Quantum Annealing and Quantum Phase Transitions

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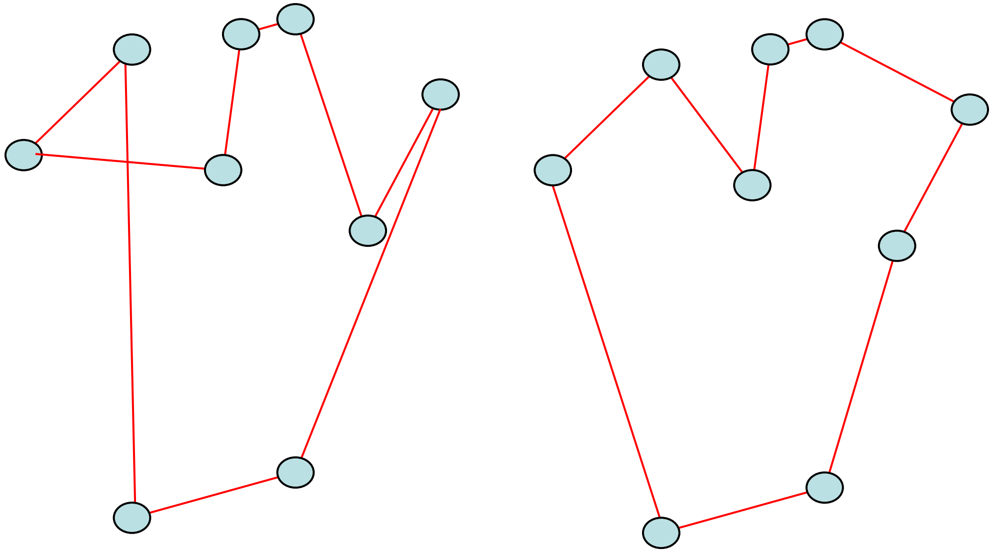
Goal : Optimization

- Travelling Salesman Problem (TSP)



Configuration 1 Configuration 2

Minimize the cost function (=*tour length*)

Optimization (2)

Ground state of Ising spin glass

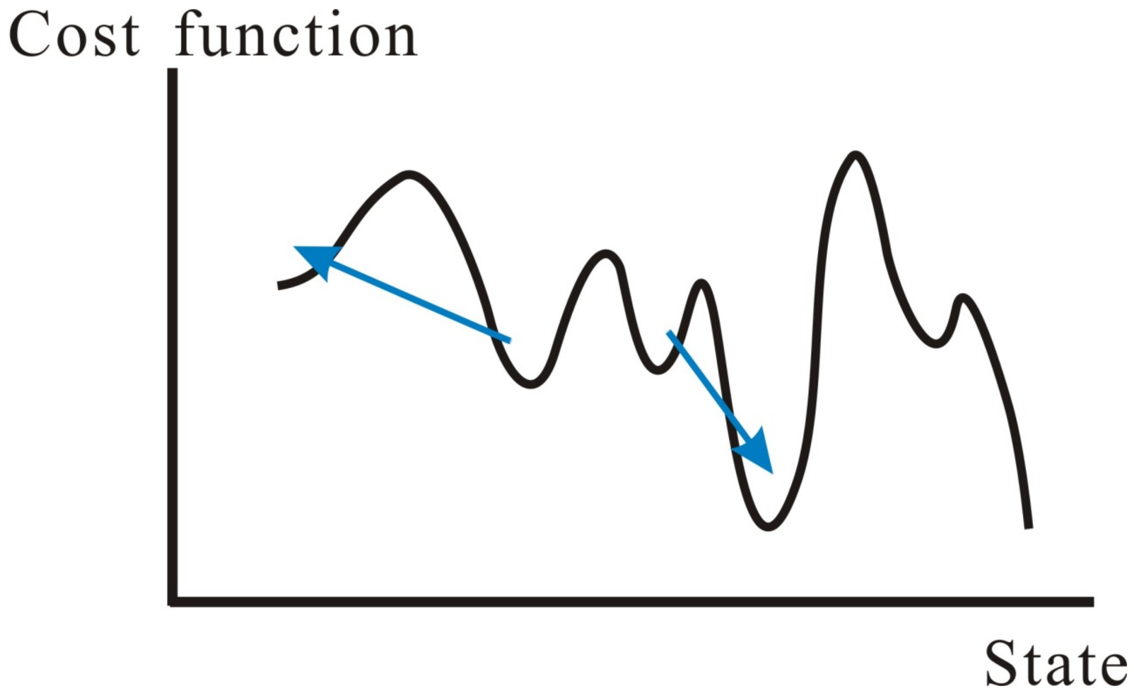
$$H = -\sum J_{ij}\sigma_i\sigma_j$$

Cost function : Multivariable & Single-valued

Machine learning, Protein folding, etc.

Quantum Annealing (QA)

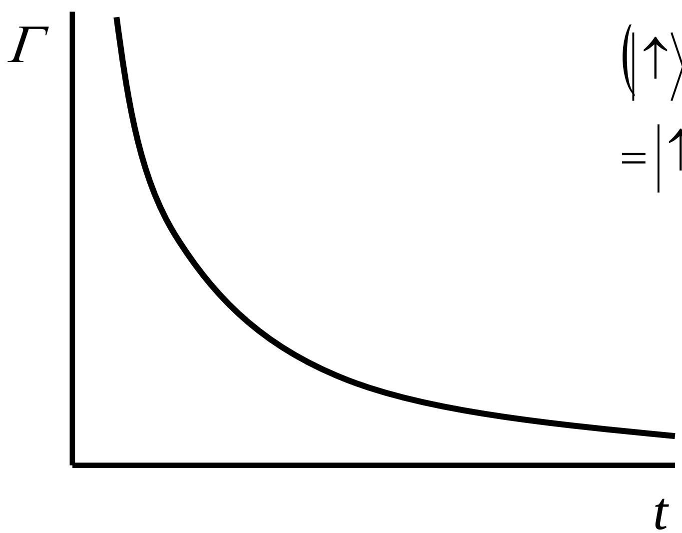
Phase-space search by quantum fluctuations



Formulation of quantum annealing

$$H_{\text{classical}} = -\sum J_{ij}\sigma_i^z\sigma_j^z$$

$$H(t) = H_{\text{classical}} + H_{\text{quantum}} = -\sum J_{ij}\sigma_i^z\sigma_j^z - \Gamma(t)\sum\sigma_i^x$$



H_{quantum} Ground state

$$\left(\left|\uparrow\right\rangle_1 + \left|\downarrow\right\rangle_1\right)\otimes\left(\left|\uparrow\right\rangle_2 + \left|\downarrow\right\rangle_2\right)\otimes\cdots\otimes\left(\left|\uparrow\right\rangle_N + \left|\downarrow\right\rangle_N\right)$$

$$= \left|\uparrow_1\uparrow_2\uparrow_3\cdots\uparrow_N\right\rangle + \left|\uparrow_1\uparrow_2\uparrow_3\cdots\downarrow_N\right\rangle + \cdots + \left|\downarrow_1\downarrow_2\downarrow_3\cdots\downarrow_N\right\rangle$$

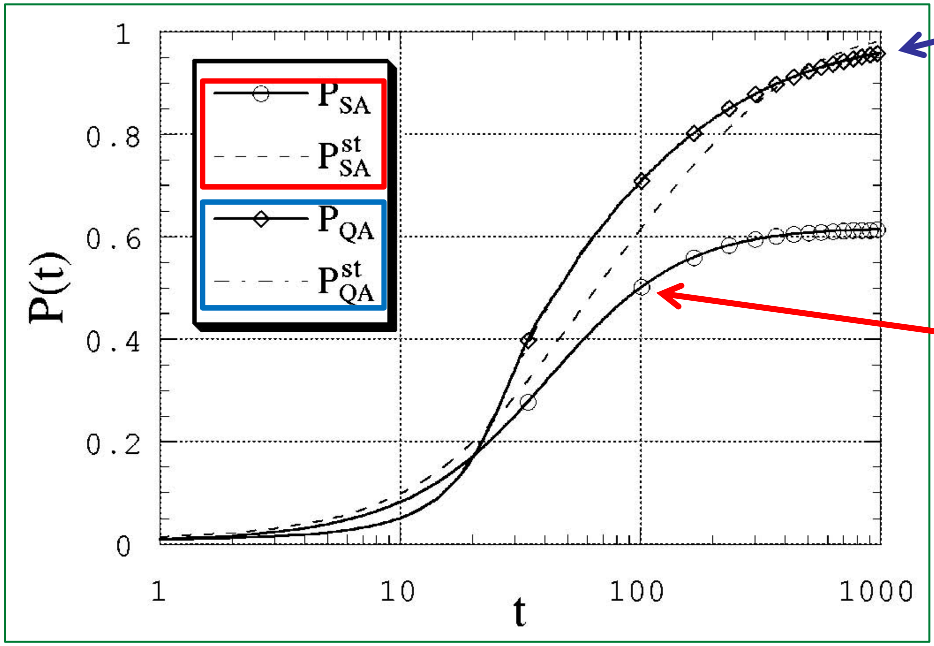
Natural Schrodinger evolution

$$\left|\uparrow\downarrow\downarrow\uparrow\uparrow\ldots\uparrow\downarrow\right\rangle$$

$H_{\text{classical}}$ Ground state = Solution

Master eqn vs. Schrödinger eqn

Spin glass (SK model) with 8 spins



$$\Gamma(t) = \frac{3}{\sqrt{t}}$$

Schrödinger

$$T(t) = \frac{3}{\sqrt{t}}$$

Master equation

Kadowaki & Nishimori (1998)

Monte Carlo for Travelling Salesman Problem (1002 cities)

$$H(t) = \frac{t}{\tau} H_{\text{classical}} + \left(1 - \frac{t}{\tau}\right) H_{\text{quantum}}$$

$$H(0) = H_{\text{quantum}} \Rightarrow H(\tau) = H_{\text{classical}}$$

cf: classical SA $T(0) = \text{large} \Rightarrow T(\tau) = 0$

Residual energy: $H(\tau) - E_{\text{true}}$

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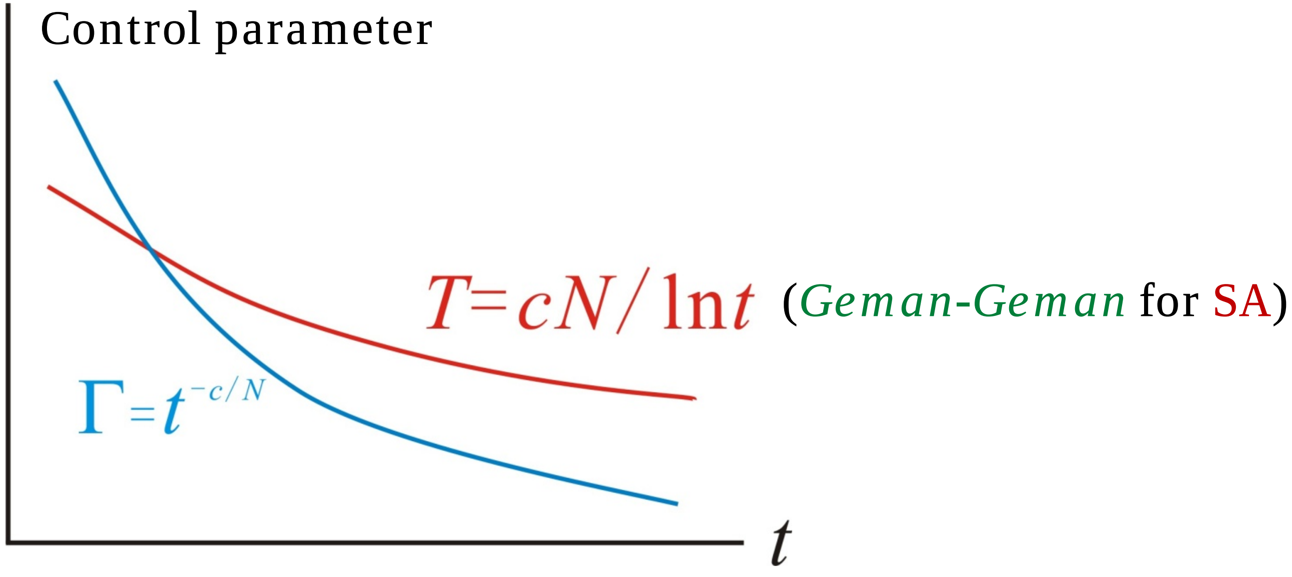
Martonak, Santoro &Tosatti (2004)

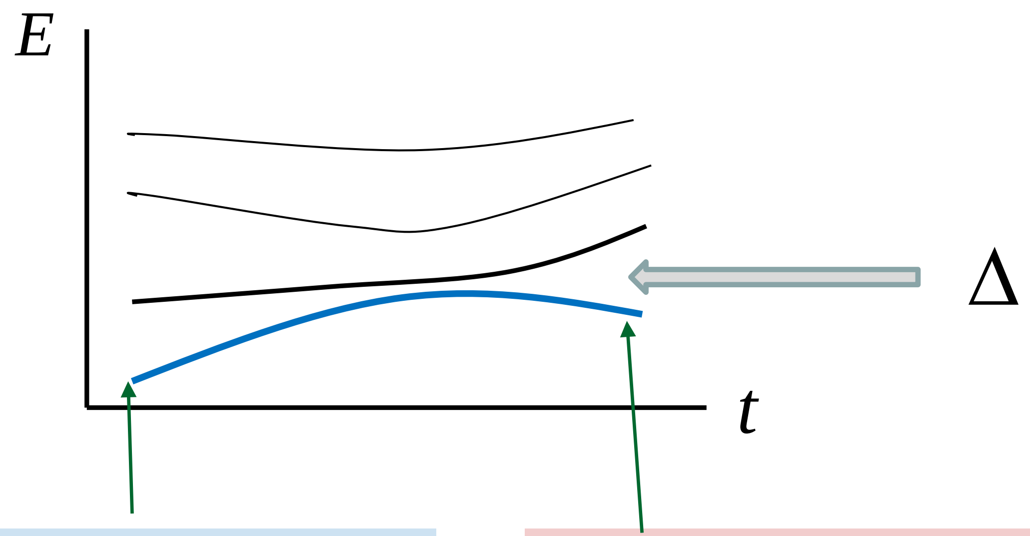
Convergence theorem

$$H = H_{\text{classical}} + H_{\text{quantum}} = -\sum J_{ij}\sigma_i^z\sigma_j^z - \Gamma(t)\sum \sigma_i^x$$

Convergence condition $\Gamma(t) = t^{-c/N}$ *Morita & Nishimori*



Quantum adiabatic computation



Trivial initial state

Non-trivial final state

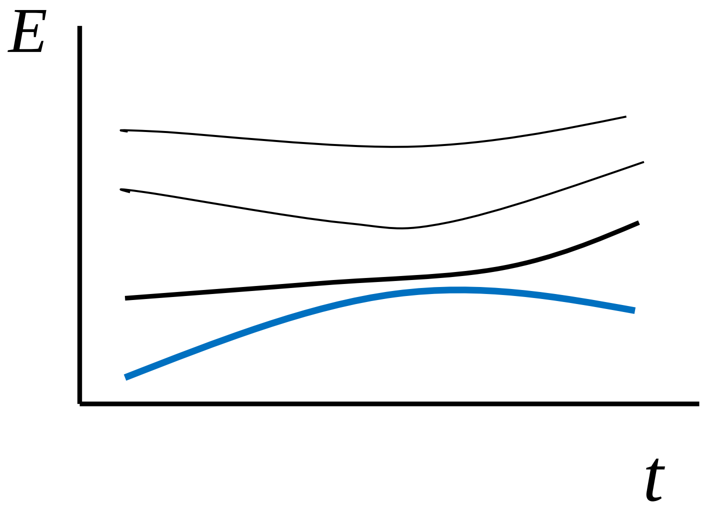
$$H(t) = -\left(1-\frac{t}{\tau}\right)\sum \sigma_i^x - \frac{t}{\tau}\sum J_{ij}\sigma_i^z\sigma_j^z$$

Computational complexity

Finite-size analysis

Adiabatic theorem $\tau \propto \Delta^{-2}$

Gap scaling $\Delta \propto \begin{cases} e^{-aN} \\ N^{-b} \end{cases}$



Complexity $\tau \propto \begin{cases} e^{2aN} & \text{(hard)} \\ N^{2b} & \text{(easy)} \end{cases}$

D-Wave Machine

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- Hardware realization of quantum annealing
- Adjustable couplings J_{ij}
- Google, NASA, Lockheed-Martin bought.

Quantumness

Energy spectrum and entanglement

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Precision of parameter setting

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Success probability increases if one allows a band of energies above the ground state

Katzgraber et al, Phys. Rev. X 2015: Seeking quantum speedup through spin glasses

Summary

- QA works fine as a generic, approximate algorithm.
- “Better” than classical simulated annealing.
- Hardware has been realized and tested.