Tracer dynamics in *E-coli* suspensions

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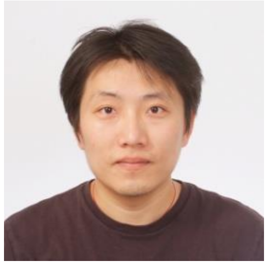

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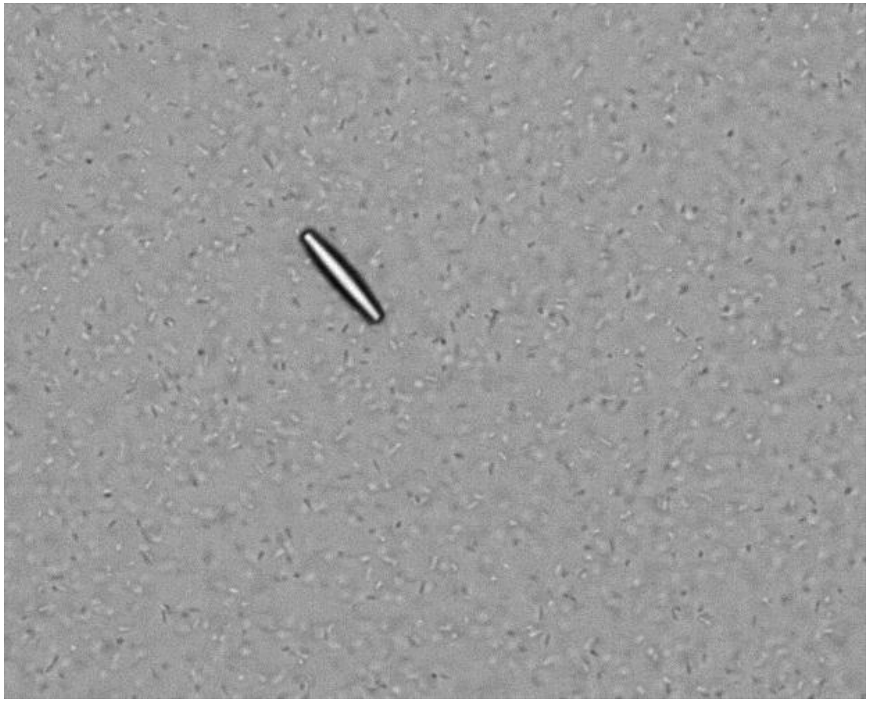
Statistical Physics
Soft Matter

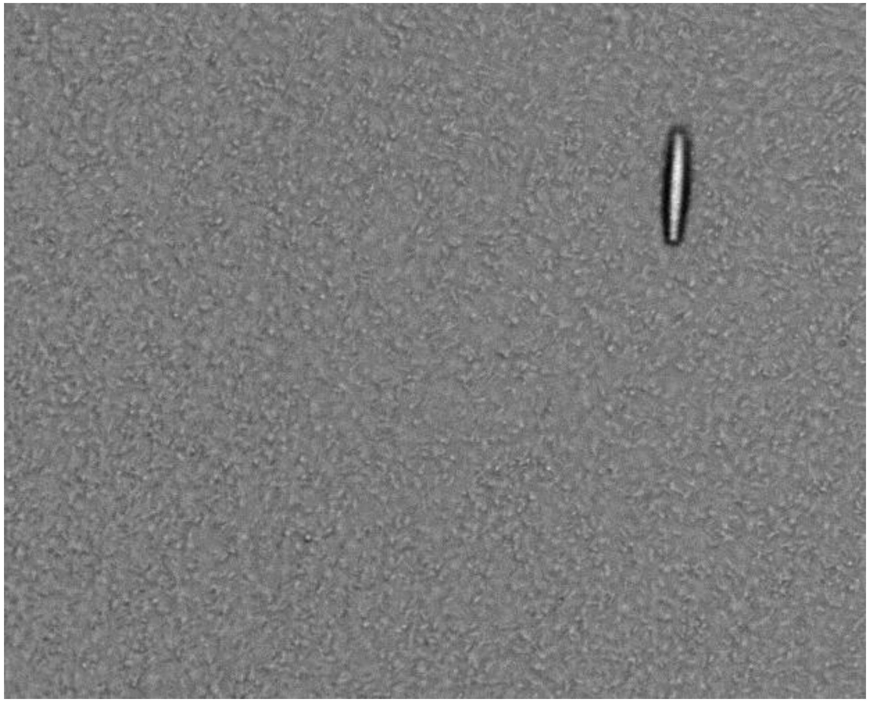
Active Matter

Non-equilibrium system consisting of self propelling individuals: animals, cells, motor proteins (natural); smart colloids, robots (man made)

- ‘flocking’ dynamics
- Light-controlled viscosity

A bit less 'free will'

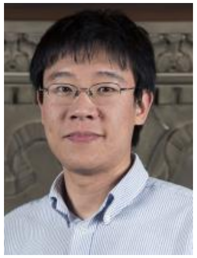




An ellipsoid in *E-coli* suspension

Free-standing films
Area: 5mm*5mm
Thickness: 10~20um

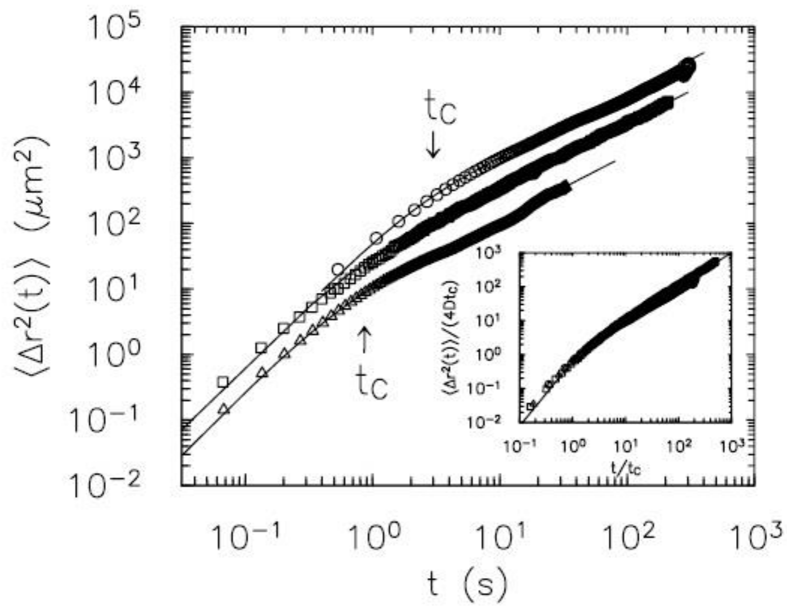
Yi Peng



Xiang Cheng
UMN, ChemE

Ellipsoid
2a~28um, 2b~5.6um

Super diffusive to diffusive

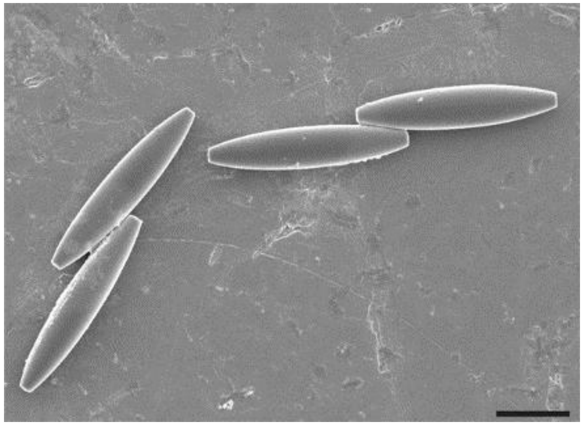


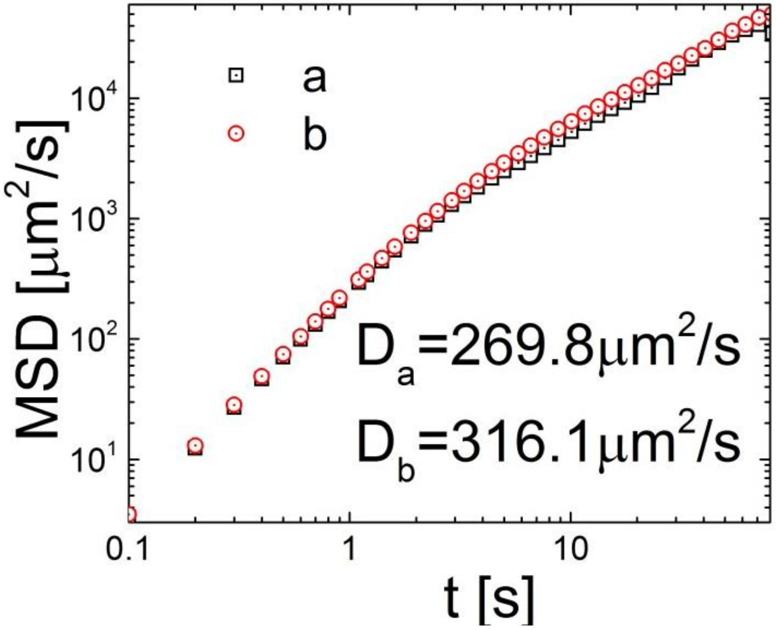
Over-damped Langevin dynamics

$$m\frac{d\vec{v}}{dt} = -\gamma\vec{v} + \vec{f}(t) \qquad t_0 = m/\gamma \simeq 10^{-5}\ \mathrm{s}$$

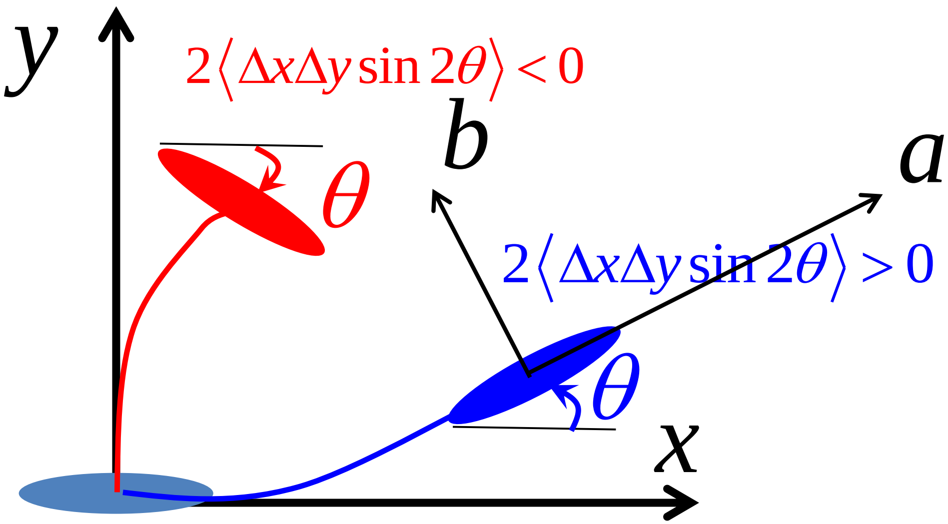
$$\langle \vec{f}(t) \rangle = 0, \ \langle \vec{f}(0) \cdot \vec{f}(t) \rangle = \frac{4D\gamma^2}{t_C}\exp(-t/t_C)$$

Wu & Libchaber, PRL (2000)





Anisotropic diffusion

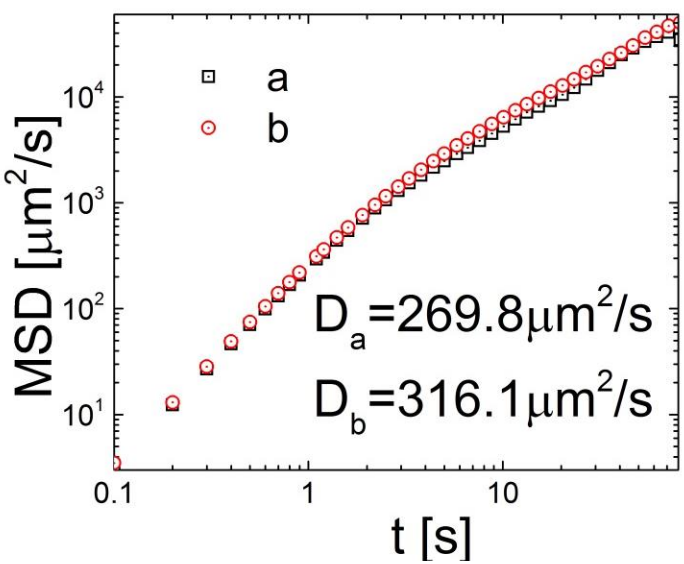


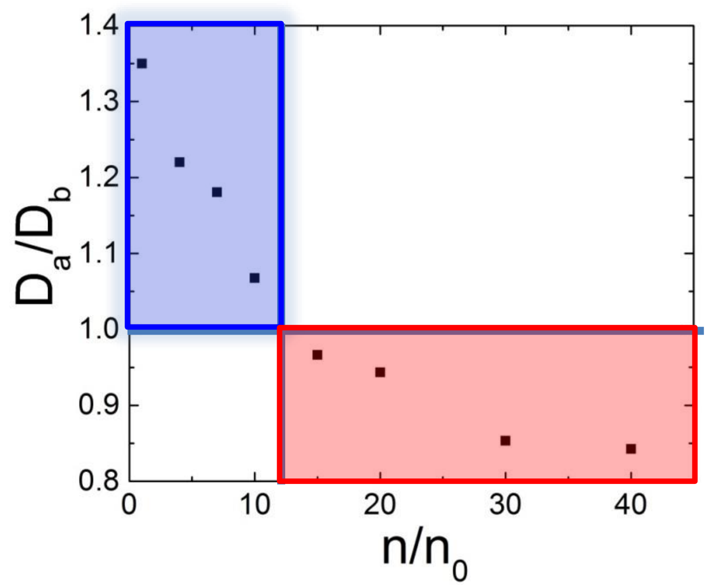
x – y lab frame

a – b body frame

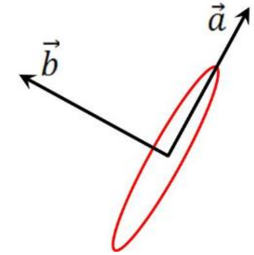
$$D_a = \left\langle \Delta a(t)^2 \right\rangle / 2t$$

$$D_b = \left\langle \Delta b(t)^2 \right\rangle / 2t$$

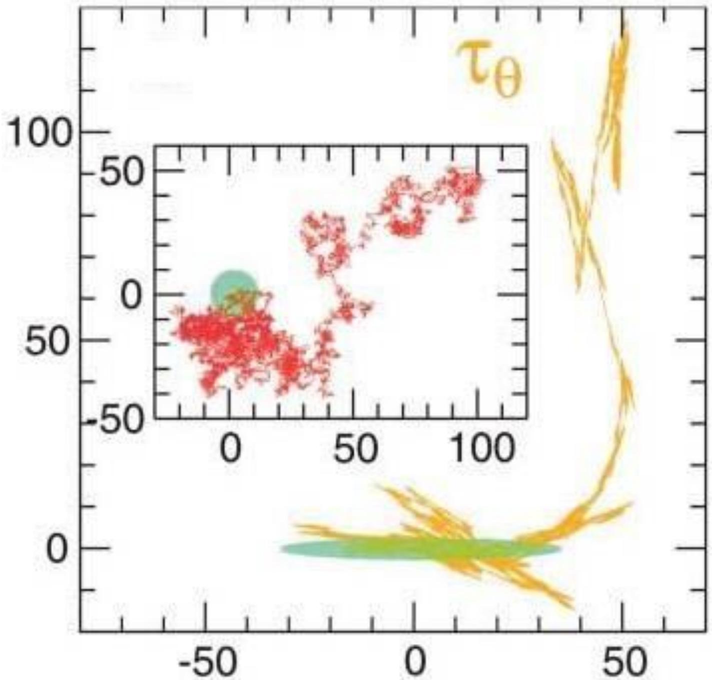




Previous understanding



Conventional fluids



Han et al., Science (2006)

Equal partition theorem

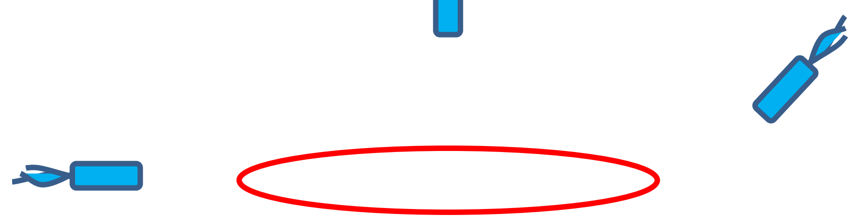
Einstein relation $D_a/D_b > 1$

Active suspension

Traditional theory:

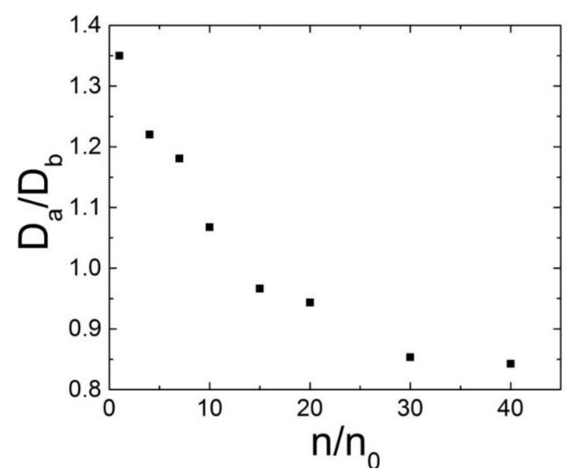
Tracer treated as point like

$D_a/D_b = 1$

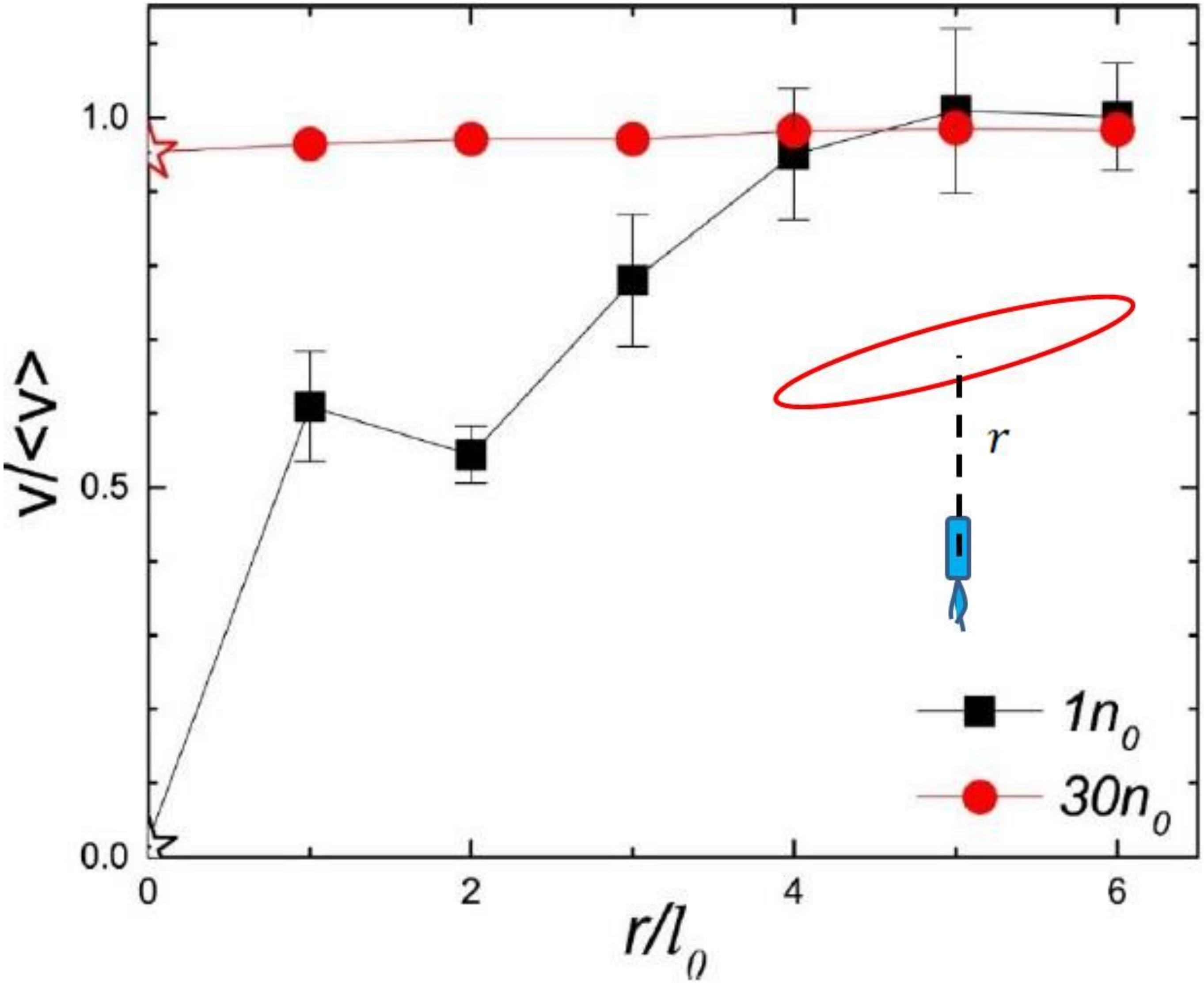


Lin et al. JFM (2011)

Morozov, Soft Matter (2014)



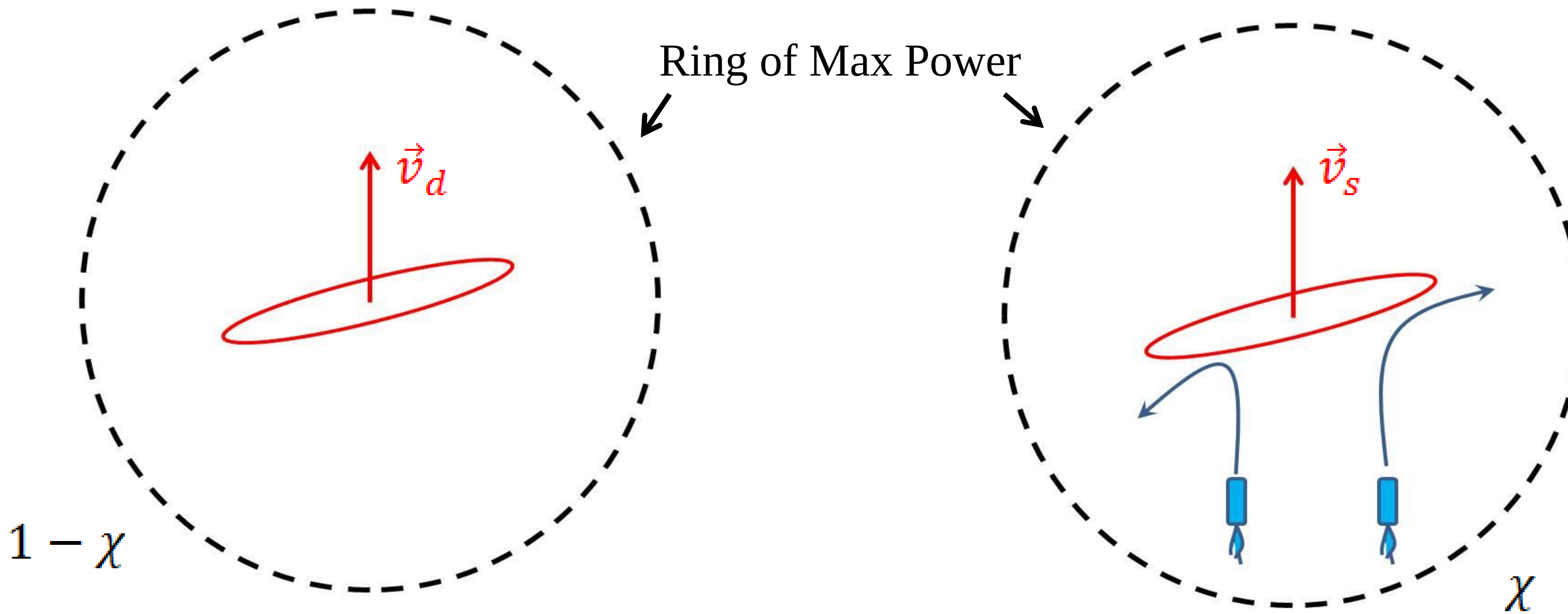
Mechanism 1: max power



Two types of interaction events

Dipolar flow events: Isotropic velocity

Scattering events: Isotropic energy

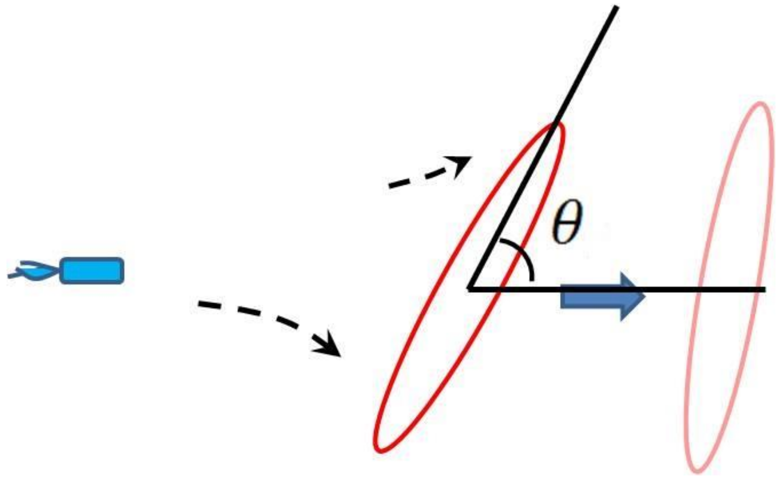


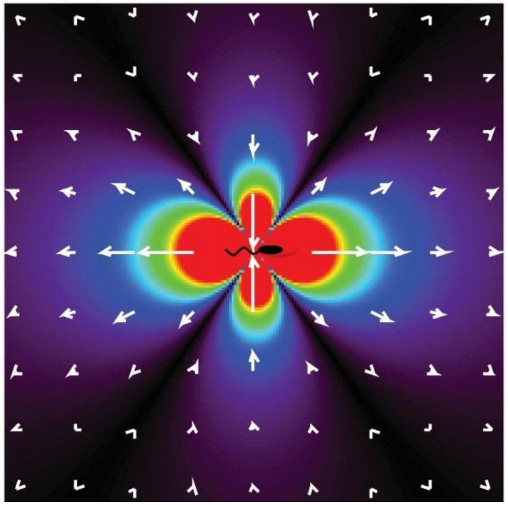
$$\vec{v} = \vec{v}_d + \vec{v}_s$$

$$|\vec{v}_d|/|\vec{v}_s| = (1-\chi):\chi$$

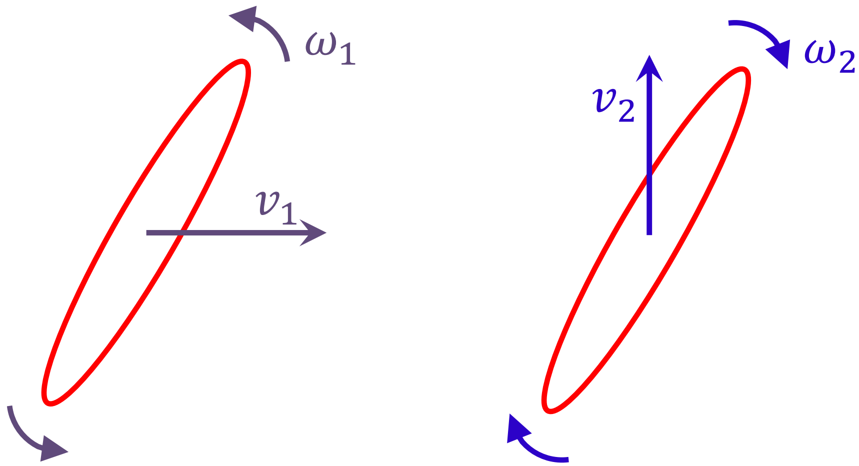
$$D_a/D_b \begin{cases} = 1 \ (if\ \chi = 0) \\ > 1 \ (if\ \chi = 1) \end{cases}$$

Mechanism 2: straining field





Examples of negative S:



Dipolar flow:

$$\boldsymbol{v} = -\kappa(\boldsymbol{r}/r^3 - 3\,x^2\boldsymbol{r}/r^5)$$

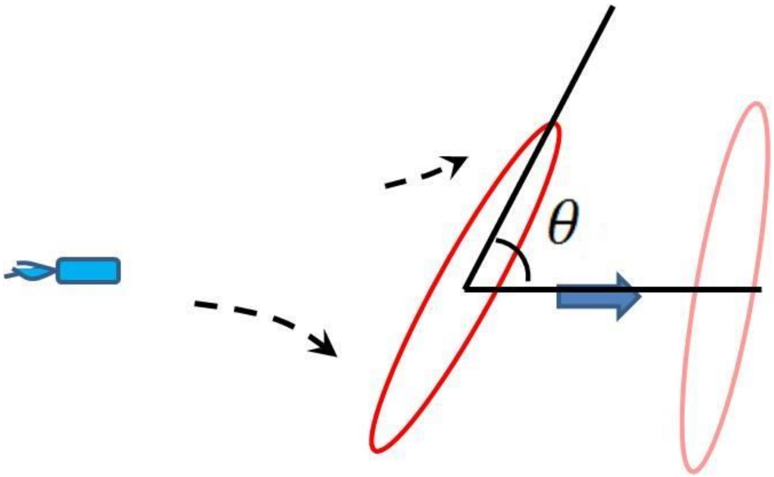
Ellipsoid rotation:

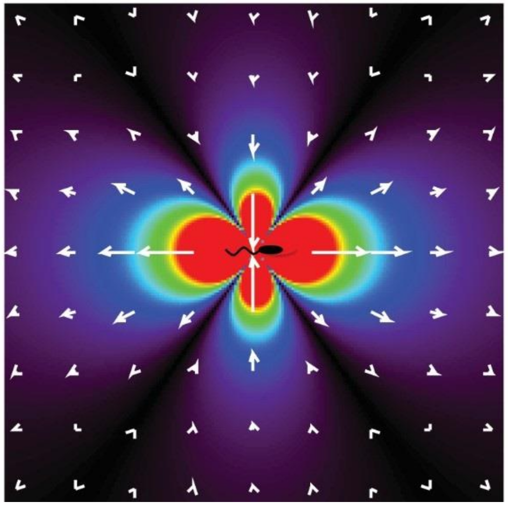
$$\boldsymbol{\omega} = \frac{1}{2}\vec{\nabla}\times\boldsymbol{v} + \frac{p^2-1}{p^2+1}\hat{\boldsymbol{a}}\times(\boldsymbol{\varepsilon}\cdot\hat{\boldsymbol{a}})$$

Intrinsic correlation:

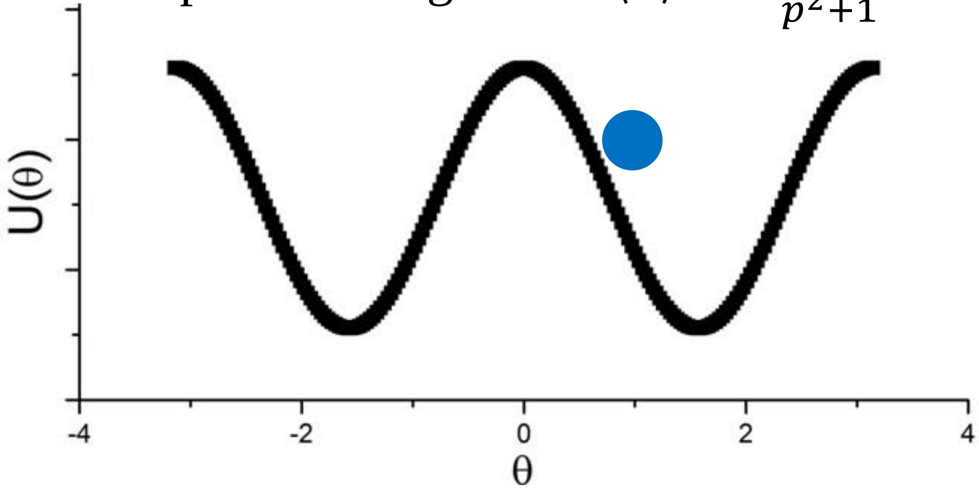
$$S = \boldsymbol{\omega}\cdot\left(\frac{\hat{\boldsymbol{a}}\times\boldsymbol{v}}{|\hat{\boldsymbol{a}}\times\boldsymbol{v}|}\right)$$

Mechanism 2: straining field





The spatial average of S: $\langle S \rangle \sim -\frac{p^2-1}{p^2+1} < 0$



Controlled by single parameter: $U/k_B T_{eff}$

Dipolar flow:

$$\boldsymbol{u} = -\kappa(\boldsymbol{r}/r^3 - 3\,x^2\boldsymbol{r}/r^5)$$

Ellipsoid rotation:

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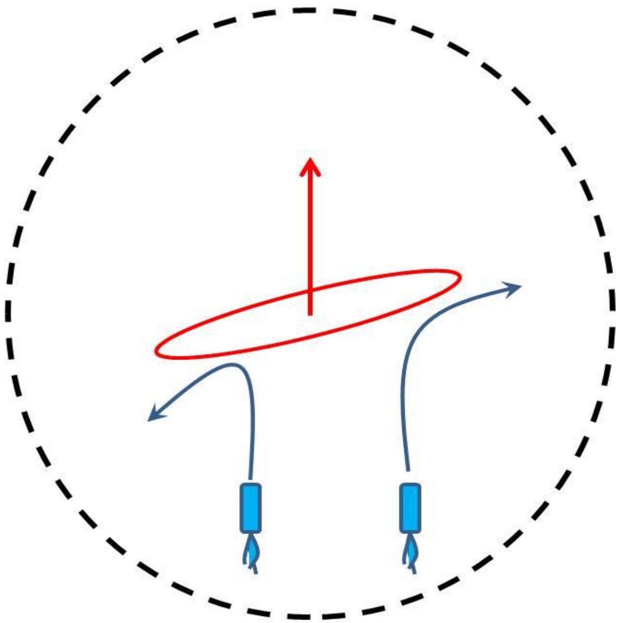
Intrinsic correlation:

$$S = \boldsymbol{\omega} \cdot \left(\frac{\hat{\boldsymbol{a}} \times \boldsymbol{v}}{|\hat{\boldsymbol{a}} \times \boldsymbol{v}|}\right)$$

Coupled Langevin dynamics

Biological limit on power output

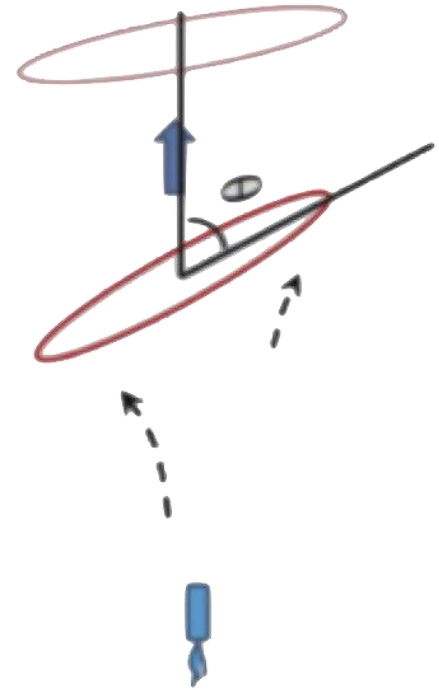


$$\gamma \frac{d\theta}{dt} = -\frac{dU(\theta)}{d\theta} + \xi(dt)$$

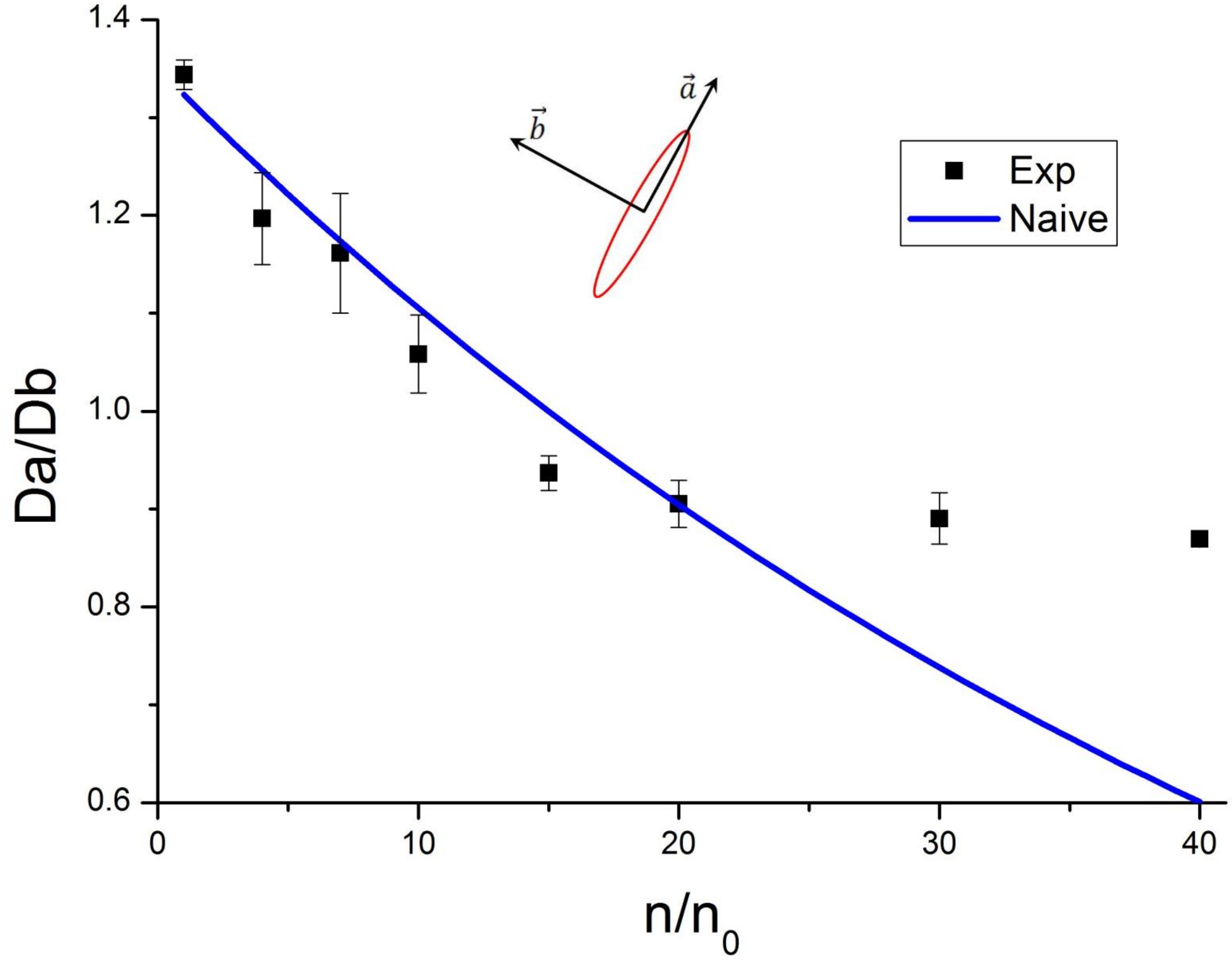
$$\vec{v} = \vec{v}_d + \vec{v}_s$$

$$|\vec{v}_d|/|\vec{v}_s| = (1-\chi):\chi$$

Straining contribution

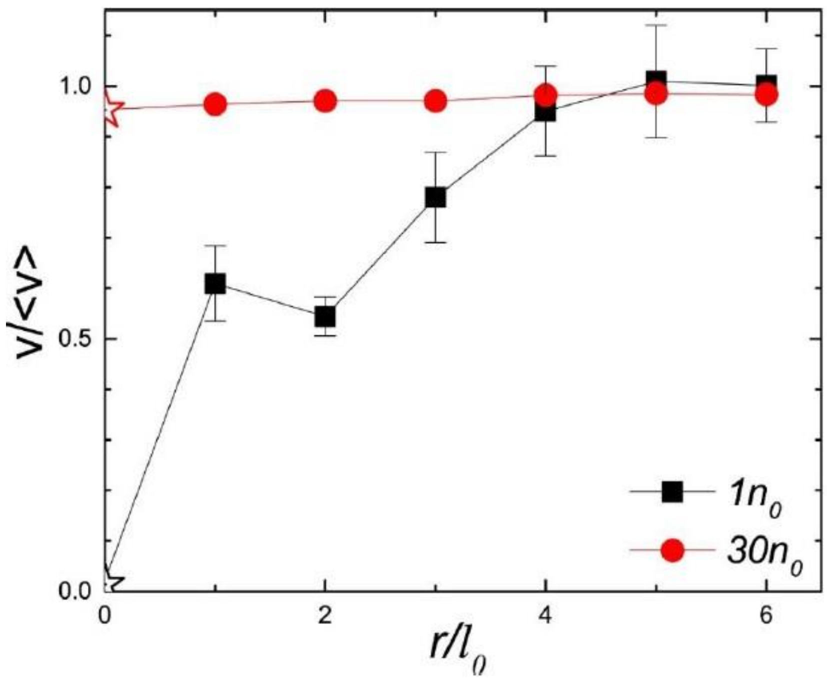


Naïve model: χ independent of n

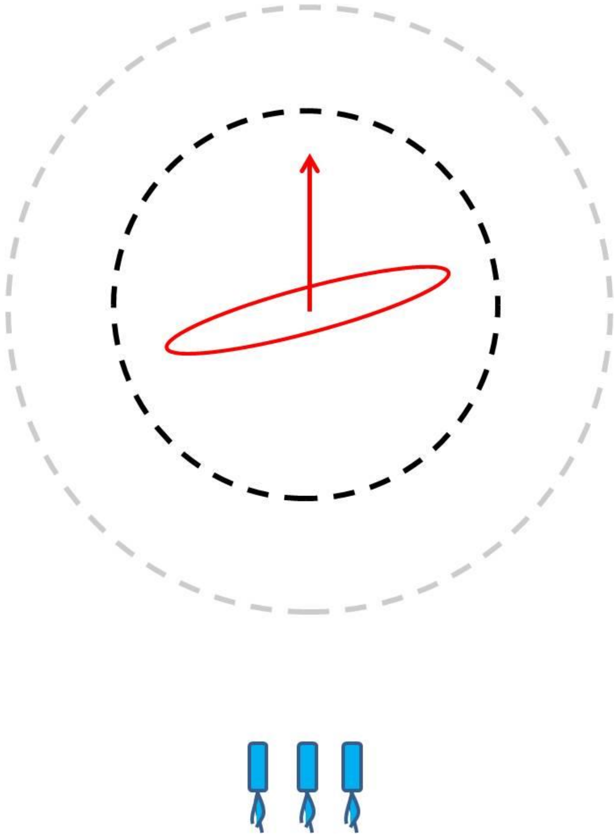


Realistic model

Experimental observation



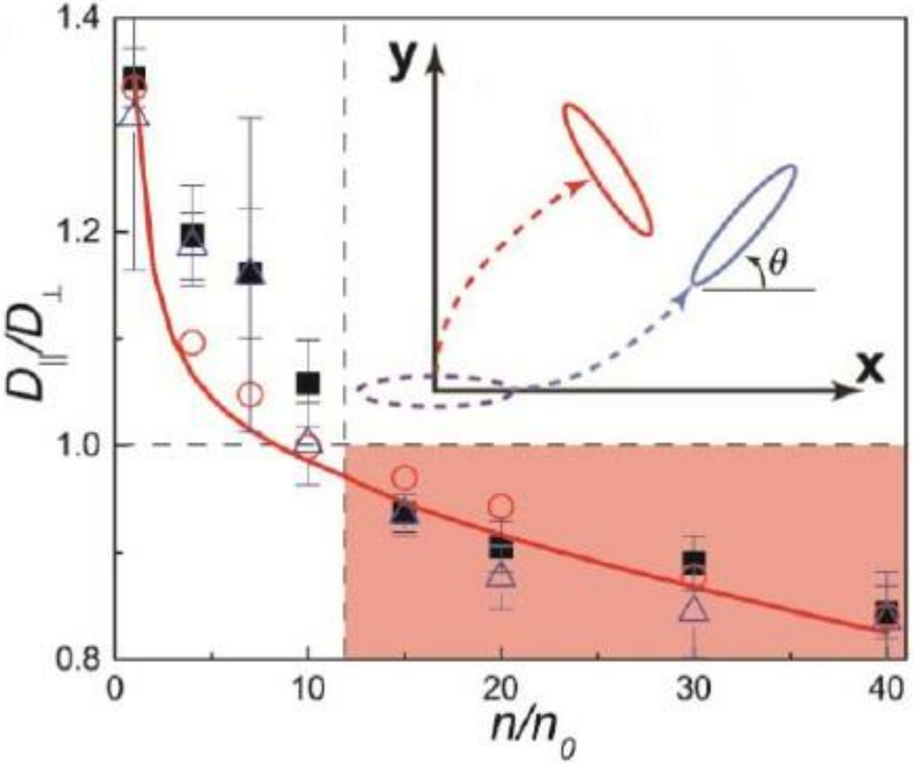
Number fluctuation



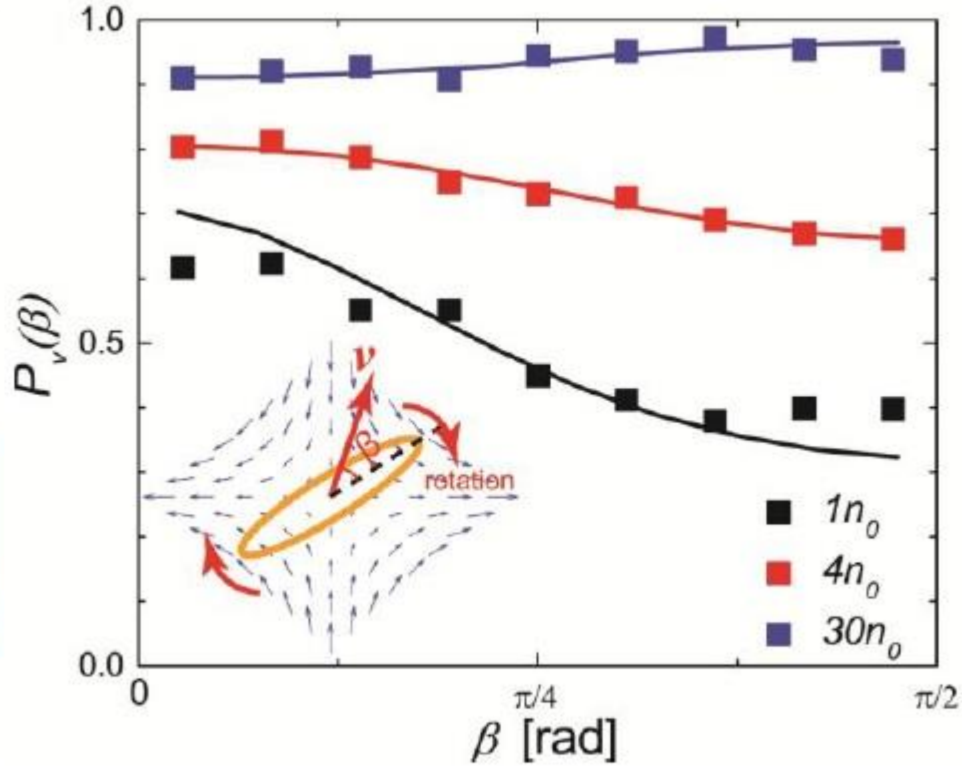
$$\chi(n) * D(n) = \chi(n_0) * D(n_0)$$

Theory vs. Experiment

Anisotropic diffusion



Angular distribution

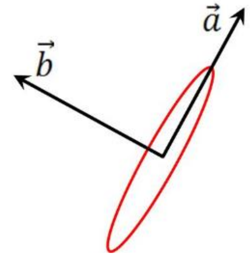


$$D_a = \int \langle v_a(0) v_a(t) \rangle dt \approx \langle v_a^2 \rangle \int \langle n_a(0) n_a(t) \rangle dt = \frac{\langle \Delta x_a^2 \rangle}{\Delta t^2} * \tau_a$$

Controlled by single parameter: $U(n)/k_B T_{eff} \sim n$

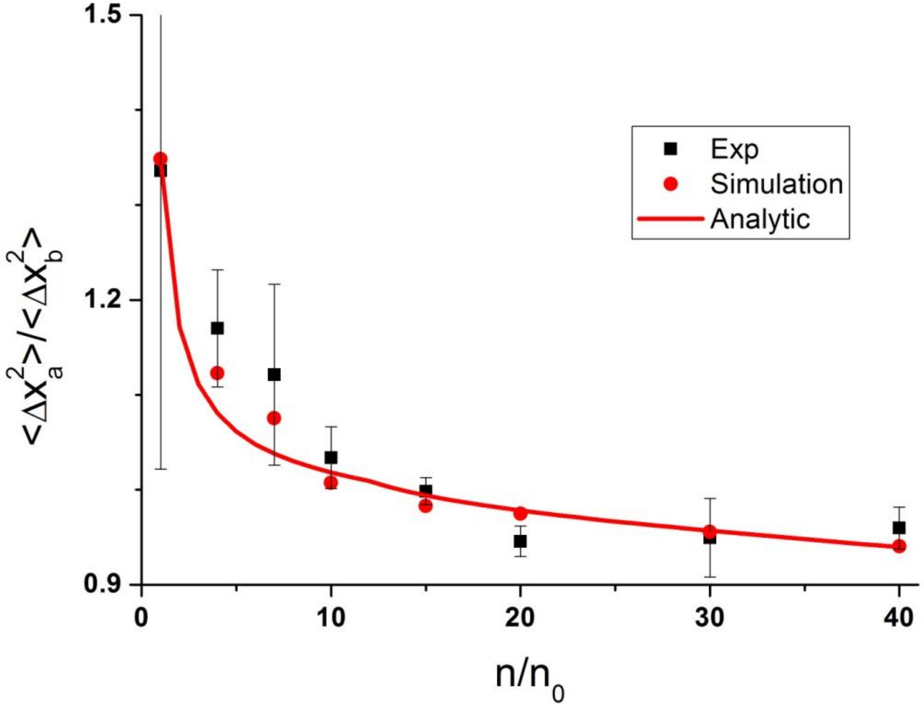
Contributions to anisotropic diffusion

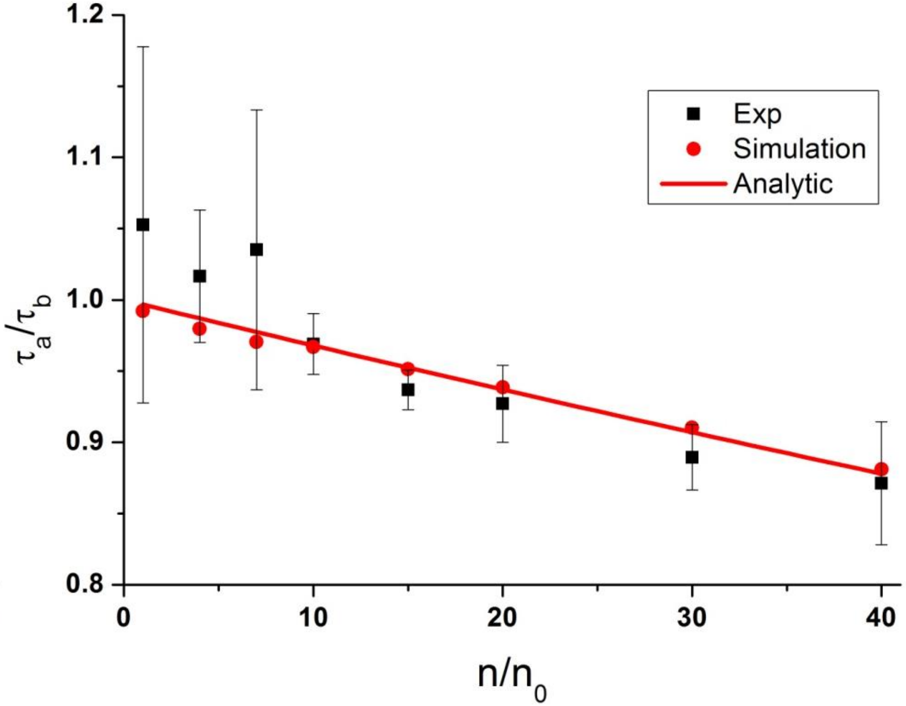
$$D_a = \int \langle v_a(0) v_a(t) \rangle dt \approx \langle v_a^2 \rangle \int \langle n_a(0) n_a(t) \rangle dt = \frac{\langle \Delta x_a^2 \rangle}{\Delta t^2} * \tau_a$$



Step size ratio

Persistent time ratio

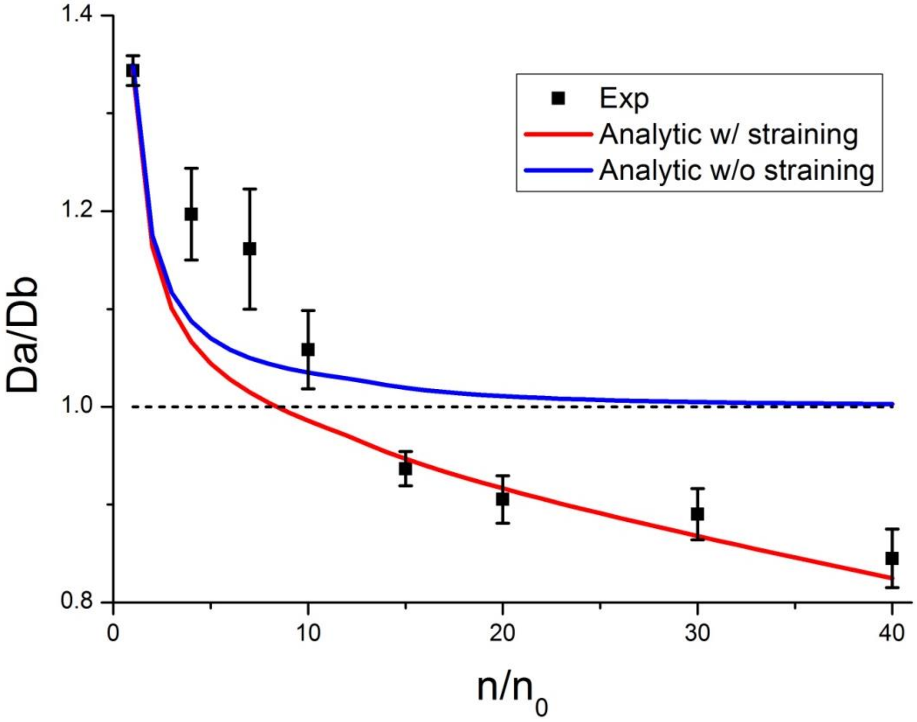




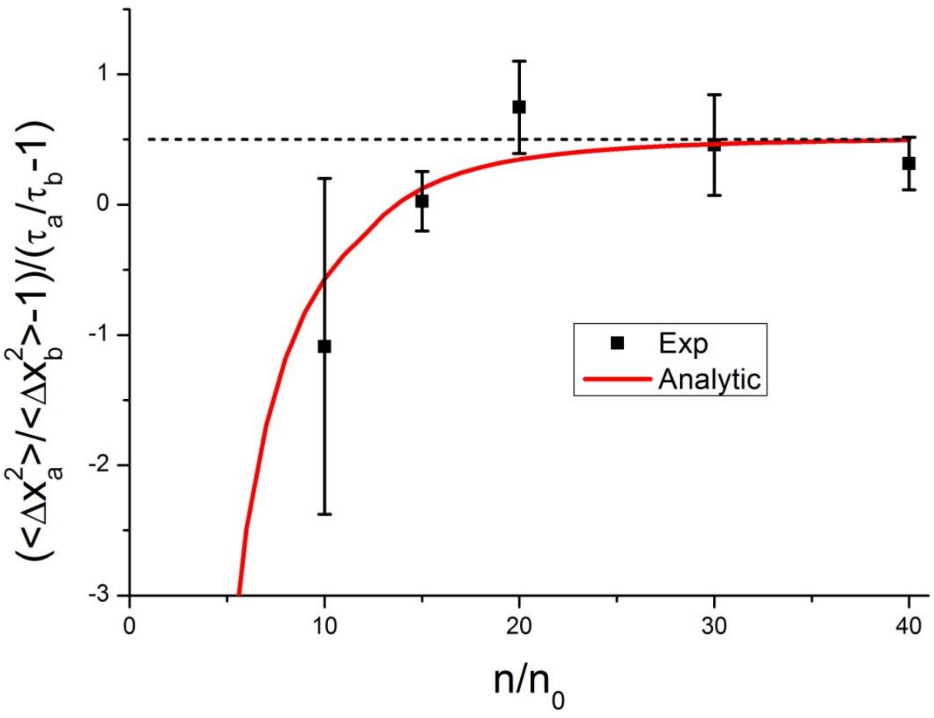
Controlled by single parameter: $U(n)/k_B T_{eff} \sim n$

More comparisons

Without straining mechanism



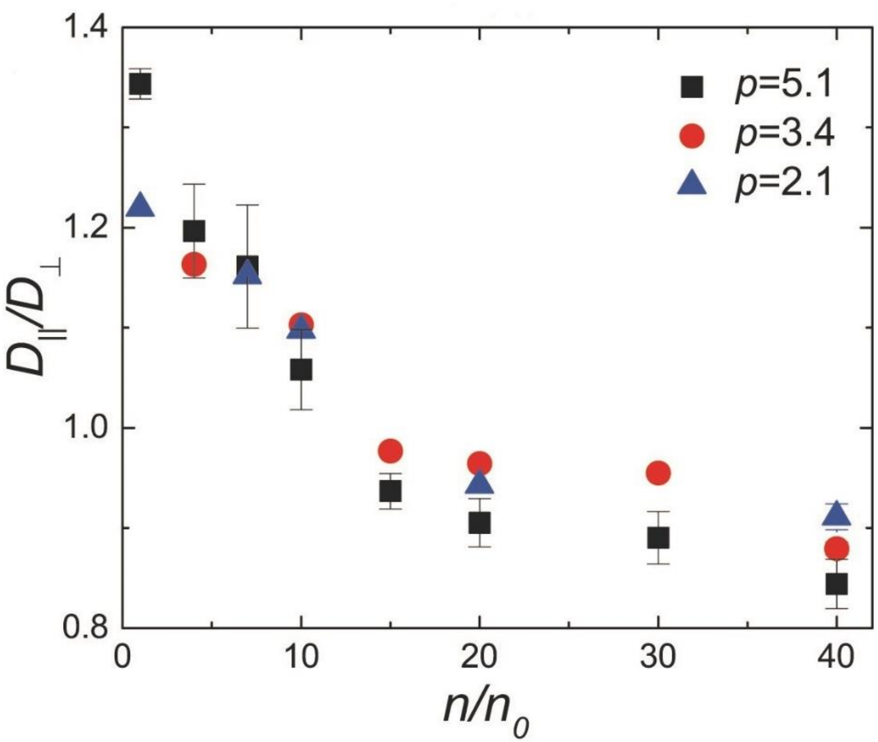
Parameter free results



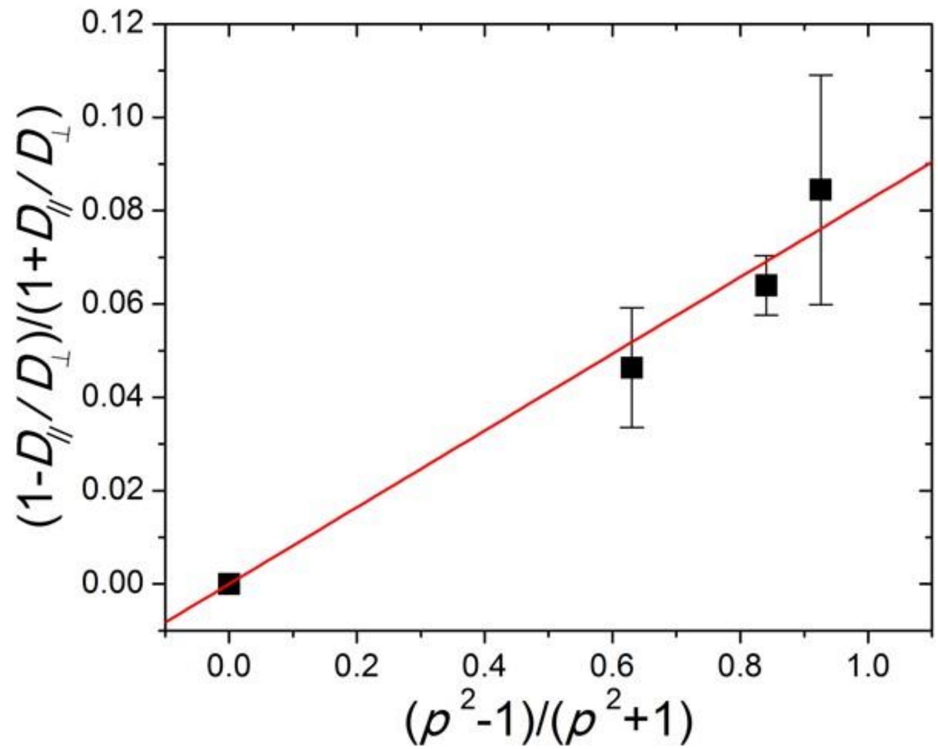
Controlled by single parameter: $U(n)/k_B T_{eff} \sim n$

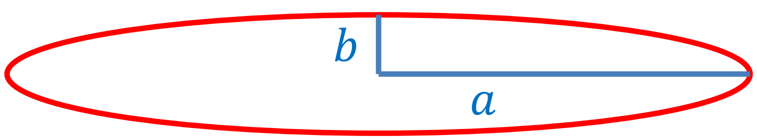
Aspect ratio dependence

Anisotropic diffusion



p dependence





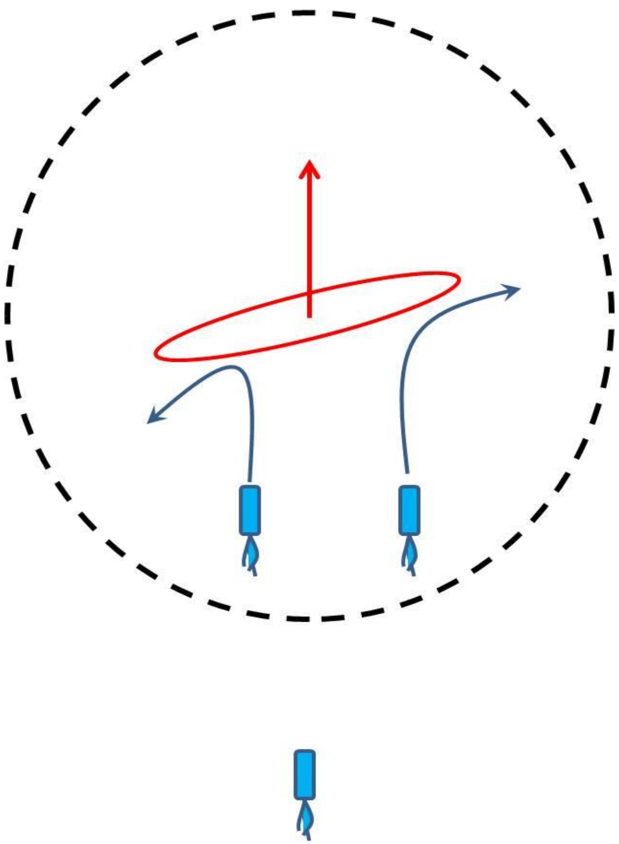
Aspect ratio $p = a/b$

Model prediction $\frac{1-D_\parallel/D_\perp}{1+D_\parallel/D_\perp} \sim \frac{p^2-1}{p^2+1}$

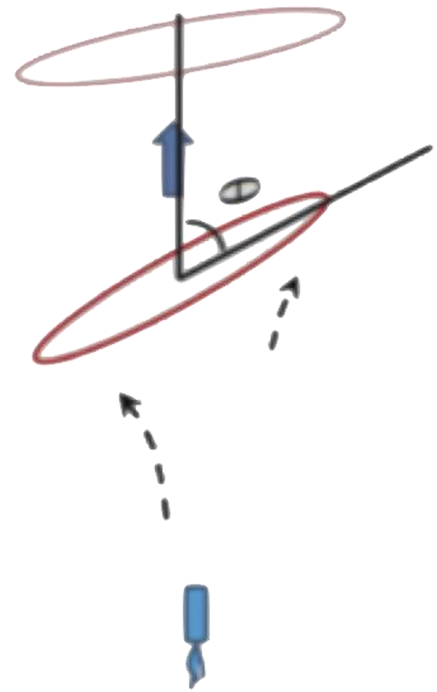
$= 0$ for spheres

Summary

Biological limit on power output



Straining contribution



Thanks for your attention

for more info, please see

www.csrc.ac.cn/~xlxu