

# Neutrinos, DM, $(g-2)_\mu$ in extended Ma model

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KIAS Workshop “Exploring the Dark Sector”

Mar 16-20, 2015

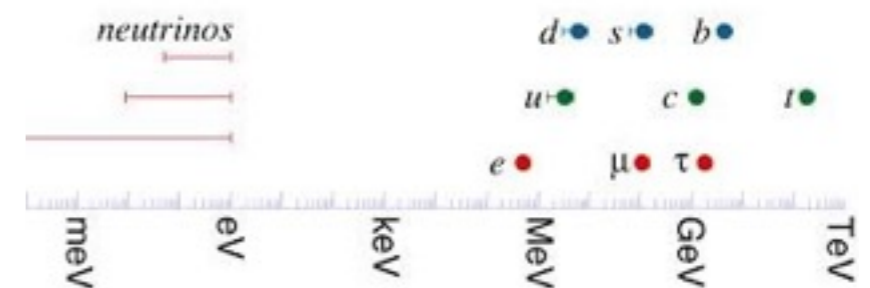
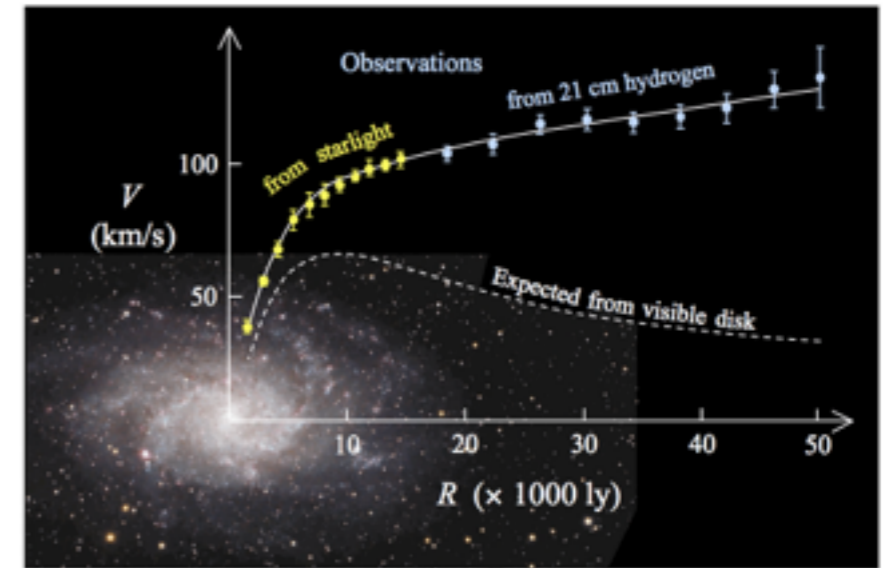
based on SB, H. Okada, K. Yagyu,  
arXiv:1501.01530 (to appear in JHEP)

# Outline

- Brief introduction to dark matter and neutrino
- Ma model and its extension with  $U(1)_{\mu-\tau}$
- $(g-2)_\mu$ , constraints on the model, and prediction on the observables in the neutrino sector
- Conclusions

# Two of Unsolved Problems in Particle Physics

- What is the nature of **dark matter**?
- Why are the **neutrino** masses are so small? Are they Dirac or Majorana? Is the mass hierarchy normal or inverted? Are the CP violating phases non-zero? ...



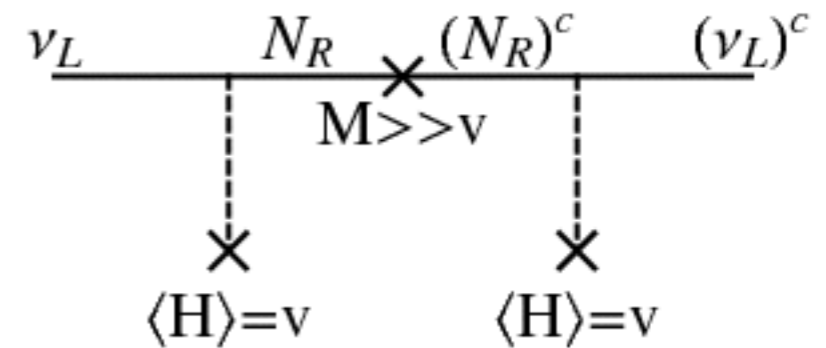
# Neutrinos

- The most natural scenario for neutrino mass is seesaw mechanism

$$\mathcal{L} = -y_\nu LH N_R - \frac{1}{2} M N_R N_R$$

$$m_\nu \sim \frac{y_\nu^2 v^2}{M}$$

- Difficult to test at colliders.



# Radiative Neutrino Masses: Ma's Scotogenic Model

- Alternative scenario for the generation of neutrino masses
- Small masses are due to loop suppression. DM is inside the loop (scotogenic).

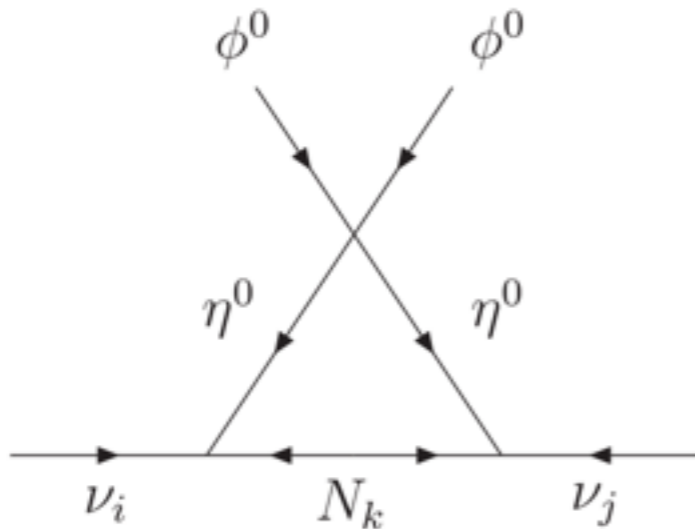


FIG. 1. One-loop generation of neutrino mass.

Under  $SU(2)_L \times U(1)_Y \times \tilde{Z}_2$ , the particle content is given by

$$(\nu_i, l_i) \sim (2, -1/2; +), \quad l_i^c \sim (1, 1; +), \quad N_i \sim (1, 0; -), \quad (3)$$

$$(\phi^+, \phi^0) \sim (2, 1/2; +), \quad (\eta^+, \eta^0) \sim (2, 1/2; -). \quad (4)$$

$$\mathcal{L}_Y = f_{ij}(\phi^- \nu_i + \bar{\phi}^0 l_i) l_j^c + h_{ij}(\nu_i \eta^0 - l_j \eta^+) N_j + \text{H.c.}$$

$$\frac{1}{2} M_i N_i N_i + \text{H.c.}$$

E. Ma, PRD73 (2006)

$$\frac{1}{2} \lambda_5 (\Phi^\dagger \eta)^2 + \text{H.c.}$$

- Colliders may produce new particles at the ew scale
- The model does not predict neutrino masses and mixings

# Relic density and direct search of $N_1$ -DM

D. Schmidt, T. Schwetz, T. Toma (2012)

- Coannihilation with  $N_2$  or  $\eta$  is necessary

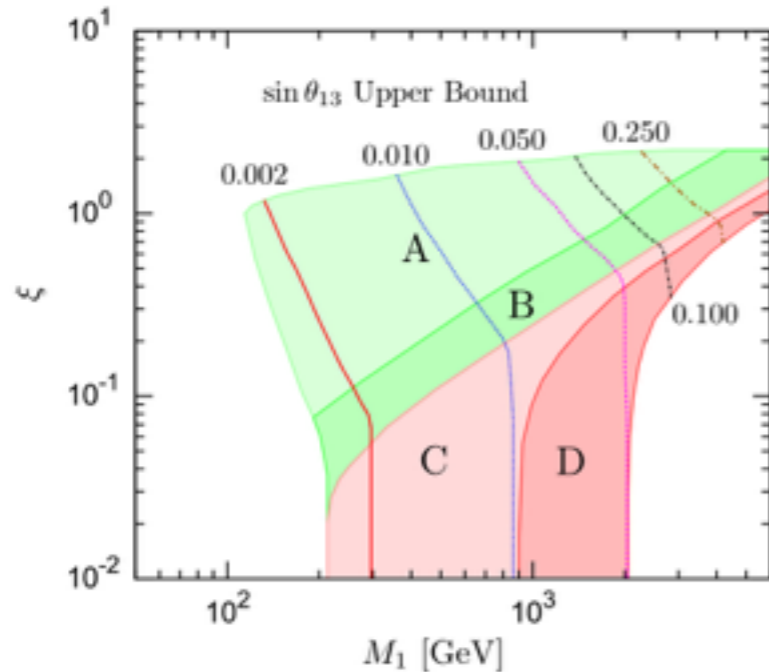


FIG. 2 (color online). Region in the space of DM mass  $M_1$  and  $\xi = |h_1 h_2| \sin(\varphi_1 - \varphi_2)$  consistent with neutrino masses and mixing, lepton-flavor violation, perturbativity, and the relic density of DM. The regions with different color shadings denoted by A, B, C, D correspond to different assumptions on  $M_\eta$ , with A:  $2.0 < M_\eta/M_1 < 9.8$ , B:  $1.2 < M_\eta/M_1 < 2.0$ , C:  $1.05 < M_\eta/M_1 < 1.20$ , D:  $1.0 < M_\eta/M_1 < 1.05$ . The curves show the upper bound on  $\sin\theta_{13}$  from  $\mu \rightarrow e\gamma$  when the Yukawa matrix Eq. (5) is extended to Eq. (14).

- Direct detection @ 1-loop

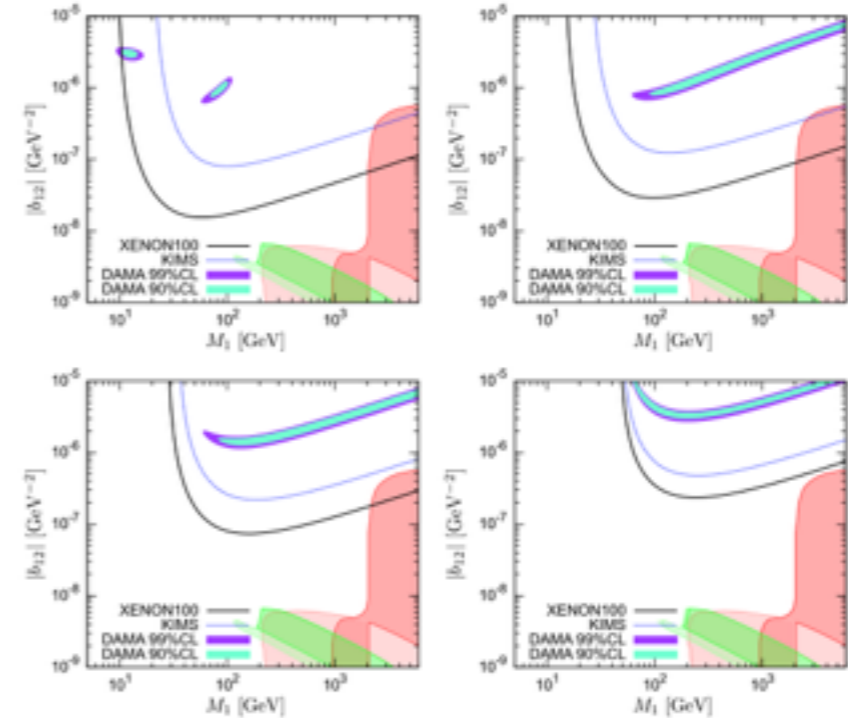
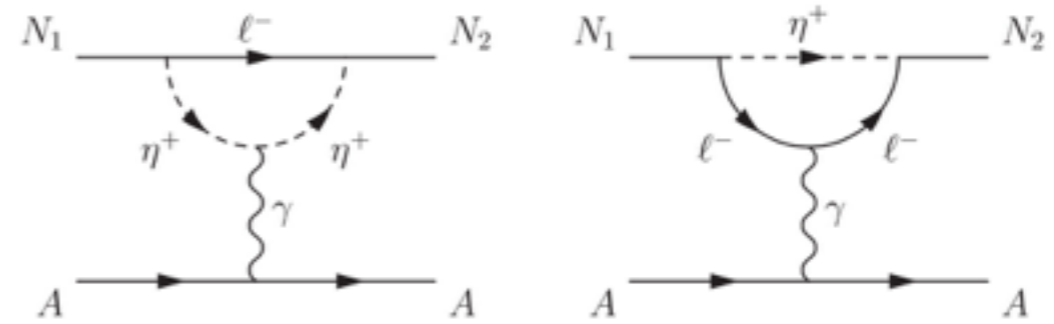


FIG. 5 (color online). Bounds from XENON100, KIMS, and allowed regions for DAMA in the  $(M_1, |b_{12}|)$  plane (charge-charge interaction). The mass difference  $\delta$  is taken as 0 keV (the top-left panel), 40 keV (the top-right panel), 80 keV (the bottom-left panel), and 120 keV (the bottom-right panel). The shaded regions correspond to the values of  $b_{12}$  predicted in the allowed parameter space of the model, as shown in Fig. 2, with the same color shading for different values of the ratio  $M_\eta/M_1$ .

# Constraint on Ma model

- Parity problem [Merle, Klatscher, 1502.03098](#)

$$V = m_1^2 \phi^\dagger \phi + m_2^2 \eta^\dagger \eta + \frac{\lambda_1}{2} (\phi^\dagger \phi)^2 + \frac{\lambda_2}{2} (\eta^\dagger \eta)^2$$

$$+ \lambda_3 (\phi^\dagger \phi) (\eta^\dagger \eta) + \lambda_4 (\eta^\dagger \phi) (\phi^\dagger \eta) + \frac{\lambda_5}{2} ((\eta^\dagger \phi)^2 + h.c.).$$

$$\mathcal{D}m_2^2 = 6\lambda_2 m_2^2 + 2(2\lambda_3 + \lambda_4) m_1^2 + m_2^2 \left[ 2T_\nu - \frac{3}{2} (g_1^2 + 3g_2^3) \right] - 4 \sum_{i=1}^3 M_i^2 (h h^\dagger)_{ii}, \quad \mathcal{D} \equiv 16\pi^2 \mu \frac{d}{d\mu}.$$

- Large  $M_i$  can make  $m_2^2$  negative, violating the  $Z_2$ -parity.

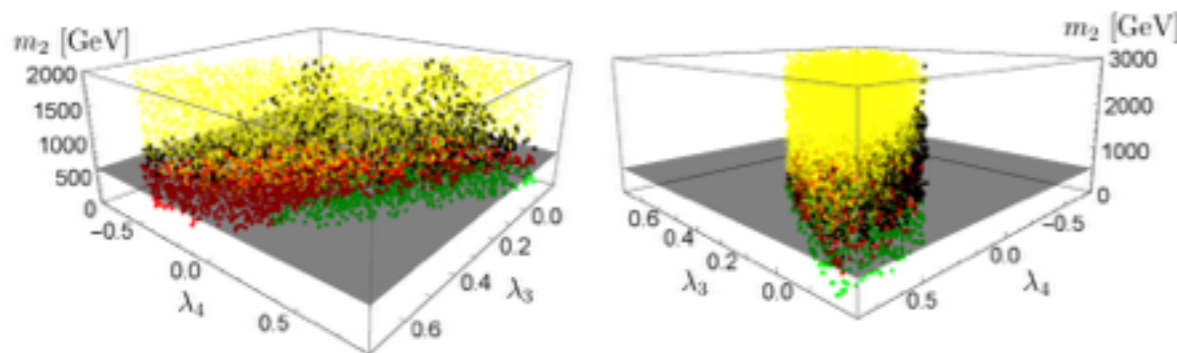


Figure 2: *Scenario a)* Scan of the parameter space of the scotogenic model. Black dots are valid input parameter values, yellow dots violate EWSB, red ( $m_R^2 < 0$ ) and green ( $m_L^2 < 0$ ) dots are excluded due to SSB of  $Z_2$ . The shaded gray area represents the “lower bound” on  $m_2$ . We have reduced the mass range in the left plot for better visibility.

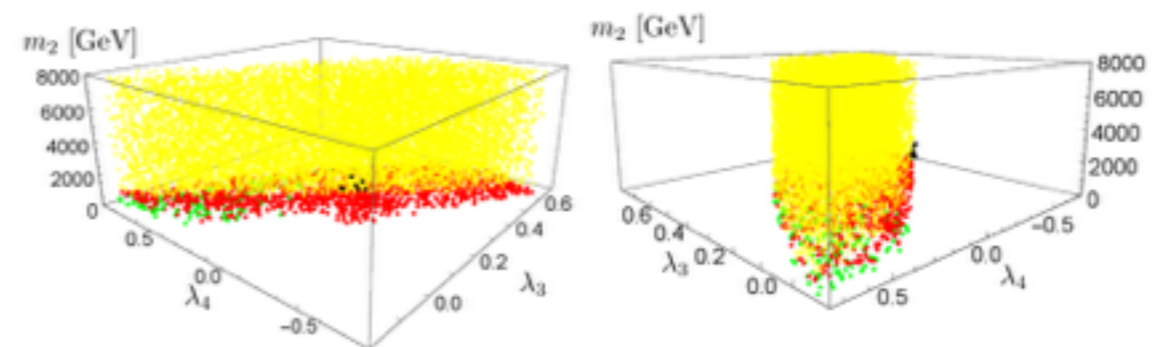


Figure 3: *Scenario b)* Parameter scan for Majorana masses up to 10 TeV. The resulting valid parameter space is very limited, see the few scattered black points. The strong tension for scenario b) would be missed if the parity problem was not taken into account.

# U(1) Extension of the SM

- Anomaly free gauged U(1) extension w/o extending the fermion sector of the SM

$$L_e - L_\mu, L_e - L_\tau, \text{ and } L_\mu - L_\tau$$



# The Model

- In addition to the Ma model, only  $S$  is introduced.
- $\eta, N_R$ : for Ma mechanism
- $S$ : break  $U(1)_{\mu-\tau}$

	Lepton Fields			Scalar Fields		
	$L_L^i = (\nu_L^i, e_L^i)^T$	$e_R^i$	$N_R^i$	$\Phi$	$\eta$	$S$
$SU(2)_L$	<b>2</b>	<b>1</b>	<b>1</b>	<b>2</b>	<b>2</b>	<b>1</b>
$U(1)_Y$	-1/2	-1	0	+1/2	+1/2	0
$Z_2$	+	+	-	+	-	+

	$(L_L^e, e_R, N_R^e)$	$(L_L^\mu, \mu_R, N_R^\mu)$	$(L_L^\tau, \tau_R, N_R^\tau)$	$S$
$U(1)_{\mu-\tau}$	0	+1	-1	+1

$$\begin{aligned}
 -\mathcal{L}_Y &= \frac{1}{2} M_{ee} \overline{N_R^{ec}} N_R^e + \frac{1}{2} M_{\mu\tau} (\overline{N_R^{\mu c}} N_R^\tau + \overline{N_R^{\tau c}} N_R^\mu) + \text{h.c.} \\
 &+ y_e \overline{L_L^e} \Phi e_R + y_\mu \overline{L_L^\mu} \Phi \mu_R + y_\tau \overline{L_L^\tau} \Phi \tau_R + \text{h.c.} \\
 &+ h_{e\mu} (\overline{N_R^{ec}} N_R^\mu + \overline{N_R^{\mu c}} N_R^e) S^* + h_{e\tau} (\overline{N_R^{ec}} N_R^\tau + \overline{N_R^{\tau c}} N_R^e) S + \text{h.c.} \\
 &+ f_e \overline{L_L^e} (i\sigma_2) \eta^* N_R^e + f_\mu \overline{L_L^\mu} (i\sigma_2) \eta^* N_R^\mu + f_\tau \overline{L_L^\tau} (i\sigma_2) \eta^* N_R^\tau + \text{h.c.}
 \end{aligned}$$

# Scalar masses

- Scalar potential:

$$\begin{aligned} \mathcal{V} = & \mu_\Phi^2 |\Phi|^2 + \mu_\eta^2 |\eta|^2 + \mu_S^2 |S|^2 \\ & + \frac{1}{2} \lambda_1 |\Phi|^4 + \frac{1}{2} \lambda_2 |\eta|^4 + \lambda_3 |\Phi|^2 |\eta|^2 + \lambda_4 |\Phi^\dagger \eta|^2 + \frac{1}{2} \lambda_5 [(\Phi^\dagger \eta)^2 + \text{h.c.}] \\ & + \lambda_S |S|^4 + \lambda_{S\Phi} |S|^2 |\Phi|^2 + \lambda_{S\eta} |S|^2 |\eta|^2, \end{aligned}$$

$$\Phi = \begin{bmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + \varphi_H + iG^0) \end{bmatrix}, \quad \eta = \begin{bmatrix} \eta^+ \\ \frac{1}{\sqrt{2}}(\eta_H + i\eta_A) \end{bmatrix}, \quad S = \frac{1}{\sqrt{2}}(v_S + S_H + iG_S),$$

- Scalar masses:

$$\begin{aligned} m_{\eta^\pm}^2 &= \mu_\eta^2 + \frac{v_S^2}{2} \lambda_{S\eta} + \frac{v^2}{2} \lambda_3, & \mathcal{M}_H^2 &= \begin{pmatrix} v^2 \lambda_1 & v v_S \lambda_{S\Phi} \\ v v_S \lambda_{S\Phi} & 2v_S^2 \lambda_S \end{pmatrix} \\ m_{\eta_A}^2 &= \mu_\eta^2 + \frac{v_S^2}{2} \lambda_{S\eta} + \frac{v^2}{2} (\lambda_3 + \lambda_4 - \lambda_5), \\ m_{\eta_H}^2 &= \mu_\eta^2 + \frac{v_S^2}{2} \lambda_{S\eta} + \frac{v^2}{2} (\lambda_3 + \lambda_4 + \lambda_5). & \begin{pmatrix} \varphi_H \\ S_H \end{pmatrix} &= \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} h \\ H \end{pmatrix} \quad \tan 2\alpha = \frac{2(\mathcal{M}_H^2)_{12}}{(\mathcal{M}_H^2)_{11} - (\mathcal{M}_H^2)_{22}} \end{aligned}$$

# Fermion masses

- Charged leptons, right-handed neutrinos

$$\mathcal{M}_\ell = \frac{v}{\sqrt{2}} \text{diag}(|y_e|, |y_\mu|, |y_\tau|), \quad \mathcal{M}_N = \begin{pmatrix} |M_{ee}| & \frac{v_S}{\sqrt{2}} |h_{e\mu}| & \frac{v_S}{\sqrt{2}} |h_{e\tau}| \\ \frac{v_S}{\sqrt{2}} |h_{e\mu}| & 0 & |M_{\mu\tau}| e^{i\theta_R} \\ \frac{v_S}{\sqrt{2}} |h_{e\tau}| & |M_{\mu\tau}| e^{i\theta_R} & 0 \end{pmatrix}$$

- Diagonalization of right-handed neutrino mass matrix

$$V^T \mathcal{M}_N V = \mathcal{M}_N^{\text{diag}} \equiv \text{diag}(M_1, M_2, M_3)$$

- Light neutrino masses from one-loop

$$-\mathcal{L}_Y = + f_e \bar{L}_L^e (i\sigma_2) \eta^* N_R^e + f_\mu \bar{L}_L^\mu (i\sigma_2) \eta^* N_R^\mu + f_\tau \bar{L}_L^\tau (i\sigma_2) \eta^* N_R^\tau + \text{h.c.}$$

$$(\mathcal{M}_\nu)_{ij} = \frac{1}{32\pi^2} \sum_{k=1-3} (f_i V_{ik}) M_{N_k} (f_j V_{jk}) \left( \frac{m_{\eta_H}^2}{M_k^2 - m_{\eta_H}^2} \ln \frac{m_{\eta_H}^2}{M_k^2} - \frac{m_{\eta_A}^2}{M_k^2 - m_{\eta_A}^2} \ln \frac{m_{\eta_A}^2}{M_k^2} \right)$$

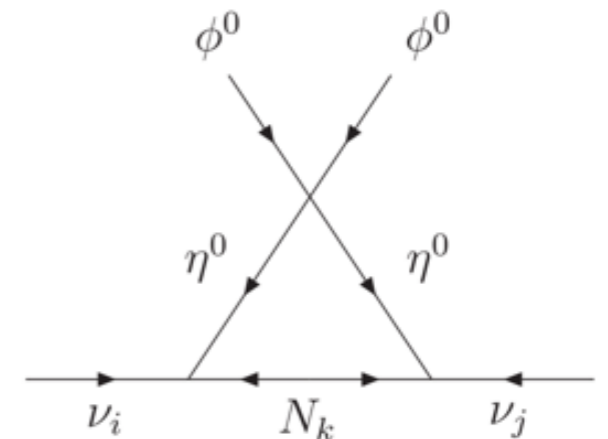


FIG. 1. One-loop generation of neutrino mass.

# Neutrino masses and PMNS

- For  $m_0^2 \equiv (m_{\eta_H}^2 + m_{\eta_A}^2)/2 \gg M_k^2$ , **two-zero texture** form is obtained from  $U(1)_{\mu-\tau}$ !!

$$\mathcal{M}_\nu = \begin{pmatrix} f_e^2 M_{11} & f_e f_\mu M_{12} & f_e f_\tau M_{13} \\ f_e f_\mu M_{12} & 0 & f_\mu f_\tau M_{23} e^{i\theta_R} \\ f_e f_\tau M_{13} & f_\mu f_\tau M_{23} e^{i\theta_R} & 0 \end{pmatrix},$$

- 5-indep. parameters  $\rightarrow$  9 observables (3 masses, 3 mixing angles, 3 CPV phases) are predicted.

# Neutrino masses and PMNS

- PMNS matrix

$$U_{\text{PMNS}} = UP, \quad U \equiv \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad P \equiv \text{diag}(e^{i\rho}, e^{i\sigma}, 1),$$

$$\mathcal{M}_\nu = U \text{diag}(\tilde{m}_1, \tilde{m}_2, \tilde{m}_3) U^T,$$

where  $\tilde{m}_1 = m_1 e^{2i\rho}$ ,  $\tilde{m}_2 = m_2 e^{2i\sigma}$  and  $\tilde{m}_3 = m_3$ .

- From the condition,  $[M_\nu]_{22} = [M_\nu]_{33} = 0$ , we get

$$\frac{\tilde{m}_1}{\tilde{m}_3} = \frac{c_{12}c_{13}^2}{s_{13}} \frac{c_{12}(c_{23}^2 - s_{23}^2)e^{i\delta} - 2s_{12}s_{23}s_{23}c_{23}}{2s_{12}c_{12}s_{23}c_{23}(e^{2i\delta} + s_{13}^2) - s_{13}(c_{12}^2 - s_{12}^2)(c_{23}^2 - s_{23}^2)e^{i\delta}} e^{2i\delta},$$

$$\frac{\tilde{m}_2}{\tilde{m}_3} = -\frac{s_{12}c_{13}^2}{s_{13}} \frac{s_{12}(c_{23}^2 - s_{23}^2)e^{i\delta} - 2s_{12}s_{23}s_{23}c_{23}}{2s_{12}c_{12}s_{23}c_{23}(e^{2i\delta} + s_{13}^2) - s_{13}(c_{12}^2 - s_{12}^2)(c_{23}^2 - s_{23}^2)e^{i\delta}} e^{2i\delta}.$$

# Neutrino masses and PMNS

- Using,  $\theta_{13} \ll 1$

$$R_{13} \simeq \left[ 1 - \frac{2 \cot \theta_{12}}{\sin \theta_{13}} \cot 2\theta_{23} \cos \delta + \left( \frac{\cot \theta_{12}}{\sin \theta_{13}} \cot 2\theta_{23} \right)^2 \right]^{1/2}, \quad R_{13} \equiv \frac{m_1}{m_3}$$

$$R_{23} \simeq \left[ 1 + \frac{2 \tan \theta_{12}}{\sin \theta_{13}} \cot 2\theta_{23} \cos \delta + \left( \frac{\tan \theta_{12}}{\sin \theta_{13}} \cot 2\theta_{23} \right)^2 \right]^{1/2}. \quad R_{23} \equiv \frac{m_2}{m_3}$$

- From the requirement  $m_1 < m_2$ ,  $\cot 2\theta_{23} \cos \delta > 0$ , which leads to  $R_{23} > 1$ , i.e., only **inverted hierarchy** is allowed!
- Neutrino masses in terms of  $R_{13}$ ,  $R_{23}$ , and mass-squared diff

$$m_3 = \frac{\sqrt{\Delta m_{21}^2}}{\sqrt{R_{23}^2 - R_{13}^2}}, \quad m_1 = m_3 R_{13}, \quad m_2 = m_3 R_{23}.$$

# Neutrino masses and PMNS

- Ratio of two mass-squared differences

$$R_\nu \equiv \frac{\Delta m_{21}^2}{|\Delta m_{31}^2|} = \frac{m_2^2 - m_1^2}{|m_3^2 - m_1^2|}$$

$$R_\nu = \frac{R_{23}^2 - R_{13}^2}{R_{13}^2 - 1} \simeq \frac{2}{\cos^2 \theta_{12}} \frac{\cot 2\theta_{12} \cot 2\theta_{23} - \sin \theta_{13} \cos \delta}{2 \sin \theta_{13} \cos \delta - \cot \theta_{12} \cot 2\theta_{23}}$$

- Now we can obtain neutrino masses and PMNS, from 5 experimental data

$$s_{12}^2 = 0.323 \text{ (0.278-0.375)}, \quad s_{23}^2 = 0.573 \text{ (0.403-0.640)}, \quad s_{13}^2 = 0.0229 \text{ (0.0193-0.0265)},$$

$$\Delta m_{21}^2 = 7.60 \text{ (7.11-8.18)} \times 10^{-5} \text{ eV}^2, \quad |\Delta m_{31}^2| = 2.38 \text{ (2.20-2.54)} \times 10^{-3} \text{ eV}^2,$$

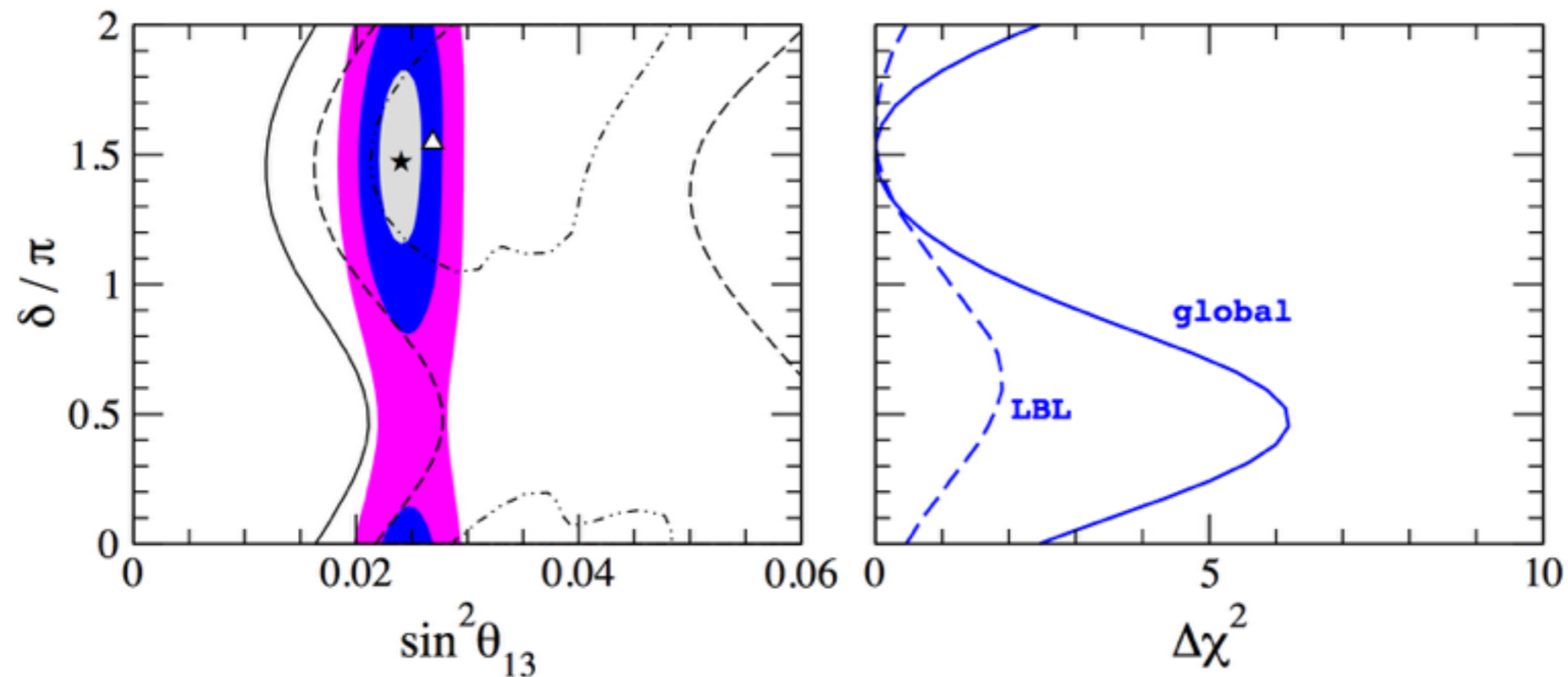
- $R_\nu \rightarrow \delta \rightarrow R_{13}, R_{23} \rightarrow m_1, m_2, m_3 \rightarrow \rho, \sigma$

# Neutrino masses and PMNS

$$\delta = \pm 1.96 \text{ (BF)}, \pm 2.07 \text{ (+3}\sigma\text{)}, \pm 0.774 \text{ (-3}\sigma\text{)}.$$

- Negative  $\delta$  is preferred for IH.

D. V. Forero, M. Tortola, J. W. F. Valle, 1405.7540





# Neutrino masses and PMNS

$$(m_1, m_2, m_3) [\text{eV}] = 0.0583, 0.0589, 0.0420, (\rho, \sigma) = (0.956, -1.34) \quad (\text{BF}),$$

$$(m_1, m_2, m_3) [\text{eV}] = 0.0533, 0.0540, 0.0262, (\rho, \sigma) = (0.759, -1.25) \quad (+3\sigma),$$

$$(m_1, m_2, m_3) [\text{eV}] = 0.0585, 0.0591, 0.0339, (\rho, \sigma) = (-0.596, 1.39) \quad (-3\sigma).$$

$$\mathcal{M}_\nu \simeq \begin{pmatrix} 0.0408 & -0.0186 & -0.0378 \\ -0.0186 & 0 & -0.0420 - 0.00631i \\ -0.0378 & -0.0420 - 0.00631i & 0 \end{pmatrix} \text{eV} \quad (\text{BF}),$$

$$\mathcal{M}_\nu \simeq \begin{pmatrix} 0.0249 & -0.0228 & -0.0410 \\ -0.0228 & 0 & -0.0270 - 0.00352i \\ -0.0410 & -0.0270 - 0.00352i & 0 \end{pmatrix} \text{eV} \quad (+3\sigma),$$

$$\mathcal{M}_\nu \simeq \begin{pmatrix} 0.0321 & 0.0399 & 0.0271 \\ 0.0399 & 0 & -0.0344 + 0.00252i \\ 0.0271 & -0.0344 + 0.00252i & 0 \end{pmatrix} \text{eV} \quad (-3\sigma),$$

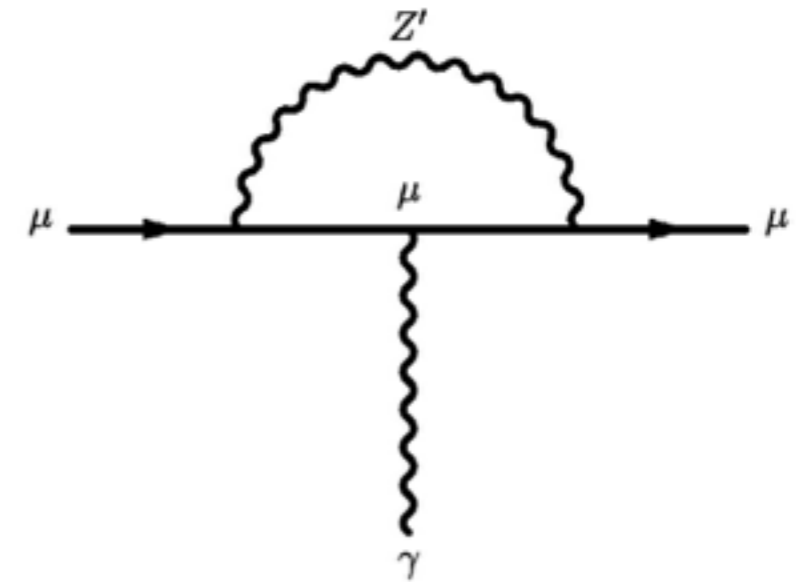
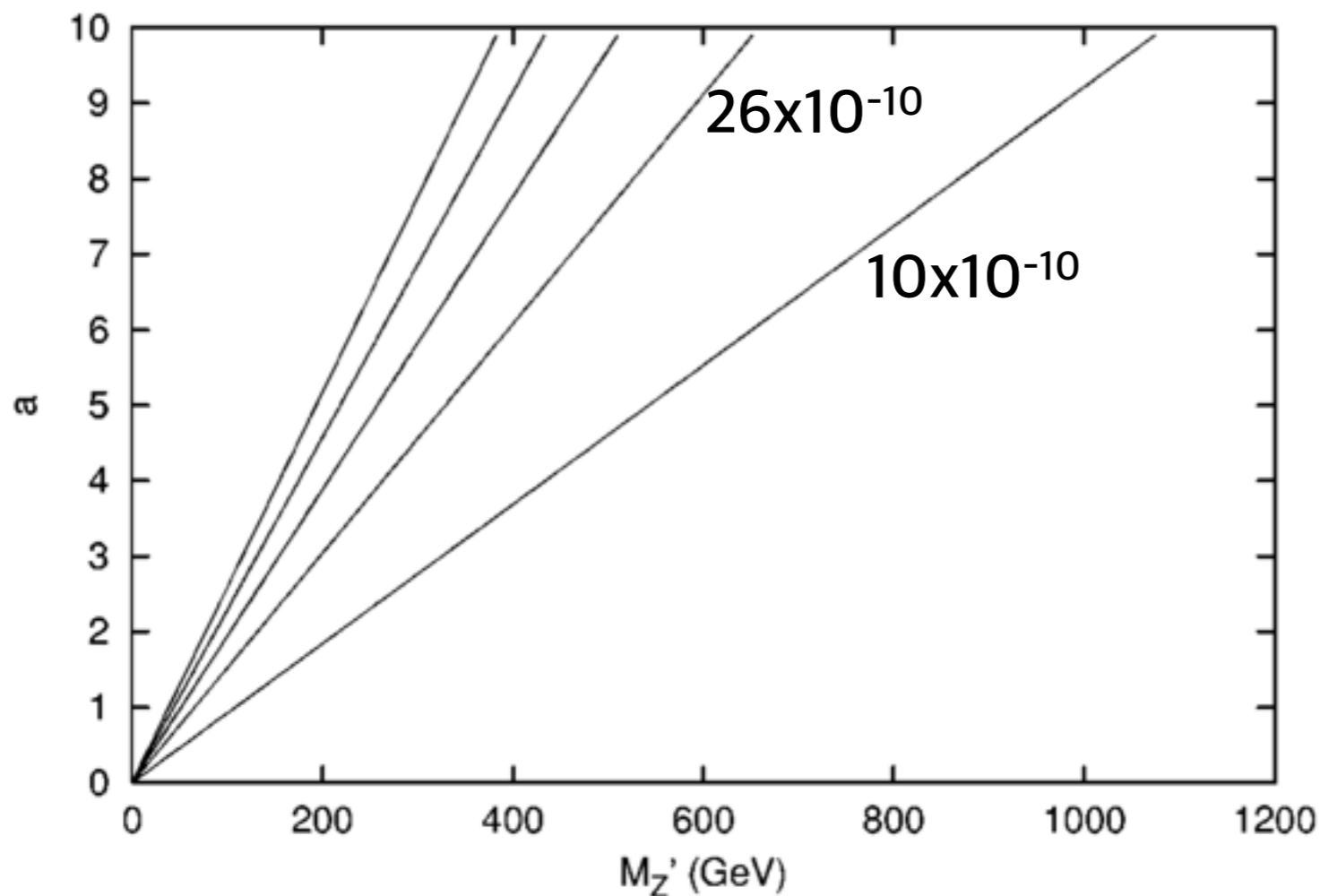
# $(g-2)_\mu$

- $\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (29.0 \pm 9.0 \text{ to } 33.5 \pm 8.2) \times 10^{-10}$ .

F. Jegerlehner, A. Nyffeler (2009);  
M. Benayoun, et.al.(2012)

- U(1) $_{\mu-\tau}$  explanation of  $(g-2)_\mu$**

SB, N. G. Deshpande, X. G. He, P. Ko (2001)



$$L = \frac{ea}{c_W} (\bar{\mu} \gamma^\mu \mu - \bar{\tau} \gamma^\mu \tau) Z'_\mu$$

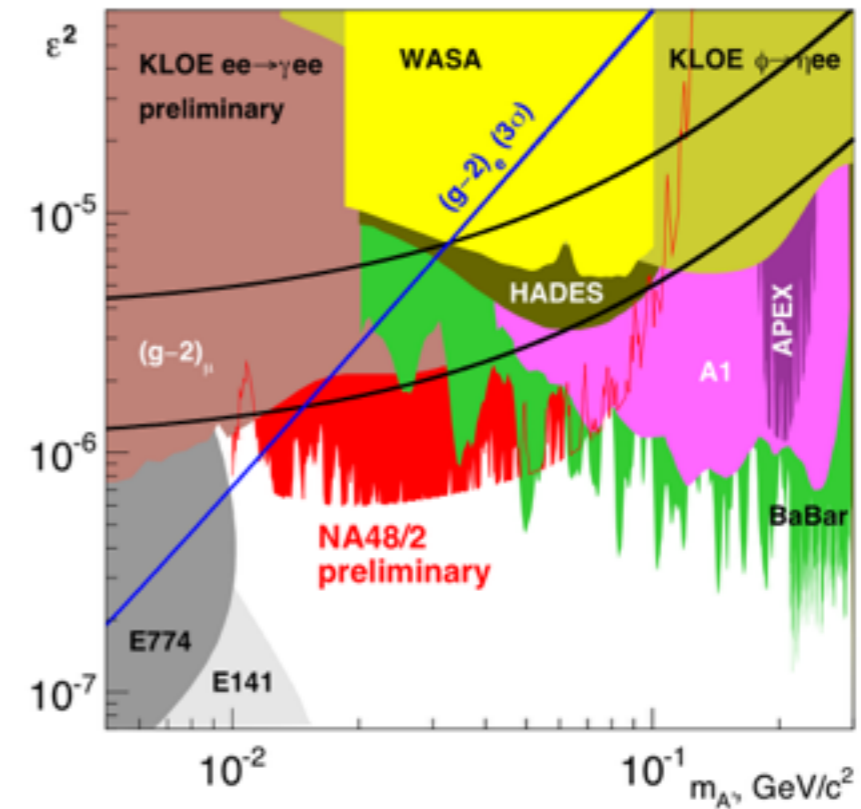
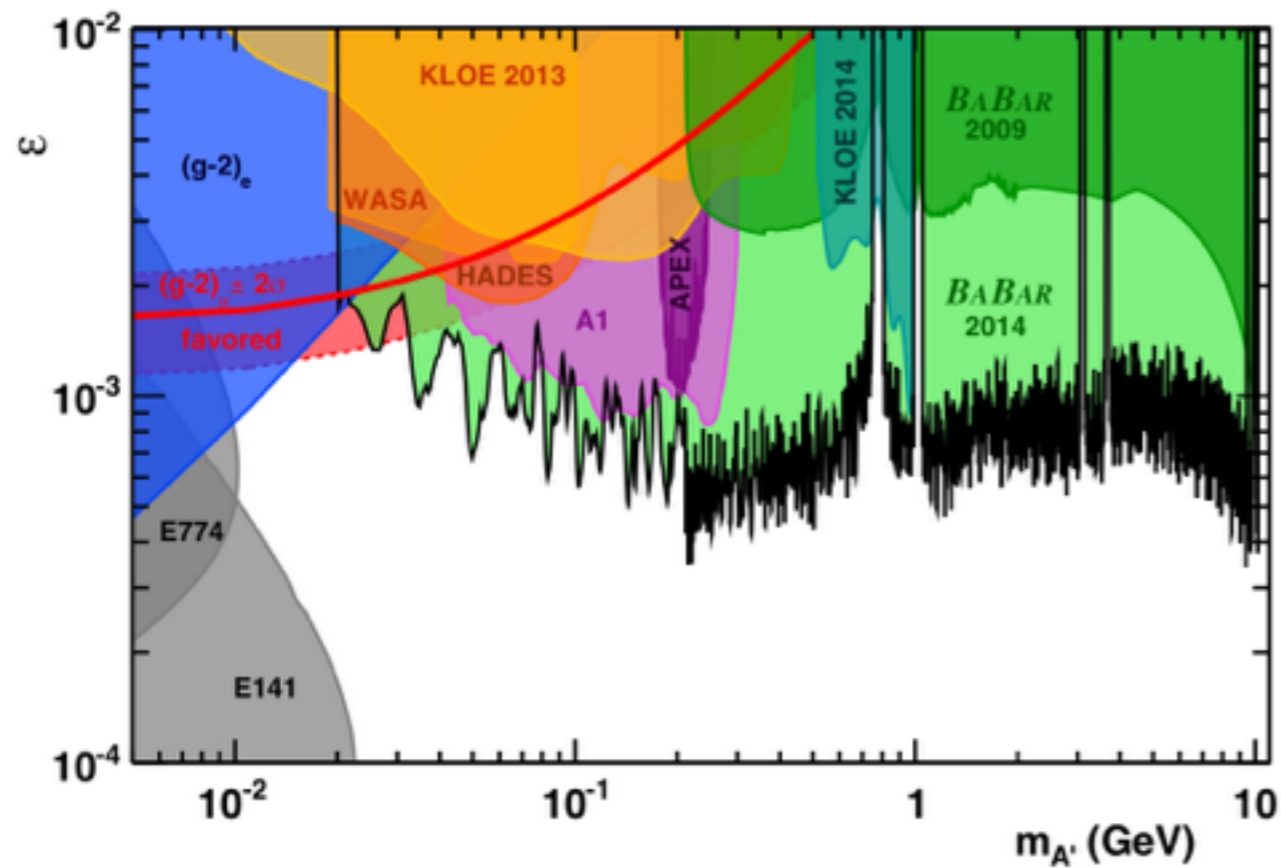
# Constraint on $U(1)_{\mu-\tau}$

BaBar (2014)

$$e^+e^- \rightarrow \gamma A', A' \rightarrow e^+e^-, \mu^+\mu^-$$

NA48/2 (2014)

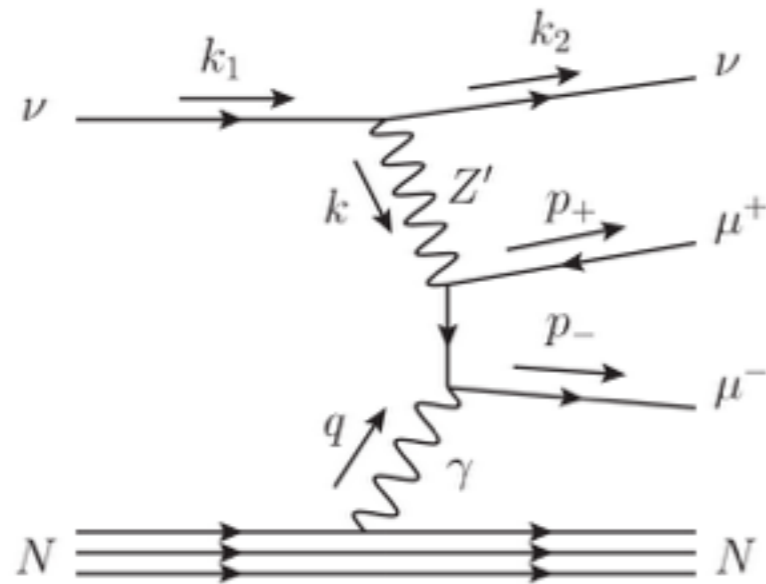
$$K^\pm \rightarrow \pi^\pm \pi^0, \pi^0 \rightarrow \gamma A', A' \rightarrow e^+e^-.$$



- Dark photon searches are NOT applicable to  $U(1)_{\mu-\tau}$

# Constraint on $U(1)_{\mu-\tau}$

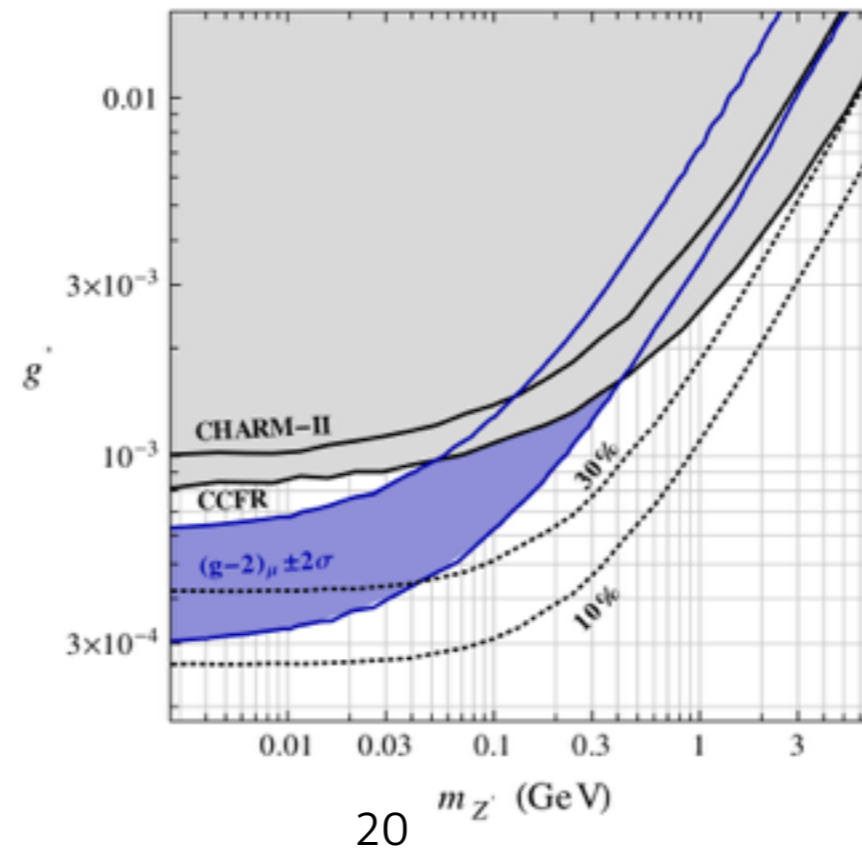
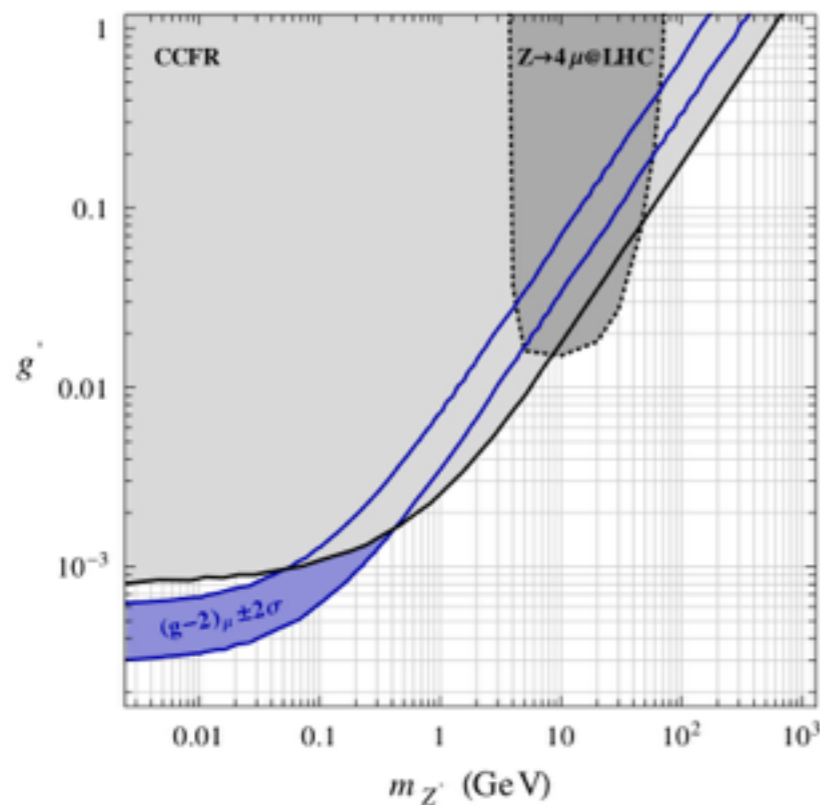
- Neutrino trident production W. Altmannshofer, et.al. (2014)



The  $Z'$  contribution is constructive with the SM

$$\sigma_{\text{CHARM-II}}/\sigma_{\text{SM}} = 1.58 \pm 0.57, \quad (1990)$$

$$\sigma_{\text{CCFR}}/\sigma_{\text{SM}} = 0.82 \pm 0.28. \quad (1991)$$



$M_{Z'} < 400 \text{ MeV}$   
can account for  
the discrepancy  
in  $(g-2)_{\mu}$

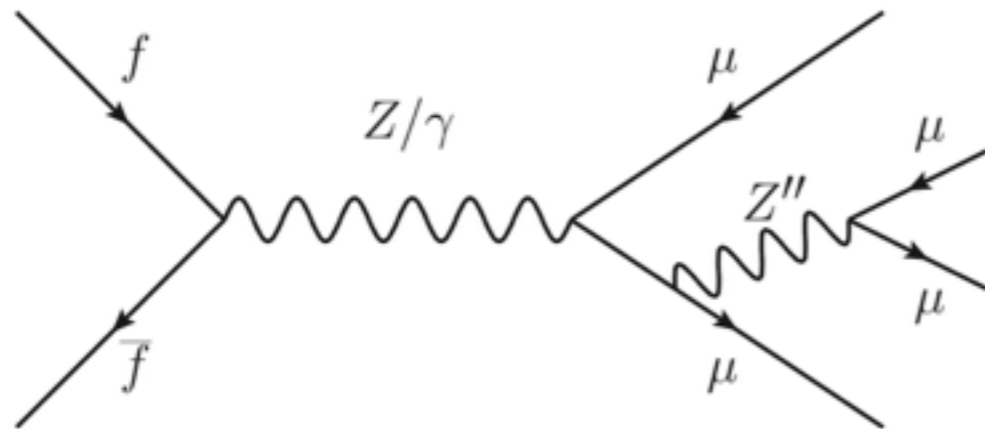
# Constraint on $U(1)_{\mu-\tau}$

- MEG exp:  $\mathcal{B}(\mu \rightarrow e\gamma) < 5.7 \times 10^{-13}$  MEG (2013)

$$\mathcal{B}(\mu \rightarrow e\gamma) \simeq (900 \text{ GeV}^2)^2 \times \left| \sum_{i=1-3} \frac{f_e f_\mu}{2m_{\eta^\pm}^2} V_{1i} V_{2i}^* G \left( \frac{M_i^2}{m_{\eta^\pm}^2} \right) \right|^2$$

- With  $\sum_{i=1-3} f_e f_\mu V_{1i} V_{2i}^* \lesssim \mathcal{O}(10^{-3})$  and  $m_{\eta^\pm} = \mathcal{O}(1) \text{ TeV}$ ,  
we can avoid the MEG constraint.

# Collider search



$$ff \rightarrow \mu\mu\mu\mu, \mu\mu\tau\tau$$

K. Harigaya, et.al., 1311.0870

- If we do not consider  $(g-2)_\mu$ , EW scale  $Z'$  is still allowed.
- 14 TeV LHC w/  $300 \text{ fb}^{-1}$  can observe  $Z'$  in  $4\mu$  channel for  $M_{Z'}=80-100 \text{ GeV}$  and  $g'=0.3$

# Conclusions

- Radiative neutrino models can be alternative to seesaw mechanism. DM can be naturally included.
- Considered  $U(1)_{\mu-\tau}$  extension of Ma model
- Right-handed neutrino as DM is a viable scenario
- $(g-2)_\mu$  can be explained with light  $Z'$ ,  $m_{Z'} \sim O(100)$  MeV
- Neutrino mass matrix has two-zero texture: predictive  
5 experimental data predicts 9 neutrino parameters: 3 masses,  
3 mixing angles, 3 CPV phases. Inverted mass hierarchy
- LHC can search for  $Z'$  with  $pp \rightarrow \mu\mu\mu\mu, \mu\mu\tau\tau$