Anomaly constraints on QCD phase transition

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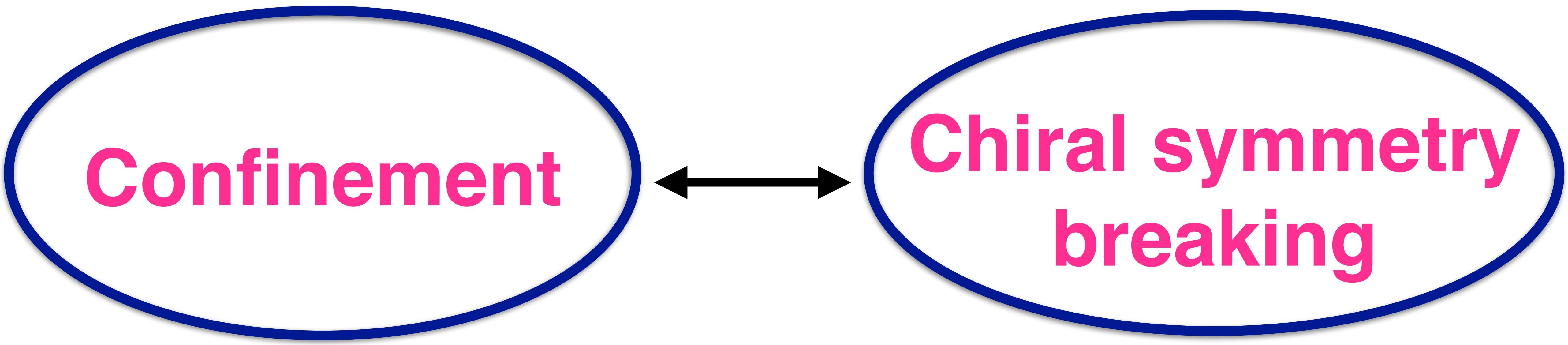
Based on

- [1706.06104] with Hiroyuki Shimizu (IPMU)

Introduction



- **The biggest mysteries in strongly coupled gauge theories such as QCD!**

- **What is the relation between them?**

Introduction

’t Hooft anomaly matching ['t Hooft,1980]

UV: gauge fields + fermions with global symmetry F

↓ confinement

IR: ???

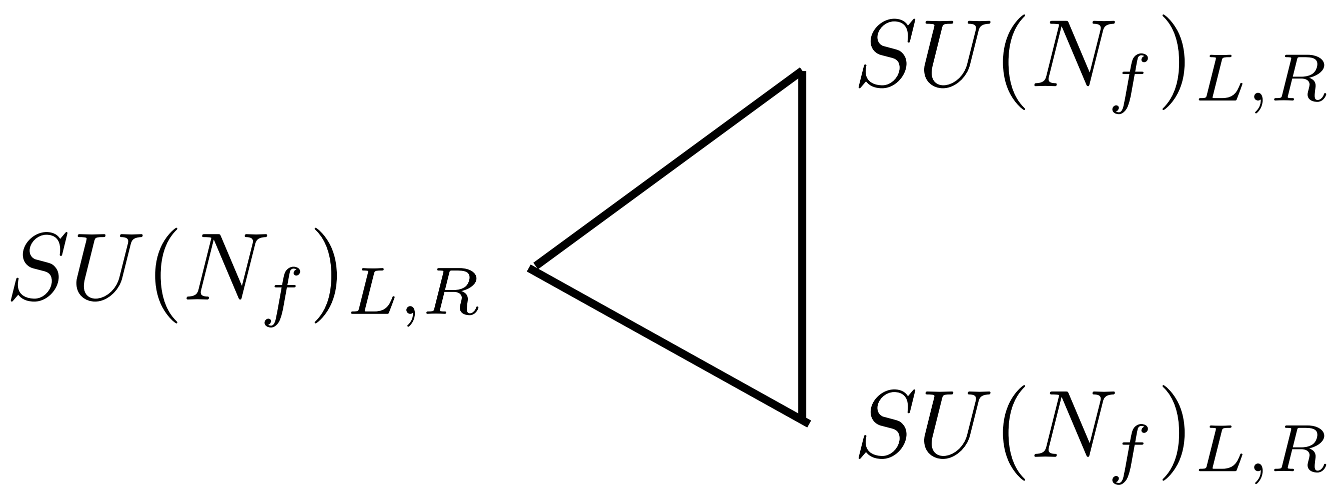
Anomaly of F in UV = Anomaly of F in IR

Introduction

’t Hooft anomaly matching in QCD

In QCD, there exist perturbative triangle anomalies.

UV:



confinement

IR:

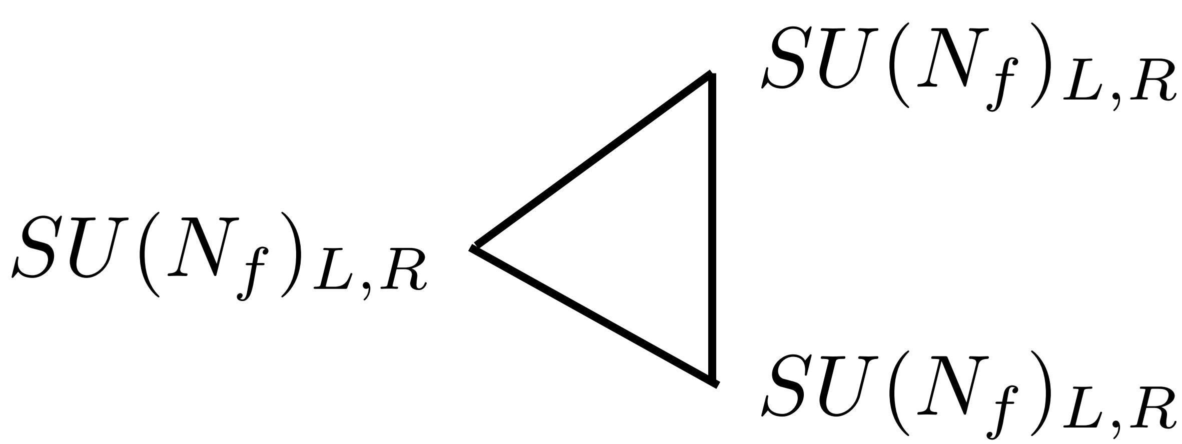
After confinement, if there is no chiral fermion, the chiral symmetry must be broken.

Introduction

How about finite temperature?

spacetime: $R^4 \to R^3 \times S^1$ S^1 : temporal direction

Unfortunately, the perturbative anomaly vanishes on R^3 :



$$\xrightarrow{S^1 \ \text{reduction}} 0$$

Main message

We show that there exists more subtle anomaly.
This anomaly survives S^1 compactification.

Implications of this subtle anomaly:

**If the theory confines at a given temperature *T*,
the chiral symmetry must be broken at that *T*.**

up to more exotic possibilities which I don't discuss in this talk.

Ref.

- Our work follows the paper on pure Yang-Mills:

  [Gaiotto-Kapustin-Komargodski-Seiberg,2017]

- For adjoint fermions, the same conclusion was obtained in

  [Komargodski-Sulejmanpasic-Unsal,2017]

- In a holographic dual of QCD, the same result in

  [Aharony-Sonnenschein-Yankielowicz,2006]

  .....

Contents

Gauge theories

I will talk about gauge theories with $SU(N_c)$ gauge group with massless fermions in either

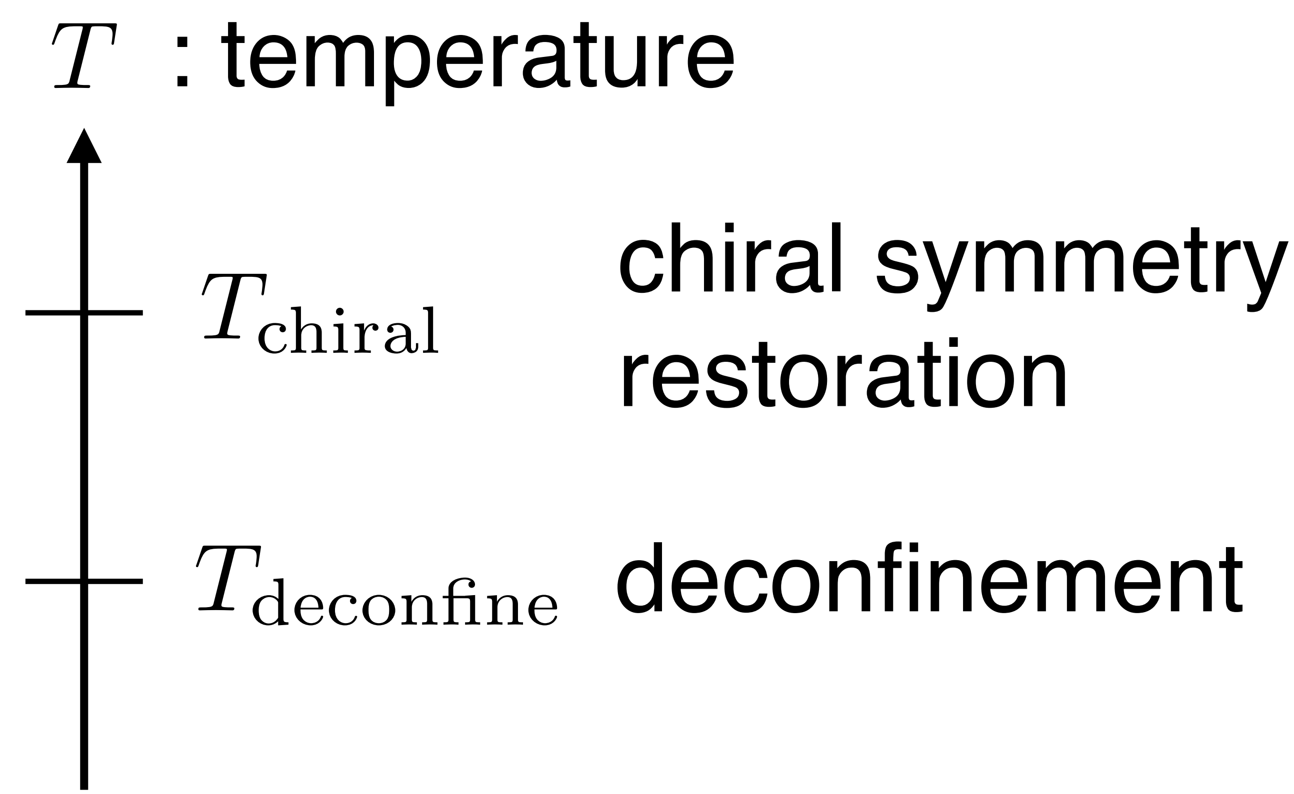
- n_f flavors of adjoint representations, or
- N_f flavors of fundamental representations with

$$\gcd(N_c, N_f) \neq 1 \qquad \text{(technical condition)}$$

$\gcd(N_c, N_f)$: greatest common divisor

I assume that flavor numbers are small enough so that the theory confines at zero temperature.

Result



The result:

$$T_{\mathrm{deconfine}} \leq T_{\mathrm{chiral}}$$

The equality is possible only for 1st order transition

Critical temperatures

What are the definitions of T_{chiral} and $T_{\mathrm{deconfine}}$?

Chiral temperature

(1) chiral phase transition T_{chiral} **.**

Axial $Z_M^{\mathrm{axial}} \subset U(1)_A$ rotation of fermions

$$Z_M^{\mathrm{axial}}: \ \psi \to e^{2\pi i/M} \psi$$

M : such that it is not broken by instantons

(Explicitly, $M = 2n_f N_c$ for adj. and $M = 2N_f$ for fund.)

T_{chiral} : temperature of Z_M^{axial} breaking/restoration

Deconfinement temperature

(2-1) deconfinement $T_{\text{deconfine}}$ **.**

Adjoint fermions:

$Z_{N_c}^{\text{center}}$ center symmetry

$$W = \mathrm{tr} P \exp(i \oint_{S^1} A_\mu dx^\mu)$$: Polyakov loop

$$Z_{N_c}^{\text{center}} : \; W \to e^{2\pi i/N_c} W$$

$T_{\text{deconfine}}$: temperature of $Z_{N_c}^{\text{center}}$ breaking/restoration

$\langle W \rangle \neq 0$: deconfinement, $\langle W \rangle = 0$: confinement

Deconfinement temperature

(2-2) deconfinement $T_{\mathrm{deconfine}}$ **.**

Fundamental fermions:

Let us introduce imaginary baryon chemical potential μ_B

$$\mu_B \simeq \mu_B + 2\pi$$ [Roberge-Weiss,1986]

- If $\mu_B = \pi$, there exists a center symmetry

$$\mathbb{Z}_2^{\mathrm{center}} \subset \mathbb{Z}_{N_c}^{\mathrm{center}} \rtimes (\mathrm{Parity\ on\ } S^1)$$

$T_{\mathrm{deconfine}}$: temperature of $\mathbb{Z}_2^{\mathrm{center}}$ breaking/restoration

Deconfinement temperature

(2-3) deconfinement $T_{\mathrm{deconfine}}$.

- For $0 \leq \mu_B < \pi$, the definition of $T_{\mathrm{deconfine}}$ is very subtle. Please see our paper.

- Intuitively, it is determined by whether the Polyakov loop is well-fluctuating or not in the path integral.

Remark on large N

Things become much simpler in the large N limit

$$N_c \to \infty$$

- The condition on $\gcd(N_c, N_f)$ is (kind of) satisfied:

$$\text{“}\gcd(\infty, N_f) = N_f \neq 1\text{”}$$

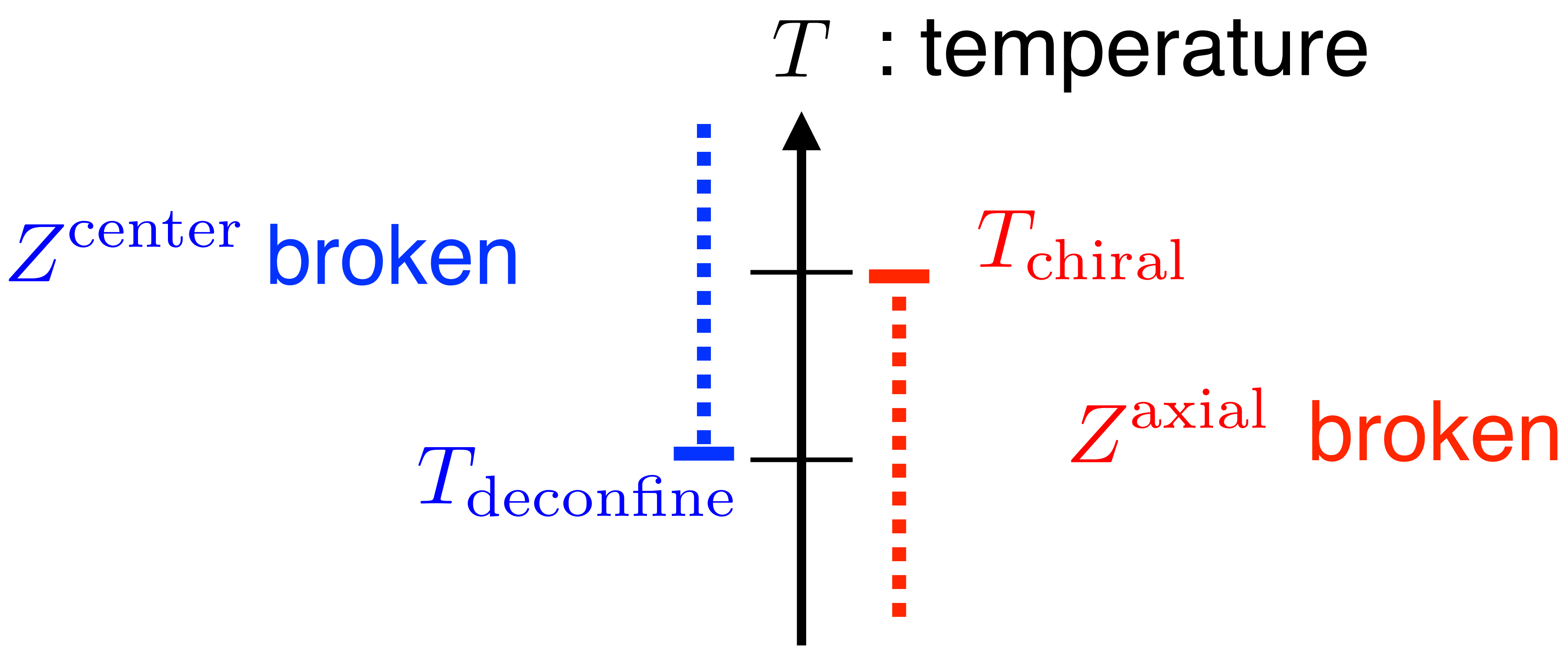
$$(\text{e.g. } N_c = K N_f, \quad K \to \infty)$$

- The effects of imaginary chemical potential vanish:

$$\frac{\text{Effects of } \mu_B}{\text{Gluon contributions}} \sim \mathcal{O}(\frac{1}{N_c^2}) \to 0$$

Deconfinement is well-defined in the large N limit without any condition.

Result again



$$T_{\text{deconfine}} \leq T_{\text{chiral}}$$

The equality is possible only for 1st order transition

What are the implications for QCD?

Implications

QCD chiral phase transition is often assumed to be described by Landau-Ginzburg (LG) effective action.

$\Phi \sim \psi\psi$: quark bilinear composite (a kind of Cooper pair)

$$\mathcal{L}_{\mathrm{LG}} = |\partial_i \Phi|^2 + (T - T_{\mathrm{chiral}})|\Phi|^2 + \cdots$$

It was successful for superconductors.
Is it really the case in gauge theories???

Implications

$$T_{\text{deconfine}} \leq T_{\text{chiral}}$$

The equality is possible only for 1st order transition

- If $T_{\text{deconfine}} < T_{\text{chiral}}$, it is counter-intuitive to use the effective action of the composite $\Phi \sim \psi\psi$.
  The theory is already deconfined around T_{chiral}

- If $T_{\text{deconfine}} = T_{\text{chiral}}$, the phase transition is 1st order.
  There is no justification for using Landau-Ginzburg.

In either case, Landau-Ginzburg is questionable, contrary to what has been believed for long time.

Contents

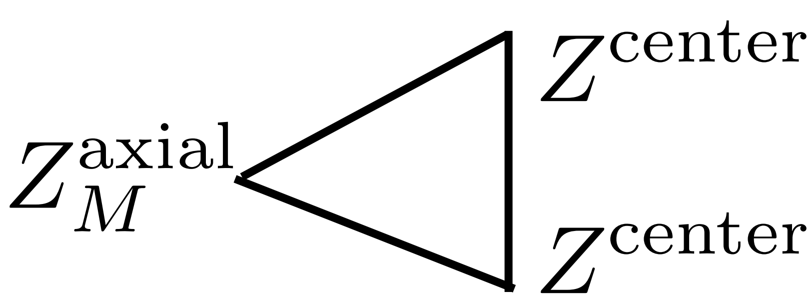
Overview

The argument will be as follows:

(1) There exists nontrivial gauge configuration with fractional instanton number by using the center of the gauge group.

(2) The axial symmetry Z_M^{axial}: $\psi \to e^{2\pi i/M}\psi$ is anomalous under the fractional instatnon.

(3) The anomaly is schematically



Center of gauge group

Let us consider $SU(N_c)$ with adjoint fermions.

Center of the gauge group:

$$Z_{N_c}^{\text{center}} \subset SU(N_c)$$

The center acts trivially on adjoint fermions.

Thus we can consider nontrivial bundles of

$$SU(N_c)/Z_{N_c}^{\text{center}}$$

SU(2) v.s. SO(3)

A simple example is given by

$$SU(2) \longrightarrow SO(3) = SU(2)/Z_2^{\text{center}}$$

An adjoint fermion is a triplet of $SO(3)$

What is the difference between $SU(2)$ and $SO(3)$ bundles?

Dirac quantization of magnetic flux will be different.

SU(2) v.s. SO(3)

$A_\mu = A_\mu^a \sigma_a$: gauge field of $su(2) = so(3)$ (Lie algebra)

For illustration, let us consider a simple embedding

$$u(1) \subset su(2) = so(3)$$

$$A_\mu = \begin{pmatrix} a_\mu & 0 \\ 0 & -a_\mu \end{pmatrix}$$

a_μ : $u(1)$ gauge field

Field strength is denoted as

$$f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$$

SU(2) v.s. SO(3)

Dirac quantization condition for $\mathbf{u(1)} \subset \mathbf{su(2)} = \mathbf{so(3)}$:

	$SU(2)$	$SO(3)$
$u(1)$ electric charge	Z	$2Z$
$u(1)$ magnetic flux	Z	$\frac{1}{2}Z$

Example:

$SU(2)$ doublet has $u(1)$ charge ± 1

$SO(3)$ triplet has $u(1)$ charge $\pm 2,\ 0$

Dirac quantization of flux

	$SU(2)$	$SO(3)$
$u(1)$ magnetic flux	Z	$\frac{1}{2}Z$

M : spacetime manifold such as T^4

S : two dimensional submanifold of M such as $T^2 \subset T^4$

Dirac quantization implies that magnetic flux are quantized:

$$\boxed{\int_S \frac{f_{\mu\nu} dx^\mu dx^\nu}{4\pi} \in \begin{cases} Z & : \ SU(2) \\ \frac{1}{2}Z & : \ SO(3) \end{cases}}$$

Fractional instanton number

We will see:
Fractional flux leads to fractional instanton numbers.

Fractional instanton number

Concrete situation

Manifold: $M = T^4 = T^2_A \times T^2_B$

Include flux:
$$\frac{1}{4\pi}\int_{T^2_A} f_{\mu\nu} dx^\mu dx^\nu = n_A$$
$$\frac{1}{4\pi}\int_{T^2_B} f_{\mu\nu} dx^\mu dx^\nu = n_B$$

Instanton number is given by

$$N_{\text{instanton}} = \int_M \frac{1}{32\pi^2} \mathrm{tr} F_{\mu\nu} F_{\rho\sigma} \epsilon^{\mu\nu\rho\sigma}$$
$$= 2 n_A n_B$$

Instanton number

$$\frac{1}{4\pi}\int_{T^2_{A,B}} f_{\mu\nu}dx^\mu dx^\nu = n_{A,B}$$

$$N_{\mathrm{instanton}} = 2n_A n_B$$

- $SU(2)$: $\quad n_{A,B} \in Z \longrightarrow N_{\mathrm{instanton}} \in 2Z$

  (Remark: it is an artifact of considering only $u(1) \subset su(2)$)

- $SO(3)$: $\quad n_{A,B} \in \frac{1}{2}Z \longrightarrow N_{\mathrm{instanton}} \in \frac{1}{2}Z$

Fractional instanton number!

Anomaly

Axial symmetry $\quad \mathbb{Z}_M^{\text{axial}} : \psi \to e^{2\pi i/M}\psi$

Path integral measure changes as

$$[\mathcal{D}\psi] \to \exp(2\pi i N_{\text{instanton}})[\mathcal{D}\psi]$$

- For integer instanton numbers, it is trivial.
- For fractional instanton numbers, it is anomalous.

$$\mathbb{Z}_M^{\text{axial}} : \ \psi \to e^{2\pi i/M}\psi$$

Broken by fractional instanton: anomaly

Magnetic flux mod 1

Fractional instanton was possible due to ['t Hooft,1979]
2-form magnetic flux modulo 1:

$$B_2 := \frac{1}{4\pi} f_{\mu\nu} dx^\mu dx^\nu \text{ flux} \mod 1$$

It is known that this flux mod 1 can be mathematically defined not only for the special case $u(1) \subset su(2) = so(3)$ but also for more general gauge field configurations.

Magnetic flux mod 1

Magnetic flux for $SU(N_c)/Z_{N_c}^{\rm center}$ bundle

$$B_2 \in H^2(M, Z_{N_c})$$

(More generally, $B_2 \in H^2(M, \pi_1(G))$ for G group)

Mathematically, it is a characteristic class of fiber bundle.
An analog of 2nd Stiefel-Whitney class.

Anomaly with magnetic flux

$$N_{\text{instanton}} = \frac{N_c}{2}\int_M B_2 \wedge B_2 \mod 1$$

See e.g. [Witten,2000]

Anomaly:

$$\exp(2\pi i N_{\text{instanton}}) = \exp(\pi i N_c \int_M B_2 \wedge B_2)$$

Center symmetry background

Subtle point:

Our gauge theory is

$SU(N_c)$ gauge theory (instead of $SU(N_c)/Z_{N_c}^{\mathrm{center}}$)

But we are considering

$SU(N_c)/Z_{N_c}^{\mathrm{center}}$ bundle with flux B_2

The 2-form flux B_2 is interpreted as a background field for 1-form center symmetry.

[Kapustin-Seiberg,2014]
[Gaiotto-Kapstin-Seiberg-Willett,2014]

1-form symmetry

[Gaiotto-Kapstin-Seiberg-Willett,2014]

	acts on …	coupled to …
0-form (ordinary) symmetry	local operator	1-form background field
1-form symmetry	line operator (e.g.Wilson line)	2-form background field

1-form center symmetry

- 1-form center symmetry in $SU(N_c)$ Yang-Mills theory

$$U = \exp(i\int A_\mu dx^\mu)$$: Wilson line operator

$$U \to e^{2\pi i/N_c} U$$

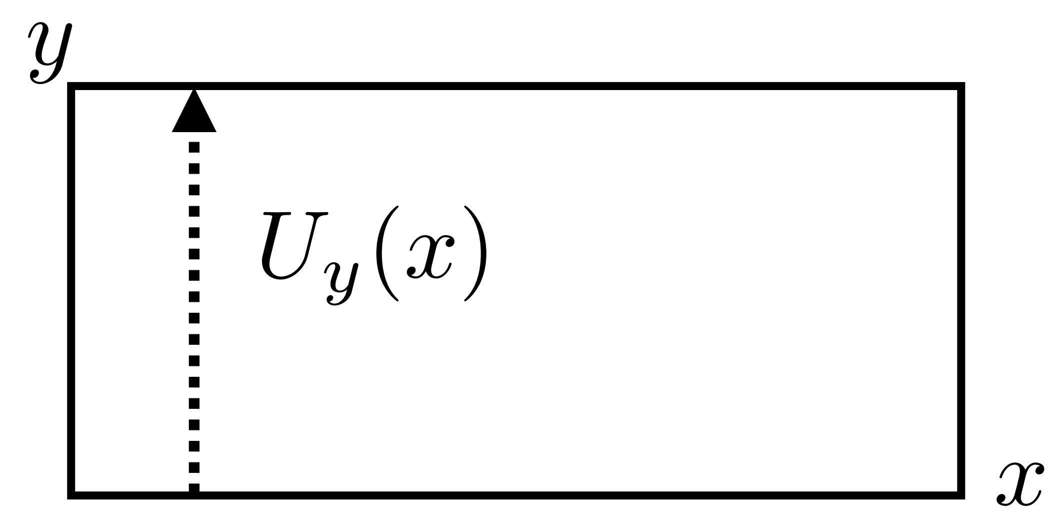
Fact:

- The 2-form background field of the 1-form center symmetry is the magnetic flux $B_2 \in H^2(M, Z_{N_c})$

2-form background

Brief explanation

Consider 2-torus T^2 :



A nontrivial background for center symmetry:

$$U_y(x+2\pi) = e^{2\pi i/N_c} U_y(x)$$

$$\downarrow$$

$$a_\mu dx^\mu \sim \frac{1}{N_c} x dy \qquad \text{(topologically)}$$

$$\downarrow$$

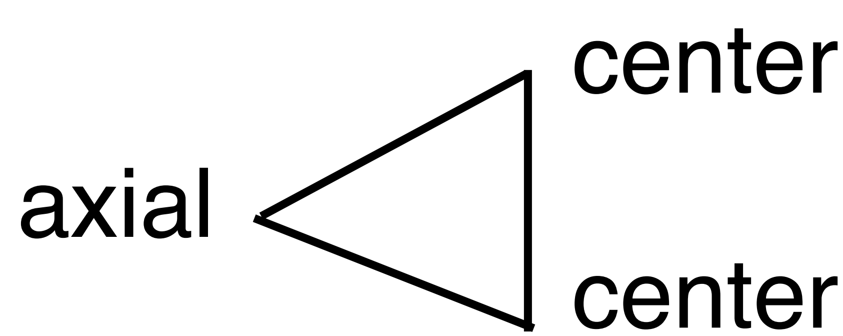
$$B_2 \sim f_{\mu\nu} dx^\mu dx^\nu \sim \frac{1}{N_c} dx \wedge dy$$: flux on the torus

Summary of anomaly

In the presence of background 2-form flux B_2 ,
the discrete axial symmetry Z_M^{axial} has anomaly

$$\exp(2\pi i N_{\mathrm{instanton}}) = \exp(\pi i N_c \int_M B_2 \wedge B_2)$$

This is a 't Hooft anomaly between axial and center symmetries



Finite temperature

Let us apply the results to finite temperature situation.

We need to compactify the theory on a circle
for finite temperature. Does the anomaly survive?

Finite temperature

Under dimensional reduction

$$S^1 \times N \to N \qquad N: \text{3-manifold}$$

$$\text{1-form center symm.} \longrightarrow \begin{cases} \text{1-form center symm.} \\ \oplus \\ \text{0-form center symm.} \end{cases}$$

The 2-form flux splits as

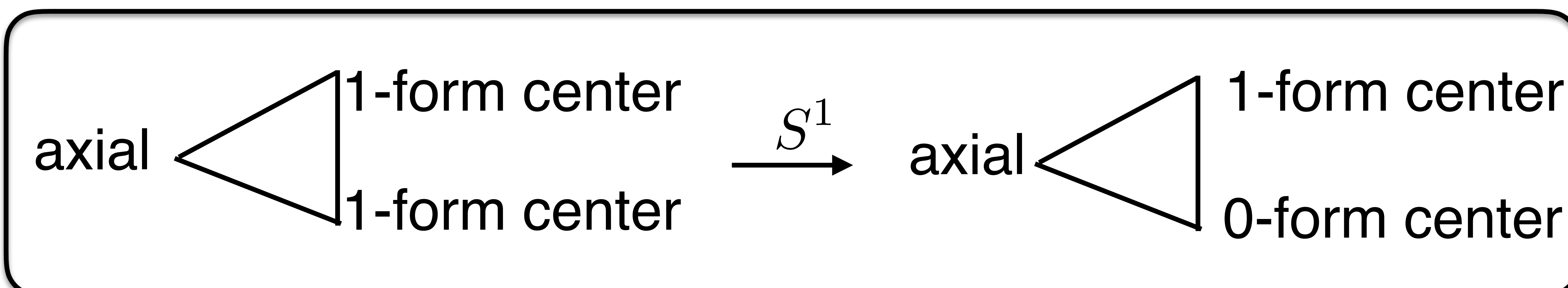
$$B_2 \to B_2' \oplus B_1'$$

Finite temperature

Anomaly:

$$\exp(\pi i N_c \int_{S^1 \times N} B_2 \wedge B_2) \to \exp(2\pi i N_c \int_N B_2' \wedge B_1')$$

This is still nonzero. The anomaly survives.



Consequence of anomaly

The existence of ’t Hooft anomaly between center and axial symmetry implies either

1. One of the symmetries is spontaneously broken

or

2. There is some degrees of freedom which match the anomaly

More detailed investigation suggests that the second possibility is rather exotic. See

[Gaiotto-Kapustin-Komargodski-Seiberg,2017]
[Shimizu-KY]

Consequence of anomaly

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Assume this

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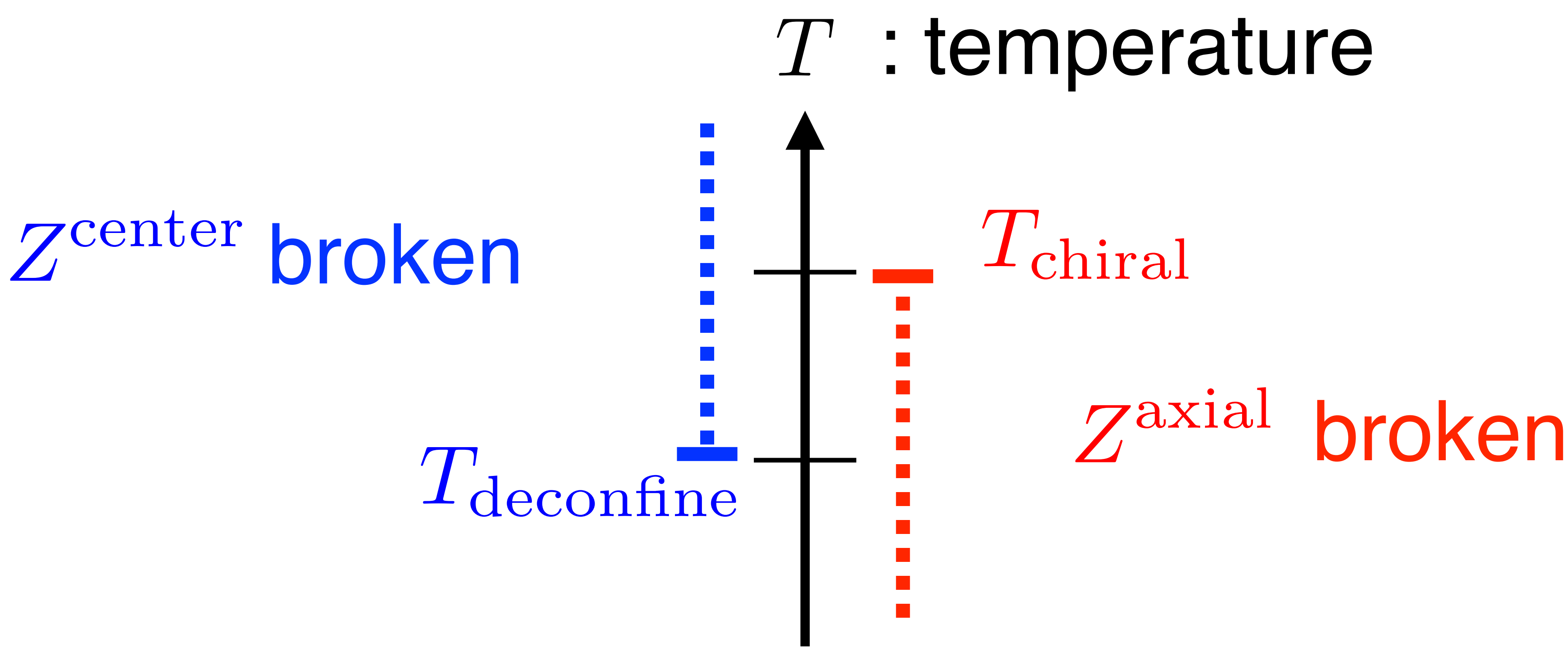
Consequence of anomaly

At least one of the symmetries must be broken
at any temperature!



$$T_{\mathrm{deconfine}} \leq T_{\mathrm{chiral}}$$

The equality is possible only for 1st order transition

Contents

Center-flavor mixing

Let us consider $SU(N_c)$ with fundamental fermions.

Center of the gauge group:

$$Z_{N_c}^{\mathrm{center}} \subset SU(N_c)$$

The center acts nontrivially on fundamental fermions.

It is not possible to couple the fermions to the bundle

$$SU(N_c)/Z_{N_c}^{\mathrm{center}}$$

because Dirac quantization condition is violated.

Center-flavor mixing

Instead, we consider

$$Z_n \subset SU(N_c) \times SU(N_f)$$
$$n = \mathrm{gcd}(N_c, N_f)$$

This Z_n acts trivially on fermions.
Therefore, we can consider a bundle for

$$[SU(N_c) \times SU(N_f)]/Z_n$$

Center-flavor mixing

By using

$$[SU(N_c)\times SU(N_f)] \to [SU(N_c)\times SU(N_f)]/Z_n$$

Conceptually: similar to the case of adjoint fermions
Technically: much more involved

$\boxed{\text{Fractional instanton number by nontrivial flux} \\ \to \text{Anomaly of } Z_M^{\mathrm{axial}}:\ \psi \to e^{2\pi i/M}\psi}$

Fractional instanton number by nontrivial flux
→ Anomaly of Z_M^{axial}: $\psi \to e^{2\pi i/M}\psi$

For technical details, please see our paper.

Contents

Summary

- There exists a subtle 't Hooft anomaly between center symmetry and axial symmetry.

- The subtle anomaly implies (up to exotic possibilities)

$$T_{\mathrm{deconfine}} \le T_{\mathrm{chiral}}$$

The equality is possible only for 1st order transition

- Landau-Ginzburg description of QCD phase transition is questionable: more work is necessary!