5d SCFTs and Shrinkable CY3

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Based on

arXiv : 1705.05836 with Patrick Jefferson, Cumrun Vafa, Gabi Zarir
arXiv : 1709.xxxxx with Patrick Jefferson, Sheldon Katz, Cumrun Vafa

Plan of talk

- Review of 5d gauge theories and Coulomb branch physics.
- Full classification of 5d gauge theories with simple gauge groups.
- Construction of Shrinkable Calabi-Yau threefolds.
- Rank 2 Shrinkable Calabi-Yau threefolds.

5d supersymmetric gauge theories

Five-dimensional $\mathcal{N}=1$ theories with gauge group G

- Preserve 8 supercharges.
- Matter content
  - Vector multiplet $(A_\mu, \phi; \lambda)$
  - Hypermultiplet $(q^A; \psi)$ $(A = 1, 2 \ : \ SU(2)_R \text{ doublet})$
- $SO(1,4)$ Lorentz symmetry + $SU(2)_R$ R-symmetry.
- $\mathcal{N}=2$ theory : vector multiplet + adjoint hypermultiplet

5d gauge theories are non-renormalizable. But certain class of SUSY theories admit non-trivial UV CFT fixed points. [Seiberg 96], [Morrison, Sieberg 96], [Intriligator, Morrison, Seiberg 97]

Interesting 5d gauge theories can be constructed by

1. (p,q) five-brane web in Type IIB [Aharony, Hanany 97], [Aharony, Hanany, Kol 97], [DeWolfe, Iqbal, Hanany, Katz 99], …
2. M-theory on Calabi-Yau 3-fold [Morrison, Seiberg 96], [Douglas, Katz, Vafa 96], [Katz, Klemm, Vafa 96], [Intriligator, Morrison, Seiberg 97], …

Effective Prepotential

Coulomb branch is parametrized by the real scalar ϕ_i in vector multiplet.

- With generic ϕ_i , gauge group will be broken to $U(1)^r$. $\quad r$: rank of G
- Low energy abelian theory is characterized by prepotential $\mathcal{F}(\phi_i)$.

Prepotential is at most cubic polynomial in ϕ_i and it is 1-loop exact :

$$\mathcal{F} = \frac{1}{2g_0^2} h_{ij}\phi^i\phi^j + \frac{\kappa}{6} d_{ijk}\phi^i\phi^j\phi_k + \frac{1}{12}\left(\sum_{e\in\mathrm{root}} |e\cdot\phi|^3 - \sum_f \sum_{w\in\mathbf{r}_f} |w\cdot\phi + m_f|^3 \right)$$

g_0 : gauge coupling, m_f : masses, $h_{ij} = \mathrm{Tr}(T_iT_j)$, $d_{ijk} = \mathrm{Tr}T_{(i}T_jT_{k)}$, κ : CS − level for $SU(N>2)$

[Witten 96], [Seiberg 96], [Intriligator, Morrison, Seiberg 97]

- Effective coupling : $\tau_{ij} = \partial_i\partial_j\mathcal{F}$
- Metric on Coulomb branch : $ds^2 = \tau_{ij}d\phi^i d\phi^j$
- Tension of magnetic monopole string : $T_{M_i} \sim \phi_{Di} \equiv \partial_i\mathcal{F}$

Intriligator-Morrison-Seiberg (IMS) classification

5d gauge theories with non-trivial fixed points were classified using the condition that metric on Coulomb branch is non-negative <u>everywhere</u>.

- $\text{eigen}(\tau_{ij}(\phi)) > 0$ [Seiberg 96], [Intriligator, Morrison, Seiberg 97]
- At strong coupling limit $g_0^2 \to \infty$, we expect interacting CFT fixed point.
- This condition provides a classification of non-trivial 5d QFTs.

Intriligator-Morrison-Seiberg (IMS) bounds :

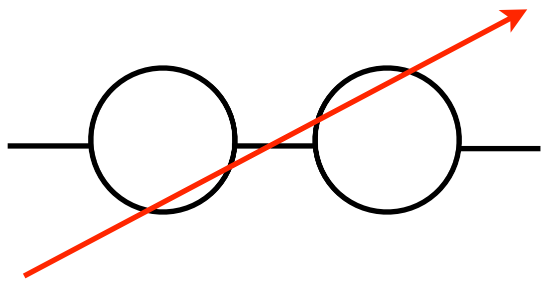
Ex)

$SU(N)$ gauge group : $n_A = 0,\ n_f + 2|\kappa| \leq 2N$
$n_A = 1,\ n_f \leq 8 - N - 2|\kappa|$

$Sp(N)$ gauge group : $n_A = 0,\ n_f \leq 2N + 4$
$n_A = 1,\ n_f \leq 8$

n_A : # of antisymmetric hypers
n_f : # of fund. hypers

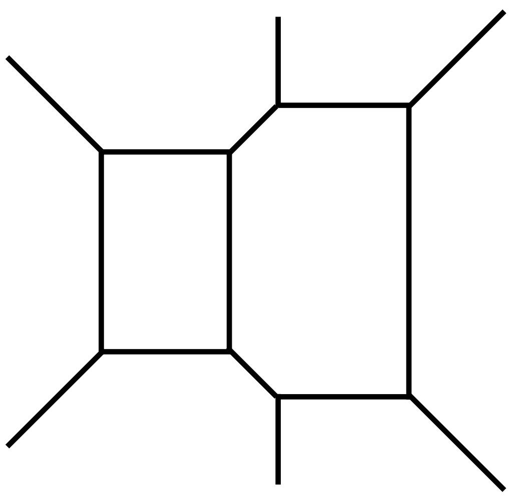
<u>This condition rules out quiver-type gauge theories.</u>



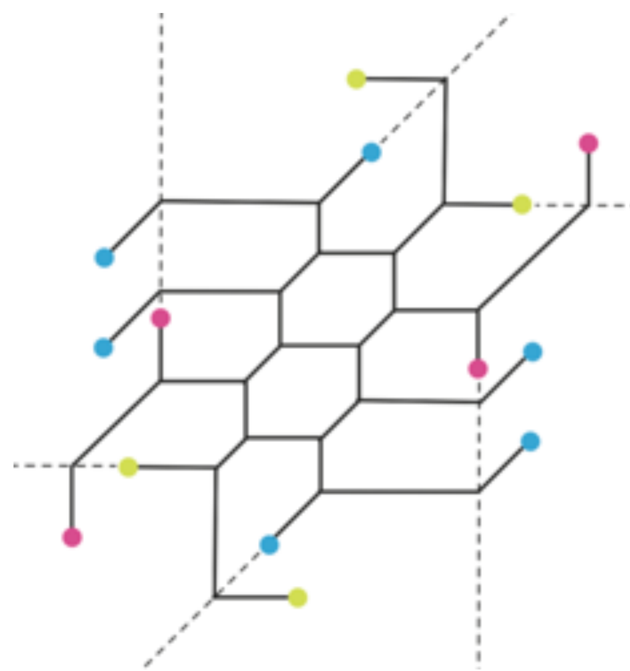
Beyond IMS classifications

However, string theory and CY3-compactification predict huge class of non-trivial gauge theories beyond IMS bounds, in particular quiver gauge theories.

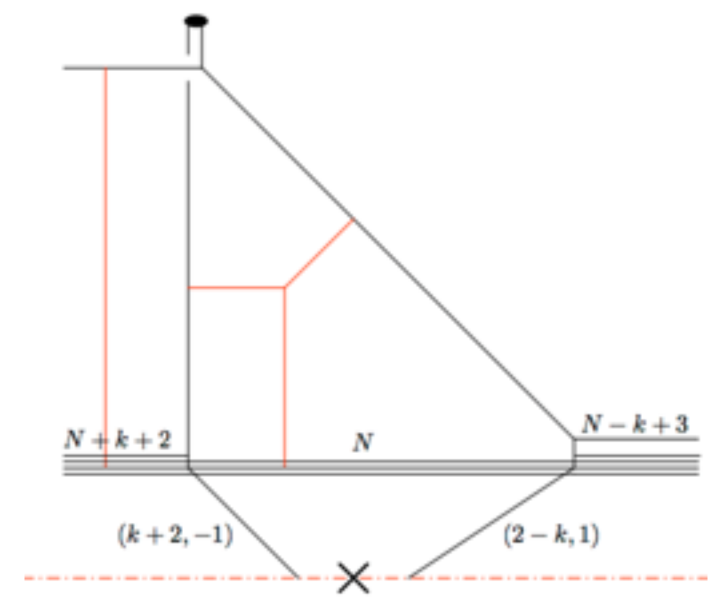
Aharony, Hanay, Kol, DeWolfe, Vafa, Katz, Mayr, Leung, Iqbal, Bergman, Rodrguez-Gomez, Zafrir, Tachikawa, Yonekura, Hayashi, S-S. Kim, K. Lee, Taki, Yagi, Gaiotto, H-C. Kim, ….



$SU(2) \times SU(2)$



$SU(3)\ w/\ N_f = 10$



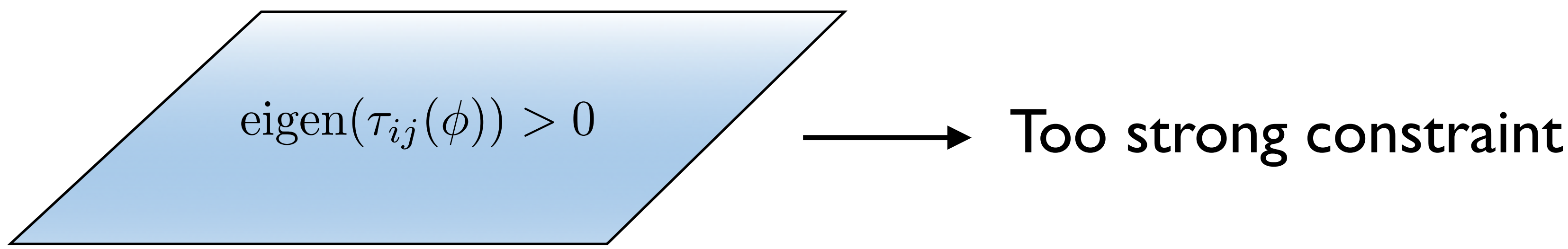
$Sp(N)\ w/\ N_f = 2N + 5$

[Yonekura 15], [Hayashi, Kim, Lee, Taki, Yagi 15], [Zafrir 15]

This implies that IMS bounds are too strong.

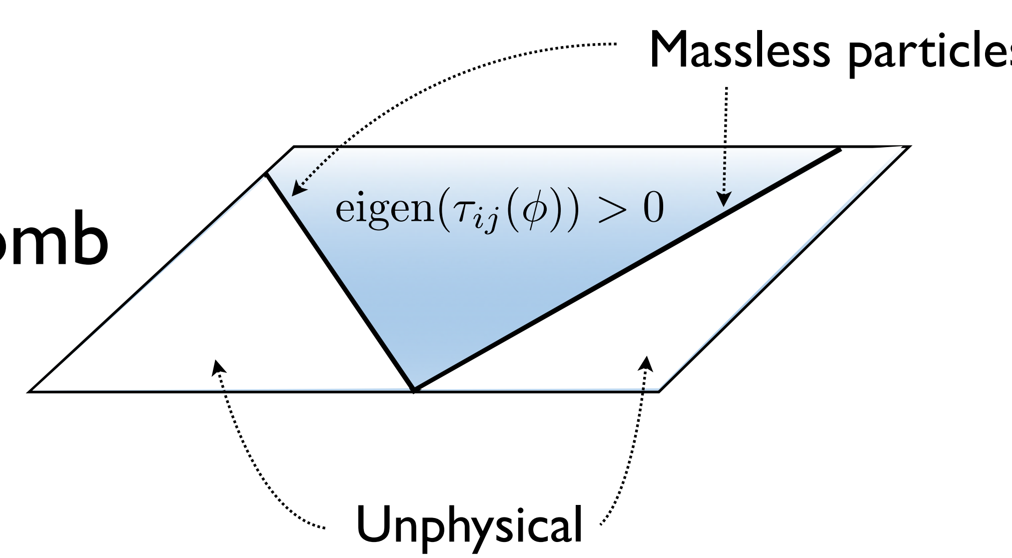
Effective Coulomb branch

The condition that metric should be non-negative in entire Coulomb branch turns out to be too strong.



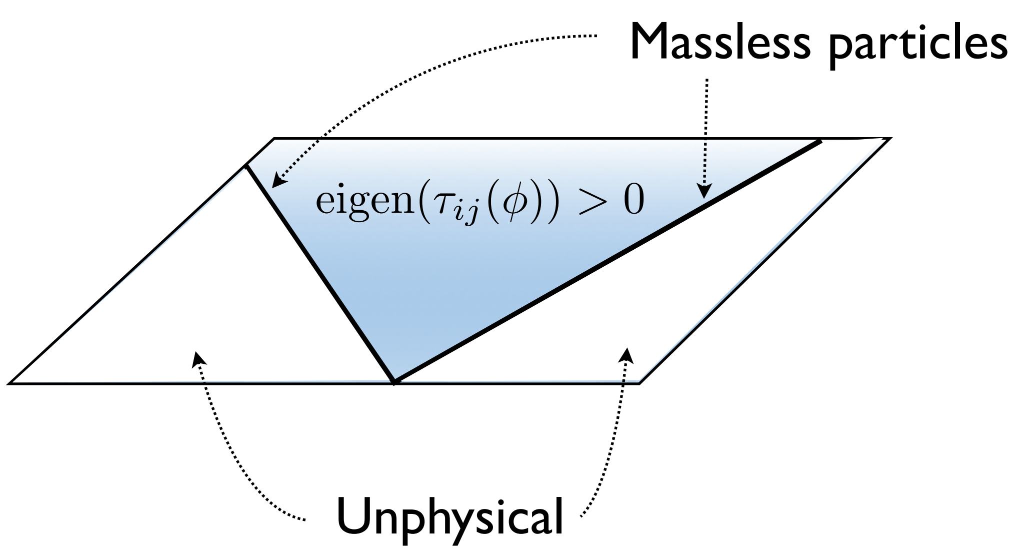
There could be non-perturbative massless particles (instantons) or tensionless strings at some point in Coulomb branch. Effective gauge theory description breaks down beyond the point. In this case, physical Coulomb branch is a smaller sub-region.

Therefore we should impose the condition only within the physical sub-region in Coulomb branch.



New criteria on physical Coulomb branch

We propose that non-trivial 5d gauge theories must have non-negative metric on the physical Coulomb branch.



Our Conjecture :

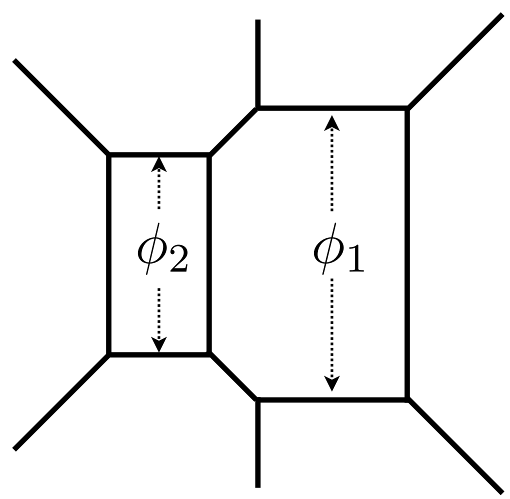
Non-trivial 5d gauge theories have positive metric on a subset of the Coulomb branch $\mathcal{C}_{\rm phys} \subseteq \mathcal{C}$ at infinite classical coupling. Namely,

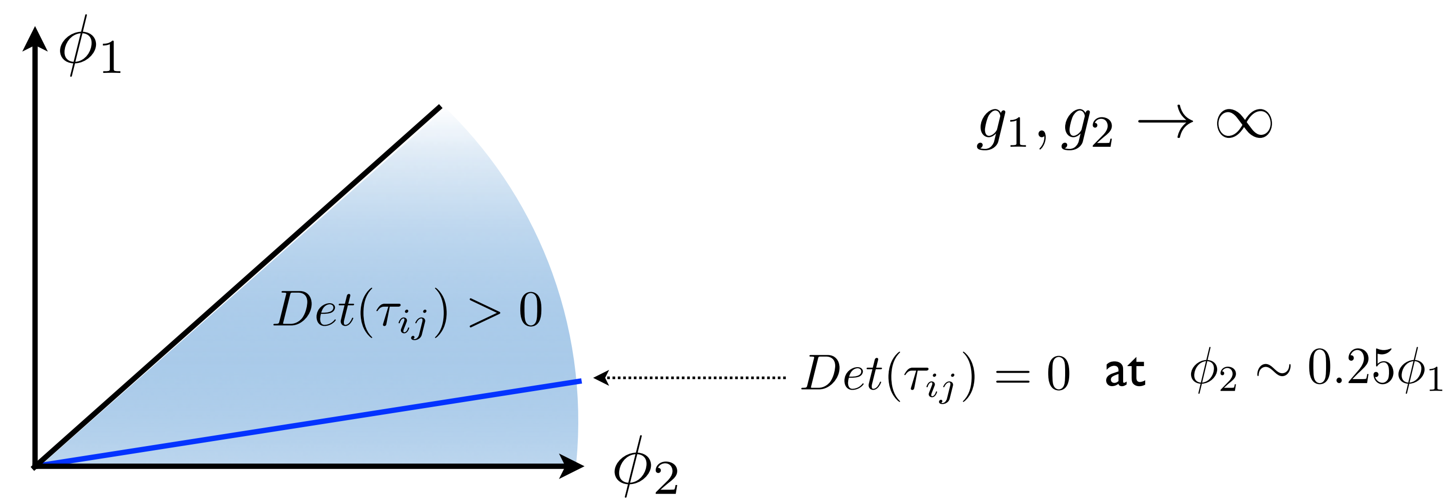
$$\tau_{\rm eff}(\phi) > 0 \ , \quad \phi \in \mathcal{C}_{\rm phys}$$

where $\mathcal{C}_{\rm phys} = \{\phi \in \mathcal{C}, |T(\phi) > 0, m(\phi) > 0\}$, when $1/g^2 \to 0$.

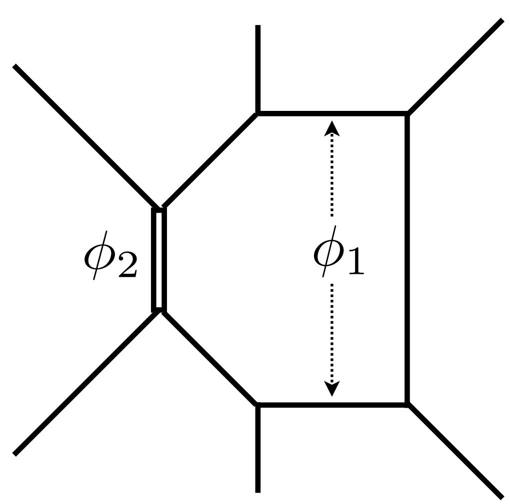
Example

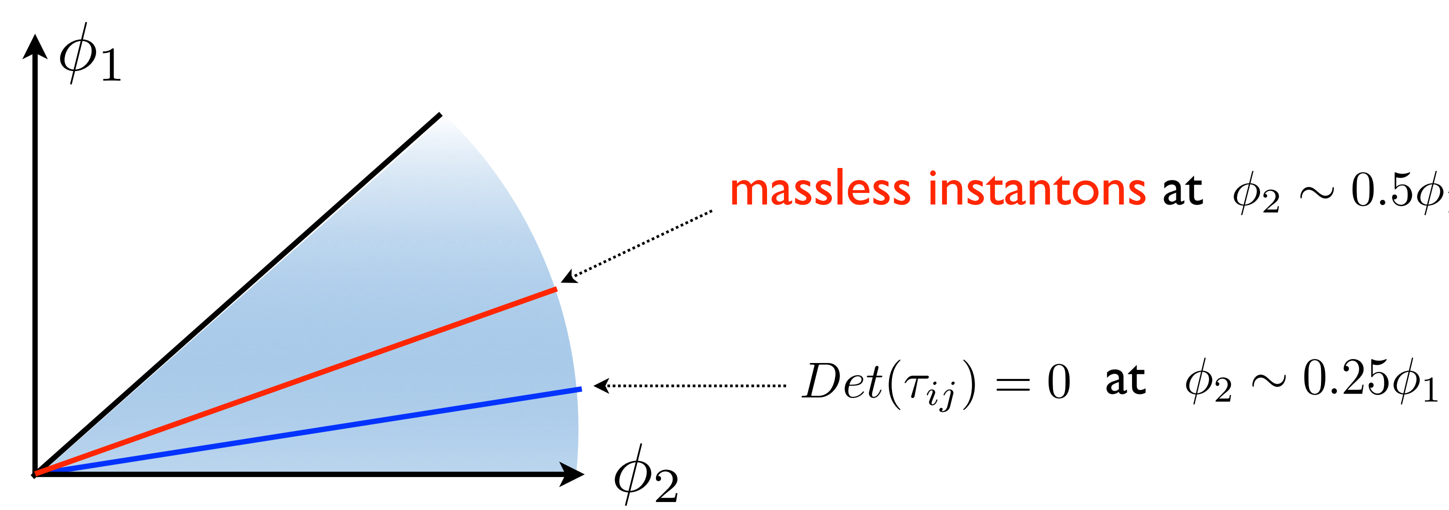
$SU(2) \times SU(2)$ gauge theory has two-dimensional Coulomb branch parametrized by ϕ_1, ϕ_2 with $\phi_1 > \phi_2 > 0$.





As we decrease ϕ_2 , we meet massless instantons.





Thus, this is a good theory with positive metric in a chamber with physical states.

Our Conjecture :

Non-trivial 5d gauge theories have positive metric on a subset of the Coulomb branch $\mathcal{C}_{\rm phys} \subseteq \mathcal{C}$ at infinite classical coupling. Namely,

$$\tau_{\rm eff}(\phi) > 0 \ , \quad \phi \in \mathcal{C}_{\rm phys}$$

where $\mathcal{C}_{\rm phys} = \{\phi \in \mathcal{C}, |T(\phi) > 0, m(\phi) > 0\}$, when $1/g^2 \to 0$.

However, <u>we need to know all spectrum of physical states</u> including instantons!

Classification :

We will instead use the following criteria for classification:

1. All $T(\phi) > 0$ <u>somewhere</u> in $\mathcal{C}$.
2. Prepotential $\mathcal{F} > 0$ <u>everywhere</u> in $\mathcal{C}$.

Note that the condition 1. is necessary condition and the condition 2. is new conjecture (based on convergence of sphere partition function).

Full Classification of rank 2 simple gauge theory

$SU(3)$

N_s	N_f	κ	F
1	0	$\frac{3}{2}$	$SU(2)\times U(1)$
1	1	0	$A_1^{(1)}\times U(1)$
0	10	0	$D_{10}^{(1)}$
0	9	$\frac{3}{2}$	$E_8^{(1)}\times SU(2)$
0	6	4	$SO(12)\times U(1)$
0	3	$\frac{13}{2}$	$U(3)\times U(1)$
0	0	9	$U(1)$

$Sp(2)$

N_a	N_f	F
3	0	$A_5^{(2)}$
2	4	$A_{11}^{(2)}$
1	8	$E_8^{(1)}$
0	10	$D_{10}^{(1)}$

G_2

N_f	F
6	$A_{11}^{(2)}$

κ : CS level, F : Global symmetry

N_s : # of symmetric hypers, N_a : # of antisymmetric hypers, N_f : # of fundamental hypers,

$A_n^{(1)}, B_n^{(1)}, C_n^{(1)}, D_n^{(1)}, E_n^{(1)}$: affine gauge group

- These theories are <u>"marginal" theories</u> which may uplift 6d theories in UV.
- All their descendants (by integrating out heavy matters) have 5d CFT fixed points.
- Global symmetry is from 1-instanton analysis, so it could be different from the full fixed point symmetry.

Full Classification of all simple gauge groups

We can fully classify non-trivial 5d gauge theories with simple gauge groups (or single gauge nodes).

We found standard theories which exist for arbitrary rank gauge groups and finite number of exceptional theories which exist only at lower rank $r_G \leq 8$.

Standard theories :

N_s	N_a	N_f	$\|\kappa\|$	F
1	1	0	0	$A_1^{(1)} \times U(1)$
1	0	$N-2$	0	$A_{N-2}^{(1)} \times U(1)$
1	0	0	$\frac{N}{2}$	$SU(2) \times U(1)$
0	2	8	0	Even N : $E_7^{(1)} \times A_1^{(1)} \times A_1^{(1)} \times SU(2)$ Odd N : $D_8^{(1)} \times A_1^{(1)} \times SU(2)$
0	2	7	$\frac{3}{2}$	Even N : $D_8^{(1)} \times SU(2)$ Odd N : $E_7^{(1)} \times SU(2) \times SU(2)$
0	1	$N+6$	0	$A_{N+6}^{(1)} \times U(1)$
0	1	8	$\frac{N}{2}$	$SO(16) \times U(1)^2$
0	0	$2N+4$	0	$D_{2N+4}^{(1)}$

Marginal $SU(N)_\kappa$ theories with N_s symmetric, N_a anti-symmetric, and N_f fundamental hypermultiplets.

N_a	N_f	F
1	8	$E_8^{(1)} \times SU(2)$
0	$2N+6$	$D_{2N+6}^{(1)}$

Marginal $Sp(N)$ theories with N_a anti-symmetric and N_f fundamental hypermultiplets

N_f	F
$N-2$	$A^{(2)}{}_{2N-5}$

Marginal $SO(N>4)$ theories with N_f fundamental hypermultiplets

Standard theories

N_s	N_a	N_f	$\|\kappa\|$	F
1	1	0	0	$A_1^{(1)} \times U(1)$
1	0	$N-2$	0	$A_{N-2}^{(1)} \times U(1)$
1	0	0	$\frac{N}{2}$	$SU(2) \times U(1)$
0	2	8	0	Even N : $E_7^{(1)} \times A_1^{(1)} \times A_1^{(1)} \times SU(2)$ Odd N : $D_8^{(1)} \times A_1^{(1)} \times SU(2)$
0	2	7	$\frac{3}{2}$	Even N : $D_8^{(1)} \times SU(2)$ Odd N : $E_7^{(1)} \times SU(2) \times SU(2)$
0	1	$N+6$	0	$A_{N+6}^{(1)} \times U(1)$
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Marginal $SU(N)_\kappa$ theories with N_s symmetric, N_a anti-symmetric, and N_f fundamental hypermultiplets.

N_a	N_f	F
1	8	$E_8^{(1)} \times SU(2)$
0	$2N+6$	$D_{2N+6}^{(1)}$

Marginal $Sp(N)$ theories with N_a anti-symmetric and N_f fundamental hypermultiplets

N_f	F
$N-2$	$A^{(2)_{2N-5}}$

Marginal $SO(N>4)$ theories with N_f fundamental hypermultiplets

All of these marginal theories are 6d SCFTs. For example,

- $SU(N)_0 + (2N+4)F$, $Sp(N-1) + (2N+4)F$ theories are 6d (D_{N+4}, D_{N+4}) conformal matter theories.
- $SU(N)_0 + 1AS + (N+6)F$ theories are 6d $SU(N-1) + 1AS + (N+7)F$.
- $SU(N)_{\frac{N}{2}} + 1AS + 8F$, $Sp(N-1) + 1AS + 8F$ theories are 6d rank-N E-string theories.
- Their descendants are 5d SCFTs.

Exceptional theories at low rank < 9

Full classification of simple gauge groups involves finite number of exceptional theories only at low rank $r_G \leq 8$ which are not in the list of standard theories.

Rank-4 theories

N_a	N_f	$\|\kappa\|$	F
3	3	0	$C_3^{(1)} \times SU(3)$
3	2	$\frac{3}{2}$	$E_6^{(2)} \times U(1)$
3	1	3	$SU(4) \times SU(2) \times U(1)$
0	5	$\frac{11}{2}$	$SO(10) \times U(1)$

(a) Marginal $SU(5)_\kappa$ gauge theories with N_a anti-symmetric, N_f fundamental matters.

N_{Λ^3}	N_f	F
$\frac{1}{2}$	4	$SO(8) \times U(1)$

(b) Marginal $Sp(4)$ gauge theories with N_{Λ^3} rank-3 anti-symmetric, N_f fundamental matters.

N_f	F
3	$Sp(4) \times SU(2)$

(c) Exceptional $F4$ gauge theories with N_f fundamental matters.

N_s	N_c	N_f	F
5	2	0	$A_{14}^{(2)}$
5	1	1	$A_{13}^{(2)}$
4	4	0	$E_6^{(2)} \times E_6^{(2)}$
4	3	1	$F_4^{(1)} \times Sp(4)$
4	2	2	$A_7^{(2)} \times D_7^{(2)}$
3	3	2	$C_6^{(1)} \times Sp(2)$

(d) Marginal $SO(8)$ gauge theories with N_s spinor, N_c conjugate spinor, N_f fundamental matters.

N_s	N_f	F
4	1	$E_6^{(2)} \times SU(2)$
3	3	$C_6^{(1)} \times Sp(3)$
2	5	$A_9^{(2)} \times A_4^{(2)}$
1	6	$A_{14}^{(2)}$

(e) Marginal $SO(9)$ gauge theories with N_s spinor, N_f fundamental matters.

Geometric construction of 5d CFTs

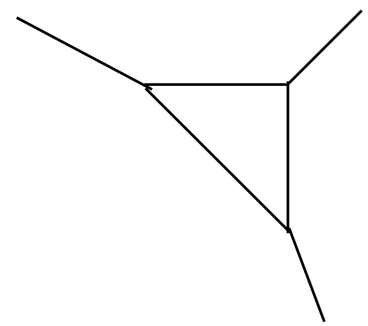
M-theory on Calabi-Yau threefold

11d M-theory compactified on a ‘contractible’ Calabi-Yau threefold X_6 will engineer a 5d SCFT.

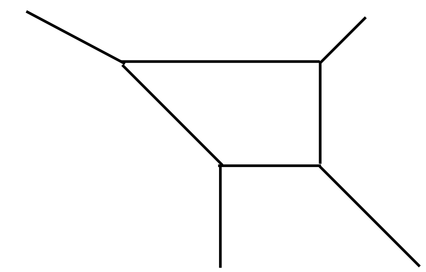
[Morrison, Sieberg 96], [Douglas, Katz, Rafa 96], [Intriligator, Morrison, Seiberg 97]

- Contractible CY3 : Surfaces $S_i \subset X_6$ can contract to a singular point.

Ex : del Pezzo surfaces dP_n in CY3



$dP_0 = \mathbb{P}^2$



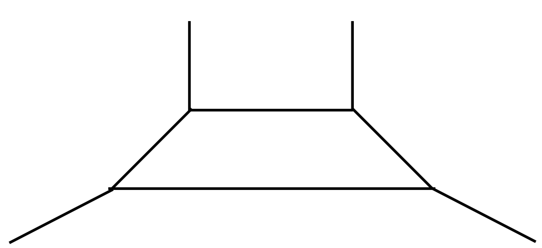
$dP_1 \to SU(2)_{\theta=\pi}$

- In fact, ‘contractible CY3’ can be generalized to ‘shrinkable CY3’.

- Shrinkable CY3 : $S_i \subset X_6$ can contract to a point or non-compact 2-cycles.

[Jefferson, Katz, H-C Kim, Vafa 2017]

Ex :

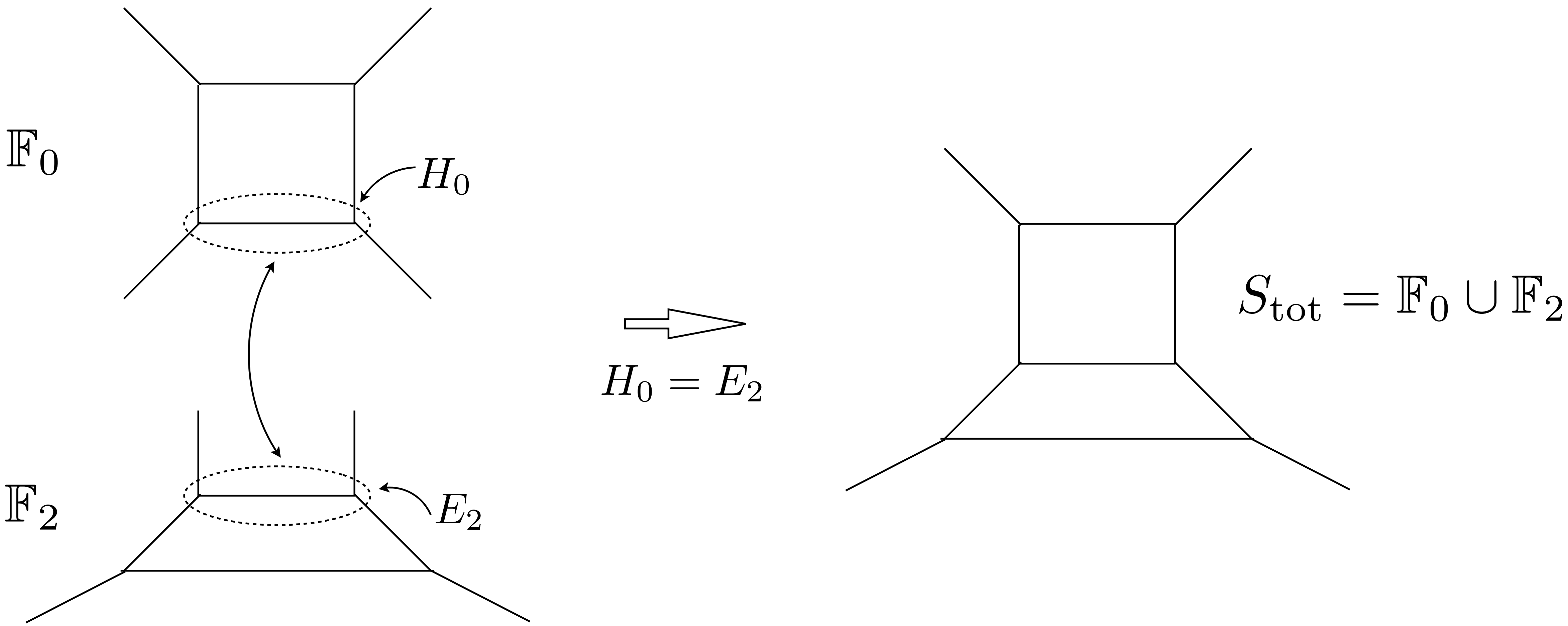


$\mathbb{F}_2 \to SU(2)_{\theta=0}$

Gluing surfaces in Calabi-Yau threefold

Generic shrinkable CY3 can be constructed by gluing rank 1 surfaces.

- Gluing two surfaces $\mathbb{F}_0$ and $\mathbb{F}_2$ yields a rank 2 surface $S_{\text{tot}} = \mathbb{F}_0 \cup \mathbb{F}_2$.



- We glue a curve class H_0 in $\mathbb{F}_0$ and another curve class E_2 in $\mathbb{F}_2$.
- Final geometry $S_{\text{tot}} = \mathbb{F}_0 \cup \mathbb{F}_2$ is a smooth CY3 corresponding to $SU(3)$ gauge theory with CS-level $\kappa = 1$.

Construction algorithm of 'Shrinkable CY3's

All shrinkable CY3 are constructed by a gluing $S_{\rm tot} = \cup_i S_i$.

1. Building blocks S_i

   a. Hirzebruch surfaces and their blowups $Bl_p(\mathbb{F}_n)$.

   b. del Pezzo surfaces dP_n.

2. Two surfaces S_i and S_j are glued along a curve $C_g = S_1 \cap S_2$

   a. C_g is a smooth irreducible rational curve.

   b. $(C_g|_1)^2 + (C_g|_2)^2 = -2$.

3. All 2-cycles have non-negative volumes (when all masses are turned off).

   $$Vol(C) = -C \cdot J \geq 0 \ , \quad C \subset S_{\rm tot} \qquad\qquad J = \sum_i \phi_i S_i , \quad \phi_i > 0$$

4. At least one 4-cycle has positive volume.

- Dimension (or rank) of Coulomb branch = number of compact surfaces
- Number of mass parameters = number of blowups

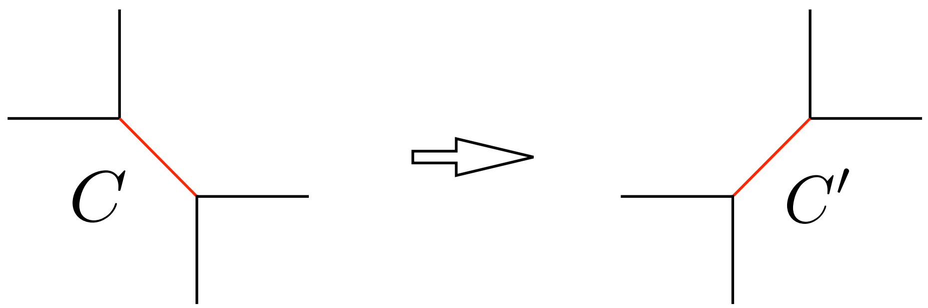
Deformation Equivalence of CY3's

Different geometries can give the same SCFT (up to decoupled free sector) when all Kahler parameters are turned off.

We claim that geometries are 'Deformation Equivalent' if they are related by

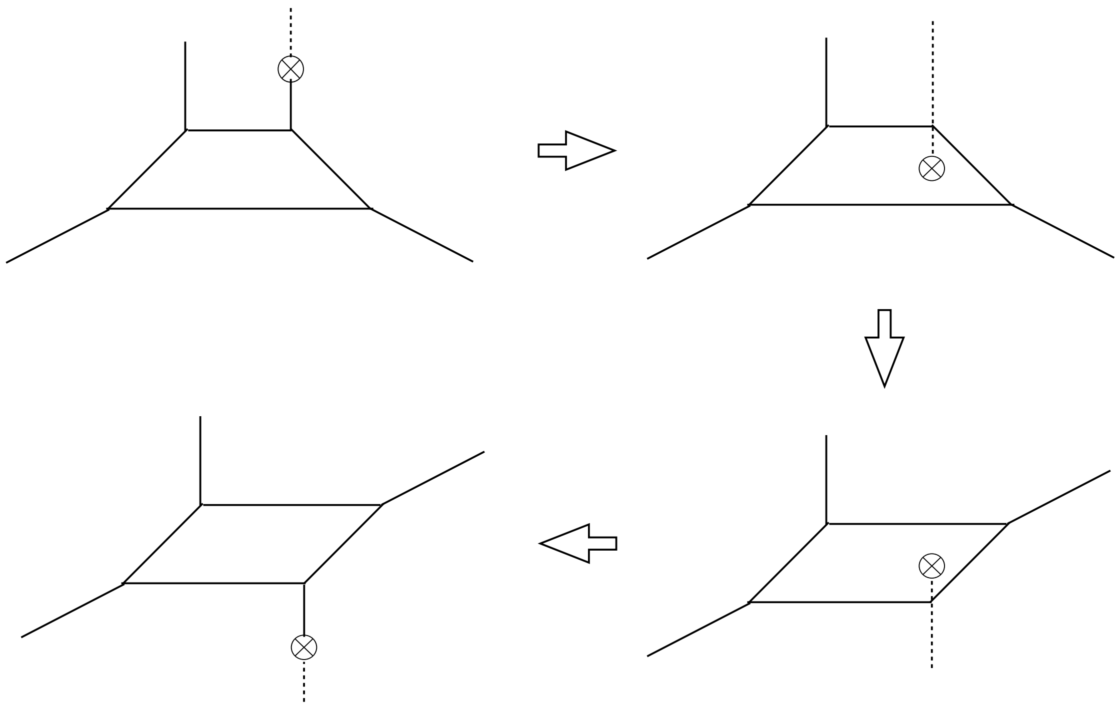
1. Flop : $Vol(C) = -Vol(C')$

   $C^2 = C'^2 = -1$



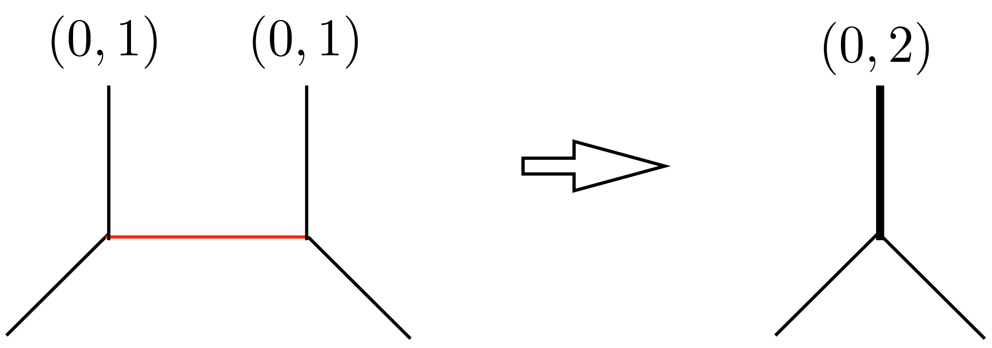
2. Hanany-Witten (HW) transition : a complex structure deformation

ex) $\mathbb{F}_2 \rightarrow \mathbb{F}_0$



Deformation Equivalence of CY3's

3. Complex structure deformation by tuning mass parameters.



- Deformation equivalent CY3's give rise to same SCFT up to decoupled free fields.

Rank 1 classification

All rank 1 SCFTs are engineered by CY3s of del Pezzo surfaces $dP_{n\leq 7}$ and a Hirzebruch surface $\mathbb{F}_0$.

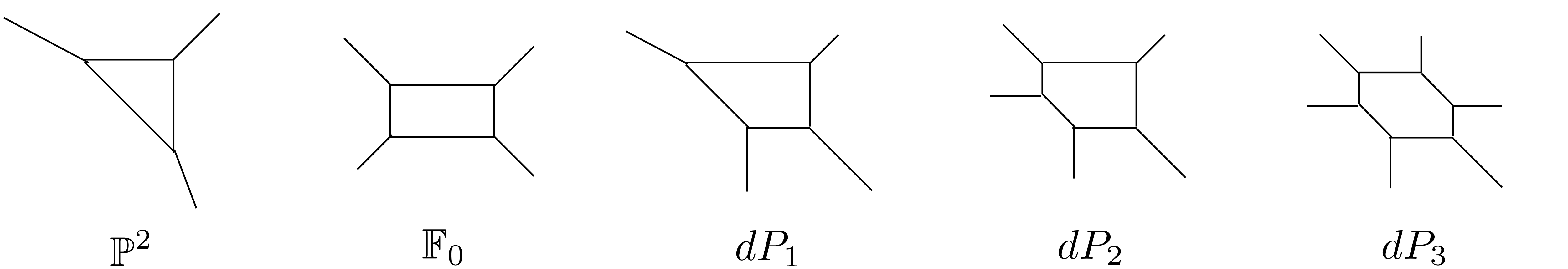
[Morrison, Seiberg 96], [Douglas, Katz, Vafa 96], [Intriligator, Morrison, Seiberg 97]

- Classification :

S	G	M
$\mathbb{P}^2$	$\cdot$	0
$\mathbb{F}_0$	$SU(2)_{\theta=0}$	1
$dP_1 = \mathbb{F}_1$	$SU(2)_{\theta=\pi}$	1
$dP_{n>1}$	$SU(2), N_f = n-1$	n

M : # of mass parameters

- Brane constructions



Rank 2 classification

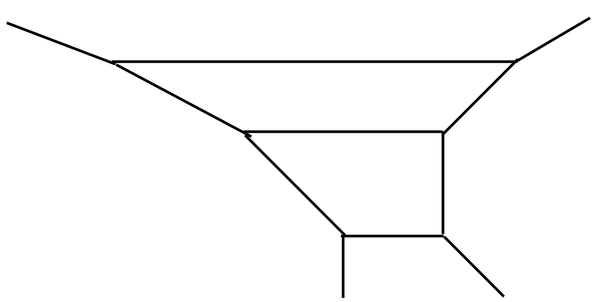
We claim that

> All rank 2 shrinkable CY3 can be realized as $S = S_1 \cup S_2$ for which $S_1 = Bl_p\mathbb{F}_m$ and $S_2 = dP_n$ or $\mathbb{F}_0$.

1. $Bl_p\mathbb{F}_m$ is a blowup of $\mathbb{F}_m$ at p *generic* points.
2. Two surfaces are glued along rational curves $C_1 \subset S_1,\ C_2 \subset S_2$.
3. Gluing curves satisfy $C_1^2 + C_2^2 = -2$.
4. $C_1 = E,\ C_1^2 = -m$.

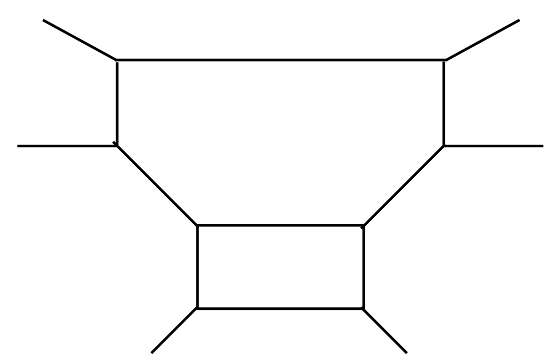
Ex)

1. $SU(3)_2$



$\mathbb{F}_3 \cup dP_1\,,\ C_2 = H,\ H^2 = 1$

2. $SU(3)_1 + 2F$



$Bl_2\mathbb{F}_2 \cup \mathbb{F}_0,\ C_2 = H,\ H^2 = 0$

Geometric RG-flow and Endpoint geometries

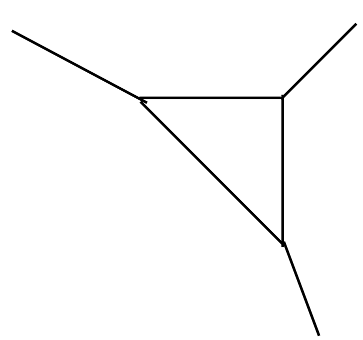
We can blow down exceptional curves with self-intersection '-1' which do not intersect with gluing curves. This defines an RG-flow to a new geometry and thus a new SCFT with one mass parameter less than the original geometry.

In field theory, such deformations are 'rank-preserving mass deformation'.

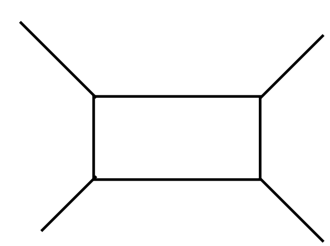
If no such deformation exists, we will call the geometry as endpoint geometry.

Rank 1 examples :

Endpoints

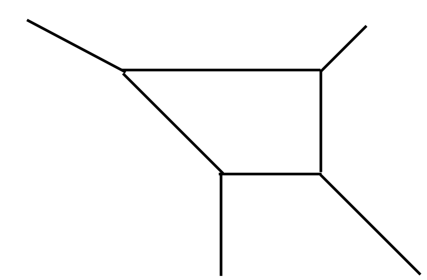


$\mathbb{P}^2$



$\mathbb{F}_0 \rightarrow SU(2)_{\theta=0}$

Not an Endpoint



$\mathbb{F}_1 \rightarrow SU(2)_{\theta=\pi}$

All other geometries are blown-ups of these endpoint geometries with same rank.

Endpoint classification : Rank 2

Rank 2 endpoint geometries have only $M = 0, 1$.

$S_1 \cup S_2$	C_1	C_2
$\mathbb{P}^2 \cup \mathbb{F}_3$	ℓ	E
$\mathbb{P}^2 \cup \mathbb{F}_6$	2ℓ	E

(a) Endpoint geometries with $M = 0$.

$S_1 \cup S_2$	C_1	G	(n_1, n_2)	C_1	G
$\mathbb{F}_0 \cup \mathbb{F}_2$	F_1	$SU(3)_1$	$\mathbb{F}_0 \cup \mathbb{F}_8$	$F_1 + 3H_1$	$SU(3)_7, G_2$
$\mathbb{F}_0 \cup \mathbb{F}_4$	$F_1 + H_1$	$SU(3)_3$	$\mathbb{F}_1 \cup \mathbb{F}_1$	E_1	$SU(3)_0$
$\mathbb{F}_0 \cup \mathbb{F}_6$	$F_1 + 2H_1$	$SU(3)_5, Sp(2)_{\theta=0}$	$\mathbb{F}_1 \cup \mathbb{F}_7$	$2F_1 + H_1$	$SU(3)_6$

(b) Endpoint geometries with $M = 1$. Here $C_2 = E_2$.

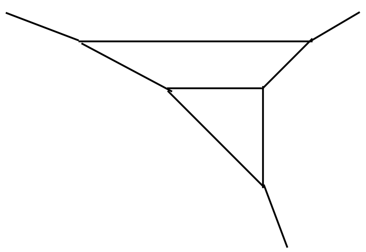
$S_1 \cup S_2$	C_1	G	Endpoint
$\mathbb{F}_1 \cup \mathbb{F}_2$	F_1	$SU(2)\hat{\times}SU(2)$	$\mathbb{P}^2 \cup \mathbb{F}_3$
$\mathbb{F}_1 \cup \mathbb{F}_3$	H_1	$SU(3)_2$	$\mathbb{P}^2 \cup \mathbb{F}_3$
$\mathbb{F}_1 \cup \mathbb{F}_5$	$F_1 + H_1$	$SU(3)_4$	$\mathbb{P}^2 \cup \mathbb{F}_6$
$\mathbb{F}_1 \cup \mathbb{F}_6$	$2H_1$	$Sp(2)_{\theta=\pi}$	$\mathbb{P}^2 \cup \mathbb{F}_6$
$\mathbb{F}_1 \cup \mathbb{F}_{10}$	$F_1 + 4H_1$	$SU(3)_9$	6d?

(c) Other geometries of $\mathbb{F}_{n_1} \cup \mathbb{F}_{n_2}$ with M=1.

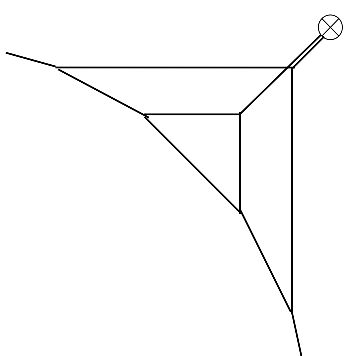
All other rank 2 geometries are blown-ups of these endpoint geometries.

Some brane constructions

$M = 0$

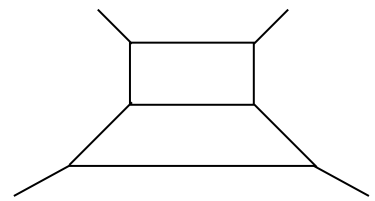


$\mathbb{P}^2 \cup \mathbb{F}_3$

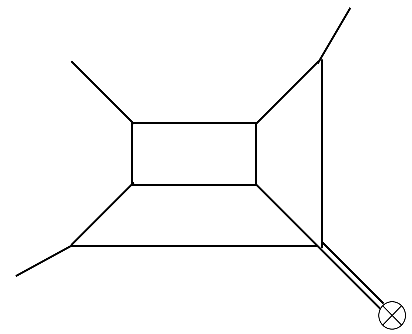


$\mathbb{P}^2 \cup \mathbb{F}_6$

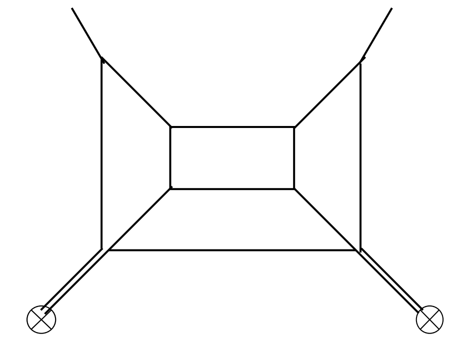
$M = 1$



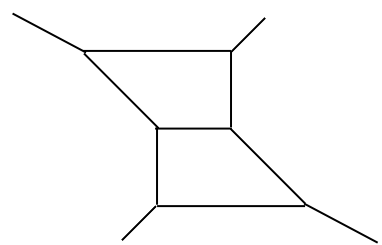
$\mathbb{F}_0 \cup \mathbb{F}_2$



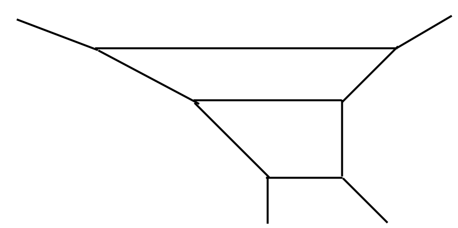
$\mathbb{F}_0 \cup \mathbb{F}_4$



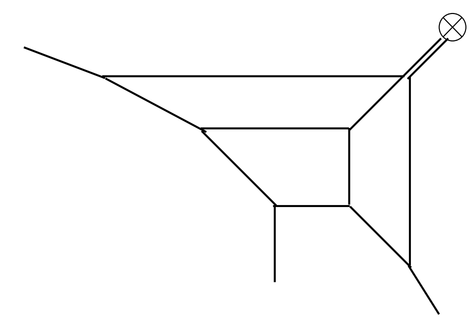
$\mathbb{F}_0 \cup \mathbb{F}_6$



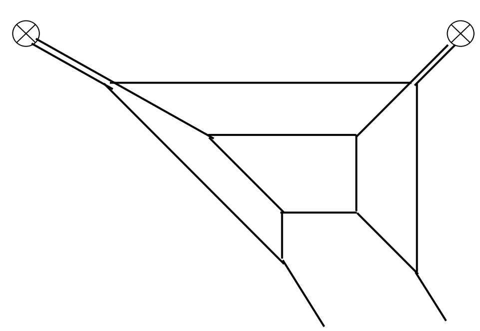
$\mathbb{F}_1 \cup \mathbb{F}_1$



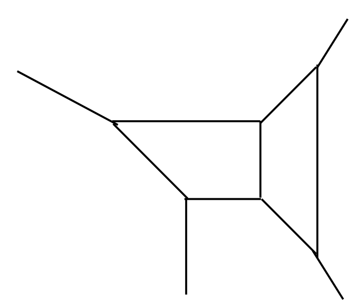
$\mathbb{F}_1 \cup \mathbb{F}_3$



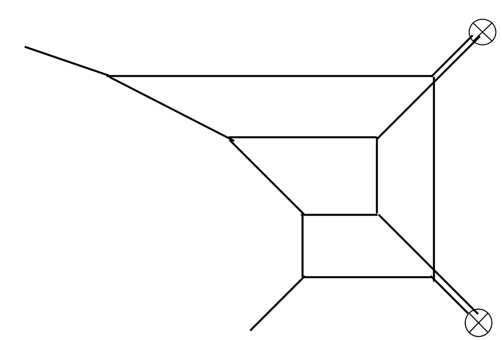
$\mathbb{F}_1 \cup \mathbb{F}_5$



$\mathbb{F}_1 \cup \mathbb{F}_7$



$\mathbb{F}_1 \cup \mathbb{F}_2$



$\mathbb{F}_1 \cup \mathbb{F}_6$

Geometry and gauge theory : rank 2 with M=1

Gauge theory analysis predicts

$SU(3)_\kappa, 0 \leq \lvert\kappa\rvert \leq 9$
$Sp(2)$,$\theta = 0, \pi$
G_2

[Jefferson, H-C. Kim, Vafa, Zafrir 2015]

Geometric classification :

$S_1 \cup S_2$	C_1	G	$S_1 \cup S_2$	C_1	G
$\mathbb{F}_0 \cup \mathbb{F}_2$	F_1	$SU(3)_1$	$\mathbb{F}_0 \cup \mathbb{F}_8$	$F_1 + 3H_1$	$SU(3)_7, G_2$
$\mathbb{F}_0 \cup \mathbb{F}_4$	$F_1 + H_1$	$SU(3)_3$	$\mathbb{F}_1 \cup \mathbb{F}_1$	E_1	$SU(3)_0$
$\mathbb{F}_0 \cup \mathbb{F}_6$	$F_1 + 2H_1$	$SU(3)_5, Sp(2)_{\theta=0}$	$\mathbb{F}_1 \cup \mathbb{F}_7$	$2F_1 + H_1$	$SU(3)_6$

(a) Endpoint geometries with $M = 1$. Here $C_2 = E_2$.

$S_1 \cup S_2$	C_1	G	Endpoint
$\mathbb{F}_1 \cup \mathbb{F}_2$	F_1	$SU(2)\hat{\times}SU(2)$	$\mathbb{P}^2 \cup \mathbb{F}_3$
$\mathbb{F}_1 \cup \mathbb{F}_3$	H_1	$SU(3)_2$	$\mathbb{P}^2 \cup \mathbb{F}_3$
$\mathbb{F}_1 \cup \mathbb{F}_5$	$F_1 + H_1$	$SU(3)_4$	$\mathbb{P}^2 \cup \mathbb{F}_6$
$\mathbb{F}_1 \cup \mathbb{F}_6$	$2H_1$	$Sp(2)_{\theta=\pi}$	$\mathbb{P}^2 \cup \mathbb{F}_6$
$\mathbb{F}_1 \cup \mathbb{F}_{10}$	$F_1 + 4H_1$	$SU(3)_9$	6d?

(b) Other geometries of $\mathbb{F}_{n_1} \cup \mathbb{F}_{n_2}$ with M=1.

Note that geometry cannot engineer $SU(3)_8$ gauge theory!

Dualities from geometry

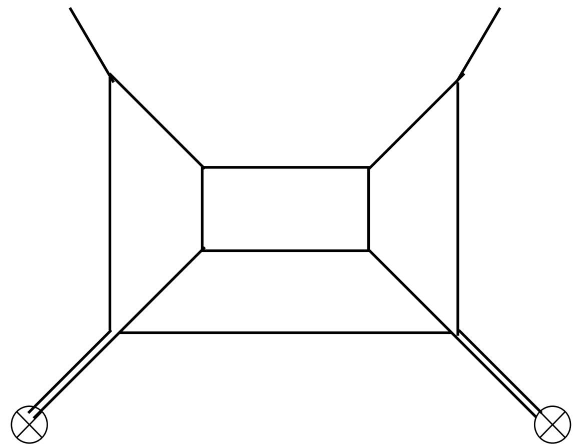
Geometric duality can lead to dualities between gauge theories.

- Fiber class W_i with $W_i^2 = 0$ in each surface $S_i \subset S = \cup_i S_i$ can form a Cartan matrix $A_{ij}(G)$ of Lie group G. Namely,

$$-W_i S_j = A_{ij}(G)$$

[Intriligator, Morrison, Seiberg 97]

- Choice of fiber classes is not unique.
- Different choices correspond to different gauge theory descriptions.
- Ex :



$\mathbb{F}_0 \cup \mathbb{F}_6\,,\ C_1 = H_1 + 2F_1$
$,\ C_2 = E_2 \subset \mathbb{F}_6$

Since $F_1^2 = H_1^2 = 0$, we have two choices :

1. $W_1 = F_1,\ W_2 = F_2 \ \rightarrow\ SU(3)_5$
2. $W_1 = H_1,\ W_2 = F_2 \ \rightarrow\ Sp(2)_{\theta=0}$

Thus, $F_1 \leftrightarrow H_1$ leads to $SU(3)_5 \leftrightarrow Sp(2)$ duality.

[Gaiotto, H-C. Kim 2015]

New dualities from geometry

- $SU(3)_7 \leftrightarrow G_2$ duality from $\mathbb{F}_0 \cup \mathbb{F}_8$
  - Gluing curves are $C_1 = H_1 + 3F_1 \subset \mathbb{F}_0,\ C_2 = E_2 \subset \mathbb{F}_8$.
  - Two fiber class choices : $1.\ W_1 = F_1,\ W_2 = F_2 \ \rightarrow \ SU(3)_7$
    $2.\ W_1 = H_1,\ W_2 = F_2 \ \rightarrow \ G_2$

- $SU(3)_6, N_f = 2 \ \leftrightarrow \ G_2, N_f = 2 \ \leftrightarrow \ Sp(2)_{\theta=0}, N_A = 2$ duality from $dP_3 \cup \mathbb{F}_6$
  - dP_3 has three exceptional curves X_1, X_2, X_3 with self-intersection '-1'.
  - Gluing curves are $C_1 = 3l - X_1 - 2X_2,\ C_2 = E_2 \subset \mathbb{F}_6$.
  - Three fiber class choices : $1.\ W_1 = l - X_1,\ W_2 = F_2 \ \rightarrow Sp(2), N_A = 2$
    $2.\ W_1 = l - X_2,\ W_2 = F_2 \ \rightarrow SU(3)_6, N_f = 2$
    $3.\ W_1 = l - X_3,\ W_2 = F_2 \ \rightarrow G_2, N_f = 2$

Summary and future directions

- We propose that QFT with 5d CFT fixed points should have positive metric on the physical Coulomb branch at infinite coupling.
- We classified 5d SCFTs with simple gauge group.
- We propose a systematic way to construct shrinkable Calabi-Yau threefolds which give 5d SCFTs.
- Geometric constructions confirm gauge theory predictions and also this constructions provide new dualities.

Future directions

- Topologial string partition functions   on progress with H. Hayashi and K. Ohmori
- Gauge theory classification including non-perturbative analysis.
- Full classification of 5d SCFTs and shrinkable Calabi-Yau threefolds.

Thank you very much !