ON SINGULARITIES of Super Conformal Field Theories

Autumn Symposium KIAS 2017/09/13

KOREA INSTITUTE FOR ADVANCED STUDY

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INTRO

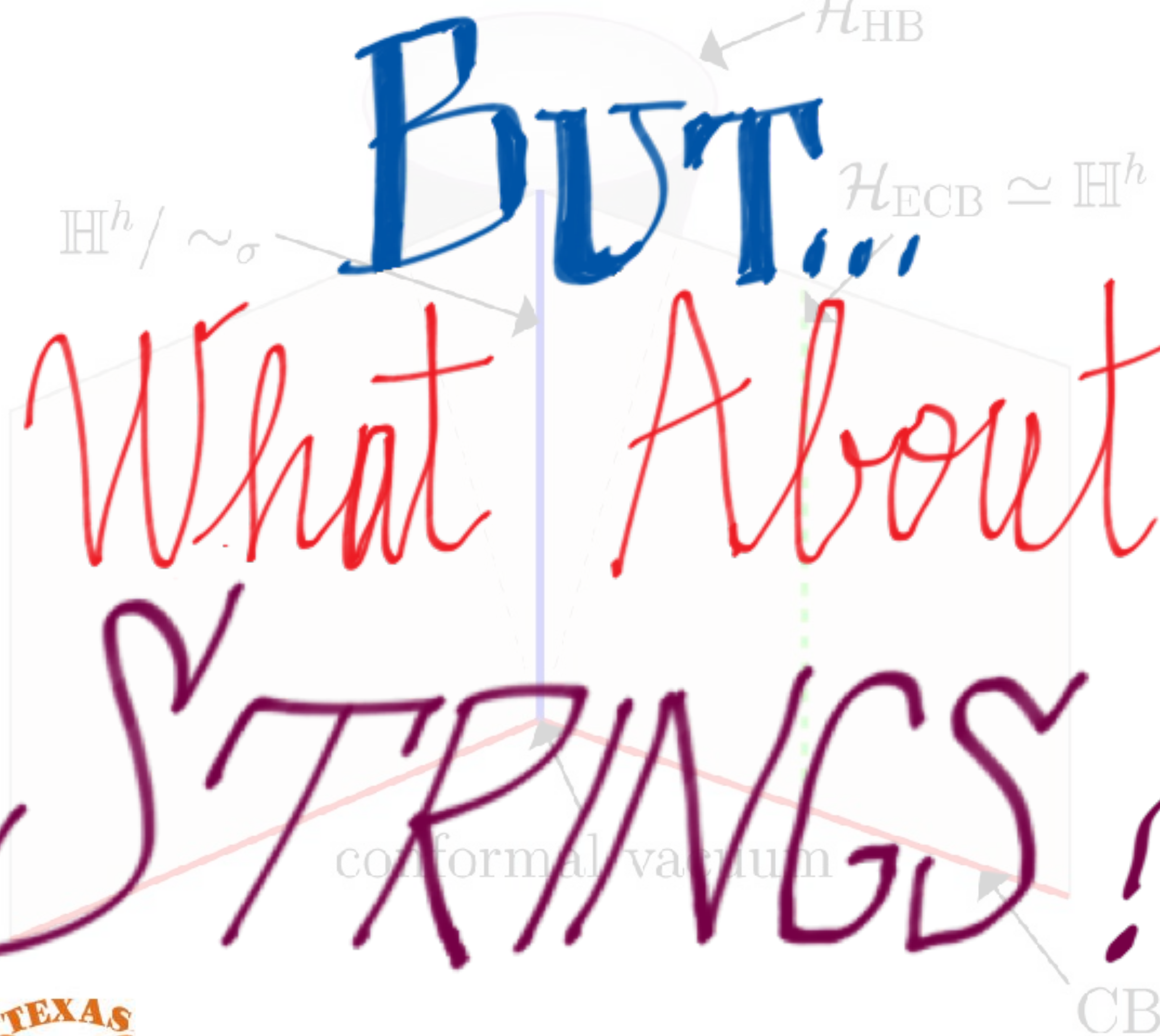
"General Considerations"

Goal of the Talk

UNDERSTAND the SINGULARITY structure of the CB/MODULI SPACE of 4d SCFTs with rank HIGHER than 1 & 8 or MORE supercharges

But...

What About STRINGS?



the study of SINGULARITIES can tell us a lot about the SPACE of EXISTING 4d SCFTs setting a TARGET for STRING constructions

PART I

"Setting Things Up"

VECTOR MULTIPLET

$$V^{\mathcal{N}=2}: \qquad \overbrace{\lambda_\alpha \longleftrightarrow A_\mu}^{\mathcal{N}=1 \text{ Vector Multiplet}}$$

$$\underbrace{\Phi \longleftrightarrow \tilde{\lambda}_\alpha}_{\mathcal{N}=1 \text{ Chiral Multiplet}}$$

Φ is a scalar which can acquire a VEV:

$$\boxed{\Phi = \begin{pmatrix} a_1 & & & \\ & a_2 & & \\ & & \ddots & \\ & & & a_N \end{pmatrix}, \quad \sum_i^N a_i = 0}$$

$\Phi \in \mathbb{C}^{N-1}$ & on the CB $SU(N) \to U(1)^{N-1}$

In general $(a_1, \ldots, a_N)$ are NOT "gauge" invariant & we need to impose further identifications (e.g. Weyl Groups)

- $\dim_{\mathbb{C}} \mathcal{CB} :=$ Rank of the Theory
- $(u_1, \ldots, u_r) :=$ Coulomb Branch coordinates.
- The geometry is Singular @ $\vec{u}$'s where massless states appear.

$\mathbb{C}^*$-Action

i] Φ has non-zero $U(1)_R$ charge.

ii] In the scale-invariant case $\exists\ \mathbb{R}^+$ Action.

i & ii Combine into a $\mathbb{C}^*$-Action

Classification Steps

1. Topology
2. Geometry
3. Deformation

TOPOLOGY

The SINGULAR Locus $\mathcal{V}$ HAS to be A Complex CO-DIMENSION 1 $\mathbb{C}^*$ INVARIANT Set.

GEOMETRY

- Define the Special Coordinates:

$$\left(a^i(\vec{u}), a^D_j(\vec{u})\right) \Big| \frac{\partial a^D_j}{\partial a_i} = \tau_{ij}$$

- There is a Metric on the CB:

$$ds^2 = \mathrm{Im}\left(da^D_i\, d\bar{a}^i\right)$$

- Imposing Special Khäler conditions: $\boxed{\mathrm{Im}\,\tau_{ij} > 0,\ \tau_{ij} = \tau_{ji}}$

- Looping around the singularity:

$$\begin{pmatrix} \vec{a} \\ \vec{a}^D \end{pmatrix} \xrightarrow[\gamma]{} M \begin{pmatrix} \vec{a} \\ \vec{a}^D \end{pmatrix},\quad M \in Sp(2r, \mathbb{Z})$$

Deformation

Hardest of All.

Listing the set of Physically Allowed Mass Deformations

Part II

"Recap On Rank-1"

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[hep-th/1609.04404] [hep-th/1611.08602] [hep-th/1704.05110]

Rank-1 Recap

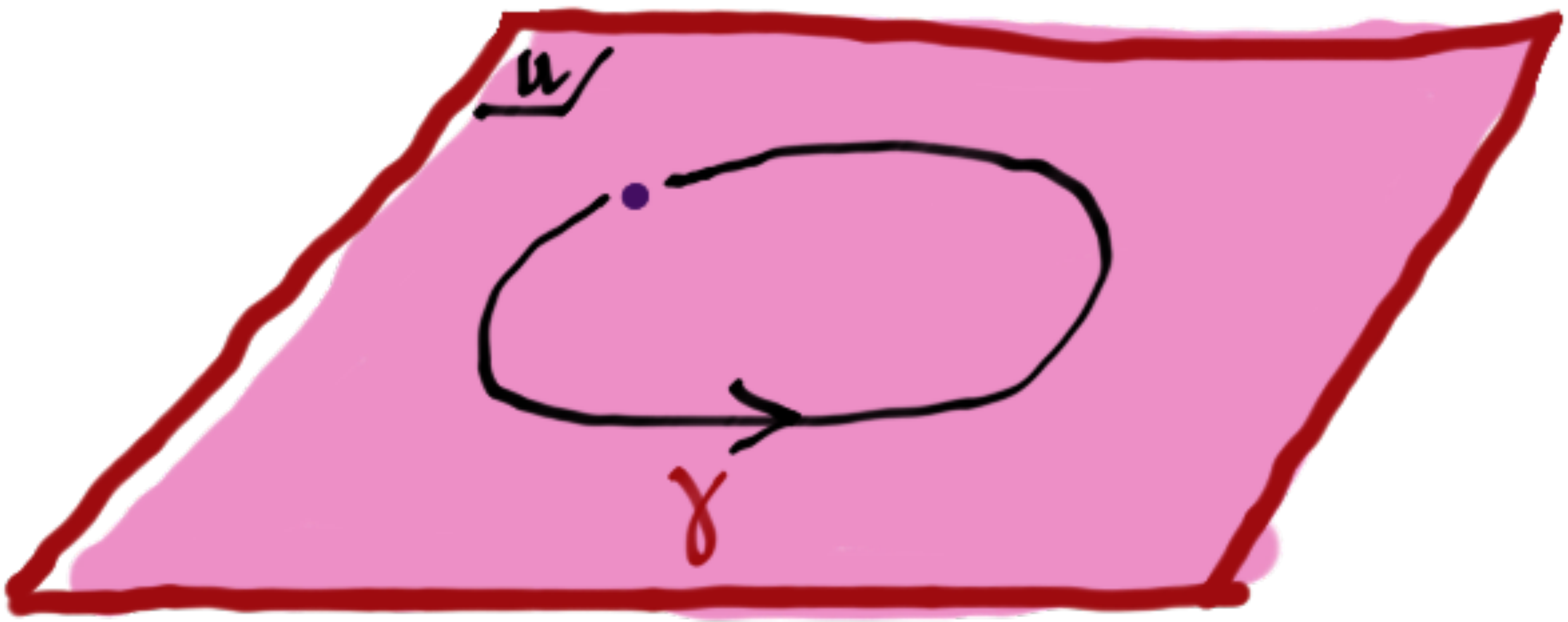
for SU(2) $\Phi = \begin{pmatrix} a & \\ & -a \end{pmatrix}$

- a is not gauge invariant ✗
- $u = \frac{1}{2}\langle \mathrm{tr}\,\Phi^2 \rangle = a^2$ is ✓

The CB is 1 Complex Dimensional

Rank-1 Recap

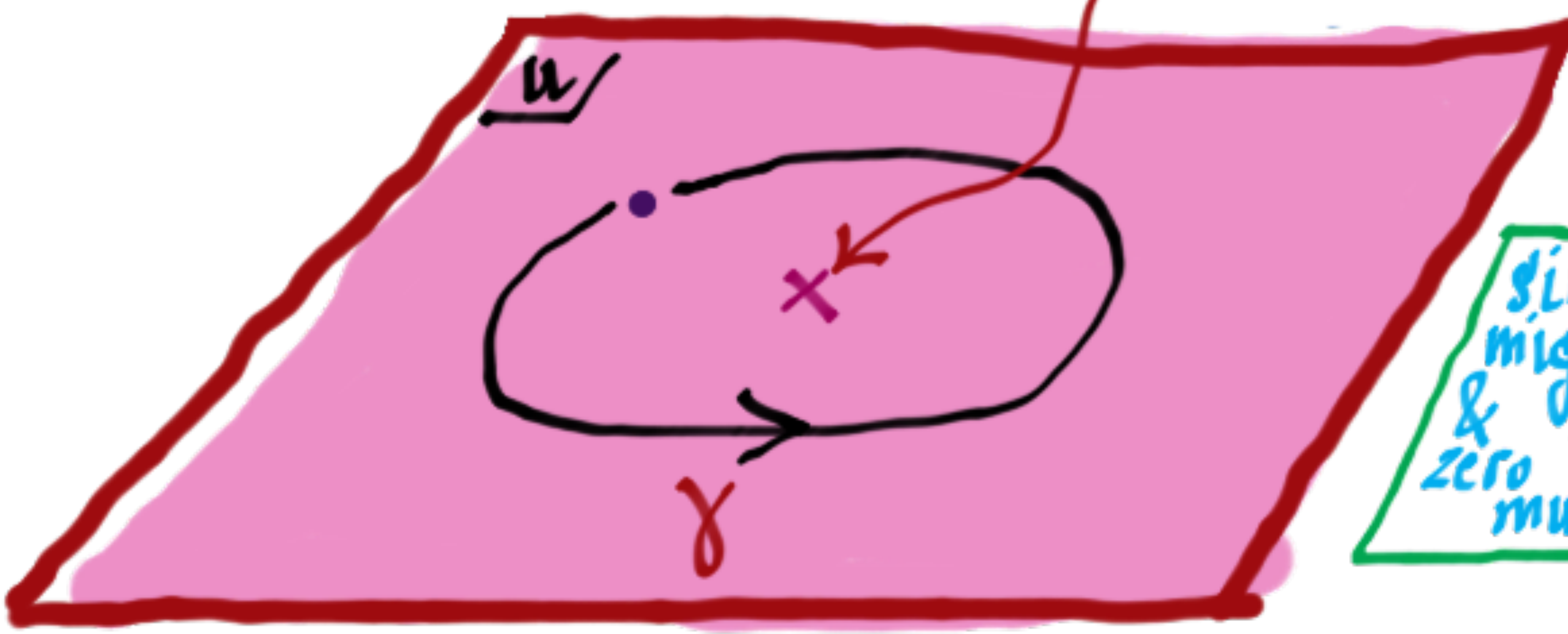
$$\begin{pmatrix} a \\ a_D \end{pmatrix} \xrightarrow[\gamma]{SL(2,\mathbb{Z})} M \begin{pmatrix} a \\ a_D \end{pmatrix} \qquad \Bigg| \qquad M = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{Z})$$



Rank-1 Recap

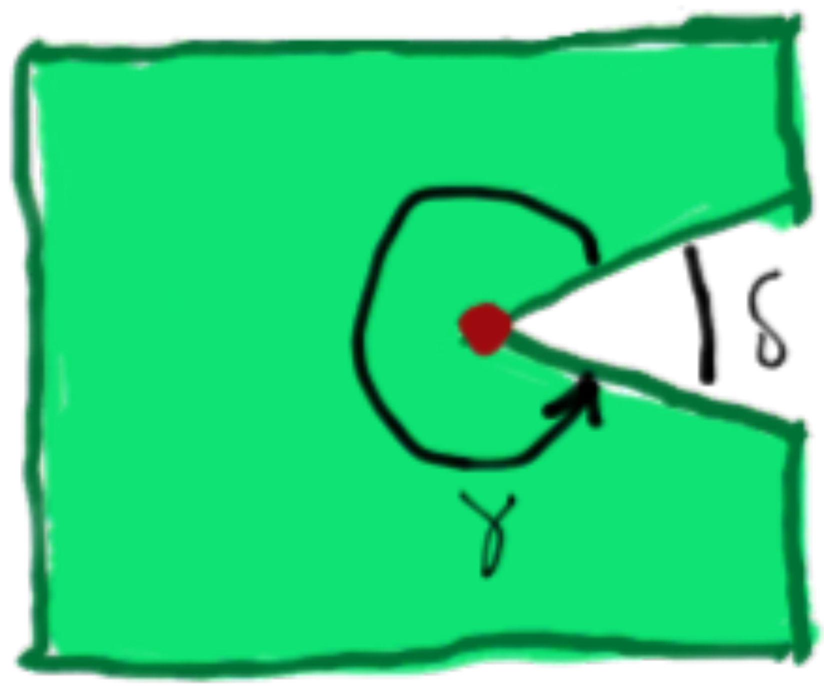
singularities appear for values of the CB parameter which allow extra massless states in the theory.

there needs to be a singularity within the curve



singularities might overlap & have non-zero multiplicity

Requiring $\mathbb{C}^*$-Invariance: $\boxed{\mathcal{V} \equiv \{0\}}$



$\mathbb{C}$ w/ deficit angle δ

- Special Kähler conditions are trivially satisfied.
- E-M Duality only Allows 7 Values for the opening angle:

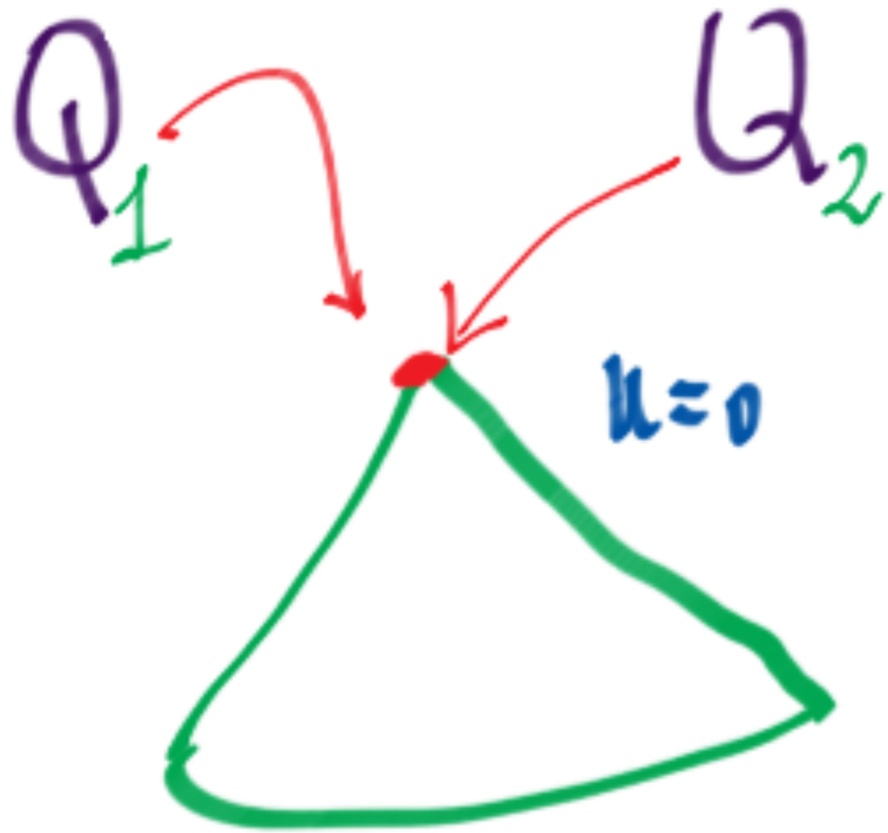
δ	$\pi/3$	$\pi/2$	$2\pi/3$	π	$4\pi/3$	$3\pi/2$	$5\pi/3$	2π (cusp)
$\Delta(a)$	6	4	3	2	$3/2$	$4/3$	$6/5$	$\mathbb{N}$

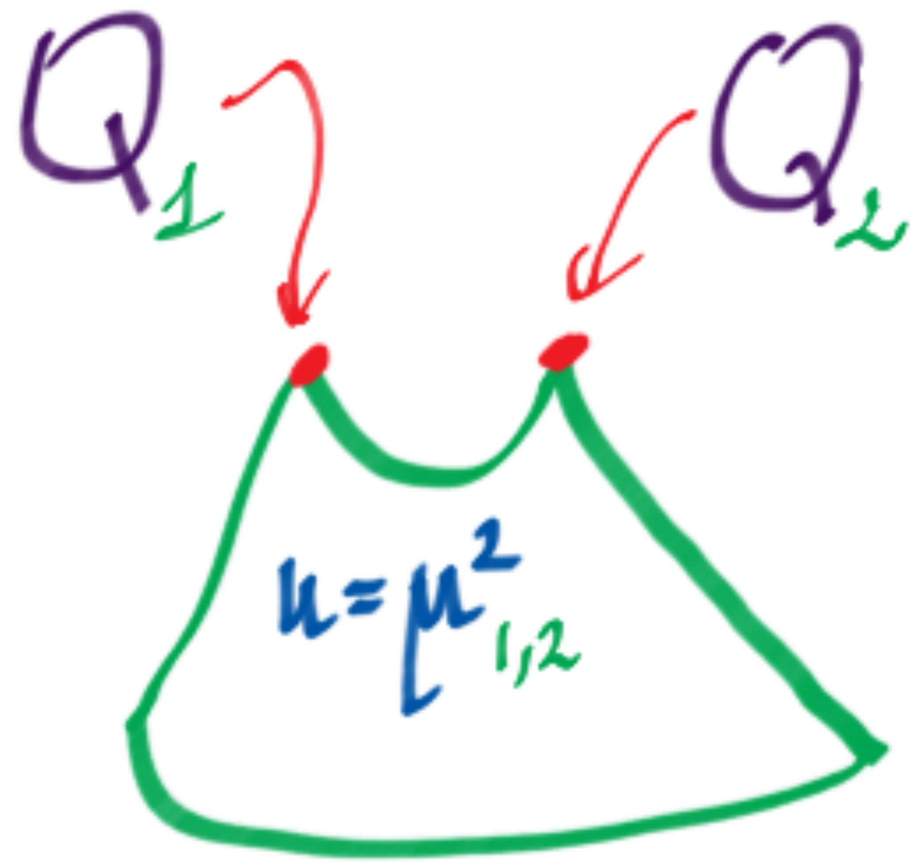
They correspond to the KODAIRA class.

Name	curve ($y^2=\dots$)	$\Delta(u)$	M_0	#
II^*	x^3+u^5	6	ST	10
III^*	x^3+u^3x	4	S	9
IV^*	x^3+u^4	3	$-(ST)^{-1}$	8
I_0^*	$x^3+u^2x+gu^2$	2	$-\mathbb{1}$	6
IV	x^3+u^2	$3/2$	$-ST$	4
III	x^3+ux	$4/3$	S^{-1}	3
II	x^3+u	$6/5$	$(ST)^{-1}$	2
$I^*_{n>0}$	$x^3+ux^2+\Lambda^{-2n}u^{n+3}$	2	$-T^{-n}$	$n+6$
$I_{n>0}$	$(x-1)(x^2+\Lambda^{-n}u^n)$	1	T^{-n}	n

MASS DEFORMATIONS

$\mu = 0$ | $\mu \neq 0$





Classification of Rank-1 $\mathcal{N}$=2 SCFTs I

– Through a systematic analysis of mass def^ns of scale inv. 1 dim. special Kähler geometries: –

We discover over 10 new $\mathcal{N}$=2 SCFTs

IMPORTANT

- Most theories are Non-Lagrangian
- Many don't have an S-class description!

Part III

"Exploring Rank-2"

Rank 2: $\dim \mathcal{CB} = 2$, $\dim \mathcal{V} = 1$

$$(u,v) \to (\lambda^p u, \lambda^q v)$$

Thus $\mathcal{V}$ can be:

- $u = 0$
- $v = 0$
- $u^p / v^q = \omega = \omega_0$

To help visualizing:

$$X_\rho := V \cap S_\rho \,, \quad S_\rho := \{ |u|^\rho + |v|^\rho = 2\varepsilon^\rho \}$$

e.g.: • $\{u = 0\} \cap S_0 := K_0(u,v)$

$$K_0(u,v) := \{ (u,v) \in \mathbb{C}^2 \,|\, u = 0, v = e^{i\phi} \}$$

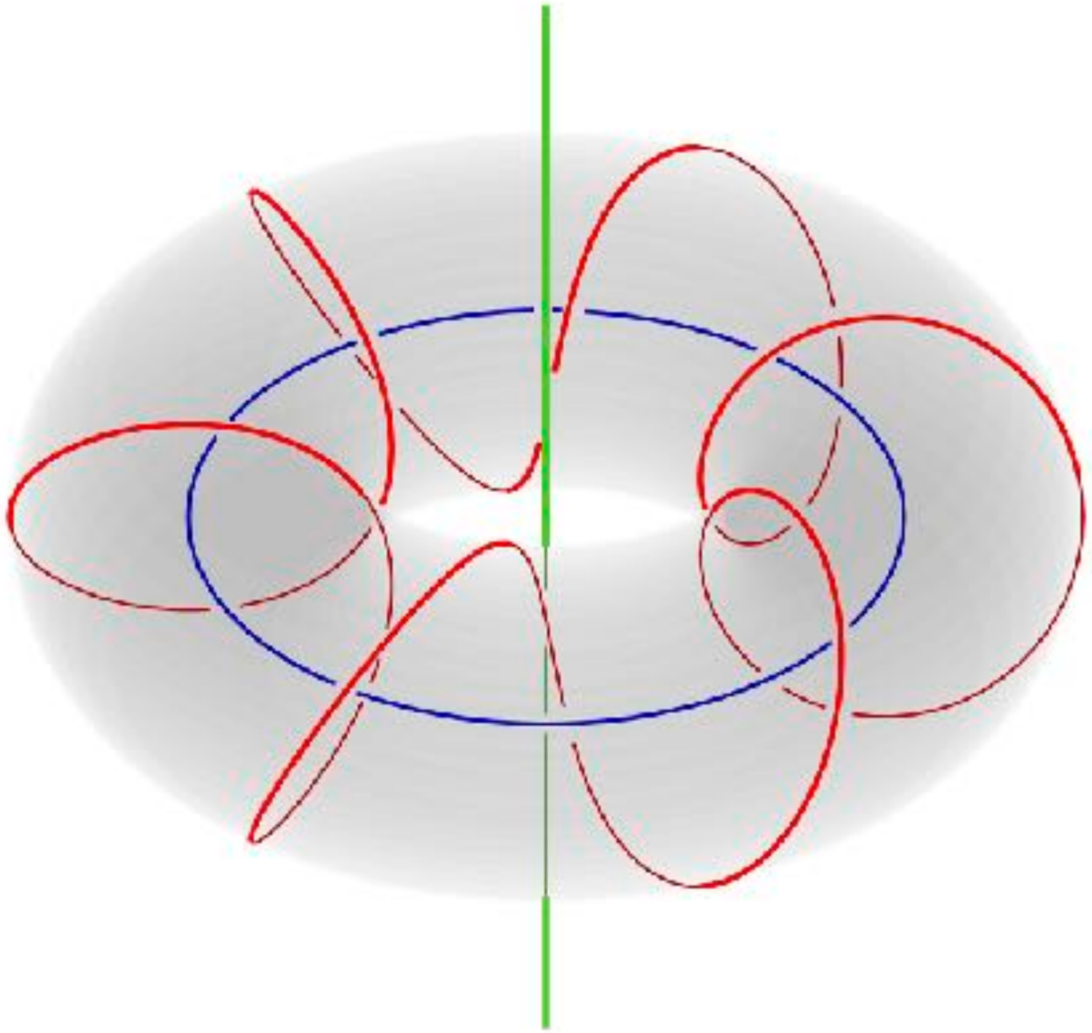
If $\gcd(p,q)=1$:

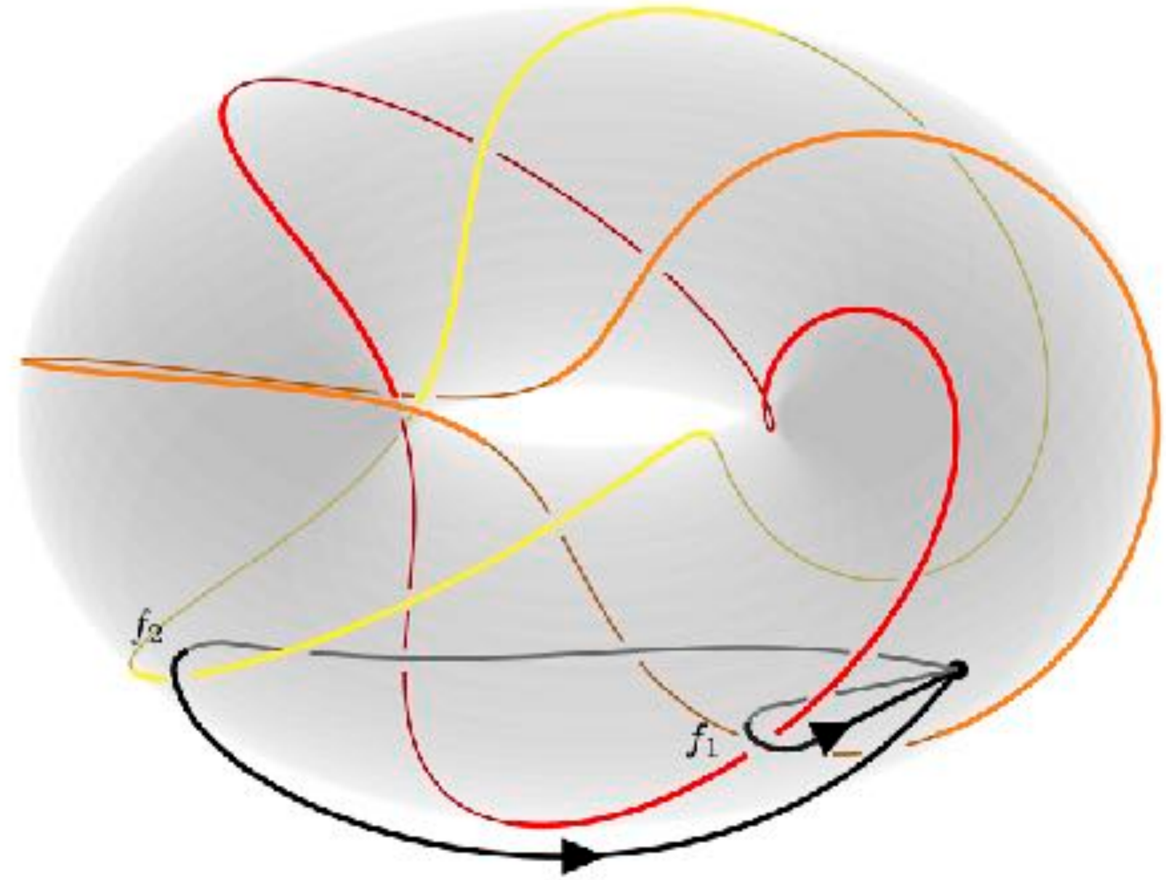
- $\{u^p/v^q=1\}\cap S_0 := K_{(p,q)}(u,v)$

On $K_{(p,q)}(u,v) \longrightarrow |u|=|v|=1$

Thus:

$$K_{(p,q)} := \{u=e^{i\theta}, v=e^{i\varphi} \,|\, p\theta=q\varphi\}$$





X_p is a Torus (p,q) link.
with Unknots.

Special Kähler conditions strongly constrain the possible $Sp(4,\mathbb{Z})$ Monodromies!

representative	eigenvalues	dim(eigenspaces)
$E_\theta \odot E_\theta$	$\{e^{i\theta}, e^{i\theta}, e^{-i\theta}, e^{-i\theta}\}$	$2 \oplus 2$
$\alpha I \odot E_\theta$	$\{\alpha, \alpha, e^{i\theta}, e^{-i\theta}\}$	$2 \oplus 1 \oplus 1$
$\alpha I \odot H_a$	$\{\alpha, \alpha, a, 1/a\}$	$2 \oplus 1 \oplus 1$
$\alpha I \odot -\alpha I$	$\{\alpha, \alpha, -\alpha, -\alpha\}$	$2 \oplus 2$
$\alpha I \odot \alpha I$	$\{\alpha, \alpha, \alpha, \alpha\}$	4
$\alpha I \odot -\alpha P_\gamma$	$\{\alpha, \alpha, -\alpha, -\alpha\}$	$2 \oplus 1$
$\alpha I \odot \alpha P_\gamma$	$\{\alpha, \alpha, \alpha, \alpha\}$	3
$\alpha P_\beta \odot \alpha P_\gamma$	$\{\alpha, \alpha, \alpha, \alpha\}$	2
$A_\alpha \oplus A_\alpha^{-T}$	$\{\alpha, \alpha, \alpha, \alpha\}$	2

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \odot \begin{pmatrix} A & B \\ C & D \end{pmatrix} := \begin{pmatrix} a & 0 & b & 0 \\ 0 & A & 0 & B \\ c & 0 & d & 0 \\ 0 & C & 0 & D \end{pmatrix}.$$

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \; P_\alpha = \begin{pmatrix} 1 & 0 \\ \alpha & 1 \end{pmatrix}, \; A_\alpha = \begin{pmatrix} 0 & 1 \\ -1 & 2\alpha \end{pmatrix};$$

$$E_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}, \; H_a = \begin{pmatrix} a & 0 \\ 0 & 1/a \end{pmatrix},$$

- Putting All together we have a **Full** Classification of Geometries with $\mathcal{V}$ a **Single Knot**.

The Analysis of **MASS** deformations is **VERY HARD**

PART IV

"An Example in Details"

SU(3)

$$\Phi = \begin{pmatrix} a & & \\ & b & \\ & & c \end{pmatrix}, \quad a+b+c=0$$

Invariant Coordinates

$$u = ab+bc+ac$$

$$v = abc$$

Where are the Singularities?

SU(3) w/ 1 adj. [$\mathcal{N}=4$]

Unbroken SU(2): $\Phi = \begin{pmatrix} a & & \\ & a & \\ & & -a \end{pmatrix}$

$$\boxed{u^3 = v^2}$$

In the $\mathcal{N}=4$ case the singularity is a Single (3,2) knot.

SU(3) w/ 6 Flavor.

i] Unbroken SU(2) @ $u^3 = v^2$

ii] Massless Quarks @ $v = 0$

What about Non Perturbative Effects?

i] In the Deep-IR Quarks decouple.

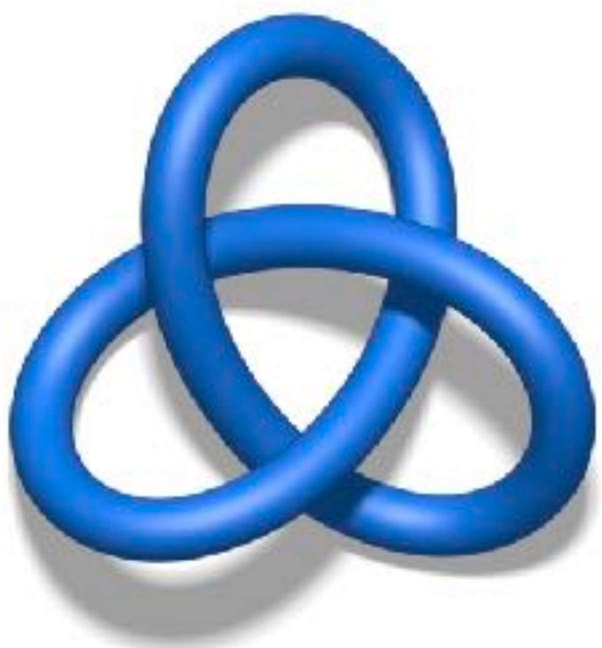
ii] The SU(2) has a Monopole-Dyon singularity!

iii] The Splitting Depends on the scale a!

$$\boxed{u^p = v^q \longrightarrow u^p = \omega_{1,2}\, v^q}$$

IMPORTANT

i] $\mathcal{N}=4$ $\mathcal{V}$ IS An Orbifold



$\simeq \mathbb{C}^2/S_3$

ii] $\mathcal{N}=2$ $\mathcal{V}$ IS NOT!

PART V

"$\mathcal{N}=3$ & $\mathcal{N}=4$ Susy"

For 9 or more supercharges the Moduli space $\mathcal{M}$ is FLAT.

Can we prove that

$$\mathcal{M} \simeq \mathbb{C}^{3r}/\Gamma$$

Let's Assume it for now...

On a $\mathcal{N}=2$ Coulomb Branch Slice

- Freely generated CB Chiral Ring:

Γ COMPLEX REFLECTION

- Flatness $\Longrightarrow \begin{pmatrix} \vec{a} \\ \vec{a}_D \end{pmatrix} \sim \begin{pmatrix} \vec{z} \\ \vec{z}' \end{pmatrix}$, $(z, z') \in \mathbb{C}^{2r}$
- Monodromies in $Sp(4, \mathbb{Z})$:

Γ CRYSTALLOGRAPIC

CRYSTALLOGRAPIC COMPLEX REFLECTION GROUPS

Have been Fully classified so we have a full list of $\mathcal{N}=3$ Geometries!

- We find many NEW Geometries already at Rank-2.

Still working out the physical properties.

STAY TUNED.

CONCLUSIONS

1. The classification of $\mathcal{N}=2$ CB seems to be extensible to Higher Ranks.
2. For $\mathcal{N}=3$ & $\mathcal{N}=4$ there seems to be a Full answer for All Ranks.
3. Getting away from the scale-invariant limit is the Hard part.

THANK YOU!