5d/6d DE instantons from trivalent gluing of web diagrams

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arXiv:1702.07263 w/ Hirotaka Hayashi

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Introduction

How to compute top. string?

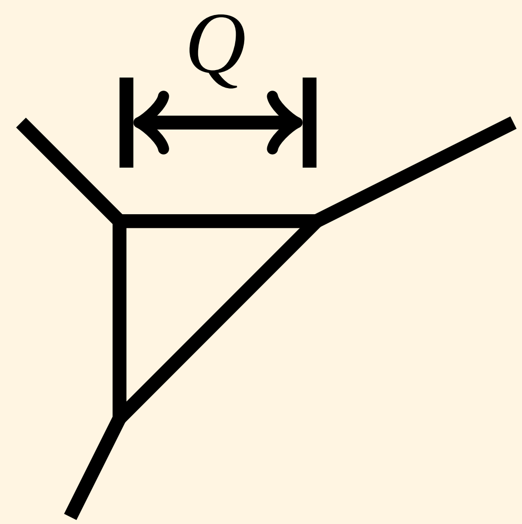
- genus 0: localization, Picard-Fuchs etc.
- For general genus, general CY3: dream
- B-model: BCOV (but holomorphic part is nontrivial)
- A-model: when CY3 is toric (hence non-compact)
  → topological vertex
- A-model on non-compact CY3 → Nekrasov Z of a 5d SQFT
- Some non-compact non-toric CY3 → Non-SU SQFT
- “Geometric-ish” computation for such CY3’s?

⇨ “trivalent gluing” of topological vertices

Toric CY and Topological Vertex

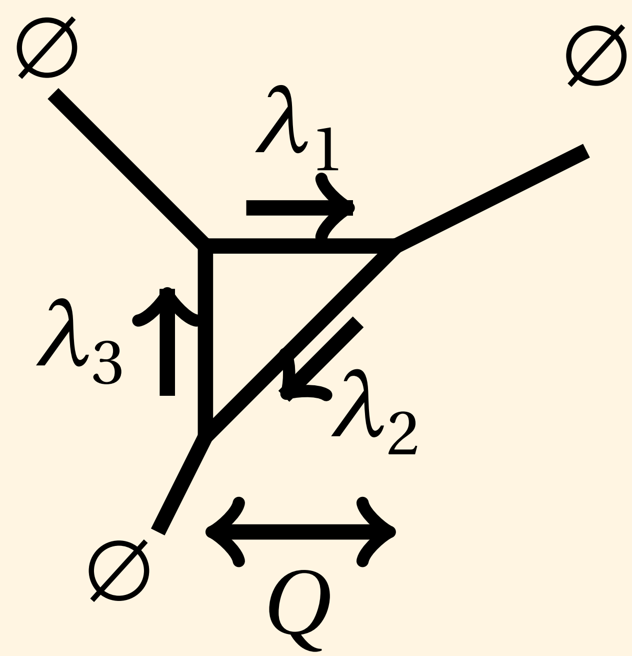
M theory on Toric CY $\rightleftarrows$ Type IIB (p,q) Web

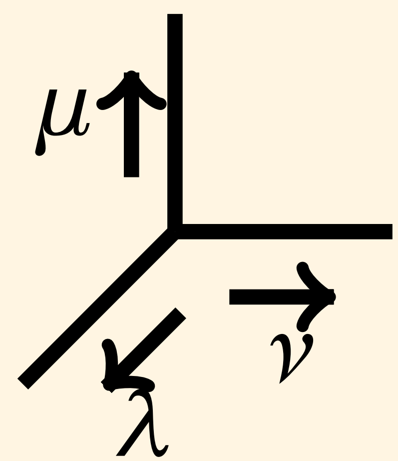
$$\widetilde{\mathbb{C}^3/\mathbb{Z}_3} \simeq \mathcal{O}_{\mathbb{P}^2}(-3)$$



$$Q = \mathrm{e}^{-t}$$

$$Z^{\mathrm{top}}_{\mathcal{O}_{\mathbb{P}^2}(-3)} = \sum_{\lambda_1,\lambda_2,\lambda_3} Q^{\sum_i |\lambda_i|}$$





$$= C_{\mu\nu\lambda}(q)$$

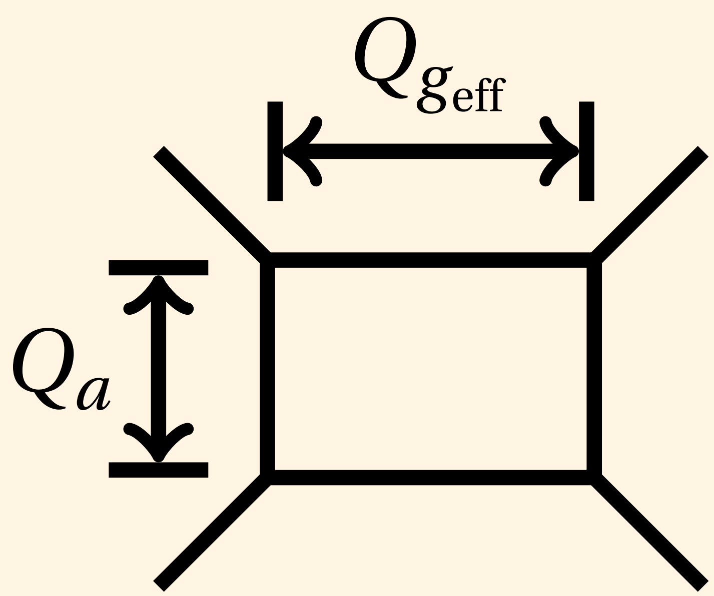
$$q = \mathrm{e}^{g_{\mathrm{st}}}$$

Gauge theory and Top. Vertex

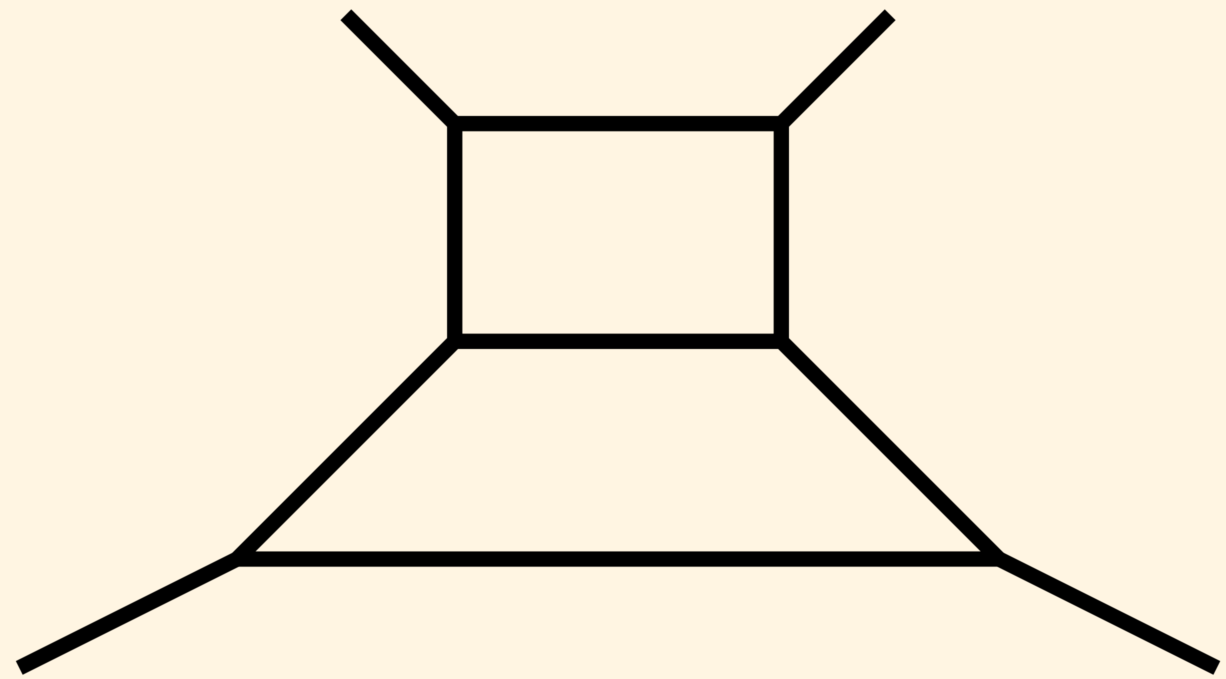
5d SU(N) quiver gauge theories w/ (bi)fundundamentals
can be engineered by toric CY3 / (p,q) web.

pure $\mathrm{SU}(2)_{\theta=0}$

local $\mathbb{P}^1 \times \mathbb{P}^1$



pure $\mathrm{SU}(3)_{k=1}$

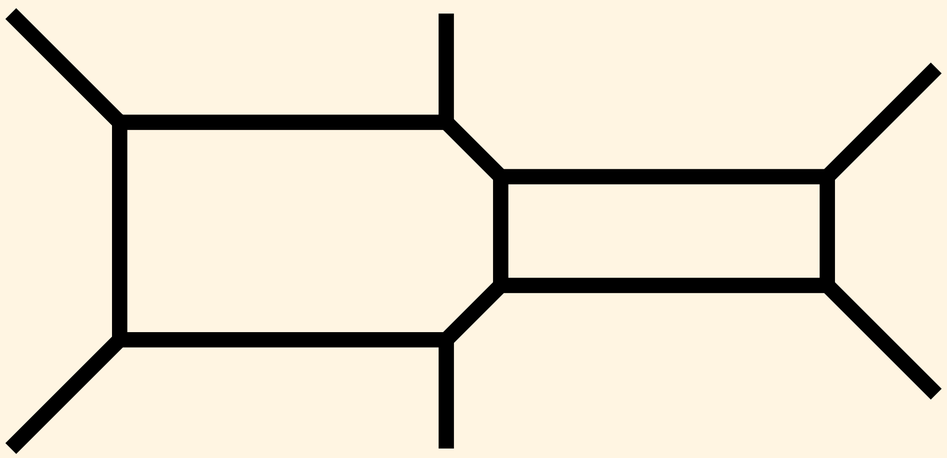


$$Z^{\mathrm{top}}(\mathrm{CY}_3) = Z^{\mathrm{Nek}}(\text{gauge theory})|_{\text{unrefined}}$$

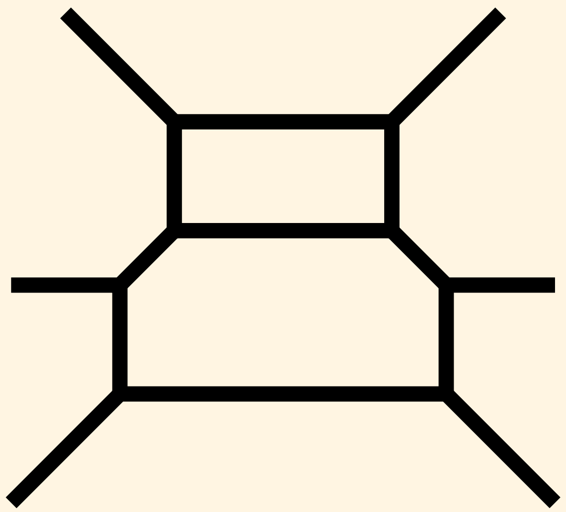
※ every theory (in my talk) has 8 supercharges

Base-Fiber duality

$\mathrm{SU}(2) - \mathrm{SU}(2)$



$\mathrm{SU}(3)_{k=0}$ with 2 flavors



These two gauge theories have common UV SCFT

Different mass def.s of the SCFT gives different IR gauge theories

Z^{Nek} of the two coincides in 5d

Non-SU gauge theory?

Can a 5d non-SU gauge theory be engineered?

SO/Sp: (p,q) web with O-planes [Zafrir '15,Hayashi-Kim-Lee-Taki-Yagi '15]

SO/E: M-theory on a D,E-orbifolded of the resolved conifold

$\mathbb{C}^2/\Gamma_{D,E}$ singular locus wrapping a sphere

non-toric

Non-Lagrangian base-fiber dual:

pure D,E gauge theory $\rightleftarrows$ SU(2) w/ non-Lagrangian matters

some 6d SCFT on circle $\rightleftarrows$ SU(2) w/ non-Lagrangian matters

[DelZotto-Vafa-Xie 15', DelZotto-Heckman-Morrison 17']

"trivalent gluing" enables us to compute RHS

For some of 6d theories,
this is the first systematic way to compute high string number part.

Outline

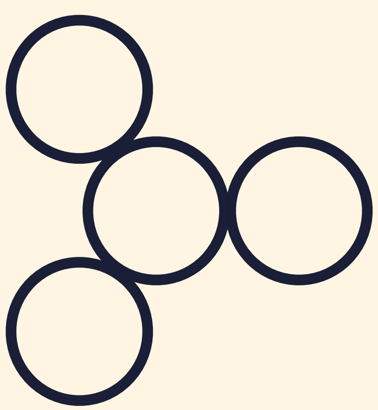
- Base-fiber duality for pure DE gauge theories
- Trivalent gluing
- Generalization to 6d SCFTs
- other generalizations

Base-fiber duality
for DE gauge theories

The orbifolded resolved conifold

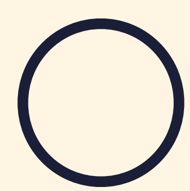
$$\begin{array}{ccc} \widetilde{\mathbb{C}^2/\Gamma_G} & \longrightarrow & (\mathscr{O}(-1)\widetilde{\oplus\mathscr{O}(-1)})/\Gamma_G \\ & & \big\downarrow \\ & & \mathbb{P}^1 \end{array}$$

G=SO(8)



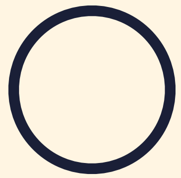
$$\longrightarrow (\mathscr{O}(-1)\widetilde{\oplus\mathscr{O}(-1)})/\Gamma_{\mathrm{SO}(8)}$$

$$\big\downarrow$$

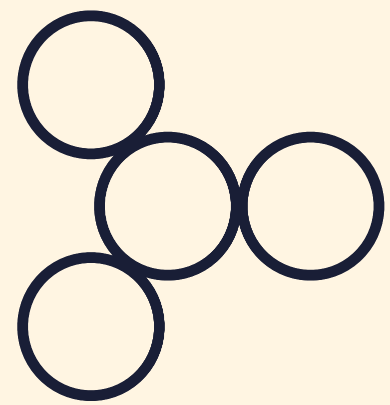


D4 singularity wrapping sphere → pure SO(8)

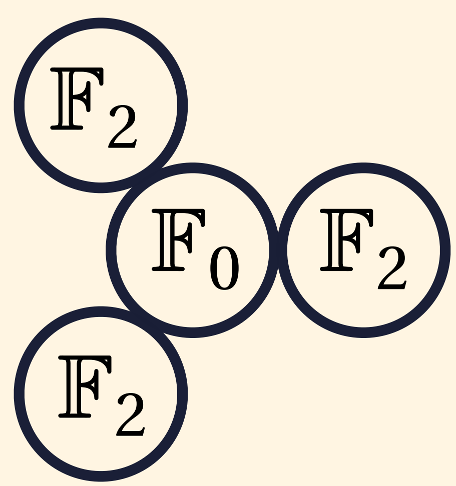
Swapping base and fiber

“  $\longrightarrow (\mathscr{O}(-1)\widetilde{\oplus \mathscr{O}(-1)})/\Gamma_{SO(8)}$ ”

$$\downarrow$$

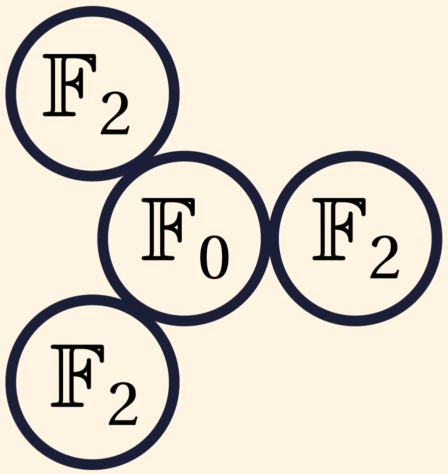


Near each sphere in the “new base”,
the geometry contains a sphere bundle over the sphere.



[DelZotto-Heckman-Morrison 17’]
[Hee-Cheol’s talk]

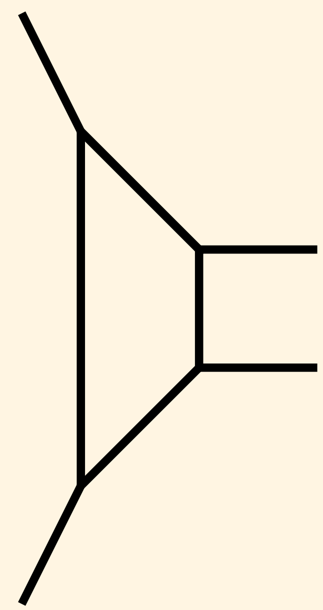
Dual description



$$\text{local } \mathbb{F}_0 = \text{local } \mathbb{P}^1 \times \mathbb{P}^1$$

→ SU(2) vector multiplet

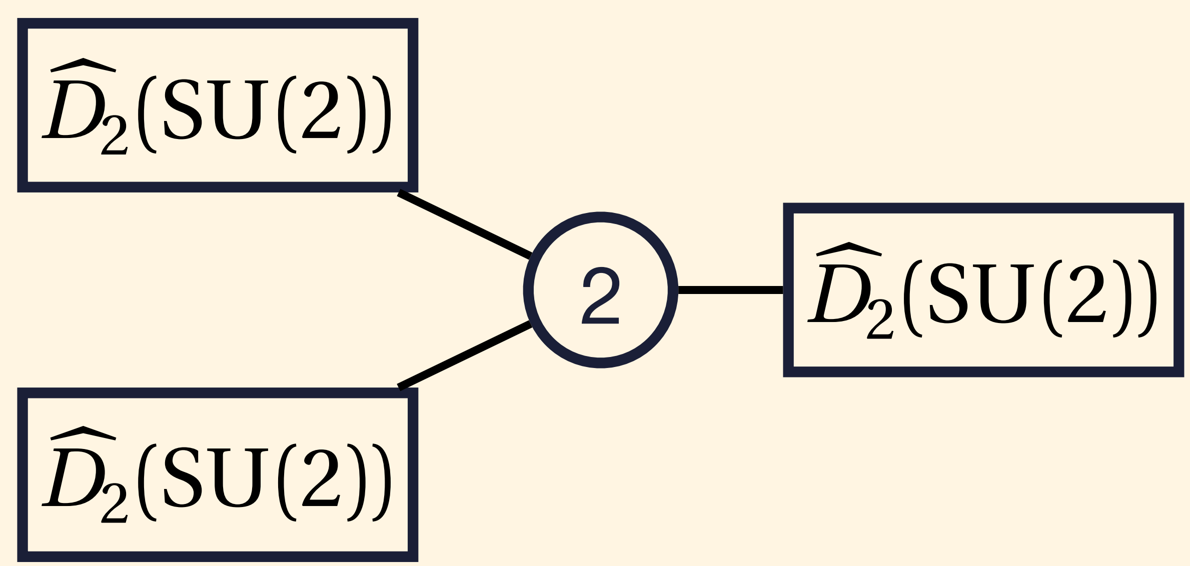
local $\mathbb{F}_2$:



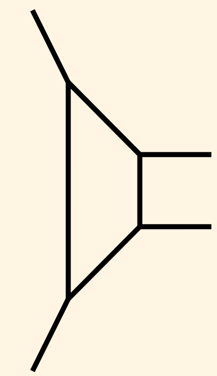
$\widehat{D}_2(\mathrm{SU}(2))$

(dual to pure SU(2))

pure SO(8) gauge theory $\rightleftarrows$

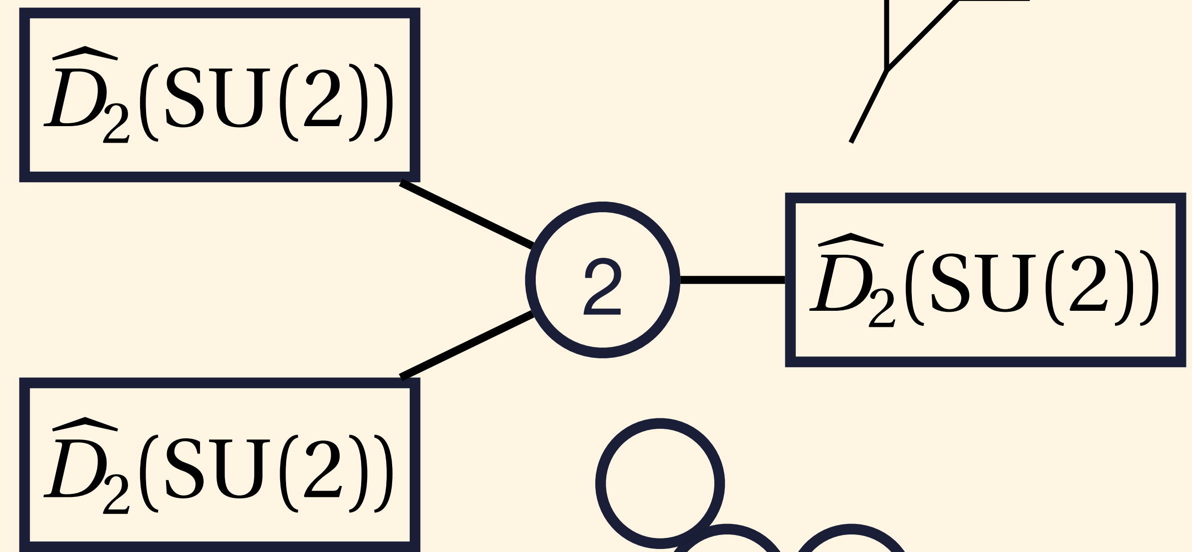


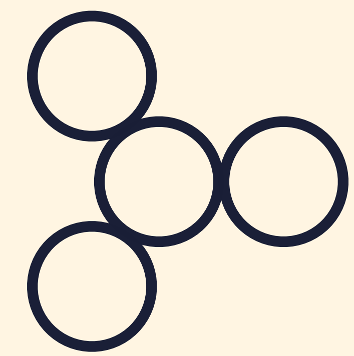
Dual description of E-type

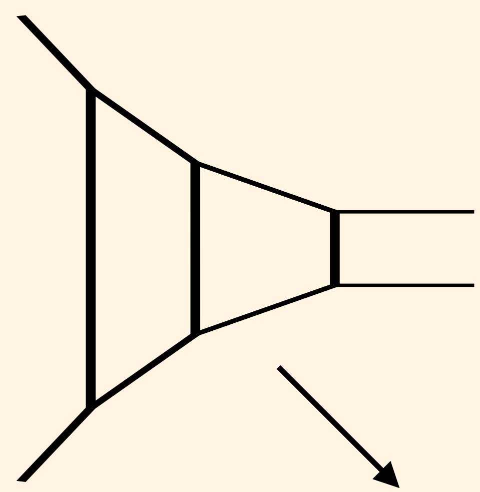


pure SO(8) gauge theory $\rightleftarrows$

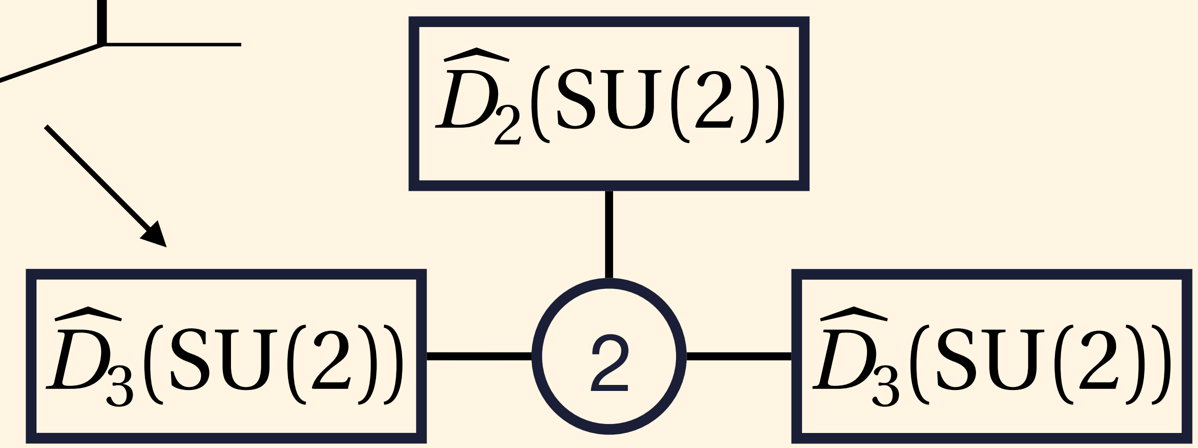
(immediate generalization to SO(N))

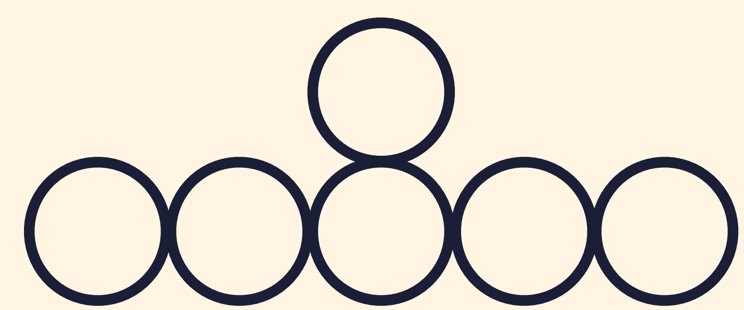






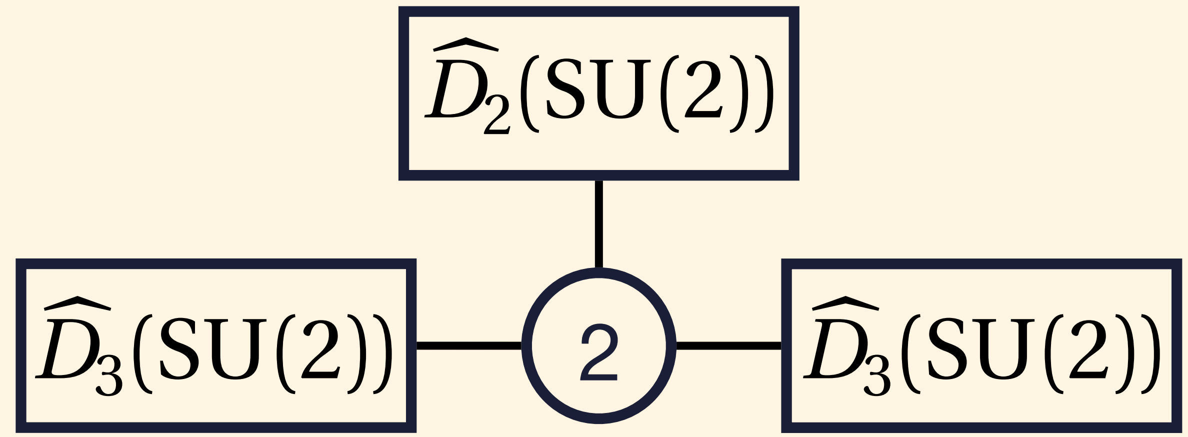
pure E_6 gauge theory $\rightleftarrows$





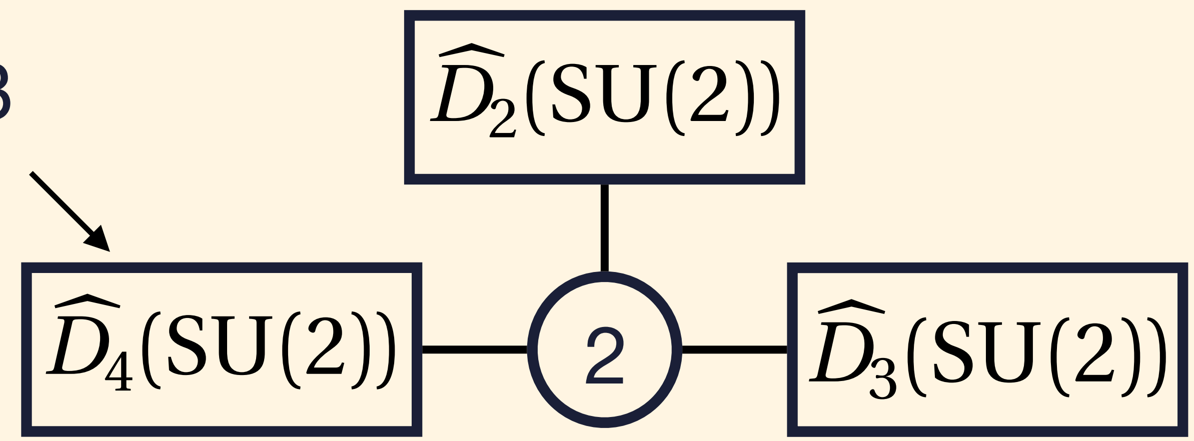
Dual description of E-type

pure E_6 gauge theory $\rightleftarrows$



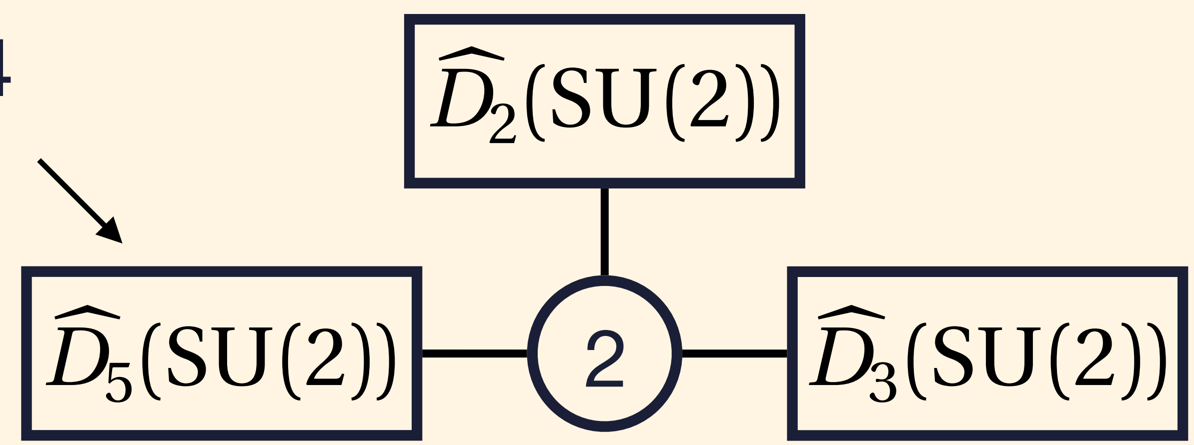
rank 3

pure E_7 gauge theory $\rightleftarrows$



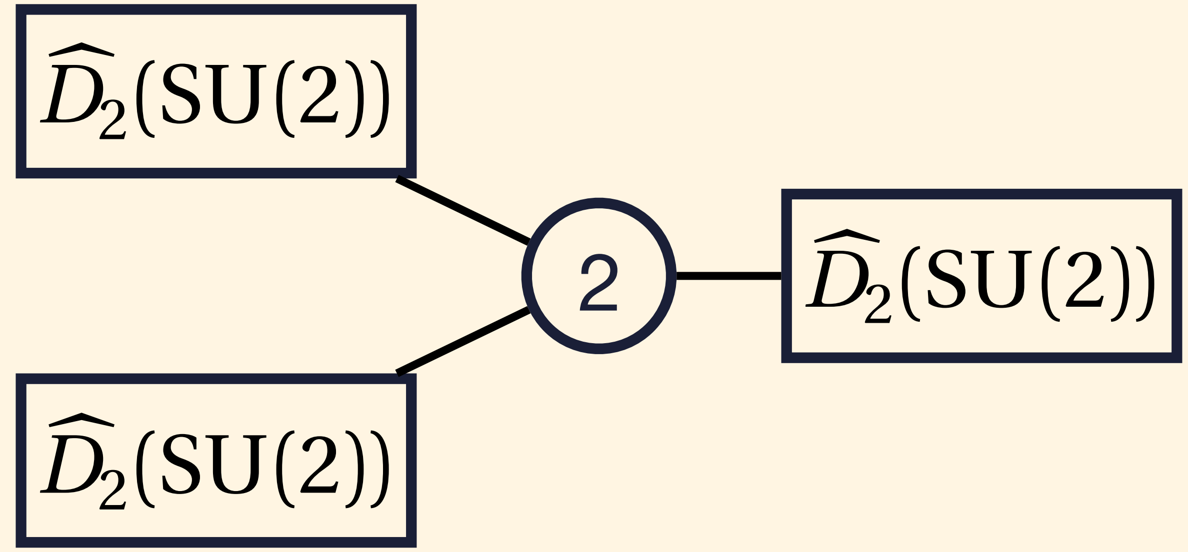
rank 4

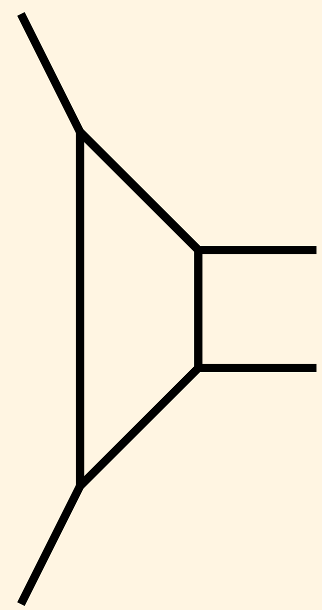
pure E_8 gauge theory $\rightleftarrows$



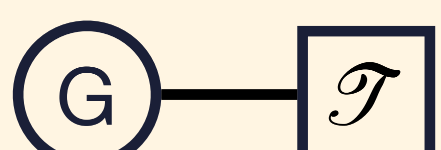
Trivalent Gluing

top. str. partition function?

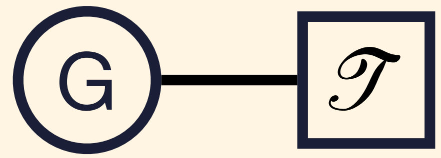
pure SO(8) gauge theory $\rightleftarrows$ 

$\widehat{D}_2(\mathrm{SU}(2))$: 

How the SU(2) gauging can be implemented?

More general: 

Gauging 4d SCI and Nekrasov Z



$$I_{\mathrm{SCI}} = \int_G \mathrm{d}\mu_G(g) I_{\mathscr{T}}(g) I_{\mathrm{vec}}(g)$$

For Nekrasov Z (and 5d SCI), not this easy.

In the localization context, G integration above comes from remaining saddle locus direction (holonomies abound the circle).

For Nekrasov Z, corresponding integration is integration over the moduli space, which is complicated.

Nevertheless, for G=(S)U(N), the original symplectic localization still holds, so that the integration reduces to sums over Young diagrams.

Nekrasov part. func. with matter

SU(2) with hyper:

$$Z = \sum_{\lambda,\mu} Q_g^{|\lambda|+|\mu|} Z_{\lambda,\mu}^{\text{hyper}}(Q_g, Q_m) Z_{\lambda,\mu}^{\text{SU(2) vector}}(Q_g)$$

$Z_{\lambda,\mu}^{\text{hyper}}(Q_g, Q_m)$: Z of hyper with flavor instanton background (λ,μ)

Not all instanton backgrounds are necessary.

SU(2) with any $\mathscr{T}$:

$$Z = \sum_{\lambda,\mu} Q_g^{|\lambda|+|\mu|} Z_{\lambda,\mu}^{\mathscr{T}}(Q_g, Q_{\text{other}}) Z_{\lambda,\mu}^{\text{SU(2) vector}}(Q_g)$$

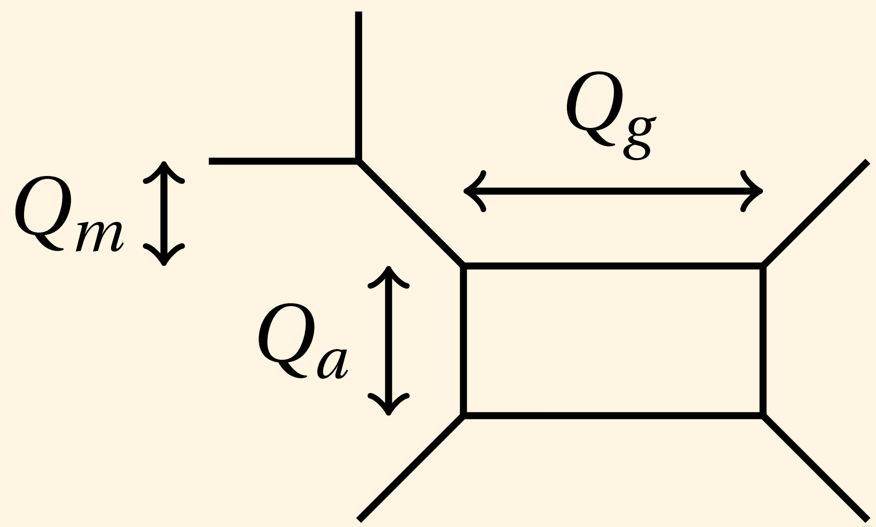
$Z_{\lambda,\mu}^{\mathscr{T}}(Q_g, Q_{\text{other}})$: Z of $\mathscr{T}$ with flavor instanton background (λ,μ)

How to compute? → Topological vertex!

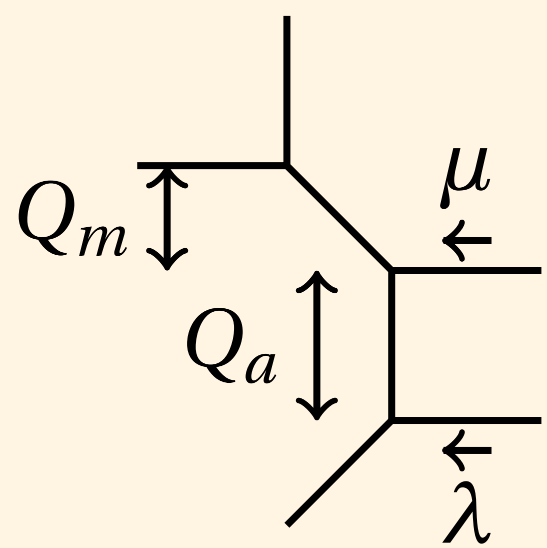
$Z^{\text{hyper}}_{\lambda,\mu}(Q_g, Q_m)$ from top. vertex

(for simplicity, we use unrefined vertices)

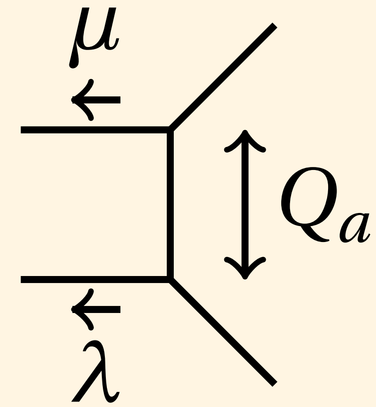
SU(2) with hyper: $Z = \sum_{\lambda,\mu} Q_g^{|\lambda|+|\mu|} \, Z^{\text{hyper}}_{\lambda,\mu}(Q_g, Q_m) Z^{\text{SU(2) vector}}_{\lambda,\mu}(Q_g)$



$= \sum_{\lambda,\mu} Q_g^{|\lambda|+|\mu|} f_{\lambda,\mu} \times$



$\times$

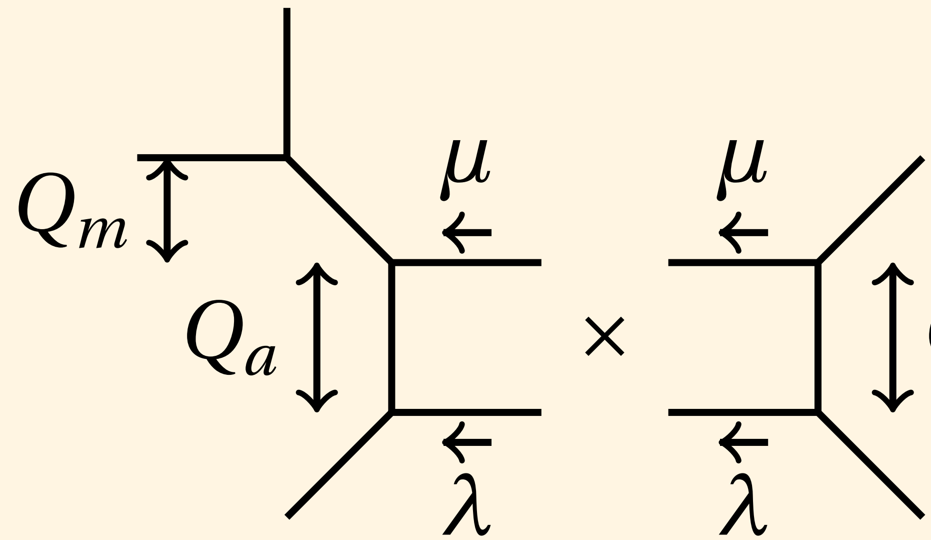


,

$Z^{\text{hyper}}_{\lambda,\mu}(Q_g, Q_m)$ from top. vertex

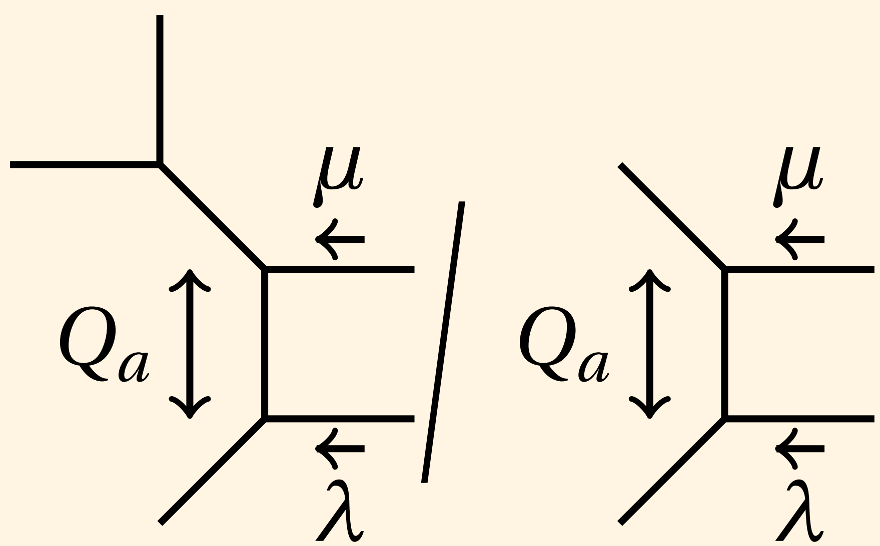
SU(2) with hyper: $Z = \sum_{\lambda,\mu} Q_g^{|\lambda|+|\mu|} Z^{\text{hyper}}_{\lambda,\mu}(Q_g, Q_m) Z^{\text{SU(2) vector}}_{\lambda,\mu}(Q_g)$

$= \sum_{\lambda,\mu} Q_g^{|\lambda|+|\mu|} f_{\lambda,\mu} \times$

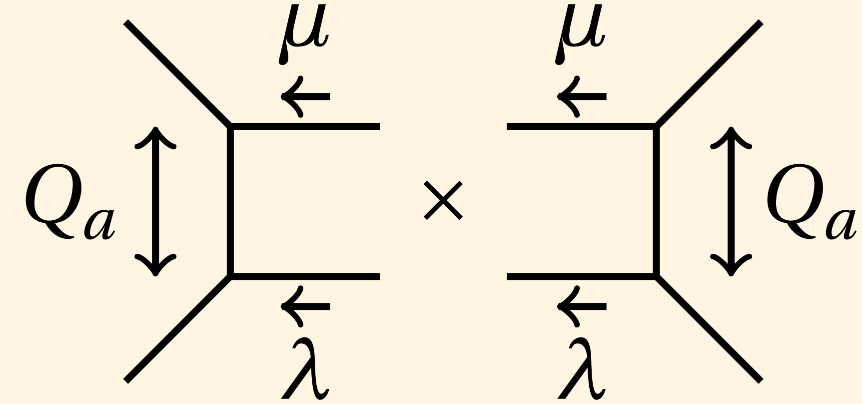


,

$= \sum_{\lambda,\mu} Q_g^{|\lambda|+|\mu|}$



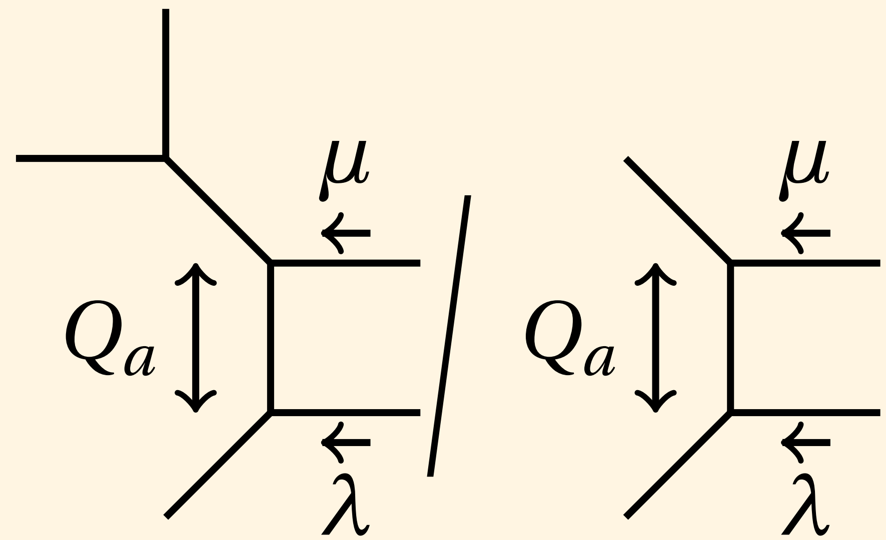
$f_{\lambda,\mu} \times$



$Z^{\text{hyper}}_{\lambda,\mu}$ $Z^{\text{vec}}_{\lambda,\mu}$

$Z^{\text{hyper}}_{\lambda,\mu}(Q_g, Q_m)$ from top. vertex

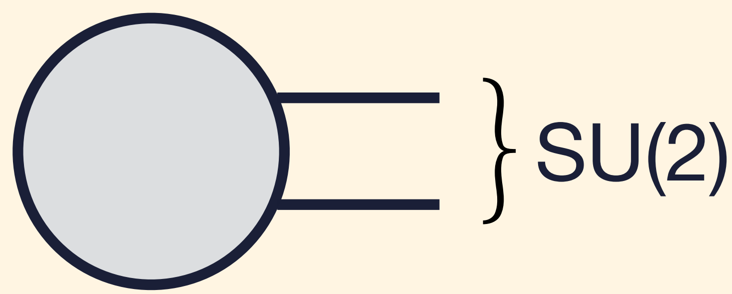
$$Z^{\text{hyper}}_{\lambda,\mu} \quad =$$



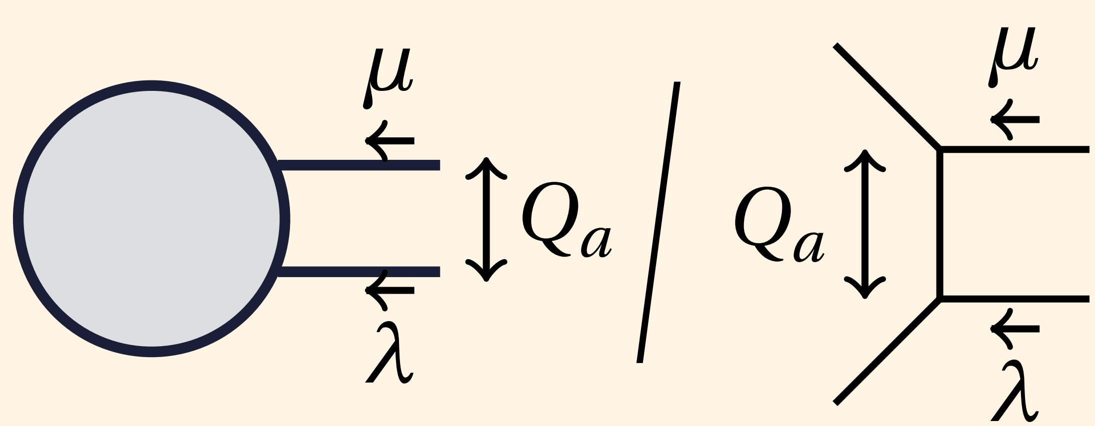
Numerator is what's called "extra factor" in the literature,
Which counts strings bridging parallel external branes.

$Z^{\mathscr{T}}_{\lambda,\mu}$ from top. vertex

Assumption: $\mathscr{T}$ is engineered by a web w/ manifest SU(2) sym.

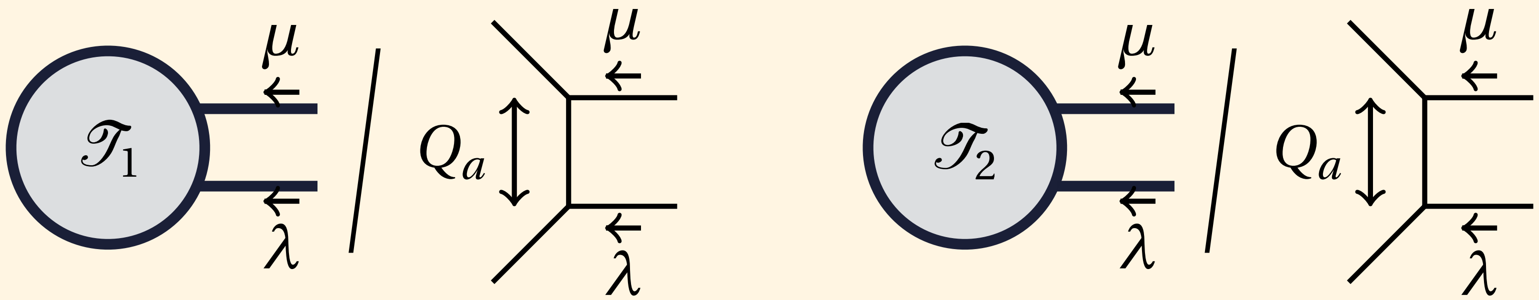


$Z^{\mathscr{T}}_{\lambda,\mu}$ =

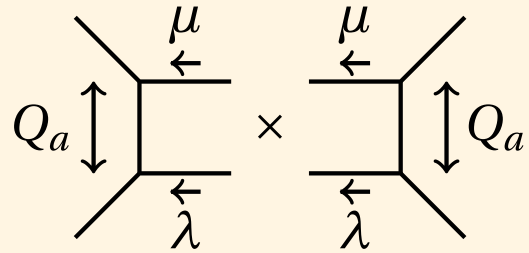


Gauging two non-Lag. theories

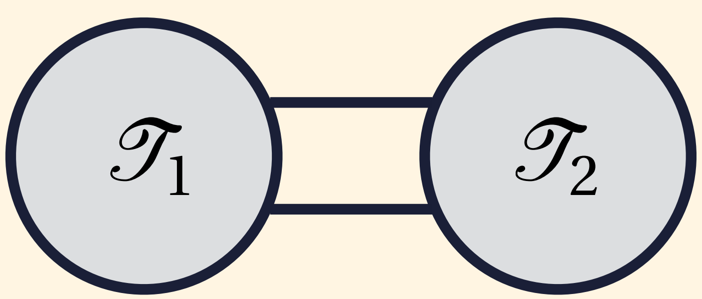
$$Z = \sum_{\mu,\lambda} Q_g^{|\mu|+|\lambda|}$$



$\times \; f_{\lambda,\mu} \times$

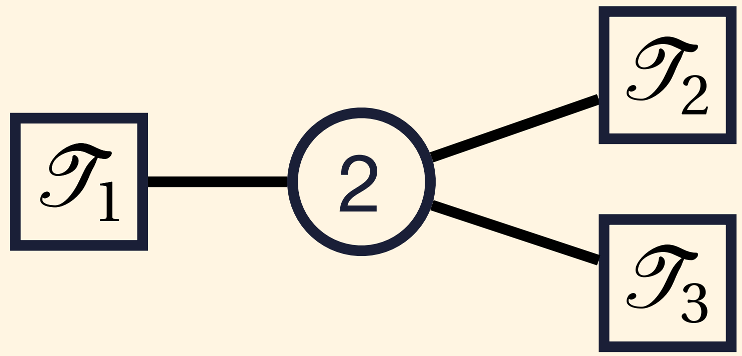


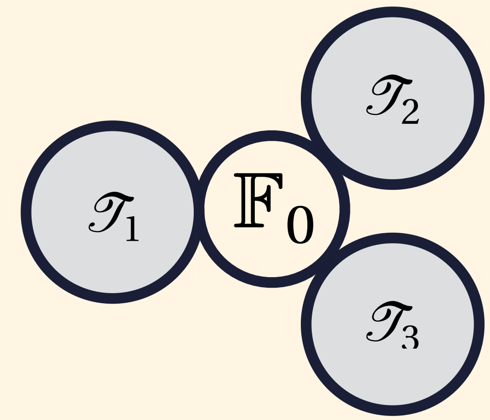
$=$

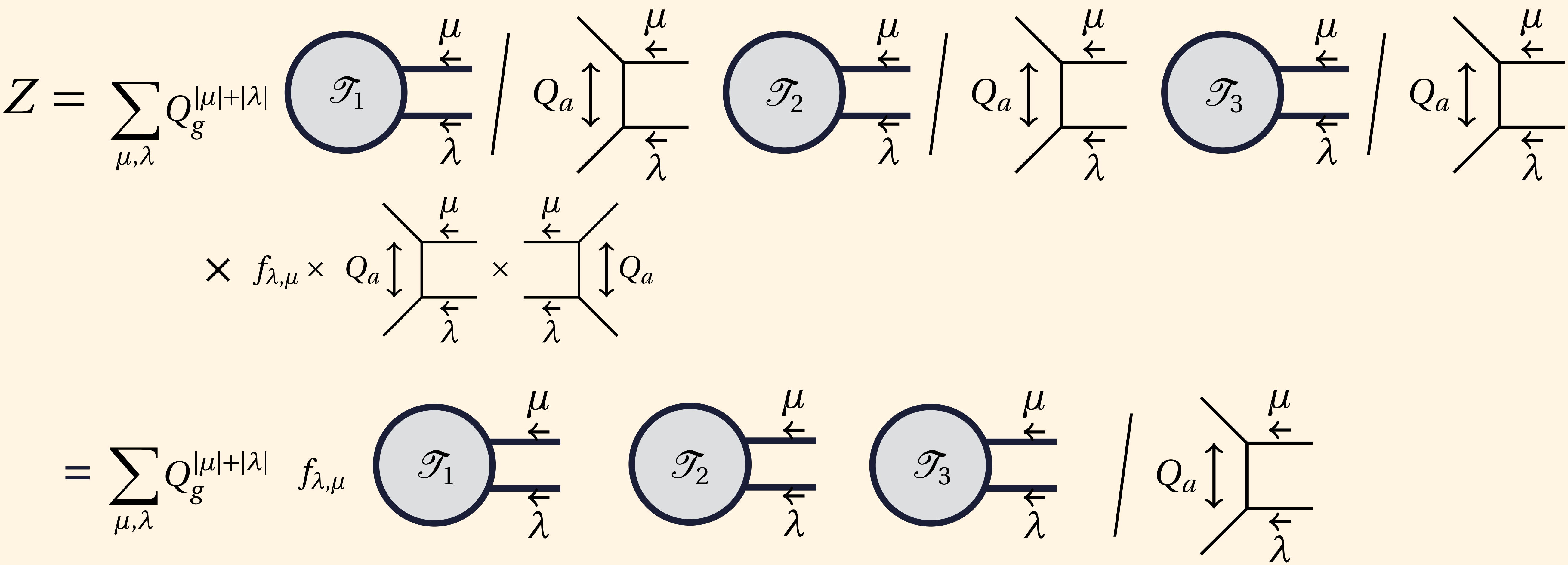


Just connecting two web diagram getting a usual web diagram.
(nothing new)

Gauging three non-Lag. theories





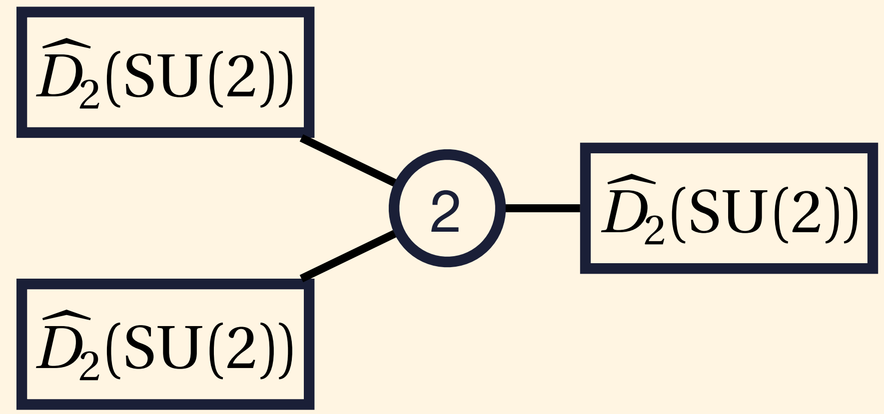


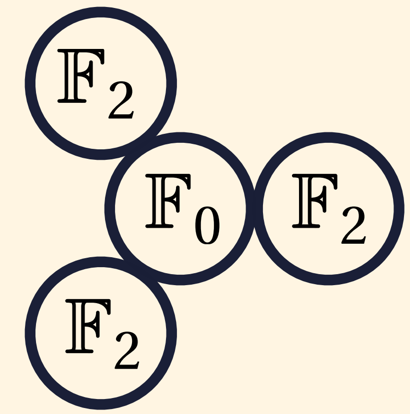
Trivalent glueing (=trivalent gauging)

Caveat: in general modification is needed

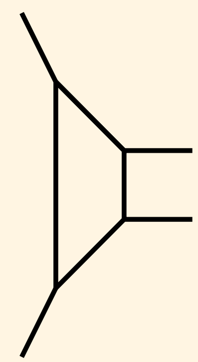
Checking SO(8) duality

pure SO(8) $\rightleftarrows$

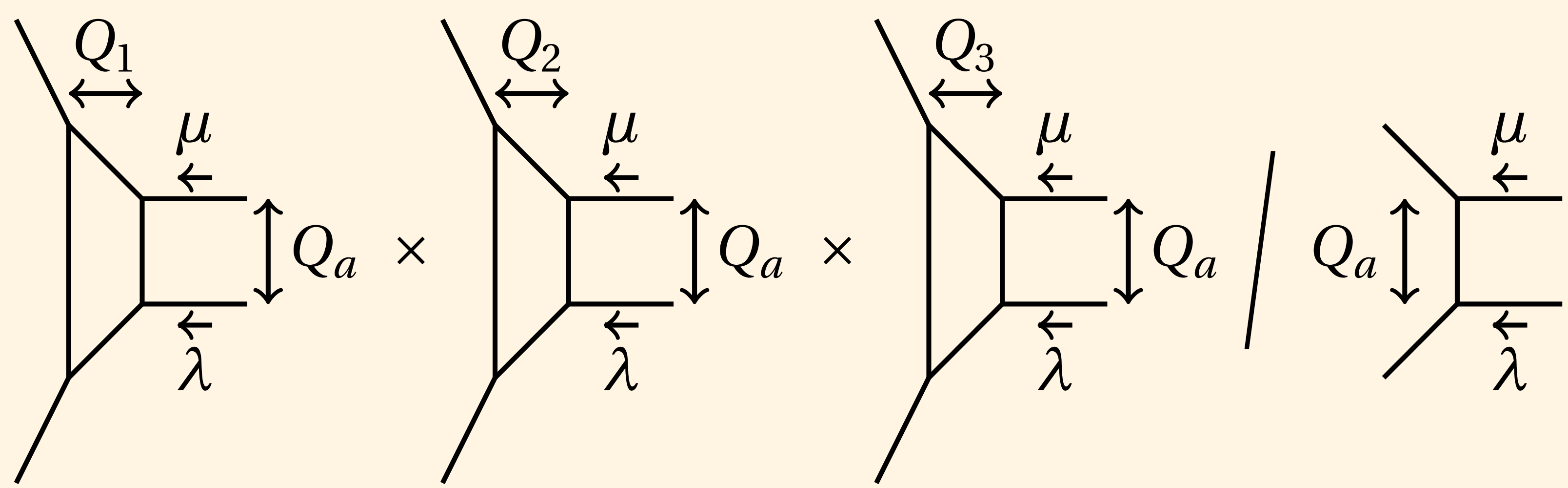




$\widehat{D}_2(\mathrm{SU}(2))$:



$$Z_{\mathrm{SO}(8)} = \sum_{\lambda,\mu} Q_g^{|\lambda|+|\mu|} f_{\lambda,\mu} \times$$



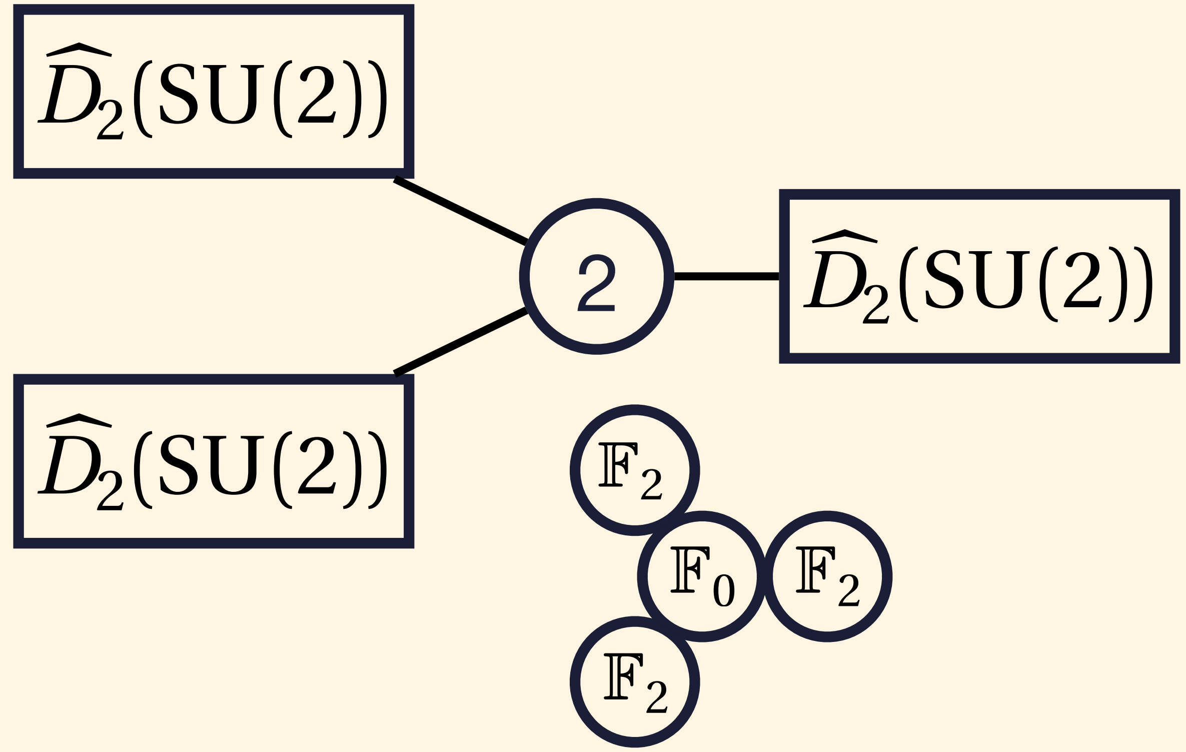
Checked explicitly at 0,1,2- instantons
and a high order of Coulomb branch parameters.

Duality for E-type theories also works w/ Hilbert series conjecture
[Keller,Mekareeya,Song,Tachikawa ’12]

Generalization to 6d SCFTs

Dual description of 6d “O(-4)”

pure SO(8) gauge theory $\rightleftarrows$



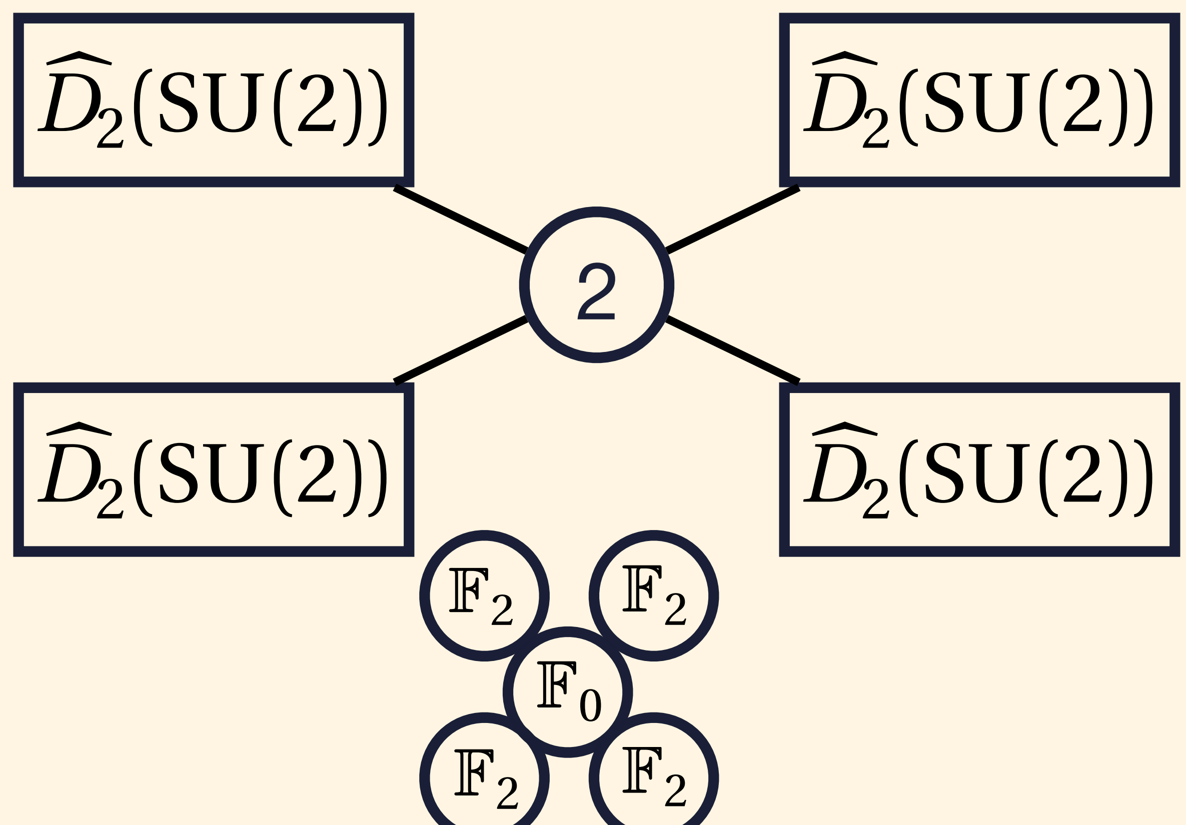
[DelZotto-Vafa-Xie 15’]

6d “O(-4)” theory on S^1 $\rightleftarrows$

tensor branch $\downarrow$

tensor+ SO(8) vector

“affine SO(8) gauge theory”

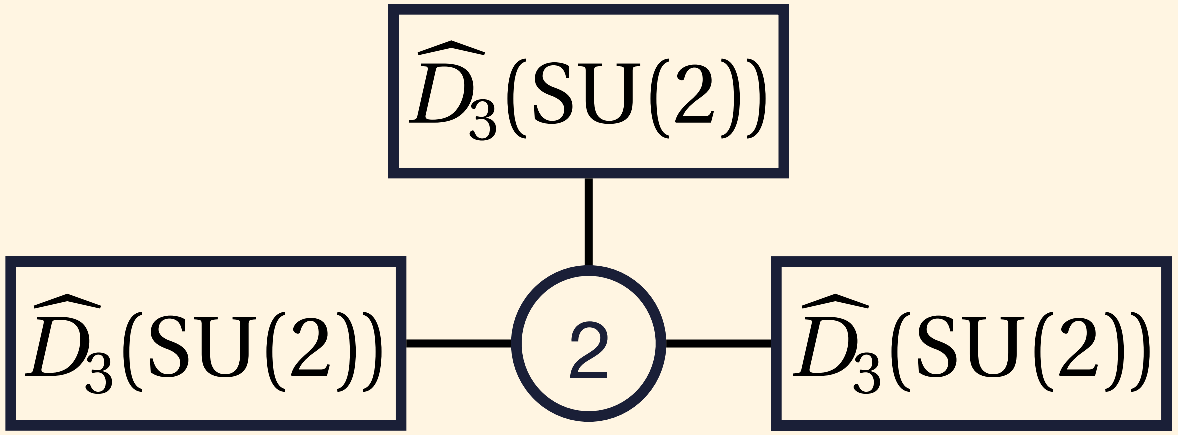


Dual description of 6d NHCs

6d SCFTs: [DelZotto-Vafa-Xie 15']

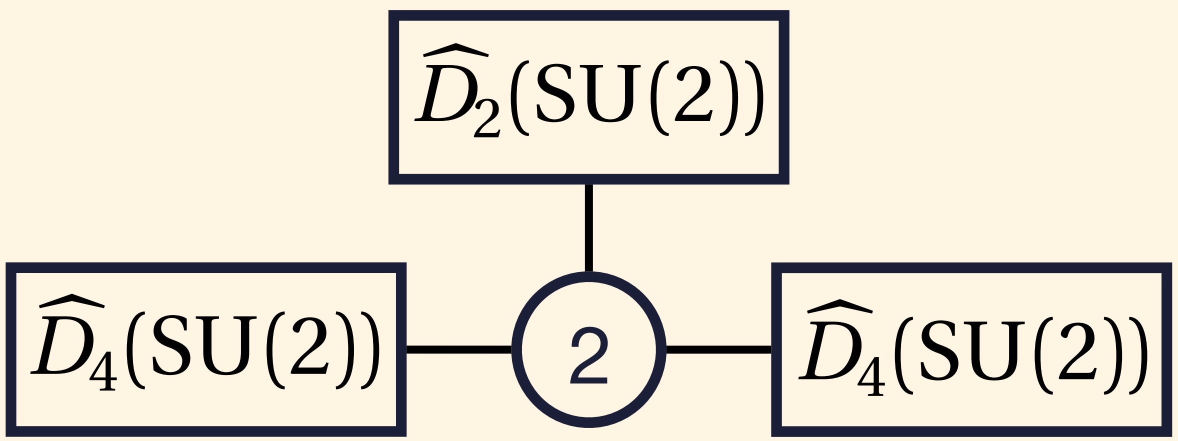
O(-6) theory $\rightleftarrows$

"$\hat{E}_6$"



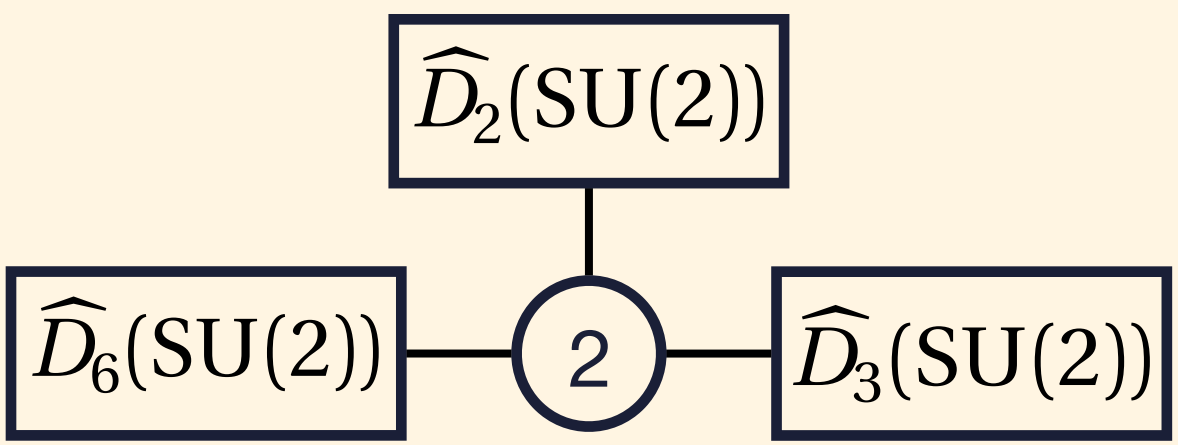
O(-8) theory $\rightleftarrows$

"$\hat{E}_7$"



O(-12) theory $\rightleftarrows$

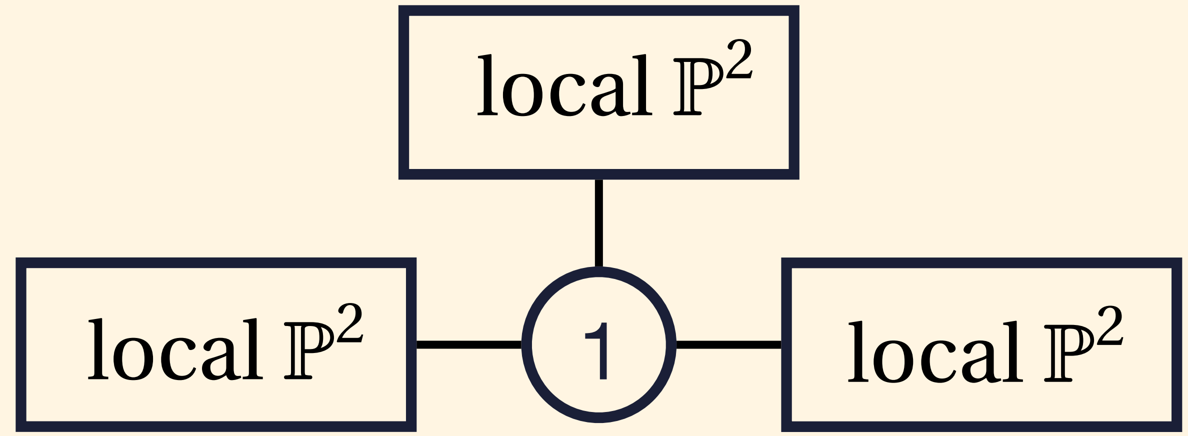
"$\hat{E}_8$"



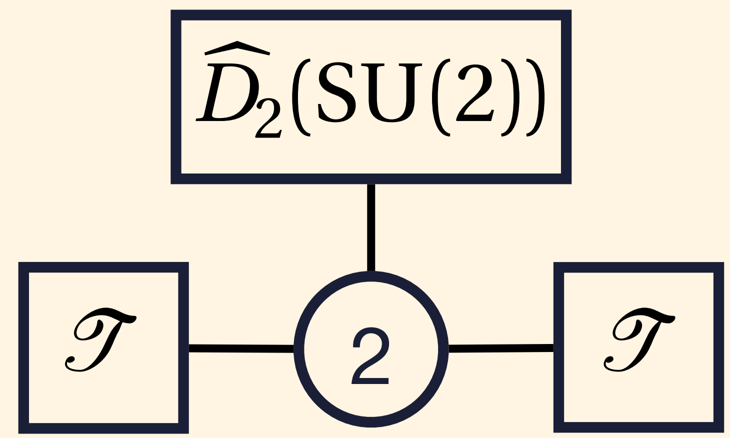
Dual description of 6d NHCs

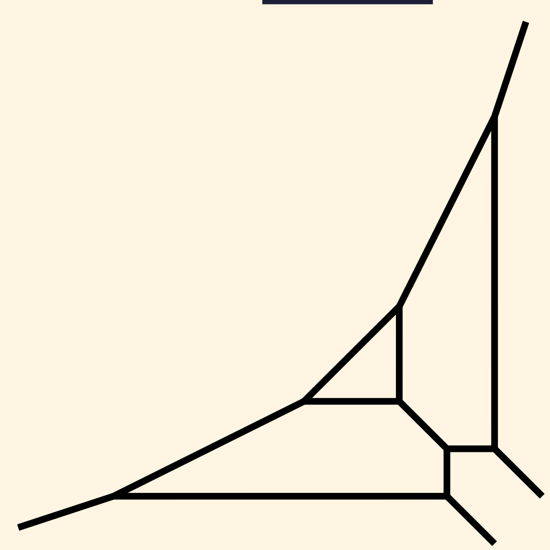
6d SCFTs: [DelZotto-Vafa-Xie 15']

O(-3) theory $\rightleftarrows$



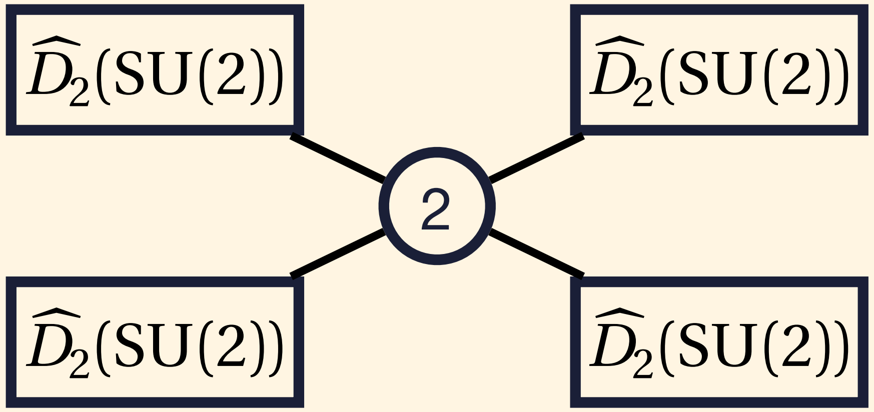
“232 NHC” $\rightleftarrows$

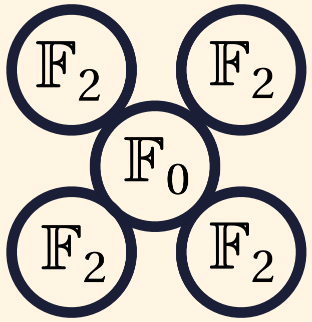




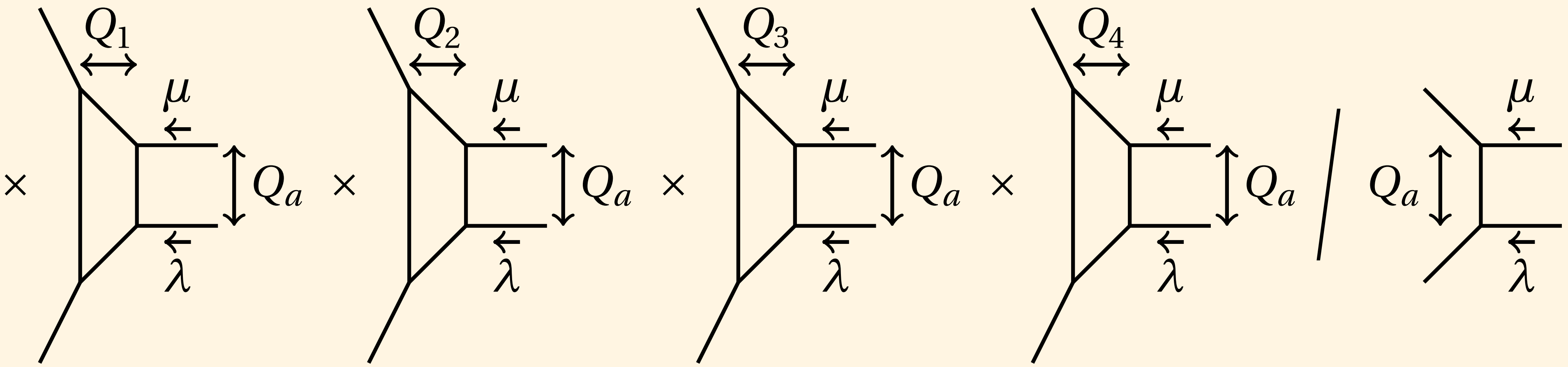
Checking O(-4) duality

6d O(-4) ⇄





$$Z_{\mathcal{O}(-4)} = \sum_{\lambda,\mu} Q_g^{|\lambda|+|\mu|} f_{\lambda,\mu} \times$$



string fugacity = $Q_a \times \mathrm{dressing}$

Checked explicitly at 0,1 - string sectors

For other NHCs, the vertex is the first method w/ general parameters.

c.f. [Del-Zotto, Lockhart 16']

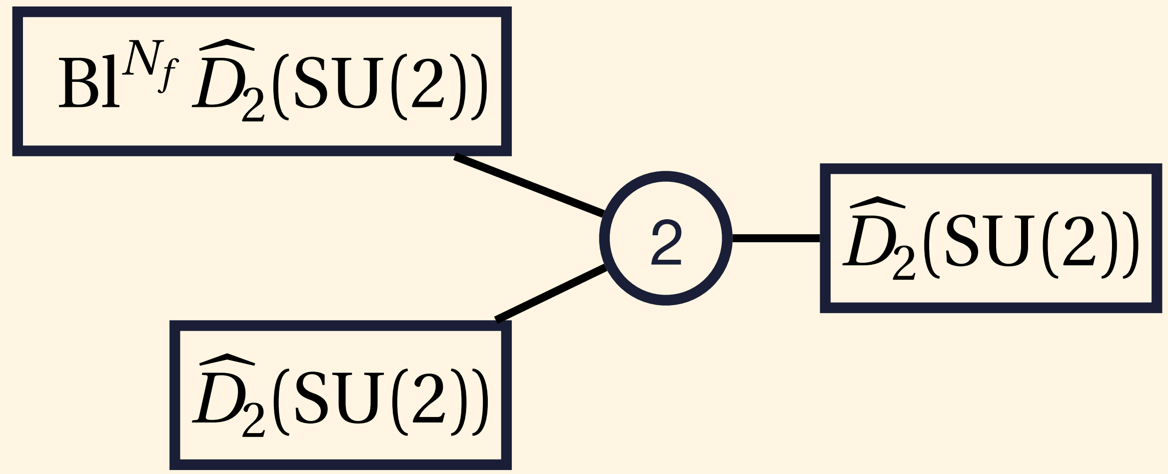
Other generalizations

in progress
w/ Hirotaka Hayashi
w/ Hirotaka Hayashi, Hee-Cheol Kim

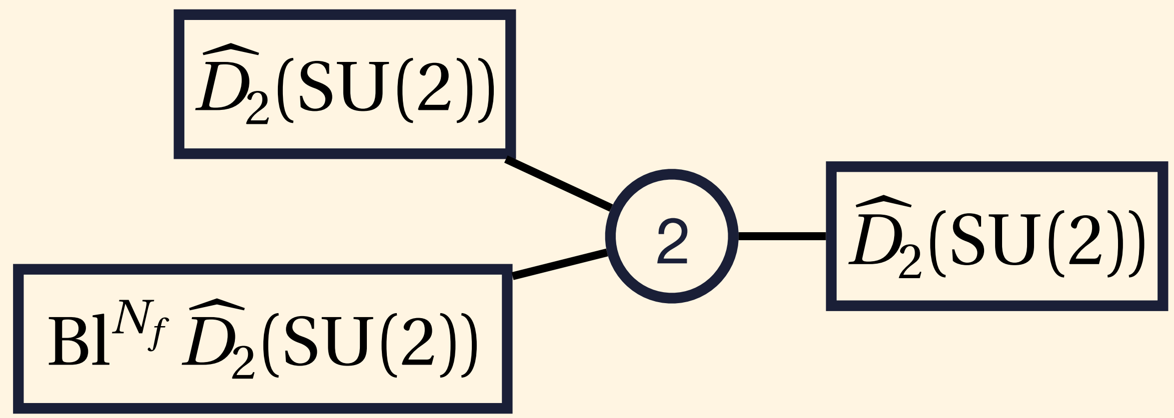
Adding matter to 5d SO(8)

SO(8) w/ Nf vectors ⇄

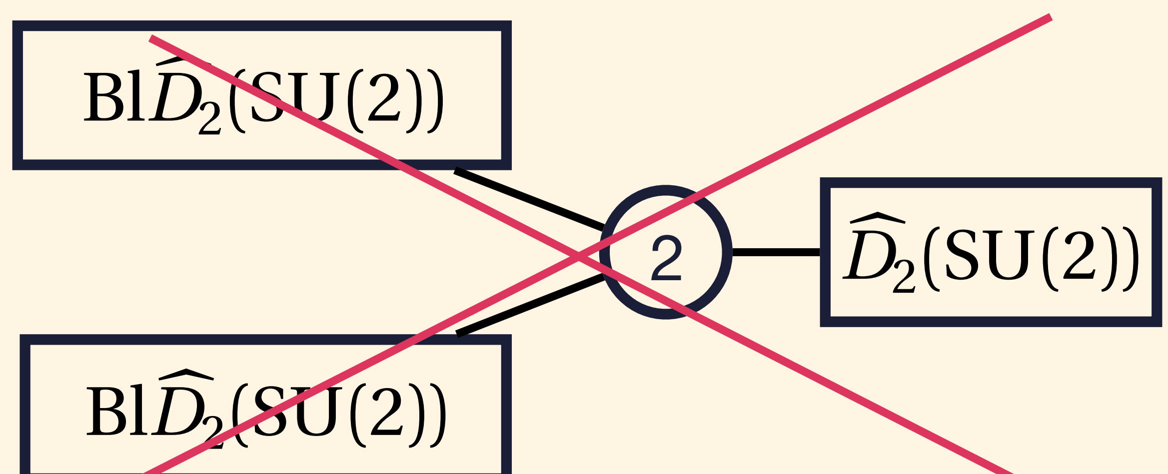
Nf=1,2,3,4,5



SO(8) w/ Nf spinors ⇄

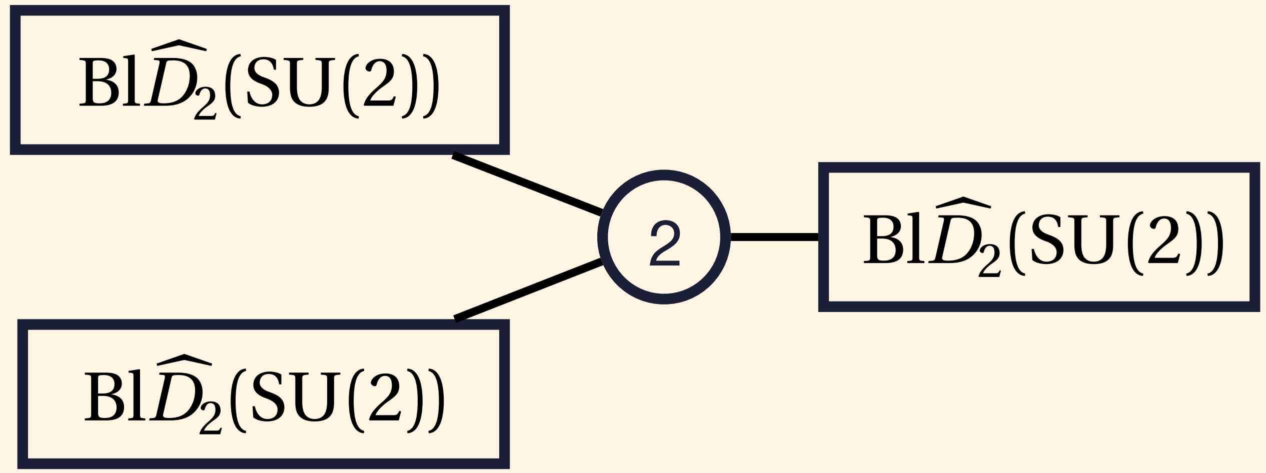


SO(8) w/ 1v+ 1s ⇄



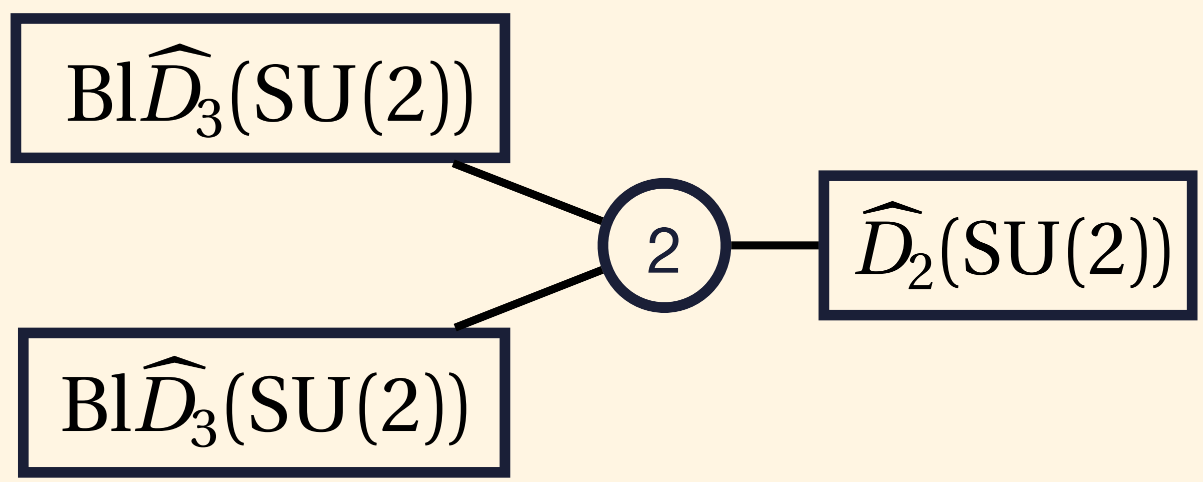
Constraint is needed

SO(8) w/ 1v+ 1s $\rightleftarrows$



w/ particular constraint on parameters

E6 w/ 27 rep $\rightleftarrows$

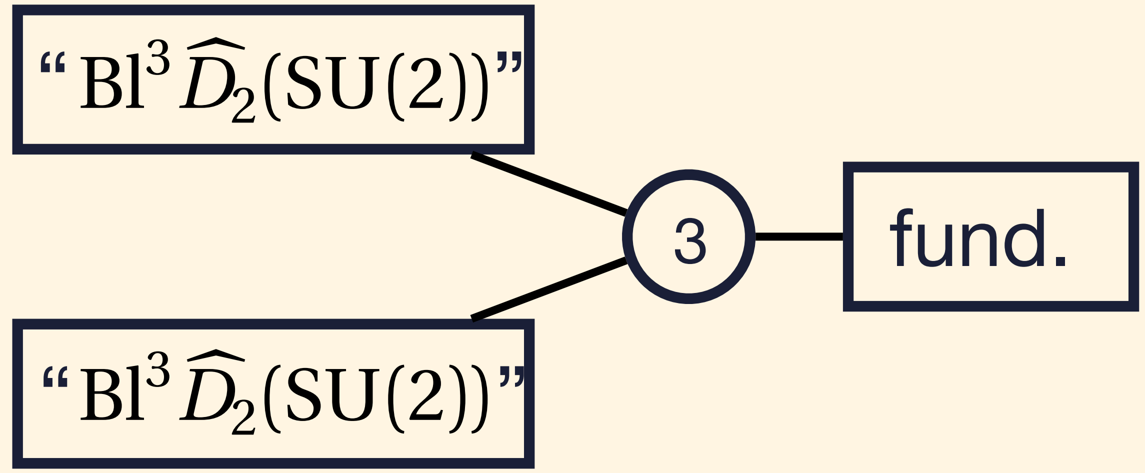


w/ particular constraint on parameters

(as far as I know) first explicit formula

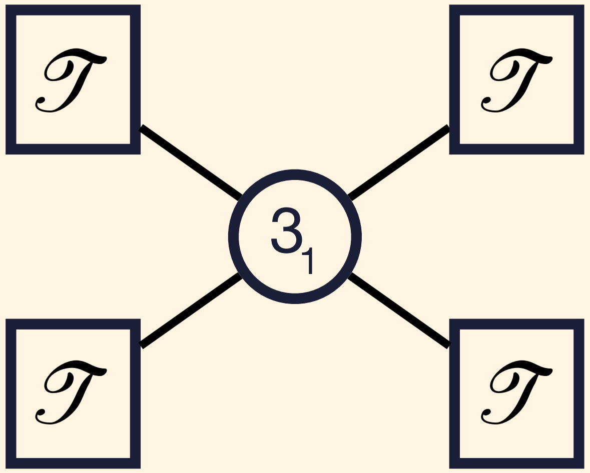
Quiver

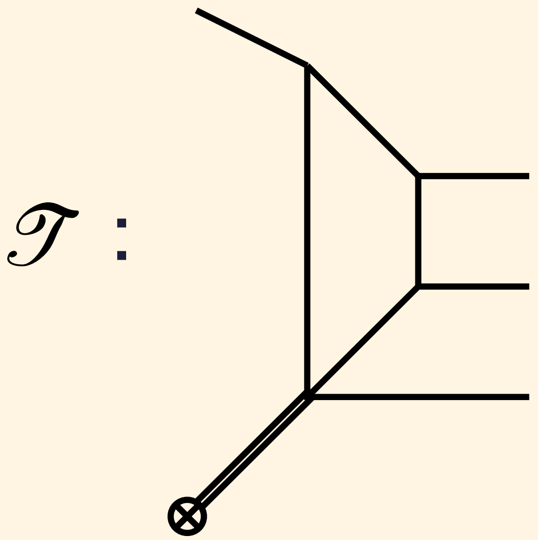
SO(6)-Sp(1)-[SO(4)] $\rightleftarrows$



What quiver theory w/ E-type possible?

Hee-Cheol’s SU(3) k=9





Thank you!