

Aspects of Anomalies and (S)CFTs and RG flows

Ken Intriligator (UCSD)
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Based on works with spectacular collaborators

Clay Córdova and **Thomas Dumitrescu**

Papers on the arXiv + some work in progress.

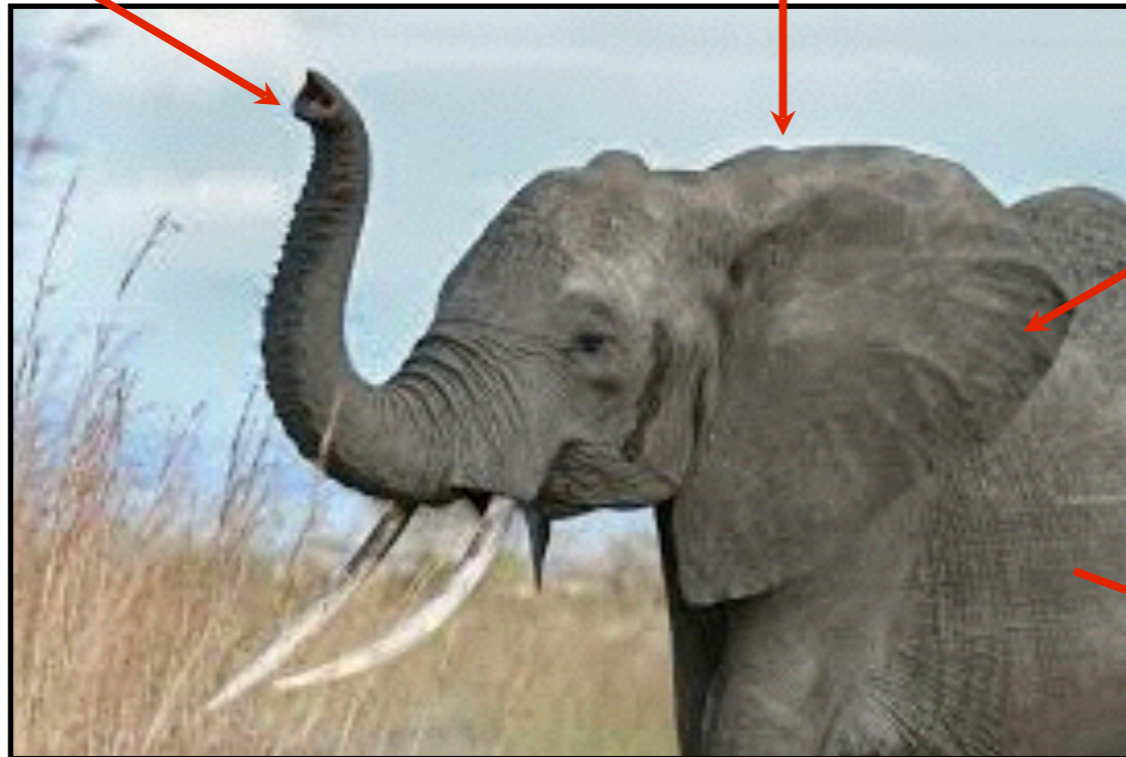
with Cordova & Dumitrescu

- Exact a -anomaly for 6d SCFTs in terms of 't Hooft anomalies, a -theorem. on tensor branch.
- Classify possible susy preserving op. deformations, incl. irrelevant, for all $d > 2$ SCFTs.
- Exhibit all possible multiplets of $d > 2$ SCFTs. Zoo with some strange animals, esp in 3d and 4d.
- To appear: (1) Aspects of mixed anomalies / associated symmetry. (2) 6d vector-tensor theories, Higgs branch, $a > 0$ and a -theorem.

“What is QFT?”

Perturbation theory
around free field
Lagrangian theories

**(S)CFTs +
perturbations**



**5d & 6d SCFTs *
+ deformations,
compactifications**

**(?unexplored...something
crucial for the future?)**

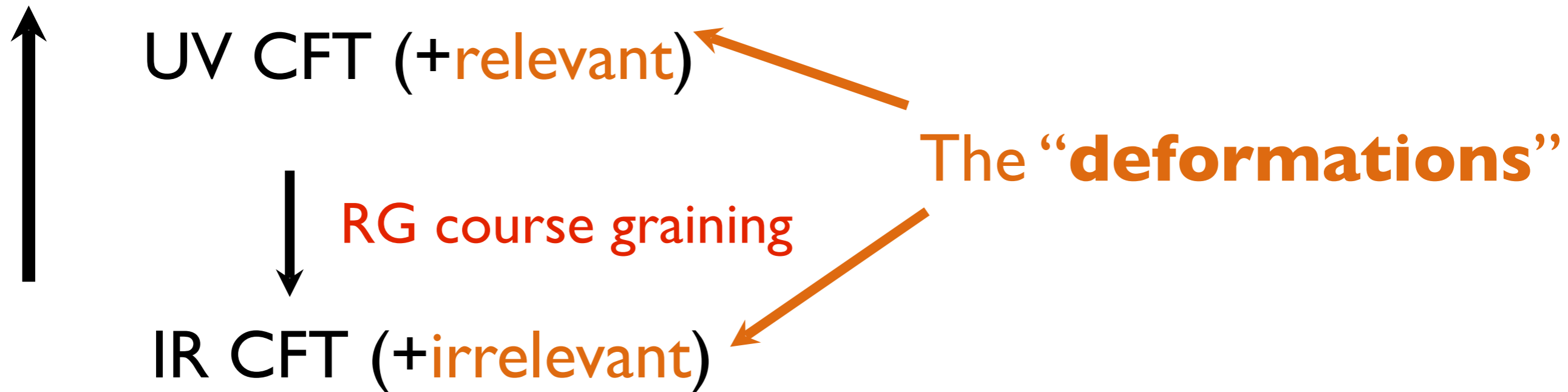
* (Above 4d, starting from free theory, added interactions all look IR free. “That’s irrelevant!”

“That’s the answer! There’s a whole lot of relevants in the circus!”)



RG flows

“# d.o.f.”



- “ $\delta\mathcal{L}$ ” = $\sum_i g_i \mathcal{O}_i$ (OK even if SCFT is non-Lagrangian)
- Move on the moduli space of (susy) vacua.
- Gauge a (e.g. UV or IR free) global symmetry.

RG flow constraints

- 't Hooft anomaly matching for global symmetries + gravity. They must be constant on RG flows; match at endpoints.
- Reducing # of d.o.f. intuition. For $d=2,4$ (& $d=6?$) : a-theorem

$$a_{UV} \geq a_{IR} \quad a \geq 0 \quad \text{For any unitary theory}$$

d=even: $\langle T_{\mu}^{\mu} \rangle \sim aE_d + \sum_i c_i I_i$

(d=odd: via sphere partition function / entanglement entropy.)

- Additional power from supersymmetry. Supermultiplets and supermultiplets of anomalies.

Anomaly polynomial

Alvarez-Gaume,
Witten; Alvarez-
Gaume+Ginsparg.

$$\mathcal{I}_{d+2} = \mathcal{I}_{d+2}^{\text{gauge}} + \mathcal{I}_{d+2}^{\text{gravity+global}} + \mathcal{I}_{d+2}^{\text{mixed}}$$

$$I_{d+2} = dI_{d+1}^{(0)}$$

$$\delta I_{d+1}^{(0)} = dI_d^{(1)}$$

$$\delta S = \int I_d^{(1)}$$

Must cancel,
restricts G & matter

“t Hooft anomalies”
Const. on RG flows.
Matching. Useful!

We’ll discuss these more.

E.g. 6d:
N=(2,0):

$$\mathcal{I}_g = r_g \mathcal{I}_{u(1)} + \frac{k_g}{24} p_2(F_{SO(5)_R})$$

A,D,E group G

Free (2,0) tensor mult

Interaction part

Duff, Liu, Minasian;
Witten; Freed,
Harvey, Minasian,
Moore

N M5s+inflow: $k_{su(N)} = N^3 - N$ Harvey, Minasian, Moore

Other methods: $k_g = h_g^\vee d_g$ KI; Yi; Ki-Myeong Lee et. al.
Ohmori, Shimizu, Tachikawa,
Yonekura.

Exact info about mysterious SCFTs (and “L”STs).

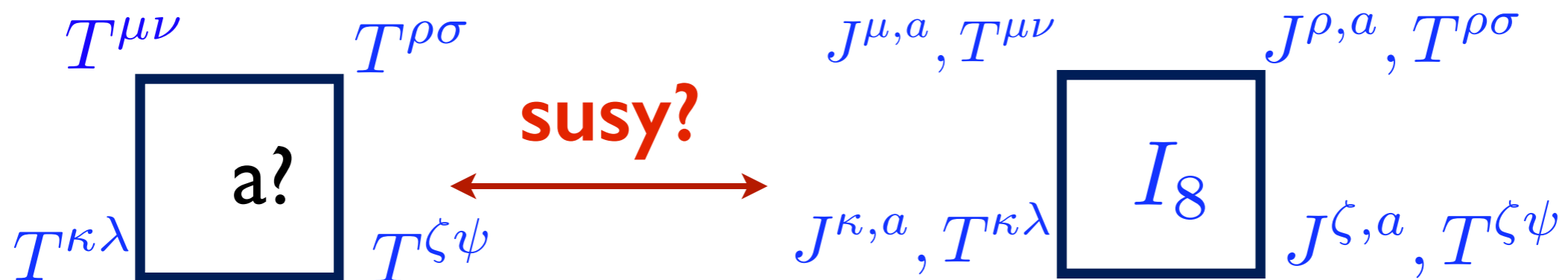
susy and anomalies

SUSY multiplet of anomalies: conformal anomaly is related to 't Hooft anomalies for the superconformal R-symmetry in the energy-momentum tensor supermultiplets.

$$T^{\mu\nu} \leftrightarrow J_R^{\mu,a} \quad \text{Stress-tensor supermultiplet}$$

$$g_{\mu\nu} \leftrightarrow A_{R,\mu}^a \quad \text{Source: bkgrd SUGRA supermultiplet}$$

Known in 2d & 4d. We showed it (indirectly) in 6d:



4-point fn with too many indices. Hard to get a (and to compute).

Easier to isolate anomaly term.
Enjoys anomaly matching

Recall 't Hooft matching

$\mathcal{I}_{d+2}^{\text{global+gravity}}$ = obstruction to gauging = RG flow const.

Intuitive idea: imagine canceling these with decoupled spectators. Then “weakly gauge” the global symmetries. OK at all scales so if the spectator contribution is constant then so were the original 't Hooft anomalies. They're invariant under any deformation that does not explicitly break the symmetries. Spontaneous breaking can lead to apparent mismatches, which must be cancelled by some interactions of the NG bosons in the low-energy theory.

E.g. 6d N=(2,0) on its tensor branch, breaking $g \rightarrow h + u(1)$

$\Delta\mathcal{I}_8 = \frac{1}{24}(k_g - k_h)p_2(F_{SO(5)_R})$ Cancelled by Hopf WZW term

$$S_{eff} \supset -\frac{(k_g - k_h)}{6} \int_{W_7} \eta_3 \wedge d\eta_3 \quad \text{KI '00}$$

dilaton & a-matching

Spontaneous conf'l symm breaking: dilaton has derivative interactions to give Δa anom matching **Schwimmer, Theisen; Komargodski, Schwimmer**

6d case: $\mathcal{L}_{\text{dilaton}} = \frac{1}{2}(\partial\varphi)^2 - b\frac{(\partial\varphi)^4}{\varphi^3} + \Delta a\frac{(\partial\varphi)^6}{\varphi^6}$ (schematic)

Maxfield, Sethi; Elvang, Freedman, Hung, Kiermaier, Myers, Theisen.

Can show that $b > 0$ ($b=0$ iff free) but b 's physical interpretation was unclear; no conclusive restriction on sign of Δa .

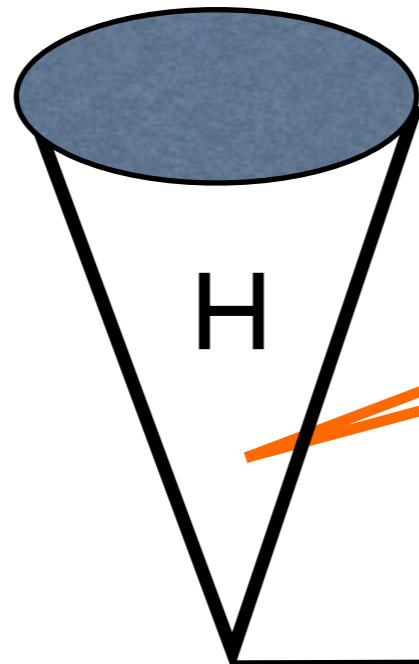
Cordova, Dumitrescu, Kl: for 6d $N=(1,0)$ susy, this matching is related by susy to 't Hooft anomaly matching term. Implies

$$\Delta a \sim b^2 > 0. \quad a^{\text{origin}} = \frac{16}{7}(\alpha - \beta + \gamma) + \frac{6}{7}\delta$$

$$\mathcal{I}_8^{\text{gravity+global}} \supset \alpha c_2(R)^2 + \beta c_2(R)p_1(T) + \gamma p_1^2(T) + \delta p_2(T)$$

6d (1,0) susy moduli

Deform SCFT
by moving on
its vacuum
manifold:



Hypermultiplet “Higgs branch”
(SU(2) R symmetry broken)

Interacting 6d
SCFT at origin

Tensor multiplet branch
SU(2) R symmetry unbroken*

* Easier case. Just dilaton, no NG bosons. Dilaton = tensor multiplet. More on Higgs branch, to appear.

(1,0) 't Hooft anomalies

$$\mathcal{I}_8^{\text{gravity+global}} \supset \alpha c_2(R)^2 + \beta c_2(R)p_1(T) + \gamma p_1^2(T) + \delta p_2(T)$$

Exactly computed for many (1,0) SCFTs **Ohmori, Shimizu, Tachikawa; & +Yonekura (OST, OSTY)**
E.g. for theory of N small E8 instantons:

$$\mathcal{E}_N : (\alpha, \beta, \gamma, \delta) = (N(N^2 + 6N + 3), -\frac{N}{2}(6N + 5), \frac{7}{8}N, -\frac{N}{2})$$

N=(1,0) tensor branch anomaly matching:

KI; Ohmori, Shimizu, Tachikawa, Yonekura

$\Delta \mathcal{I}_8 \equiv \mathcal{I}_8^{\text{origin}} - \mathcal{I}_8^{\text{tensor branch}} \sim X_4 \wedge X_4$ must be a **perfect square**,
match \mathfrak{g}_8 via X_4 sourcing B:

$$\mathcal{L}_{GSWS} = -iB \wedge X_4 \quad dH \sim X_4$$

$$X_4 \equiv 16\pi^2 (x c_2(R) + y p_1(T)) \quad \mathbf{x, y = integer coefficients}$$

6d (1,0) tensor LEEFT

Elvang
et. al.

$$\mathcal{L}_{\text{dilaton}} = \frac{1}{2}(\partial\varphi)^2 - b\frac{(\partial\varphi)^4}{\varphi^3} + \Delta a\frac{(\partial\varphi)^6}{\varphi^6}$$

Our deformation classification implies that b=D-term and

$\Delta a = \frac{98304\pi^3}{7}b^2 > 0$ **Proves the 6d a-theorem** for susy tensor branch flows.

b-term susy-completes to terms in $X_4 = \sqrt{\Delta I_8}$
 $b=(y-x)/2$ $X_4 \equiv 16\pi^2(xc_2(R) + yp_1(T))$

By recycling a 6d SUGRA
analysis from Bergshoeff,
Salam, Sezgin '86

Upshot:

$$a^{\text{origin}} = \frac{16}{7}(\alpha - \beta + \gamma) + \frac{6}{7}\delta$$

So exact 't Hooft anomaly coefficients give the **exact conformal anomaly**, useful! E.g. using this and **OST** for the anomalies:

$$a(\mathcal{E}_N) = \frac{64}{7}N^3 + \frac{144}{7}N^2 + \frac{99}{7}N \quad (\text{N M5s @ M9 Horava-Witten wall.})$$

Classification of SCFT algebras= super-algebras:

Nahm '78

$d > 6$ no SCFTs can exist

$d = 6$ $OSp(6, 2|\mathcal{N}) \supset SO(6, 2) \times Sp(\mathcal{N})_R$ $(\mathcal{N}, 0)$ $8\mathcal{N}Q_s$

$d = 5$ $F(4) \supset SO(5, 2) \times Sp(1)_R$ $8Q_s$

$d = 4$ $Su(2, 2|\mathcal{N} \neq 4) \supset SO(4, 2) \times SU(\mathcal{N})_R \times U(1)_R$ $4\mathcal{N}Q_s$

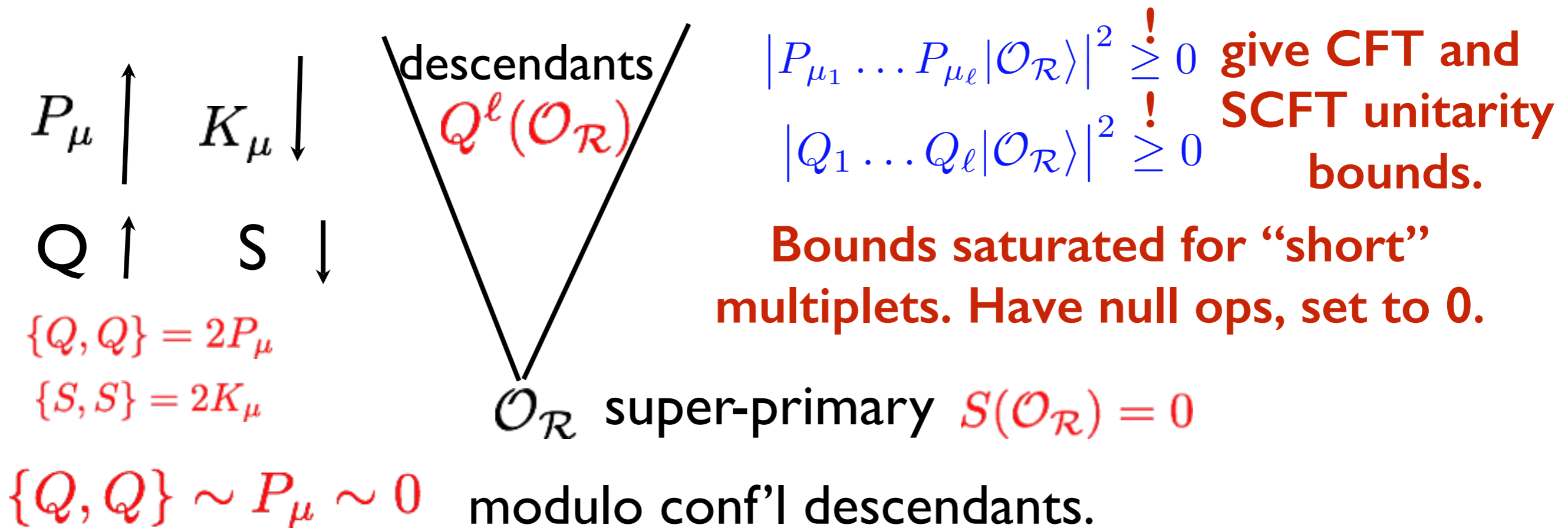
$d = 4$ $PSU(2, 2|\mathcal{N} = 4) \supset SO(4, 2) \times SU(4)_R$

$d = 3$ $OSp(4|\mathcal{N}) \supset SO(3, 2) \times SO(\mathcal{N})_R$ $2\mathcal{N}Q_s$

$d = 2$ $OSp(2|\mathcal{N}_L) \times OSp(2|\mathcal{N}_R)$ $\mathcal{N}_L Q_s + \mathcal{N}_R \bar{Q}_s$

Unitary SCFTs: operators in unitary reps of the s-algs

Dobrev and Petkova PLB '85 for 4d case. Shiraz Minwalla '97 for $d=3,4,5,6$.



Grassmann algebra.

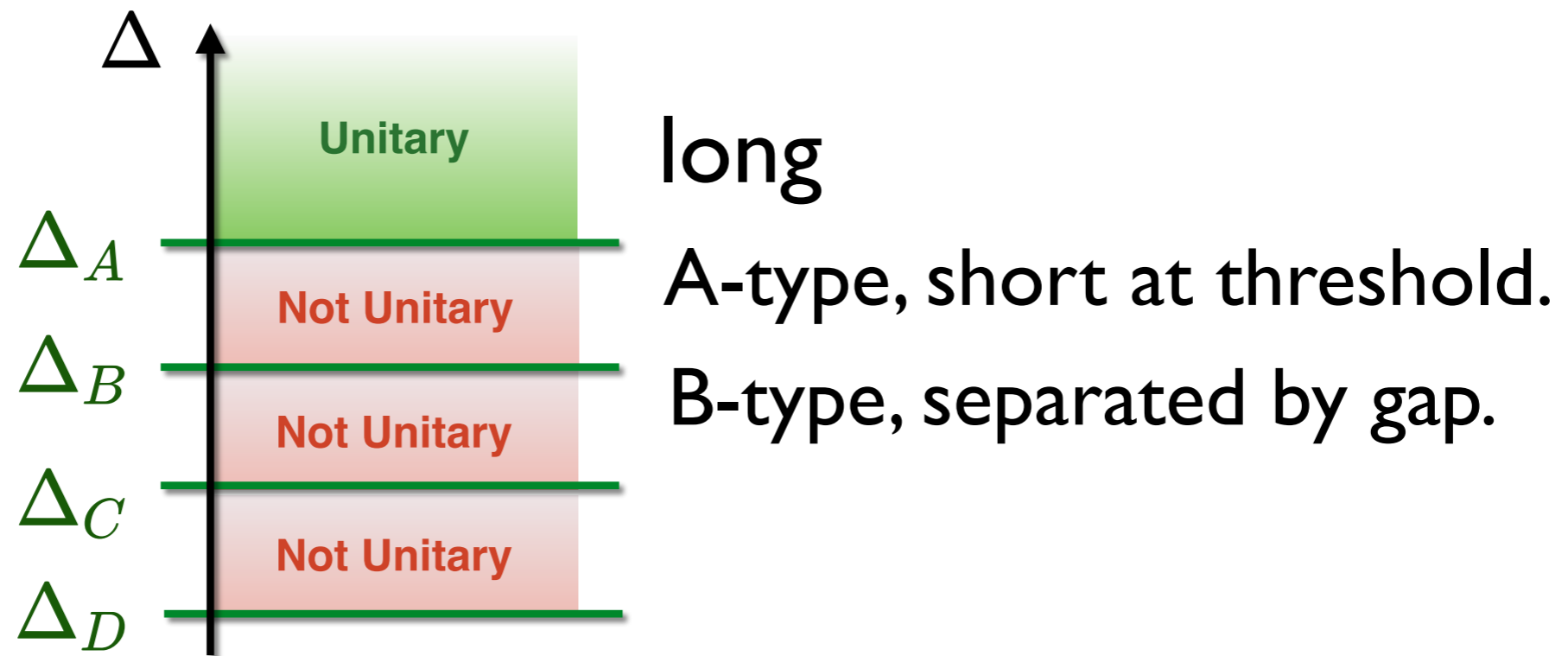
Level $Q^{\wedge \ell}(\mathcal{O}_R)$ $\ell = 0 \dots \ell_{max} \leq N_Q$

Multiplet is “long” iff

$$\ell_{max} = N_Q$$

otherwise, it’s “short”

Unitarity constraints:

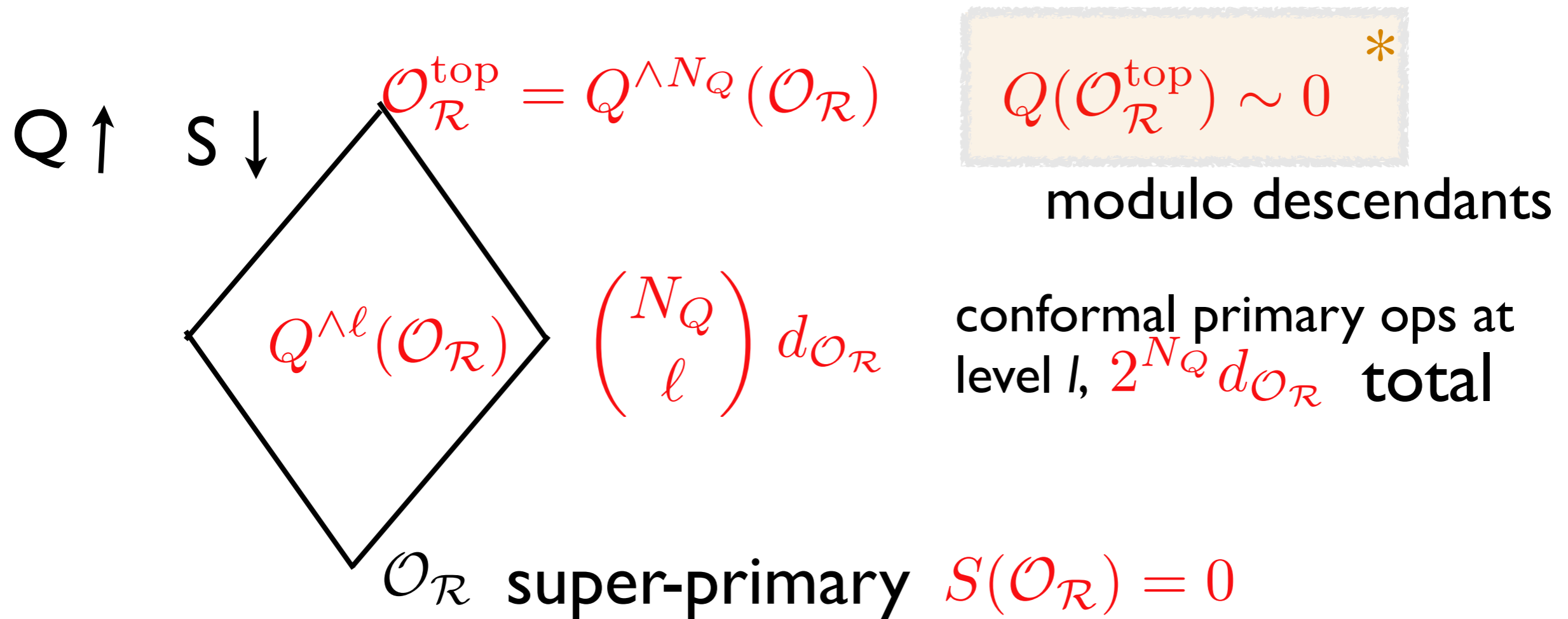


$$\Delta_{A,B,C,D} = f(L\nu) + g(R\nu) + \delta_{A,B,C,D}$$

E.g. in $d=6$: $f(L) = \frac{1}{2}(j_1 + 2j_2 + 3j_3)$ $g(R) = 2R$

$$\delta_{A,B,C,D} = 6, 4, 2, 0$$

Long generic multiplets:



Can generate multiplet from bottom up, via Q , or from top down, via S . **Reflection symmetry**. Unique op at bottom, so unique op at the top. Operator at top = susy preserving deformation (irrelevant for all d and N except for $3d, N=1$) if Lorentz scalar. D-terms. This is the easy case. Some short multiplets have exotic shapes.

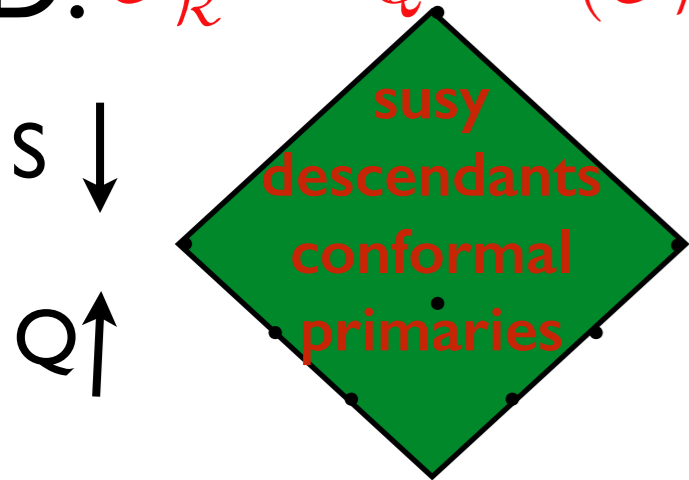
Classify SCFT multiplets and all susy deformations

Cordova, Dumitrescu, KI

$$Q(\mathcal{O}_{\mathcal{R}}^{\text{top}}) \sim 0$$

$$\{Q, Q\} \sim P \sim 0$$

$$D: \mathcal{O}_{\mathcal{R}}^{\text{top}} = Q^{\wedge N_Q}(\mathcal{O}_{\mathcal{R}})$$

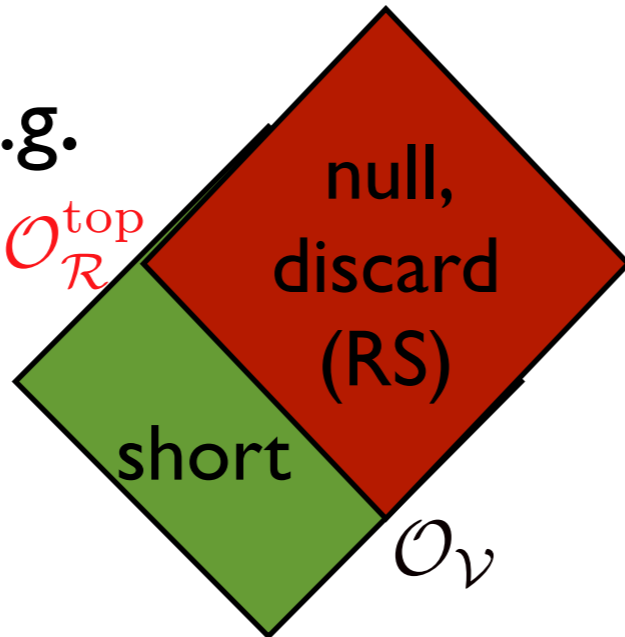


primary: $\mathcal{O}_{\mathcal{R}}$

Generic long

E.g.

$$F: \mathcal{O}_{\mathcal{R}}^{\text{top}}$$



primary: $\mathcal{O}_{\mathcal{R}}$

Generic short

Non-Generic Short
(small R-symm quant #s)

= a **ZOO** of sporadic cases.

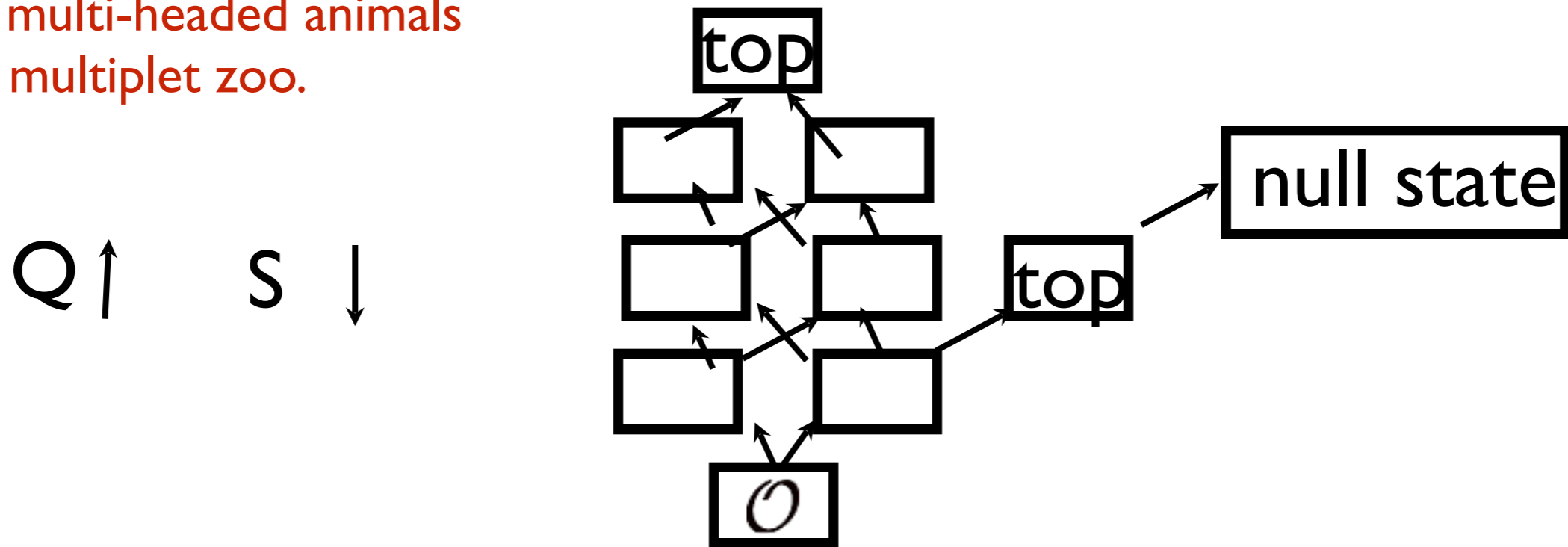
(See Dolan + Osborn for some 4d N=2,4 cases.)

We eventually found a working, conjectural, general d algorithm to properly eliminate null op descendants. Highly tested.

We use then find the op. dim. constraints on the top components. As we increase d or N, fewer or none relevant deformations.

Exotic zoo: e.g. cases ($d=3$) with mid-level susy top

Some multi-headed animals
in the multiplet zoo.



E.g. 3d $\mathcal{N} \geq 4$ $T_{\mu\nu}$ multiplet: the stress-tensor is at top, at level 4.

Another top, at level 2, Lorentz scalar. Gives susy-preserving “universal mass term” relevant deformations. First found in 3d $\mathcal{N}=8$ (KI '98, Bena & Warner '04; Lin & Maldacena '05). Special to 3d. Indeed, they give a deformed susy algebra that is special to 3d (non-central extension).

SUSY forbids some short operators of CFTs

Our classification of all SCFT multiplets includes any with short reps of the bosonic conformal group, i.e. conserved currents and higher spin generalizations, generalized free fields, etc. Some reps, which were allowed by $SO(d,2)$, cannot fit into any rep of the superconformal algebra, hence they cannot appear in any SCFT. Some forbidden operators are interesting: e.g. in 6d, no conserved 2-form current $j^{\mu\nu}$! All known 6d SCFTs have tensor branches with a $B_{\mu\nu}$. Their associated currents cannot be gauge invariant operators in the SCFT, indeed they're "non-Abelian."

Also clarifies maximal Susy

In $d=6,4,3,(+2)$, superconformal algebras exist for any N . Free-field methods (particle spectrum) show that there are higher-spin particles if more than 16 supercharges. Question: **Can this be evaded with interacting SCFTs?** We show that the answer is **no**. For $d=4$ and $d=6$, the algebra for more than 16 Q s has a short multiplet with a conserved stress-tensor, but it is not Q closed (mod P). Also higher spin currents. Free theory with wrong algebra. For $d=3$, stress-tensor is a mid-level “top” operator for all N , and higher spin currents. So $d=3$ (only) has free field SCFT realizations for any N , no upper bound.

a-theorem, and sign, for SCFTs with gauge fields

A free gauge field not conformal for $d > 4$. It is unitary, but it can be regarded as a subsector of a **non-unitary CFT**. **El-Showk, Nakayama, Rychkov**

Applying our formula to a free 6d (1,0) vector multiplet gives

$$a^{\text{origin}} = \frac{16}{7}(\alpha - \beta + \gamma) + \frac{6}{7}\delta \quad \longrightarrow \quad a(\text{vector}) = -\frac{251}{210}$$

negative.. (Same value later found for non-unitary, higher derivative, (1,0) SCFT version by **Beccaria & Tseytlin**)

Many 6d unitary SCFTs built from coupled vectors + tensors. We show that they satisfy the a-theorem, and have $a > 0$, thanks to the matter and tensor multiplet contributions.

Vectors + B-fields

$S_{eff,4d} \supset \int \left(\frac{\phi}{\Lambda} F \wedge F + cB \wedge F \right)$ axion and /or 2-form B coupled to Abelian magnetic 2-form current

$$S_{eff,6d} \supset \int (\phi F \wedge *F + cB \wedge F \wedge F)$$

6d: With no usual gauge kinetic terms, now this can be conformal. Need $\langle \phi \rangle \sim g_{eff}^{-2} > 0$. Two form B couples to instanton string current density $j^{\mu\nu} = \star c_2(F_{gauge})$ which is conserved. Hence this 2-form current must have canonical operator dimension, 4. Many SCFTs based on such a description away from the origin, on their tensor branch, starting 21 years ago with N. Seiberg's paper. The reps know that j cannot exist as an operator in the spectrum. B is non-Abelian (also seen via compactification).

6d susy tensor + vectors

$$S_{eff,6d} \supset \sqrt{c}(\phi F \wedge \star F + B \wedge c_2(F))$$

Last term contributes to reducible gauge anomalies, so its coeff is completely fixed by gauge anomaly cancellation. Susy relates the two terms. The sqrt emphasizes there is a sign condition. Rules out theories whose hyper contrib to reducible gauge anomaly is $>$ than the vectors, e.g. no $U(1)$ gauge theory + tensor SCFTs.

Example: $SU(N)$ with $2N$ hypers + tensor multiplet. As in this example, 6d $(1,0)$ SCFTs typically have global symmetries.

$$S_{eff,6d} \supset \int B \wedge (n_F c_2(F_{global}) + n_R c_2(F_R) + n_T p_1(T))$$

$$dH = \sqrt{c} c_2(F_{gauge}) + n_F c_2(F_{global}) + n_R c_2(F_R) + n_T p_1(T)$$

Gauge or global
bkgd instantons =
string sources for
B field. n's in \mathbb{Z} .

Mixed anomalies

$$S_{eff,6d} \supset \int B \wedge X \quad , \quad X = n_G c_2(F_{gauge}) + n_F c_2(F_{global}) + n_R c_2(F_R) + n_T p_1(T)$$

$\Delta I = \frac{\kappa}{2} X \wedge X$ n_G is fixed by gauge anomaly cancellation. What about the others? They contribute to mixed gauge + gravity anomalies, and to 't Hooft anomalies.

The excellent paper of [Ohmori, Shimizu, Tachikawa, and Yonekura](#) assumed mixed anomalies must be cancelled in their determination of the lg for 6d theories. We showed that this is not obvious, and that non-conformal theories can have non-zero mixed anomalies, e.g. 4d QED with massless fermion flavors, and the 6d (1,0) LST related to small $SO(32)$ instantons. For (S)CFTs, we show that the mixed anomalies actually must vanish, so the **OSTY** for lg are correct. The extra contributions from $\Delta\alpha = \frac{\kappa}{2} n_R^2$ are often crucial for ensuring that the SCFT has $a > 0$.

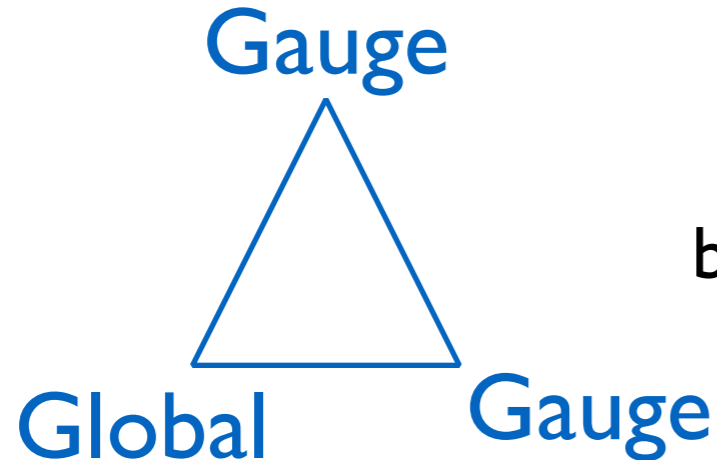
a, for 6d SCFTs with gauge flds:

E.g. $SU(N)$ gauge group, $2N$ flavors, 1 tensor + anomaly cancellation for reducible gauge + mixed gauge + R-symmetry anomalies. Use $a^{\text{origin}} = \frac{16}{7}(\alpha - \beta + \gamma) + \frac{6}{7}\delta$

$$a_{SCFT} = \overset{\text{V}}{(N^2 - 1)\left(-\frac{251}{210}\right)} + \overset{\text{H}}{2N^2\left(\frac{11}{210}\right)} + \overset{\text{T}}{\frac{199}{210}} + \overset{\text{AC :interactions}}{\frac{96}{7}N^2} > 0.$$

We verify that other generalizations likewise have positive a . Also, that Higgs branch flows satisfy the 6d a -theorem.

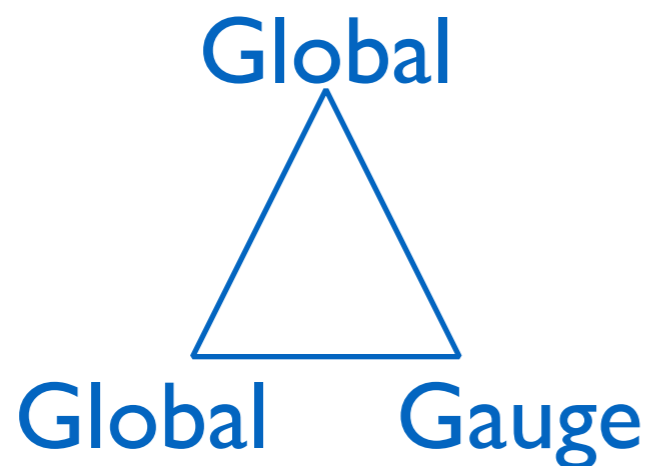
Mixed anomalies in 4d



Axial U(1) ABJ anomaly: global symmetry broken (to discrete subgroup) by instantons:

$$\partial_\mu j_{\text{global}}^\mu = n c_2(F_{\text{gauge}})$$

Can compensate for anomalous global phase by shifting the theta-angle. If we try to gauge the global symmetry, need an axion that shifts and the upshot is the gauge field is massive. This case is not of interest for us here.



Now the global current is conserved, unless it is in a non-trivial background. Under a background gauge transformation for the global symmetry, the action changes by $\delta_\lambda S = n c_2(F_{\text{global}})^{(1)} \wedge F_{\text{gauge}}$
 Example: QED with N_f massless Dirac flavors has such mixed anomalies for the global $SU(N_f)_{L,R}$

Notation: $c_2(F) = d c_2(F)^{(0)}$, $\delta c_2(F)^{(0)} = d c_2(F)^{(1)}$

4d mixed “anomaly” cont

$$\delta_\lambda S = \int nc_2(F_{\text{global}})^{(1)} \wedge F_{\text{gauge}}$$

Here F_{gauge} is a 2-form global current, which can be coupled to a background 2-form B-field. The anomalous transformation becomes a symmetry if regarded as combined with a shift of B

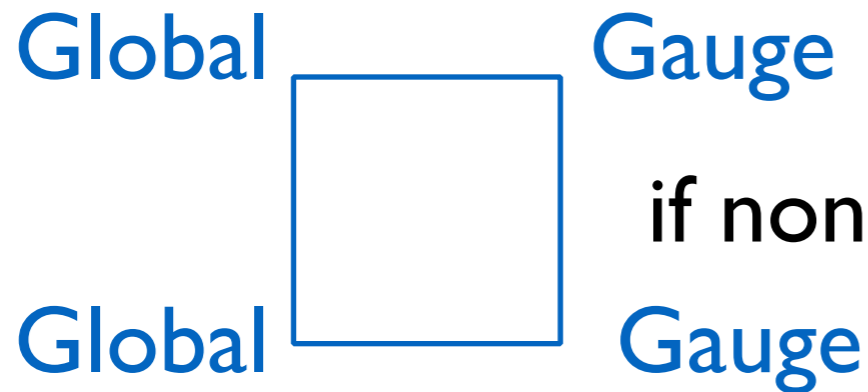
$$\delta_\lambda B = -nc_2(F)^{(1)}$$

“Two-group” symmetry where both the background 1-form and 2-form gauge fields shift under gauge transf.

Writing the anomaly as $D_\mu J_{\text{global}}^{a,\mu} = n \star F_{\text{global}}^a \wedge F_{\text{gauge}}$

we see that it implies a contact term in $\langle J_{\text{global}}^{a,\mu}(x_1) J_{\text{global}}^{b,\nu}(x_2) F^{\rho\sigma}(x_3) \rangle$ but we show it's zero in CFTs.

6d mixed anomalies



if nonzero:

Under global background gauge field transformation:

$$\delta_\lambda S = \int n c_2(F_{\text{global}})^{(1)} \wedge c_2(F_{\text{gauge}})$$

$\star c_2(F_{\text{gauge}})$ = global 2-form current, which couples to a background 2-form gauge field B . The “anomaly” is a symmetry if the 0-form charge symmetry also shifts B $\delta_\lambda B = -n c_2(F_{\text{global}})^{(1)}$ note then

$dB + n c_2(F_{\text{global}})^{(0)}$ is invariant (2nd is Chern-Simons 3-form).

Again, such mixed anomalies do occur in non-CFTs, e.g. the small $SO(32)$ instanton little string theory. But we argue they cannot occur in CFTs. For 6d SCFTs, follows simply from our multiplet classification, since no conserved 2-form current to play role of

$\star c_2(F_{\text{gauge}})$

6d mixed anomalies, cont

In the 6d SCFTs with vector + tensor + hypers, the box diagram with two gauge and two global currents is non-zero, so it **must** be cancelled by a GS mechanism with the dynamical 2-form gauge field. Otherwise, the global current conservation in a background gauge field would be violated by an operator that cannot exist:

$$\partial_\mu J_{\text{global}}^\mu \neq n F_{\text{global}}^{\rho\sigma} \wedge j_{\rho\sigma} \quad \text{since} \quad j_{\rho\sigma} \neq \star c_2(F_{\text{gauge}})$$

The apparent existence of such an operator is an illusion: it sources the dynamical H and becomes some sort of non-gauge invariant quantity, not a legit operator in the CFT. The multiplet classification knew about that. Similar for mixed gauge + gravity anomalies. We also have longer arguments that similar results hold without assuming susy.

Mixed “anomalies” cont

Generalities: if non-zero, it is an obstruction to interacting conformal invariance. The theory can only RG flow to being IR free, as in the 4d QED and 6d little string examples. Also, unlike 't Hooft anomalies, it is not generally an obstruction to an IR mass gap: a non-zero value does not have to be matched in the IR. On the other hand, if the coefficient is zero, it has to remain zero along the entire RG flow, since then there is no obstruction to gauging the global symmetry. Such theories can RG flow into an interacting CFT. We're exploring various other aspects in work in progress, to appear.

Conclude

- QFT is vast, expect still much to be found.
- susy QFTs and RG flows are rich, useful testing grounds for exploring QFT. Strongly constrained: unitarity, a-thm., etc. Can rule out some things. Exact results for others.
- Mixed “anomalies” are interesting.
- Thank you !

감사합니다
[kamsahamnida]