Project: Z' boson in E_6 GUT model

In the supersymmetric E_6 model, there are two additional $U(1)$ factors beyond the SM gauge group since the rank of E_6 group is 6. The canonical decomposition of ψ and χ models are as follows:

$$E_6 \to SO(10) \times U(1)_\psi, \tag{1}$$
$$SO(10) \to SU(5) \times U(1)_\chi. \tag{2}$$

After the extra $U(1)$'s are spontaneously broken by heavy Higgs fields (which we would not care in this project), the gauge bosons Z_ψ and Z_χ of $U(1)_\psi \times U(1)_\chi$, respectively, got masses and the mass eigenstates are linear combinations of the two bosons. We call the lighter one Z' and represent it as a linear combination of Z_ψ and Z_χ parametrized by a mixing angle θ_E:

$$Z'(\theta_E) = \cos\theta_E Z_\chi + \sin\theta_E Z_\psi, \tag{3}$$

which is assumed to be in TeV range and mix with the standard model Z boson sizably. (Here assume the heavier boson, Z'' is much heavier than 1 TeV so that it does not affect the low energy phenomenology)

There are three particular cases we are mostly interested in:

- ψ model: $Z' = Z'(0) = Z_\psi$
- χ model: $Z' = Z'(\frac{\pi}{2}) = Z_\chi$
- η model: $Z' = Z'(\tan^{-1}(-\sqrt{\frac{5}{3}}) = Z_\eta$

The interaction Lagrangian for Z' with the SM fermion field f is described by

$$\mathcal{L}_{Z'} = -\frac{g_{Z'}}{2}\sum_f \overline{\Psi}_f \gamma^\mu \left(f_V - f_A\gamma_5\right)\Psi_f Z'_\mu, \tag{4}$$

where model parameters in each model are

	ψ		χ		η	
	$f_V/\sqrt{\frac{5}{72}}$	$f_A/\sqrt{\frac{5}{72}}$	$2\sqrt{6}f_V$	$2\sqrt{6}f_A$	$12f_V$	$12f_A$
ν	0	1	4	-1	6	-4
e	0	1	2	1	3	-1
u	0	1	2	1	3	-1
d	0	1	-2	1	-3	-1

Table 1: Z' charge assignment for the standard model fermions

After the electroweak symmetry breaking, the gauge sector with neutral gauge bosons $\hat{A}, Z, Z'$ are given by

$$\mathcal{L} = -\frac{1}{4}\left(\hat{F}_{\mu\nu}\hat{F}^{\mu\nu} + Z_{\mu\nu}Z^{\mu\nu} + Z'_{\mu\nu}Z'^{\mu\nu}\right) - \frac{1}{2}\left(m_Z^2 Z^\mu Z_\mu + m_{Z'}^2 Z'^\mu Z'_\mu\right) - \frac{\sin\epsilon}{2}Z'_{\mu\nu}B^{\mu\nu} + \delta M^2 Z'_\mu Z^\mu \tag{5}$$

where $\hat{F}_{\mu\nu}, Z_{\mu\nu}, Z'_{\mu\nu}$ are the usual field strength tensor for $\hat{A}, Z$ and Z', respectively. The fields $\hat{A}$ and Z are obtained by usual weak mixing angle as

$$\begin{pmatrix} Z \\ \hat{A} \end{pmatrix} = \begin{pmatrix} c_W W_3 - s_W B \\ s_W W_3 + c_W B \end{pmatrix} \tag{6}$$

where $c_W = \cos\theta_W$ and $s_W = \sin\theta_W$. The kinetic mixing with $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ (for the hypercharge gauge boson) is generically allowed and also mass mixing ($\sim \delta M^2$).

I. MASS EIGENSTATES

Show that the mass eigenstates (A, Z_1, Z_2) are obtained by rotation

$$\begin{pmatrix} A \\ Z_1 \\ Z_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & c_W s_\chi \\ 0 & c_\zeta & -c_\zeta s_W s_\epsilon + s_\zeta c_\epsilon \\ 0 & -s_\zeta & c_\zeta c_\epsilon + s_\zeta s_W s_\epsilon \end{pmatrix} \begin{pmatrix} \hat{A} \\ Z \\ Z' \end{pmatrix} \tag{7}$$

where

$$\tan 2\zeta \equiv \frac{-2c_\epsilon(\delta M^2 + m_Z^2 s_W s_\epsilon)}{m_{Z'}^2 - m_Z^2 s_W^2 s_\epsilon^2 + 2\delta M^2 s_W s_\epsilon}, \tag{8}$$

with $s_\chi = \sin\chi$, $c_\chi = \cos\chi$, and $s_\zeta = \sin\zeta$, $c_\zeta = \cos\zeta$. Find mass eigenvelues (M_A, M_1, M_2).

II. INTERACTION LAGRANGIAN WITH MASS EIGENSTATES

Recast the interaction Lagrangian of neutral gauge bosons with the SM fermions in terms of mass eigenstates (A, Z_1, Z_2):

$$\begin{aligned} -\mathcal{L} &= eJ^\mu_{em}\hat{A}_\mu + g_Z \sum_f \overline{\psi}_f \gamma^\mu \left(T_3 P_L - s_W^2 Q_f\right) \psi_f Z_\mu - \mathcal{L}_{Z'} \\ &\equiv eJ^\mu_{em} A_\mu + \sum_f \overline{\psi}_f \gamma_\mu \left(V_1^f - A_1^f \gamma_5\right) \psi_f Z_1^\mu + \sum_f \overline{\psi}_f \gamma_\mu \left(V_2^f - A_2^f \gamma_5\right) \psi_f Z_2^\mu \end{aligned} \tag{9}$$

where $P_L = (1-\gamma_5)/2$ and $J^\mu_{em} = \sum_f \overline{\psi}_f \gamma^\mu Q \psi_f$.

Hint: You can write the expression as

$$\begin{aligned} -\mathcal{L}_{Z_i f\bar{f}} &= \frac{e}{2 s_W c_W} \Bigg[\left(1 + \frac{\alpha T}{2}\right) \sum_f \bar{\Psi}_f \gamma^\mu [(g_V^f + \zeta \tilde{f}_V^f) - (g_A^f + \zeta \tilde{f}_A^f)\gamma_5] \Psi_f Z_{1\mu} \\ &\quad + \sum_f \bar{\Psi}_f \gamma^\mu [(h_V^f - \zeta g_V^f) - (h_A^f - \zeta g_A^f)\gamma_5] \Psi_f Z_{2\mu} \Bigg] \end{aligned} \tag{10}$$

and find $g^f_{V,A}, h^f_{V,A}$ and corrections $\tilde{f}^f_{V,A}$ with the Peskin-Takeuchi variable S and T are given by

$$\begin{aligned} \alpha S &= 4\zeta {c_W}^2 s_W \tan\epsilon, \\ \alpha T &= \zeta^2 \left(\frac{M_2^2}{M_1^2} - 1\right) + 2\zeta s_W \tan\epsilon, \end{aligned} \tag{11}$$

up to the leading order of ζ.

III. DISCUSSION

- Discuss the differences and similarities in ψ, χ and η models.
- Discuss how to find Z' at the LHC and also at other low energy experiments.