Abstracts (40 Minutes Talk)

Minimal Legendrian surfaces in the 5-dimensional Heisenberg group

Kazuo Akutagawa (Tokyo Institute of Technology)

This talk is based on a joint work with Reiko Aiyama (University of Tsukuba). We consider *Legendrian surfaces* in the 5-dimensional Heisenberg group $\mathfrak{H}^5$. We give a *representation formula* for Legendrian surfaces in $\mathfrak{H}^5$, in terms of spinors. For especially *minimal* Legendrian surfaces, such data are holomorpic. We can regard this formula as an analogue (in Contact Geometry) of *Weierstrass representation* for minimal surfaces in $\mathbb{R}^3$. Hence for minimal ones in $\mathfrak{H}^5$, there are many similar results to those for minimal surfaces in $\mathbb{R}^3$. In particular, we will prove a *Halfspace Theorem* for *properly immersed* minimal Legendrian surfaces in $\mathfrak{H}^5$.

Let $\mathfrak{H}^5$ be the 5-dimensional *Heisenberg group*, that is, $\mathfrak{H}^5$ is $\mathbb{R}^5 = \mathbb{C}^2 \times \mathbb{R} = \{(\mathbf{z} = (z^1, z^2), t) \mid z^i = x^i + \sqrt{-1}y^i \in \mathbb{C}, t \in \mathbb{R}\}$ as a C^∞ manifold, and it has the group structure as $(\mathbf{z}, t) \cdot (\mathbf{z}', t') = (\mathbf{z} + \mathbf{z}', t + t' + 2\,\mathrm{Im}(\mathbf{z} \cdot \overline{\mathbf{z}'}))$. Here, $\mathbf{z}_1 \cdot \mathbf{z}_2$ denotes the standard $\mathbb{C}$-linear quadratic form. It has also a natural right invariant *contact 1-form* η defined by $\eta = dt - \frac{1}{2}(\mathbf{x} \cdot d\mathbf{y} - \mathbf{y} \cdot d\mathbf{x})$, that is, $\eta \wedge (d\eta)^2 \neq 0$ everywhere on $\mathfrak{H}^5$ and $(R_p)^*\eta = \eta$ for $p = (\mathbf{z}, t) \in \mathfrak{H}^5$. The *contact structure* $H := \mathrm{Ker}\,\eta$ is a codimension one totally non-integrable subbundle of the tangent bundle $T\mathfrak{H}^5$, which is spanned by the basis $\{T_i := \partial_{x^i} - \frac{1}{2}y^i\partial_t, T_{2+i} := \partial_{y^i} + \frac{1}{2}x^i\partial_t \mid i = 1, 2\}$. Associated with η, there exists a unique vector field $\xi \in \mathfrak{X}(\mathfrak{H}^5)$ with $\eta(\xi) = 1$ and $d\eta(\xi, \cdot) = 0$, the so-called *Reeb vector field*. In this case, $\xi = \partial_t$. Then, $T\mathfrak{H}^5$ has a natural decomposition

$$T\mathfrak{H}^5 = H \oplus \mathbb{R}\,\xi.$$

Let J be the almost complex structure on H defined by $J(T_i) = T_{2+i}, J(T_{2+i}) = -T_i$. We then get an inner product $g_H = d\eta \circ (J \otimes \mathbf{1})$ on H. Since the triple (η, g_H, J) induces the natural Riemannian metric $g = g_\eta$ on $\mathfrak{H}^5$ as

$$g_\eta = \pi_H^* g_H + \eta^2,$$

the triple is called a *contact Riemannian structure* on $\mathfrak{H}^5$. g_η is also called the standard *Sasakian metric* (or *Webster metric*) on $\mathfrak{H}^5$. Here, $\pi_H : T\mathfrak{H}^5 \to H$ is the natural projection associated with the above decomposition. A surface M of $\mathfrak{H}^5$ is said to be *Legendrian* if $T_pM \subset H_p$ for any $p \in M$, which is an integral submanifold of the distribution H with maximal dimension.

With these understandings, the below is the representation formula for minimal Legendrian surfaces in $(\mathfrak{H}^5, g_\eta)$.

Representation Formula. *Let M be a (simply connected) Riemann surface with an isothermal coordinate $w = u + \sqrt{-1}\,v$ around each point. Let $F = (F_1, F_2) : M \to \mathbb{C}^2$ be a holomorphic map satisfying $|S_1|^2 + |S_2|^2 \neq 0$ everywhere on M, where $S_1 := (F_2)_w$ and $S_2 := -(F_1)_w$. For a constant β, set $f : M \to \mathbb{C}^2 \times \mathbb{R}$ as*

$$\begin{cases} f = \left(\frac{1}{\sqrt{2}}e^{\sqrt{-1}\beta/2}(F_1 - \sqrt{-1}\ \overline{F_2}), \frac{1}{\sqrt{2}}e^{\sqrt{-1}\beta/2}(F_2 + \sqrt{-1}\ \overline{F_1}), t(w)\right), \\ t(w) = -\frac{1}{2}\,\mathrm{Re}\int (F_1S_1 + F_2S_2)dw\,. \end{cases}$$

Then, f is a minimal Legendrian conformal immersion from M to $(\mathfrak{H}^5, g_\eta)$. The induced metric ds^2 on M by f and its Gauss curvature K are respectively given by

$$ds^2 = (|S_1|^2 + |S_2|^2)|dw|^2, \qquad K = -2\,\frac{|S_1(S_2)_w - S_2(S_1)_w|^2}{(|S_1|^2 + |S_2|^2)^3}.$$

Conversely, every minimal Legendrian immersion $f : M \to (\mathfrak{H}^5, g_\eta)$ is congruent with the one constructed as above.

Remark. For a minimal Legendrian conformal immersion $f : M \to (\mathfrak{H}^5, g_\eta)$, $\pi_{\mathbb{C}^2} \circ f : M \to \mathbb{C}^2$ is a minimal Lagrangian immersion, where $\pi_{\mathbb{C}^2} : \mathbb{C}^2 \times \mathbb{R} \to \mathbb{C}^2$ denotes the natural projection. In the above representation formula, the constant β is the Lagrangian angle of $\pi_{\mathbb{C}^2} \circ f$. The Gauss map g of $\pi_{\mathbb{C}^2} \circ f$ is given by

$$g = [-S_2; S_1] = (-S_2/S_1) : M \to \mathbb{C}P^1 = \widehat{\mathbb{C}} \cong S^2.$$

Here, by the identification of S^2 with $S^2(1) \times \{(e^{\sqrt{-1}\beta/2}, 0)\} \subset \mathbb{R}^3 \times \mathbb{R}^3$, g can be regarded as the generalized Gauss map for the Lagrangian surface $(\pi_{\mathbb{C}^2} \circ f)(M)$ in $\mathbb{R}^4 = \mathbb{C}^2$. Set a holomorphic 1-form $hdw := S_1 dw$ on M. In terms of the Gauss data (hdw, g) of M, the induced metric ds^2 and the Gauss curvature K can be rewritten respectively by

$$ds^2 = |h|^2(1 + |g|^2)|dw|^2, \qquad K = -2\Big(\frac{|g_w|}{|h|(1 + |g|^2)^{3/2}}\Big)^2.$$

Set $\mathfrak{H}^5_{\geq t_0} = \{p \in \mathfrak{H}^5 \mid t(p) \geq t_0\}$ and $\mathfrak{H}^5_{\leq t_0} = \{p \in \mathfrak{H}^5 \mid t(p) \leq t_0\}$ for $t_0 \in \mathbb{R}$. The following is a Legendrian version of the Halfspace Theorem for minimal surfaces in $\mathbb{R}^3$.

Halfspace Theorem. *Any properly immersed minimal Legendrian surface in $(\mathfrak{H}^5, g_\eta)$ contained in an upper half space $\mathfrak{H}^5_{\geq t_0}$ (or a lower half space $\mathfrak{H}^5_{\leq t_0}$) is a Legendrian plane contained in $\{p \in \mathfrak{H}^5 \mid t(p) = t_1\} = \mathbb{C}^2 \times \{t_1\}$ for some $t_1 \in \mathbb{R}$. It is also a Lagrangian plane in $\mathbb{C}^2 = \mathbb{C}^2 \times \{t_1\}$.*

Heat flow and perimeter in Euclidean space

Michiel Van den Berg (University of Bristol)

Let Ω be an open set in Euclidean space $\mathbb{R}^m$ with finite perimeter $\mathcal{P}(\Omega)$, and with m-dimensional Lebesgue measure $|\Omega|$. It was shown by M. Preunkert that if $T(t)$ is the heat semigroup on $L^2(\mathbb{R}^m)$ and that if the boundary of Ω is of class $C^{1,1}$, then $H_\Omega(t) := \int_\Omega T(t)\mathbf{1}_\Omega(x)dx = |\Omega| - \pi^{-1/2}\mathcal{P}(\Omega)t^{1/2} + o(t^{1/2}),\ t \downarrow 0$. $H_\Omega(t)$ represents the amount of heat in Ω if Ω is at initial temperature 1 and if $\mathbb{R}^m \setminus \Omega$ is at initial temperature 0. In this talk we will
(i) compare the quantitative behaviour of $H_\Omega(t)$ with the usual heat content $Q_\Omega(t)$ associated to the Dirichlet heat semigroup on Ω,
(ii) present new results in the case where Ω is convex, bounded and $\partial\Omega$ is polygonal,
(iii) present a sufficient geometric condition on Ω with $|\Omega| = +\infty$ such that $H_\Omega(t) < \infty$ for all $t > 0$. We analyze $H_\Omega(t)$ for open sets of the form $\Omega(\alpha, \Sigma) = \{(x, x') \in \mathbb{R}^m : x' \in (1 + x)^{-\alpha}\Sigma, x > 0\}$, where $\alpha > 0$, and where Σ is an open set in $\mathbb{R}^{m-1}$ with finite perimeter in $\mathbb{R}^{m-1}$, which is star-shaped with respect to 0.

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Stable capillary hypersurfaces in a wedge

Jaigyoung Choe (KIAS)

Let Σ be an immersed stable hypersurface of constant mean curvature in a wedge bounded by two hyperplanes in $\mathbb{R}^n$. Suppose that Σ meets those two hyperplanes in constant contact angles and is disjoint from the edge of the wedge. We will show that if $\partial\Sigma$ is embedded for $n = 3$, or if $\partial\Sigma$ is convex for $n = 4$, then Σ is part of the sphere.

On gradient Ricci solitons with symmetries

Eduardo García-Río (University of Santiago de Compostela)

A *gradient Ricci soliton* is a triple (M, g, f) of a pseudo-Riemannian manifold (M, g) and a real-valued function f such that

$$\mathrm{Hes}_f + \rho = \lambda g,$$

where λ is a real constant, ρ denotes the Ricci tensor and Hess_f is the Hessian of the potential function f. The Ricci soliton is said to be *expanding*, *steady* or *shrinking* depending on whether $\lambda < 0$, $\lambda = 0$ or $\lambda > 0$.

A gradient Ricci soliton is said to be *rigid* if it locally decomposes as a product of an Einstein manifold and an Euclidean factor. Moreover, in such case, the potential function of the soliton is essentially determined by the norm of the projection on the Euclidean factor. The purpose of this lecture is to examine some symmetry conditions on the underlying manifold (constant scalar curvature, homogeneity, local symmetry) to characterize rigidity of the corresponding gradient Ricci soltions.

A Riemannian homogeneous gradient Ricci soliton is rigid. We explore two extensions of this rigidity result as follows. Firstly we consider Riemannian gradient Ricci solitons with constant scalar curvature and show that rigidity of the soliton structure follows from this assumption in some cases. Secondly we consider Lorentzian homogeneous gradient Ricci solitons, with special attention to the irreducible but not idecomposable ones. A complete classification of three-dimensional Lorentzian homogeneous gradient Ricci solitons is given.

The Chern-Gauss-Bonnet theorem for metrics of indefinite signature

Peter Gilkey (University of Oregon)

The Gauss-Bonnet theorem expresses a topological invariant (the Euler characteristic) of a Riemann surface in terms of curvature. It is named after Carl Friedrich Gauss (who first investigated the Gaussian curvature κ) and Pierre Ossian Bonnet (who published a special case of the Gauss-Bonnet theorem in 1848); the general case seems to be due to von Dyck (1888) who discussed the global version:

Theorem 1. *Let (M,g) be a 2-dimensional Riemannian manifold with piecewise smooth boundary ∂M. Let $\kappa = R_{1221}$ be the Gaussian curvature, let κ_g be the geodesic curvature, and let α_i be the interior angles at which consecutive smooth boundary components meet. Then* $2\pi \cdot \chi(M) = \int_M \kappa + \int_{\partial M} \kappa_g + \sum_i (\pi - \alpha_i)$.

This has been generalized by Chern [4] to higher dimensions (see related work in Allendoerfer and Weil [1]). The Riemannian measure and the Euler form (or Pfaffian) are defined by setting:

$$| \operatorname{dvol}_g | = | \det(g_{ij})|^{1/2} dx^1 \dots dx^m,$$
$$E_{2\ell}(g) := \frac{1}{(8\pi)^\ell \ell!} R_{i_1 i_2 j_2 j_1} \dots R_{i_{\ell-1} i_\ell j_\ell j_{\ell-1}} g(e^{i_1} \wedge \dots \wedge e^{i_\ell}, e^{j_1} \wedge \dots \wedge e^{j_\ell})\,.$$

Theorem 2. *Let M be a compact Riemannian manifold of dimension $m = 2\ell$ without boundary. Then* $\int_M E_{2\ell}(g) \operatorname{dvol}_g = \chi(M)$.

Theorem 2 has been generalized to the pseudo-Riemannian setting by Avez [3] and Chern [5]:

Theorem 3. *Let (M,g) be a compact pseudo-Riemannian manifold of signature (p,q) without boundary of dimension $m \equiv 0 \mod 2$. If $p \equiv 1 \mod 2$, then $\chi(M)$ vanishes. If $p \equiv 0 \mod 2$, then* $\chi(M) = (-1)^{p/2} \int_M E_m | \operatorname{dvol}_g |$.

We shall derive Theorem 3 from Theorem 2 directly by considering complex metrics and using analytic continuation (see Gilkey and Park [6]). We shall also derive the corresponding result for manifolds with boundary (reproving a result of Alty [2]) and related Euler-Lagrange equations (see Gilkey, Park, and Sekigawa [7]) in the indefinite setting from the corresponding results in the Riemannian setting. This is a joint work with J.H. Park.

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Spectral analysis for separable partial differential equations

Klaus Kirsten (Baylor University)

The eigenvalue spectrum $\{\lambda_k\}_{k\in\mathbb{N}}$ of certain interesting, mostly Laplace type, differential operators, encode many properties of physical systems or Riemannian manifolds. These properties are analyzed by considering suitable functions of the spectrum. Areas where these so-called spectral functions make their appearance are quantum field theory or quantum mechanics under external conditions and global analysis. A detailed understanding of properties of spectral functions is often warranted.

Probably the most intelligent organization of the spectrum is the zeta function, $\zeta(s) = \sum_{k=1}^{\infty} \lambda_k^{-s}$ with $s \in \mathbb{C}$ and $\Re s$ large enough, which is directly related to topics such as analytic torsion, the heat kernel, Casimir energies and effective actions. In some detail, the residues of the zeta function at certain values of s modulo a multiplicative constant equal the heat kernel coefficients in the small-time asymptotic behavior of the heat kernel. The derivative of the zeta function at $s = 0$ describes the analytic torsion of a manifold as introduced by Ray and Singer as well as effective actions in quantum field theories. Finally, the properties of $\zeta(s)$ at $s = -1/2$ contain information about the Casimir effect. These connections make it very desirable to have effective analytical tools for the complete analysis of zeta functions available. In particular, the analytic continuation of the above sum defining the zeta function to the relevant s-range needs to be constructed. The obvious problem is that an explicit knowledge of the eigenvalues is in general not given. For cases where the spectrum is not known explicitly only few methods for the analysis of properties of zeta functions are available.

In this talk, I will present the essential tools to analyze zeta functions resulting from separable partial differential equations. Examples considered comprise of generalized cones, spherical suspensions, warped manifolds and surfaces of revolution. This class of situations can be handled because an implicit eigenvalue equation is known. In this case a suitable integral representation of the zeta function involving the implicit eigenvalue equation is an excellent starting point. The implicit eigenvalue equation involves solutions to an initial value problem of an ordinary differential equation resulting from the process of separating variables. For the zeta function analysis relevant are certain uniform asymptotic properties of these solutions that follow from WKB analysis. Analytic expressions for residues, values and derivatives of the zeta function are given.

Gap theorems for locally conformally flat manifolds via Yamabe flows

Li Ma (Henan Normal University)

In this report, we consider the gap theorems for locally conformally flat Riemannian manifolds. We show that there is a remarkable relation between solubility of Poisson equation and global Yamabe flows. By using the well-known result of B.Chow that the Yamabe flows on the locally conformally flat manifolds with non-negative Ricci curvature have the differential Harnack inequality, we can show that the solutions to the Yamabe flows have the scalar curvature decay property, which gives the gap results based on the understanding of Poisson equations on such manifolds.

Nonstrongly isospectral lens spaces that are Hodge isospectral

Roberto Miatello (National University of Cordoba)

To every lens space L we associate a congruence lattice in $\mathbb{R}^m$, showing that two lens spaces are isospectral on functions if and only if the associated lattices are isospectral with respect to the 1-norm. We prove that the lens spaces are isospectral on p-forms for every p if and only if the associated lattices are 1-isospectral and satisfy an additional geometric condition. By constructing such congruence lattices we give infinitely many pairs of 5-dimensional lens spaces that are p-isospectral for all p but are not strongly isospectral. We also give such examples in arbitrarily high dimensions. We show that 1-isospectrality can be detected by calculation of finitely many numbers, which allows us to get all such examples for low values of the parameters using computer programs. This is a joint work with Emilio Lauret and Juan Pablo Rossetti.

Variations of almost Hermitian structures

Kouei Sekigawa (Niigata University)

In this talk, we mainly discuss the following two topics concerning one parameter deformations of almost Hermitian structures :

I. Critical almost Hermitian structures of the functional introduced by T.Koda which is an almost Hermitian extension of the Einstein-Hilbert functional.

II. Curvature identities on compact almost Hermitian surfaces. In the topic I, we show the almost Hermitian extension of the result of the Hilbert for the characterization of Einstein metrics and provide critical almost Hermitian structures of the Koda functional for several special classes of almost Hermitian manifolds. In the topic II, we derive curvature identities on compact almost complex surfaces from the integral formulas for the first Pontrjagin number and the first Chern number. This is a joint work with Yunhee Euh, Jungchan Lee and JeongHyeong Park.

Abstracts (20 Minutes Talk)

Conformal Killing-Yano tensors on solvable Lie groups

Isabel Dotti (National University of Cordoba)

A skew-symmetric (0,2)-tensor ω on an n-dimensional Riemannian manifold (M,g) is called *conformal Killing-Yano* if it satisfies the following equation:

$$(\nabla_X \omega)(Y,Z) = \frac{1}{3} d\omega(X,Y,Z) - \frac{1}{n-1}(X^* \wedge d^*\omega)(Y,Z),$$

where X, Y, Z are arbitrary vector fields on M. If moreover ω is co-closed, then ω is called *Killing-Yano*.

Killing-Yano forms were first introduced by Kentaro Yano in 1952 as a natural generalization of ordinary Killing vectors to forms. A Killing vector field is preserved by the geodesic flow and furthermore, the corresponding quantum mechanical operator commutes with the wave and the Dirac operators. Killing forms also allow to define operators which commute or anti-commute with the Dirac operator on the manifold.

In this talk we will show that, on a Lie group, conformal Killing tensors can only exist in odd dimensions, we will give a classification in dimension three and will construct families of examples in dimension five.

Γ-Gamma Extensions of the spectrum of an orbifold

Carla Farsi (University of Colorado)

The famous question 'Can you hear the shape of a manifold?' has been recently answered in the negative for both manifolds and orbifolds. Generalized spectra have been proposed to provide additional spectral invariants. In this talk, after reviewing some basic concepts of orbifold spectral theory, I will introduce the Γ-spectrum of an orbifold (here Γ is a finitely generated discrete group). This spectrum is the union of the Laplace spectra of the Γ-sectors of the orbifold, and hence constitutes an invariant that is directly related to the singular set of the orbifold. Many known examples of isospectral orbifolds need not be Γ-isospectral. We additionally present a version of Sunada's theorem that allows us the construction of pairs of orbifolds that are Γ-isospectral for any choice of Γ. This is a joint work with E. Proctor and C. Seaton.

Lefschetz fixed point formula on a compact manifold with boundary for some boundary condition

Rung-Tzung Huang (National Central University)

The Lefschetz fixed point formula on a closed manifold is a cohomological criterion for a map to have a fixed point, which uses the deRham complex. Brenner and Shubin obtained the Lefschetz fixed point formula on a compact manifold with boundary using absolute and relative de Rham complexes. In this talk, I will first introduce a de Rham complex on a compact manifold with boundary by introducing a pair of new boundary conditions. Then I will discuss the Lefschetz fixed point formula on a compact manifold with boundary using this new de Rham complex. This is a joint work with Yoonweon Lee.

A new de Rham complex associated to a well-posed boundary condition for the odd signature operator

Yoonweon Lee (Inha University)

In this talk I'd like to introduce a new de Rham complex on a compact oriented Riemannian manifold with boundary by using a well-posed boundary condition for the odd signature operator. The peculiar feature of this complex is to compute the relative cohomology and absolute cohomology in an alternating way. More precisely, this complex computes the relative cohomologies for even degrees and the absolute cohomologies for odd degrees. The advantage of using this complex is to be able to define the refined analytic torsion on a compact manifold with boundary, which is originally introduced by M. Braverman and T. Kappeler on a closed manifold. This is a joint work with R-T Huang.

Transversally harmonic and holomorphic maps on foliated manifolds

Seoung Dal Jung (Jeju National University)

Let $\phi : (M, \mathcal{F}) \to (M', \mathcal{F}')$ be a smooth foliated map, i.e., ϕ is a leaf-preserving map. A smooth foliated map ϕ is said to be *transversally harmonic* if ϕ is a solution of the transversal tension field $\tau_b(\phi) = 0$. For any non-zero basic function f, we define a transversal f-energy functional $E_f(\phi; \Omega)$ by

$$E_f(\phi; \Omega) = 1\overline{2} \int_\Omega |f d_T \phi|^2 \mu_M,$$

where $d_T\phi$ is the differential map of ϕ restricted to the normal bundle of $\mathcal{F}$. Then a transversally harmonic map is a critical point of the transversal f_κ-energy functional $E_{f_\kappa}(\phi; \Omega)$ of ϕ over a compact domain $\Omega \subset M$, where f_κ is a basic function such that $\kappa = d(\ln f^2)$ and κ is the mean curvature form of $\mathcal{F}$.

Let $(M, g, \mathcal{F}, J)$ and $(M', g', \mathcal{F}', J')$ be two Riemannian manifolds with Kähler foliations $\mathcal{F}$ and $\mathcal{F}'$, respectively. Then $\phi : (M, \mathcal{F}) \to (M', \mathcal{F}')$ is transversally holomorphic (anti-holomorhic) if $d_T\phi \circ J = J' \circ d_T\phi$ ($d_T\phi \circ J = -J' \circ d_T\phi$). Then we have the following theorem.

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Isospectral Riemannian surfaces

Hyunsuk Kang (KIAS)

We review the progress in isospectral manifolds, one of the topics in spectral geometry. The isospectrality here refers to the possession of the same eigenvalue spectrum of the Laplacian acting on smooth functions defined on Riemannian manifolds. The common technique to construct isospectral manifolds is the Sundada's covering method. In particular, isospectral Riemann surfaces of low genera will be discussed. This is a joint work with Dennis Barden.

Scalar curvature decrease of Riemannian metrics

Jongsu Kim (Sogang University)

In 1974, Muto in [3] has studied the behavior of the total scalar curvature and shown that for any Riemannian metric g on a compact manifold, the total scalar curvature can decrease as one deforms g to $g + th$ for some symmetric (2,0)-tensor h. Moreover, Muto could choose h supported in any ball.

A related scalar-curvature deformation result can be found in Lohkamp's paper [1], where it is shown that for any metric g in M and a ball $B \subset M$, and a function f such that $f = s(g)$ outside B and $f < s(g)$ inside B, for any $\varepsilon > 0$ there exists a deformed Riemannain metric g_ε whose scalar curvature $s(g_\varepsilon)$ lies $f - \varepsilon \leq s(g_\varepsilon) \leq f$ on B_ε and $g_\varepsilon = g$ on $M \setminus B_\varepsilon$, where B_ε is a ε-neighborhood of B. From the construction, these metrics can be close to g in C^0 sense, but not in C^∞ topology.

In another paper [2], Lohkamp has made the following conjecture in Riemannian geometry.

Conjecture. *Let (M^k, g_0), $k \geq 3$, be a manifold and $B \subset M$ an open ball. Then there is a C^∞-continuous path of Riemannian metrics g_t, $0 \leq t \leq \varepsilon$ on M with*
(i) *Ricci curvature of g_t is strictly decreasing in t on B.*
(ii) *$g_t \equiv g_0$ on $M \backslash B$.*

Concerning this conjecture, not even the scalar-curvature case is properly studied, although one may believe that for generic metrics such a scalar-curvature-decreasing path of metrics should exist.

Note that the conjecture is concerned with *local* deformation of metrics on a ball; on a compact Riemannian manifold, Muto's integral decrease and conformal deformation argument (Yamabe solutions) would produce a scalar-curvature decrease on the whole manifold. Here we prove the following result.

Theorem. *Given a Riemannian metric g on a manifold M of dimension≥ 3 and a ball B in M, we obtain C^∞-continuous paths of Riemannian metrics g_t, $0 \leq t < \varepsilon$ on M with $g_0 = g$ such that the scalar curvatures $s(g_t)$ strictly decrease, i.e. $s(g_{t_1}) > s(g_{t_2})$ for $t_1 < t_2$ on B, $g_t = g$ on $M \setminus B$.*

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Homogeneity of Lorentzian three-manifolds with recurrent curvature

S. Nikčević Stana Nikcevic (SANU, Serbia)

$k-$Curvature homogeneous three-dimensional Walker metrics are described for $k = 0, 1, 2$. This allows a complete description of locally homogeneous three-dimensional Walker metrics, showing that there exist exactly three isometry classes of such manifolds. As an application one obtains a complete description of all locally homogeneous Lorentzian manifolds with recurrent curvature. Moreover, potential functions are constructed in all the locally homogeneous manifolds resulting in steady gradient Ricci and Cotton solitons.

The geometry of Cotton tensor:conformal symmetry in dimension three

Ramón Vázquez-Lorenzo (University of Santiago de Compostela)

It is well-known that an n-dimensional pseudo-Riemannian manifold is locally conformally flat if the Weyl curvature tensor W vanishes, provided that $n \geq 4$. A pseudo-Riemannian manifold (of dimension ≥ 4) is conformally symmetric if $\nabla W = 0$. Non-trivial examples (i.e, those which are neither locally symmetric nor locally conformally flat) only exist in the strictly pseudo-Riemannian setting (see for example [2, 3] for further information on conformally symmetric manifolds).

Dimension three is exceptional due to the fact that the Weyl curvature tensor vanishes, and the conformal structure of the manifold is codified by the Cotton tensor [4]. The aim of this lecture is to report on recent investigations on conformally symmetric three-dimensional pseudo-Riemannian manifolds [1]. A complete description of three-dimensional manifolds with parallel Cotton tensor is given, with special attention to the examination of the isometry classes.

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The logarithmic singularities of the Green Functions of the conformal powers of the Laplacian

Raphaël Ponge (Seoul National University)

Motivated by the analysis of the singularity of the Bergman kernel of a strictly pseudoconvex domain, Charles Fefferman launched in the late 70s the program of determining all local biholomorphic invariants of strictly pseudoconvex domains. This program has since evolved to include other (parabolic) geometries such as conformal geometry. Green functions play an important role in conformal geometry at the interface of PDEs and geometry. In this talk, I shall explain how to compute explicitly the logarithmic singularities of the Green functions of the conformal powers of the Laplacian. These operators include the Yamabe and Paneitz operators, as well as the conformal fractional powers of the Laplacian arising from the scattering theory for asymptotically hyperbolic Einstein metrics. The results are formulated in terms of explicit conformal invariants defined by means of the ambient metric of Fefferman-Graham. As an application we obtain a spectral theoretic characterisation of the conformal classes of the round spheres. Although the problems and the final formulas only refer to analysis and geometry, the computations actually involves a significant amount of representation theory and ultimately boils down to some elaboration on Schur's duality.

L^2 harmonic 1-forms and first eigenvalue estimates of complete minimal submanifolds

Keom Kyo Seo (Sookmyung Women's University)

A minimal hypersurface in a Riemannian manifold is called *stable* if the second variation of its volume is always nonnegative for any normal variation with compact support. More precisely, an n-dimensional minimal hypersurface M in a Riemannian manifold N is called *stable* if it holds that for any compactly supported Lipschitz function f on M

$$\int_M |\nabla f|^2 - \Big(|A|^2 + \overline{\mathrm{Ric}}(\nu,\nu)\Big) f^2 dv \geq 0,$$

where ν is the unit normal vector of M, $\overline{\mathrm{Ric}}(\nu,\nu)$ denotes the Ricci curvature of N in the ν direction, $|A|^2$ is the square length of the second fundamental form A, and dv is the volume form for the induced metric on M.

Utilizing this analytic aspect of a stable minimal hypersurface, we estimate the smallest spectral value of the Laplace operator on a complete noncompact stable minimal hypersurface in a Riemannian manifold under the assumption on L^2 norm of the second fundamental form. Moreover, we obtain various vanishing theorems for L^2 harmonic 1-forms on minimal hypersurfaces.

Evolution of closed sections of vector bundles

Luigi Vezzoni (University of Torino)

In the talk it will be showed a general criterium to establish if a geometric flow involving closed sections of a vector bundle over a compact manifold has a short-time solution.

The criterium works in the case of the *Laplacian flow* and the *modified Laplacian coflow* in G_2-geometry introduced and firstly studied by Bryant-Xu and Grigorian, respectively. Moreover the criterium will be applied in order to study a *Calabi-type flow* in balanced geometry.

Abstracts (15 Minutes Talk)

The harmonicity and minimality of the characteristic vector field

Sunhyang Chun(Chosun University)

Every smooth unit vector field on Riemannian manifold determines a mapping between the Riemannian spaces and their unit tangent sphere bundle. We define the energy of unit vector fields as the energy of the corresponding map and the volume of unit vector fields as the volume of the immersion. In this way, one defines two functionals-energy functional and volume functional on the space of unit vector fields on the Riemannian manifold. A unit vector field which is critical for the energy functional is called a harmonic vector field; one which is critical for the volume functional is called a minimal vector field. In this talk, we introduce the geometric properties of the unit tangent sphere bundle whose characteristic vector field is harmonic or minimal. This is a joint work with J.H. Park and K. Sekigawa.

A curvature identity on a 6-dimensional Riemannian manifold and its applications

Yunhee Euh (National Institute for Mathematical Sciences)

We describe a curvature identity which holds on any 4-dimensional Riemannian manifold and its applications. Next, we give a curvature identity explicitly which holds for the case of dimension 6 using similar methods to those used in 4-dimensional case based on the Chern-Gauss-Bonnet theorem. As its application, we provide additional curvature identities on 5- and 6- dimensional harmonic manifolds focusing on an approach to the Lichnerowicz conjecture. Finally we give an another proof of the Lichnerowicz conjecture on 4-dimensional harmonic spaces using the obtained curvature identity.

Bertrand curves in the 3-dimensional sphere

Chanyong Kim (Sungkyunkwan University)

A Frenet curve C is called a Bertrand curve if there exists another Frenet curve $\bar{C}$, distinct from C, and a bijection f between C and $\bar{C}$ such that the same principal normal lines of C and $\bar{C}$ at corresponding points coincide. Here, $\bar{C}$ is called a Bertrand mate of C. It is well-known that a Frenet curve C in $\mathbb{R}^3$ is a Bertrand curve if and only if there exists a linear relation $a\kappa(s) + b\tau(s) = 1$ for all $s \in I$, where a and b are non-zero constant real numbers. In this talk, We show many examples of curves on the unit 2-sphere $S^2(1)$ in $\mathbb{R}^3$ and the unit 3-sphere $S^3(1)$ in $\mathbb{R}^4$. We study whether its curves are Bertrand curves or spherical Bertrand curves and provide some examples illustrating the resultant curves.

Abstracts (Poster Session)

Bundle-like foliations and submersions in quaternion-like geometries

Gabriel Eduard Vîlcu (University of Bucharest)

The paraquaternionic structures, firstly named quaternionic structures of second kind, have been introduced in geometry by P. Libermann [C.R. Acad. Sc. Paris 234 (1952)]. The theory of paraquaternionic manifolds parallels the theory of quaternionic manifolds, but uses the algebra of paraquaternionic numbers, in which two generators have square 1 and one generator has square -1. Accordingly, such manifolds are equipped with a subbundle of rank 3 in the bundle of the endomorphisms of the tangent bundle, locally spanned by two almost product structures and one almost complex structure. From the metric point of view, the almost paraquaternionic Hermitian manifolds have neutral signature [E. Garcia-Rio, Y. Matsushita, R. Vasquez-Lorenzo, Rocky Mt. J. Math. 31 (2001)]. The counterpart in odd dimension of paraquaternionic geometry was introduced by S. Ianuş, R. Mazzocco and G.E.Vîlcu [Mediterranean J. Math. 3(2006)]. It is called mixed 3-structure and appears in a natural way on lightlike hypersurfaces in paraquaternionic manifolds. I will investigate some classes of submanifolds in manifolds endowed with paraquaternionic and mixed 3-structures which naturally come equipped with some canonical foliations. In particular, some characterizations are provided for these foliations to become semi-Riemannian, i.e. with bundle-like metric. Moreover, we introduce a new class of semi-Riemannian submersions from a manifold endowed with a metric mixed 3-structure onto an almost paraquaternionic Hermitian manifold. We obtain some fundamental properties and discuss the transference of structures and the geometry of the fibers. In particular we obtain that such a submersion is a harmonic map, provided that the total space is mixed 3-cosymplectic or mixed 3-Sasakian. Finally, some non-trivial examples are given.

A generalization of contact metric manifolds

Wonmin Shin(Sungkyunkwan University)

We study contact metric manifolds and their generalizations. An almost contact metric manifold is called a quasi contact metric manifold if the corresponding almost Hermitian cone is a quasi Kähler manifold. As a natural generalization of the contact metric manifold, quasi contact metric manifolds are discussed. We show that a quasi contact metric manifold is a contact manifold.