

5d/6d DE instantons from trivalent gluing of web diagrams

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arXiv:1702.07263 w/ Hirotaka Hayashi

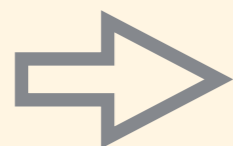
Autumn Symposium on String Theory @KIAS

Sep. 14th 2017

Introduction

How to compute top. string?

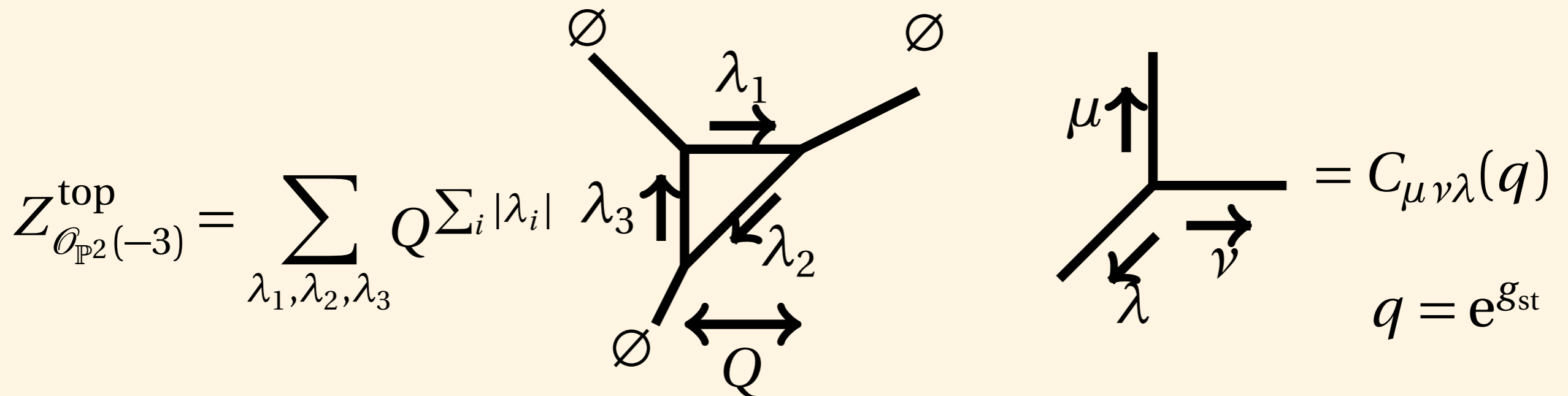
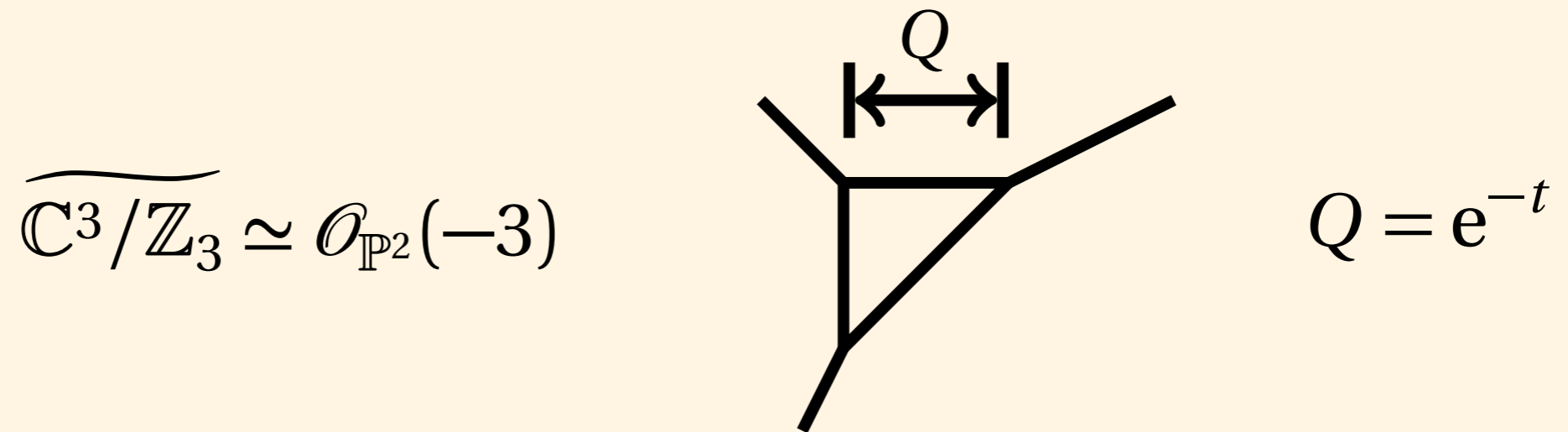
- genus 0: localization, Picard-Fuchs etc.
- For general genus, general CY3: dream
- B-model: BCOV (but holomorphic part is nontrivial)
- A-model: when CY3 is toric (hence non-compact)
→ topological vertex
- A-model on non-compact CY3 → Nekrasov Z of a 5d SQFT
- Some non-compact non-toric CY3 → Non-SU SQFT
- “Geometric-ish” computation for such CY3’s?



“trivalent gluing” of topological vertices

Toric CY and Topological Vertex

M theory on Toric CY \Leftrightarrow Type IIB (p,q) Web

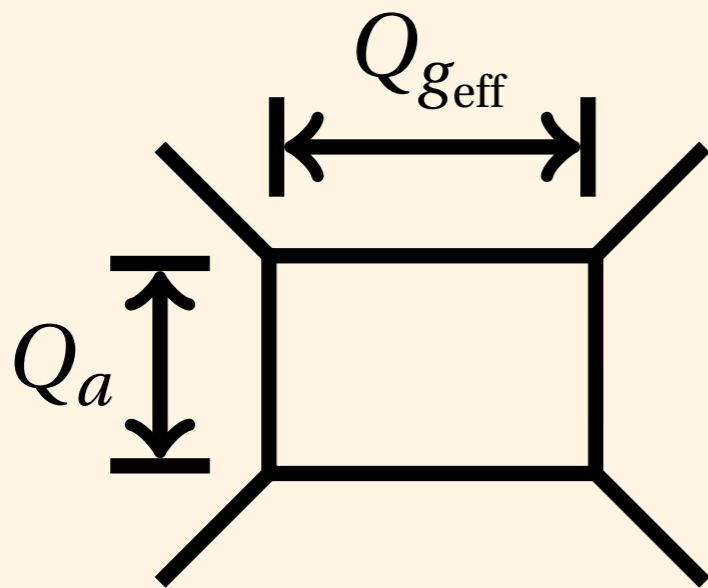


Gauge theory and Top. Vertex

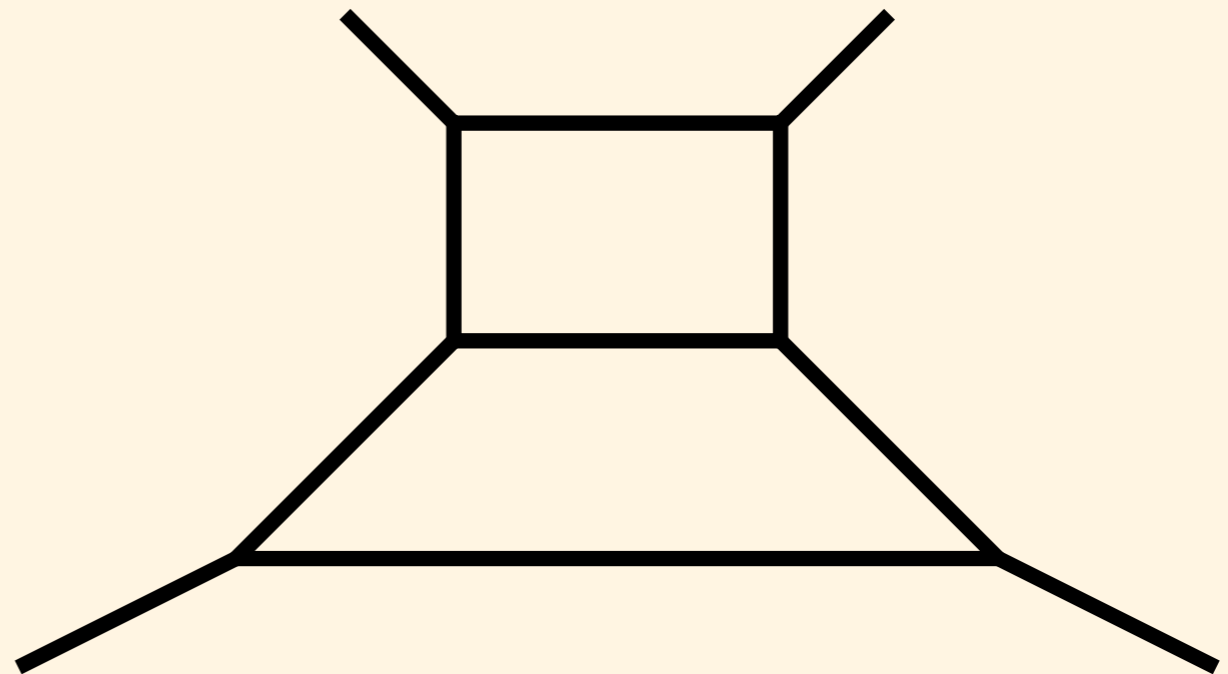
5d $SU(N)$ quiver gauge theories w/ (bi)fundamentals can be engineered by toric CY3 / (p,q) web.

pure $SU(2)_{\theta=0}$

local $\mathbb{P}^1 \times \mathbb{P}^1$



pure $SU(3)_{k=1}$

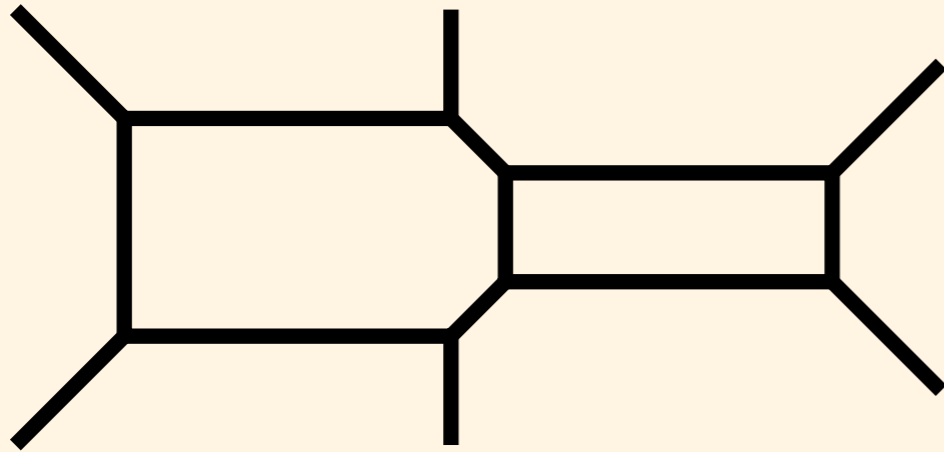


$$Z^{\text{top}}(\text{CY}_3) = Z^{\text{Nek}}(\text{gauge theory})|_{\text{unrefined}}$$

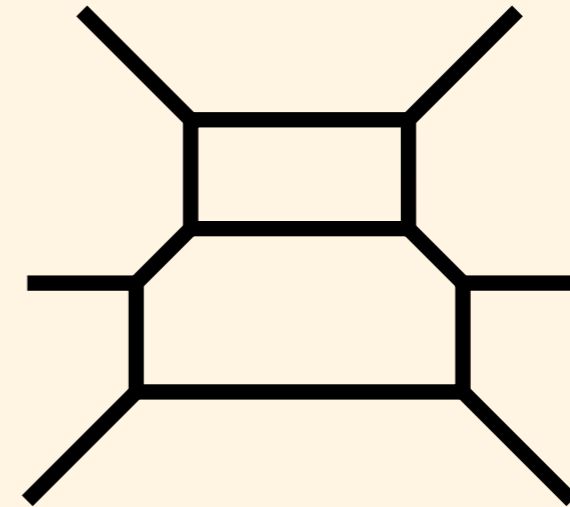
✧ every theory (in my talk) has 8 supercharges

Base-Fiber duality

$SU(2) - SU(2)$



$SU(3)_{k=0}$ with 2 flavors



These two gauge theories have common UV SCFT

Different mass def.s of the SCFT gives different IR gauge theories

Z^{Nek} of the two coincides in 5d

Non-SU gauge theory?

Can a 5d non-SU gauge theory be engineered?

SO/Sp: (p,q) web with O-planes [Zafrir '15, Hayashi-Kim-Lee-Taki-Yagi '15]

SO/E: M-theory on a D,E-orbifolded of the resolved conifold

$\mathbb{C}^2/\Gamma_{D,E}$ singular locus wrapping a sphere

non-toric

Non-Lagrangian base-fiber dual:

pure D,E gauge theory \Leftrightarrow SU(2) w/ non-Lagrangian matters

some 6d SCFT on circle \Leftrightarrow SU(2) w/ non-Lagrangian matters

[DelZotto-Vafa-Xie 15', DelZotto-Heckman-Morrison 17']

“trivalent gluing” enables us to compute RHS

For some of 6d theories,
this is the first systematic way to compute high string number part.

Outline

- Base-fiber duality for pure DE gauge theories
- Trivalent gluing
- Generalization to 6d SCFTs
- other generalizations

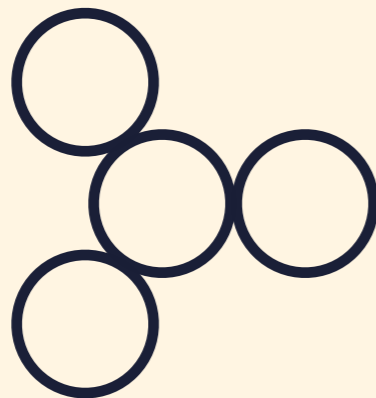
Base-fiber duality for DE gauge theories

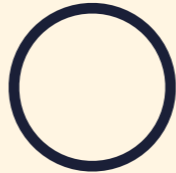
The orbifolded resolved conifold

$$\widetilde{\mathbb{C}^2/\Gamma_G} \longrightarrow (\mathcal{O}(-1) \oplus \mathcal{O}(-1))/\Gamma_G$$

\downarrow
 \mathbb{P}^1

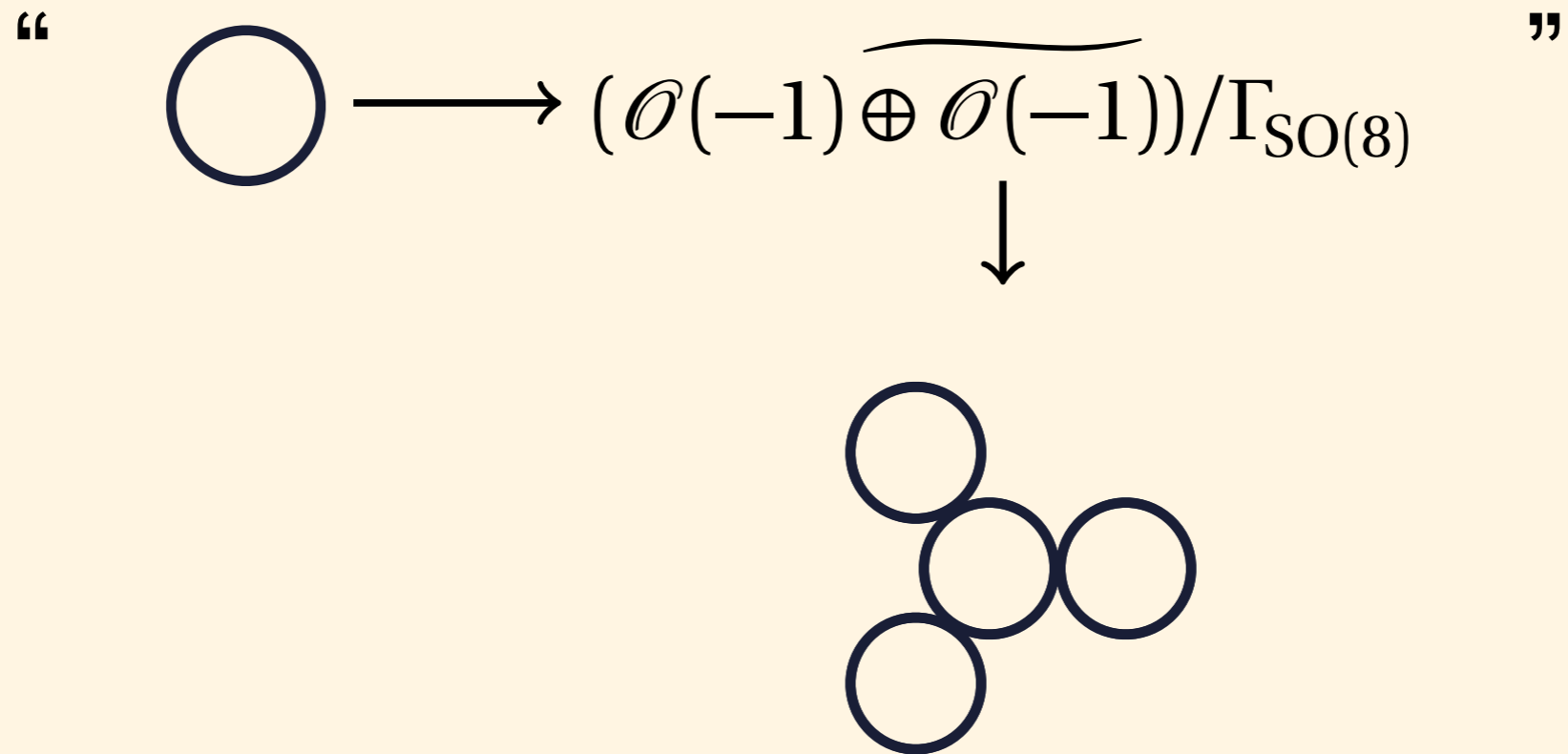
$G = \text{SO}(8)$


$$\longrightarrow (\mathcal{O}(-1) \oplus \mathcal{O}(-1))/\Gamma_{\text{SO}(8)}$$

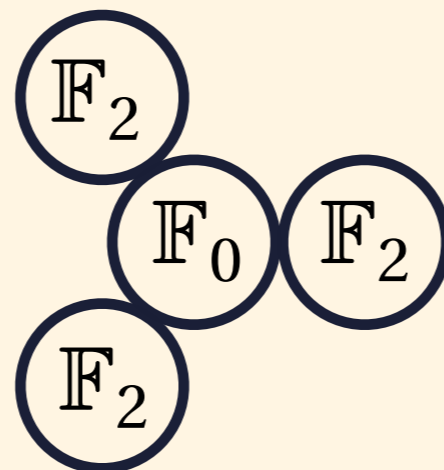
\downarrow


D4 singularity wrapping sphere \rightarrow pure $\text{SO}(8)$

Swapping base and fiber

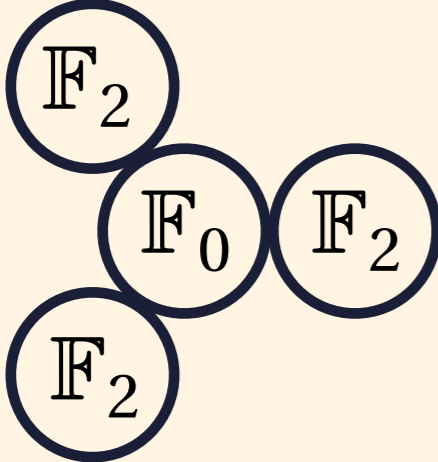


Near each sphere in the “new base”,
the geometry contains a sphere bundle over the sphere.



[DelZotto-Heckman-Morrison 17']
[Hee-Cheol's talk]

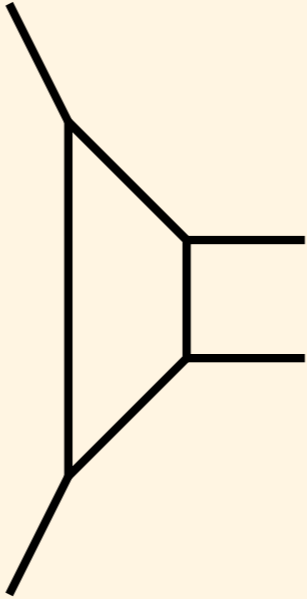
Dual description



local $\mathbb{F}_0 = \text{local } \mathbb{P}^1 \times \mathbb{P}^1$

→ SU(2) vector multiplet

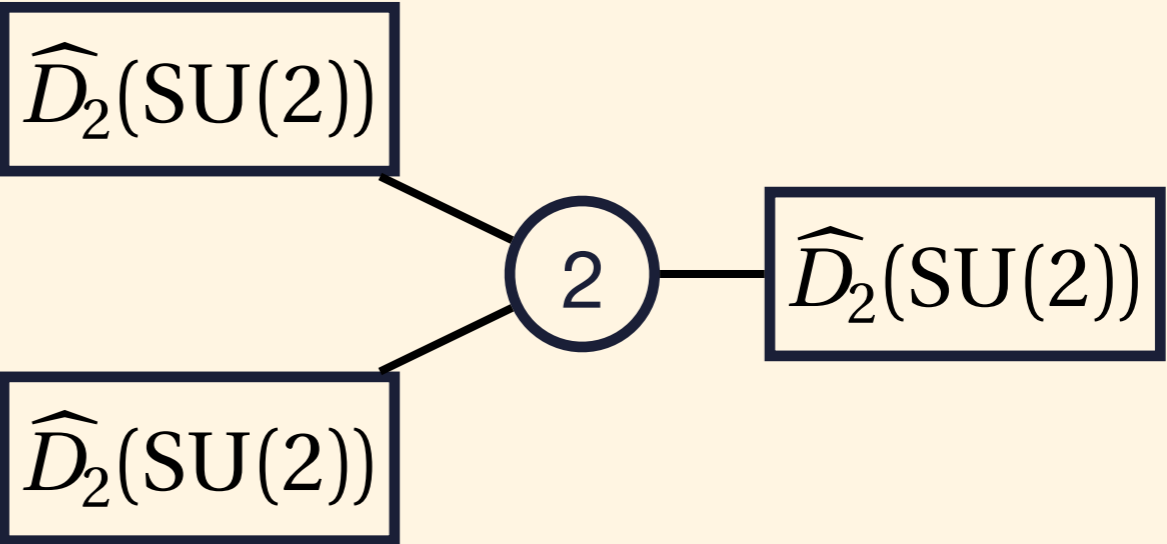
local \mathbb{F}_2 :



$\widehat{D}_2(\text{SU}(2))$

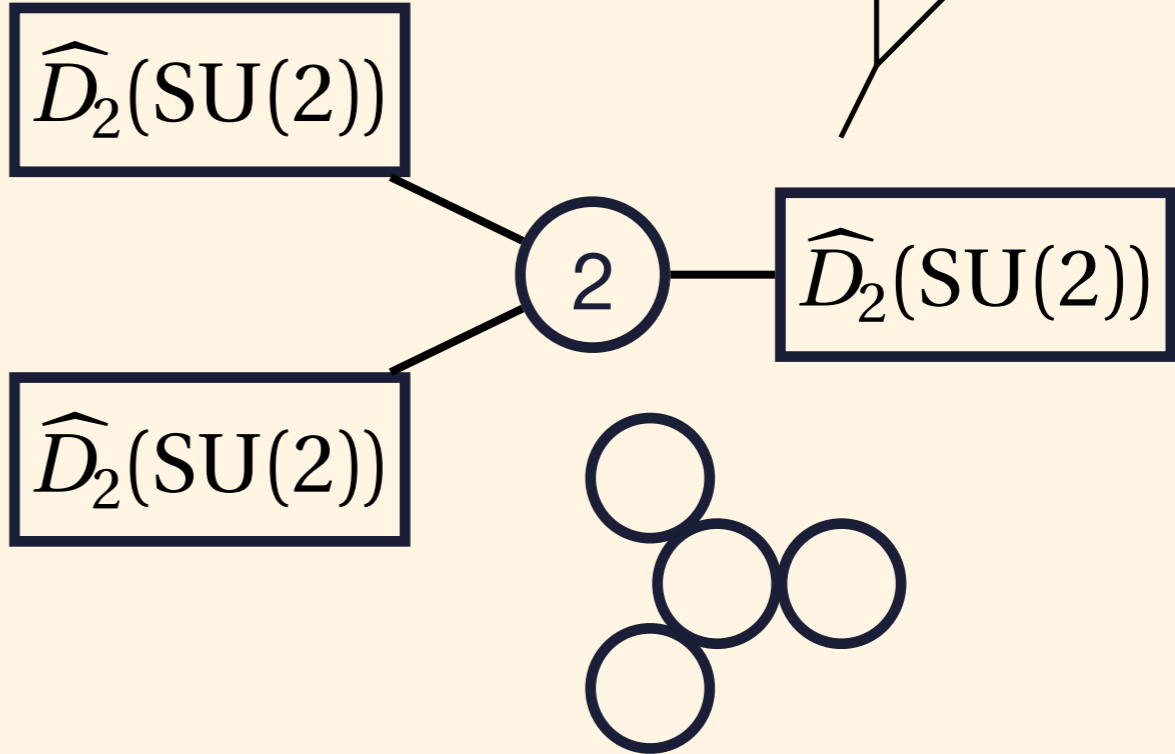
(dual to pure SU(2))

pure SO(8) gauge theory \iff

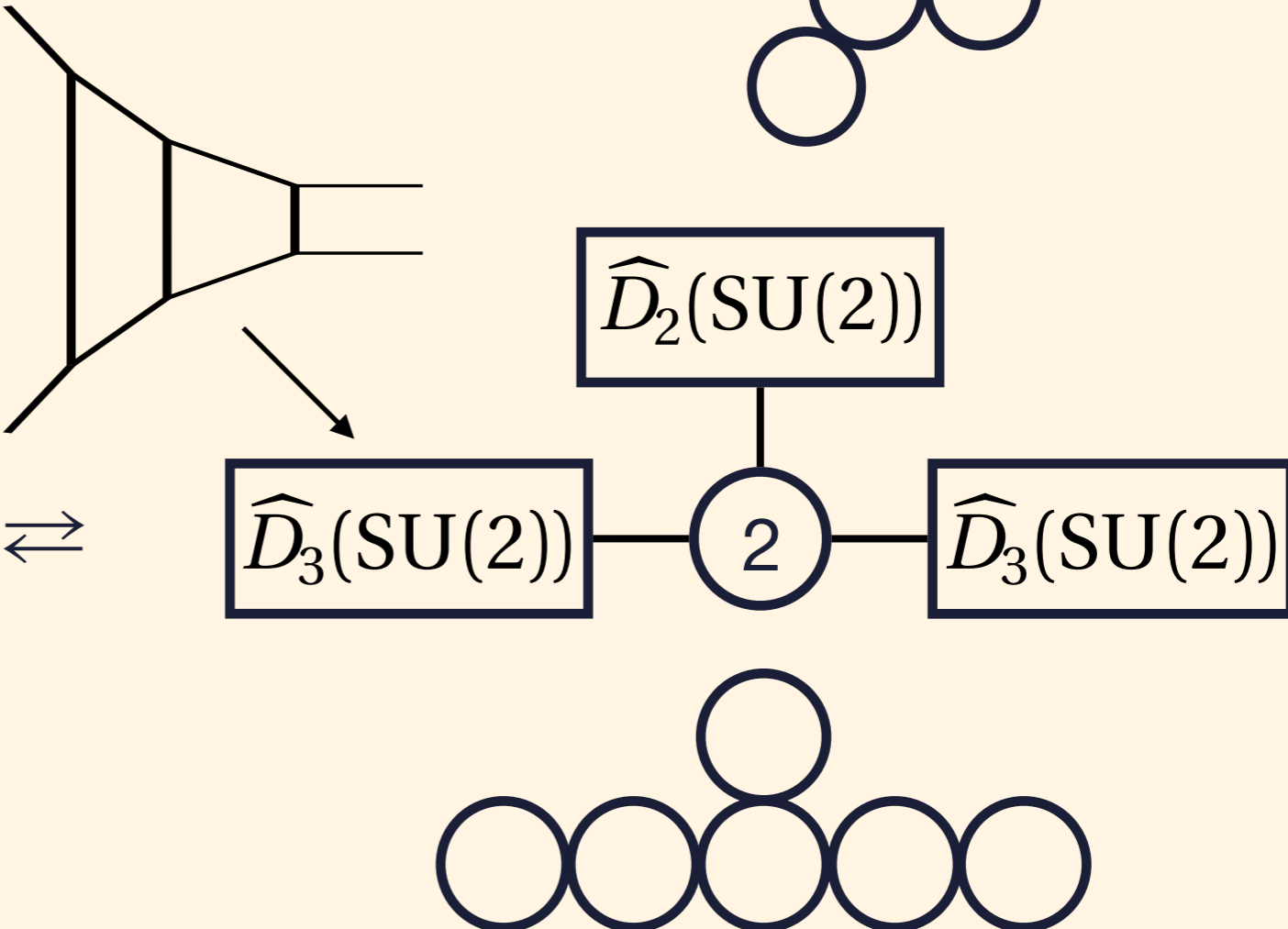


Dual description of E-type

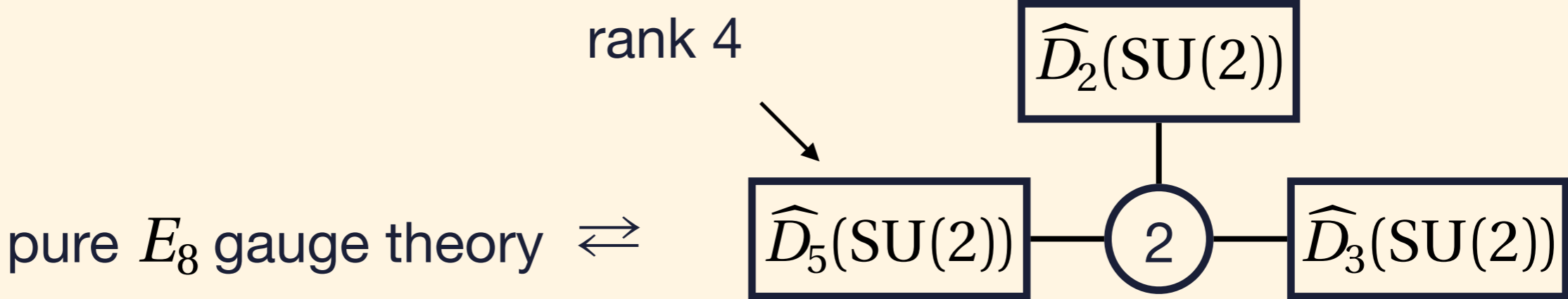
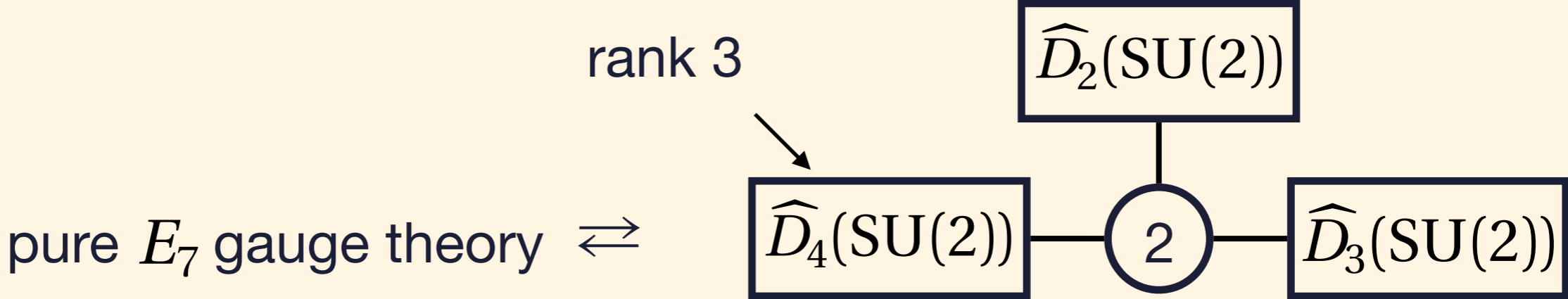
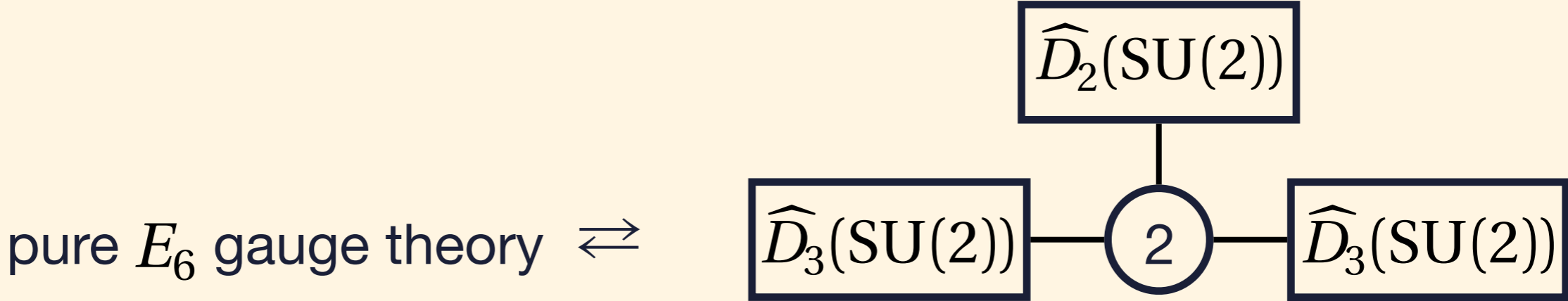
pure $SO(8)$ gauge theory \Leftrightarrow
 (immediate generalization to $SO(N)$)



pure E_6 gauge theory \Leftrightarrow



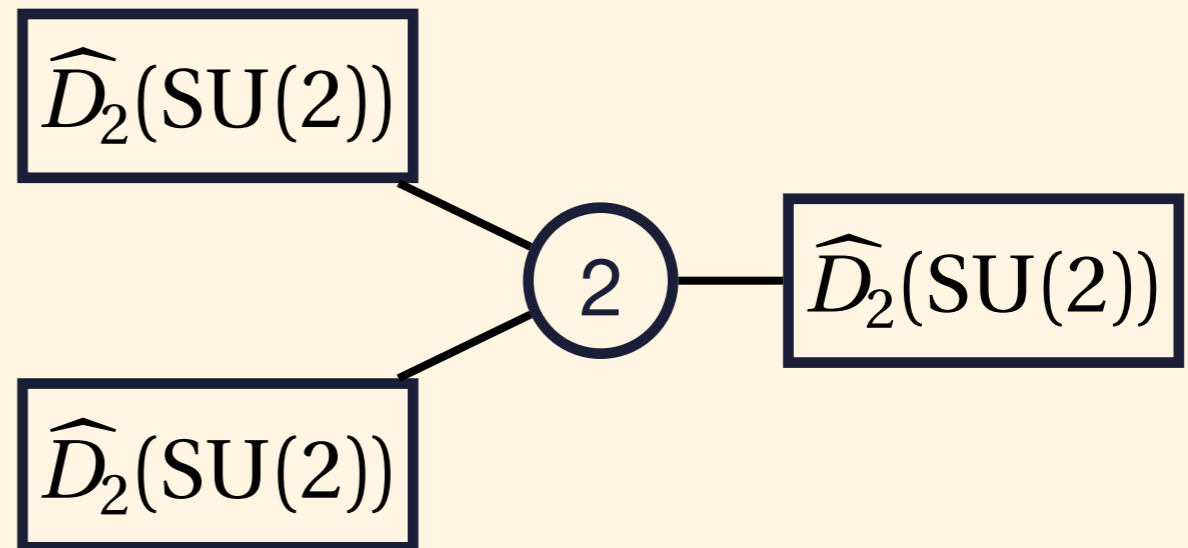
Dual description of E-type



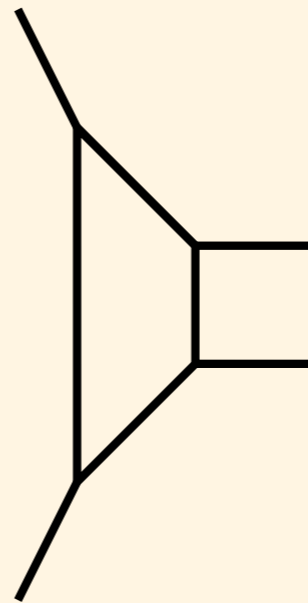
Trivalent Gluing

top. str. partition function?

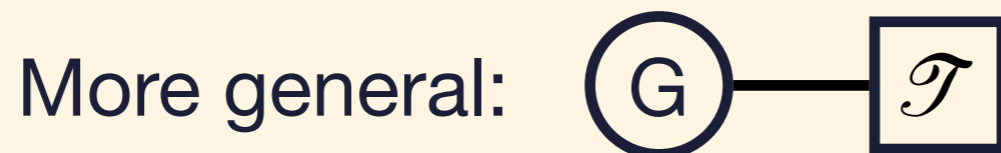
pure SO(8) gauge theory \Leftrightarrow



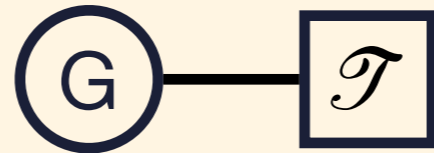
$\widehat{D}_2(SU(2)) :$



How the SU(2) gauging can be implemented?



Gauging 4d SCl and Nekrasov Z



$$I_{\text{SCl}} = \int_G d\mu_G(g) I_{\mathcal{T}}(g) I_{\text{vec}}(g)$$

For Nekrasov Z (and 5d SCl), not this easy.

In the localization context, G integration above comes from remaining saddle locus direction (holonomies around the circle).

For Nekrasov Z, corresponding integration is **integration over the moduli space**, which is complicated.

Nevertheless, for $G=(S)U(N)$, the original symplectic localization still holds, so that the integration reduces to sums over Young diagrams.

Nekrasov part. func. with matter

SU(2) with hyper:

$$Z = \sum_{\lambda, \mu} Q_g^{|\lambda|+|\mu|} Z_{\lambda, \mu}^{\text{hyper}}(Q_g, Q_m) Z_{\lambda, \mu}^{\text{SU}(2) \text{ vector}}(Q_g)$$

$Z_{\lambda, \mu}^{\text{hyper}}(Q_g, Q_m)$: Z of hyper with flavor instanton background (λ, μ)

Not all instanton backgrounds are necessary.

SU(2) with any \mathcal{T} :

$$Z = \sum_{\lambda, \mu} Q_g^{|\lambda|+|\mu|} Z_{\lambda, \mu}^{\mathcal{T}}(Q_g, Q_{\text{other}}) Z_{\lambda, \mu}^{\text{SU}(2) \text{ vector}}(Q_g)$$

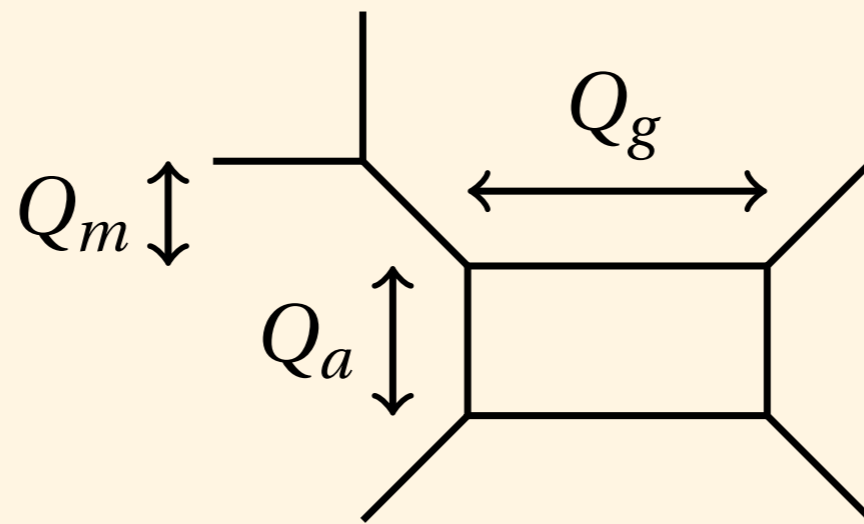
$Z_{\lambda, \mu}^{\mathcal{T}}(Q_g, Q_{\text{other}})$: Z of \mathcal{T} with flavor instanton background (λ, μ)

How to compute? → Topological vertex!

$Z_{\lambda,\mu}^{\text{hyper}}(Q_g, Q_m)$ from top. vertex

(for simplicity, we use unrefined vertices)

$$\text{SU}(2) \text{ with hyper: } Z = \sum_{\lambda,\mu} Q_g^{|\lambda|+|\mu|} Z_{\lambda,\mu}^{\text{hyper}}(Q_g, Q_m) Z_{\lambda,\mu}^{\text{SU}(2) \text{ vector}}(Q_g)$$



$$= \sum_{\lambda,\mu} Q_g^{|\lambda|+|\mu|} f_{\lambda,\mu} \times$$

$Z_{\lambda,\mu}^{\text{hyper}}(Q_g, Q_m)$ from top. vertex

SU(2) with hyper: $Z = \sum_{\lambda,\mu} Q_g^{|\lambda|+|\mu|} Z_{\lambda,\mu}^{\text{hyper}}(Q_g, Q_m) Z_{\lambda,\mu}^{\text{SU}(2) \text{ vector}}(Q_g)$

$$= \sum_{\lambda,\mu} Q_g^{|\lambda|+|\mu|} f_{\lambda,\mu} \times$$

$$= \sum_{\lambda,\mu} Q_g^{|\lambda|+|\mu|}$$

$$\times$$

$$\times$$

$Z_{\lambda,\mu}^{\text{hyper}}$

$Z_{\lambda,\mu}^{\text{vec}}$

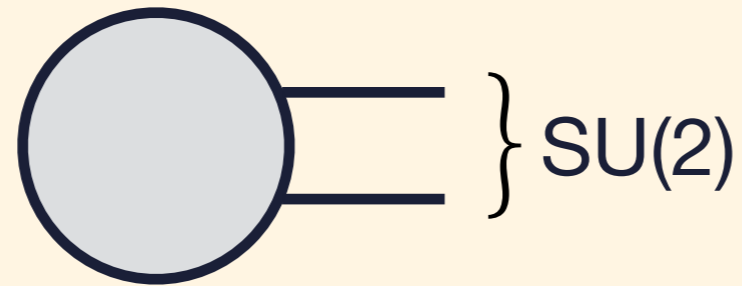
$Z_{\lambda,\mu}^{\text{hyper}}(Q_g, Q_m)$ from top. vertex

$$Z_{\lambda,\mu}^{\text{hyper}} = \frac{\text{Diagram 1}}{\text{Diagram 2}}$$

Numerator is what's called "extra factor" in the literature, Which counts strings bridging parallel external branes.

$Z_{\lambda,\mu}^{\mathcal{T}}$ from top. vertex

Assumption: \mathcal{T} is engineered by a web w/ manifest SU(2) sym.



$$Z_{\lambda,\mu}^{\mathcal{T}} = \text{Diagram 1} \Big/ \text{Diagram 2}$$

The diagram shows the definition of $Z_{\lambda,\mu}^{\mathcal{T}}$ as a ratio of two diagrams. The numerator is a light blue circle with two horizontal lines extending to the right. The top line has an arrow pointing left labeled μ , and the bottom line has an arrow pointing left labeled λ . A vertical double-headed arrow labeled Q_a is positioned to the right of the circle. The denominator is a square-like shape with a vertical double-headed arrow labeled Q_a on its left side. The top-left corner of the square is a vertex where two lines meet: one horizontal line pointing left labeled μ , and one diagonal line pointing up and to the left. The bottom-right corner is a vertex where two lines meet: one horizontal line pointing left labeled λ , and one diagonal line pointing down and to the left.

Gauging two non-Lag. theories

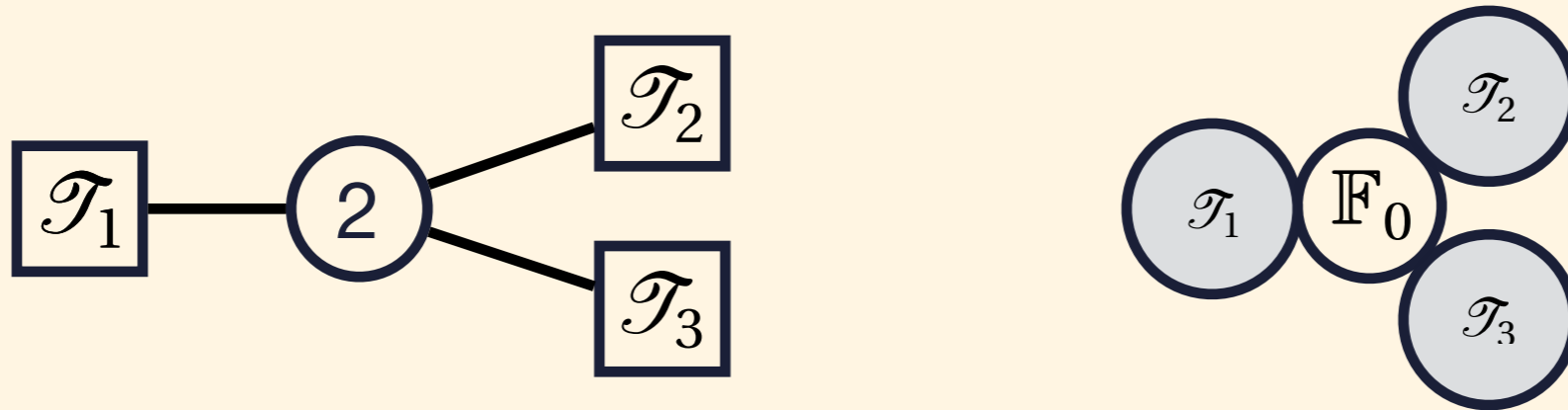


$$\begin{aligned}
 Z &= \sum_{\mu, \lambda} Q_g^{|\mu|+|\lambda|} \left(\text{web diagram } \mathcal{T}_1 \right) / Q_a \left(\text{web diagram } \square \right) \left(\text{web diagram } \mathcal{T}_2 \right) / Q_a \left(\text{web diagram } \square \right) \\
 &\quad \times f_{\lambda, \mu} \times Q_a \left(\text{web diagram } \square \right) \times Q_a \left(\text{web diagram } \square \right) \\
 &= \text{web diagram } \mathcal{T}_1 \text{ --- } \mathcal{T}_2
 \end{aligned}$$

The diagram illustrates the derivation of a web diagram for the gauged theory. It starts with the partition function Z as a sum over μ, λ of $Q_g^{|\mu|+|\lambda|}$ multiplied by two web diagrams: one for \mathcal{T}_1 and one for \mathcal{T}_2 . Each web diagram is divided by a factor Q_a and a square web diagram. The square web diagram has four external lines: two horizontal lines labeled μ and λ , and two diagonal lines. A vertical double-headed arrow labeled Q_a is placed between the horizontal lines. The product of these terms is then simplified to a single web diagram where \mathcal{T}_1 and \mathcal{T}_2 are connected by two parallel horizontal lines.

Just connecting two web diagram getting a usual web diagram.
(nothing new)

Gauging three non-Lag. theories

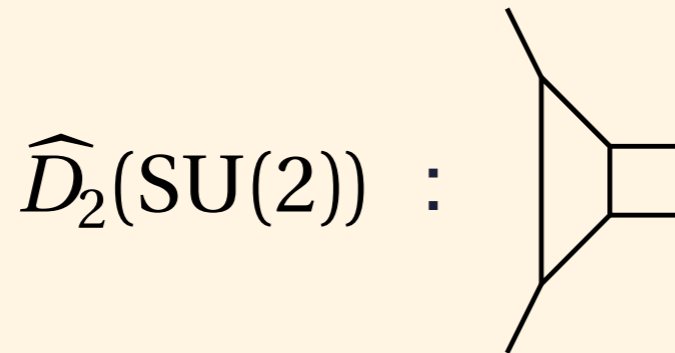
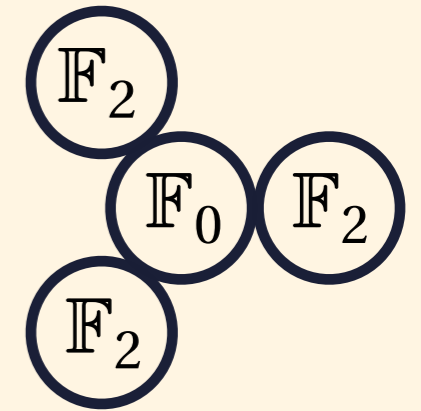
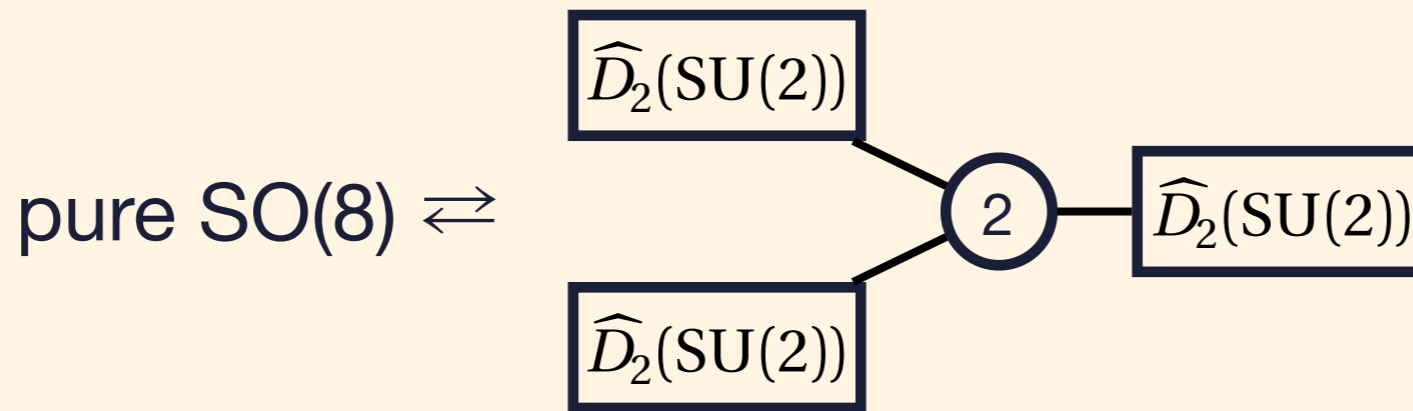


$$\begin{aligned}
 Z &= \sum_{\mu, \lambda} Q_g^{|\mu|+|\lambda|} \left(\text{circle } \mathcal{T}_1 \text{ with } \mu \text{ and } \lambda \text{ legs} \right) / \left(\text{square with } \mu \text{ and } \lambda \text{ legs and } Q_a \text{ arrow} \right) \\
 &\quad \left(\text{circle } \mathcal{T}_2 \text{ with } \mu \text{ and } \lambda \text{ legs} \right) / \left(\text{square with } \mu \text{ and } \lambda \text{ legs and } Q_a \text{ arrow} \right) \\
 &\quad \left(\text{circle } \mathcal{T}_3 \text{ with } \mu \text{ and } \lambda \text{ legs} \right) / \left(\text{square with } \mu \text{ and } \lambda \text{ legs and } Q_a \text{ arrow} \right) \\
 &\quad \times f_{\lambda, \mu} \times \left(\text{square with } \mu \text{ and } \lambda \text{ legs and } Q_a \text{ arrow} \right) \times \left(\text{square with } \mu \text{ and } \lambda \text{ legs and } Q_a \text{ arrow} \right) \\
 &= \sum_{\mu, \lambda} Q_g^{|\mu|+|\lambda|} f_{\lambda, \mu} \left(\text{circle } \mathcal{T}_1 \text{ with } \mu \text{ and } \lambda \text{ legs} \right) \left(\text{circle } \mathcal{T}_2 \text{ with } \mu \text{ and } \lambda \text{ legs} \right) \left(\text{circle } \mathcal{T}_3 \text{ with } \mu \text{ and } \lambda \text{ legs} \right) / \left(\text{square with } \mu \text{ and } \lambda \text{ legs and } Q_a \text{ arrow} \right)
 \end{aligned}$$

Trivalent glueing (=trivalent gauging)

Caveat: in general modification is needed

Checking SO(8) duality



$$Z_{\text{SO}(8)} = \sum_{\lambda, \mu} Q_g^{|\lambda|+|\mu|} f_{\lambda, \mu} \times$$

Checked explicitly at 0,1,2- instantons

and a high order of Coulomb branch parameters.

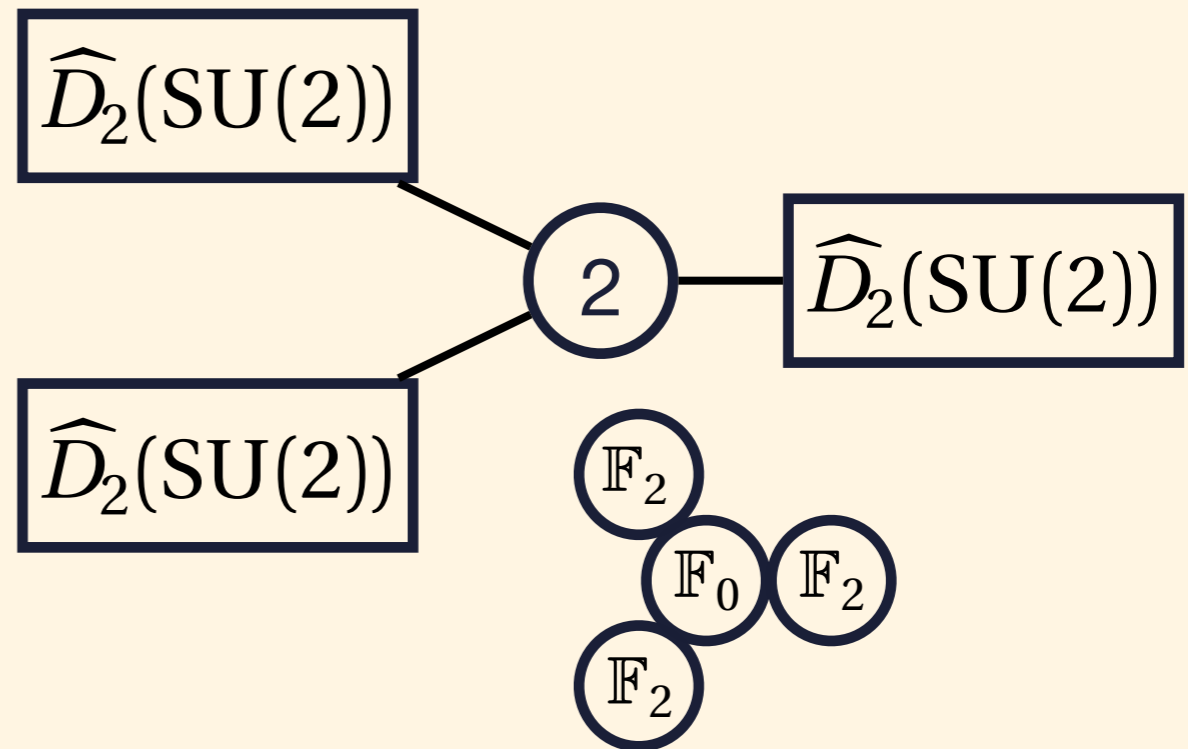
Duality for E-type theories also works w/ Hilbert series conjecture

[Keller, Mekareeya, Song, Tachikawa '12]

Generalization to 6d SCFTs

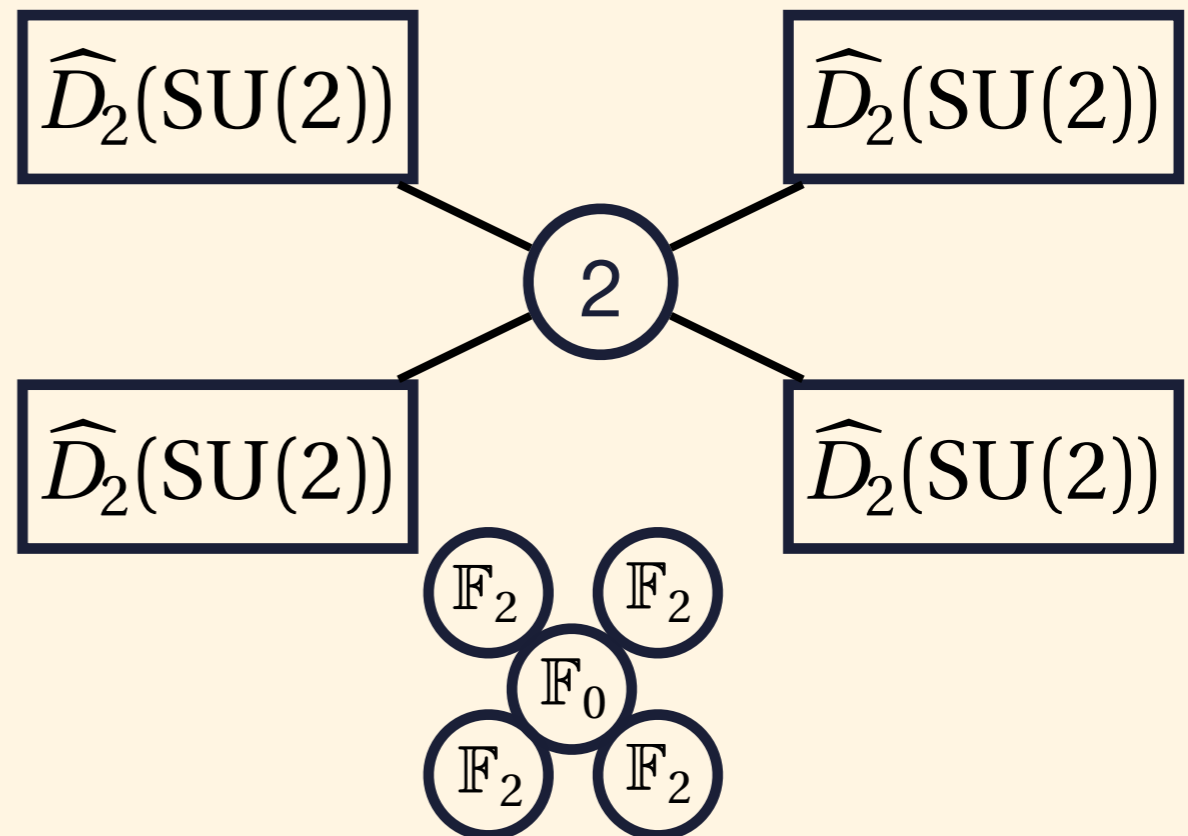
Dual description of 6d “O(-4)”

pure SO(8) gauge theory \Leftrightarrow



[DelZotto-Vafa-Xie 15']

6d “O(-4)” theory on $S^1 \Leftrightarrow$



tensor branch
↓

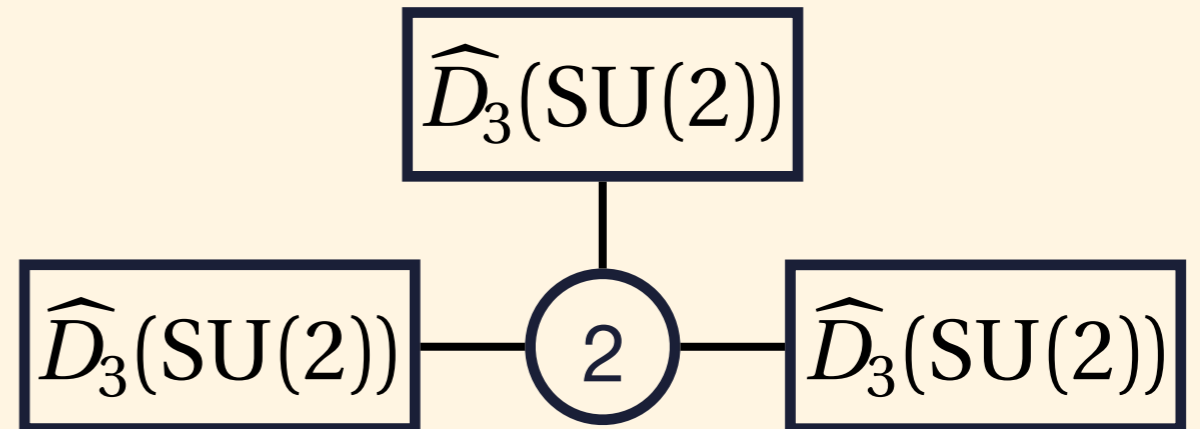
tensor+ SO(8) vector

“affine SO(8) gauge theory”

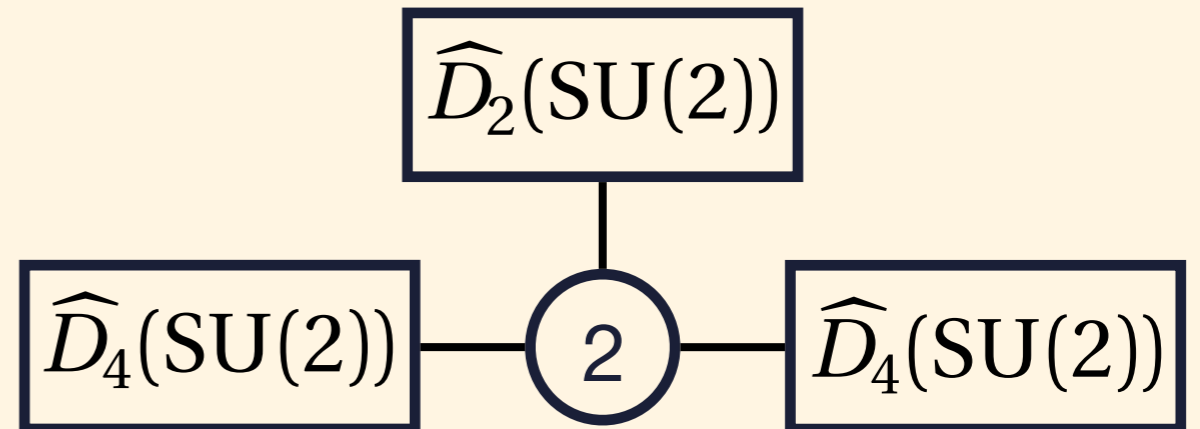
Dual description of 6d NHCs

6d SCFTs: [DelZotto-Vafa-Xie 15']

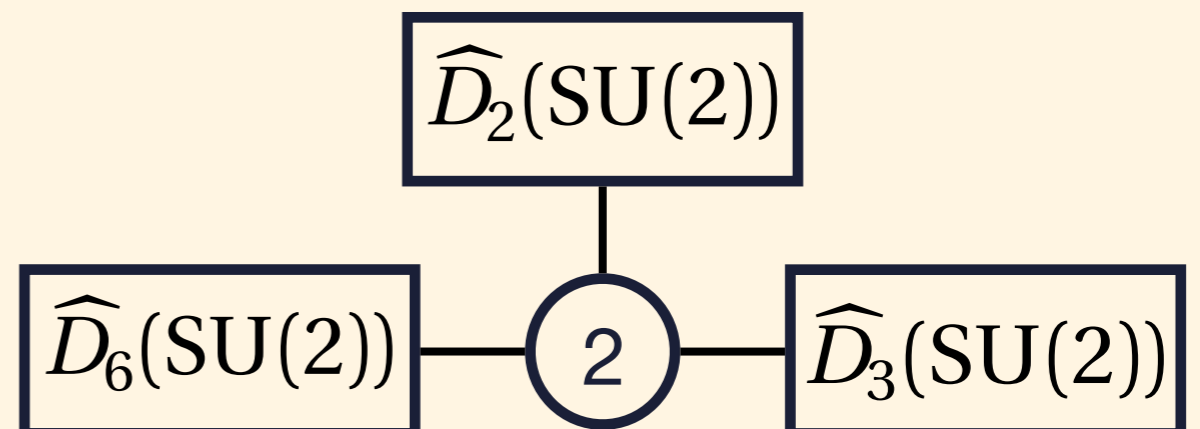
O(-6) theory \Leftrightarrow
 “ \hat{E}_6 ”



O(-8) theory \Leftrightarrow
 “ \hat{E}_7 ”



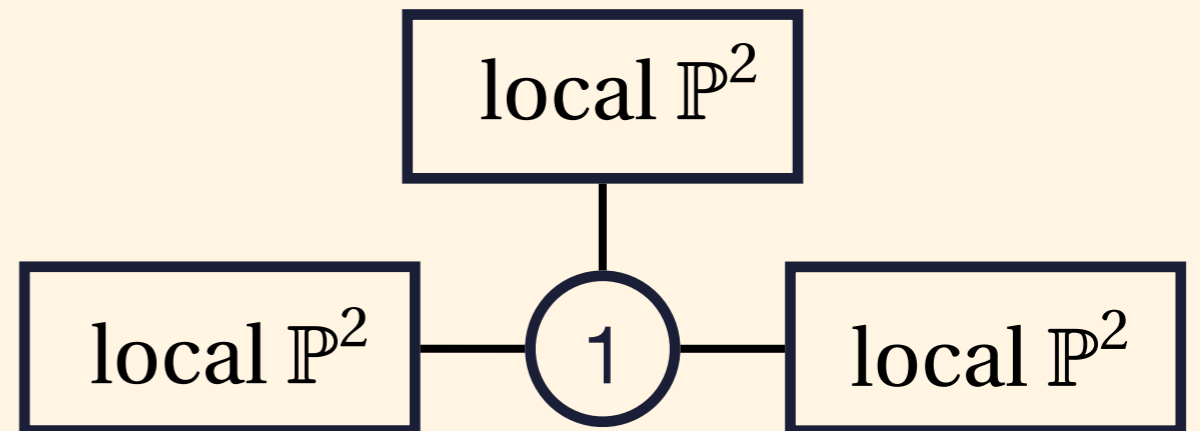
O(-12) theory \Leftrightarrow
 “ \hat{E}_8 ”



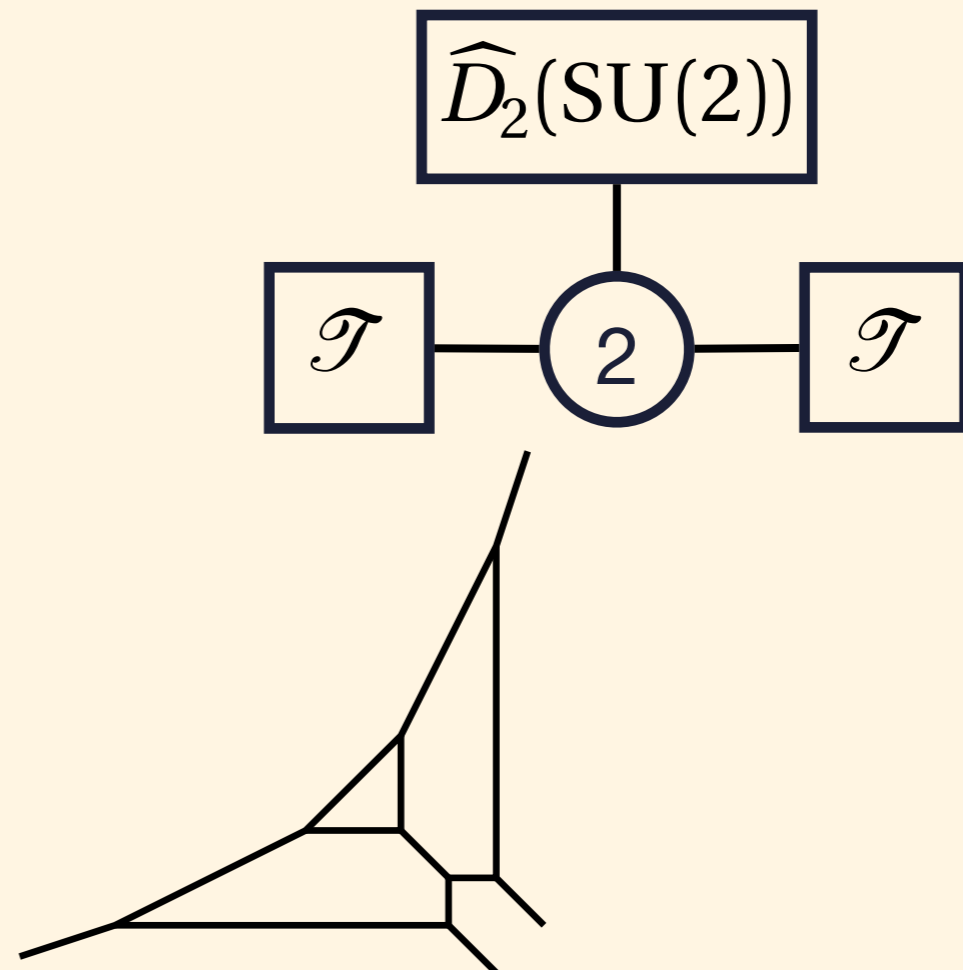
Dual description of 6d NHCs

6d SCFTs: [DelZotto-Vafa-Xie 15']

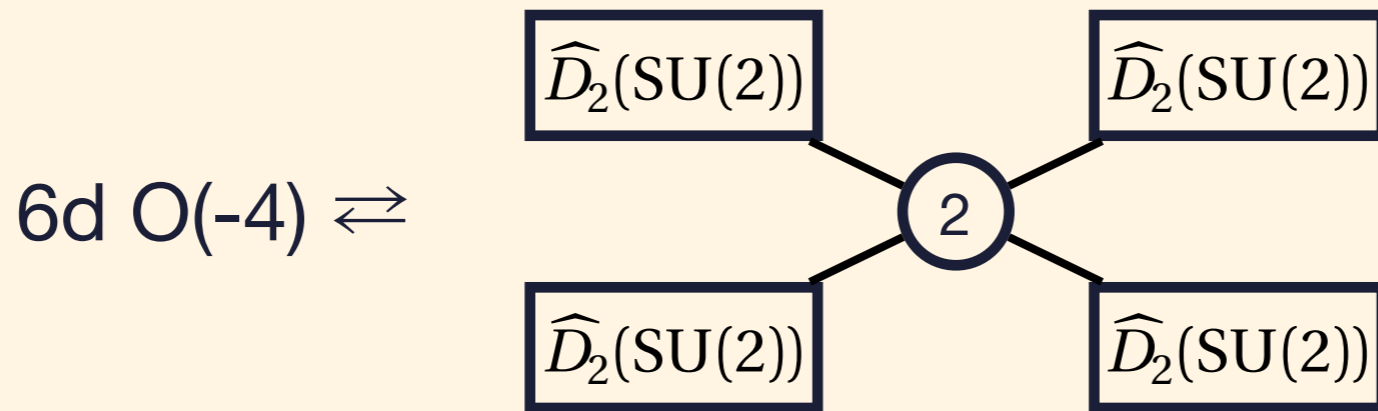
O(-3) theory \Leftrightarrow



"232 NHC" \Leftrightarrow



Checking 0(-4) duality



$$Z_{\mathcal{O}(-4)} = \sum_{\lambda, \mu} Q_g^{|\lambda|+|\mu|} f_{\lambda, \mu} \times$$

string fugacity = $Q_a \times$ dressing

Checked explicitly at 0,1 - string sectors

For other NHCs, the vertex is the first method w/ general parameters.

c.f. [Del-Zotto, Lockhart 16']

Other generalizations

in progress

w/ Hirotaka Hayashi

w/ Hirotaka Hayashi, Hee-Cheol Kim

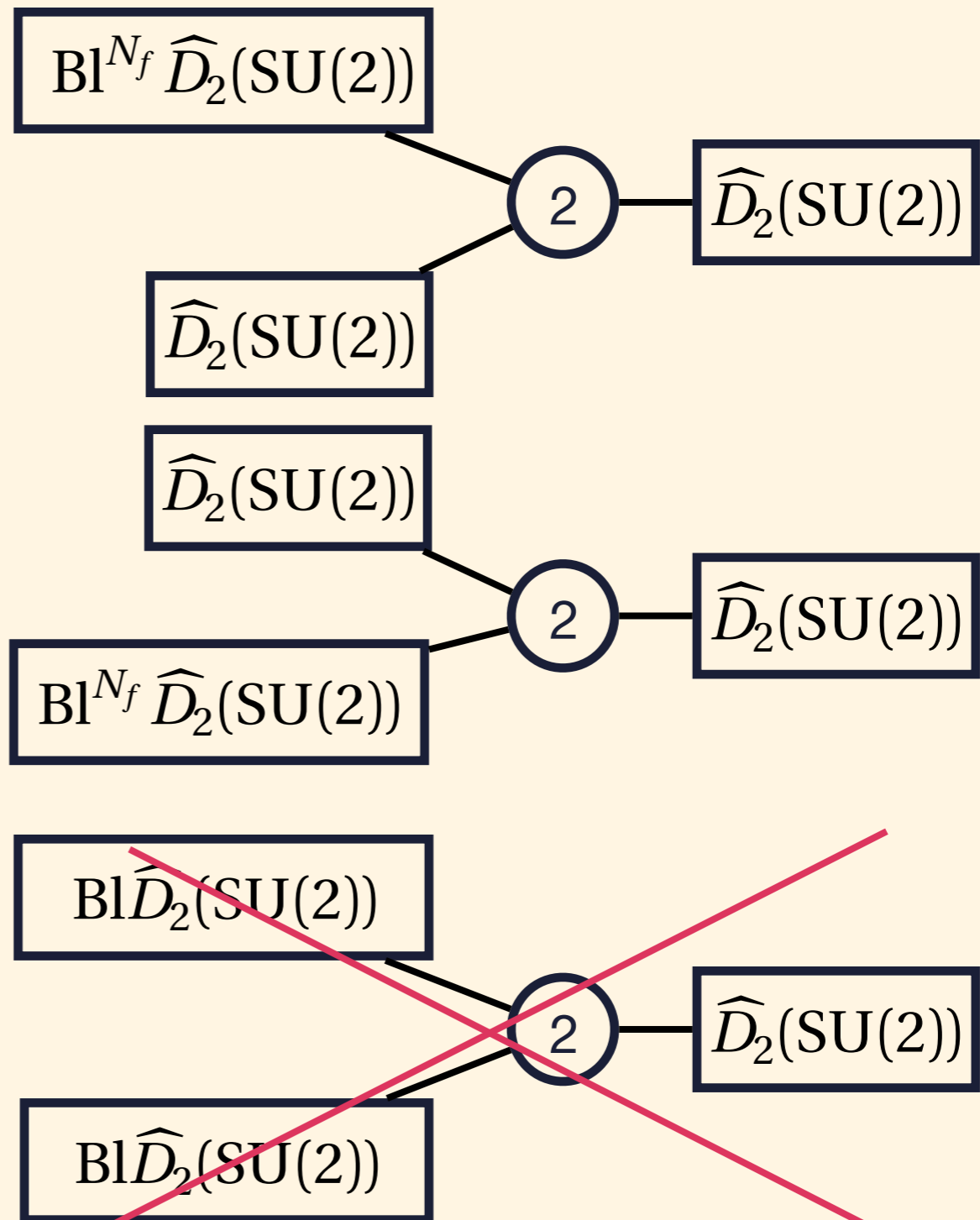
Adding matter to 5d SO(8)

SO(8) w/ N_f vectors \Leftrightarrow

$N_f=1,2,3,4,5$

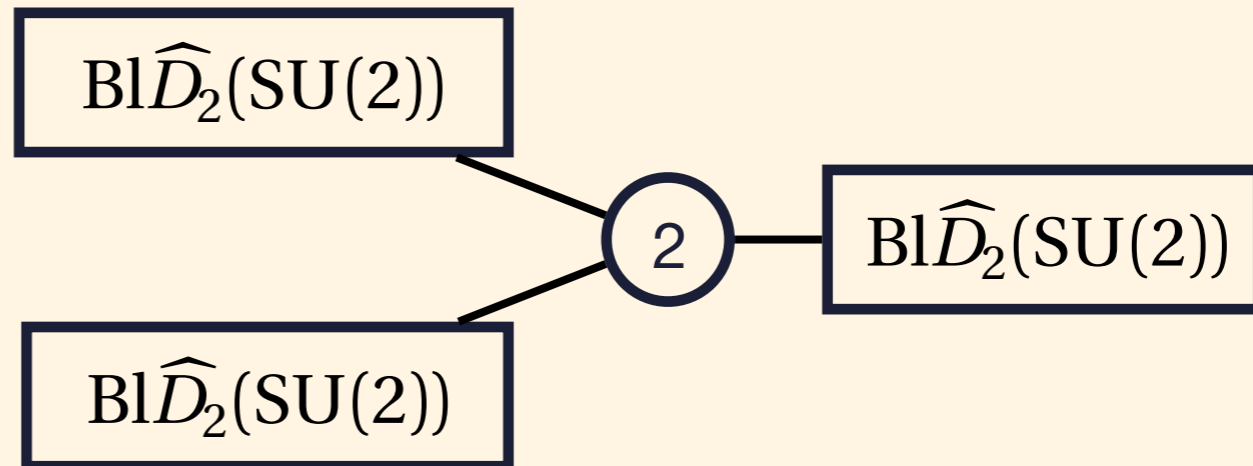
SO(8) w/ N_f spinors \Leftrightarrow

SO(8) w/ $1v+1s$ \Leftrightarrow



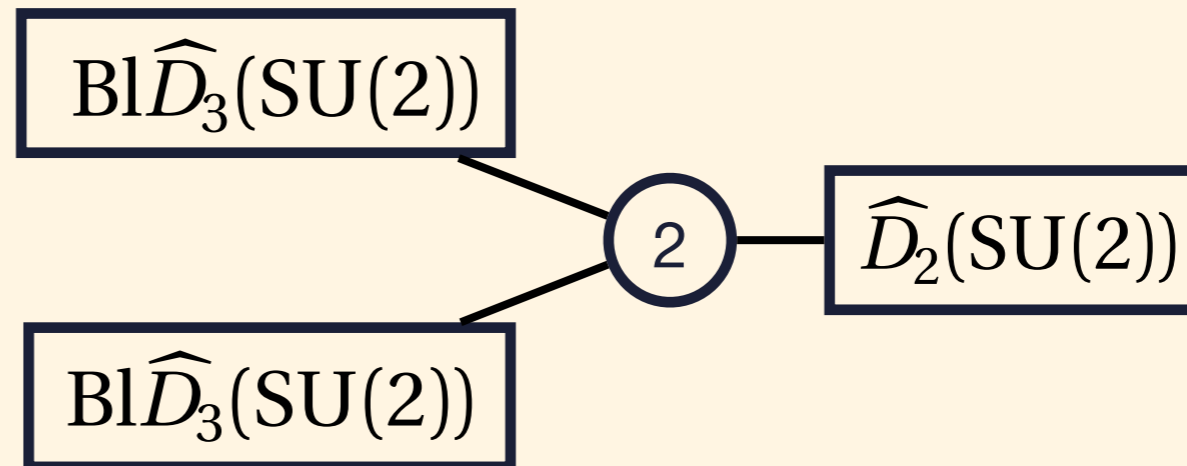
Constraint is needed

$SO(8)$ w/ $1v+1s \iff$



w/ particular constraint on parameters

E_6 w/ 27 rep \iff

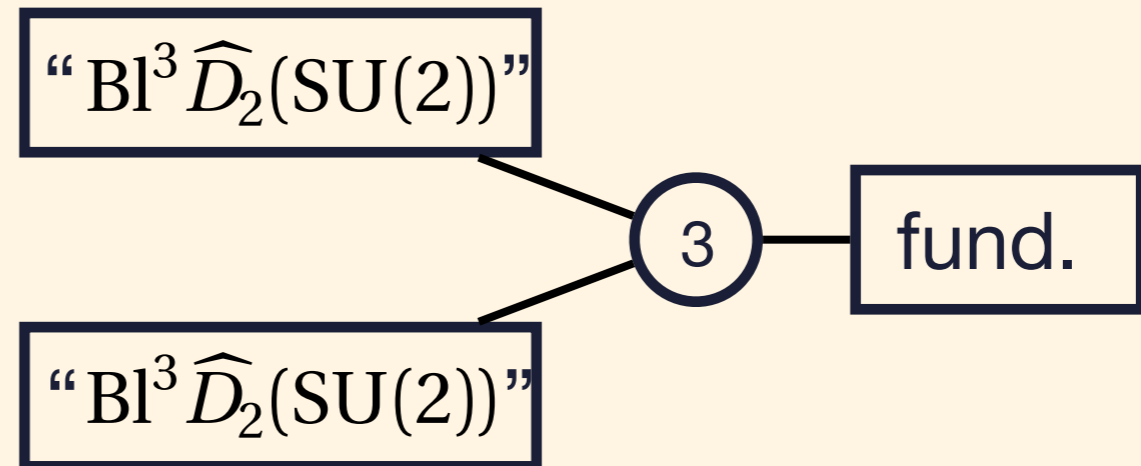


w/ particular constraint on parameters

(as far as I know) first explicit formula

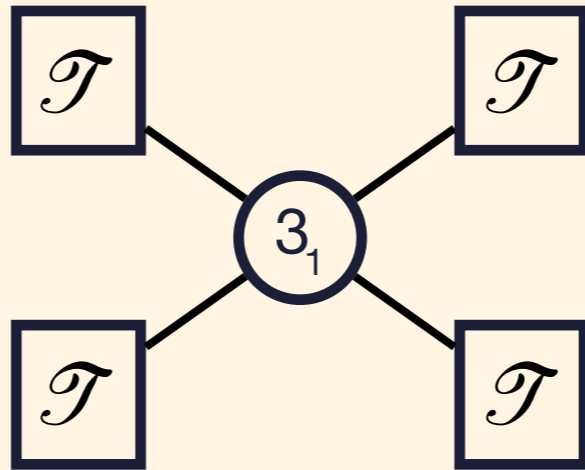
Quiver

$SO(6)-Sp(1)-[SO(4)] \rightleftharpoons$

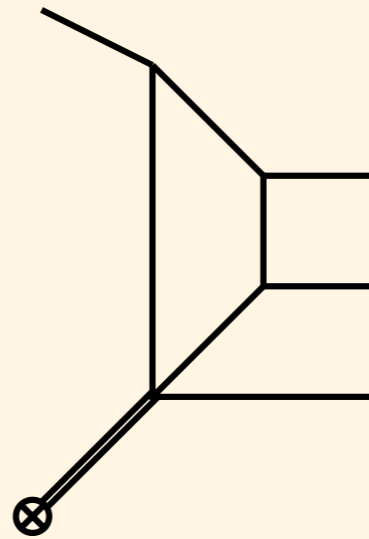


What quiver theory w/ E-type possible?

Hee-Cheol's $SU(3)$ $k=9$



\mathcal{T} :



Thank you!