The Frozen Phase of F-Theory

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where f and g are functions on the base (locally).The fiber develops singularities over a codimension one locus defined by vanishing of the discriminant of the above equation:

$$\Delta = 4f^3 + 27g^2 = 0$$

Compactification to 6d $\mathcal{N} = (1, 0)$ (contd.)

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Physically, a type of singular fiber in Kodaira’s list corresponds to a type of 7-brane wrapped on the curve. Each 7-brane carries an 8d gauge algebra which contributes in a particular way to the 6d gauge algebra visible at low energies.

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This correspondence between simple factors of 6d gauge algebra and curves is critical for the anomaly cancellation of the 6d theory as we now explain.

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$$\mathrm{Tr}_{adj} F_a^2 - \mathrm{Tr}_{\rho_a} F_a^2 = 6(K \cdot D_a)\ \mathrm{tr}\, F_a^2 \tag{1}$$

$$\mathrm{Tr}_{adj} F_a^4 - \mathrm{Tr}_{\rho_a} F_a^4 = -3(D_a \cdot D_a)\ (\mathrm{tr}\, F_a^2)^2 \tag{2}$$

$$\mathrm{Tr}_{\rho_{ab}} (F_a^2 \otimes F_b^2) = (D_a \cdot D_b)\ \mathrm{tr}\, F_a^2\ \mathrm{tr}\, F_b^2 \tag{3}$$

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- Each curve is associated either to a **semi-simple** factor of the 6d gauge algebra or to no gauge algebra, and
- A simple factor of 6d gauge algebra can be associated to **multiple** curves.

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This 7-brane is not included in traditional F-theory compactifications. We will see that including $O7^+$ forces the violation of the correspondence between curves and simple factors of 6d gauge algebra.

Goal

In this talk, our goal will be:

1. To understand why the correspondence is violated in the presence of $O7^+$,
2. To understand how 6d anomaly cancellation works in the presence of $O7^+$, and
3. To see that one can construct new 6d SCFTs using F-theory compactifications involving $O7^+$.

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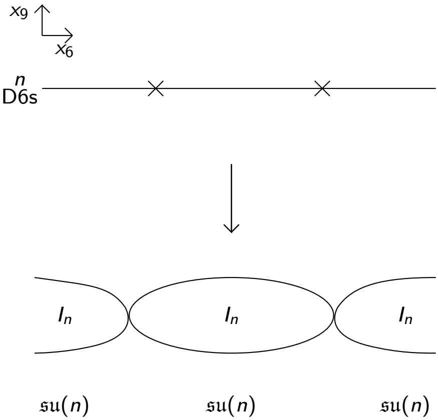

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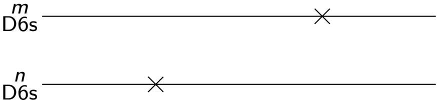

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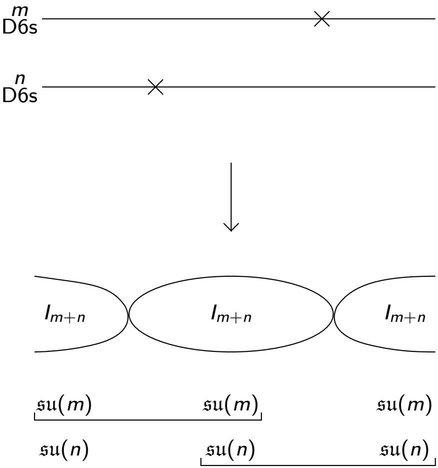

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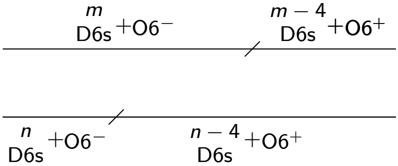

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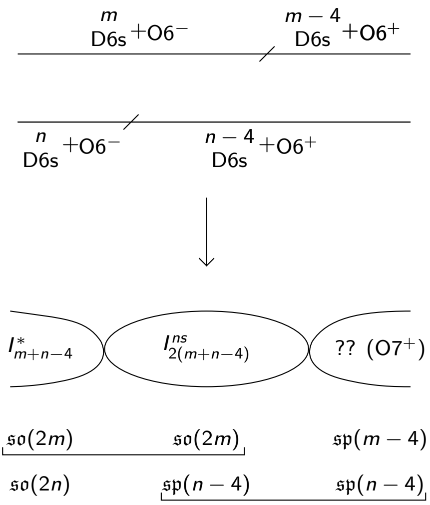

More about O7$^+$

The RR-charge of O7$^+$ is equal to that of O7$^- + 8$ D7 branes. Also, both of them induce the same $\mathbb{Z}_2$ orbifolding action on the geometry. These two facts imply that the Weierstrass equation governing both O7$^+$ and O7$^- + 8$ D7 branes is the same, and corresponds to I_4^* Kodaira singularity.

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Witten further showed that the full F-theory geometry corresponding to $O7^+$ is such that the singularity is not resolved. Hence, we have two physical situations with exactly the same F-theory geometry. It is still unknown which physical quantity distinguishes them in F-theory.

Frozen singularities

One difference between the two situations is that the I_4^* singularity corresponding to $O7^+$ cannot be resolved because there are no D7 branes in the Type IIB description that can be moved around. In other words, I_4^* corresponding to $O7^+$ is **frozen**.

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We can stack more D7 branes to find that each $I*_{n\geq 4}$ singularity in F-theory has two physical descriptions: the traditional one being $O7^- + (n+4)$ D7 branes and the partially frozen one being $O7^+ + (n-4)$ D7 branes. In what follows, we will denote a (partially) frozen I_n^* singularity as $\hat{I}_n^*$.

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In a recent work, Tachikawa argued that these are the only examples of partially frozen singularities in F-theory.

Anomaly Cancellation

Now we describe anomaly cancellation conditions for most general F-theory compactifications involving shared gauge algebras and frozen singularities. We will modify the original argument of Sadov to achieve this.

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Consider an F-theory compactification with irreducible components of discriminant locus being D_a. Each D_a carries an 8d gauge algebra $\mathfrak{g}_a$. Call the simple factors in the 6d gauge algebra as $\mathfrak{h}_i$. Each $\mathfrak{h}_i$ is shared between some D_a. Define $n_{i,a} = 1$ if $\mathfrak{h}_i$ is shared with D_a and define $n_{i,a} = 0$ otherwise.

Gravitational Coupling without $O7^+$

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When there are no $O7^+$, the stack of 7-branes on D_a has a ten dimensional coupling to gravity given by

$$\int B^{(4)} \left(-\frac{N_a}{24}\, \mathrm{tr} R^2 \right) \delta^{(2)}(D_a)$$

where $B^{(4)}$ is the chiral 4-form of type IIB, N_a is the order of vanishing of discriminant Δ on D_a, and $\delta^{(2)}(D_a)$ is the delta-function supported on D_a.

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In particular, a D7 brane contributes $-1/24$ and an $O7^-$ contributes $-2/24$.

Gravitational Coupling with $O7^+$

It is known that the contribution of $O7^+$ to this gravitational coupling is of opposite sign to that of $O7^-$, that is $2/24$. Since an $\hat{I}_k^*$ singularity corresponds to $O7^+ + (k-4)D7$ branes, it contributes $\frac{2}{24} - \frac{(k-4)}{24} = -\frac{k-6}{24}$ but $-\frac{N_a}{24} = -\frac{k+6}{24}$.

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Hence, in the presence of O7$^+$, the above ten dimensional coupling can be written as

$$\int B^{(4)} \left(-\frac{N_a - 12 s_a}{24} \, \mathrm{tr} R^2 \right) \delta^{(2)}(D_a)$$

where $s_a = 1$ when the curve D_a carries an O7$^+$ and $s_a = 0$ when it does not.

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In our case, there are some holonomies on D_a breaking $\mathfrak{g}_a$ to $\oplus_i n_{i,a}\mathfrak{h}_i$. Thus the gauge coupling becomes

$$\int B^{(4)} \left(2 \sum_i n_{i,a} o_{i,a}\,\mathrm{tr} F_{i,a}^2\right) \delta^{(2)}(D_a)$$

where $F_{i,a}$ is the field strength valued in $\mathfrak{h}_i$ and $o_{i,a}$ describes how the Casimir of $\mathfrak{g}_a$ is decomposed into Casimirs of $\mathfrak{h}_i$.

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$$\int B^{(4)} \sum_i \left(2\,\mathrm{tr} F_i^2\right) \delta^{(2)}(D_i)$$

where $D_i = \sum_a n_{i,a} o_{i,a} D_a$ is the **gauge divisor** corresponding to $\mathfrak{h}_i$.

6d Counterterm

The full six dimensional coupling relevant for Green-Schwarz mechanism is then

$$\int_B B^{(4)} \left(\sum_a \frac{1}{2}(c_1(B) + s_a \delta^{(2)}(D_a))\mathrm{tr}R^2 + \sum_i 2\delta^{(2)}(D_i)\mathrm{tr}F_i^2 \right)$$

where the integral is performed only over the base B, $B^{(4)}$ has two legs on the base B and we have used the Calabi-Yau condition $c_1(B) = -\frac{1}{12}N_a\delta^{(2)}(D_a)$.

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The contribution to anomaly polynomial is then a square of the coefficient of $B^{(4)}$:

$$I_{8,\mathrm{GS}} = -\frac{1}{2} \int_B \left(\frac{1}{2}(c_1(B) + \sum_a s_a \delta^{(2)}(D_a))\mathrm{tr}R^2 + 2 \sum_i \delta^{(2)}(D_i)\mathrm{tr}F_i^2 \right)^2$$

Anomaly Cancellation Conditions

Expanding the square, we find the anomaly cancellation conditions:

$$\mathrm{Tr}_{adj} F_i^2 - \mathrm{Tr}_{\rho_i} F_i^2 = 6[(K + F) \cdot D_i] \ \mathrm{tr}\, F_i^2 \tag{4}$$

$$\mathrm{Tr}_{adj} F_i^4 - \mathrm{Tr}_{\rho_i} F_i^4 = -3(D_i \cdot D_i) \ (\mathrm{tr}\, F_i^2)^2 \tag{5}$$

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Notice that there are two differences from the traditional case:

1. On the right hand side, the gauge divisor $D_i = \sum_a n_{i,a} o_{i,a} D_a$ appears instead of a single curve D_a.
2. The canonical divisor K is replaced by $K + F$ where $F = \sum_a s_a D_a$ is the divisor corresponding to the location of frozen singularities.

Potential SCFT 1

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Consider a 6d $\mathcal{N} = (1, 0)$ gauge theory having gauge group $\mathsf{SU}(n)\times\mathsf{SO}(n+8)$ with a hypermultiplet charged in the bifundamental and $n-8$ hypermultiplets charged in the fundamental of $SU(n)$.

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The gauge anomaly can be cancelled by the Green-Schwarz mechanism and hence this theory can appear as a low-energy effective theory on the tensor branch of a 6d $\mathcal{N} = (1, 0)$ SCFT.

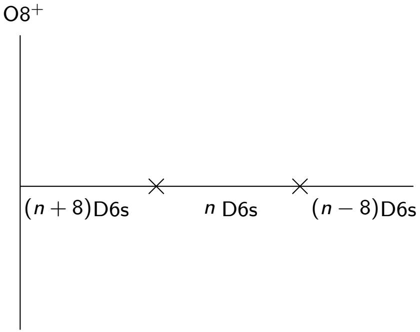
SCFT is Missing

If this SCFT were to have a traditional F-theory realization without $O7^+$, the anomaly cancellation conditions will tell us that $D_{SU} \cdot D_{SO} = 2$.

However, it is impossible for two components of discriminant locus to intersect twice in an F-theory configuration manufacturing a 6d SCFT.

Thus this SCFT does not appear in the classification of Heckman, Morrison, Rudelius and Vafa based on traditional F-theory compactifications.

SCFT is Found Using O7$^+$



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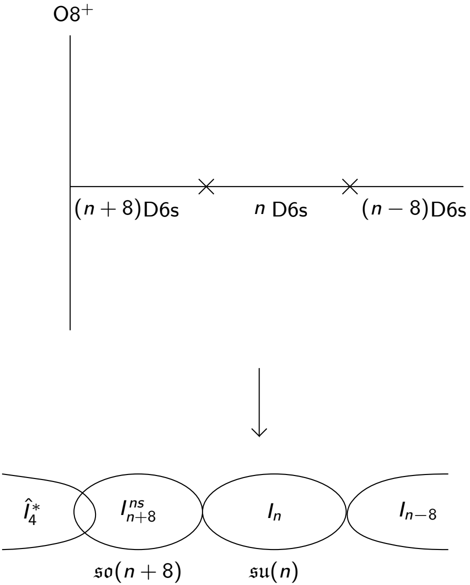

SCFT Satisfies Anomaly Cancellation

Label the curves carrying $\hat{I}_4^*$, I_{n+8}^{ns}, I_n and I_{n-8} as D_1, D_2, D_3 and D_4 respectively.

The frozen locus is $F = D_1$. The gauge divisors are $D_{SU} = D_3$ and $D_{SO} = 2D_2$. The factor of 2 comes from the fact that $\operatorname{tr} F^2_{\mathfrak{su}(k)} = 2 \operatorname{tr} F^2_{\mathfrak{so}(k)}$.

Using these divisors, it can be checked that the anomaly cancellation presented in this talk is satisfied. In particular, $D_{SU} \cdot D_{SO} = 2$ as expected from gauge theory.

A More Non-Trivial Example which is Compact

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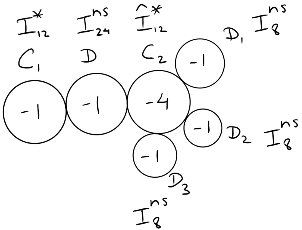
Consider an F-theory model with base $\mathbb{P}^1 \times \mathbb{P}^1$ with coordinates z, w. Say we have an I_{12}^* singularity at $z = 0$ and an $\hat{I}_{12}^*$ singularity at $w = 0$. This model has a perturbative Type IIB dual in terms of O7 planes on T^4. The perturbative spectrum is:

- Gauge algebra $\mathfrak{su}(8) \oplus \mathfrak{usp}(8)_1 \oplus \mathfrak{usp}(8)_2$.
- A hypermultiplet in the bifundamental of each pair of gauge algebras.
- Two hypermultiplets in 2-index antisymmetric of $\mathfrak{su}(8)$.
- 1+4 tensor multiplets.

F-Theory Construction

Since we have 4 tensor multiplets, we have 4 exceptional divisors.

The F-theory geometry is as follows:



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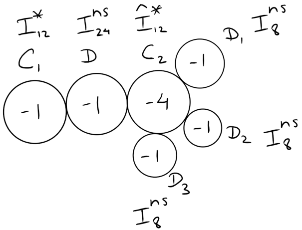

The gauge divisors are:

- $2(C_1 + D)$ for $\mathfrak{su}(8)$.
- $D + \frac{1}{2}C_2 + D_1$ for $\mathfrak{usp}(8)_1$.
- $D_2 + \frac{1}{2}C_2 + D_3$ for $\mathfrak{usp}(8)_2$.

Thank you for your attention.