Scattering in large N matter Chern Simons Theory

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Last lecture

- Pure Chern Simons theory. Gauge invariant. Coupling inverse of an integer.
- No local degrees of freedom. All degrees of freedom topological. No states on S^2. Finite dimensional Hilbert space on T^2.
- Expectation value unknotted Wilson loops effectively factorize
- Theoy non trivial when we add matter. Can easily be made conformally invariant.

Chern Simons theory coupled to fermionic matter

- Simple example of conformal theories.

$$S_{\rm matter} = \int \bar{\psi} \left(D_\mu \gamma^\mu + m_F \right) \psi$$

  where ψ is a fermion in any representation of the gauge group.
- Atleast at small enough λ_F this theory is conformal at $m_F = 0$ in dim reg. Proof. Gauge coupling cant run. No other relevant or marginal operator.
- This talk. Fermion taken to be in the fundamental representation. Will be able to solve theory at all λ_F at large N because effectively vector model (no dof in gauge field)

Chern Simons - Bosonic matter theories

- The second theory we study in this talk is the Chern Simons theory coupled to bosonic matter

$$\int \left(D_\mu\bar{\phi}D^\mu\phi + m_B^2\bar{\phi}\phi + \frac{1}{2N_B}b_4(\bar{\phi}\phi)^2 \right)$$

- We will be particularly interested in a scaling limit of the bosonic theory. To explain this we rewrite the theory as

$$\int \left(D_\mu\bar{\phi}D^\mu\phi + m_B^2\bar{\phi}\phi + \frac{b_4}{2N_B}(\bar{\phi}\phi)^2 - \frac{N_B}{2b_4}\left(\sigma - \frac{b_4}{N_B}\bar{\phi}\phi - m_B^2\right)^2 \right)$$
$$\int \left(D_\mu\bar{\phi}D^\mu\phi + \sigma\bar{\phi}\phi + N_B\frac{m_B^2}{b_4}\sigma - N_B\frac{\sigma^2}{2b_4} \right) \tag{1}$$

Chern Simons coupled to critical bosons

- The so called critical limit of the bosonic theory is defined by the limit

$$b_4 \to \infty, \quad m_B \to \infty, \quad \frac{4\pi m_B^2}{b_4} = m_B^{\rm cri} = {\rm fixed}. \qquad (2)$$

- In this limit

$$S = \int \left(D_\mu \bar{\phi} D^\mu \phi + \sigma \bar{\phi} \phi + N_B \frac{m_B^{\rm cri}}{4\pi} \sigma \right)$$

- Atleast in the large N limit, this theory is also conformal at $m_B^{\rm cri} = 0$. $m_B^{\rm cri} = 0$ is a relevant deformation of this critical theory.

Conjectured Duality

- It has been conjectured that the Fermionic Chern Simons theory is dual in the large N limti to the critical boson Chern Simons theory under the duality map

$$\begin{aligned} \kappa_F &= -\kappa_B, \\ N_F &= |\kappa_B| - N_B, \\ \lambda_B &= \lambda_F - \mathrm{sgn}(\lambda_F), \\ m_F &= -m_B^{\mathrm{cri}} \lambda_B. \end{aligned} \tag{3}$$

- The duality map is stated in the t' Hooft large N limit and could receive subleading corrections at finite N and κ. Note that the map for the t' Hooft coupling is precisely that of level rank duality.
- Prior to this talk: three kinds of concrete evidence.

Evidence for duality: Spectrum

- Specialize to $m_F = m^{\text{cri}} = 0$. Spectrum of single trace operators (better single sum operators) easy to compute. Method: compute in free theory and then prove non renormalization theorem.
- Lets start with the fermions. Compute partition function

$$Z_{ss} = \mathrm{tr} x^{\Delta} \mu_3^J$$

over single trace fermionic operators. Operators are schematically $(\partial\psi)(\partial\bar{\psi})$. It follows that $Z_{ss}^F = (Z_L^F)^2$.

- $$Z_L^F = \frac{x(\mu^{\frac{1}{2}} + \mu^{-\frac{1}{2}})}{(1-\mu x)(1-x)(1-\frac{x}{\mu})} - \frac{x^2(\mu^{\frac{1}{2}} + \mu^{-\frac{1}{2}})}{(1-\mu x)(1-x)(1-\frac{x}{\mu})} \frac{x(\mu^{\frac{1}{2}} + \mu^{-\frac{1}{2}})}{(1-\mu x)(1-\frac{x}{\mu})} \quad (4)$$

- Can decompose the trace into a sum over characters of representations of the conformal algebra. Representations labled by scaling dimension Δ and spin J. From unitarity

$$\delta \geq J + 1$$

Representations with $\Delta = J + 1$ are short.

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$$Z^L_{\Delta,J} = \frac{x^\delta \chi_J(\mu)}{(1-\mu x)(1-x)(1-\frac{x}{\mu})} \qquad (5)$$

$$Z^s_J = Z^L_{J+1,J} - Z^L_{J+2,J-1}$$

Null states $\partial_\mu J^{\mu\mu_1\mu_2\ldots\mu_{j-1}}$

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$$Z^F_{ss} = Z_{2,0} + \sum_{J=1}^{\infty} Z_{J+1,J}$$

$$\text{Spectrum} = (2,0) + \sum^{\infty}(j+1,j)$$

Spectrum: Non Renormalization

- $(2, 1)$ and $(3, 2)$ protected operators (conserved charge and stress tensor). What about $(4, 3)$. Short rep. Can only become long by combining with $(5, 2)$. But unique spin 2 rep cannot be renormalized. No $(5, 2)$ to combine with. Implies $(4, 3)$ not renormalized. Similarly implies $(5, 4)$ is not renormalized. And so on. Shows *Large N* non renormalization of all operators except $(2, 0)$.
- Can show
$$\partial . J_S = \frac{JJ}{\kappa} + \frac{JJJ}{\kappa^2}$$
Nonlinear identity plus nonrenormalization of all other J then implies nonrenormalization of J_0 as well.
- Similar analysis for scalars. For free scalar
$$\text{Spectrum} = (1, 0) + \sum_{j=1}^{\infty} (j, j + 1)$$
But critical theory $(1, 0) \to (2, 0)$. Matches Fermions.

Three point functions and thermal partition functions

- $$\partial \langle JJJ \rangle \sim \langle JJ \rangle \langle JJ \rangle$$
- This structure used by Maldacena and Zhibedeov to greatly constrain the structure of 3 pt functions. Then Aharony and collaborators demonstrted matching under duality map
- Very impressive. But consequence of symmetry. Some map had to work. Indepdent evidence? Computation of thermal partition functions. Relation to susy duality.

Scattering amplitudes and bosonization

- Bose-Fermi or bosonization duality. Sounds interesting. Would like to understand in more detail. Best case: bosonisation formula like $\psi = e^{\phi}$. Too much to ask for in current context as ψ and ϕ are not gauge invariant.
- While arbitrary insertions of ψ and ϕ are not meaningful, insertions that are taken to infinity along lines of equations of motion - i.e. S matrices - are well defined. For this reason the map between S matrices is a ‘poor mans bosonization’ map between these two theories.
- This talk: We indpependently compute the S matrix for the bosons and fermions, and examine their interrelationship under duality. Also uncover interesting structural features along the way.

Scattering Amplitudes: Colour Kinematics

- In either the bosonic or fermionic theory we refer to a quantum the transforms in the fundamental of $U(N)$ as a particle and a quantum that tranforms in the antifundamental of $U(N)$ as an antiparticle.
- We study the most general 2×2 scattering process. There are three such processes

$$\begin{aligned} P_i + P_m &\to P_j + P_n \\ P_i + A^j &\to P_n + A^n \\ A^j + A^n &\to A^i + A^j \end{aligned} \tag{7}$$

- All these S matrices are contained in appropriate on shell limits of the single correlator. In e.g. the bosonic theory this is

$$< \phi_i \phi_m \bar{\phi}^j \bar{\phi}^n >$$

Color Kinematics

- It follows from $U(N)$ invariance that

$$< \phi_i \phi_m \bar{\phi}^j \bar{\phi}^n >= a\delta_i^j \delta_m^n + b\delta_i^n \delta_m^j$$

- Consequently, in each channel we need to compute 2 distinct scattering amplitudes. For particle-particle scattering it is convenient to work in the following basis for $U(N)$ singlets

$$< \phi_i \phi_m \bar{\phi}^j \bar{\phi}^n >= T_{sym} \left(\delta_i^j \delta_m^n + \delta_i^n \delta_m^j \right) + T_{as} \left(\delta_i^j \delta_m^n - \delta_i^n \delta_m^j \right)$$

$T_{Sym} = \frac{a+b}{2}$ is the S matrix in the channel of symmetric exchange, while $T_{as} = \frac{a-b}{2}$ is the S matrix in the channel of antisymmetric exchange.

Colour Kinematics, $\frac{1}{N}$

- On the other hand for particle - antiparticle exchange the following basis is most useful

$$< \phi_i \phi_m \bar{\phi}^j \bar{\phi}^n >= T_{Sing} \frac{\delta_i^j \delta_m^n}{N} + T_{Adj} \left(\delta_i^n \delta_m^j - \frac{\delta_i^j \delta_m^n}{N} \right)$$

  $T_{Sing} = Na + b$ is the scattering matrix in the singlet channel, while $T_{Adj} = b$ is the scattering matrix in the adjoint channel.

- It is not difficult to convince oneself that, at leading order in the $\frac{1}{N}$ expansion

$$a \sim b \sim T_{sym} \sim T_{as} \sim T_{Adj} \sim \frac{1}{N}$$

  On the other hand

$$S_{Sing} \sim \mathcal{O}(1)$$

- We have chosen the colour structures multiplying T_{Sing} etc to be simply projectors onto the exchange representations. As projectors square to unity it follows from unitarity that

$$T_c - T_c^\dagger = T_c T_c^\dagger$$

  T_c is any one of the four T matrices described above. We have used the fact that e.g. $2 \to 4$ production contributes to unitarity only at subleading order in $\frac{1}{N}$.
- The unitarity equation is rather trivial for T_{sym}, T_{as}, T_{adj}, as the RHS is subleading in $\frac{1}{N}$. However it imposes a highly nontrivial nonlinear constraint on T_{sing}, the most nontrivial of the four scattering matrices.

Non Relativistic Limit

- The S matrix for non relativistic particles interacting via Chern Simons exchange was worked out in the early 90s, most notably by Bak, Jackiw and collaborators.
- The main result is strikingly simple. Consider the scattering of two particles in representations R_1 and R_2, in exchange channel R. It was demonstrated that the S matrix equals the scattering matrix of a $U(1)$ charged particle of unit charge scattering off a point like flux tube of magnetic field strength $\nu = \frac{c_2(R_1)+c_2(R_2)-c_2(R)}{\kappa}$.
- This quantum mechanical S matrix was computed originally by Aharonov and Bohm and generalized by Bak and Camillio to take account of possible point like interactions between the scattering particles.

Non Relativistic Scattering Amplitude

- $$T_{NR} = -16\pi i c_B \left(\cos\left(\pi\nu\right) - 1\right) \delta(\theta) + 8 i c_B \sin(\pi\nu) \mathrm{Pv} \left(\cot\frac{\theta}{2}\right)$$
$$+ 8 c_B |\sin \pi\nu| \frac{1 + e^{i\pi|\nu|} \frac{A_{NR}}{k^{2|\nu|}}}{1 - e^{i\pi|\nu|} \frac{A_{NR}}{k^{2|\nu|}}},$$
$$A_{NR} = \frac{-1}{w} \left(\frac{2}{R}\right)^{2|\nu|} \frac{\Gamma(1+|\nu|)}{\Gamma(1-|\nu|)}. \tag{8}$$

- Here $wR^|2\nu|$ is a measure of the strength of the Bak-Camillio contact interaction between the scattering particles. In the limit $w(kR)^|2\nu| \to 0$ $A_{NR} \to \infty$ and the second line of the scattering amplitude simplifies to
$$8 c_B |\sin \pi\nu|,$$
the original Aharonov Bohm result

The δ function and its physical interpretation

- The non relativistic amplitude above has a very unusual feature, a piece in the scattering amplitude proportional to the $\delta(\theta)$.
- This term in the scattering amplitude was missed in the original paper of Aharonov and Bohm. The amplitude was corrected with the addition of this piece in the early 80s. In the early 90s Jackiw and collabortors emphasized that is term is necessary to unitarize Aharonov Bohm scattering.
- Infact this term has a simple physical interpretation (white board). The physical interpretation makes no reference to the non relativistic limit, so we assume that the δ function piece is present unmodified even in relativistic scattering.

Effective value of ν

- In the large N limit there is a simple formula for the quadratic Casimir of representations with a finite number of boxes plus a finite number of antiboxes.

$$c_2(R) = \frac{N(n_b + n_a)}{2}$$

- using this result in the formula for ν we find

$$\nu_{sym} \sim \nu_{as} \sim \nu_{Adj} \sim \mathcal{O}(1/N)$$

  On the other hand

$$\nu_{Sing}| = \lambda$$

- Now the non relativistic limit is obtained by taking $k \to 0$ at fixed ν. If $\nu \sim \mathcal{O}(1/N)$ and if we work to leading order in $\frac{1}{N}$ we effectively take $\nu \to 0$ first. In other words the results of Aharonov Bohm Bak Camillio yield a sharp prediction for the non relativistic limit only of T_{sing}.

Exact Propagators: method

- Work in Lorentzian space. Set $A_- = 0$. Main advantage: $\int A^3 = 0$. No gauge boson self interactions. Also no IR divergences (contrast with $A_0 = 0$).
- If we want to understand scattering we first have understand free propagation.
- No gauge self interactions plus planarity gives a simple Schwinger Dyson equation (like t' Hooft model). Nonlinear integral equation. Quite remarkably admits exact solution

Exact Bosonic Propagator

- In the case of the bosons we have

$$\langle \phi_j(p) \bar{\phi}^i(-q) \rangle = \frac{(2\pi)^3 \delta^i_j \delta^3(-p+q)}{p^2 + c_B^2} \tag{9}$$

where the pole mass, c_B is a function of m_B, b_4 and λ_B, given by

$$c_B^2 = \frac{\lambda_B^2}{4} c_B^2 - \frac{b_4}{4\pi} |c_B| + m_B^2. \tag{10}$$

Exact Fermionic Propagator

- $$\left\langle \psi_j(p)\bar{\psi}^i(-q)\right\rangle = \frac{\delta^i_j (2\pi)^3 \delta^3(-p+q)}{i\gamma^\mu p_\mu + \Sigma_F(p)}, \qquad (11)$$

  where

  $$\begin{aligned} \Sigma_F(p) =& i\gamma^\mu \Sigma_\mu(p) + \Sigma_I(p) I, \\ \Sigma_I(p) =& m_F + \lambda_F \sqrt{c_F^2 + p_s^2}, \\ \Sigma_\mu(p) =& \delta_{+\mu} \frac{p_+}{p_s^2}\left(c_F^2 - \Sigma_I^2(p)\right), \\ c_F^2 =& \left(\frac{m_F}{\mathrm{sgn}(m_F) - \lambda_F}\right)^2 . \end{aligned} \qquad (12)$$

- Nontrivial (and necessary) that pole in propagator $p^2 - c_F^2 = 0$ is Lorentz invariant.

Restrictions on soln of 4 point function

- Now turn to evaluating the 4 point function needed for computing scattering. Linear integral equation. Shamefully unable to solve in general. Exact solution for the special case $q_{\pm} = 0$.
- Defect has different implications in different channels
- In the singlet exchange channel it turns out that q_μ is the centre of mass energy. If $q_{\pm} = 0$ then momentum centre of mass momentum is spacelike. Incompatable with putting onshell. Solution not useful for directly computing T_{sing}.
- In the other three channels q_μ is the momentum transfer. In the scattering of two particles of equal mass, the momentum exchange is always spacelike. Setting $q_{\pm} = 0$ is simply a choice of Lorentz frame. Assuming the answer is covariant, there is no loss of information. Can read off full results for T_{Adj}, T_{sym}, T_{as}.

Onshell 4 point function for bosons

- Evaluating the onffshell 4 pt function subject to this restriction and taking the onshell limit we find
- $$T = \frac{4\pi i}{\kappa_B}\epsilon_{\mu\nu\rho}\frac{q^\mu (p-k)^\nu (p+k)^\rho}{(p-k)^2} + j(\sqrt{q^2}). \tag{13}$$
- Where

$$j(q) = \frac{4\pi i q}{\kappa_B}\left(\frac{\left(4\pi i q\lambda_B + \widetilde{b}_4\right) + \left(-4\pi i q\lambda_B + \widetilde{b}_4\right)\left(\frac{\frac{1}{2}+\frac{c_B}{iq}}{-\frac{1}{2}+\frac{c_B}{iq}}\right)^{\lambda_B}}{\left(4\pi i q\lambda_B + \widetilde{b}_4\right) - \left(-4\pi i q\lambda_B + \widetilde{b}_4\right)\left(\frac{\frac{1}{2}+\frac{c_B}{iq}}{-\frac{1}{2}+\frac{c_B}{iq}}\right)^{\lambda_B}} \right). \tag{14}$$

Limits of $j(q)$

- In the limit $\lambda_B \to 0$ $j(q)$ reduces to

$$j = \frac{-b_4}{1 + b_4 H(q)} \tag{15}$$

where

$$H(q) = \int \frac{d^3 r}{(2\pi)^3} \frac{1}{\left(r^2 + c_B^2\right)\left((r+q)^2 + c_B^2\right)} \tag{16}$$

Matches well known exact answer for large N ϕ^4 scattering.

- If we take $b_4 \to \infty$ and then take the limit $\lambda_B \to 1$ we get the even simpler answer
-

$$j(q) = -8\pi c_B. \tag{17}$$

Matches tree Fermionic scattering amplitude!

Fermionic Results and duality

- We can repeat the computation for the fermions at all values of λ_F. Can show that the final result exactly matches with the Bosonic onshell amplitude once we substitute

$$\lambda_F = \lambda_B - \mathrm{sgn}\lambda_B$$

- This implies $T^B_{Adj} = T^F_{Adj}$
- Moreover

$$T^B_{sym} = T^F_{as}, \quad T^B_{as} = T^F_{sym}$$

This is the result we should have expected both from level rank duality as well as from basic statisitics.

Scattering in the singlet channel (bosons)

- According to standard lore we can obtain T_{sing} by analytically continuing the above results.
- Performing the naive analytic continuation gives a result that cannot be right. To start with it does not have the delta function piece that we know must be there on physical grounds. Indeed no analytic continuation can give this piece as it is not analytic.
- Clearly the rules for crossing symmetry must be modified. By playing around we have come up with the following conjecture

$$T_{sing} = \frac{\sin(\pi\lambda_B)}{\pi\lambda_B} T^{ac}_{sing} - i(\cos(\pi\lambda_B) - 1) I(p_1, p_2, p_3, p_4)$$

where T_S^{trial} is the singlet amplitude one obtaines from naive analytic continuation

Properties of conjectured S channel formula

Our conjecture for the exact scattering matrix has the following features

- It agrees with the well known result for ϕ^4 scattering as $\lambda \to 0$ (trivial check).
- Agrees with tree level scattering for Fermions at $\lambda_B \to 1$. More generally it maps under duality to a similar conjectured formula on the Fermionic side.
- Exactly obeys the nonlinear unitarity equation

$$T - T^\dagger = TT^\dagger$$

  in a highly nontrivial manner
- Agrees at one loop with explicit computation in Lorentz gauge
- Has the correct nonrelativisitic limit. More below.

Explicit Conjecture

- Our conjecture is

$$T_{sing} = 8\pi i\sqrt{s}(1-\cos(\pi\lambda_B))\delta(\theta) + 4i\sqrt{s}\sin(\pi\lambda_B)\mathrm{Pv}\left(\cot\left(\frac{\theta}{2}\right)\right) + 4\sqrt{s}\sin(\pi|\lambda_B|)F;$$

$$F = \left(\frac{\left(4\pi|\lambda_B|\sqrt{s}+\widetilde{b}_4\right) + e^{i\pi|\lambda_B|}\left(-4\pi|\lambda_B|\sqrt{s}+\widetilde{b}_4\right)\left(\frac{\frac{1}{2}+\frac{c_B}{\sqrt{s}}}{\frac{1}{2}-\frac{c_B}{\sqrt{s}}}\right)^{|\lambda_B|}}{\left(4\pi|\lambda_B|\sqrt{s}+\widetilde{b}_4\right) - e^{i\pi|\lambda_B|}\left(-4\pi|\lambda_B|\sqrt{s}+\widetilde{b}_4\right)\left(\frac{\frac{1}{2}+\frac{c_B}{\sqrt{s}}}{\frac{1}{2}-\frac{c_B}{\sqrt{s}}}\right)^{|\lambda_B|}} \tag{18}$$

Straight forward non relativistic limit

- The straight forward non relativistic limit is taken by taking $\sqrt{s} \to 2c_B$. In this limit $F = \mathrm{sgn}(\lambda_B)$ and

$$T_{sing} = 8\pi i\sqrt{s}(1 - \cos(\pi\lambda_B))\delta(\theta) + 4i\sqrt{s}\sin(\pi\lambda_B)\mathrm{Pv}\left(\cot\left(\frac{\theta}{2}\right)\right)$$
$$+ 4\sqrt{s}|\sin(\pi|\lambda_B|)| \tag{19}$$

- This is in perfect agreement with the Aharonov Bohm result. Natural quesiton: is there any way to get the Bak-Camillio modification out of our formula? Ans: yes! There is a second, more sophisiticated non relativistic limit one can take.

Tuned Non Relativistic limit

- We first note that our conjectured S matrix has a pole for $\widetilde{b}_4 \geq \widetilde{b}_4^{crit} = 8\pi c_B|\lambda_B|$ indicating the existence of a particle - antiparticle bound state in the singlet channel at these values of parameters
- As $\widetilde{b}_4$ approaches $\widetilde{b}_4^{crit}$ from above, the mass of the bound state approaches $2c_B$. Setting $\widetilde{b}_4 = \widetilde{b}_4^{crit} + \delta b_4$, it may be shown that $E_B \sim (\delta b_4)^{1/|\lambda_B|}$) at small δb_4
- This observation motivates study of the scaled non relativistic limit

$$\frac{\delta b_4}{c_B} \to 0, \quad \frac{k}{c_B} \to 0, \quad \frac{k}{c_B}\left(\frac{c_B}{\delta b_4}\right)^{\frac{1}{2|\lambda_B|}} = \text{fixed.} \qquad (20)$$

Reduction in tuned non relativistic limit

- In the tuned non relativistic limit the nontrivial term in our conjectured S matrix simplifies to

$$8c_B|\sin(\pi\lambda_B)|\frac{1+e^{i\pi|\lambda_B|}\left[\frac{\delta b_4\left(\frac{2c_B}{k}\right)^{2|\lambda_B|}}{16\pi|\lambda_B|c_B}\right]}{1-e^{i\pi|\lambda_B|}\left[\frac{\delta b_4\left(\frac{2c_B}{k}\right)^{2|\lambda_B|}}{16\pi|\lambda_B|c_B}\right]},$$

- This agrees exactly with the Bak -Camillio generalization of the Aharonov Bohm scattering provided we identify

$$-w\,(c_BR)^{2|\lambda_B|}=\frac{c_B}{\delta b_4}\left(16\pi|\lambda_B|\frac{\Gamma(1+|\lambda_B|)}{\Gamma(1-|\lambda_B|)}\right). \tag{21}$$

- I view this nontrivial match as an extremely nontrivial check of our conjecture

Possible explanation of $\frac{\sin \pi\lambda}{\pi\lambda}$

- Interesting observation: $\frac{\sin \pi\lambda}{\pi\lambda}$ equals Witten's result for the expectation value of a circular Wilson loop on S^3 in pure Chern Simons theory in the large N limit.
- This observation suggests a possible explanation of the modified crossing symmetry rules (details on white board)
- If this is right it is the tip of the iceberg. Finite N and κ.

Bosons-Fermions?

- Let us accept the conjecture for the moment. How did fermions turn into bosons? Different answers in different channels.
- Adjoint channel. Boring. No phase, no statistics. Boring.
- Sym and as channel. No phase but statistics. Level rank duality on Young Tableax of exchange representations
- Singlet channel. Most interesting. Anyonic phase. Neither bosons nor fermions. $e^{i\pi\lambda_B} = e^{i\pi\lambda_F} e^{-i\pi \mathrm{sgn}\lambda_\mathrm{F}}$ implies $\lambda_B = \lambda_F - \mathrm{sgn}\lambda_F$. Equating anyonic phases in the singlet channel gives a derivation of the anyonic phase!

Brief Summary of ongoing work

- Idea: generalize computation to theories one boson and one fermion that preserves atleast $N = 1$ sypersymmetry.
- Motivation. To check duality for these theories. To check that the conjecture for crossing symmetry continues to work for these theories. Make contact with extensive literature on scattering in $N = 6$ theories.
- Method. Have found an offshell $N = 1$ generalization of lightcone gauge. Removes gauge boson self interactions. Can repeat all calculations presented here in superspace. Have done this. Have verified that, in the on shell limit, our results are in perfect agreement with the duality map on this manifold that I conjectured with Jain and Yokoyama from the study of partition functions
- Have also verified that our results dramatically simpilfy when the $N = 1$ theories become $N = 2$ susy.
- In process of verifying that the same conjecture for S channel S matrix unitarizes scattering.

Conclusions

- Presented computations and conjectures for all orders scattering matrices in Chern Simons matter theories.
- Results in perfect agreement with duality
- Suggest novel transformations of S matrices under crossing symmetry. Current conjectures are large N results. Study of crossing at finite N and k a very interesting problem.
- Generalizations to susy theories underway. Should evenually make contact with scattering in ABJM theories.