6D SCFTs and Group Theory

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IAS

Based On

- 1502.05405/hep-th
  - with Jonathan Heckman, David Morrison, and Cumrun Vafa
- 1506.06753/hep-th
  - with Jonathan Heckman
- 1601.04078/hep-th
  - with Jonathan Heckman, Alessandro Tomasiello
- 1605.08045/hep-th
  - with David Morrison
- 1612.06399/hep-th
  - with Noppadol Mekareeya, Alessandro Tomasiello
- work in progress
  - with Darrin Frey

Outline

I. Classification of 6D SCFTs
   i. Tensor Branches/Strings
   ii. Gauge Algebras/Particles
II. 6D SCFTs and Homomorphisms
   i. $\mathfrak{su}(2) \to \mathfrak{g}_{\text{ADE}}$
   ii. $\Gamma_{\text{ADE}} \to E_8$
III. Implications for 6D SCFTs
   i. C-theorems in 6D
   ii. Classification of RG Flows

Why Study 6D SCFTs?

- Nahm: Maximal SCFT dimension is *six*
- Degrees of freedom $\neq$ particles (but it's a QFT!)
- QFT of M5-branes is a 6D SCFT
- Compactification $\Rightarrow$ 5D/4D/3D/2D Theories

Focus: $(1,0)$ SCFTs

Conformal Symmetry: $\mathfrak{so}(6,2)$

Supersymmetry: 8 Q's and 8 S's

R-symmetry: $\mathfrak{su}(2)_{\mathcal{R}}$

Studied since the 1990's

Many groups:

Witten '95; Strominger '95; Ganor and Hanany '96;

Seiberg and Witten '96; Bershadsky and Johansen '96;

Brunner and Karch '96; Blum and Intriligator '97;

Intriligator '97; Hanany Zaffaroni '97;

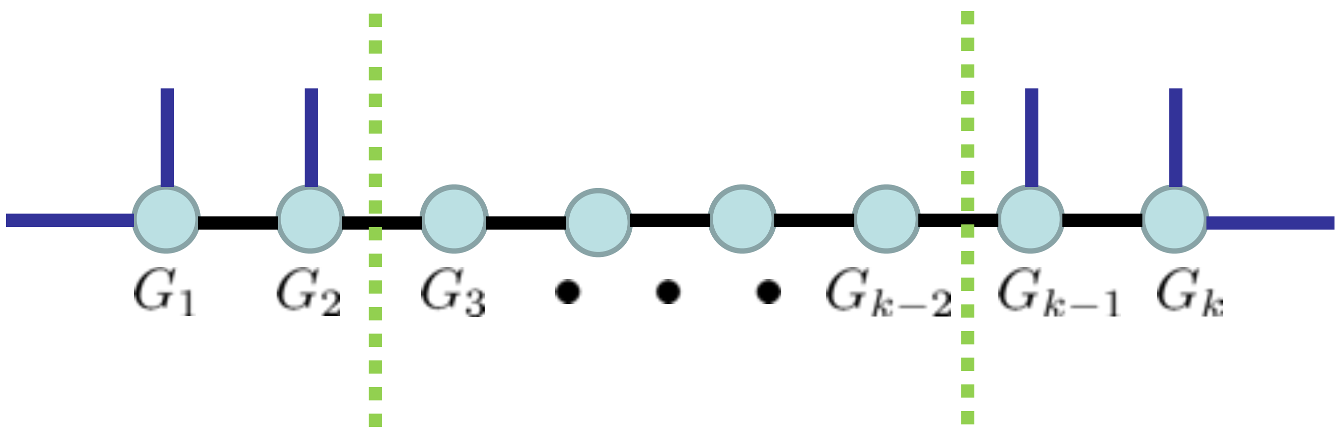
+ ....

But: Even now, still viewed as "mysterious"...

Classification of 6D SCFTs

Classification of 6D SCFTs

- 6D SCFTs can be classified via F-theory
- Nearly all F-theory conditions can be phrased in field theory terms
- 6D SCFTs = Generalized Quivers

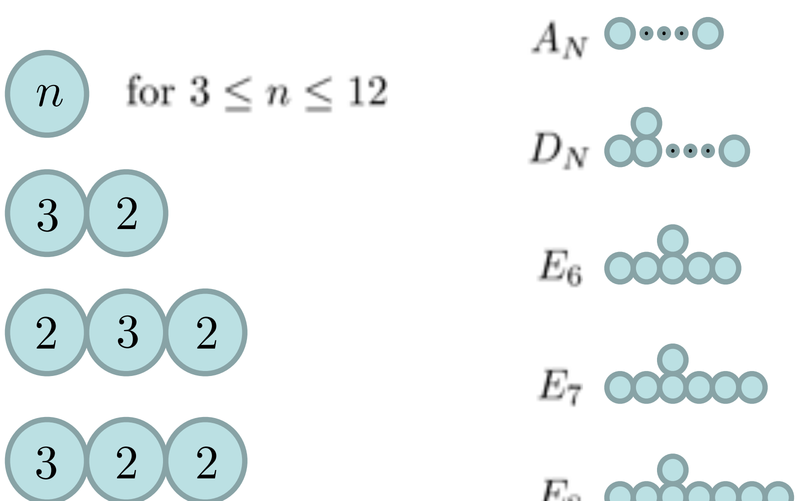


Classification of 6D SCFTs

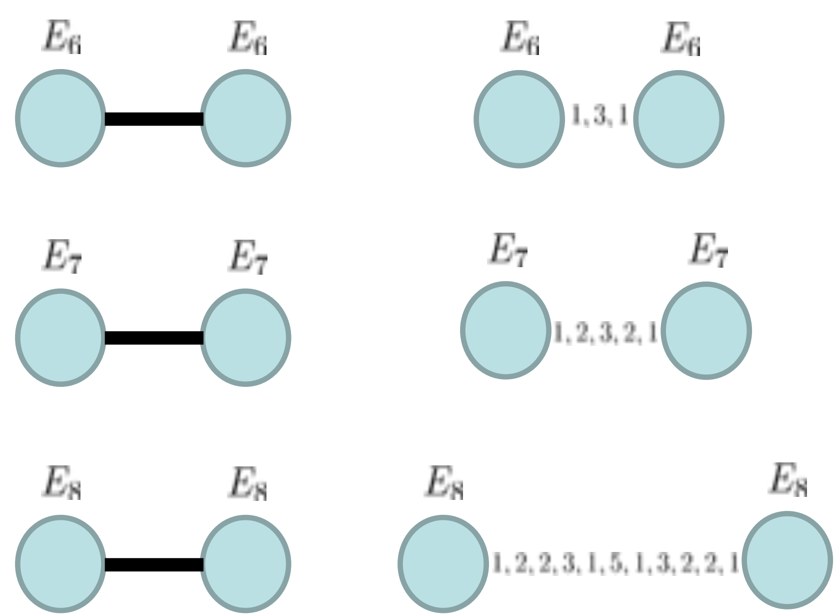
- Looks like chemistry

"Atoms"

c.f. Morrison, Taylor '12



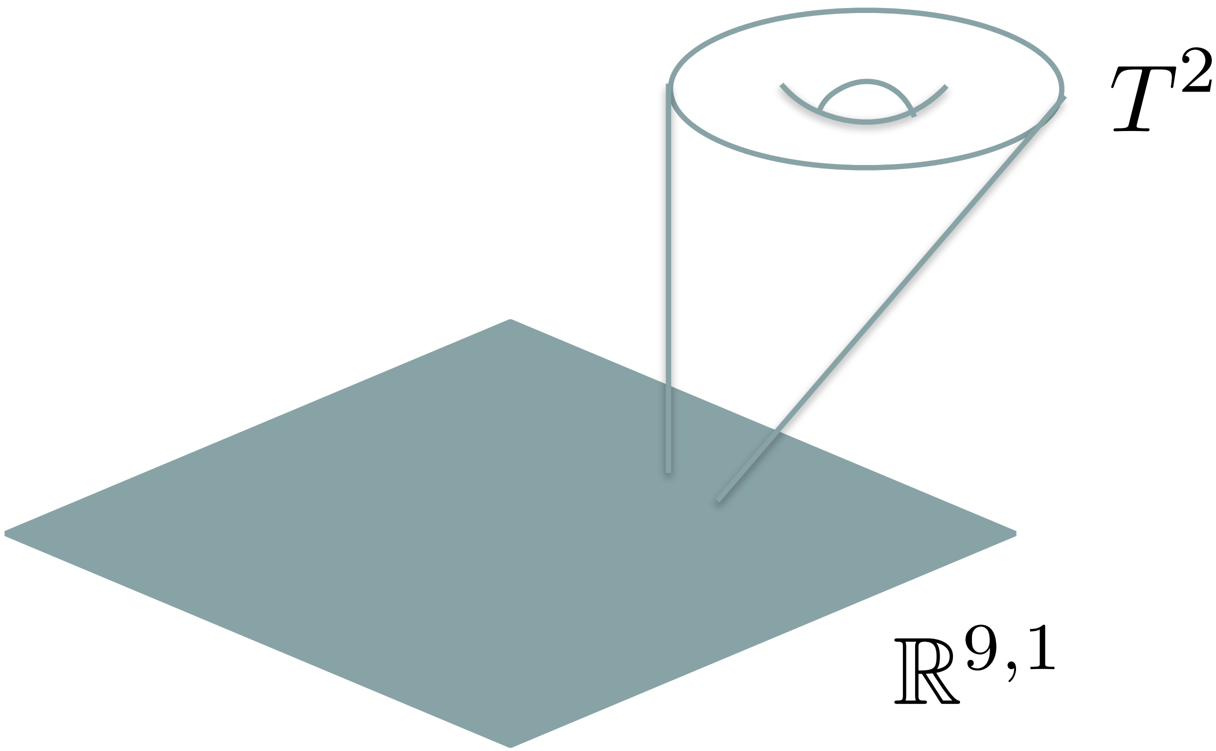
"Radicals"



What is F-theory?

Vafa '96

IIB: $\mathbb{R}^{9,1}$ with position-dependent coupling $\tau = C_0 + ie^{-\Phi}$



6D Theories and F-theory

Vafa '96, Vafa Morrison, I/II '96

All known 6D theories have F-theory avatar*

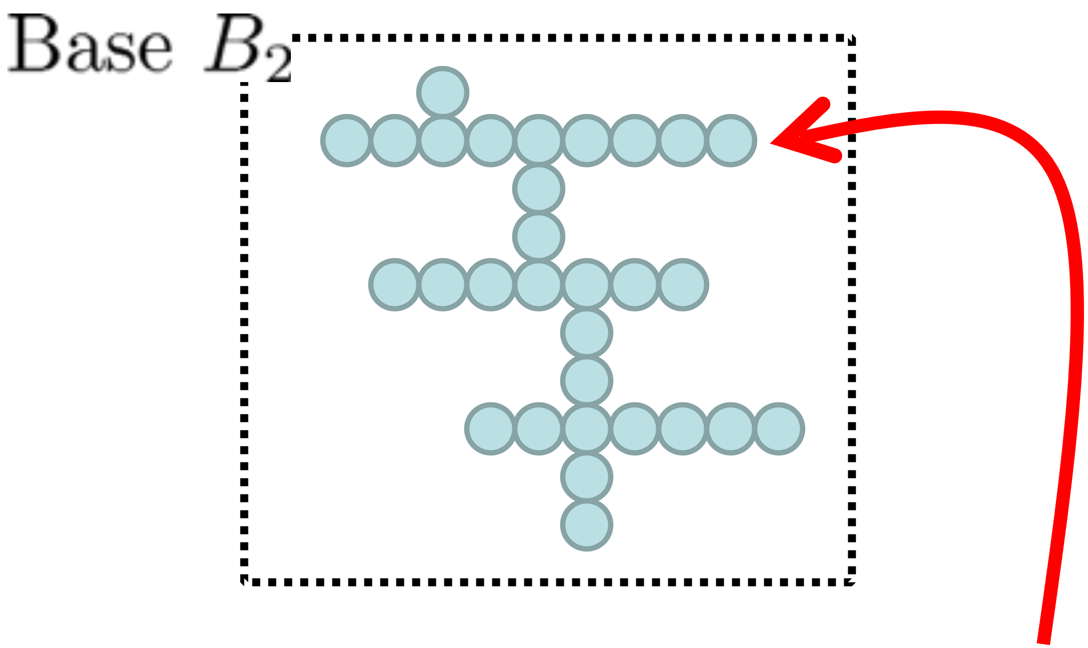
IIB: $\mathbb{R}^{5,1} \times B_2$ with pos. dep. coupling $\tau(z_B)$

F-theory on $\mathbb{R}^{5,1} \times CY_3$

$$\begin{array}{ccc} T^2 & \rightarrow & CY_3 \\ & & \downarrow \\ & & B_2 \end{array}$$

*up to subtleties involving frozen singularities, see Lakshya Bhardwaj's talk

Geometric Picture



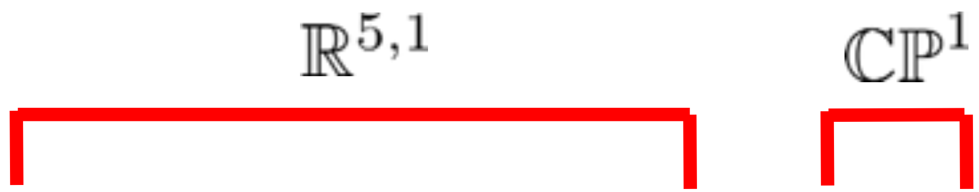
Singularities in base $\Rightarrow$ strings (D3 / $\mathbb{P}^1$)

Singularities in fiber $\Rightarrow$ particles (7-brane on $\mathbb{P}^1$)

Tensionless Strings in F-theory

- Realized by D3-brane on collapsing $\mathbb{CP}^1$

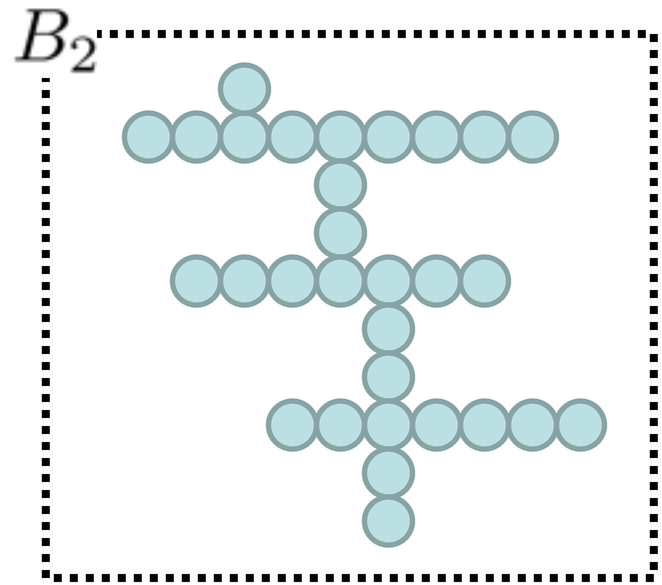
  Tension $= \mathrm{Vol}(\mathbb{CP}^1) \to 0$



	0	1	2	3	4	5	6	7	8	9
D3	×	×	•	•	•	•	×	×	•	•

SCFT Limit

Start: A smooth base B_2



End: To get a CFT, sim. contract curves of B_2

Example: All $(2,0)$ Theories

Witten '95, Strominger '95

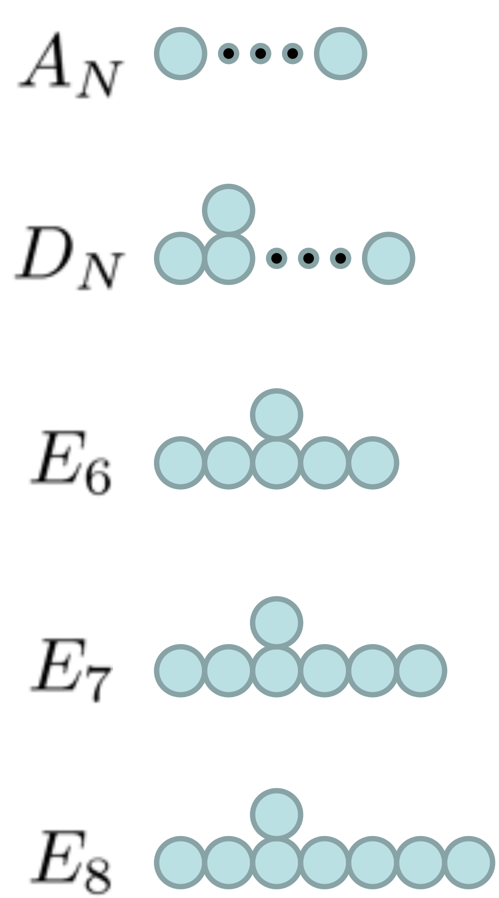
Type IIB on $\mathbb{C}^2/\Gamma_{ADE}$

Resolution Involves:

Bouquet of $\mathbb{CP}^1$'s

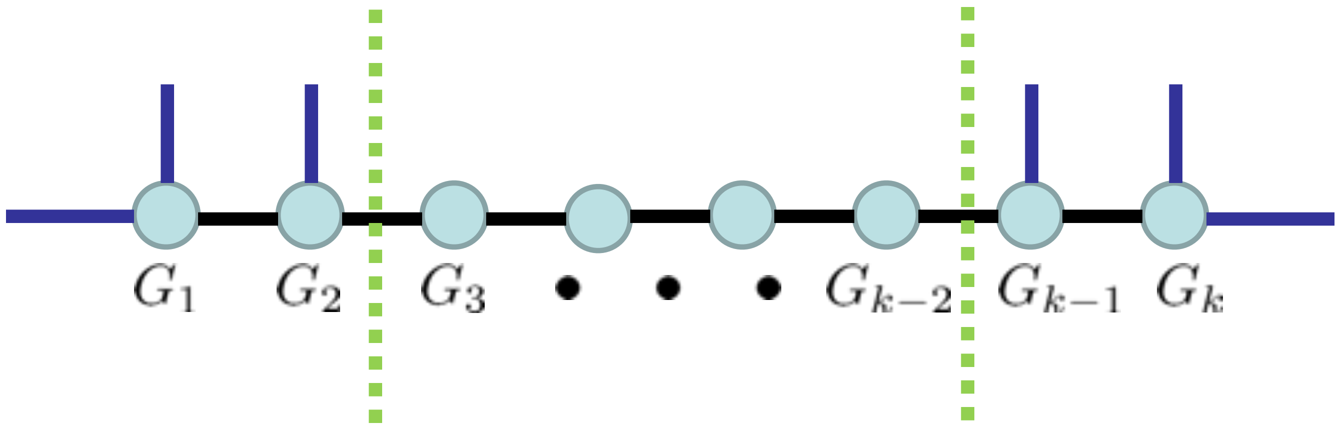
$\mathbb{CP}^1_i \cap \mathbb{CP}^1_j = -\text{Dynkin}_{ij}$

Note: $\mathbb{CP}^1_i \cap \mathbb{CP}^1_i = -2$



Classification of Bases

The Base Quivers have a *very* simple structure!

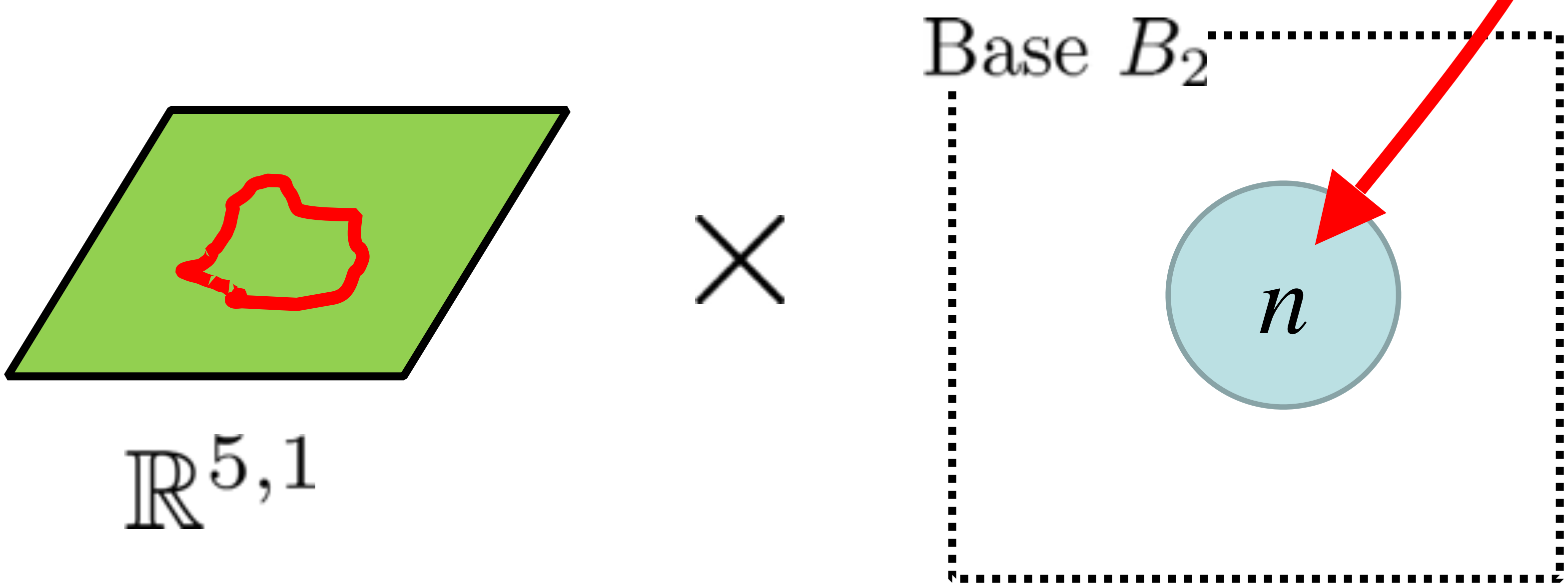


$$G_1 \subseteq G_2 \subseteq \cdots \subseteq G_m \supseteq \cdots \supseteq G_{k-1} \supseteq G_k$$

Strings from D3 on a $\mathbb{P}^1$

$$-\Sigma \cap \Sigma = \text{String Charge}$$

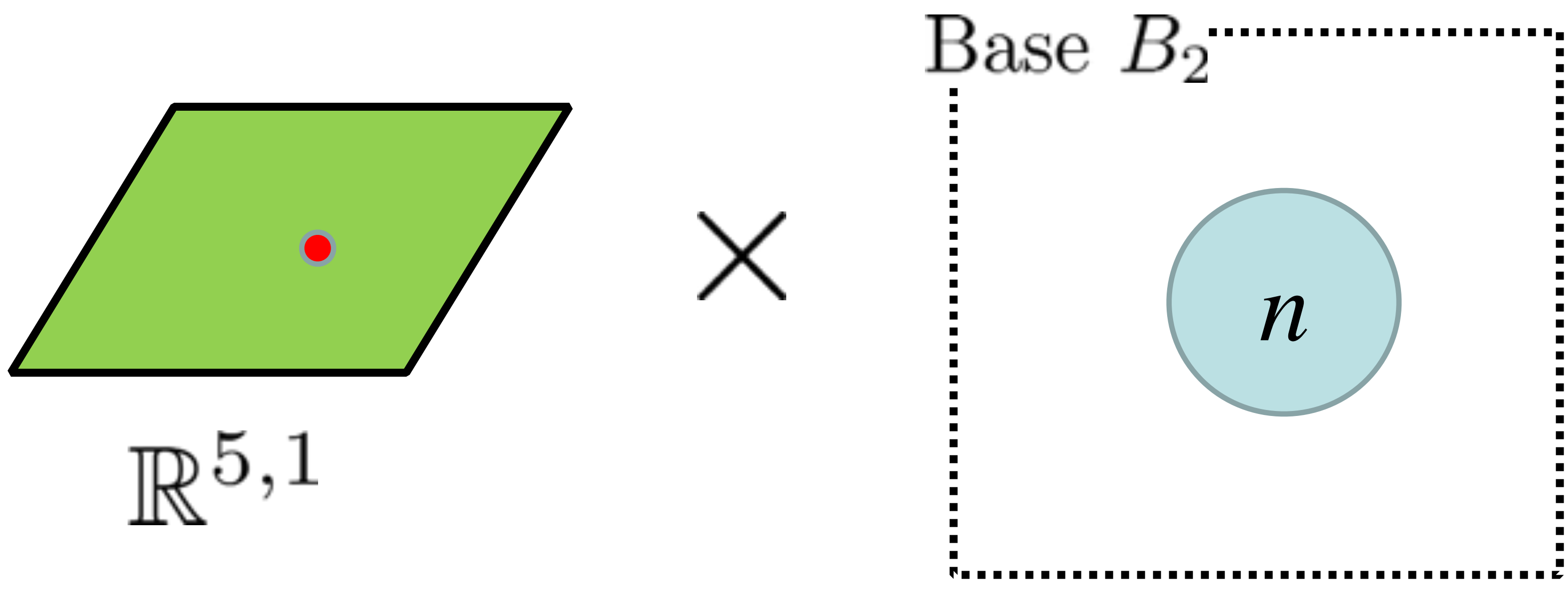
(which must be integer > 0)



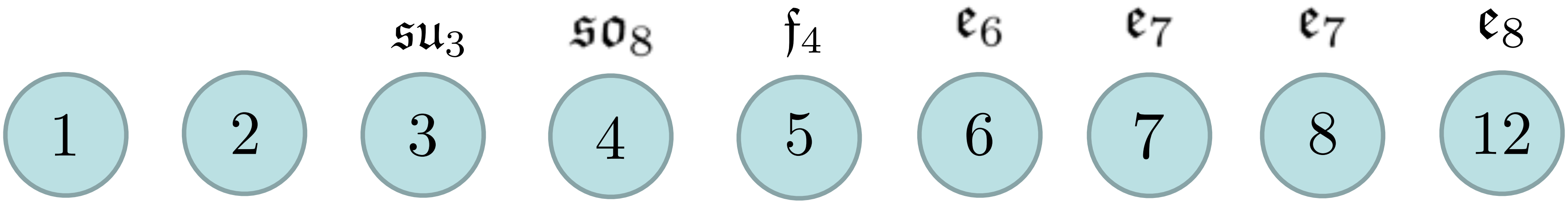
Particles from D7's on a $\mathbb{P}^1$

$3 \leq n \leq 12 \Rightarrow$ always have gauge fields

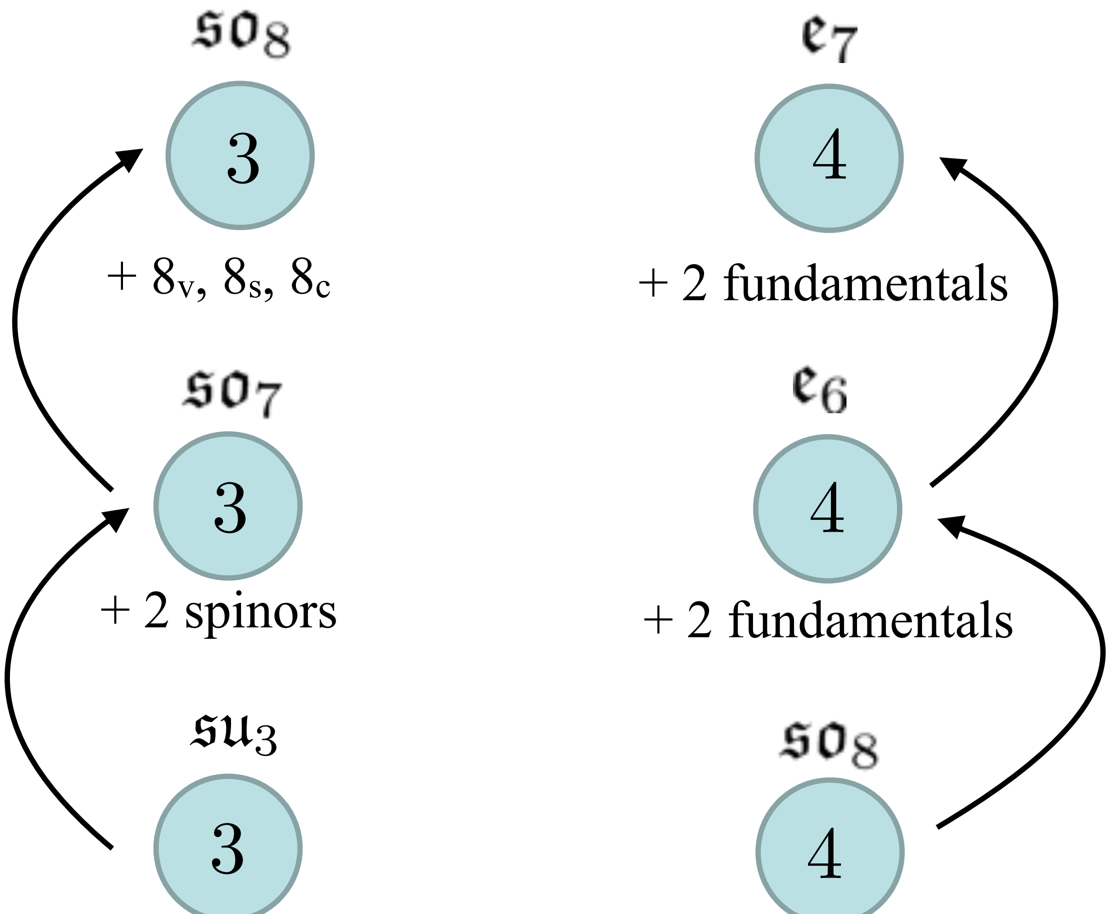
(elliptic fiber is singular: Morrison Taylor '12)



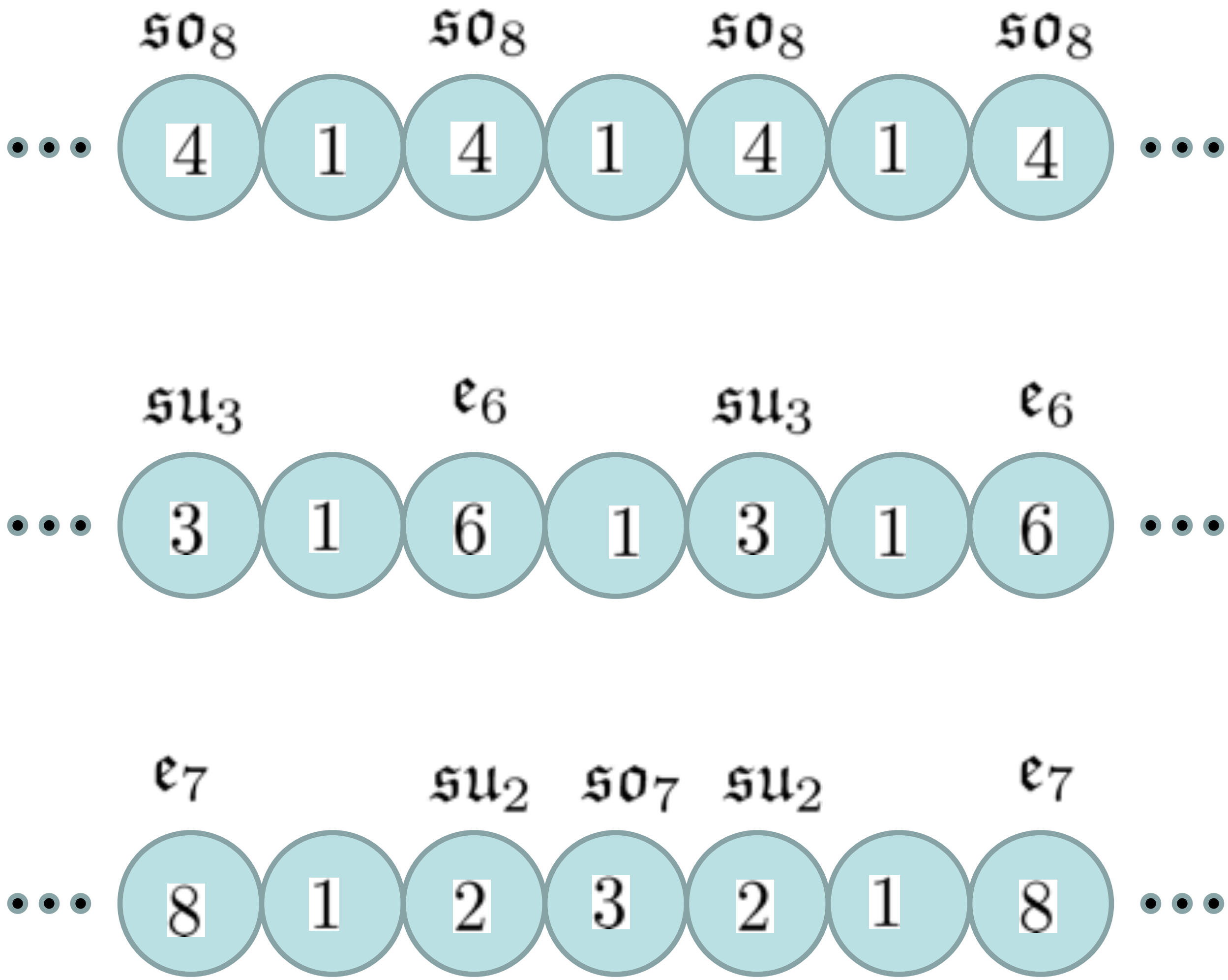
Minimal Gauge Algebras



Fiber Enhancements



Examples

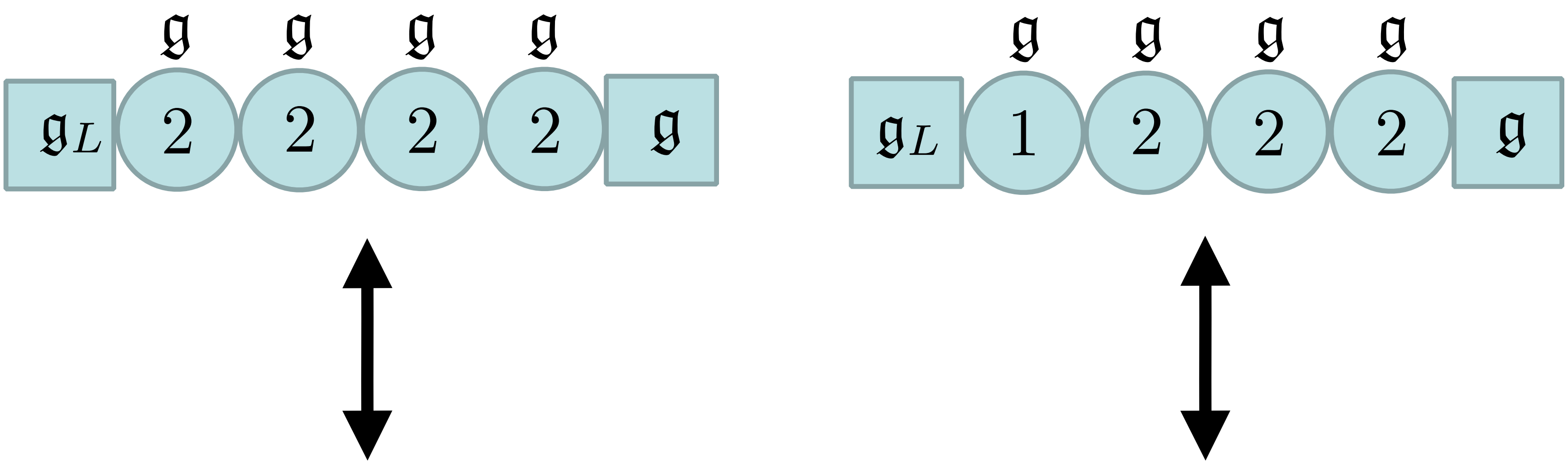


so8 so8 so8 so8
4 1 4 1 4 1 4
su3 e6 su3 e6
3 1 6 1 3 1 6
e7 su2 so7 su2 e7
8 1 2 3 2 1 8

6D SCFTs and Homomorphisms

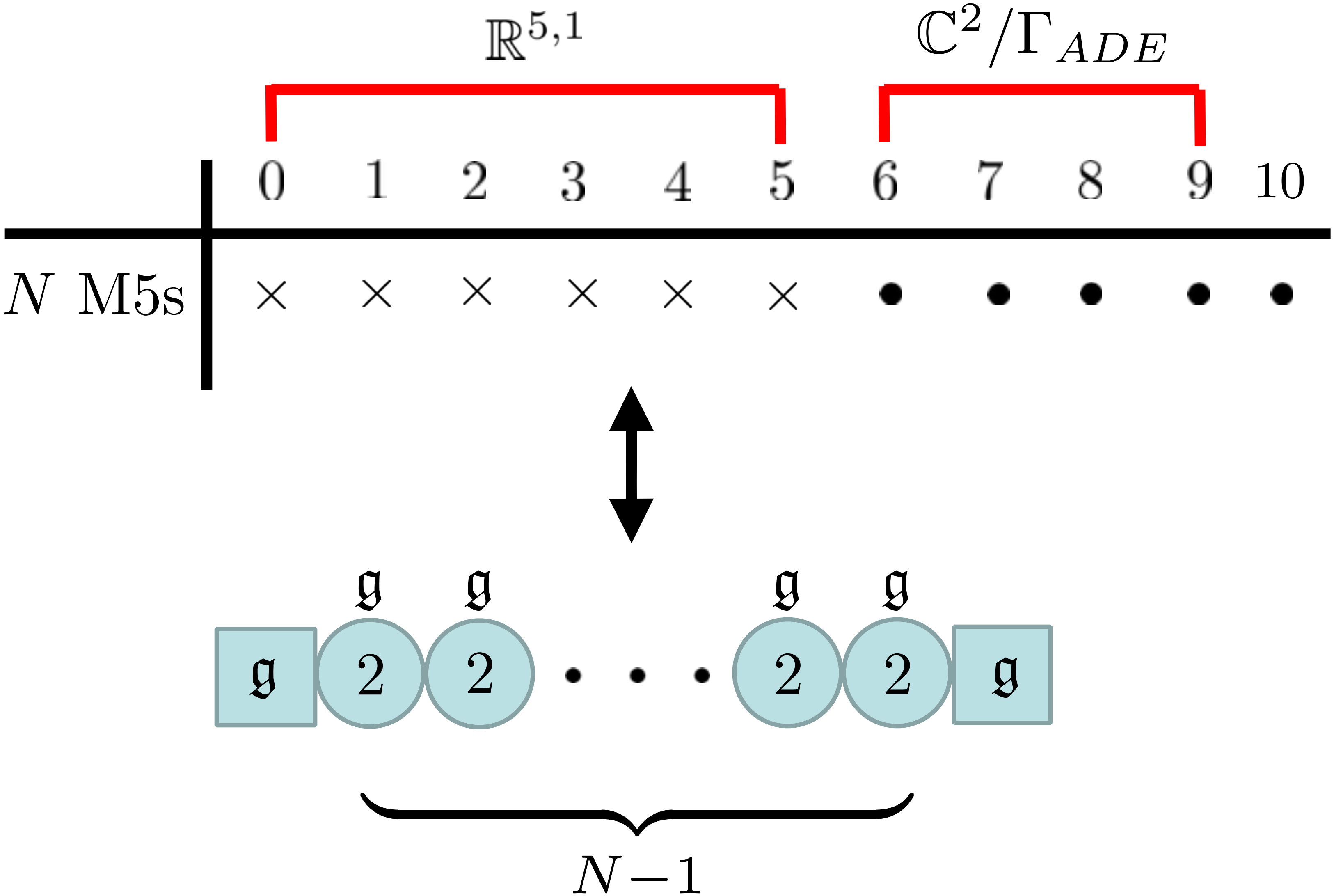
6D SCFTs and Group Theory

- Large classes of 6D SCFTs have connections to structures in group theory
- The correspondence has been verified explicitly



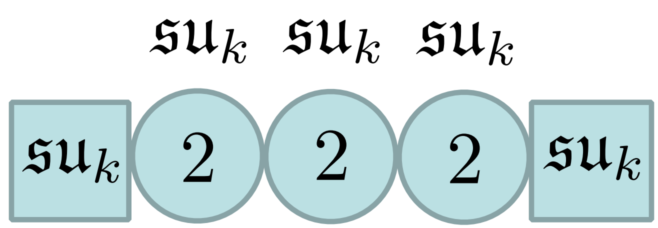
$\mathrm{Hom}(\mathfrak{su}(2), \mathfrak{g})$ $\mathrm{Hom}(\Gamma_{\mathfrak{g}}, E_8)$

M5-Branes Probing $\mathbb{C}^2/\Gamma_{ADE}$

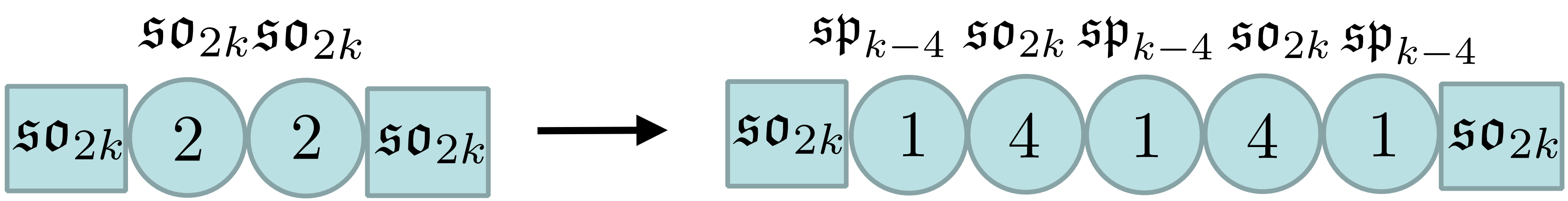


M5-Branes Probing $\mathbb{C}^2/\Gamma_{ADE}$

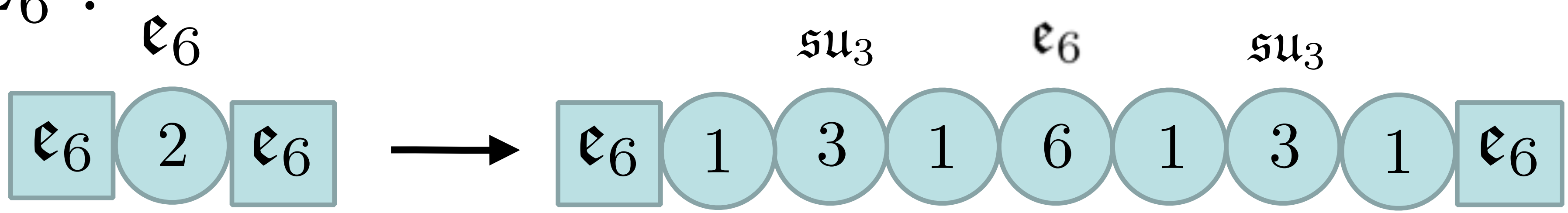
$A_{k-1}:$



$D_k:$



$E_6:$



Nilpotent Deformations

- Matrix of normal deformations Φ characterizes positions of 7-branes
- View intersection points of $\mathbb{CP}^1$ in base as marked points
- Can let adjoint field Φ have singular behavior at marked points $\Rightarrow$ Hitchin system coupled to defects:

$$\partial_A \Phi = \sum_p \mu_{\mathbb{C}}^{(p)} \delta_{(p)} \qquad F + [\Phi, \Phi^\dagger] = \sum_p \mu_{\mathbb{R}}^{(p)} \delta_{(p)}$$

Nilpotent Deformations

- Split $\mu_{\mathbb{C}} = \mu_s + \mu_n$, consider nilpotent part μ_n, get $\mathfrak{su}_2$ algebra:

$$J_+ = \mu_{\mathbb{C}} \qquad J_- = \mu_{\mathbb{C}}^\dagger \qquad J_3 = \mu_{\mathbb{R}}$$

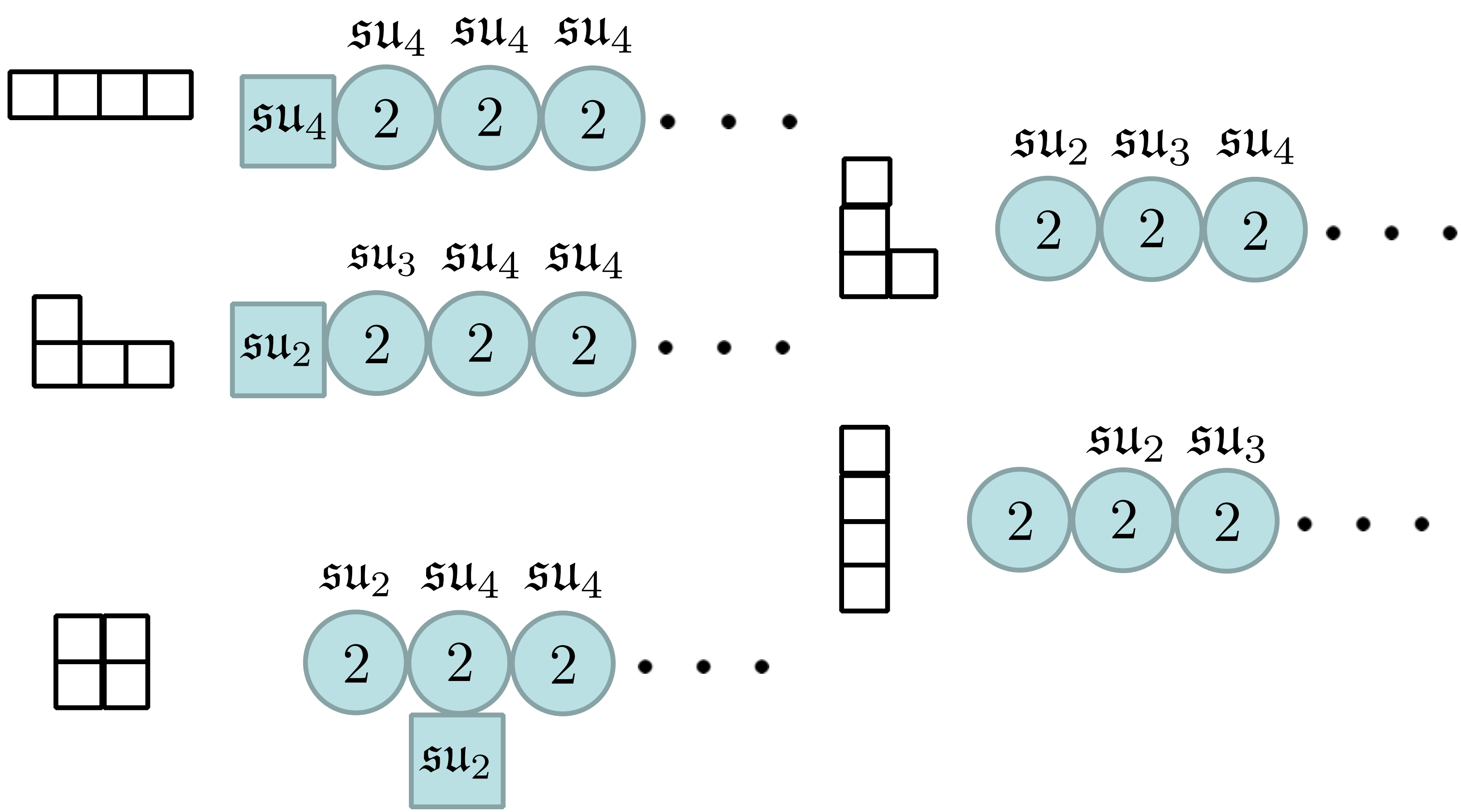
- Adjoint vevs $\Phi \sim \mu_{\mathbb{C}} \dfrac{dz}{z}$

$$\Rightarrow \text{ Classified by } \mathrm{Hom}(\mathfrak{su}(2), \mathfrak{g})$$

(equivalently, by nilpotent orbits J_+)

6D SCFTs and Hom($\mathfrak{su}(2), A_{k-1}$)

Hom($\mathfrak{su}(2), A_{k-1}$) labeled by partitions of k:

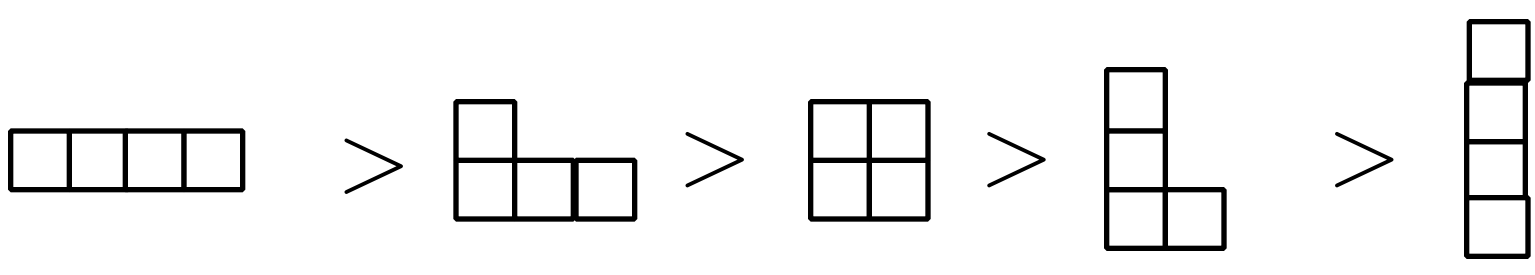


Partial Ordering of Nilpotent Orbits

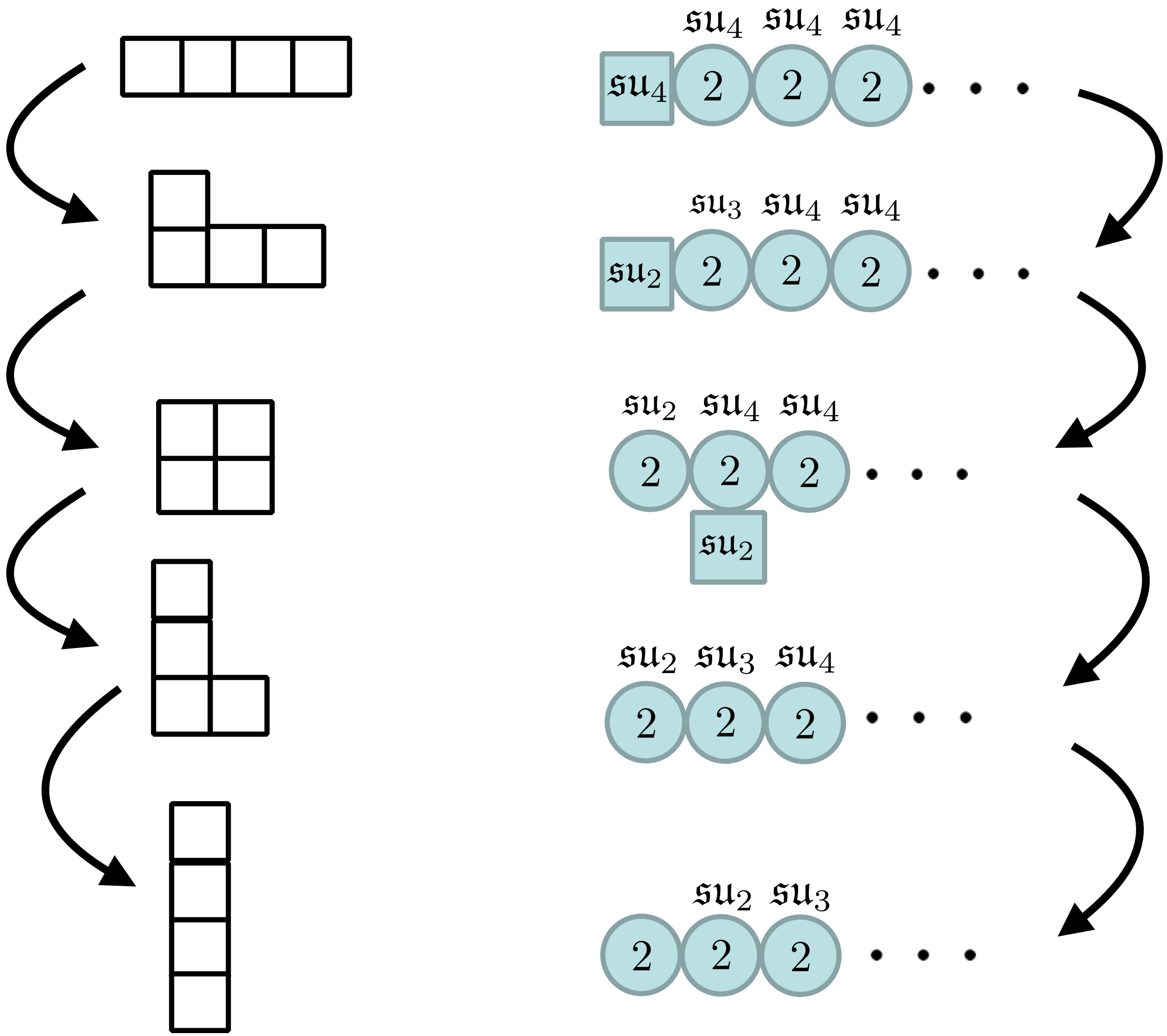
$$\mathcal{O}_\mu \geq \mathcal{O}_\nu \Leftrightarrow \bar{\mathcal{O}}_\mu \supset \mathcal{O}_\nu$$

$$\Leftrightarrow \mu \geq \nu$$

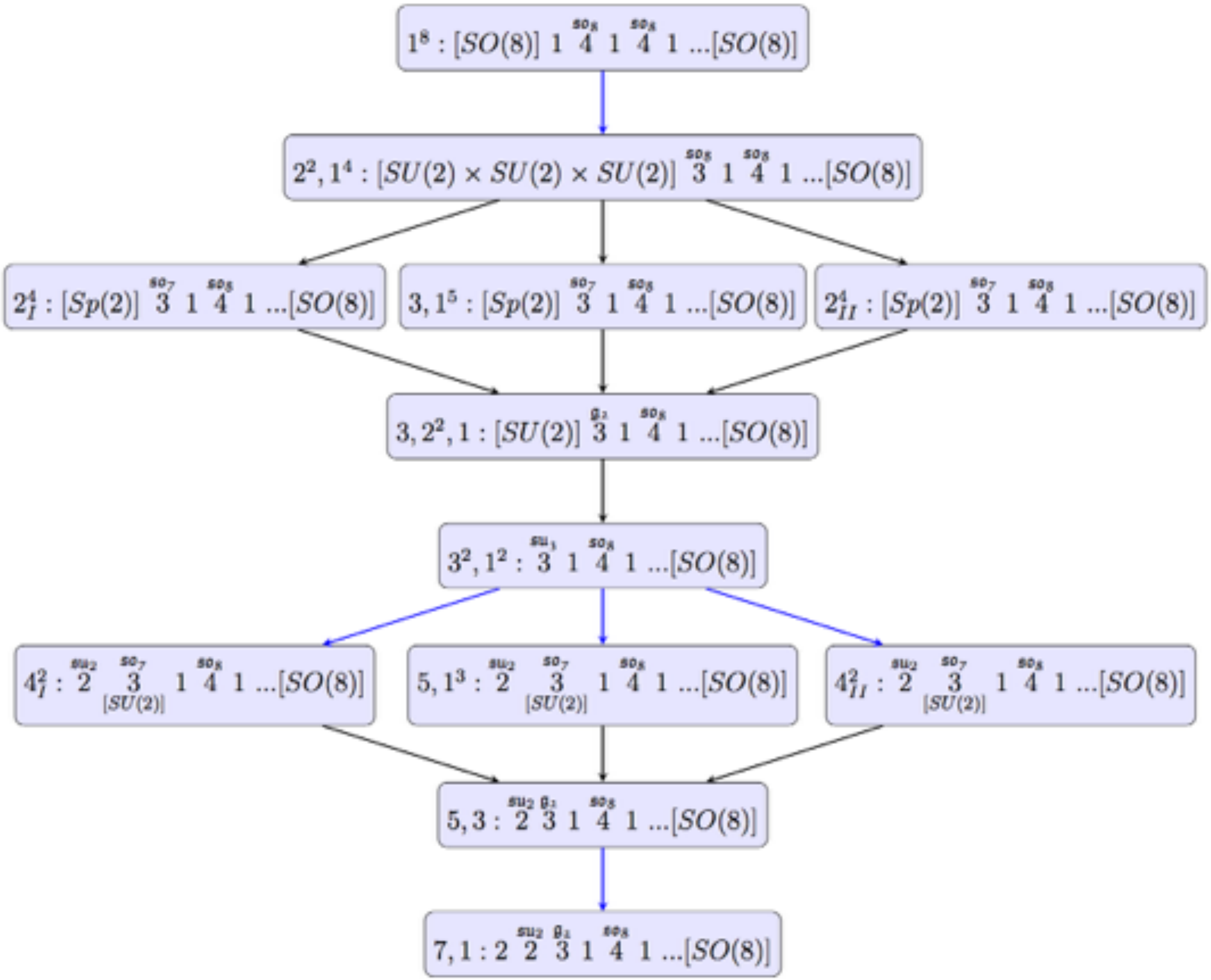
$$\Leftrightarrow \sum_{i=1}^{m} \mu_i^T \geq \sum_{i=1}^{m} \nu_i^T \ \forall m$$



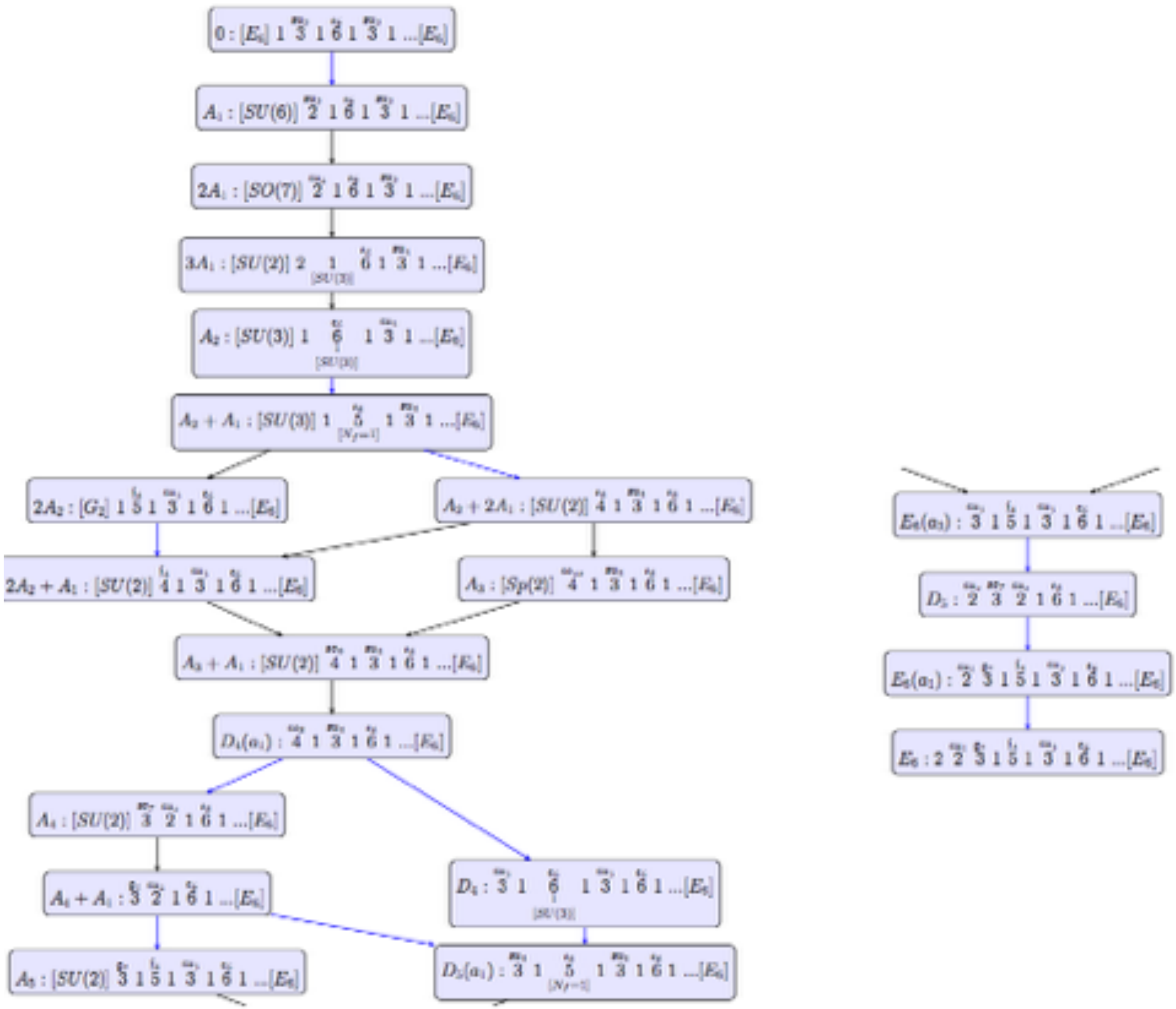
Nilpotent Hierarchy Matches RG Hierarchy!



6D SCFTs and Hom($\mathfrak{su}(2), D_k$)

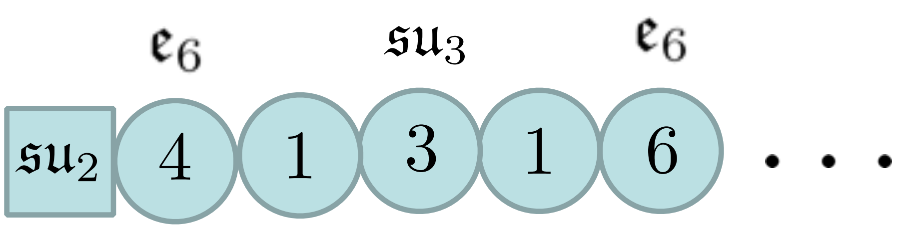


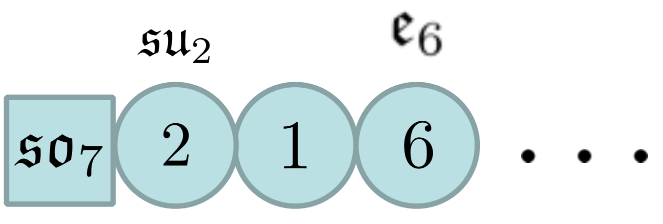
6D SCFTs and $\mathrm{Hom}(\mathfrak{su}(2), E_6)$



Nilpotent Orbits and Global Symmetries

- Consider nilpotent orbit $\mathcal{O}_\mu \in \mathfrak{g}$
- Let $F(\mu)$ be subgroup of G commuting with nilpotent element
- Claim: $F(\mu)$ is the global symmetry of the 6D SCFT associated with $\mathcal{O}_\mu$
- E.g.





6D SCFTs and Hom(Γ_{ADE},E_8)

- Consider M5-branes probing Horava-Witten wall and $\mathbb{C}^2/\Gamma_{ADE}$ singularity

$\mathbb{R}^{5,1}$ (0–5) $\quad$ $\mathbb{C}^2/\Gamma_{ADE}$ (6–9)

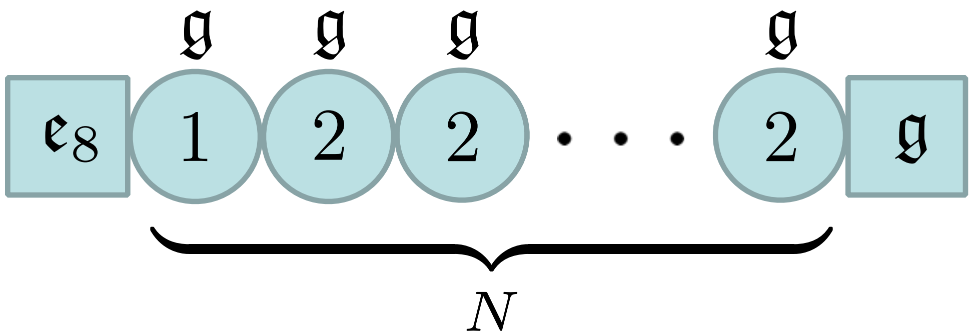
	0	1	2	3	4	5	6	7	8	9	10
N M5s	×	×	×	×	×	×	•	•	•	•	•
E_8 Wall	×	×	×	×	×	×	×	×	×	•	×

- Boundary data $\simeq$ flat E_8 connections on $S^3/\Gamma_{\mathrm{ADE}}$

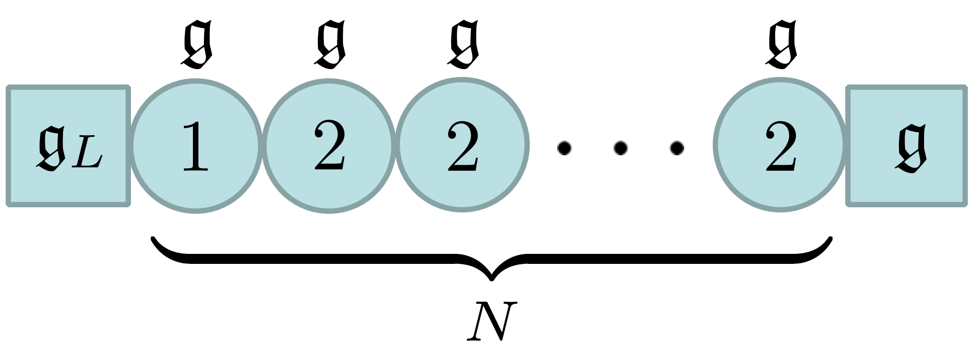
$$\simeq \mathrm{Hom}(\Gamma_{\mathrm{ADE}}, E_8)$$

6D SCFTs and $\mathrm{Hom}(\Gamma_{\mathrm{ADE}}, E_8)$

- For trivial boundary data, get 6D SCFT:

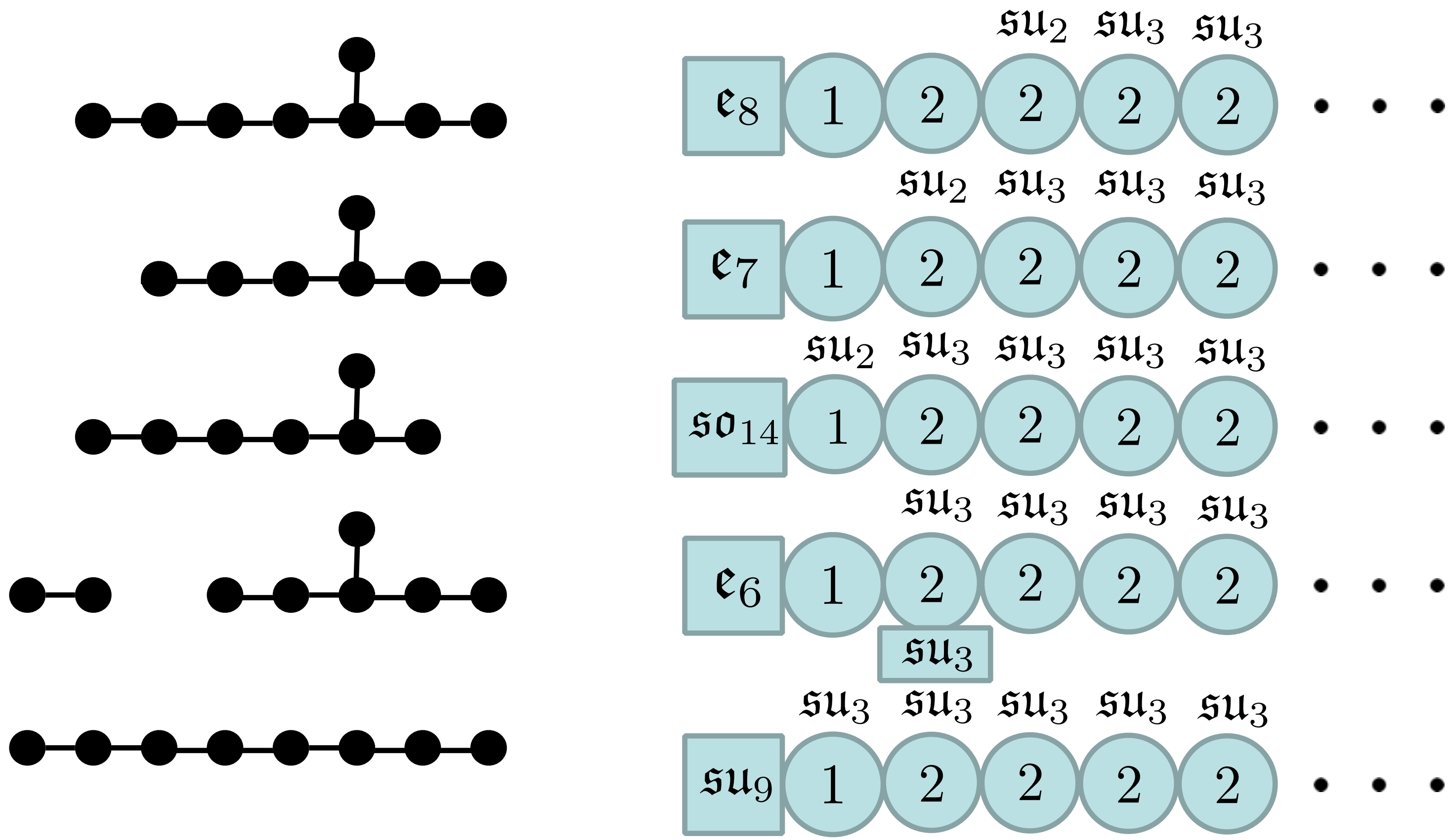


- For non-trivial boundary data, global symmetry is broken to a subgroup



6D SCFTs and Hom(Γ_{ADE},E_8)

E.g. Γ_{A_2}, Hom($\mathbb{Z}_3, E_8$):

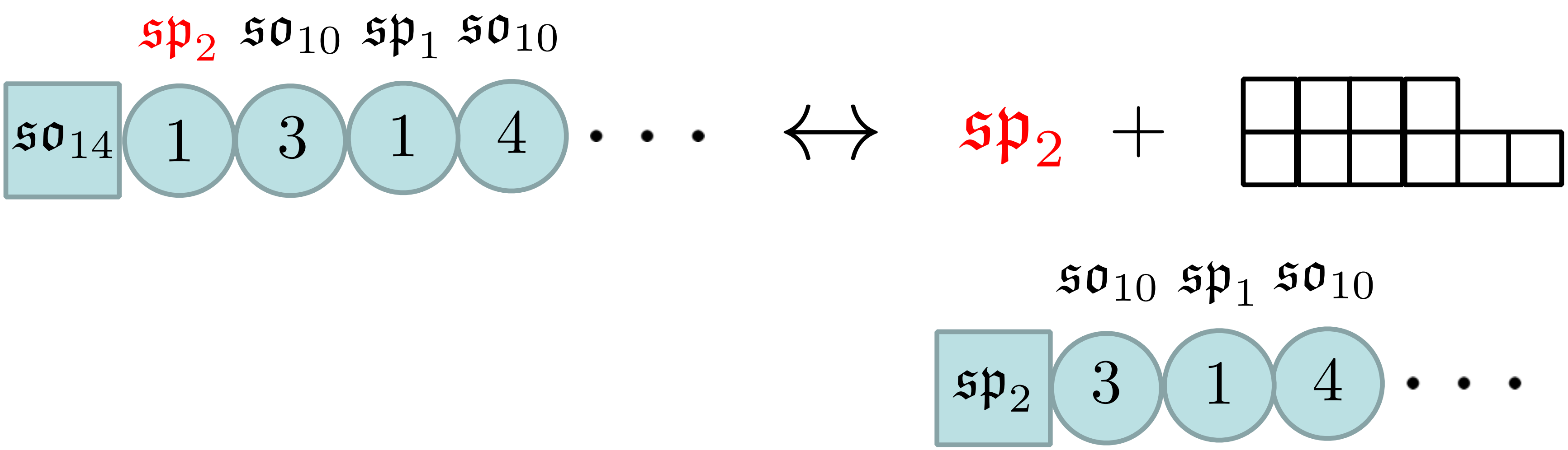


Classification of $\mathrm{Hom}(\Gamma_{\mathrm{A}DE}, E_8)$

- A_n case: done (Kac '83)
- E_8 case: done (Frey '98)
- D_n case: open!
- E_6 case: open!
- E_7 case: open!

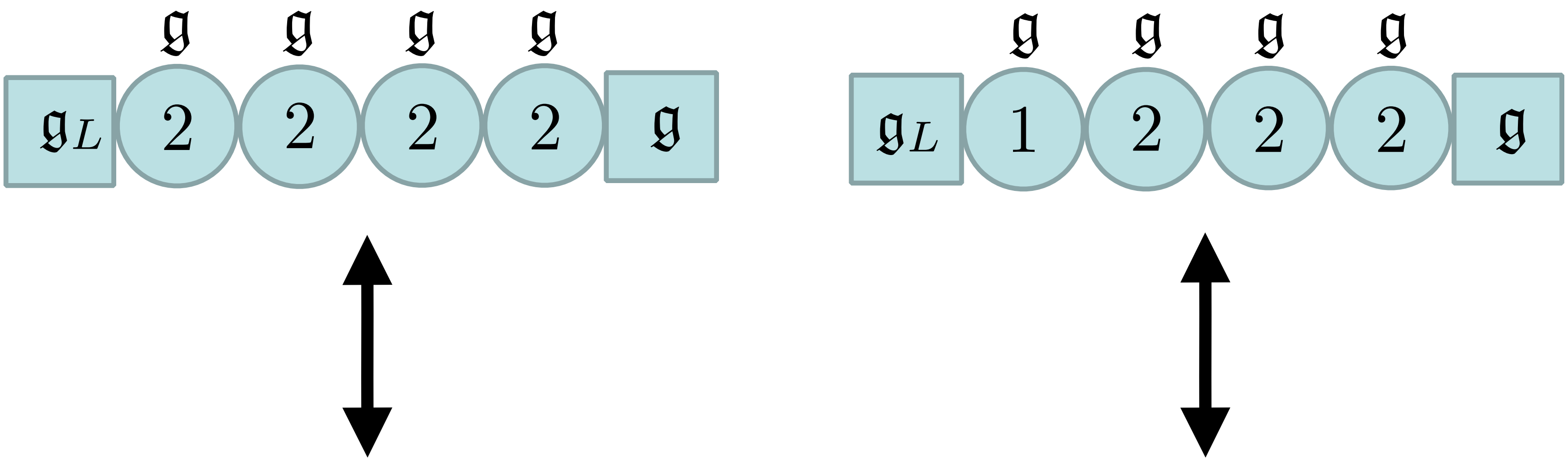
Classification of $\mathrm{Hom}(\Gamma_{D_n}, E_8)$

- $\mathrm{Hom}(\Gamma_{D_n} \simeq \mathrm{Dic}_{n-2}, E_8)$ are uniquely labeled by a nilpotent orbit of D_n together with a simple Lie algebra!
- E.g. $\Gamma_{D_5} \to E_8$:



6D SCFTs and Group Theory

- Large classes of 6D SCFTs have connections to structures in group theory
- The correspondence has been verified explicitly



$$\mathrm{Hom}(\mathfrak{su}(2), \mathfrak{g}) \qquad\qquad \mathrm{Hom}(\Gamma_{\mathfrak{g}}, E_8)$$

Implications for 6D SCFTs

Implications for 6D SCFTs

- There is significant evidence for the a-theorem (and an infinite collection of other c-theorems) in 6D SCFTs
- Connections to group theory provide a proof in certain classes of RG flows
- We speculate that a full classification of RG flows among 6D SCFTs is possible through these connections to group theory

't Hooft Anomalies in 6D SCFTs

- Anomaly polynomial calculable for any 6D SCFT

Ohmori, Shimizu, Tachikawa, Yonekura '14

$$I = \alpha c_2(R)^2 + \beta c_2(R) p_1(T) + \gamma p_1(T)^2 + \delta p_2(T) + \ldots$$

- Trace anomaly related to 6D Euler density

$$\langle T^{\mu}_{\mu} \rangle = \left(\frac{1}{4\pi} \right)^3 a E_6 + \ldots$$

- Can be expressed in terms of anomaly polynomial:

$$a = \frac{8}{3}(\alpha - \beta + \gamma) + \delta$$

Cordova, Dumitrescu, Intriligator '15

Candidate C-Functions

Linear Combinations: $C = \vec{m} \cdot \vec{\alpha}_{\text{anomaly}}$

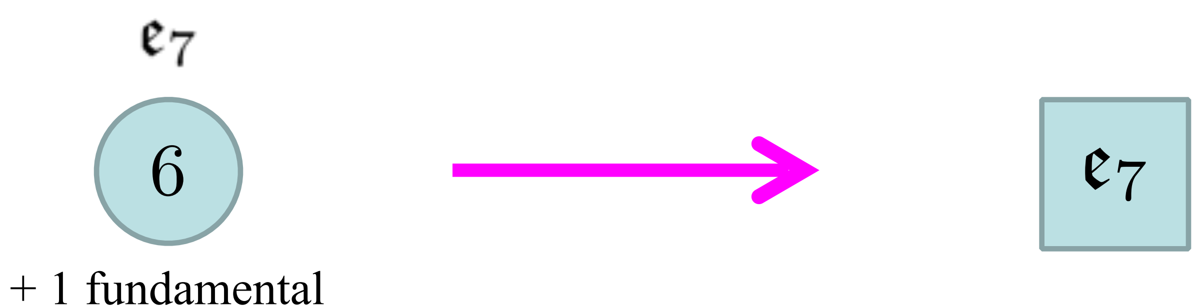
Given a CFT, look for numbers C such that:

$$C_{UV} > C_{IR}$$

Two Deformation Types

Complex Structure Deformation / Higgs Branch

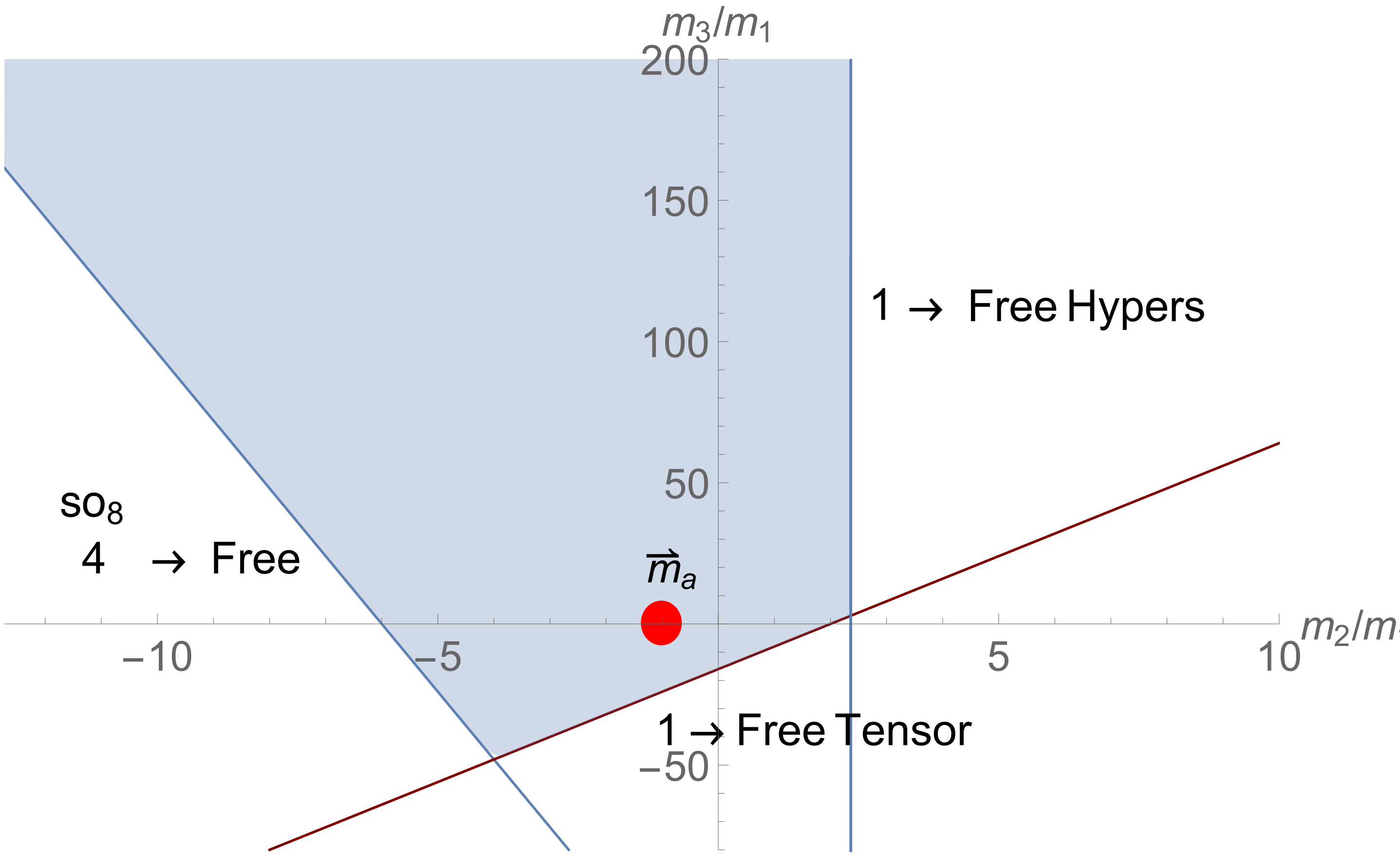
Expand a curve in base to large size / Tensor Branch



“When in doubt, use brute force.”

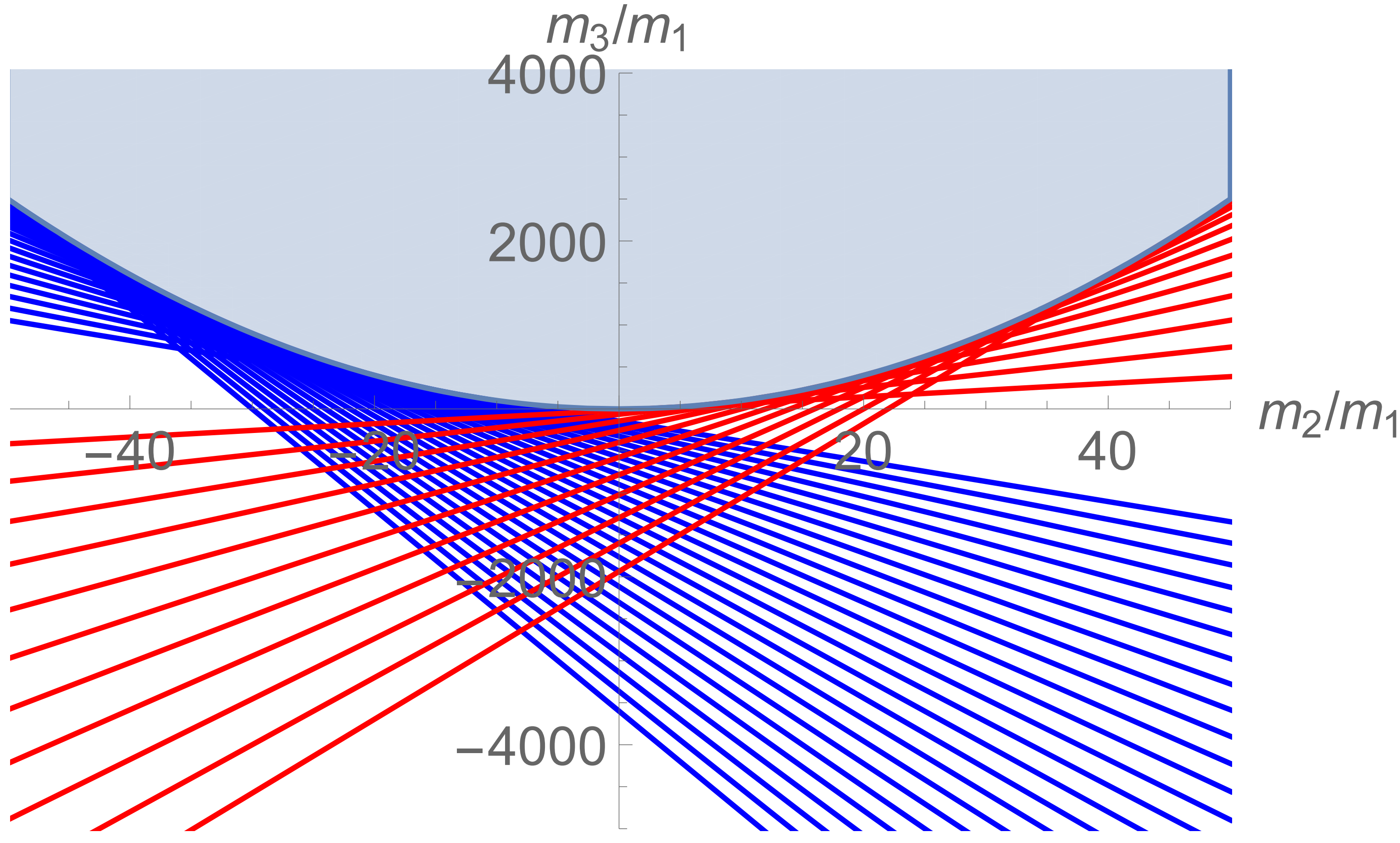
–Ken Thompson

Results of Sweep



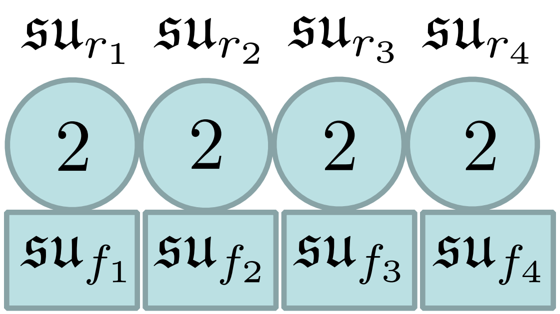
Tightest bounds from simplest theories!

Tensor Branch Sweep



Nilpotent Orbit SCFTs

- Can relate anomalies to data of nilpotent orbit



$\Rightarrow$

$$\alpha = 12 \sum_{i,j} C^{-1}_{i,j} r_i r_j + 2(N-1) - \sum_i r_i^2$$

$$\beta = N - 1 - \frac{1}{2} \sum_i r_i^2$$

$$\gamma = \frac{1}{240} \left(\frac{7}{2} \sum_i r_i f_i + 30(N-1) \right)$$

$$\delta = -\frac{1}{120} \left(\sum_i r_i f_i + 60(N-1) \right)$$

Cremonesi, Tomasiello '15

- $\Delta d_H \sim -\Delta \delta \sim -\Delta d_{\mathcal{O}}$
- Allows for proof of a-theorem for these flows

Summary and Future Research

- So far…
  - 6D SCFTs have been classified
  - There are remarkable relationships between 6D SCFTs and two classes of homomorphisms
  - There is strong evidence for c-theorems in 6D
- In the future…
  - Can we classify full set of 6D RG Flows in terms of group theory data?
  - Can we prove a-theorem in full generality?