Vortex Partition Functions in 3d Seiberg-like Dualities

Chiung Hwang (KIAS)
Autumn Symposium on String Theory
Sep 12, 2017

Based on

arXiv:1703.00213
arXiv:1506.03951
arXiv:1211.6023
and work in progress

with Hee-Cheol Kim, Hyungchul Kim, Jaemo Park, Piljin Yi, Yutaka Yoshida

Contents

3d Supersymmetric Partition Functions

3d Supersymmetric Partition Functions

- Various supersymmetric partition functions have been discussed

Kim '09, Kapustin-Willett-Yaakov '09, Hama-Hosomich-Lee '11, Benini-Zaffaroni '15, Closset-Kim '16, …

- $S^2 \times S^1$, (Squashed) S^3, (topologically twisted) $\Sigma_g \times_p S^1$
- Can be exactly computed by the supersymmetric localization

Pasquetti '11, CH-Kim-Park '12, Taki '13, …

- Angular momentum on S^2 -> factorization
- Tell us about information of strongly interacting IR theories, nontrivial tests of supersymmetric dualities, new integrable models by Gauge/Yang-Baxter correspondence, …

Localization

- Path integral can be localized by a Q-exact deformation

Fujitsuka-Honda-Yoshida ’13, Benini-Peelaers ’13

- Coulomb- vs Higgs-branch localization
  - *Coulomb* localizes the path integral to saddles where chiral multiplet scalar = 0 & vector multiplet scalar = const
  - *Higgs* localizes the path integral to saddles where vector multiplet scalar = mass & chiral multiplet scalar ~ $\xi^{1/2} e^{-in\phi}$ with $\xi \to \infty$
- Deformed Coulomb-branch localization for finite ξ = Coulomb+Higgs

- Higgs = limit of def. Coulomb unless there is a pole at infinity

Topologically Twisted Partition Functions

Witten '88

A-twisted 3d theory defined on

$$S^1 \to \mathcal{M}_{g,p} \to \Sigma_g$$

$$ds^2 = (d\phi + C(z,\bar{z}))^2 + 2g_{z\bar{z}}dzd\bar{z}, \qquad \frac{1}{2\pi}\int_{\Sigma_g} dC = p \in \mathbb{Z}$$

Benini-Zaffaroni '16, Closset-Kim '16

Perform a SUSY localization such that

$$Z[\mathcal{M}_{g,p}] = \frac{1}{|W_G|}\sum_{\mathfrak{m}\in\mathbb{Z}_p^r}\int_{C^\eta} d^r u \; \mathcal{F}(u)^p \; \Pi_a(u)^{\mathfrak{m}_a} \; e^{-2\pi i\Omega(u)} \; \mathcal{H}(u)^g$$

$$\mathcal{F}(u) = \exp\left(2\pi i\left(\mathcal{W} - u_a\frac{\partial\mathcal{W}}{\partial u_a}\right)\right),$$

$$\Pi_a(u) = \exp\left(2\pi i\frac{\partial\mathcal{W}}{\partial u_a}\right),$$

$$\mathcal{H}(u) = e^{2\pi i\Omega(u)}\det_{ab}\left(\frac{\partial^2\mathcal{W}}{\partial u_a\partial u_b}\right),$$

Twisted superpotential $\quad \mathcal{W}(u) = \frac{1}{2}ku(u+1) + \frac{1}{24}k_g + \frac{1}{(2\pi i)^2}\sum_\rho \mathrm{Li}_2(e^{2\pi i\rho(u)})$

Topologically Twisted Partition Functions

Rewrite the partition function as follows

$$Z[\mathcal{M}_{g,p}] = \sum_{\hat{u}\in\mathcal{S}_{\mathrm{BE}}} \mathcal{F}(\hat{u})^p\ \mathcal{H}(\hat{u})^{g-1} = \langle \mathcal{F}(\hat{u})^p\ \mathcal{H}(\hat{u})^g \rangle_{S^2_A\times S^1}$$

$$\mathcal{S}_{\mathrm{BE}} = \{\hat{u}_a | \Pi_a(\hat{u}) = 1, \quad \forall a, \qquad w\cdot\hat{u} \neq \hat{u}, \quad \forall w\in W_G\} / W_G$$

-> vacuum condition from the twisted superpotential $\mathcal{W}$ on Σ_g

Special cases

$$Z[T^3] = \sum_{\hat{u}\in\mathcal{S}_{\mathrm{BE}}} 1$$

$$Z[S^3] = \sum_{\hat{u}\in\mathcal{S}_{\mathrm{BE}}} \mathcal{F}(\hat{u})\ \mathcal{H}(\hat{u})^{-1} = \langle \mathcal{F}(\hat{u}) \rangle_{S^2_A\times S^1}$$

Relate to S^3 without A-twist by a large gauge transformation of A_R

-> reproduce Kapustin-Willett-Yaakov

C.f., Hama-Hosomich-Lee -> S^3 with squashing

Superconformal Index

Bhattacharya-Bhattacharyya-Minwalla-Raju '08

- Tr $(-1)^F x^{E+j} t_i^{F_i}$
- Counting chiral operators of SCFT on R^3
- Mapped to BPS states on $S^2 \times R$ by radial quantization
- Partition function on $S^2 \times S^1$ with the periodic boundary condition
- Exactly computed by the SUSY localization

Kim '09, Imamura-Yokoyama '11

$$I(x,t) = \sum_{m \in \text{monopole background}} \frac{1}{|\mathcal{W}_m|} \int \frac{dz}{2\pi i z} \, Z_{\text{classical}}(x,t;z,m) \, Z_{\text{vector}}(x;z,m) \, Z_{\text{chiral}}(x,t;z,m)$$

$$I(x,t) = \sum_{\sigma \in \text{Higgs vacua}} Z_{\text{pert}}(x,\sigma(t)) \, Z_{\text{vort}}(x,\sigma(t)) \, Z_{\text{anti-vortex}}(x,\sigma(t))$$

Accidental Symmetries?

Localization <- the symmetry group and the representations of matters

- **Usually fully determined by UV data**
- **Accidental symmetries in IR -> not visible in UV**
- **E.g., negative UV R-charge**
  - **Recall UV/IR R-charges don't need to be the same: $R_{IR} = R_{UV}+\alpha_i F_i$**
  - **α_i is fixed by the F-maximization** Jafferis '10
  - **$\Delta = R_{IR} \geq 1/2$ for a scalar operator due to the unitarity** Minwalla '97
  - **If naive $R_{IR} \leq 1/2$, the operator decouples and is freely rotated by new $U(1)_{acc}$**
  - **$R_{IR} = R_{UV}+\alpha_i F_i+\alpha_{acc} F_{acc} = 1/2$ for such an operator**

Accidental Symmetries?

$U(1)_{acc}$ cannot be seen in UV. What we can compute is

$$I_{\rm w/o\ acc} = {\rm Tr}(-1)^F x^{R_{\rm w/o\ acc}+2j \to \text{can be negative}}$$

- **$I_{w/o\ acc}$ is not analytic at $x = 0$**
- **$I_{w/\ acc}$, on the other hand, is analytic at $x = 0$ by definition**
- **Indeed, those two are related by**

$$I_{\rm w/o\ acc} = I_{\rm w/\ acc}(t_{\rm acc} x^{-\alpha_{\rm acc}})\Big|_{t_{\rm acc}=1}$$

- **The singularity of $I_{w/o\ acc}$ at $x = 0$ is not intrinsic**
- **The exact quantity of $I_{w/o\ acc}$ is meaningful in spite of its singularity $x = 0$**

Factorization of SCI

Let's perform the matrix integration explicitly

= Higgs-branch localization (if no singularity at infinity)

CH-Kim-Park '12

E.g., N=2 U(N) SQCD

$$I^{N_f>\tilde{N}_f}(x,t,\tilde{t},\tau,w) = \sum_{\substack{1\le b_1<\cdots \\ <b_N\le N_f}} Z_{pert}^{\{b_j\}}\left(x,t,\tilde{t},\tau\right) Z_{vortex}^{\{b_j\}}\left(x,t,\tilde{t},\tau,\mathfrak{w}\right) Z_{anti}^{\{b_j\}}\left(x,t,\tilde{t},\tau,\mathfrak{w}\right),$$

$$Z_{vortex}^{\{b_j\}}\left(x,t,\tilde{t},\tau,\mathfrak{w}\right) = \sum_{n=\vec{0}}^{\vec{\infty}} \mathfrak{w}^{\sum_j n_j} \mathfrak{I}_{(n_j)}^{\{b_j\}}\left(x,t,\tilde{t},\tau\right),$$

$$\mathfrak{I}_{(n_j)}^{\{b_j\}}\left(x,t,\tilde{t},\tau\right) = e^{-S_0}\prod_{j=1}^{N}\prod_{k=1}^{n_j}\frac{\prod_{a=1}^{\tilde{N}_f} 2\sinh\frac{-i\tilde{M}_a-iM_{b_j}-2i\mu+2\gamma(k-1)}{2}}{\left(\prod_{i=1}^{N} 2\sinh\frac{iM_{b_i}-iM_{b_j}+2\gamma(k-1-n_i)}{2}\right)\left(\prod_{a\in\{b_j\}^c} 2\sinh\frac{iM_a-iM_{b_j}+2\gamma k}{2}\right)}$$

- **Universal for partition functions with the S^2 isometry up to the perturbative part and the gluing rule**
- **Vortex partition function on omega deformed $R^2\times S^1$**

Supersymmetric Vortices

Supersymmetric Vortices

- **3d U(N) gauge theory has half-BPS solitons**
- **BPS equations**

$$iF_{z\bar{z}} = \frac{e^2}{2}\left(|Q|^2 - \frac{\xi}{2\pi}\right), \qquad D_z Q = 0$$

- **Solution (for large |z|)**

$$Q = \sqrt{\frac{\xi}{2\pi}} e^{in\theta} + \ldots, \qquad A_\theta = n + \ldots$$

$$2\pi n = -i \int dz^2 F_{z\bar{z}}$$

- **The moduli space of a single vortex for N_c=N_f=N,**

$$\mathcal{V}_{1,N,N} = \mathbf{C} \times \mathbb{CP}^{N-1}$$

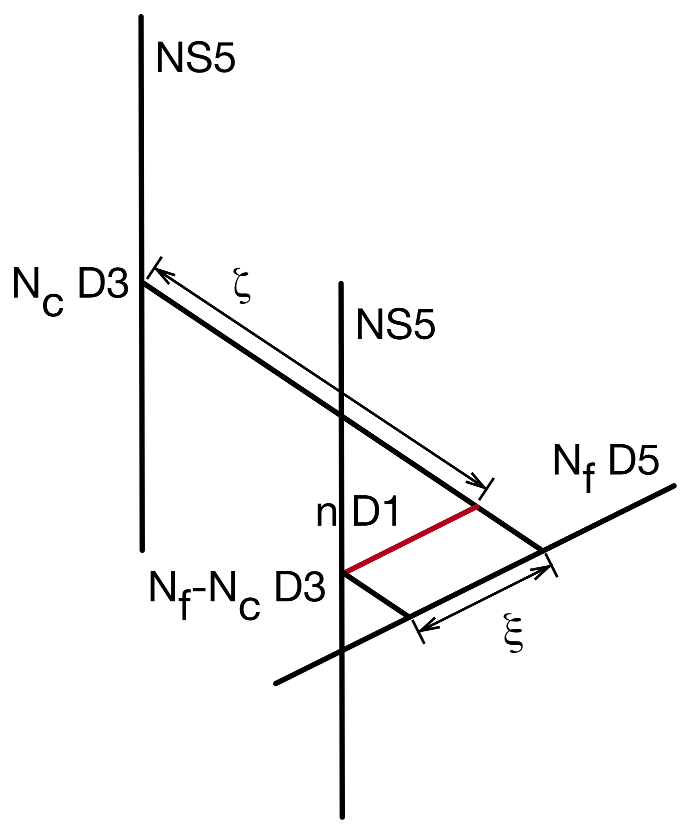
- **For N_c<N_f,** $\dim(\mathcal{V}_{n,N_c,N_f}) = 2nN_f$

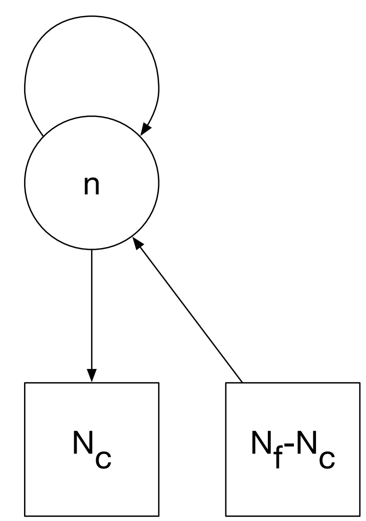
Vortex Quantum Mechanics

Let's consider the GLSM description of the vortex moduli space

Hanany-Tong '03

- **E.g., vortices of N=4 SQCD -> D1s in the type IIB setup**





1d GLSM with G=U(n),

N_c fund chirals,

N_f-N_c antifund chiral

and one adjoint chiral

- **The D1 world volume theory gives the correct twisted Witten index for the vortex moduli space**

H.-C.Kim-J.Kim-S.Kim-Lee '10

Vortex Partition Function Revisited

The twisted Witten index of 1d N=2 GLSM can be computed by the supersymmetric localization Hwang-J.Kim-S.Kim-Park '14, Cordova-Shao '14, Hori-Kim-Yi '14

$$I = \mathrm{Tr}\left[(-1)^F e^{-\beta H} e^{\sigma\cdot\mu}\right] = \frac{1}{|W|}\text{JK-Res}_{\vec{\eta}=\zeta\vec{1}}[g(u)d^r u]$$

- **Jeffrey-Kirwan residue:**

$$\text{JK-Res}_{u=0}(Q(0),\eta)\frac{d^r u}{\prod_{p=1}^r Q_{i_p}\cdot u} = \begin{cases} \frac{1}{|\det(Q_{i_1}\dots Q_{i_r})|}, & \text{if } \eta\in\mathrm{Cone}(Q_{i_1},\dots,Q_{i_r}), \\ 0, & \text{otherwise.}\end{cases}$$

- **1-loop determinants:**

$$g_{\text{vector}}(u) = \prod_{\alpha\in\Delta_G} 2\sinh\frac{-\alpha\cdot u}{2},$$

$$g_{\text{chiral}}(u) = \prod_{\rho\in R_\Phi}\prod_{\sigma\in F_\Phi}\frac{1}{2\sinh\frac{\rho\cdot u+\sigma\cdot\mu}{2}},$$

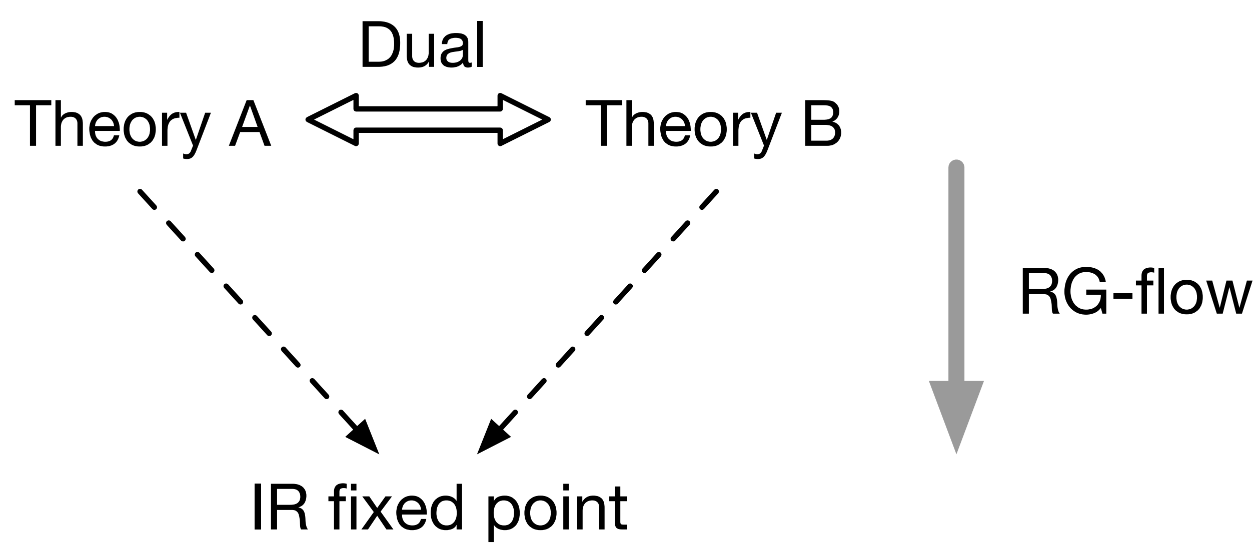
$$g_{\text{fermi}}(u) = \prod_{\rho\in R_\Psi}\prod_{\sigma\in F_\Psi} 2\sinh\frac{-\rho\cdot u-\sigma\cdot\mu}{2}$$

The twisted Witten index of vortex quantum mechanics is identified as the vortex partition function of the 3d theory

Dualities with Fundamentals

3d Seiberg-like Dualities

- **Reminiscent of 4d Seiberg duality and cousins**



- **Monopole operators, Chern-Simons couplings, … in 3d**
- **Various versions: from N=2 to N=6 SUSY, from (S)U to (S)O gauge group, from fundamental to tensor matter, …**
- **Two categories: maximally chiral (Aharony type), minimally chiral (Giveon-Kutasov type)**

Benini-Closset-Cremonesi '11

3d Seiberg-like Dualities

Intriligator-Seiberg '13

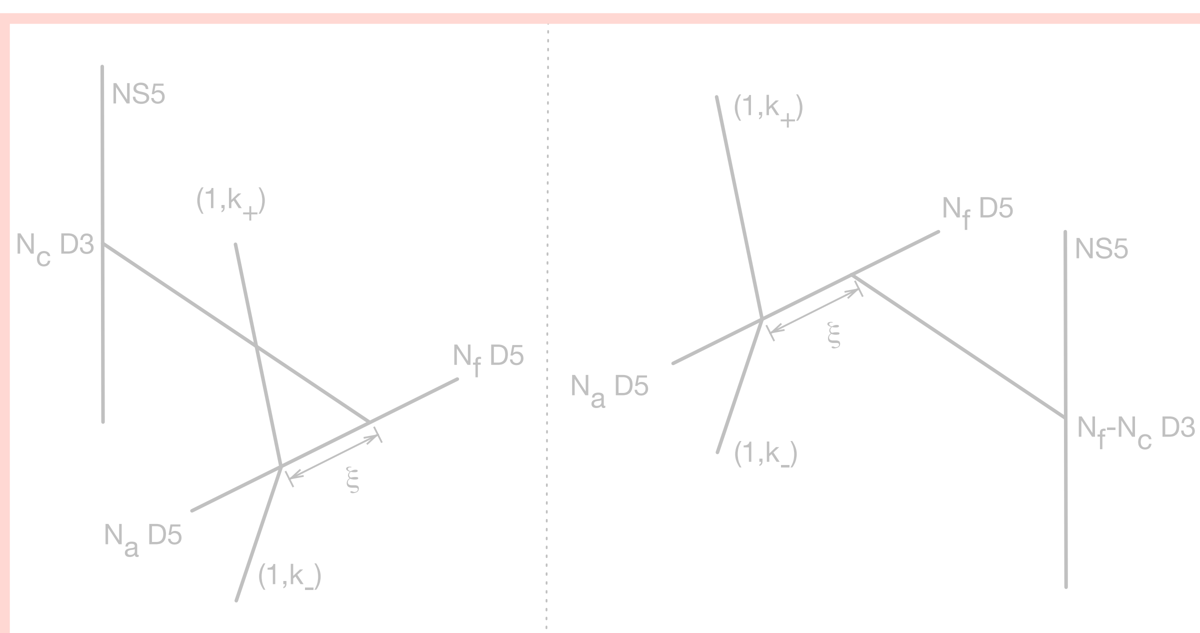
Turning on generic real masses, the semi-classical potential is given by

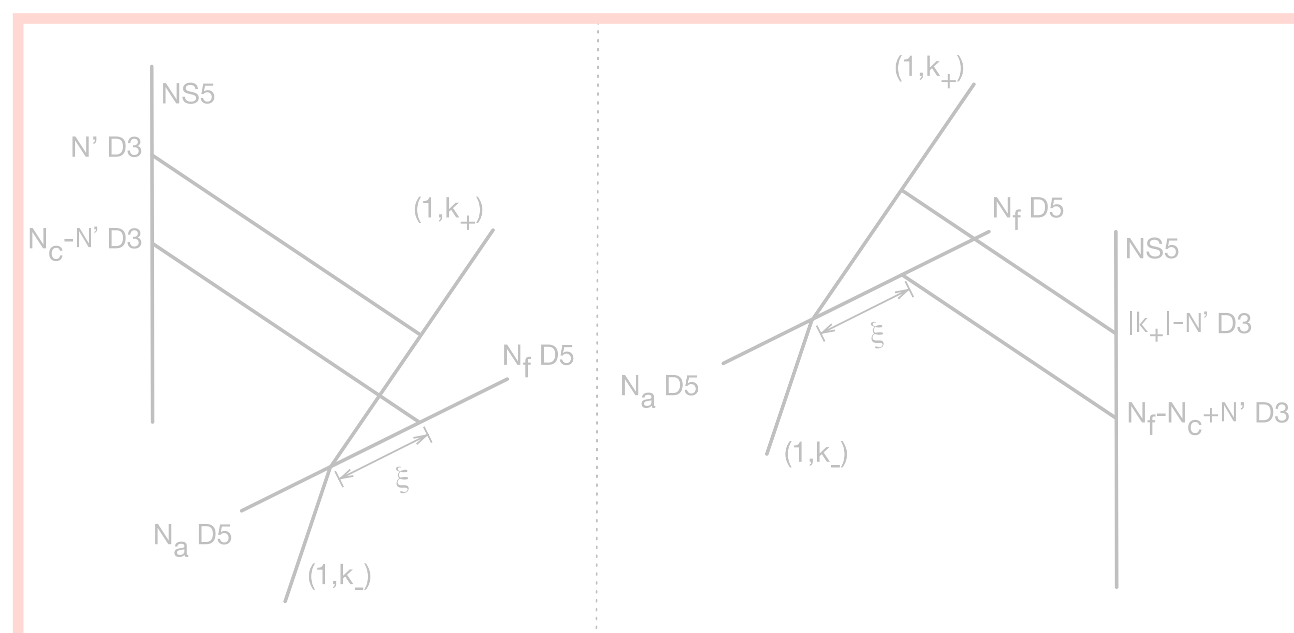
$$V = \frac{e_{\text{eff}}^2}{32\pi^2}\left(\sum_i 2\pi n_i |Q_i|^2 - \xi_{\text{eff}} - \kappa_{\text{eff}}\sigma\right)^2 + \sum_i (m_i + n_i\sigma)^2 |Q_i|^2$$

Higgs vacua: $\xi_{\text{eff}} \neq 0, \quad \kappa_{\text{eff}} = 0 \quad \Rightarrow \quad \sigma = -\frac{m_j}{n_j}, \quad |Q_i|^2 = \frac{\xi_{\text{eff}}}{2\pi n_i}\delta_{ij}$

Coulomb vacua: $\xi_{\text{eff}} = 0, \quad \kappa_{\text{eff}} = 0 \quad \Rightarrow \quad \sigma \neq 0, \quad Q_i = 0$

Topological vacua: $\kappa_{\text{eff}} \neq 0 \quad \Rightarrow \quad \sigma = -\frac{\xi_{\text{eff}}}{\kappa_{\text{eff}}}, \quad Q_i = 0$





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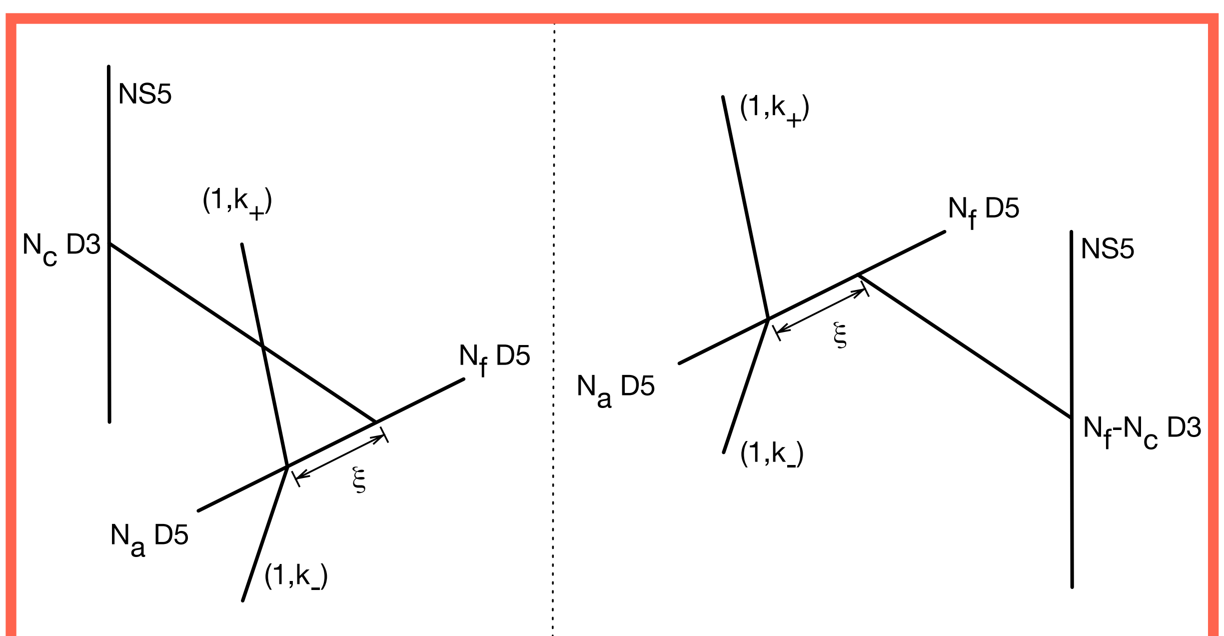

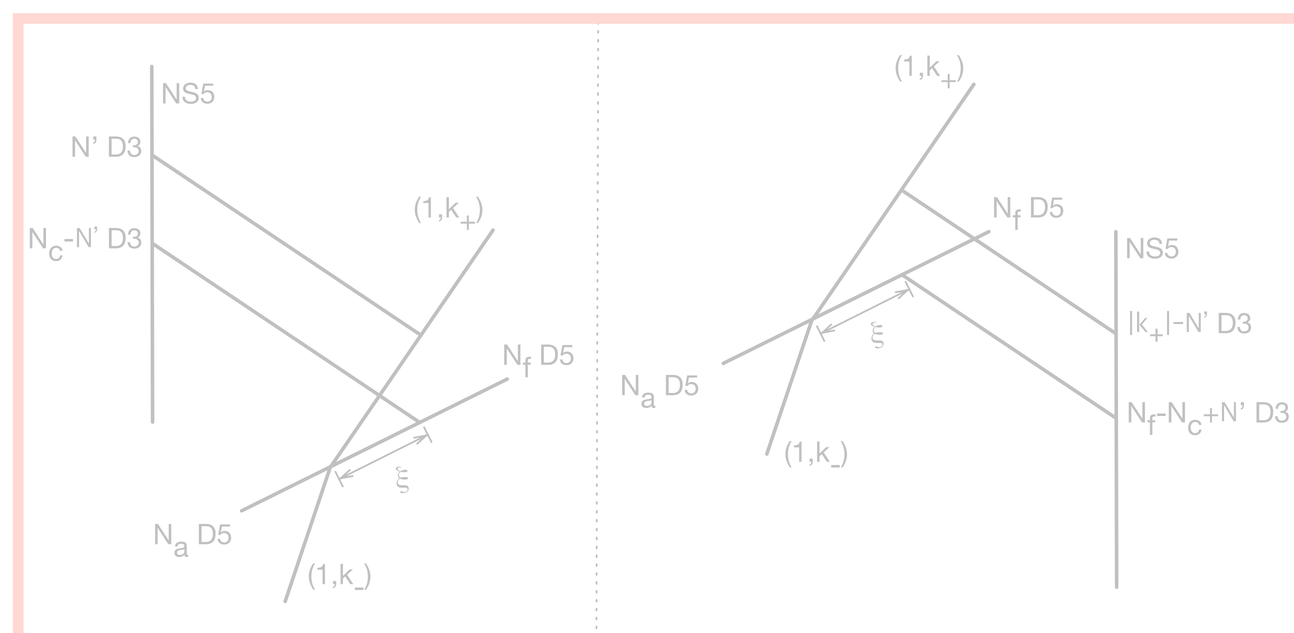

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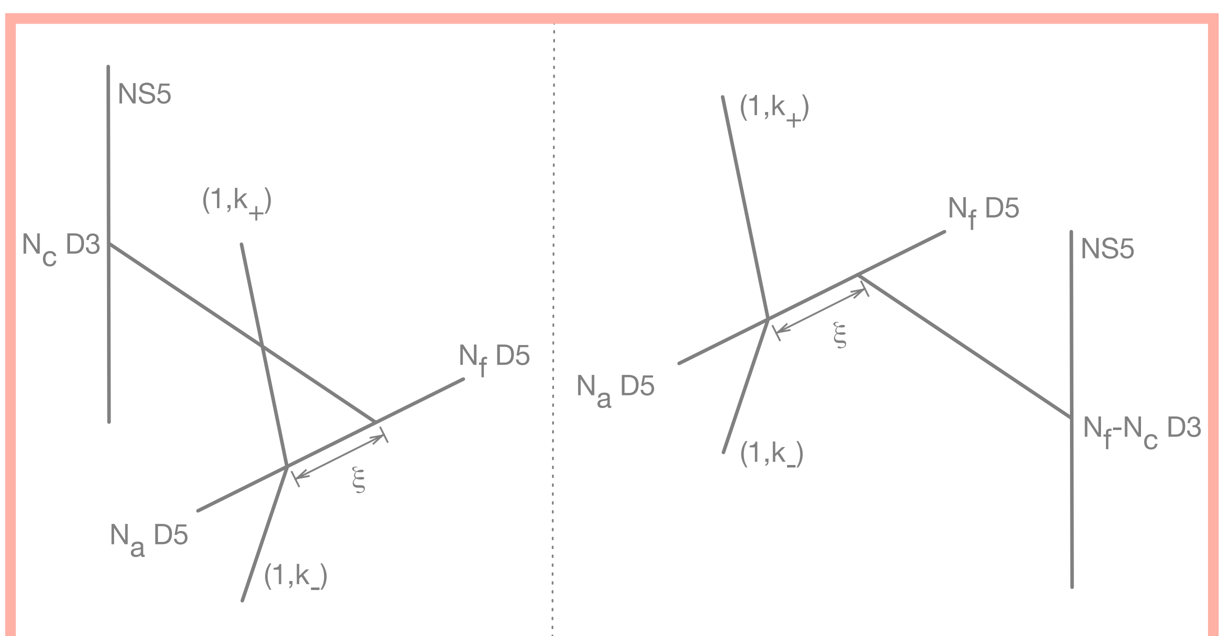

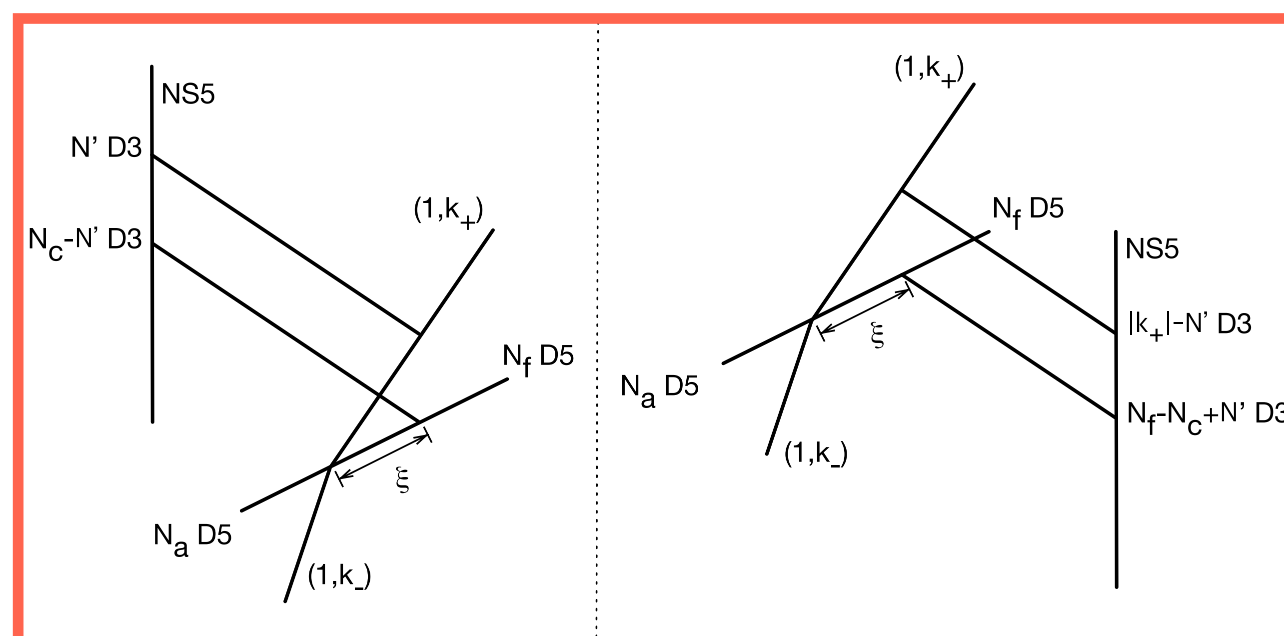

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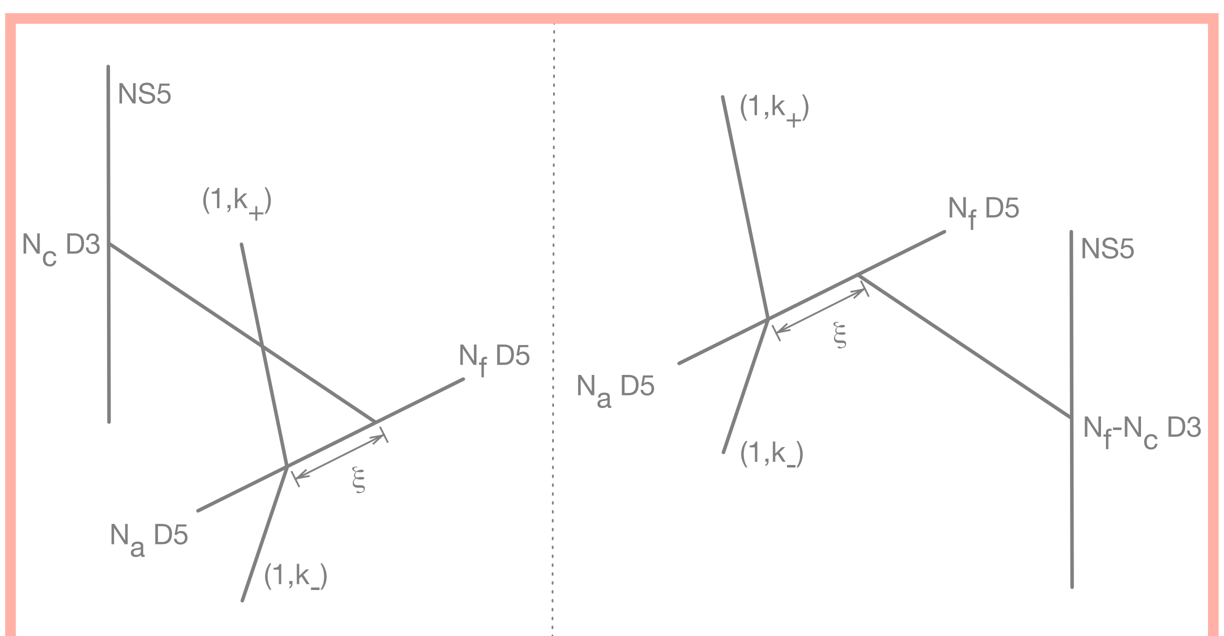

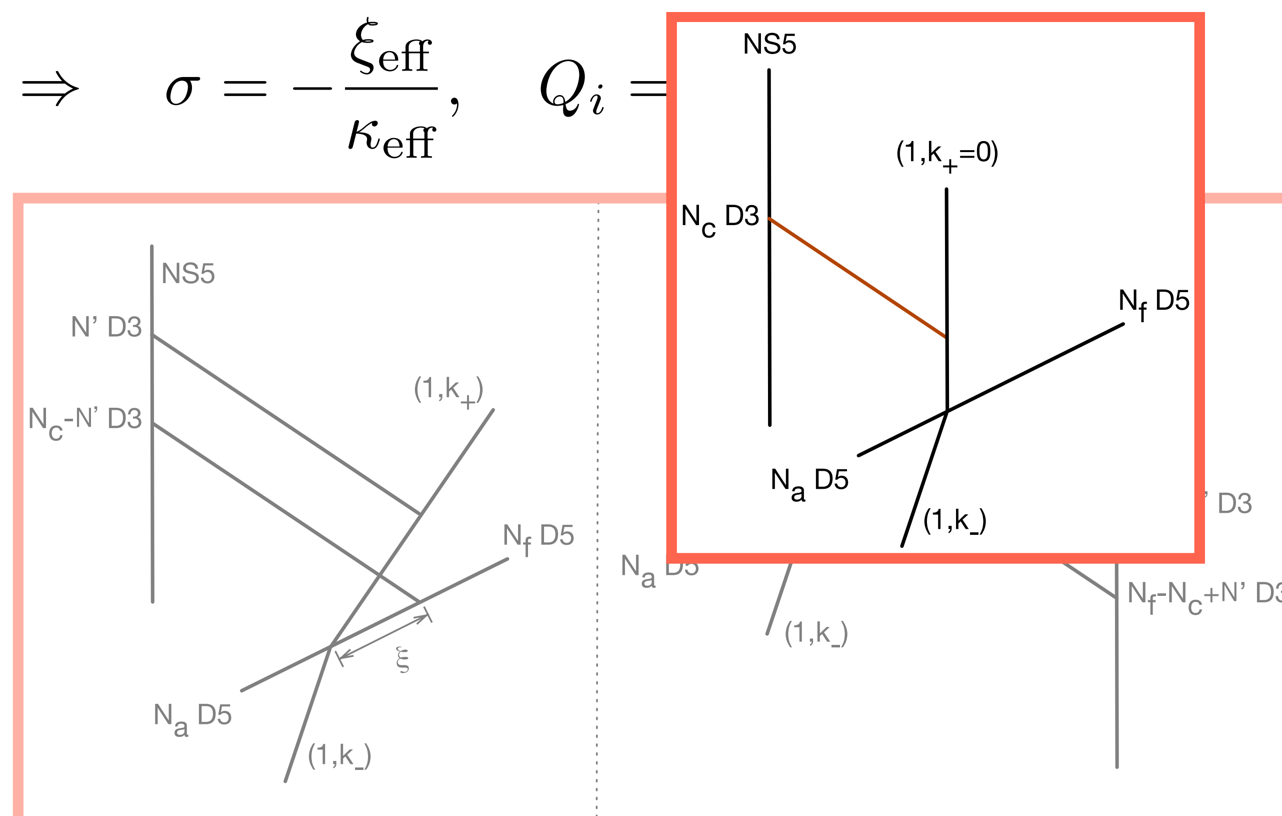

$\mathcal{N} = 2$ SQCD

Aharony '97, Benini-Closset-Cremonesi '11

N=2 Aharony(-Benini-Closset-Cremonesi) duality

- **$U(N_c)_k+(N_f, N_a)$ flavors ($|k| \le (N_f-N_a)/2$)**

  <-> $U(N_f-N_c)_{-k}+(Na, N_f)$ flavors+N_fN_a mesons $M^a{}_b$ (+$V_\pm$) with superpotentials

- **Higgs branch parametrized by $M^a{}_b$**
- **Coulomb branch for $|k| = (N_f-N_a)/2$ parametrized by $V_\pm$**
- Special case: $U(1)_{1/2}$+a fund chiral <-> a free chiral (also mirror symmetry)

  Kachru, Mulligan, Torroba, Wang '16

  - Turning on flavor D-terms -> non-SUSY duality (bosonization)

$$|D_{-a}\phi|^2 - |\phi|^4 + \frac{1}{4\pi}ada - \frac{1}{2\pi}\hat{A}da \leftrightarrow \bar{\Psi}i\gamma_\mu D^\mu_{\hat{A}}\Psi - \frac{1}{8\pi}\hat{A}d\hat{A}$$

  - Vortex states <-> elementary particle states

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N = 4 SQCD

Gaiotto-Witten ’08, Kapustin-Willett-Yaakov ’10

N=4 Seiberg-like duality

- **$U(N_c)+N_f$ funds <-> $U(N_f-N_c)+N_f$ funds$+2N_c-N_f$ free twisted hypers for $N_f=2N_c$, $2N_c-1$**

H.-C.Kim-J.Kim-S.Kim-Lee ’10, Yaakov ’13

- For $N_f < 2N_c-1$, the duality is also proposed but recently notice that the moduli spaces do not coincide; schematically

Assel-Cremonesi ’17

- Nevertheless, there is a common fixed point called symmetric vacuum
- Also, for nonzero FI, all Coulomb branches are lifted; the moduli spaces become the same

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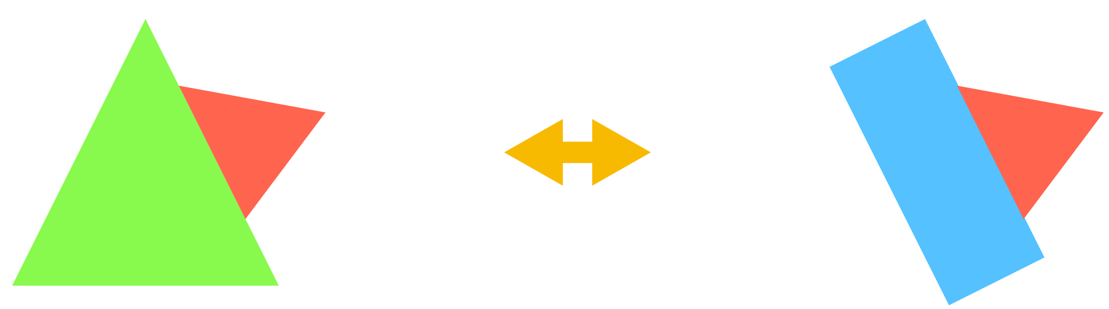

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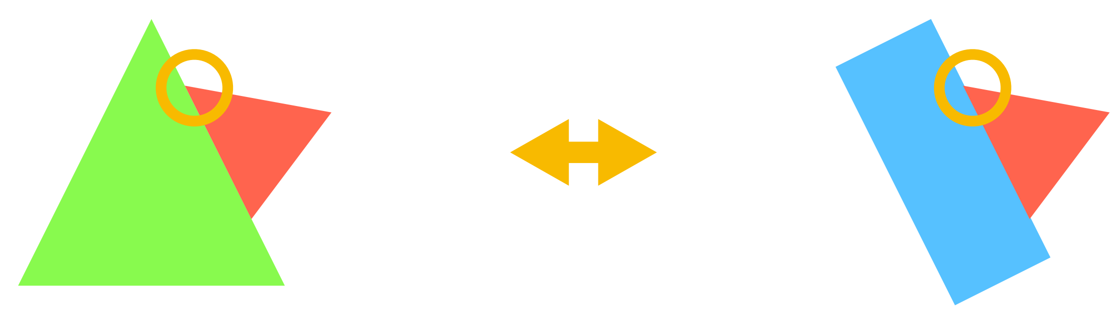

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- **Special case: U(1)+a fund hyper <-> a free twisted hyper**
  - **Flow to the previous N=2 SQCD duality by a real mass deformation**
  - **Vortex states <-> elementary particle states**
- Recall the 1d GLSM description of vortices
- What happens to vortex QM under the 3d duality?

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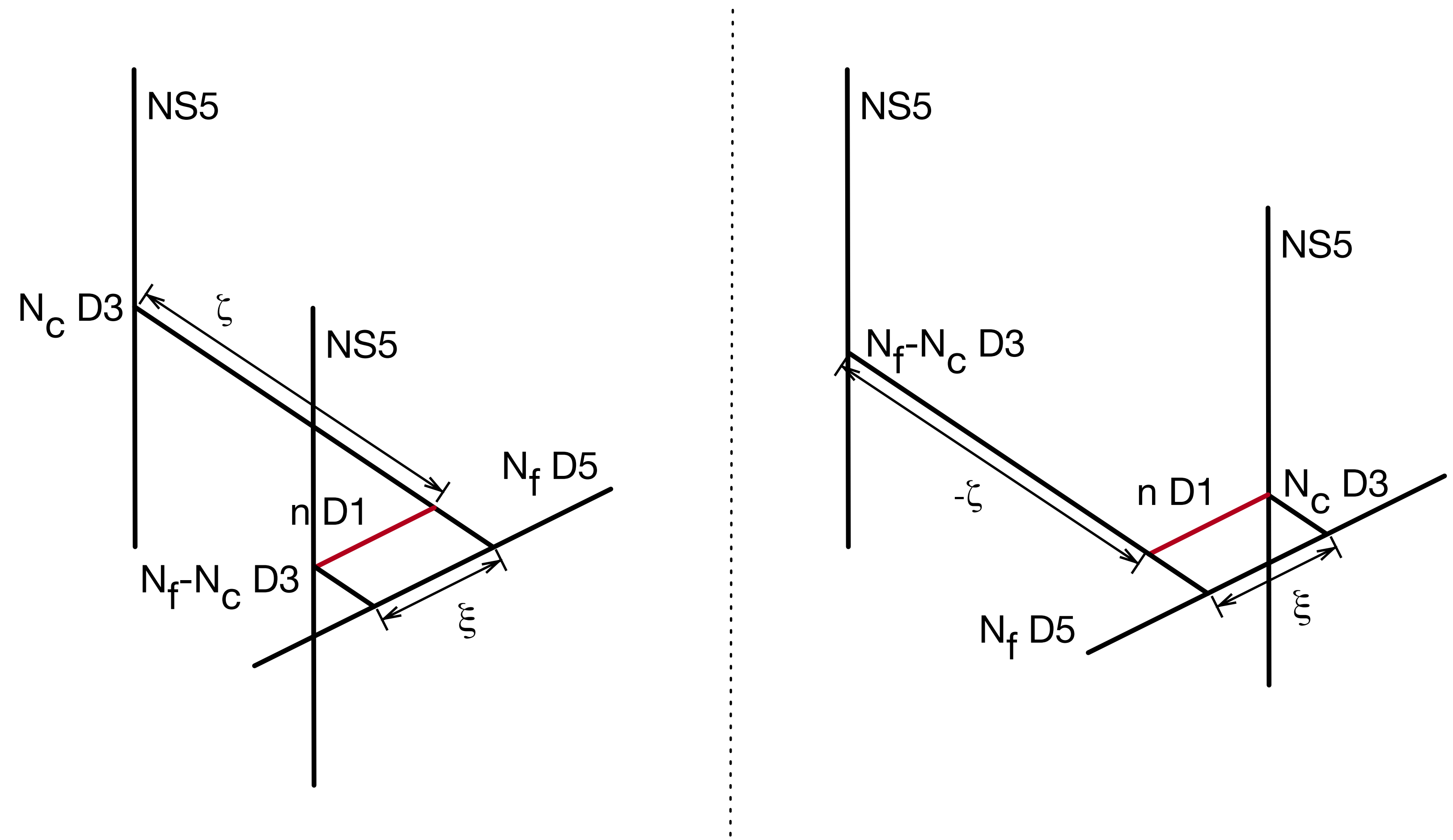
Vs Wall-Crossing of Vortex Quantum Mechanics

Vs Wall-Crossing of Vortex QM

Hanany-Tong ’03

Recall the brane setup of N=4 SQCD



N=4 U(N_c)+N_f funds

ξ ~ 3d FI

ζ ~ 3d $1/g^2$ ~ 1d FI

- **Vortices are realized as D1s**
- **What is the 1d interpretation of this brane motion?**

Vs Wall-Crossing of Vortex QM

CH-Yi-Yoshida '17

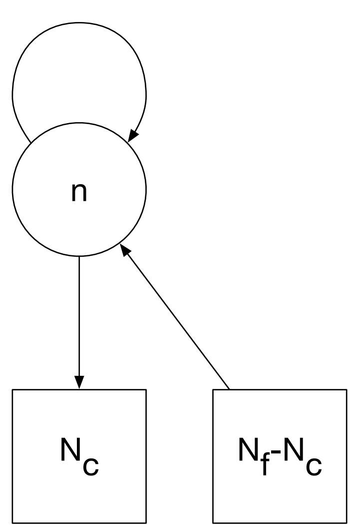
Wall-crossing controlled by 1d FI

- **A 3d N=4 Seiberg-like dual pair share the same vortex QM; the only difference is 1d FI**
- **3d N=4 Seiberg-like duality = wall-crossing of vortex QM**

Vortex QM & index

$$I^n = \frac{1}{|W|}\text{JK-Res}_{\vec{\eta}=\zeta\vec{1}}\left[g^n(u)d^n u\right]$$

$$g^n(u) = \left(\frac{1}{2\sinh\frac{-2\mu}{2}}\right)^n \left(\prod_{i\neq j}^n \frac{\sinh\frac{u_i-u_j}{2}}{\sinh\frac{u_i-u_j-2\mu}{2}}\right)\left(\prod_{i,j=1}^n \frac{\sinh\frac{u_i-u_j-2\mu-2\gamma}{2}}{\sinh\frac{u_i-u_j-2\gamma}{2}}\right)$$
$$\times\left(\prod_{i=1}^n\prod_{b=1}^{N_c}\frac{\sinh\frac{u_i-m_b-2\mu-\gamma}{2}}{\sinh\frac{u_i-m_b-\gamma}{2}}\right)\left(\prod_{i=1}^n\prod_{a=N_c+1}^{N_f}\frac{\sinh\frac{-u_i+m_a-2\mu-\gamma}{2}}{\sinh\frac{-u_i+m_a-\gamma}{2}}\right).$$



Vs Wall-Crossing of Vortex QM

QM indices for different FIs

- **The choice of FI determines the poles contributing to the JK-residue**
- **E.g., the 1-vortex indices**

Positive FI:

$$I_{\zeta>0} = \text{JK-Res}_{\eta=\zeta}\left[g(u)du\right] = \sum_{Q(u*)>0} \text{Res}_{u=u*}\left[g(u)du\right]$$

- **the vortex partition function of the original description**

Negative FI:

$$I_{\zeta<0} = \text{JK-Res}_{\eta=\zeta}\left[g(u)du\right] = -\sum_{Q(u*)<0} \text{Res}_{u=u*}\left[g(u)du\right]$$

- **the vortex partition function of the dual description**

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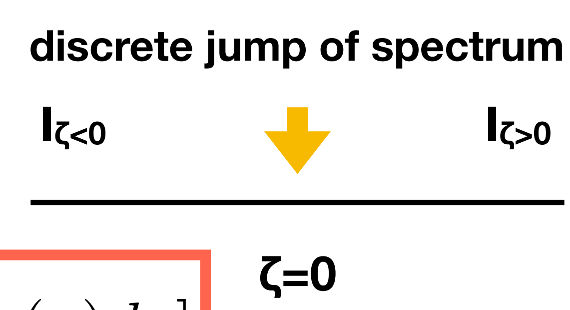

nontrivial wall-crossing at ζ=0

Vs Wall-Crossing of Vortex QM

QM indices for different FIs

- **The choice of FI determines the poles contributing to the JK-residue**
- **E.g., the 1-vortex indices**
- **For general vortex numbers, we prove**

CH-Park '15, CH-Yi-Yoshida '17

$$\sum_{n=0}^{\infty} w^n I^n_{\zeta>0} = \left(\sum_{n=0}^{\infty} w^n I^n_{\zeta<0}\right)\left(\prod_{i=1}^{2N_c-N_f} Z_{\text{hyper}}(x,\tau,w\tau^{2N_c-N_f-2i+1})\right)$$

- **The wall-crossing factor coincides with the contribution of the additional gauge singlets on the dual side**
- **SCIs also match**

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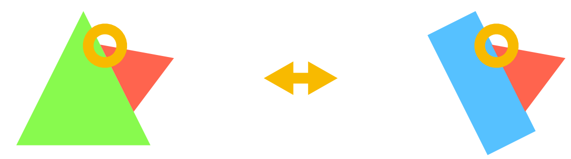
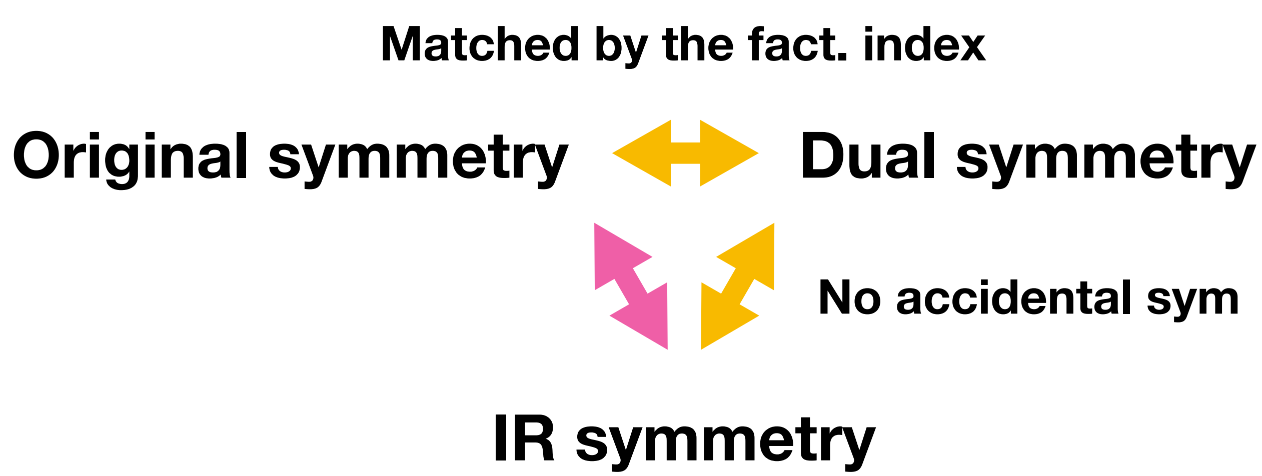
Accidental Symmetries in N=4 SQCD



- The symmetric vacuum of the original theory has accidental IR symmetries due to monopole operators of negative UV R-charges
- Those monopoles are mapped to decoupled twisted hypers on the dual side; the accidental symmetries manifest in the dual UV description
- Can identify the embedding of the original symmetry into the dual symmetry by the SCI matching <- the conserved current multiplet is captured



Accidental Symmetries in N=4 SQCD

Symmetry enhancement for the original theory CH-Park '15

$$SU(N_f) \times U(1)_T \to SU(N_f) \times U(1)_T \times Sp(2N_c - N_f)$$

- **UV charges vs IR charges**

$$R^{UV} = R^{IR} - \frac{1}{2} \sum_{i=1}^{2N_c - N_f} (2N_c - N_f - 2i + 1) B_i^{IR},$$

$$A^{UV} = A^{IR} + \sum_{i=1}^{2N_c - N_f} (2N_c - N_f - 2i + 1) B_i^{IR},$$

$$T^{UV} = T^{IR} + \sum_{i=1}^{2N_c - N_f} B_i^{IR}$$

- **One can identify the enhanced IR symmetries and their charges in terms of UV charges by the duality and the corresponding SCI matching**

More Examples—Aharony Duality

N=2 Aharony(-Benini-Closset-Cremonesi) duality

- **$U(N_c)_k+(N_f, N_a)$ flavors ($|k| \leq (N_f-N_a)/2$)**

  <-> $U(N_f-N_c)_{-k}+(N_a, N_f)$ flavors+N_fN_a mesons (+$V_\pm$) with superpotentials

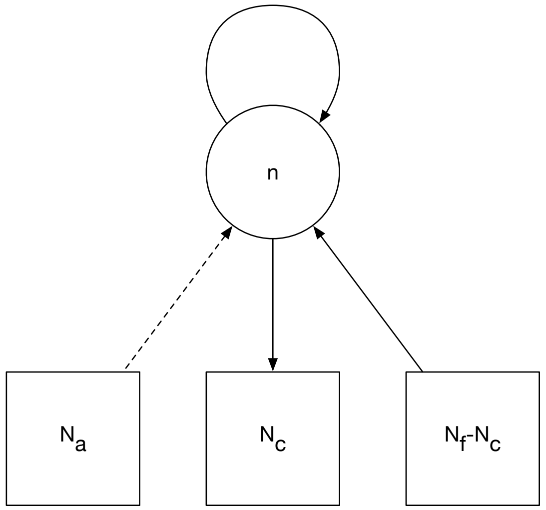
- **Coulomb branch for $|k| = (N_f-N_a)/2$ parametrized by $V_\pm$**

Fujitsuka-Honda-Yoshida '13

Vortex QM & index

$$I^n = \frac{1}{|W|}\text{JK-Res}_{\vec{\eta}=\zeta\vec{1}}\left[g^n(u)d^nu\right]$$

$$g^n(u) = \frac{e^{\kappa\sum_{i=1}^n u_i}\left(\prod_{i\neq j}^n \sinh\frac{u_i-u_j}{2}\right)\left(\prod_{j=1}^n\prod_{a=1}^{N_a}\sinh\frac{u_j-\tilde{m}_a+\mu-\gamma}{2}\right)}{\left(\prod_{i,j=1}^n \sinh\frac{u_i-u_j-2\gamma}{2}\right)\left(\prod_{i=1}^n\prod_{b=1}^{N_c}\sinh\frac{u_i-m_b-\mu-\gamma}{2}\right)\left(\prod_{j=1}^n\prod_{a=N_c+1}^{N_f}\sinh\frac{-u_j+m_a+\mu-\gamma}{2}\right)}$$

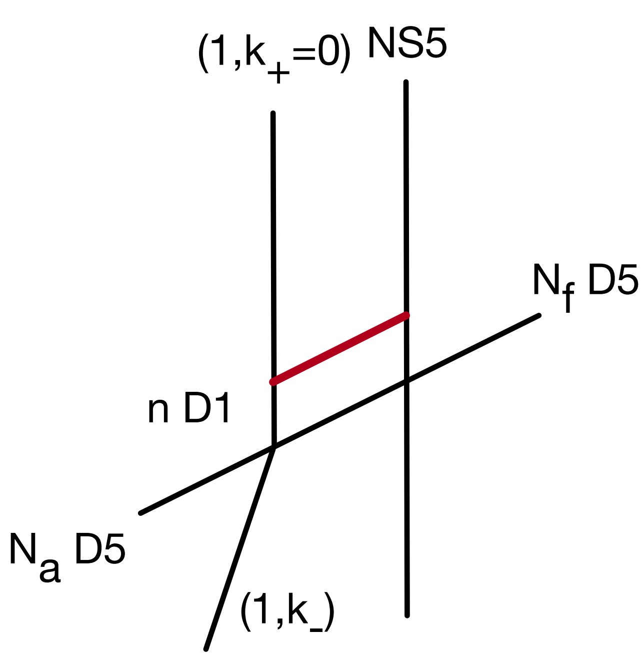


More Examples — Aharony Duality

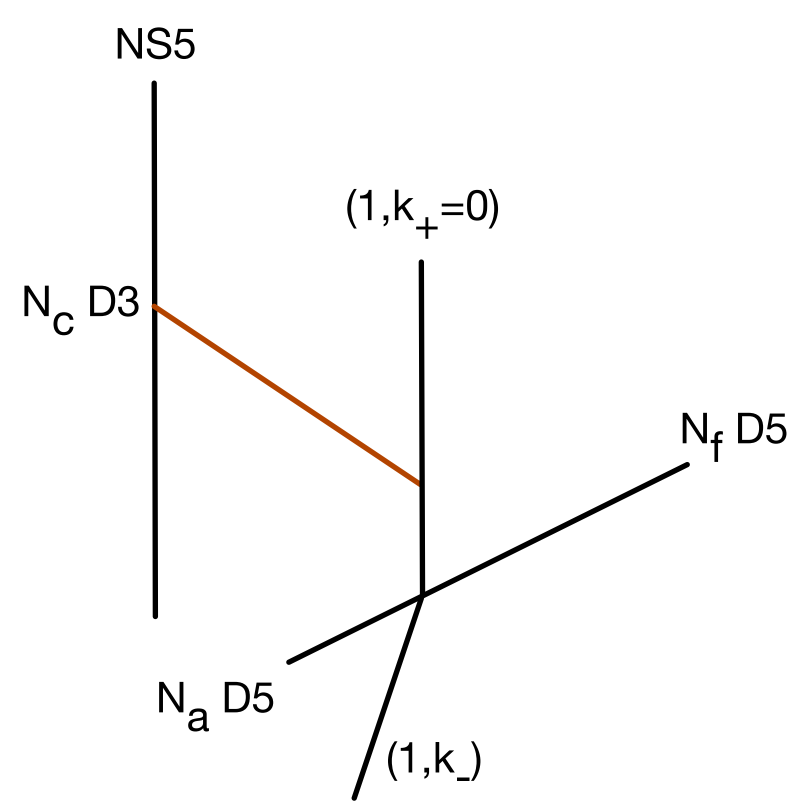
A necessary condition for the nontrivial wall-crossing, i.e., non-vanishing asymptotic contribution:

$$g^n(u) \sim e^{\left(\kappa \pm \frac{N_a - N_f}{2}\right) u_i} \quad \Rightarrow \quad \pm\kappa - \frac{N_f - N_a}{2} = 0$$

- **The branes at ζ=0**



- **C.f., the branes at ξ=0**



More Examples—Aharony Duality

CH-Kim-Park '12, CH-Yi-Yoshida '17

We prove the exact wall-crossing factor is given by

$$\sum_{n=0}^{\infty} w^n I^n_{\zeta>0} = \left(\sum_{n=0}^{\infty} w^n I^n_{\zeta<0}\right)$$
$$\times \, \mathrm{PE}\left[\frac{\delta_{2\kappa, N_a - N_f}\, \tau^{-\frac{N_f+N_a}{2}} x^{-N_c+\frac{N_f+N_a}{2}+1} - \delta_{2\kappa, N_f-N_a}\, \tau^{\frac{N_f+N_a}{2}} x^{N_c - \frac{N_f+N_a}{2}+1}}{1-x^2}\mathbf{w}\right]$$

- Compute the wall-crossing factor of vortex QM without referring the 3d duality
- Reproduce additional gauge singlets on the dual side
- Prove partition function identities for S^3, $S^2\times S^1$, twisted $S^2\times S^1$
- Accidental for simple theories?

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V_+ & V_-, which parametrize the Coulomb moduli space of the 3d theory

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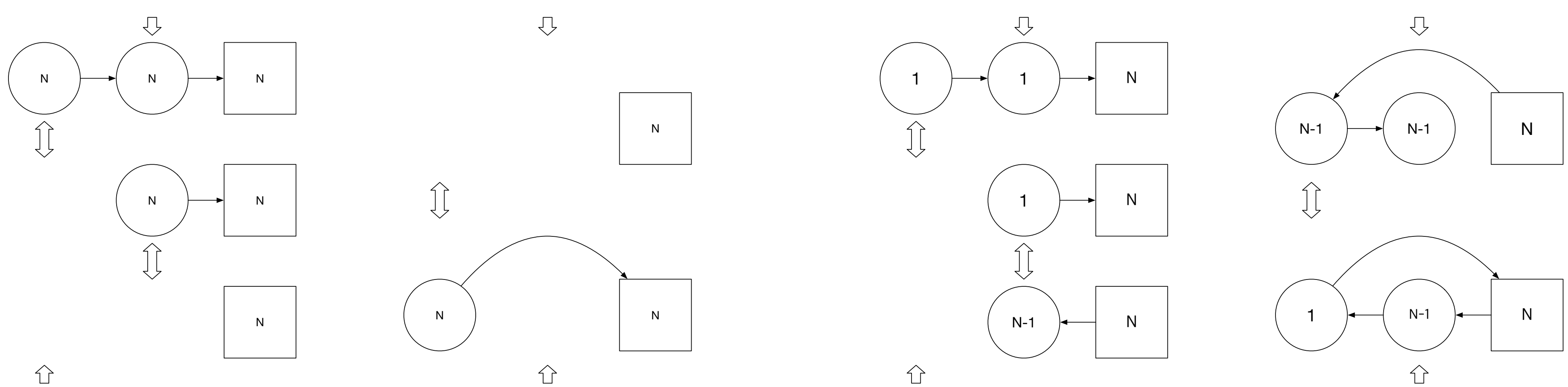
- **Accidental for simple theories?**

More Examples—Linear Quiver

N=4 linear quiver theory with bifunds—T_ρ[SU(N)]

N=2 linear quiver theory of two types: (N)-(N)-[N] & (1)-(1)-[N]

- **Unknown type IIB realizations**
- **Various CS/BF couplings**
- **The Seiberg-like dual chain**



- **Nontrivial shifts of CS/BF couplings are predicted by vortex wall-crossing**
- **Independently confirm by 3d Witten index counting**

More Dualities—with Adjoints

More Dualities — with Adjoints

Introduce a matter in the adjoint representation

- **Adjoint matter with the superpotential** $W = \tilde{Q}XQ$

  -> N=4 SQCD

- **Adjoint matter with the superpotential** $W = \mathrm{Tr}X^{n+1}$

  -> 3d version of the Kutasov-Schwimmer-Seiberg duality

Kim-Park '13, CH-Park '15

- **$U(N_c)_k+(N_f,\, N_a)$ flavors+adjoint X with $W=\mathrm{Tr}\, X^{n+1}$ ($|k| \leq (N_f-N_a)/2$)**

  <-> $U(nN_f-N_c)_{-k}+(N_a,\, N_f)$ flavors, adjoint X, mesons M_i, (+$V_i^{\pm}$) with

$$W = \mathrm{Tr}X^{n+1} + \sum_{i=0}^{n-1} M_i \tilde{q} X^{n-1-i} q + \sum_{i=0}^{n-1} (V_i^+ v_{n-1-i}^- + V_i^- v_{n-1-i}^+)$$

More Dualities—with Adjoints

CH-Park '15

Work out SCI of the adjoint theory *without* superpotential

$$I(x,t,\tilde{t},\tau,\upsilon,w)=\sum_p I^p_{\rm pert}(x,t,\tilde{t},\tau,\upsilon)Z^p_{\rm vortex}(x,t,\tilde{t},\tau,\upsilon,\mathfrak{w})Z^p_{\rm antivortex}(x,t,\tilde{t},\tau,\upsilon,\mathfrak{w})$$

$$Z^p_{\rm vortex}(x,t,\tilde{t},\tau,\upsilon,\mathfrak{w})=\sum_{\mathfrak{n}_j\geq 0}\mathfrak{w}^{\sum_{j=1}^{N_c}\sum_{n=0}^{l_j-1}\mathfrak{n}_j^n}\mathfrak{Z}^p_{(\mathfrak{n}_j)}(x,t,\tilde{t},\tau,\upsilon)$$

$$\mathfrak{Z}^p_{(\mathfrak{n}_j)}(x=e^{-\gamma},t=e^{iM},\tilde{t}=e^{i\tilde{M}},\tau=e^{i\mu},\upsilon=e^{i\nu})$$

$$=e^{-S^p_{(\mathfrak{n}_j)}(x,t,\tau,\upsilon)}\left(\prod_{a,b=1}^{N_f}\prod_{q=1}^{p_a}\prod_{\substack{r=1\\(\neq q\text{ if }a=b)}}^{p_b}\prod_{k=1}^{\sum_{n=1}^{r}\mathfrak{n}_{(b,n)}}\frac{\sinh\frac{iM_a-iM_b+i\nu(q-r)+2\gamma k}{2}}{\sinh\frac{iM_a-iM_b+i\nu(q-r)+2\gamma(k-1-\sum_{n=1}^{q}\mathfrak{n}_{(a,n)})}{2}}\right)$$

$$\times\left(\prod_{a,b=1}^{N_f}\prod_{q=1}^{p_a}\prod_{\substack{r=1\\(\neq q\text{ if }a=b)}}^{p_b}\prod_{k=1}^{\sum_{n=1}^{r}\mathfrak{n}_{(b,n)}}\frac{\sinh\frac{iM_a-iM_b+i\nu(q-r-1)+2\gamma(k-1-\sum_{n=1}^{q}\mathfrak{n}_{(a,n)})}{2}}{\sinh\frac{iM_a-iM_b+i\nu(q-r+1)+2\gamma k}{2}}\right)$$

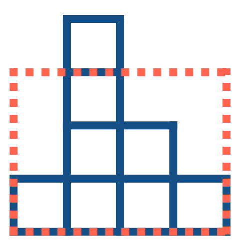
$$\times\left(\prod_{b=1}^{N_f}\prod_{r=1}^{p_b}\prod_{k=1}^{\sum_{n=1}^{r}\mathfrak{n}_{(b,n)}}\frac{\prod_{a=1}^{N_a}\sinh\frac{-i\tilde{M}_a-iM_b-2i\mu-i\nu(r-1)+2\gamma(k-1)}{2}}{\prod_{a=1}^{N_f}\sinh\frac{iM_a-iM_b-i\nu(r-1)+2\gamma k}{2}}\right)$$

p: partition of N_c into N_f nonnegative integers, υ: fugacity for $U(1)_X$, …

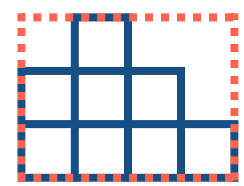
More Dualities—with Adjoints

With the superpotential $W = \mathrm{Tr}\, X^{n+1}$

- **$U(1)_X$ is explicitly broken -> $\upsilon = x^{2/(n+1)}$**
- **Partitions are restricted to those of nonnegative integers ≤ n**
- **E.g., U(8) with N_f=4, n=3**



Not allowed



Allowed

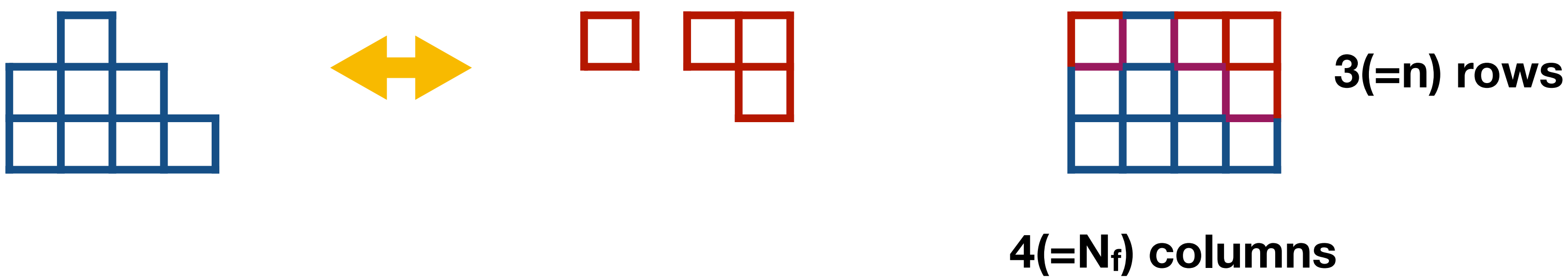
- **# of the partitions = # of Higgs vacua**
- **Partitions of N_c into N_f nonnegative integers ≤ n**

<-> partitions of $(nN_f - N_c)$ into N_f nonnegative integers ≤ n

More Dualities — with Adjoints

We find the duality map between partitions and dual partitions

- **E.g., U(8) with N_f=4, n=3 <-> U(4) with N_f=4, n=3**



For each vacuum, vortex partition functions match

$$Z^{p,N_c,N_f,N_a,\kappa}_{\text{vortex}}(x,t,\tilde{t},\tau,\mathfrak{w})$$

$$= Z^{n-p,nN_f-N_c,N_f,N_a,\kappa}_{\text{antivortex}}(x,t^{-1},\tilde{t}^{-1},\tau^{-1}x^{\delta},\mathfrak{w}^{-1}x^{-\kappa(2-\delta)}) \times \prod_{i=1}^{n} \frac{Z_{\text{chiral}}(x,w\tau^{-\frac{N_f+N_a}{2}}x^{\tilde{\Delta}_i})^{\delta_{N_f-N_a,-2\kappa}}}{Z_{\text{chiral}}(x,w\tau^{\frac{N_f+N_a}{2}}x^{2-\tilde{\Delta}_i})^{\delta_{N_f-N_a,2\kappa}}}$$

-> SCIs also match in total

More Dualities — with Adjoints

What about two adjoints?

Intriligator-Wecht '03

In 4d, SCFTs with two adjoints are classified by their superpotentials

$$W_{\hat{O}} = 0, \quad W_{\hat{A}} = \mathrm{Tr} Y^2, \quad W_{\hat{D}} = \mathrm{Tr} X Y^2, \quad W_{\hat{E}} = \mathrm{Tr} Y^3,$$
$$W_{A_n} = \mathrm{Tr}\left(X^{n+1} + Y^2\right), \quad W_{D_{n+2}} = \mathrm{Tr}\left(X^{n+1} + XY^2\right),$$
$$W_{E_6} = \mathrm{Tr}\left(Y^3 + X^4\right), \quad W_{E_7} = \mathrm{Tr}\left(Y^3 + YX^3\right), \quad W_{E_8} = \mathrm{Tr}\left(Y^3 + X^5\right)$$

Kutasov-Lin '14

Evidence is limited; e.g., SCIs are compared only in the large N limit

We are investigating those dualities by reducing them to 3d

More Dualities — with Adjoints

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Brodie '96, Kutasov-Lin '14

Dual theories are proposed

Kutasov-Lin '14

Evidence is limited; e.g., SCIs are compared only in the large N limit

We are investigating those dualities by reducing them to 3d

More Dualities—with Adjoints

Work in progress

- $U(N_c)+N_f$ flavors+adjoints X, Y with $W=\mathrm{Tr}(X^{n+1}+XY^2)$ (n=odd)

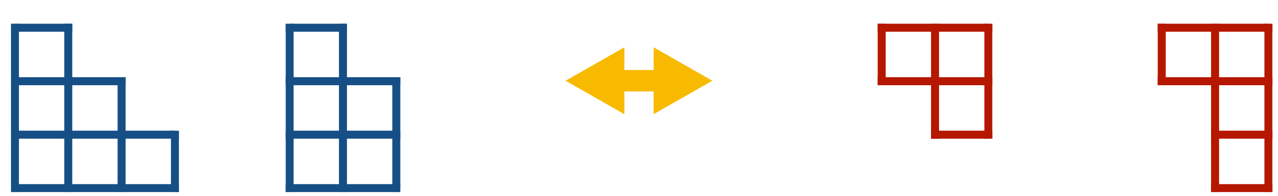
  <-> $U(3nN_f-N_c)+N_f$ flavors, adjoints X, Y, singlets M_{st}, $V_{st}^{\pm}$, $W_u^{\pm}$ with

$$W = \mathrm{Tr}\left(X^{n+1}+XY^2\right) + M_{st}\tilde{q}X^{n-1-s}Y^{2-t}q + V^{\pm}_{s,0}v^{\mp}_{n-1-s,0} + V^{\pm}_{0,t}v^{\mp}_{0,2-t} + \boxed{W^{\pm}_{u}w^{\mp}_{\frac{n-3}{2}-u}}$$

Monopole operator of topological charge 2 ~ $X^{2u}|1,1,0,\ldots>$

- Higgs vacua are labeled by *particular* 2-dim integer partitions of N_c
- E.g., U(11) with $N_f=2$, n=3 <-> U(7) with $N_f=2$, n=3
  - 11->6(≤3n=9)+5->(3+2+1)+(3+2+0) <-> 7->3+4->(2+1+0)+(3+1+0)

3(=n) rows



3 columns for $\mathrm{Tr}(X^{n+1}+XY^2)$

2(=N_f) 1-dim partitions

More Dualities — with Adjoints

- **For two adjoints without superpotential, the matrix integral for SCI has double poles**
- **$W=\mathrm{Tr}(X^{n+1}+XY^2)$ with n=odd reduces those double poles to simple ones**
- **Only a subset of 2-dim integer partitions are allowed -> growing trees determined by scanning the perturbative part**
- **For $N_f=1$, # of Higgs vacua is**

$$
\begin{aligned}
&\left[\frac{N_c}{2}\right]+1, && 1\le N_c\le n-2,\\
&\frac{n+1}{2}, && n-1\le N_c\le 2n+1,\\
&\left[\frac{3n-N_c}{2}\right]+1, && 2n+2\le N_c\le 3n
\end{aligned}
$$

★
$$\left[\frac{N_c}{2}\right]+1=\left[\frac{3n-N_c^D}{2}\right]+1$$

- **Still have double poles for $W=\mathrm{Tr}(X^{n+1}+XY^2)$ with n=even & $W=\mathrm{Tr}(Y^3+YX^3)$**

Conclusion

Conclusion

- **The moduli space of 3d vortices can be described by 1d GLSM**
- **In 3d Seiberg-like dualities, especially the Aharony-type, ∃ a nontrivial correspondence between vortex states and elementary particle states**
- **This correspondence can be described in vortex QM's point of view—the wall-crossing controlled by 1d FI**
- **The vortex partition function is also useful to study complicated Seiberg-like dualities—including accidental IR symmetries, high gauge ranks, etc**
- **SCIs match for the 3d duality with $W = \mathrm{Tr}(X^{n+1}+XY^2)$ (n=odd) (work in progress)**

Thank you