.12	…	…	…
30	…	0.13	…	…	…
31	…	0.15	…	…	…
32	…	0.17	…	…	…
33	…	0.2	…	…	…
34	…	0.25	…	…	…
35	…	0.3	…	…	…
36	…	0.4	…	…	…
37	…	0.5	…	…	…
38	…	0.7	…	…	…
39	…	1.2	…	…	…
40	…	∞	…	…	…

Environmental Dependence of Merger and Accretion

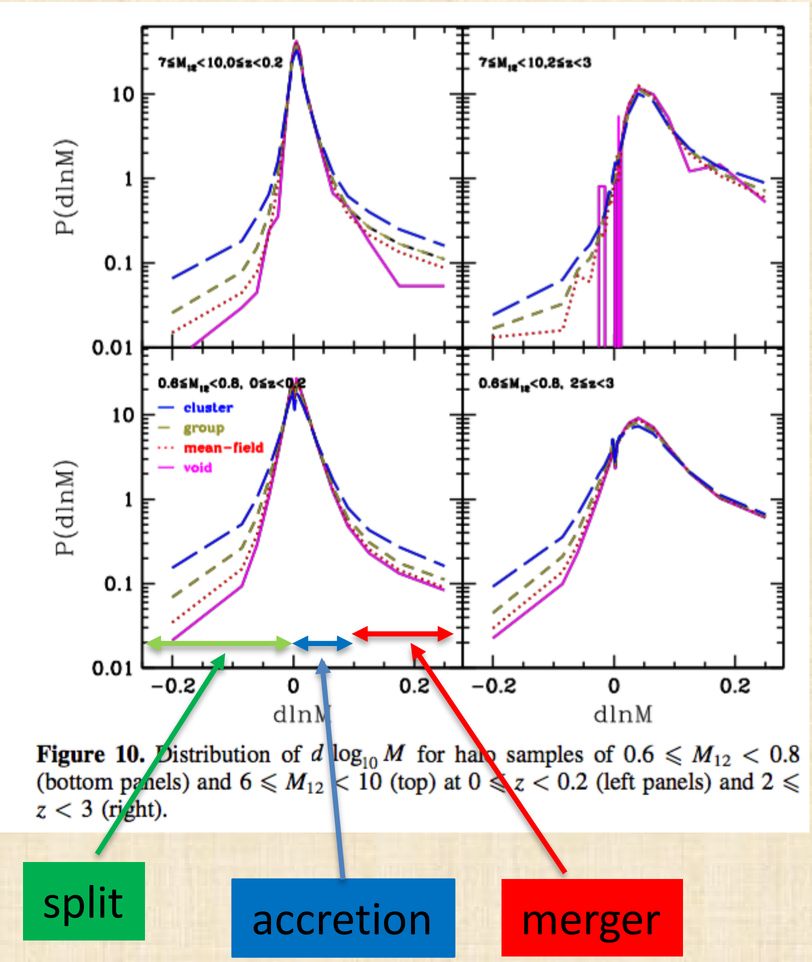


Figure 10. Distribution of $d\log_{10}M$ for halo samples of $0.6 \leqslant M_{12} < 0.8$ (bottom panels) and $6 \leqslant M_{12} < 10$ (top) at $0 \leqslant z < 0.2$ (left panels) and $2 \leqslant z < 3$ (right).

For dlnM>0 cases,

- At high z, no difference of dlnM between environments
- At low z, halos in denser region tend to have higher dlnM or frequent mergers.

Distribution of Spin Change (D)

- $D \equiv \frac{d \log |\mathbf{J}|}{d \log M}$: amplitude change of angular momentum when halo mass changes

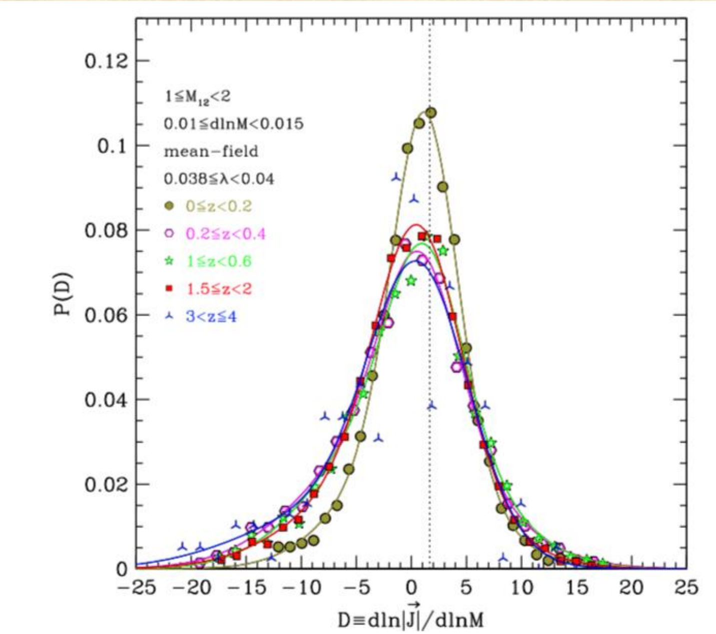
- $P(D)$: measured probability distribution of spin changes

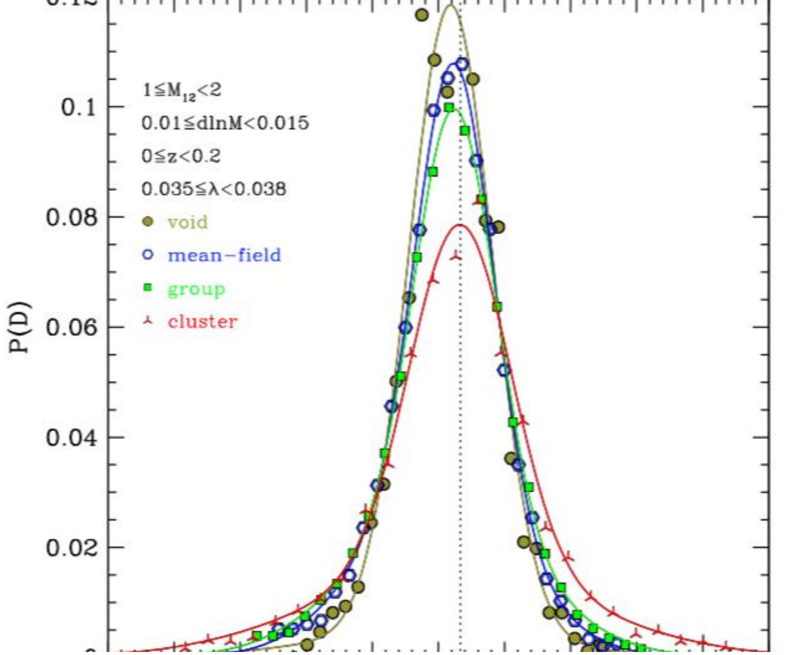
- $P(D) = P(D|\lambda, M, \Delta M, z, \rho_{10})$

- a fitting of measured $P(D)$ to $\boxed{\text{bimodal Gaussian function.}}$

$$P_{fit}(D) = \frac{f_1}{\sqrt{2\pi\sigma_1^2}} exp\left[-\frac{(D-\mu_1)^2}{2\sigma_1^2}\right] + \frac{f_2}{\sqrt{2\pi\sigma_2^2}} exp\left[-\frac{(D-\mu_2)^2}{2\sigma_2^2}\right] \quad (1)$$

where $f_1 + f_2 = 1$.





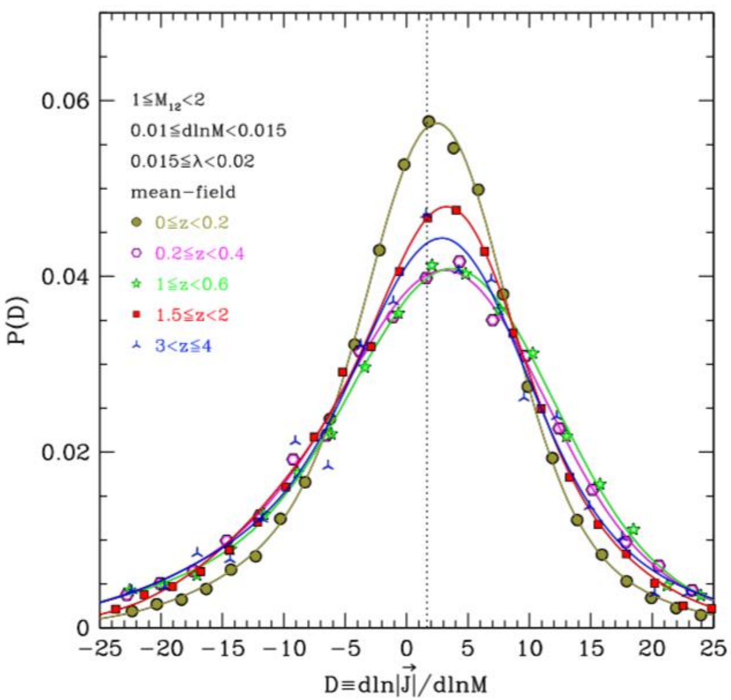


Figure 4. Same as Figure 3, but for the low-spin sample of $0.015 \leqslant \lambda < 0.02$, which is used to isolate the spin effect on the angular momentum change.

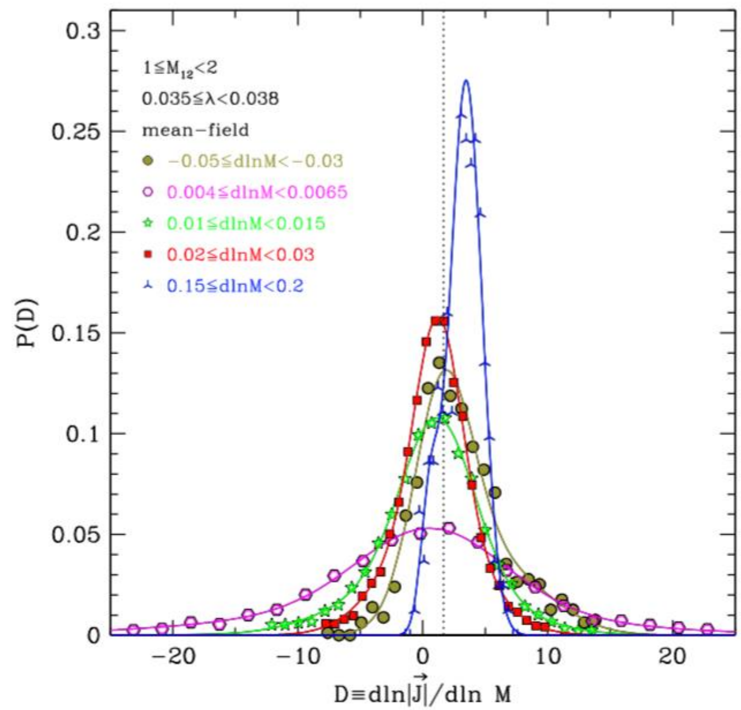
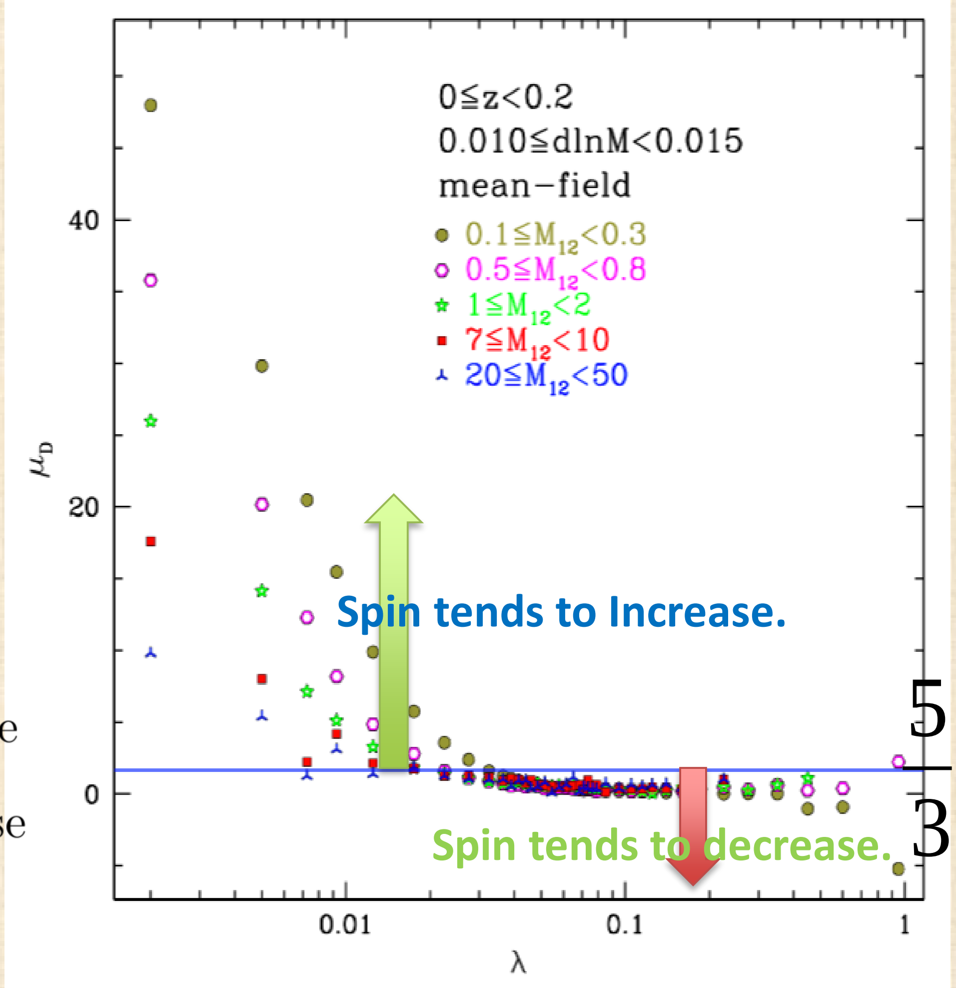


Figure 6. Dependence of $P(D)$ on the amount of mass infall ratio ($\Delta \log_{10} M$) using samples of $1 \leqslant M_{12} < 2$, $0.7 \leqslant \Delta\rho_{10} < 2$, and $0.035 \leqslant \lambda < 0.038$.

$$\mu_D = \int_{-\infty}^{\infty} P(D)dD$$

- μ_D: mean value of the distribution
- blue horizontal line: 5/3.
- If $\mu_D > 5/3$, halo spin tends to increase
- If $\mu_D < 5/3$, halo spin tends to decrease
- $\lambda \equiv \lambda_c$ when $\mu_D(\lambda) = 5/3$.
- μ_D decreases with λ crossing 5/3 somewhere around $\lambda_c \sim 0.01 - 0.05$
- $\boxed{\lambda_c \text{ represents the average spin value.}}$
- Different mass, environment, redshift samples have different λ_c.

$$\therefore \lambda_c = \lambda_c(M, \Delta M, \rho, \lambda, z)$$



DISTRIBUTION OF λ_c

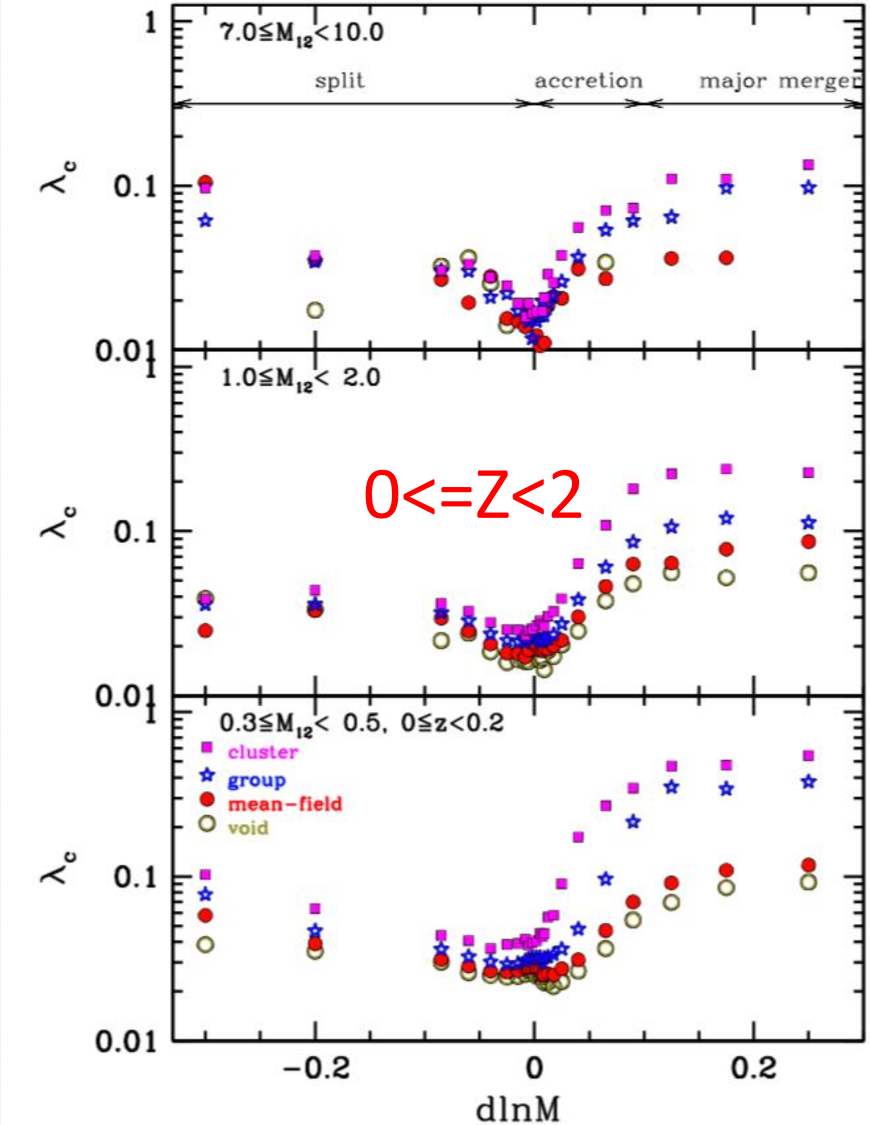


Figure 8. Dependence of λ_c on merging mass ($\Delta \log_{10} M$) between $0 \leqslant z < 0.2$ for three mass samples: $0.3 \leqslant M_{12} < 0.5$ (bottom panel), $1 \leqslant M_{12} < 2$ (middle), and $7 \leqslant M_{12} < 10$ (top). Symbols with different colors are used to distinguish the effect of local environment.

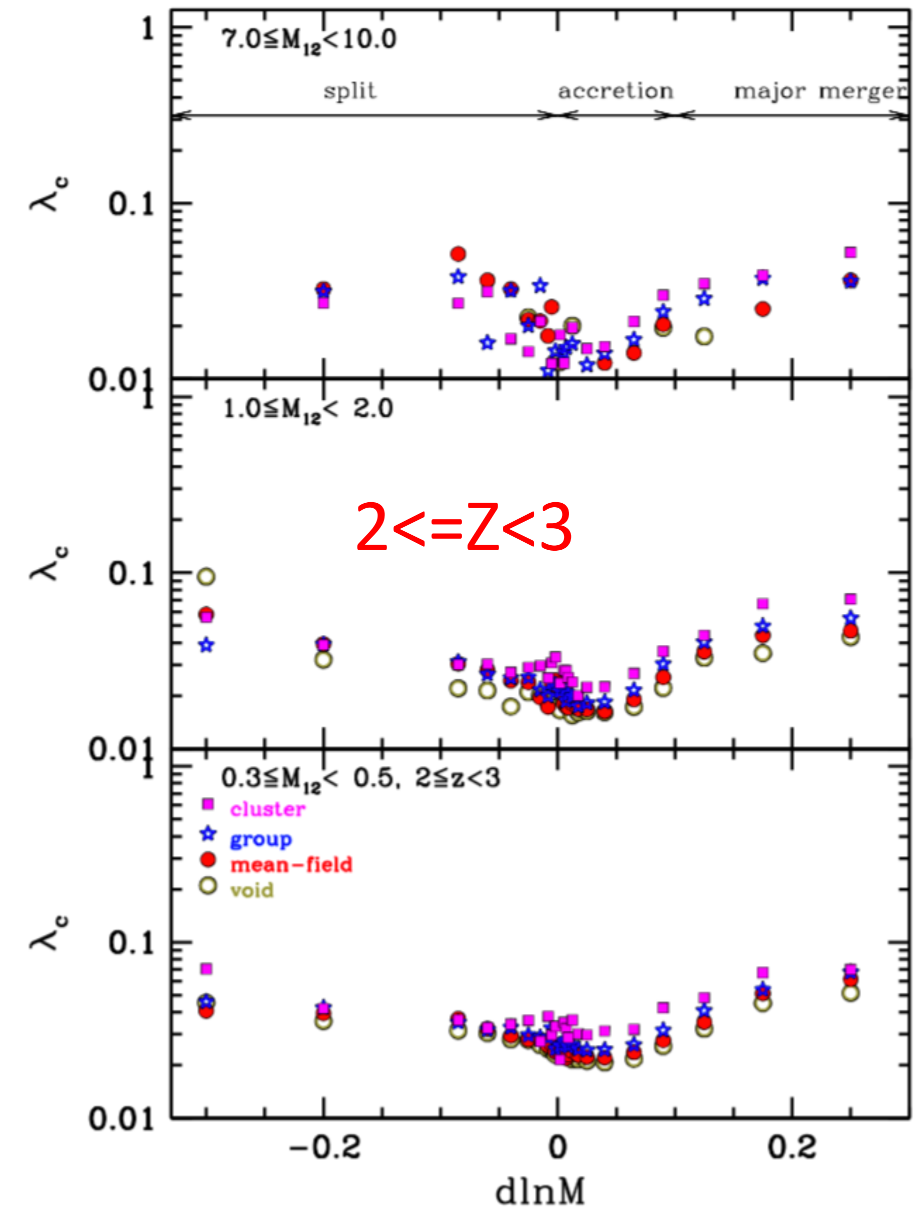


Figure 9. Same as Figure 8, but for redshift samples of $2 \leqslant z < 3$.

From the distribution of λ_c, we come to know that

- At bigger merger events, λ_c is higher.
- Or accretion tends to make halos have less spin value.
- For more massive halo, the average spin is lower than less massive halo
- In less denser region, halos tends to be have smaller spin.

Stochastic Simulation

- Random-simulated spin evolution is obtained as

$$\lambda_{i+1} = \lambda_i + \Delta\lambda_i \tag{1}$$

  where $\Delta\lambda_i$ is **randomly generated** using $P(D|M_i, \lambda_i, z_i, \Delta\rho_i, \Delta M_i)$ and ΔM_i is given from the N-body simulation. Running from $i = 0$ to $i = i_f$, we get the **target** λ_f for **each** major descendent tree found in simulation.

- We iterate the procedure (Eq. 1) for each major descendent tree and measure $P(\lambda)$.

Results from Stochastic Model

- **Red histogram**: spin distribution from N-body simulation
- **Blue histogram**: one obtained from the stochastic model
- Slight differences are found at lower z and less massive halo samples.

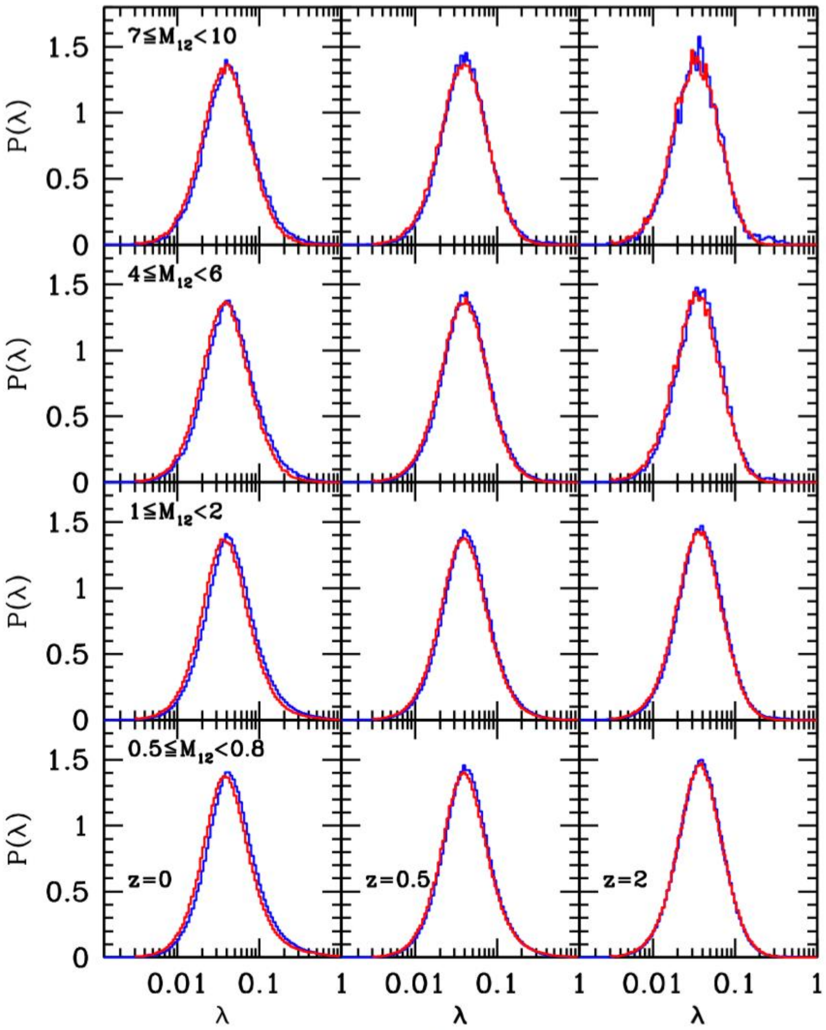


Figure 11. Spin distributions of various halo mass samples at $z = 0$, 0.5, and 2 (from left panels). The blue and red histograms are the randomly generated and *N*-body simulated spin distributions, respectively.

Local Environmental Effects (1)

Less massive halos at z=2

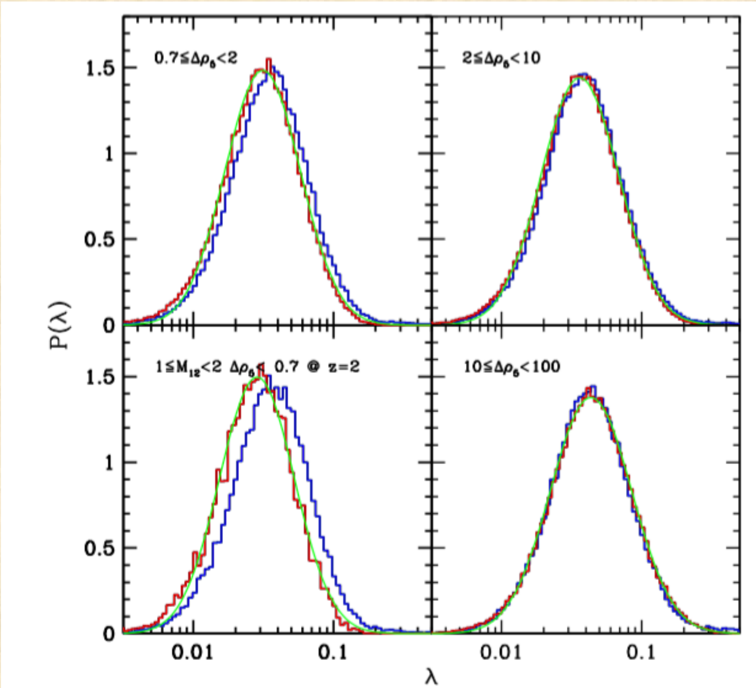


Figure 14. Spin distributions in various local environments at $z = 2$. Counter-clockwise from the bottom left panel are the spin distributions of *N*-body (red histogram) and random-generated (blue) samples of local densities of $\Delta\rho_{10} < 0.7$, $0.7 \leqslant \Delta\rho_{10} < 2$, $2 \leqslant \Delta\rho_{10} < 10$, and $10 \leqslant \Delta\rho_{10} < 100$. The green solid curve in each panel is a log-normal fit to the *N*-body spin distribution.

Less massive halos at z=0

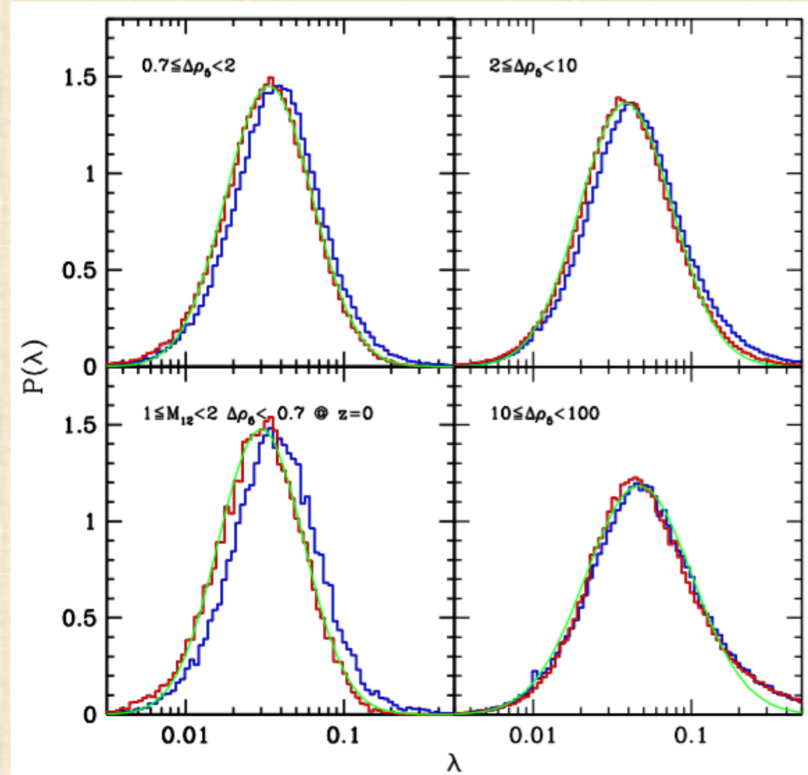


Figure 16. Same as Figure 14, but at $z = 0$.

Green histogram: log-normal fit to N-body result

Local Environmental Effects (2)

More massive halos at z=0

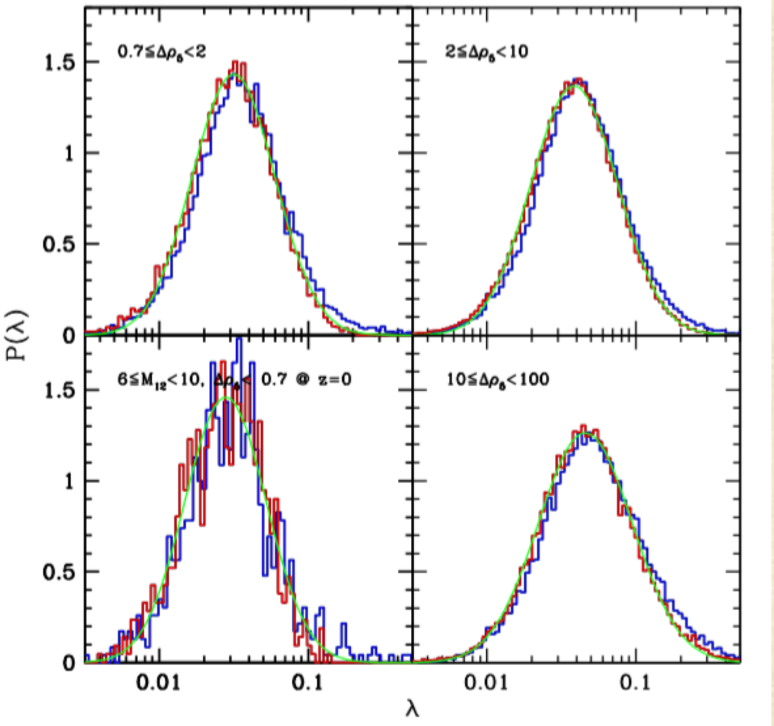


Figure 17. Spin distributions of a more massive sample of $6 \leqslant M_{12} < 10$ at $z = 0$. The green solid curve is a log-normal fit to the corresponding *N*-body distribution.

- **Less massive** halos in **field/underdense** regions tend to have **lower spin values** than expected from the stochastic evolution model.
  - Maybe correlated infall in these parameter space? Ans: we tried various test but can't found the cause of this difference yet.
- Halos in the other regions are well described by the stochastic model

Why P(D) leads to the Log Normal Distribution

- Geometric Brownian Motion (Ross 2007)

$$\frac{d\log_{10}\lambda(\tau)}{d\tau} = \theta + \sigma_c \frac{dW_\tau}{d\tau},$$

where θ is the long-term drift of the system, σ_c is set constant and W_τ is a kind of normally distributed Wiener process or $W_\tau \sim \mathcal{N}(0, \tau)$.

- Characteristics of Wiener process

$$\boxed{\frac{dW_\tau}{d\tau} = \frac{W_{\tau+d\tau} - W_\tau}{d\tau} \sim \frac{\mathcal{N}(0, d\tau)}{d\tau} = \mathcal{N}(0, 1/d\tau) \quad dW_\tau^2 \sim \mathcal{N}(0,1)d\tau \propto d\tau} \tag{1}$$

- Ito's formula (Movellan 2011):

Taylor expansion

$$d\log\lambda = \frac{d\log\lambda}{d\lambda}d\lambda + \frac{1}{2}\frac{d^2\log\lambda}{d\lambda^2}d\lambda^2 \tag{2}$$

$$= \sigma_c dW_\tau + \left(\theta - \frac{\sigma_c^2}{2}\right)d\tau \tag{3}$$

Using Eq. (1)

Why P(D) leads to the Log Normal Distribution (Cont.)

- Using the following equations,

$$
\begin{aligned}
d\lambda &= \lambda(\theta d\tau + \sigma_c dW_\tau) && (1) \\
dW_\tau^2 &= d\tau && (2)
\end{aligned}
$$

- We got

long-term drift

$$\log\left(\frac{\lambda(\tau)}{\lambda_0}\right) = \sigma_c W_\tau + \left(\theta - \frac{\sigma_c^2}{2}\right) \quad (3)$$

where $(\theta - \sigma_c^2/2)$ is a corrected long-term drift (Oksendal 2000) and σ_c is the standard deviation.

- Therefore, if W_τ is **Gaussian**, the distribution of λ is **log normal**.

Summary

- Log-normal distribution is a simple consequence of the stochasticity of the spin.
  - Predicted by Ito's formula
  - Subsequent mass merging/accretion are stochastic (Markovian: independent of previous history)
- Some deviations are observed of halos in the mean/underdense region at lower redshifts.
  - Possibly correlated mass infall events of those halos.
  - Halos in group and cluster environments have spin distributions well described by the stochastic model.