Supersymmetry in Curved Space

Outline:

1. Review Flat Space $\mathcal{N}=1$ $\mathbb{R}^4$. Including Subtleties...

2. Flat Space $\longrightarrow$

Curved Space.

$$\delta S = \int T_{\mu\nu} h^{\mu\nu} d^4x + \dots$$

$$g_{\mu\nu} = \delta_{\mu\nu} + h_{\mu\nu}$$

3. Z_{M_4} exactly.

$$\partial^\mu S_{\mu\alpha} = 0 \quad ; \quad Q_\alpha = \int d^3x \, S_{0\alpha}$$

$$\{ Q_\alpha , \bar{Q}_{\dot\alpha} \} = 2 \sigma^\mu_{\alpha\dot\alpha} P_\mu .$$

Q_α : operators on H.S.

$\longrightarrow$ differential operators

$$[\xi^\alpha Q_\alpha + \bar\xi_{\dot\alpha} \bar{Q}^{\dot\alpha} , S]$$

$$\longrightarrow i (\xi \mathcal{Q} + \bar\xi \bar{\mathcal{Q}}) \cdot S$$

$$Q_\alpha = \frac{\partial}{\partial \theta^\alpha} - i\sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_\mu$$

$$\bar Q_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} + i\sigma^\mu_{\alpha\dot\alpha}\theta^\alpha\partial_\mu$$

Chiral superfield

$$[\bar Q, \phi] = 0.$$

$$\Phi = \phi(y) + \sqrt{2}\,\theta\psi_\alpha + \theta^2 F$$

$$y = x + \theta\sigma^\mu\bar{\theta}$$

$$S(x,\theta,\bar{\theta}) = C + \theta\psi + \cdots$$
$$+ \frac{1}{2}\theta^2\bar{\theta}^2 D$$

σ-model in $d=4$

$$L = \int d^4\theta \; K(\Phi^i, \bar{\Phi}^{\bar{i}})$$

$$= g_{i\bar{j}} \partial_\mu \phi^i \partial^\mu \bar{\phi}^{\bar{j}} + \cdots$$

$$g_{i\bar{j}} = \partial_i \partial_{\bar{j}} K$$

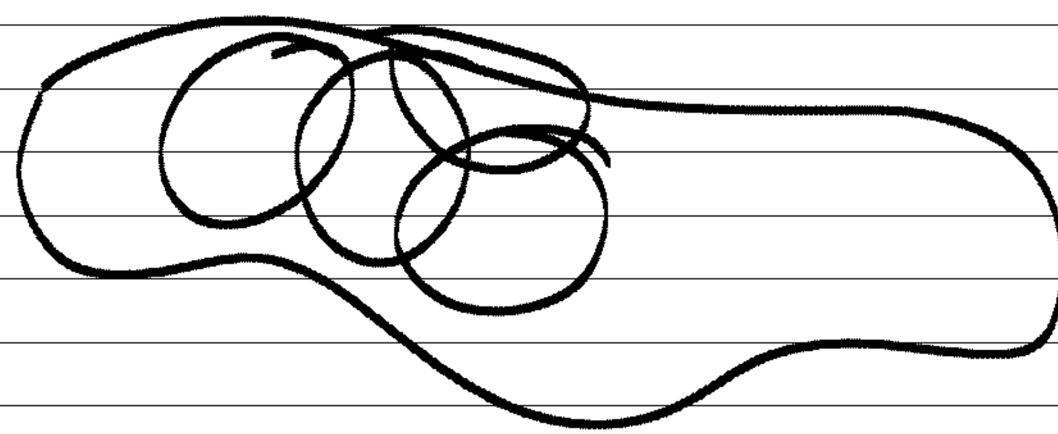
$\mathbb{R}^4 \to \mathcal{M}$

- $\Phi'^i = f^i(\Phi^j)$
- $K \to K + F(\Phi) + \bar{F}(\bar{\Phi})$

$$\Phi' = f(\Phi) \quad , \quad K \to K + F + \bar{F}$$

F: holomorphic.

M



$M = \bigcup U_\alpha$

Non-trivial

$$K_{U_\alpha} \longrightarrow K_{U_\beta} + F_{\alpha\beta} + \bar{F}_{\alpha\beta}.$$

$$S^2 = \mathbb{CP}^1 \qquad ds^2 = \frac{1}{(1+|z|^2)^2}\, dz\, d\bar{z}$$

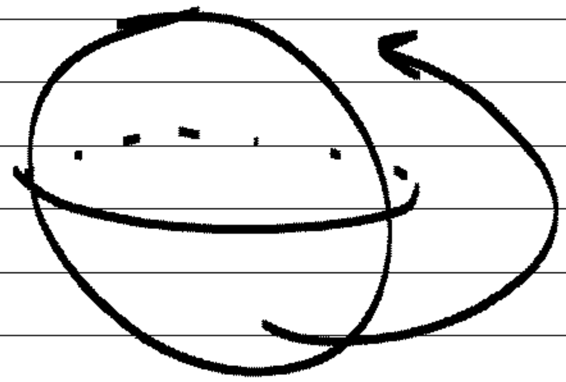
$$L = \int d^4\theta\, f_\pi^2 \log\left(1+|\Phi|^2\right)$$

$$g_{\phi\bar\phi} = f_\pi^2\, \partial_\phi \partial_{\bar\phi} \log\left(1+|\phi|^2\right)$$

$$= f_\pi^2 \Big/ (1+|\phi|^2)^2$$

$$\Phi \to 1/\Phi$$

$$F(\Phi) = \int_{\Omega}^{2} \log(\Phi) .$$



Proof No global K exists:

$$\mathrm{vol}(S^2) = \int \sqrt{g} = \int g_{\phi\bar\phi} \underset{\substack{\text{if } K \\ \text{existed}}}{=} \int \partial\bar\partial K = 0$$

contradiction.

Compact Kähler manifold

$\rightarrow$ nowhere has a globally defined

K. $\omega = i g_{i\bar{j}}\, dz^i \wedge d\bar{z}^{\bar{j}}$

$d\mathrm{Vol} = \omega^n$

$\rightarrow$ if K is well defined

ω is exact $\rightarrow$ dVol is exact $\rightarrow$ contradiction.

Gauge Fields, SUSY-breaking

$V \to V + i(\Lambda - \bar{\Lambda})$ i Λ chiral

$W_\alpha = -\frac{1}{4}\bar{D}^2 D_\alpha V$: gauge invariant

$\mathcal{L} = \frac{1}{4e^2}\int W^\alpha W_\alpha d^2\theta + c.c.$

(θ - term)

$\mathcal{L}_{FI} = \xi_{FI}\int d^4\theta\, V$.

$\partial^\mu j_\mu = 0$ (Flavor symmetry)

$Q \equiv \int d^3x\, j_0$

Multiplet ?

$[Q_\alpha, j_\mu(x)] =$ total derivatives.

{Schwinger Terms}

$\left([Q_\alpha, \underset{\int j}{Q}] = 0 \right)$

$[Q_\alpha, j_\mu(x)]$ = schwinger terms

multiplet that contains j_μ

better not have $\delta > 1$.

$$\mathcal{J} = J + (\theta^\alpha j_\alpha + c.c.) + (\theta \sigma^\nu \bar\theta)(j_\nu + \underline{der}\ ?)$$

$$+ \ldots \leftarrow \theta^3, \theta^4$$

$$D^2 \mathcal{J} = 0 \quad , \bar{D}^2 \mathcal{J} = 0$$

Linear multiplet

$$\boxed{[Q_\alpha, j_\mu(x)] = (\sigma_{\mu\nu})_\alpha{}^\beta \partial^\nu j_\beta + \ldots}$$

$\mu = 0 \rightarrow \nu$ spacelike.

$$\boxed{D^2 J = \bar{D}^2 J = 0}$$ superspace conservation

$$\delta L = \int J V \, d^4\theta + \underbrace{O(V^2)}_{\text{seagull terms}}$$

$$V \to V + i(\Lambda - \bar{\Lambda})$$

$$\delta_\Lambda \int J V = i \int J \Lambda + \text{c.c.}$$

= total der

$$d^4\theta \to \bar{D}^2$$

1

$$i\,j_\mu = \phi \overleftrightarrow{\partial} \phi^\dagger + \text{fermions} \qquad i\,\partial^\mu j_\mu = 0$$

$$\rightarrow \quad j_\mu A^\mu + (A^\mu)^2 |\phi|^2$$

$$|D\phi|^2 \supset (A^\mu)^2 |\phi|^2$$

seagull terms.

Example: $\quad K = \Phi^\dagger \Phi$

$\Phi \to e^{i\alpha} \Phi \quad ; \quad \mathcal{J} = \Phi^\dagger \Phi \Rightarrow J = |\phi|^2$.

$\partial^\mu S_{\alpha\mu} = 0 \qquad Q_\alpha = \int S_{0\alpha}\, d^3x$.

In which multiplet $S_{\alpha\mu}$ sits?

- curved space
- NoRenormalization Thm
- Proof of a nontrivial property of moduli

⋮

$$\{\bar{Q}_{\dot\alpha}, S_{\alpha\nu}(x)\} = 2\sigma^{\mu}_{\alpha\dot\alpha} T_{\mu\nu} + \text{schwinger terms.}$$

$$\left(\{\bar{Q}, Q\} = P \right)$$
$$\quad \int S \qquad \int T$$

set of schwinger terms is non-unique.

1. Ferrara - Zumino multiplet

spin ≤ 2

$$J_\mu = \cdots + (\theta\sigma^\nu\bar\theta)(T_{\mu\nu} + \cdots) + \cdots$$

$$\left\{\begin{array}{l} l_\mu = \frac{1}{4}\bar\sigma_\mu^{\dot\alpha\alpha}\, l_{\alpha\dot\alpha} \\ l_{\alpha\dot\alpha} = -2\sigma^\mu_{\alpha\dot\alpha}\, l_\mu \end{array}\right\}$$

$$\Rightarrow \boxed{\bar D^{\dot\alpha} J_{\alpha\dot\alpha} = D_\alpha X} \;, \quad \bar D X = 0$$

$$(D^2 J = 0 \,,\ \bar D^2 J = 0)$$

$$\mathcal{J}_\mu = j_\mu^{(FZ)} - i\theta\left(S_\mu + \tfrac{1}{3}\sigma_\mu\bar{\sigma}_\nu S^\nu\right) + c.c$$

$$+ \tfrac{i}{2}\theta^2\partial_\mu x^+ - \tfrac{i}{2}\bar{\theta}^2\partial_\mu x$$

$$+ \left(\theta\sigma^\nu\bar{\theta}\right)\left(2T_{\mu\nu} - \tfrac{2}{3}\eta_{\mu\nu}T + \tfrac{1}{2}\varepsilon_{\mu\nu\rho\sigma}\partial^\rho j^{(FZ)\sigma}\right)$$

$$+ \theta^3(\ ) + \theta^4(\ ).$$

$T_{\mu\nu}$ conserved | $j_\mu^{(FZ)}$ is not conserved.

$S_{\mu\alpha}$ conserved |

$$\{\bar{Q}_{\dot\alpha}, S_{\alpha\mu}\} = \sigma^{\nu}_{\alpha\dot\alpha}\Big(2T_{\mu\nu} + \underline{i\partial_{\nu} j^{(FZ)}_{\mu}} - \underline{i\eta_{\mu\nu}\partial\cdot j^{(FZ)}}$$

$$+ \underline{\tfrac{1}{2}\varepsilon_{\mu\nu\rho\sigma}\partial^{\rho} j^{(FZ)\sigma}}\Big). \quad \eta_{\mu\nu} = \begin{pmatrix} -1 & & \\ & 1 & \\ & & 1 \\ & & & 1 \end{pmatrix}$$

$$\{Q_{\beta}, S_{\mu\alpha}\} = \text{pure s.t.} =$$

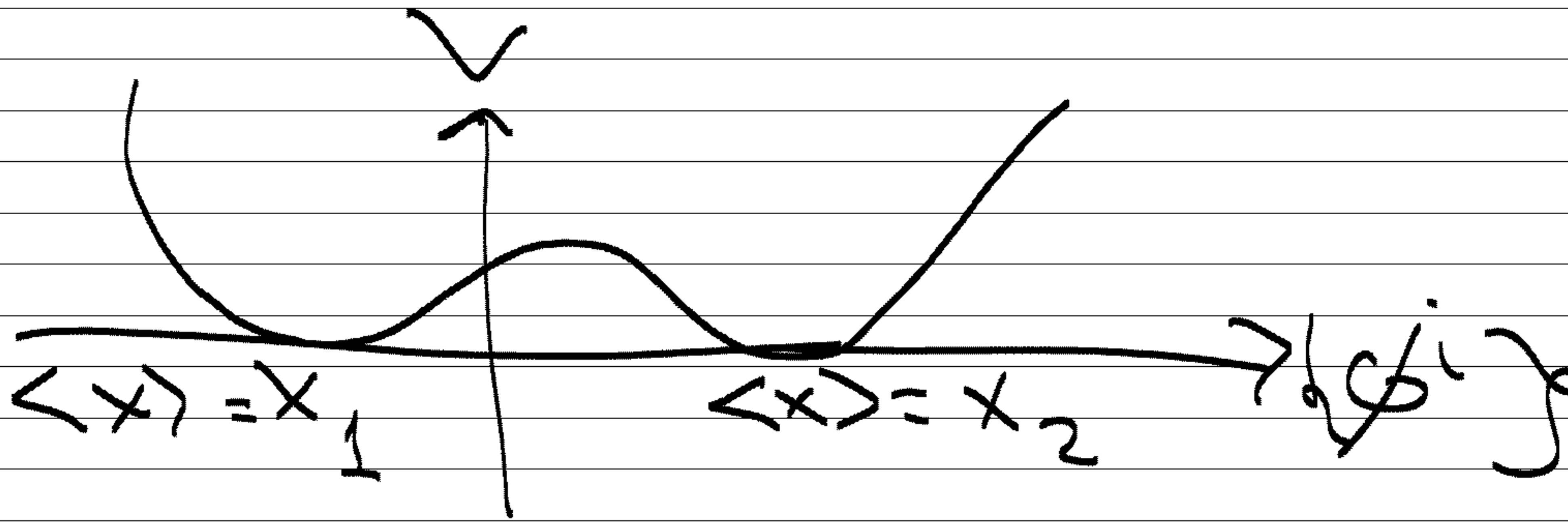
$$= 2i\,\varepsilon_{\lambda\beta}(\sigma_{\mu\rho})^{\lambda}_{\alpha}\,\partial^{\rho}X^{+}$$

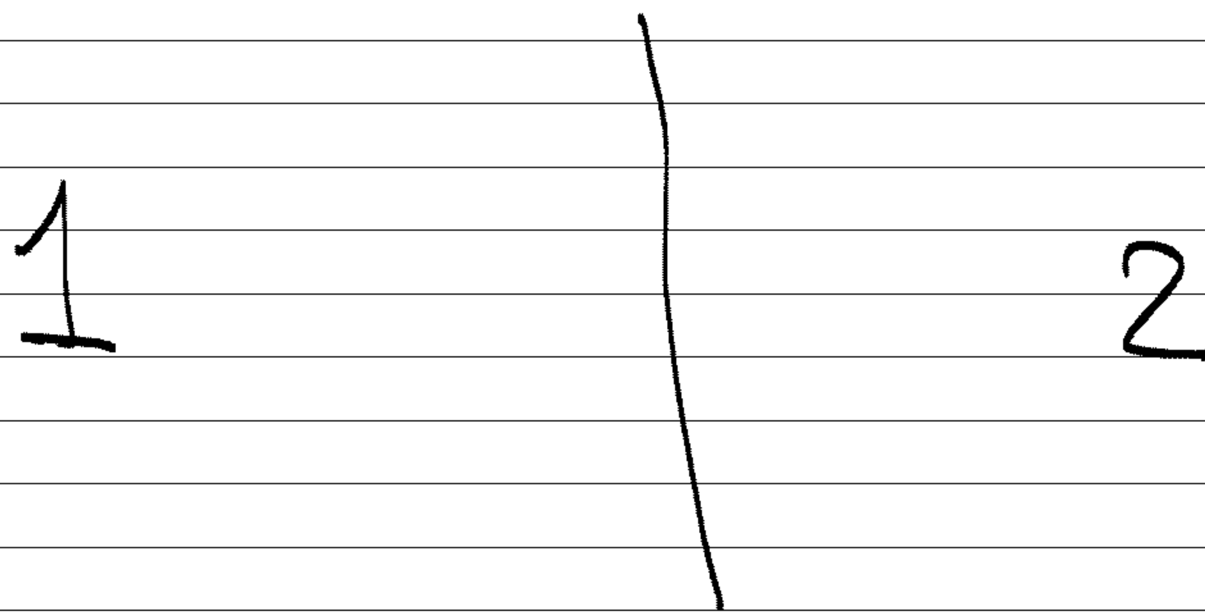
$$\{Q^2 = 0\}$$

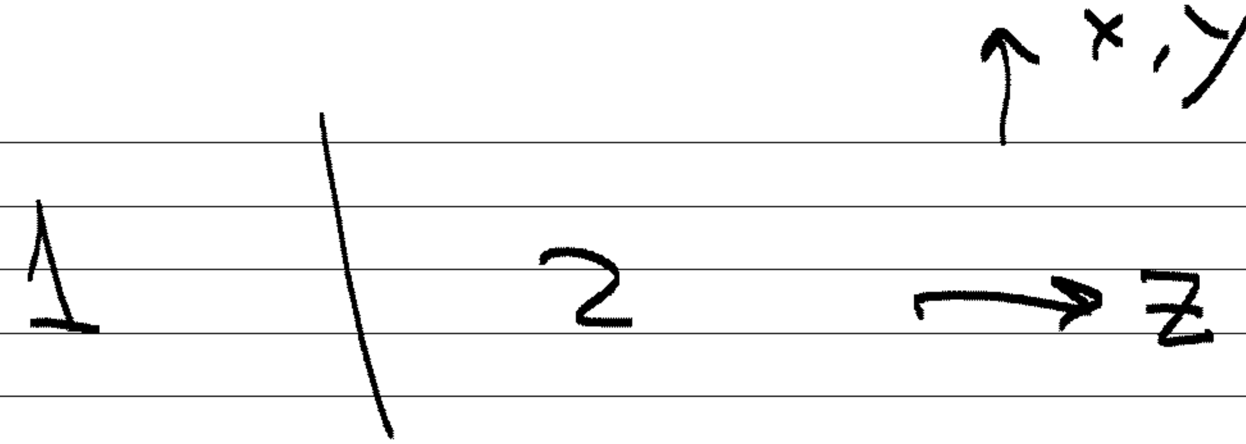
$\mu = 0 \longrightarrow \rho$ space-like

$\partial^{\mu} \longrightarrow 0$

It turns out that in some interesting situation does not integrate to 0.







$$G_{\mu\nu\rho} = \epsilon_{\mu\nu\rho\sigma}\,\partial^{\sigma} X$$

$$\partial^{\mu} G_{\mu\nu\rho} = 0$$

3-form current

$$\boxed{\text{charge} = \int G_{0\nu\rho} \sim \Delta X}$$

Domain wall breaks 1/2 SUSY

$$\{\tilde{Q}, \bar{\tilde{Q}}\} = \underbrace{E}_{\text{Tension}} + \text{charge} \Rightarrow \boxed{\text{Tension} = \Delta X}$$

Example:

σ-model $\quad L = \int d^4\theta\, K + \int d^2\theta\, W + c.c.$

$$J_{\alpha\dot\alpha} = 2 g_{i\bar j} D_\alpha \Phi^i \bar D_{\dot\alpha} \bar\Phi^{\bar j} \boxed{- \frac{2}{3}\boxed{[D_\alpha, \bar D_{\dot\alpha}]} K}$$

$$X = 4W - \frac{1}{3}\bar D^2 K$$

$$\boxed{\begin{array}{l} D^2 J = 0 \\ \bar D^{\dot\alpha} J_{\alpha\dot\alpha} = D_\alpha X \end{array}}$$

$$\langle X \rangle \sim \langle W \rangle$$

$$\boxed{\text{Tension} = \Delta W}$$

$$(\text{SYM } \Delta W \sim \Lambda^3_{QCD})$$

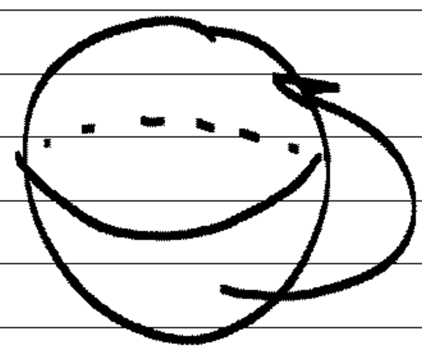
$$\langle \bar D^2 K \rangle = 0$$

$$\sim \langle \bar Q^2 K \rangle = 0$$

$$J_{\alpha\dot{\alpha}} \supset -\frac{2}{3}[D_\alpha, \bar{D}_{\dot{\alpha}}] K \, .$$

depends on K ! ?

multiplet of $S_{\mu\alpha}$ appears to be necessarily dependent on K ?!



$$\delta J_{\alpha\dot{\alpha}} \sim [D, \bar{D}] \log(\bar{\Phi}) \sim \sigma^{\mu}_{\alpha\dot{\alpha}} \partial_\mu \log \bar{\Phi} \neq 0$$

observers cannot agree on $T_{\mu\nu}$!

Non-Renorm : $4D,\ \mathcal{N}=1$

UV $\longleftarrow K = \bar{\Phi}^i \Phi^{i+} e^V \ ,\ W = \cdots$

$\Bigg\{ \downarrow$ UV-free QFT . . .

FZ : exists.

IR $\rightsquigarrow$ target space is $\boxed{\text{non-compact}}$

$\left(\begin{array}{l} \text{Kähler Class} = 0 \\ K : \text{exists} \end{array} \right)$

Conjecture

FZ multiplet exists iff

- No FI terms
- vanishing Kähler class

→ FI terms are never generated!

$$\det M = B\tilde{B} + \Lambda^{2N}$$

S^2

$$\bar{D}^{\dot{\alpha}} \mathcal{J}_{\alpha\dot{\alpha}} = D_\alpha X$$

where: $$\mathcal{J}_\mu = j_\mu^{(FZ)} + \theta S + \bar{\theta}\bar{S} + \theta\bar{\theta}\,(T_{\mu\nu}) + \theta^2 \partial x^{+} + \bar{\theta}^2 \partial x + \ldots$$

$$X = x + \sqrt{2}\,\theta\psi + \theta^2 F$$

$$\psi = \frac{i\sqrt{2}}{3}\,\sigma^\mu_{\alpha\dot{\alpha}}\,\bar{S}^{\dot{\alpha}}_\mu$$

$$F = \frac{2}{3} T^\mu_\mu + i\,\partial^\mu j_\mu^{(FZ)}$$

X order parameter for non-conformality

Review of yesterday

We considered the multiplet of $\underbrace{T_{\mu\nu}}$, $S_{\mu\alpha}$ ....

Ferrara-Zumino multiplet

$$\bar{D}^{\dot{\alpha}} J_{\alpha\dot{\alpha}} = D_\alpha X \,, \qquad \bar{D} X = 0$$

$$(j_\mu^{FZ}, X, T_{\mu\nu}) \oplus (S_{\mu\alpha})$$

$$4 \quad 2 \quad 6 \qquad\qquad 12$$

$$12 \qquad \oplus \qquad 12$$

- FI model:

$$\mathcal{J}_{\alpha\dot\alpha} = -\frac{4}{e^2} W_\alpha \bar{W}_{\dot\alpha} - \frac{2}{3} \xi_{FI} [D_\alpha, \bar{D}_{\dot\alpha}] V$$

This isn't gauge invariant

$\Rightarrow$ FZ multiplet does not exist

$\xi_{FI} = 0$ in the UV

ξ for all $U(1)$ gauge symmetries $= 0$

There is another $12 \oplus 12$ solution : R-multiplet.

$\boxed{\text{Only if } \exists \text{ R-symmetry.}}$

$$\mathcal{R}_\mu = j_\mu^{(R)} + \left(\theta^\alpha S_{\alpha\mu} + c.c.\right)$$
$$+ \left(\theta \sigma^\nu \bar{\theta}\right) \left(2 T_{\mu\nu} + \tfrac{1}{2} \varepsilon_{\mu\nu\rho\sigma} \partial^\rho j^{(R)\sigma} + \tfrac{1}{8} \varepsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}\right)$$
$$+ \cdots$$

R-multiplet:

$$\left(j_\mu^{(R)}\ ,\ F_{\mu\nu}\ ,\ T_{\mu\nu}\right)\oplus\left(S_{\mu\alpha}\right)$$

$$12\oplus 12$$

$$\partial^\mu j_\mu^{(R)} = 0$$

$$dF = 0 \quad \left(\partial_\rho F_{\mu\nu}\,\varepsilon^{\sigma\rho\mu\nu} = 0\right)$$

F: BPS - STRINGS

Since in the R-multiplet no domain wall charge

$\Rightarrow$ R-symmetry no BPS domain walls.

Flavor symmetry $\quad D^2 J = 0$

$\Rightarrow \int J V d^4\theta \quad \bar{D}^2 J = 0$

+ seagull terms

$\sim \int R_{\alpha\dot{\alpha}} \mathcal{H}^{\alpha\dot{\alpha}} d^4\theta$ + seagull terms

$\mathcal{H}_\mu \supset (\theta\sigma^\nu\bar{\theta})(h_{\mu\nu} + \cdots)$

$$\int R_{\alpha\dot{\alpha}}\, \mathcal{H}^{\alpha\dot{\alpha}}\, d^4\theta \supset T_{\mu\nu}\, h^{\mu\nu}$$

$$\boxed{\delta_L \mathcal{H}_{\alpha\dot{\alpha}}} = D_\alpha \bar{L}_{\dot{\alpha}} - \bar{D}_{\dot{\alpha}} L_\alpha$$

$$\longrightarrow \bar{D}_{\dot{\alpha}} D^2 \bar{L}^{\dot{\alpha}} = D^\alpha \bar{D}^2 L_\alpha$$

$$\boxed{\begin{array}{l} 1.\ \bar{D}^{\dot{\alpha}} R_{\alpha\dot{\alpha}} = \chi_\alpha \ , \quad \bar{D}\chi_\alpha = 0 \\ D^\alpha \chi_\alpha = \bar{D}_{\dot{\alpha}} \bar{\chi}^{\dot{\alpha}} \ . \end{array}}$$

$$\Gamma_{\dot\alpha} \supset \cdots + \theta^{\alpha} P_{\alpha\dot\alpha} + \cdots$$

$$D_{\alpha} \bar{\Gamma}_{\dot\alpha} \supset \cdots + \underbrace{\theta \sigma^{\nu} \bar\theta\, \partial P}_{} + \cdots$$

$$\delta h_{\mu\nu} = \partial_{\mu} P_{\nu} + \cdots$$

$j^{(R)}_{\mu}$	$A^{(R)}_{\mu}$	$(A^{(R)}_{\mu} \to A^{(R)}_{\mu} + \partial_{\mu}\omega)$
$F_{\mu\nu}$	$B_{\mu\nu}$	$(\delta B_{\mu\nu} = \partial_{[\mu}\omega_{\nu]}$
$T_{\mu\nu}$	$g_{\mu\nu}$	$(\delta g_{\mu\nu} = \partial_{(\mu}\xi_{\nu)})$

QFT (SUSY)

$\{Q, \bar{Q}\} = 2 P_\mu \sigma^\mu_{\alpha\dot{\alpha}}$

$\partial_\mu J_\alpha = 0$

want to put in curved space

$$\left[A^{(R)}_\mu, B_{\mu\nu}, g_{\mu\nu} \right]$$

such that some SUSY is (unpreserved

$\partial_\mu \to \nabla_\mu$ $\quad$ $\underline{(A^{(R)}_\mu = B_{\mu\nu} = 0)}$

$$\nabla_\mu J_\alpha = 0$$

CY $\quad$ $\mathbb{T}^4, K_3$

$$\to \left(\nabla_\mu - A^{(R)}_\mu\right) J_\alpha = 0 \qquad [B_{\mu\nu} = 0]$$

Kähler Manifolds $\mathcal{M}_4$

$S^2 \times \mathbb{T}^2 \ldots$

$$(\nabla_\mu - iA_\mu^{(R)})\zeta_\alpha = -iV_\mu \zeta_\alpha - iV^\nu (\sigma_{\mu\nu})_\alpha{}^\beta \zeta_\beta \quad (*)$$

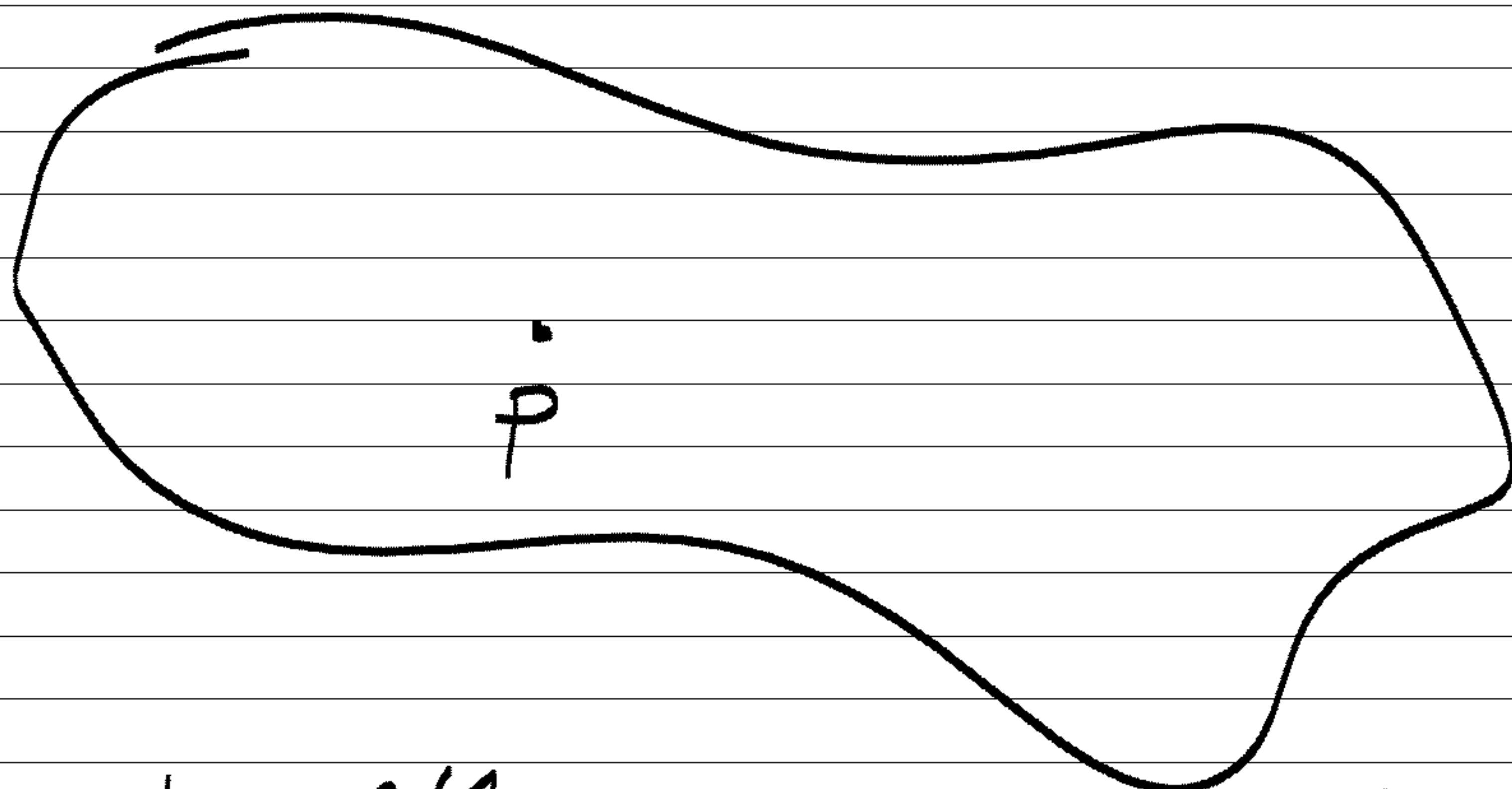
$$V_\mu \equiv \tfrac{1}{2} \varepsilon_{\mu\nu\rho\sigma} \partial^\nu B^{\rho\sigma} .$$

M_4 admits SUSY iff $\exists\, \zeta_\alpha$ satisfying $(*)$.

Not studied by mathematicians.

$$\delta_\zeta \psi_\alpha = 0 \Rightarrow \psi_{\mu\alpha} = 0$$

$\rightarrow (\nabla - iA)\xi = V_\mu \xi - \sigma_{\mu\nu} V^\nu \xi$



solution

$\xi_\alpha(p) \rightarrow$ fixed

$2_{\mathbb{C}}$ space of sols.

together with the eq. for $\bar{\xi}_{\dot\alpha}$,

$4_{\mathbb{C}}$ at most. ≤ 4 supercharges

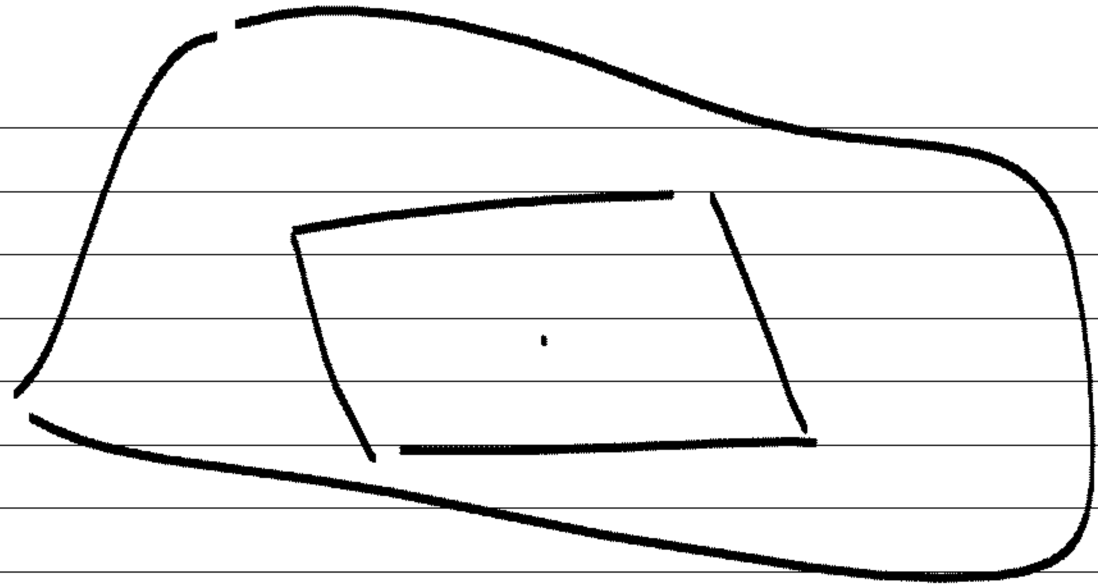
$\forall p$. $\quad \xi \xi^\dagger = |\xi|^2 > 0$

$$S = |\zeta|^2 > 0$$

$$J_{\mu\nu} = \frac{2i}{|\zeta|^2} \zeta^\dagger \sigma_{\mu\nu} \zeta$$

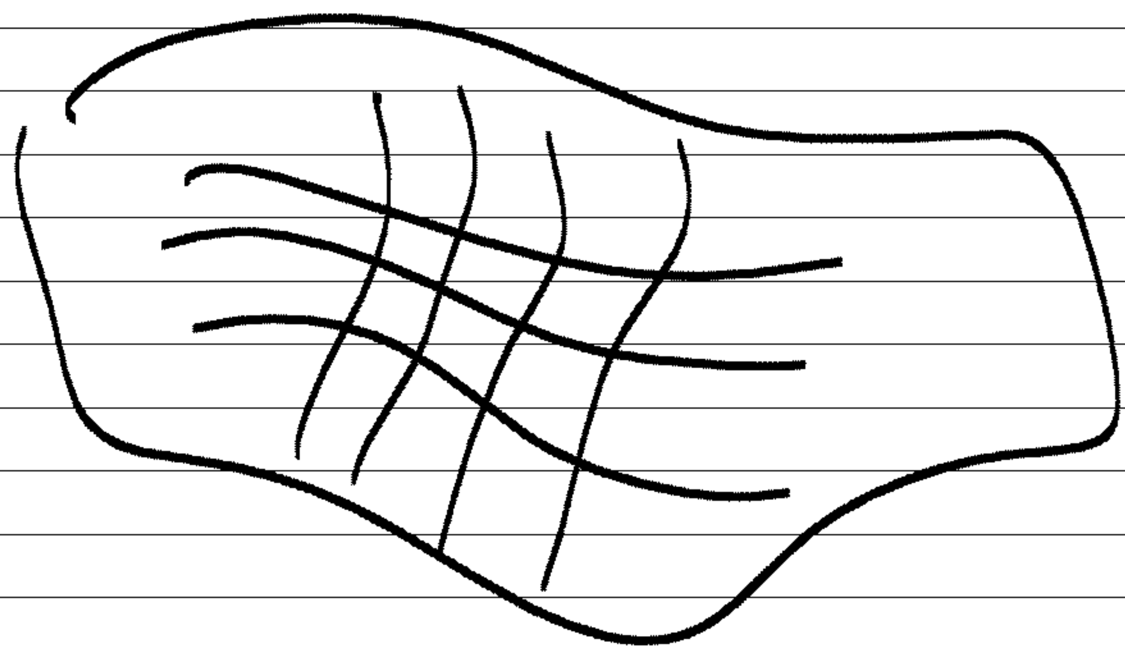
$$J^\mu{}_\nu J^\nu{}_\rho = -\delta^\mu_\rho \quad (J^2 = -1)$$

Almost-Complex Manifolds



$$J^2 = -1 \quad \begin{cases} i \\ -i \end{cases}$$

$$T_p = T_p^{+i} \oplus T_p^{-i}$$



$$z^i, \ \bar{z}^i$$

$$\frac{\partial}{\partial z^i} \in T_p^{+i}, \quad \frac{\partial}{\partial \bar{z}^i} \in T_p^{-i}$$

Manifolds for which this can be done are called Complex.

Nijenhuis TENSOR

$$N[J_{\mu\nu}] = 0$$

1950's.

$\updownarrow$

complex

$$N^{\mu}{}_{\nu\rho} = J^{\lambda}{}_{\nu} \nabla_{\lambda} J^{\mu}{}_{\rho} - J^{\lambda}{}_{\rho} \nabla_{\lambda} J^{\mu}{}_{\nu}$$

+ 2 permutations.

$$J_{\mu} = \frac{2i}{|\zeta|^2} \zeta^{\dagger} \sigma_{\mu} \zeta .$$

Miracle : Yes! $N = 0$.

SUSY can be preserved iff

$\boxed{M_4 \text{ is complex}}$,

$$ds^2 = g_{i\bar{j}}\, dz^i\, d\bar{z}^{\bar{j}}$$

1 supercharge $\Leftrightarrow$ M_4 complex

4 supercharges $\Leftrightarrow$ $\mathbb{R}^4$, $\mathbb{T}^4$

$S^3 \times S^1$

compact space

$H^3 \times S^1$

S^4 is not complex

naively J_α may exist only if spin

J_α spin $\times \underbrace{\overbrace{L}}$ = $\underline{\underline{\text{spin}^c}}$

variant of twisting

$\mathcal{N} = \underline{1}$!

$\mathcal{N} = 2$ bigger SUGRA multiplet

more manifold

OPEN PROBLEM

$g_{\mu\nu}$ doesn't need to satisfy Einstein's Equations.

$$(A_\mu^{(R)}, g_{\mu\nu}, B_{\mu\nu})$$

$$\bar{D}^{\dot{\alpha}} R_{\alpha\dot{\alpha}} = \chi_\alpha$$

$$\int R_{\alpha\dot{\alpha}} \mathcal{H}^{\alpha\dot{\alpha}} d^4\theta \supset A_\mu^{(R)} j^{(R)\mu} + g_{\mu\nu} T^{\mu\nu} + B_{\mu\nu} {}^*F^{\mu\nu} + \text{seagull terms}$$

off-shell sort!

$$\underbrace{S^3\times\mathbb{R}\;,\quad S^3\times S^1}$$

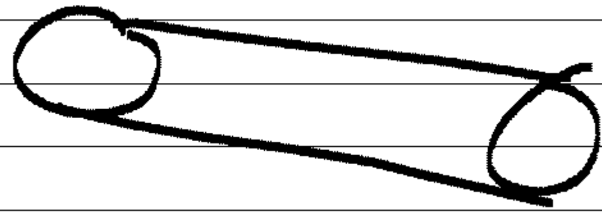
$$ds^2 = d\tau^2 + r^2 d\Omega_3^2$$

$$ds^2 = \left(dw - \frac{r\bar{z}}{1+|z|^2}dz\right)\left(d\bar{w} - \frac{rz}{1+|z|^2}d\bar{z}\right)$$

$$+\frac{r^2}{(1+|z|^2)^2}\,dz\,d\bar{z}$$

$S^3 \simeq S^1$ fibered over $S^2 \Rightarrow$ $S^3\times S^1$ torus over S^2

$$\omega \simeq \omega + 2\pi i r$$

 fiber $S^2 \simeq S^3 \times \mathbb{R}$

$$\boxed{A^{(R)} = -i/r\,(\partial\omega + \partial\bar{\omega}) \sim \frac{1}{r}\, d\tau}$$

$$V_\mu = A^{(R)}_\mu \quad \leftarrow \quad \int_{S^3} dB \neq 0$$

$$J_\alpha = \begin{pmatrix} a_1\, e^{-(\omega-\bar{\omega})/r} \\ a_2\, e^{+(\omega-\bar{\omega})/r} \end{pmatrix}, \quad \bar{J}_{\dot{\alpha}} \text{ similar}$$

Note that spinors don't depend on time (τ)!

S^3  $\leftarrow$ $\mathcal{H}(S^3)$ would sit in reps of our superalgebra.

$\xrightarrow[\tau]{\hspace{3em}}$ $S^3 \times \mathbb{R}$ $\quad A^{(R)}$ anything (flat)

$\boxed{\text{comment: } (\nabla - iA)_\mu \zeta_\alpha = \bar{\chi}_{\dot\alpha} \bar{\sigma}_\mu^{\dot\alpha}{}_{\alpha}}$

clarify!

4D

$\mathcal{N} = \underline{1}$

R ✓ $\quad$ $r = z$ ✓ $\quad$ $12 \oplus 12$

$\mathcal{S}'$? $\quad$ $16 \oplus 16$

$\mathcal{N} = \underline{2}$

SUSY in curved space?