ICAP Summer School, Seoul, S. Korea, July 19, 2016

3. Other topics in Cold Collisions: Universality, Confinement, and Chaos

Paul S. Julienne

Joint Quantum Institute
NIST and The University of Maryland

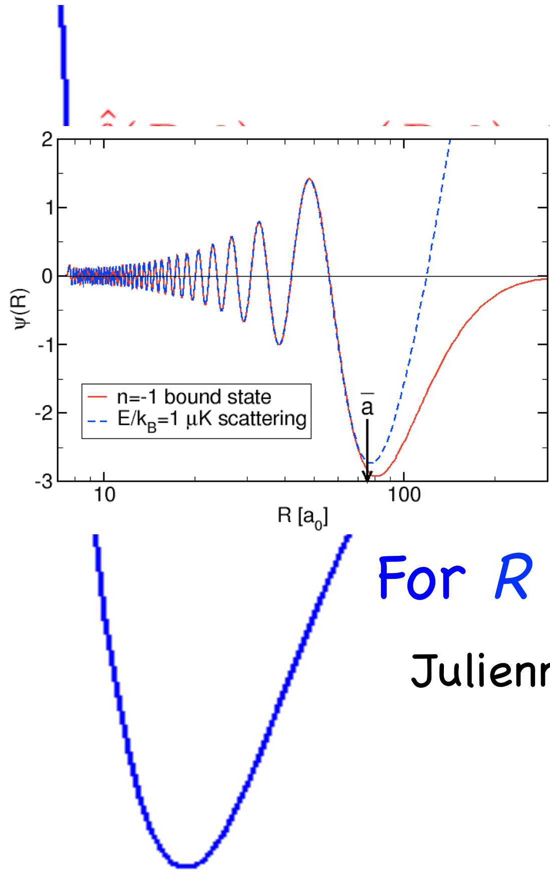
Thanks to many colleagues in theory and experiment who have contributed to this work

http://www.jqi.umd.edu/

NIST

Joint Quantum Institute

QDT Semiclassical considerations



$\ldots n\,\beta(R,0)$

$$f(R,E) \to \frac{1}{\sqrt{k}} \sin(kR + \eta(E))$$

$R_{\rm vdw}$

$$f(R,E) = C(E)^{-1}\hat{f}(R,E)$$

For $R \ll R_{vdw}$ $\quad = C(E)^{-1}\hat{f}(R,0)$

Julienne and Mies, J. Opt. Soc. Am. B 89, 2257 (1989)

If short-range coupling << R_{vdw}

$\alpha(R,E) = \ldots$

$\beta(R,E) = \ldots$

Consequently: $\quad V_n(E) = \langle n|H|E\rangle = C(E)^{-1}\hat{V}_n$

$$\Gamma_n(E) = C(E)^{-2}\hat{\Gamma}_n$$

Universal inelastic loss rates:
Molecular reactions

VOLUME 90, NUMBER 11 PHYSICAL REVIEW LETTERS week ending 21 MARCH 2003

Creation of a Dipolar Superfluid in Optical Lattices

B. Damski,[1,2] L. Santos,[1] E. Tiemann,[3] M. Lewenstein,[1] S. Kotochigova,[4] P. Julienne,[4] and P. Zoller[5]

[1]*Institut für Theoretische Physik, Universität Hannover, D-30167 Hannover, Germany*
[2]*Instytut Fizyki, Uniwersytet Jagielloński, Reymonta 4, PL-30 059 Kraków, Poland*
[3]*Institut für Quantenoptik, Universität Hannover, D-30167 Hannover, Germany*
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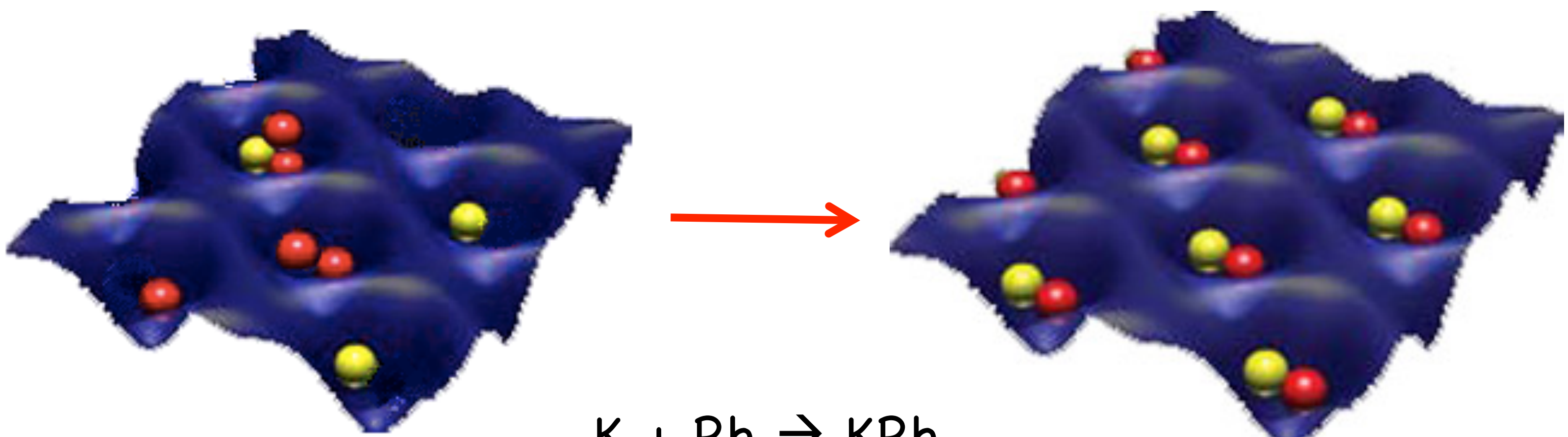
[5]*Institut für Theoretische Physik, Universität Innsbruck, A-6020 Innsbruck, Austria*

(Received 20 August 2002; published 17 March 2003)

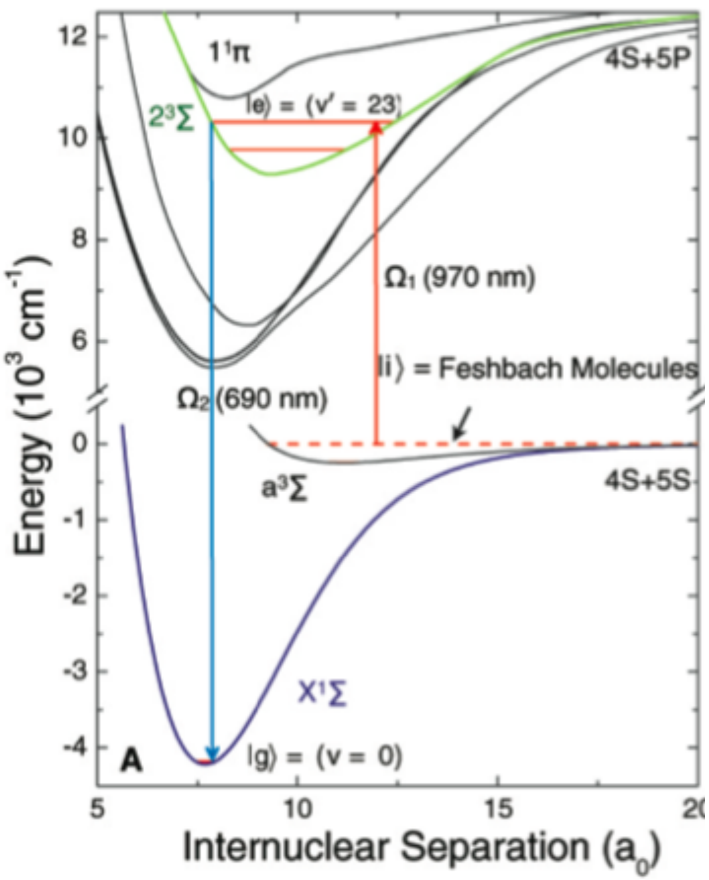
We show that, by loading a Bose-Einstein condensate of two different atomic species into an optical lattice, it is possible to achieve a Mott-insulator phase with exactly one atom of each species per lattice site. A subsequent photoassociation leads to the formation of one heteronuclear molecule with a large electric dipole moment, at each lattice site. The melting of such a dipolar Mott insulator creates a dipolar superfluid, and eventually a dipolar molecular condensate.



K + Rb → KRb

Science
AAAS

A High Phase-Space-Density Gas of Polar Molecules
K.-K. Ni, *et al.*
Science **322**, 231 (2008); Ye/Jin group, JILA



$^{40}K^{87}Rb$

20000 v=0, J=0 molecules

in single hyperfine state

200 nK trapped gas

10^{12} molecules/cm^3

Loss by chemical reaction:

$KRb + KRb \rightarrow K_2 + Rb_2$

Current (<5000 v=0 nonreactive molecules)

RbCs (Innsbruck, Durham)

NaK (MIT)

NaRb (Hong Kong)

s-wave collision summary

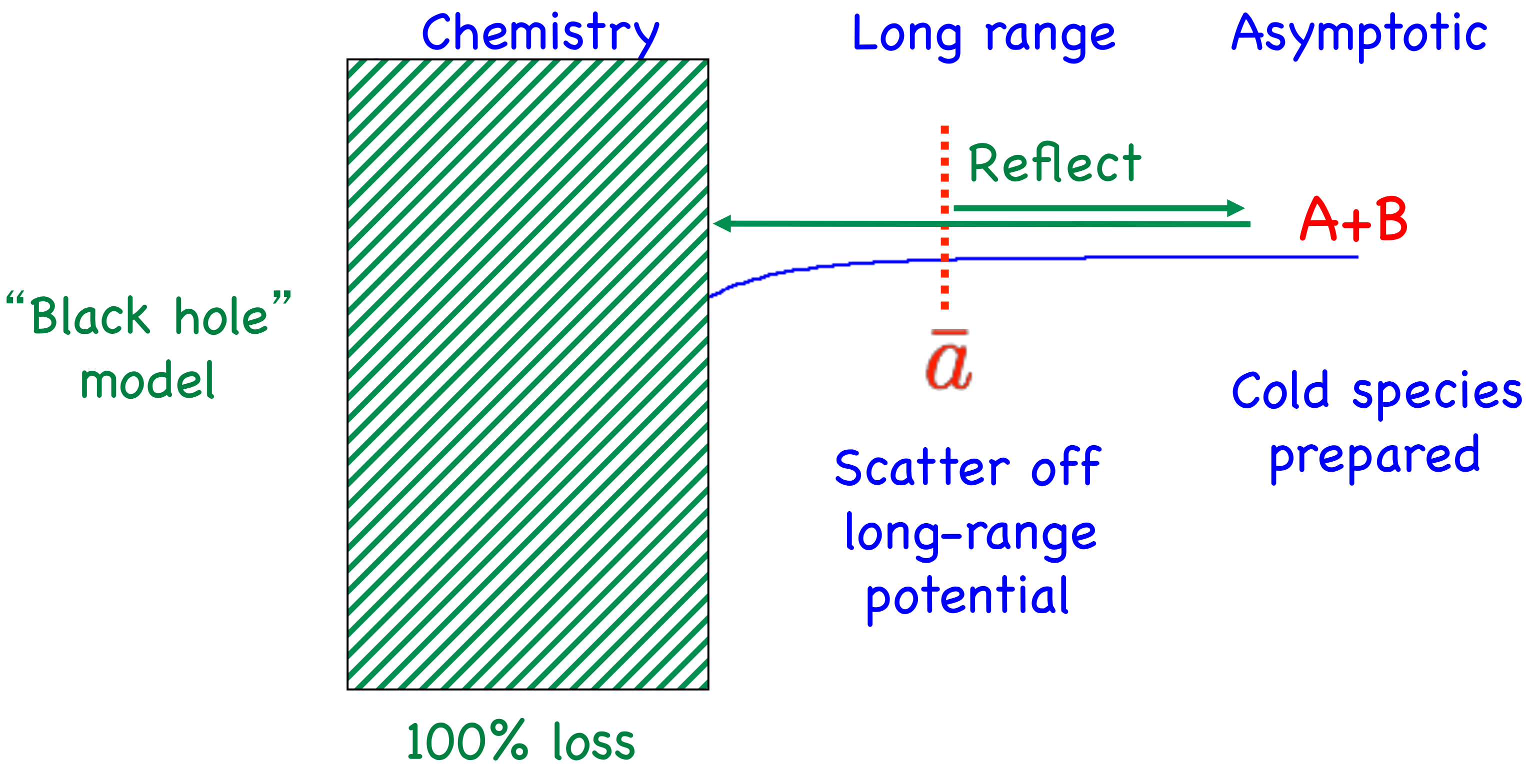
In general, $S_{\alpha\alpha} = e^{-2ik(a-ib)}$ as $k \to 0$

Complex scattering length *a-ib*

$$\sigma_{\alpha\alpha} = 4\pi(a^2 + b^2)$$

$$K_{\mathrm{loss}} = \sum_{\alpha' \neq \alpha} K_{\alpha\alpha'} = 2\frac{h}{\mu}b$$

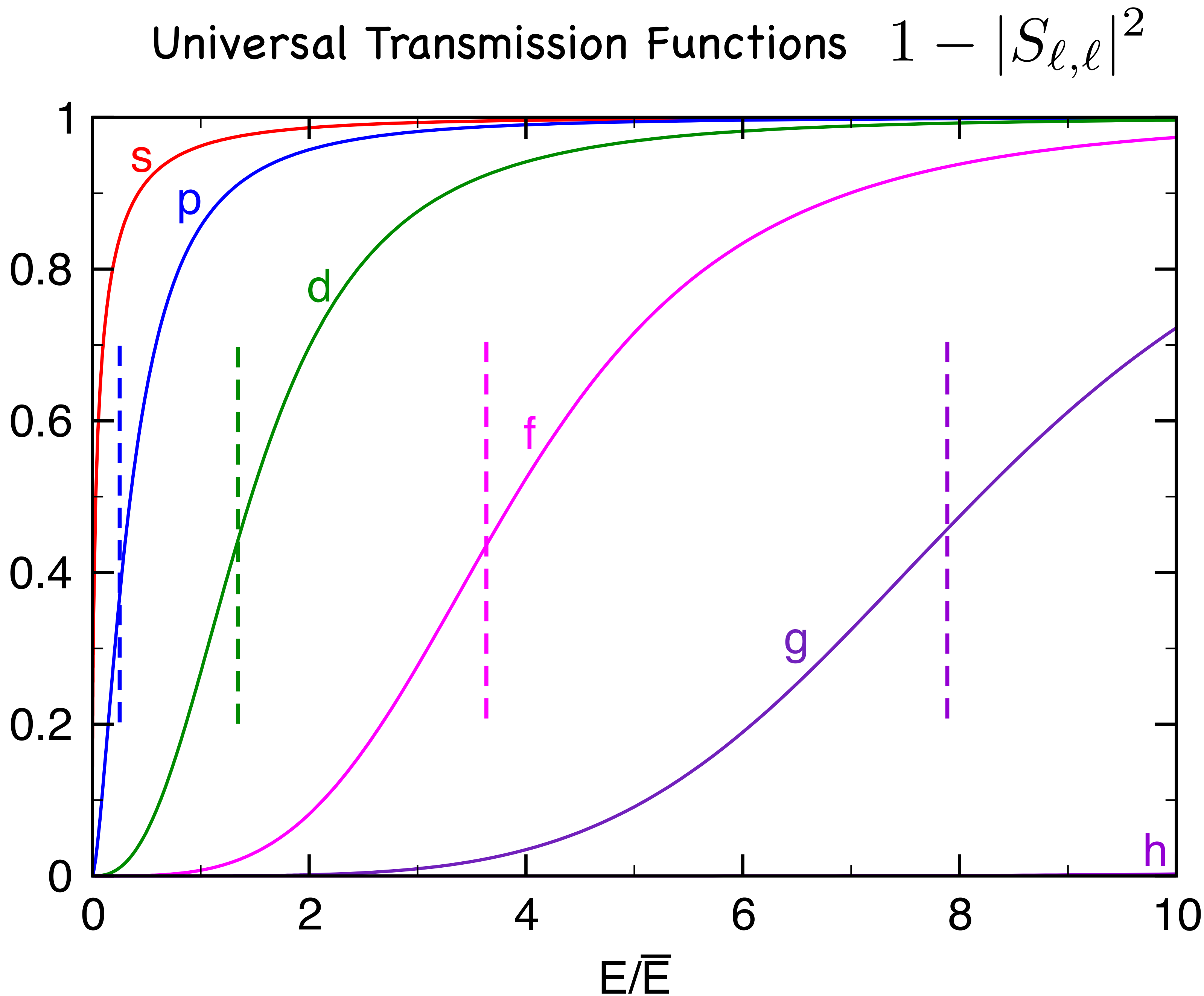
“Universal” van der Waals capture model
Quantum version of classical Langevin (1905) and Gorin (1938) models



$$\tilde{a}_0 = \bar{a}(1-i) \qquad K^{\text{loss}}_{\ell=0}(E) = 2\frac{h}{\mu}\,\bar{a}$$

Identical fermions (p-wave): $$K^{\text{loss}}_{\ell=1}(T) = 1513\,\bar{a}^3\,\frac{k_B T}{h}$$

Idziaszek & PSJ, PRL 104, 113204 (2010)
Similar: Gao, PRL 105, 263203(2010)

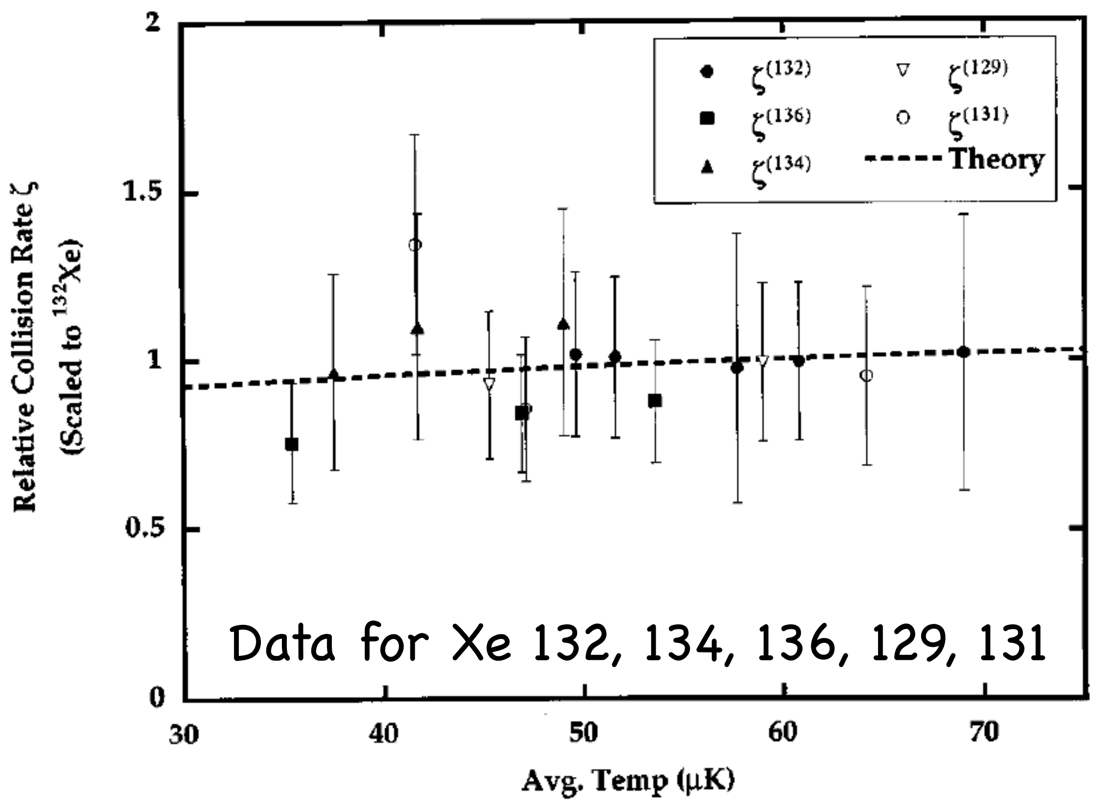

Universal Transmission Functions $1 - |S_{\ell,\ell}|^2$
s
p
d
f
g
h
0
0.2
0.4
0.6
0.8
1
0
2
4
6
8
10
E/Ē

PHYSICAL REVIEW A VOLUME 59, NUMBER 3 MARCH 1999

Spin polarization and quantum-statistical effects in ultracold ionizing collisions

C. Orzel,* M. Walhout,† U. Sterr,‡ P. S. Julienne, and S. L. Rolston
National Institute of Standards and Technology, PHY A167, Gaithersburg, Maryland 20899

$$\mathrm{Xe}(^3P_2) + \mathrm{Xe}(^3P_2) \rightarrow \text{ Penning or associative ionization}$$



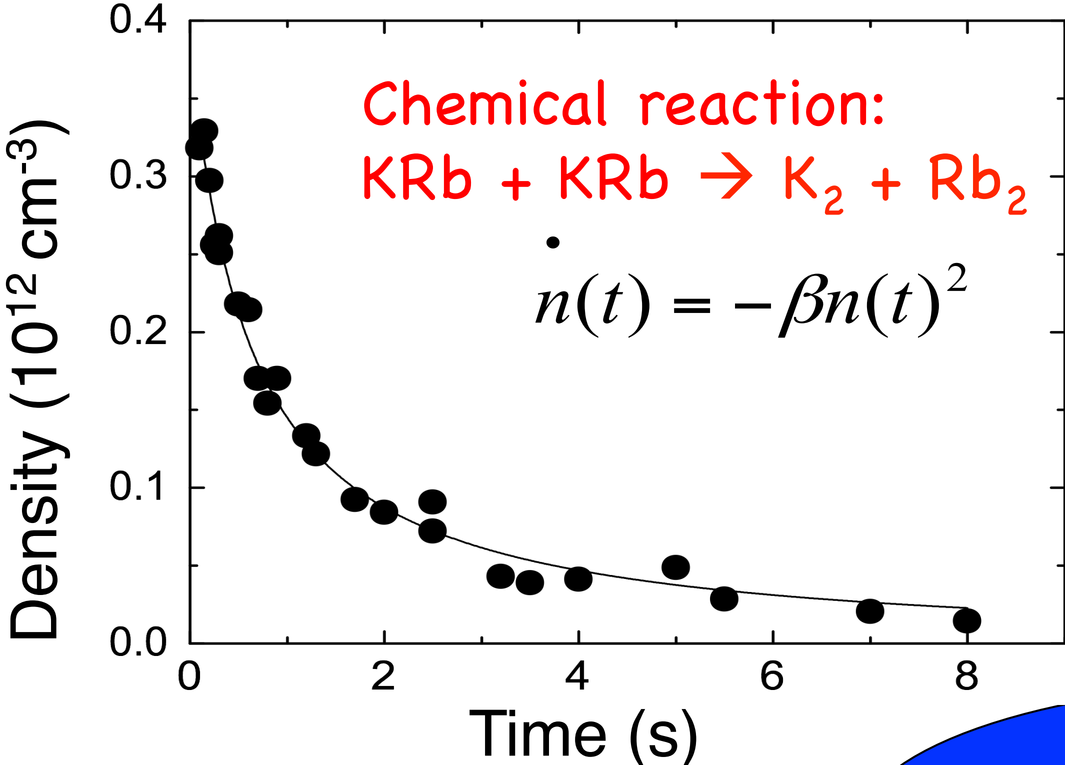
Universal
van der Waals
"black hole" model

6.5×10^{-11} cm^3/s(Theory)

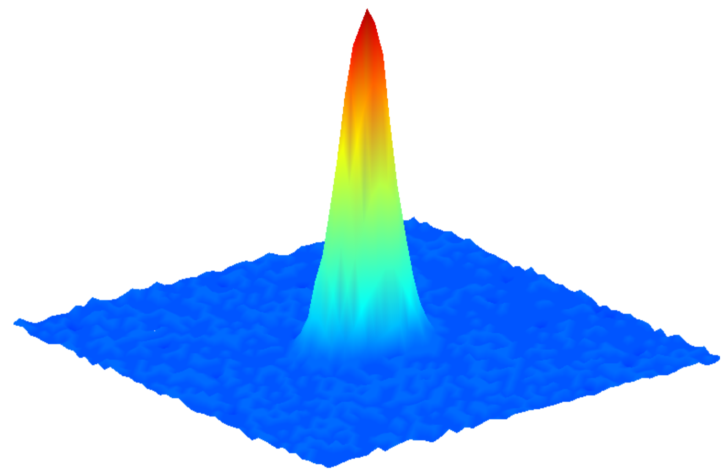
$6(2)\times10^{-11}$ cm^3/s(Exp)

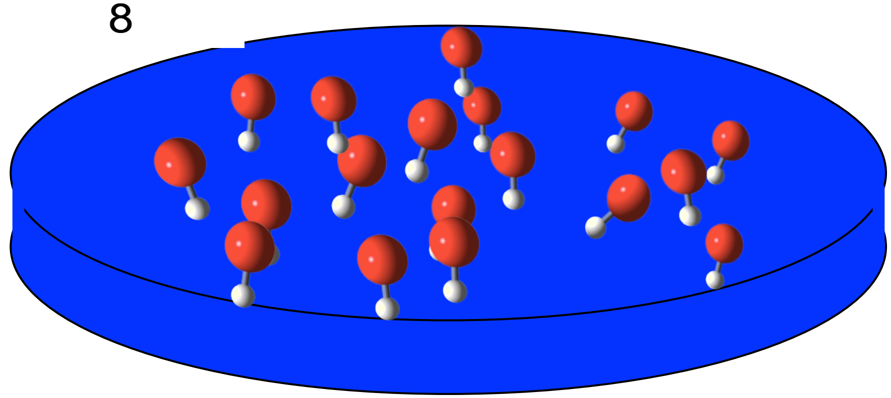
Applied to collisional quenching of excited vibrational levels of RbCs by
Hudson, Gilfoy, Kotochigova, Sage, and De Mille, Phys. Rev. Lett. 100, 203201 (2008)

Trapped $^{40}K^{87}Rb$ molecules in the lowest energy state (electronic, vibrational, rotational, hyperfine)



T = 200 nK
> 1 s lifetime (3D gas)





S. Ospelkaus et al., Science 327, 853 (2010)

Slide from Ye/Jin JILA group

Apply to $^{40}K^{87}Rb$ collisions

Universal rate coefficient, van der Waals potentials

Identical fermions (p-wave): $K^{\mathrm{loss}}_{\ell=1}(T) = 1513\,\bar{a}^3\,\frac{k_B T}{h}$

Non-identical (s-wave): $K^{\mathrm{loss}}_{\ell=0}(T) = 2\,\frac{h}{\mu}\,\bar{a}$

$\bar{a}$ = 6.2(2) nm S. Kotochigova, New J. Phys. 12, 073041(2010)

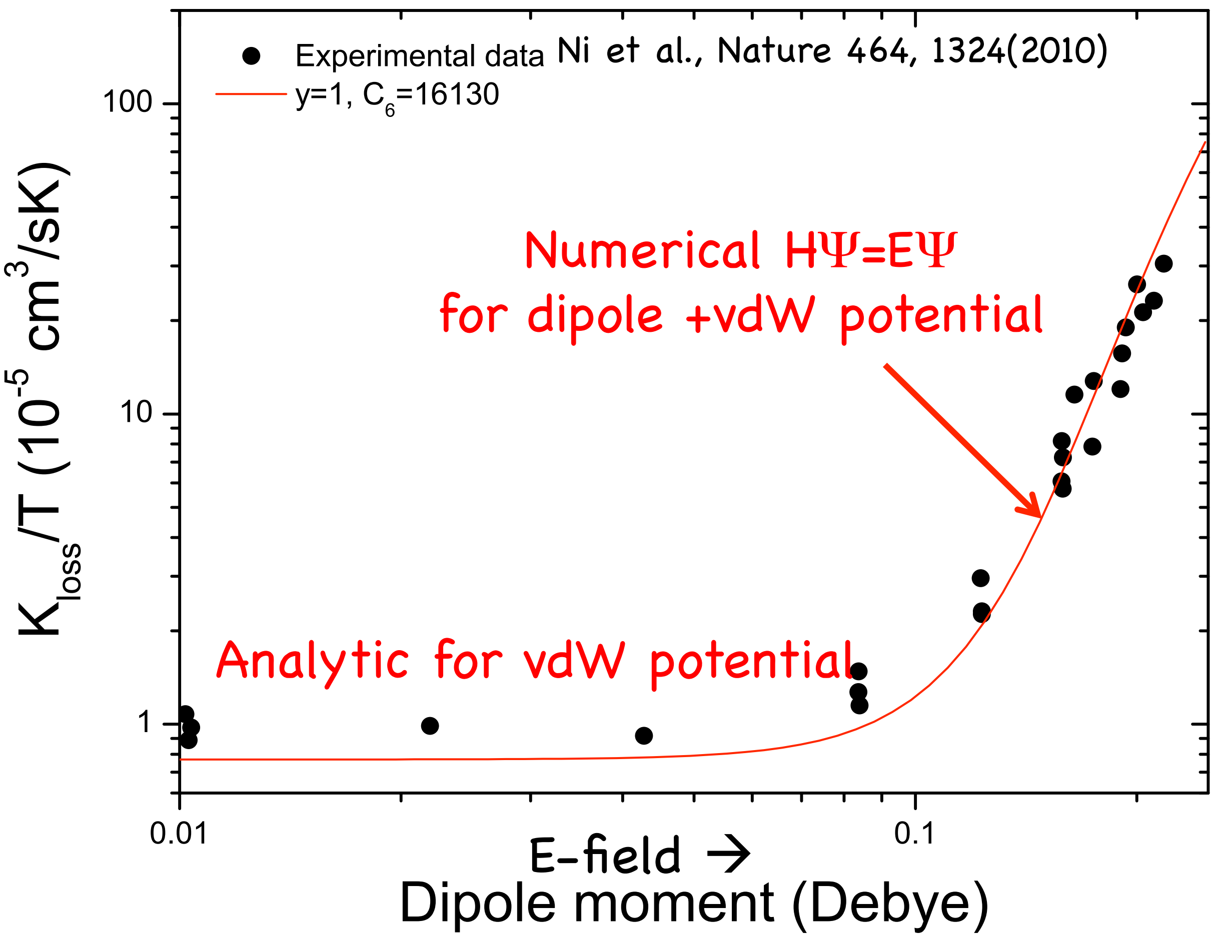
	Measured	Universal	
KRb + KRb	1.1(3)x10^{-5} cm^3/s/K	0.8(1)x10^{-5} cm^3/s/K	p-wave ≈s lifetime
KRb + KRb'	1.9(4)x10^{-10} cm^3/s	0.8x10^{-10} cm^3/s	s-wave ≈10ms lifetime
K + KRb	1.7(3)x10^{-10} cm^3/s	1.1x10^{-10} cm^3/s	s-wave

S. Ospelkaus et al., Science 327, 853 (2010)
Z. Idziaszek and PSJ, Phys. Rev. Lett. 104, 113204 (2010)

Reaction rate for identical ultracold $^{40}K^{87}Rb$ fermions

$^{40}K^{87}Rb$ has Universal "chemistry"

$KRb + KRb \rightarrow K_2 + Rb_2$

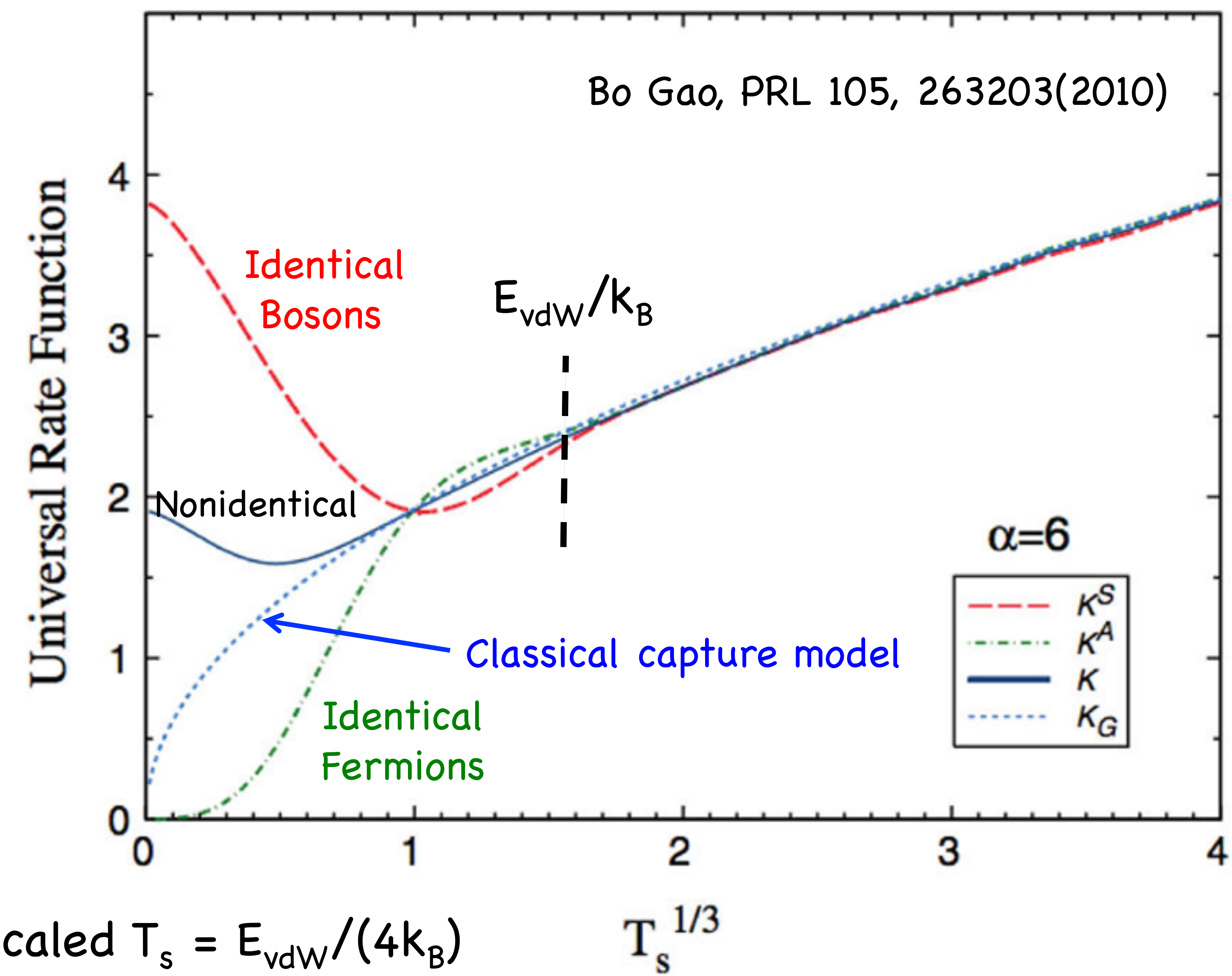


NO RESONANCES!!

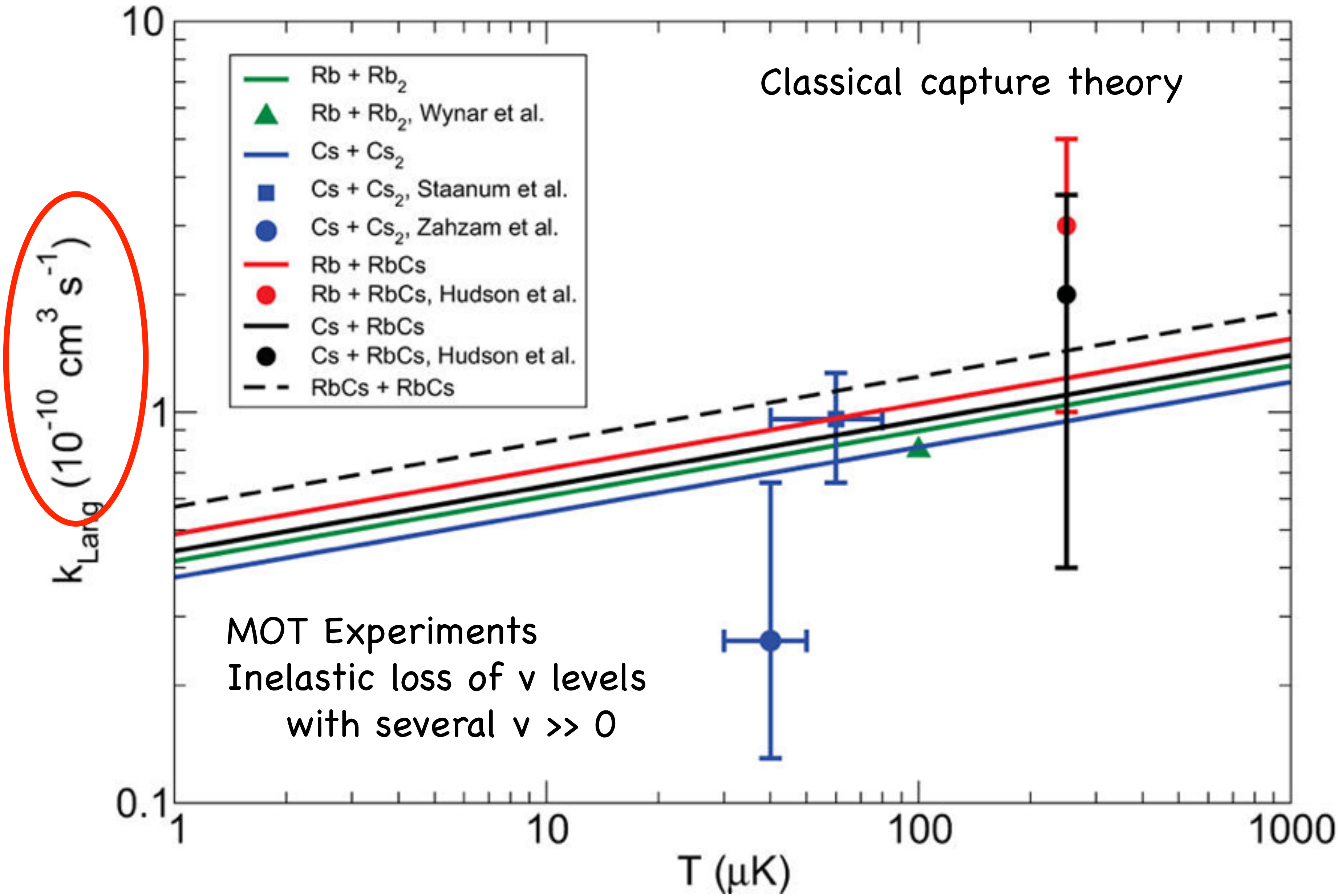
Effect of Strong 1D or 2D confinement:

Micheli, et al Phys. Rev. Lett. 105, 073202 (2010)

Universal van der Waals rate constant ("black hole")



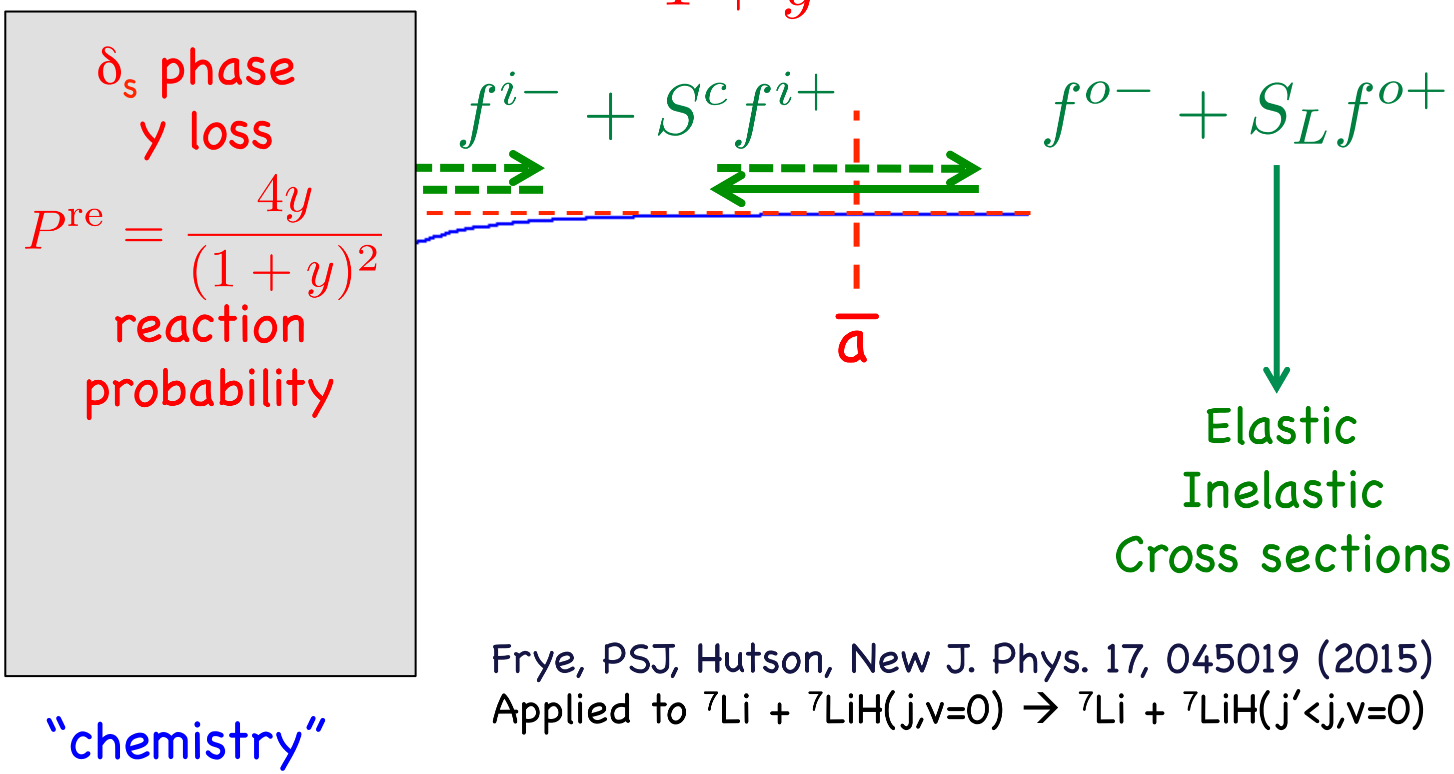
Scaled $T_s = E_{vdW}/(4k_B)$



From Quéméner and PSJ, Chem. Rev. 112, 4949 (2012)

Universal "grey hole" reaction rate theory

2 QDT parameters $S^c = \frac{1-y}{1+y} e^{i\delta_s}$ $\frac{a}{\bar{a}} = 1 + \cot\left(\delta_s - \frac{\pi}{8}\right)$



Frye, PSJ, Hutson, New J. Phys. 17, 045019 (2015)
Applied to ^{7}Li + ^{7}LiH(j,v=0) → ^{7}Li + ^{7}LiH(j'<j,v=0)

y=1 special case, "black hole," Langevin theory $P^{re} = 1$
y=0, no loss, reaction, $P^{re}=0$

Gao , PRA 78, 012702(2008); PRL 105, 263203(2010); Idziaszek and PSJ (2010)
Jachymski, Krych, Idziaszek, PSJ, Phys. Rev. Lett. 110, 213202 (2013)
Phys. Rev. A 90, 042705 (2014)

Confined collisions in reduced dimension

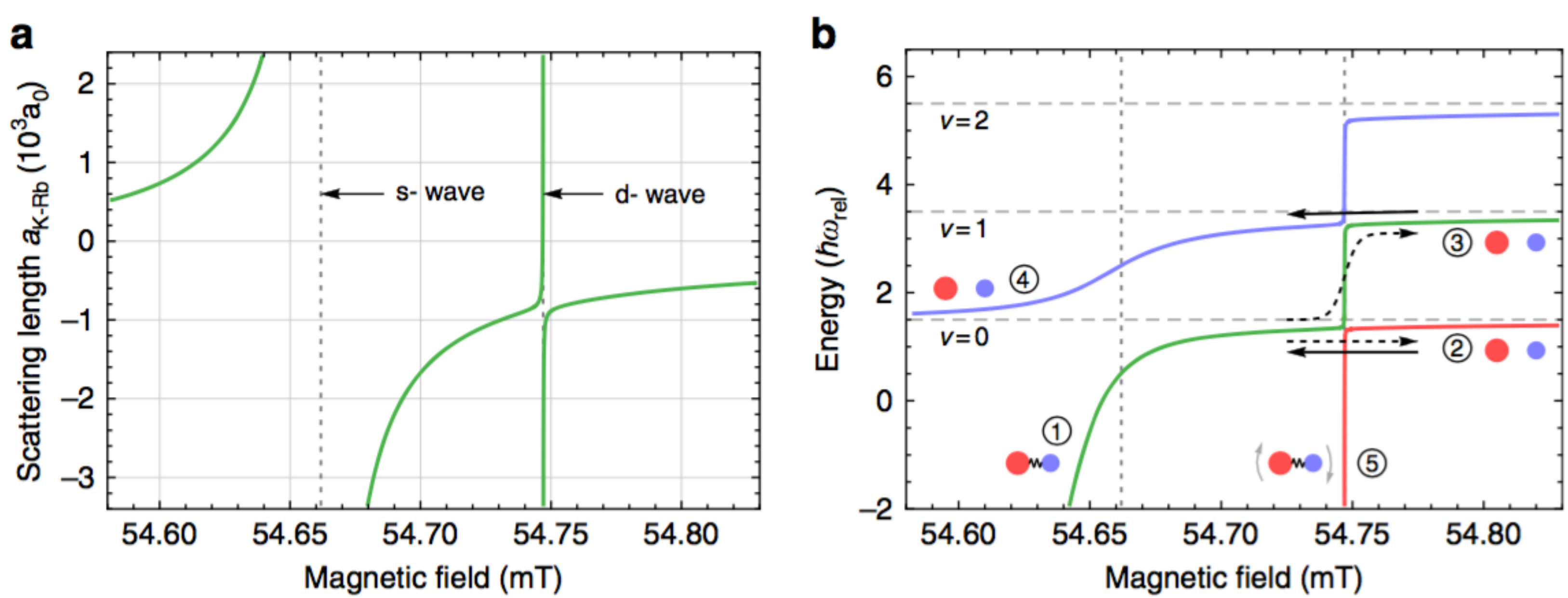
ARTICLE Nat. Comm. 7, 11279 (2016)

Received 12 Jan 2016 | Accepted 3 Mar 2016 | Published 14 April 2016 DOI: 10.1038/ncomms11279 OPEN

Doublon dynamics and polar molecule production in an optical lattice

Jacob P. Covey[1,2], Steven A. Moses[1,2], Martin Gärttner[1,2], Arghavan Safavi-Naini[1,2], Matthew T. Miecnikowski[1,2], Zhengkun Fu[1,2], Johannes Schachenmayer[1,2], Paul S. Julienne[3], Ana Maria Rey[1,2], Deborah S. Jin[1,2] & Jun Ye[1,2]

$$^{40}K + ^{87}Rb \longleftrightarrow ^{40}K^{87}Rb$$



Two atoms in a trap

3D spherically symmetric harmonic trap: $\omega = 2\pi\nu$

2-body potential: V(r)

Separate the center of mass and relative motion

Center of mass

$$\left[-\frac{\hbar^2}{2(2m)}\nabla_R^2 + \frac{1}{2}(2m)\omega^2 R^2\right]\Psi_{cm}(R) = E_{cm}\Psi_{cm}(R)$$

Relative

$$\left[-\frac{\hbar^2}{2(\mu)}\nabla_r^2 + \frac{1}{2}(\mu)\omega^2 r^2 + V(r)\right]\Psi(r) = E\Psi(r)$$

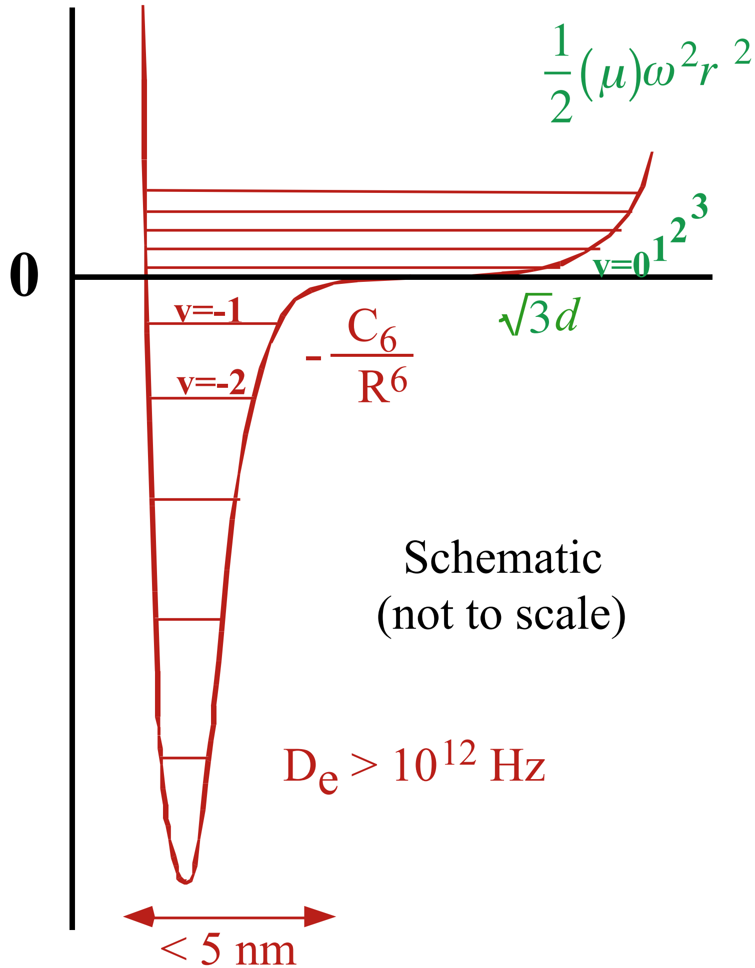
$V_{eff}(r)$

From Tiesinga, Williams, Mies, and PSJ, *Phys. Rev.* A **61**, 063416 (2000)

Scale size of trap

$$d = \sqrt{\frac{\hbar}{\mu\omega}}$$

$\approx$ 50 nm

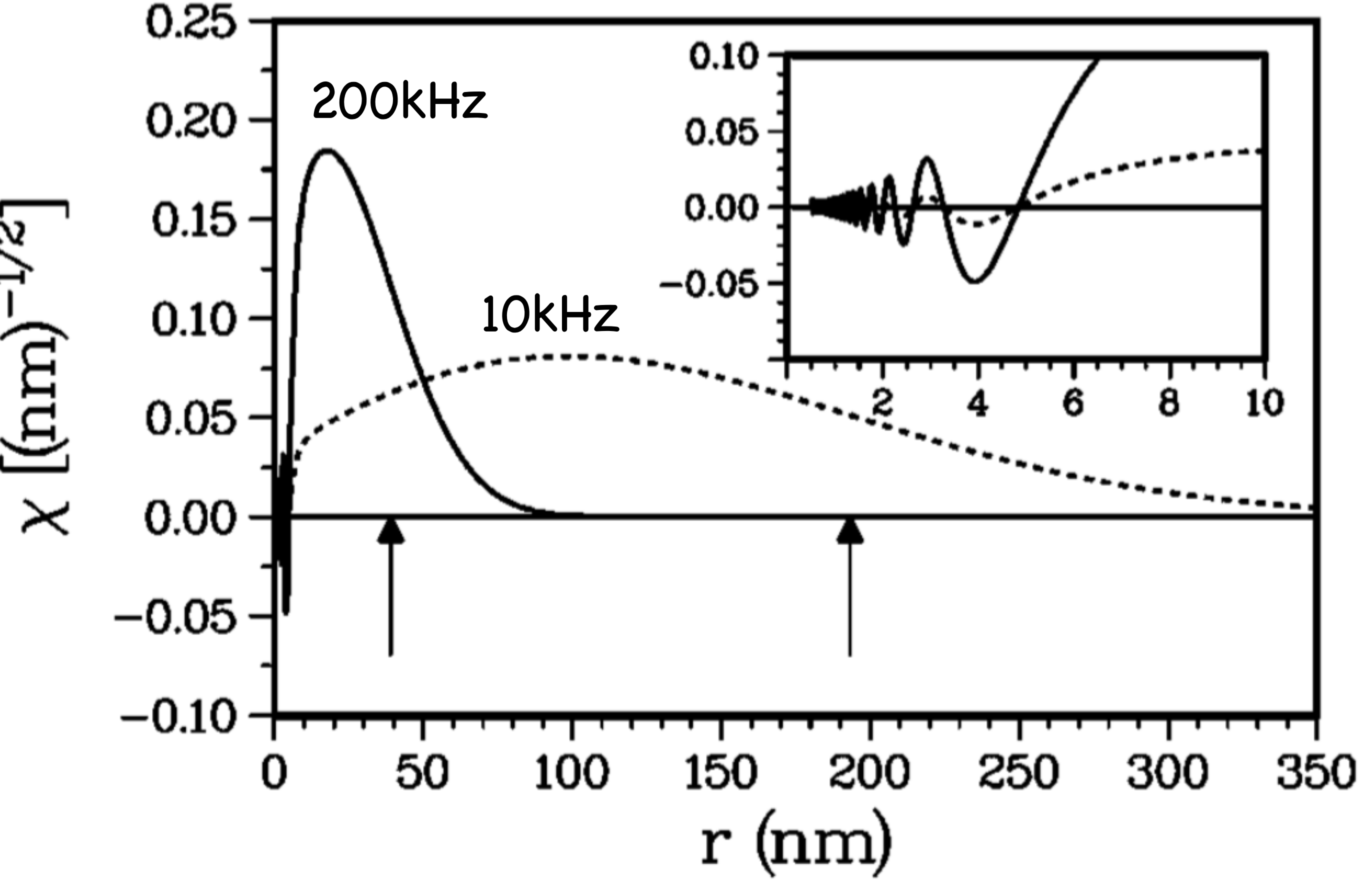


Scale size of V

R_{vdW}

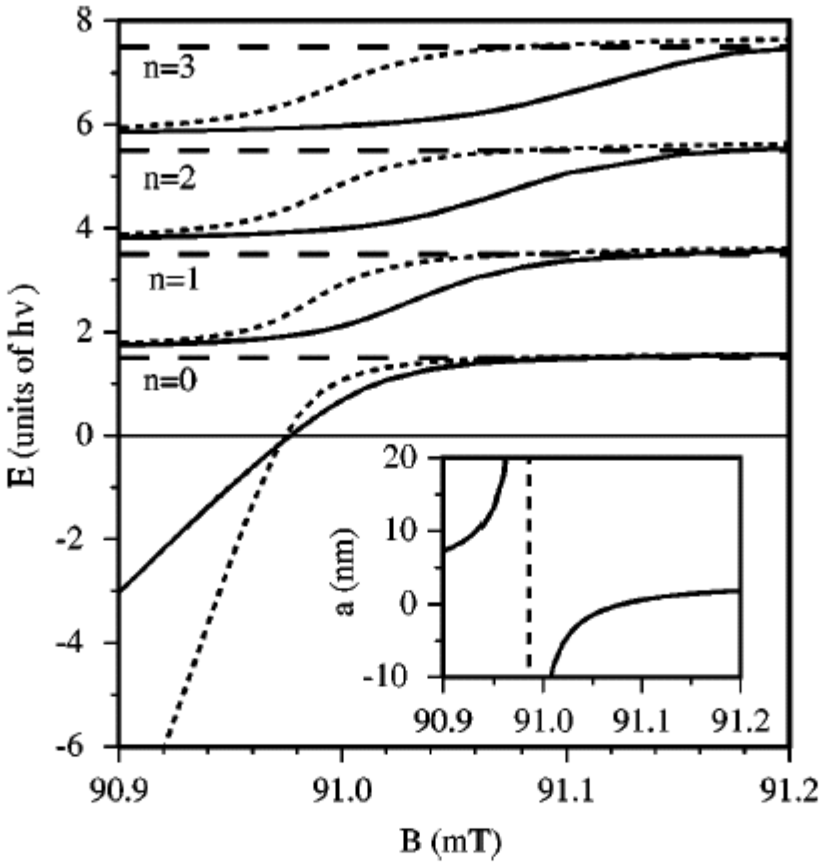
< 5 nm

Wave function in a harmonic trap
2 Cs atoms



From Tiesinga, Williams, Mies, and PSJ, *Phys. Rev.* A **61**, 063416 (2000)

Na F=1,M=+1
in a 1 MHz harmonic trap



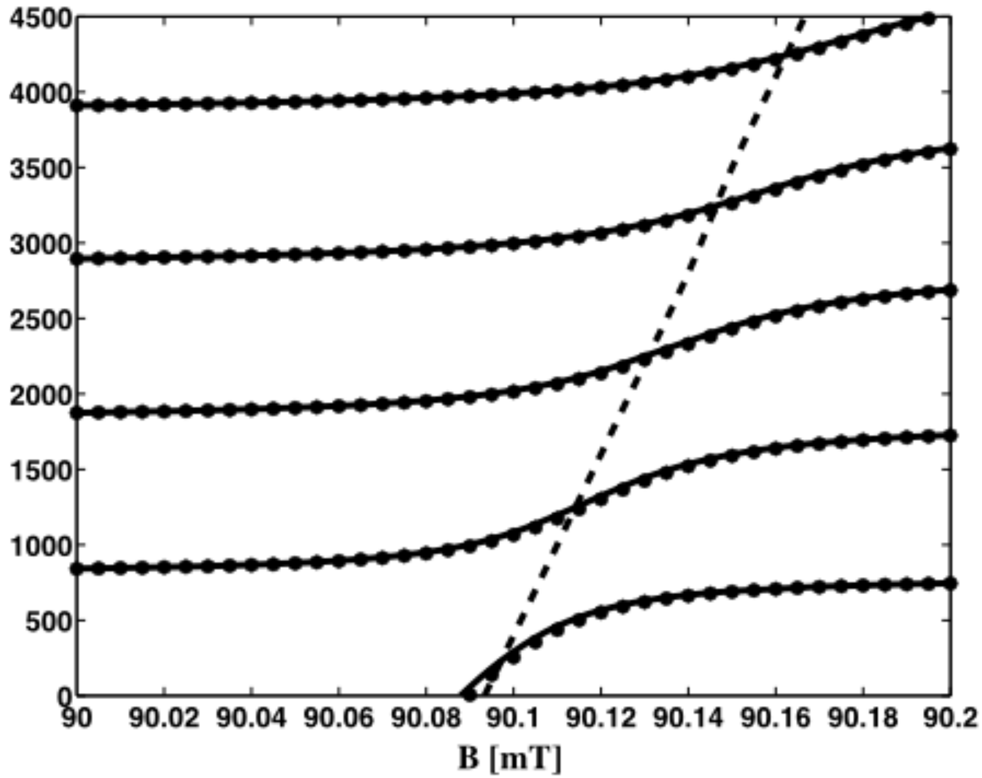
$$\hat{V} = \frac{4\pi\hbar^2}{m} A\delta(\vec{R}) \frac{\partial}{\partial R} R$$

$$A = -\lim_{k \to 0} \frac{\tan(\eta_s(E))}{k}$$

Solid: numerical
Dashed: pseudopotential

Tiesinga, Williams, Mies, and Julienne, *Phys. Rev.* A **61**, 063416 (2000)

Na F=1,M=+1
in a 1 MHz harmonic trap



Energy-dependent Pseudopotential:

$$\hat{V} = \frac{4\pi\hbar^2}{m} A(E_v)\delta(\vec{R})\frac{\partial}{\partial R}R$$

$$A(E_v) = -\frac{\tan(\eta_s(E_v))}{k}$$

$A(E_v)$ from scatttering calculation of $\eta_s(E_v)$

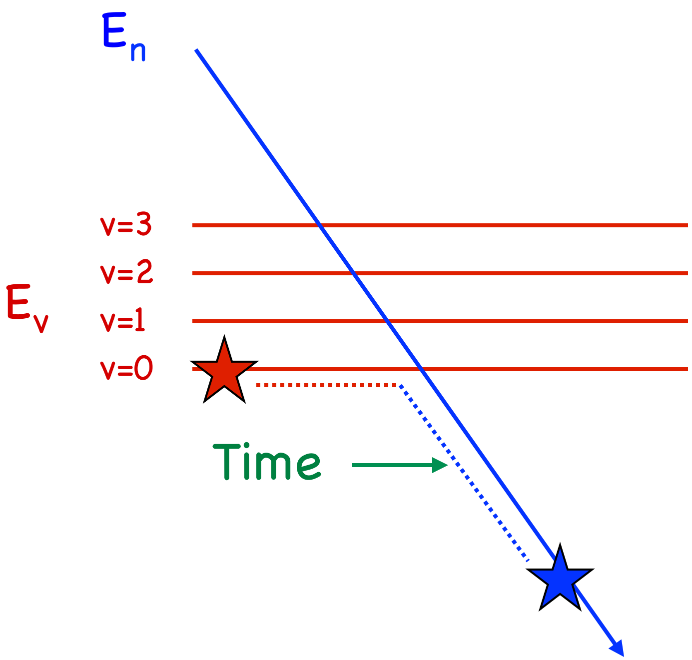
Points: numerical
Solid Line: pseudopotential

E. L. Bolda, E. Tiesinga, and P. S. Julienne, *Phys. Rev.* A **66**, 013403 (2002)
D. Blume and Chris H. Greene, Phys. *Rev.* A **65**, 043613 (2002)

Extension to quasi-1D and -2D: Bolda, Tiesinga, and PSJ, *Phys. Rev.* A**68**, 032702 (2003)
Naidon, Tiesinga, Mitchell, PSJ, New J. Phys. **9**, 19 (2007)

Making cold molecules by time-dependent resonances in a trap

F. H. Mies, E. Tiesinga, P. S. Julienne, *Phys. Rev.* A **61**, 022721 (2000)
P. S. Julienne, E. Tiesinga, T. Köhler, cond-mat/0312492; J. Mod. Opt. **51**, 1787(2004)
Adapt coupled channels free-space scattering to a trap (QDT viewpoint).



$$A_{LZ} \propto s_{\text{res}} \frac{\bar{a}}{d} \frac{\bar{E}}{\hbar\omega}$$

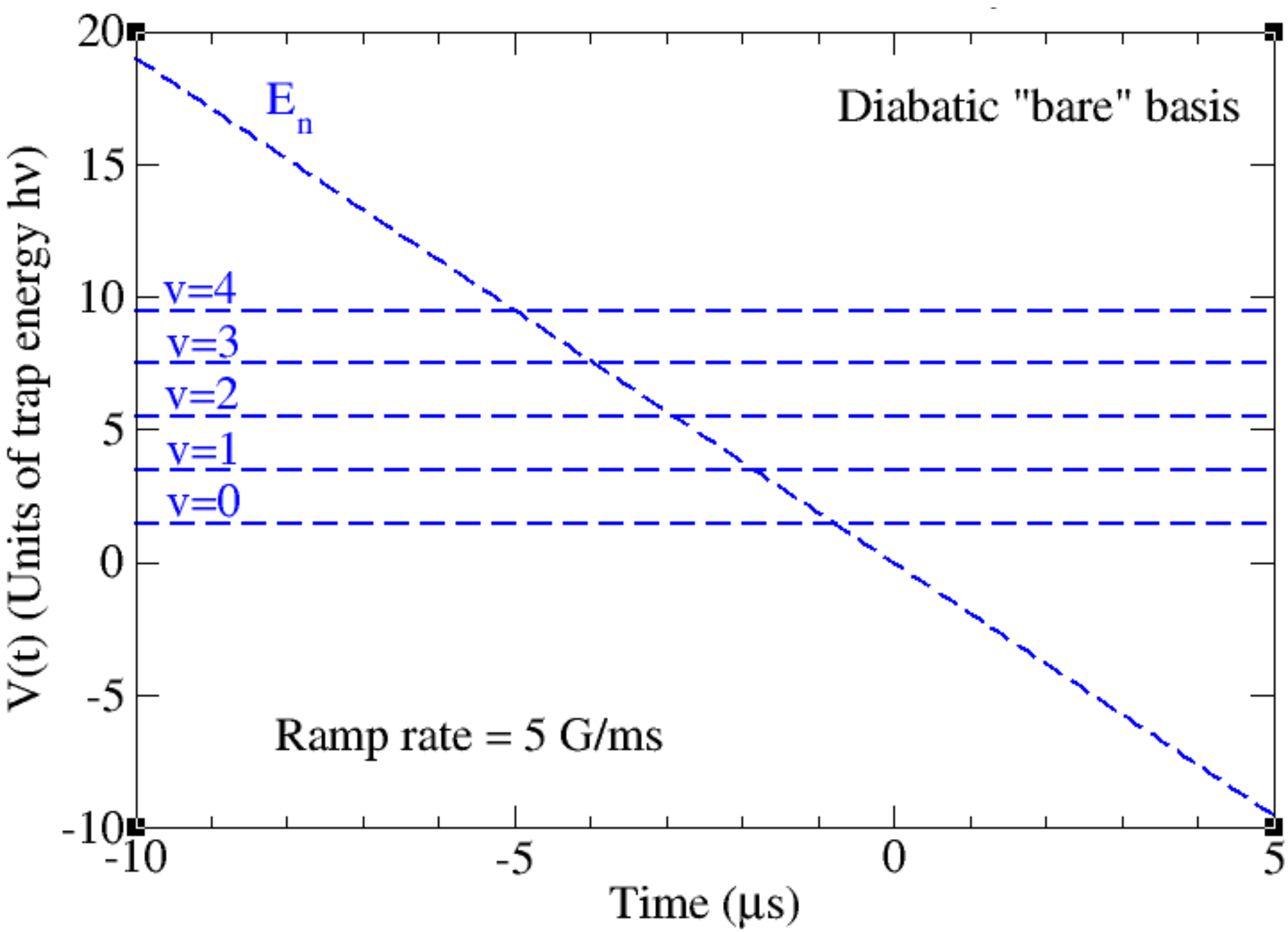
$$\frac{V_{\text{nv}}}{\hbar\omega} = \left(\frac{2\sqrt{3}}{\pi}\right)^{\frac{1}{2}} \left(\frac{A_{bg}}{d}\right)^{\frac{1}{2}} \left(\frac{s_{\text{n}}\Delta_{\text{n}}}{\hbar\omega}\right)^{\frac{1}{2}} \left(1 + \tfrac{4}{3}\,\text{v}\right)^{\frac{1}{4}}$$

Landau-Zener curve crossing:

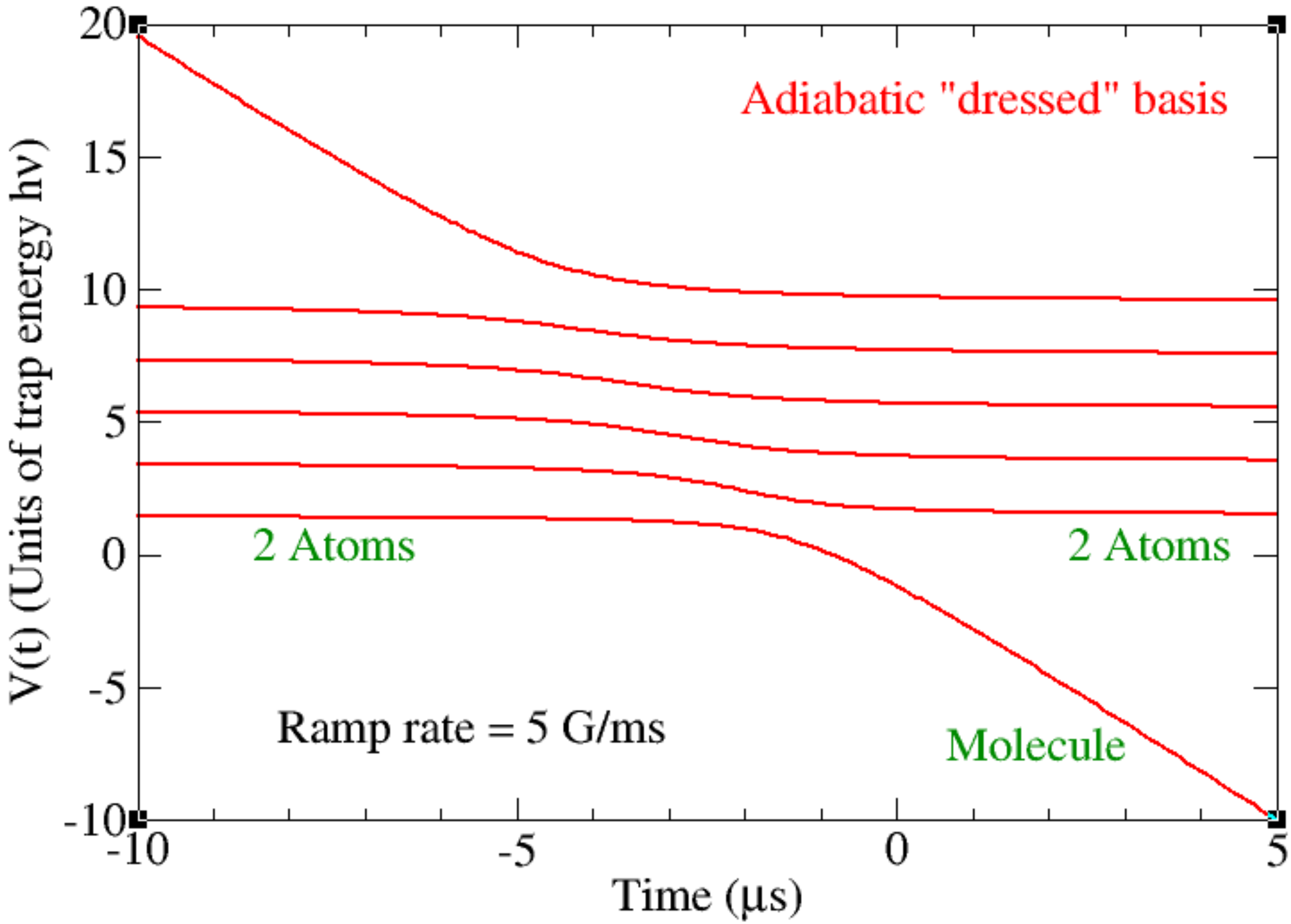
$$A_{LZ} = \frac{2\pi}{\hbar} \frac{\left|V_{\text{nv}}\right|^2}{s_{\text{n}}\dot{B}} = \frac{4\hbar}{\mu d^3} \left|\frac{A_{bg}\Delta_n}{\dot{B}}\right|$$

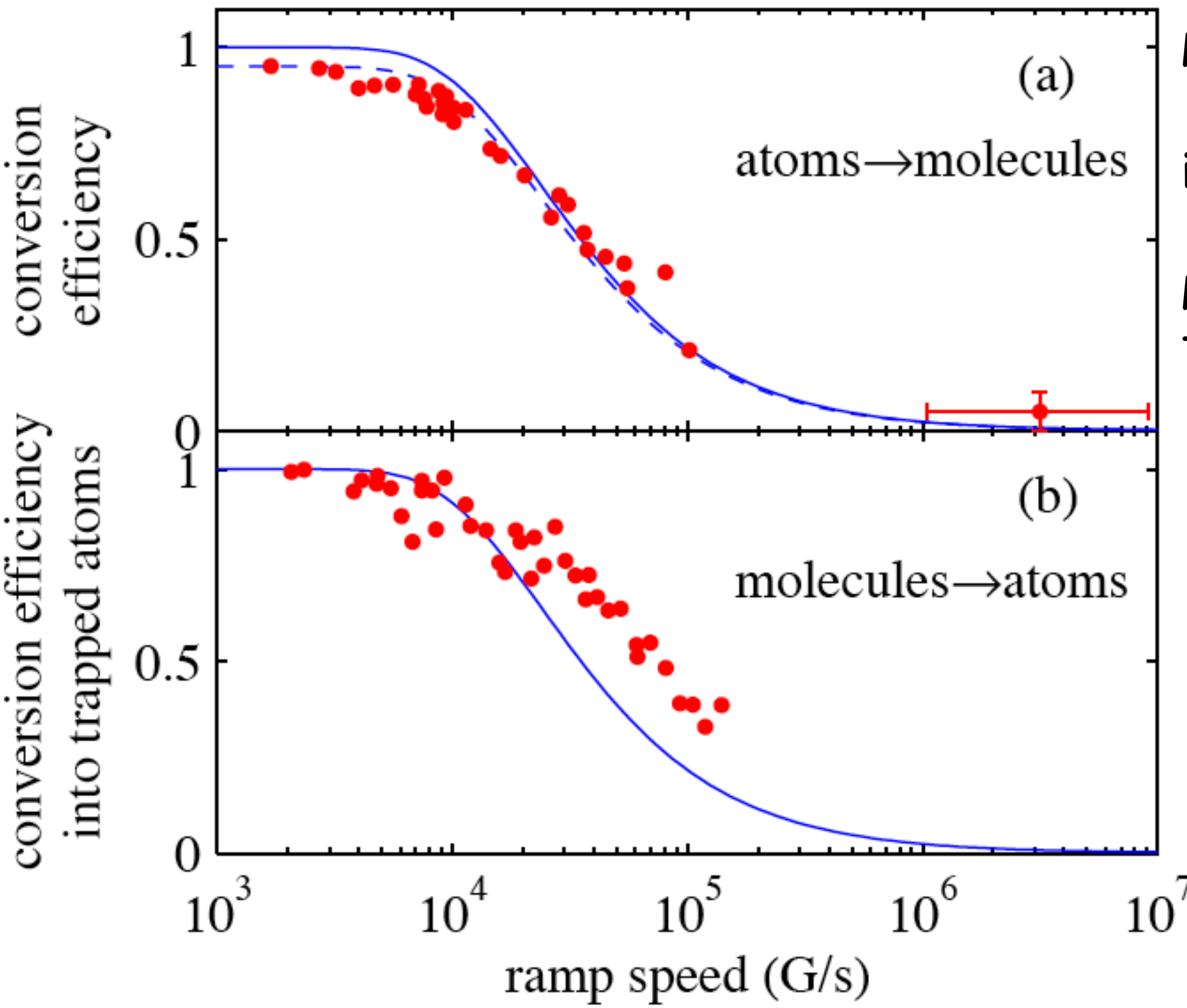
Fraction converted: $1 - e^{-A_{LZ}}$

^{87}Rb F=1, M=+1 in lattice cell near 1007 G



^{87}Rb 2 *a* atoms in lattice cell near 1007 G





Mott insulator with 2 ^{87}Rb atoms in a 3D lattice cell

Molecules: 700ms lifetime

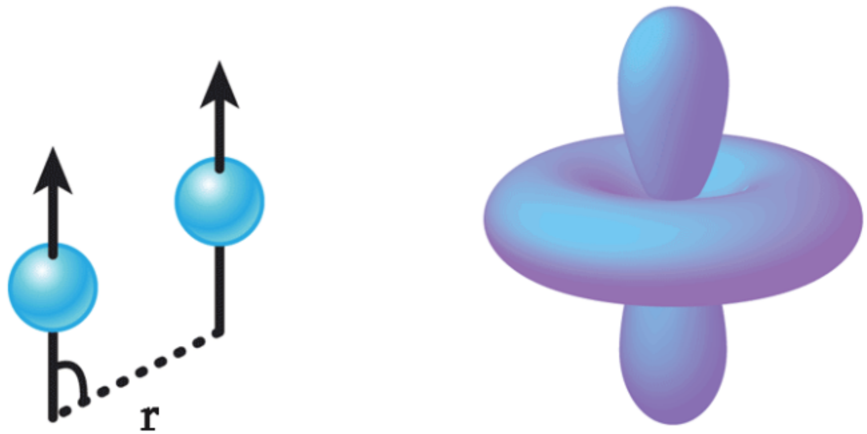
Line: LZ theory

Thalhammer, Winkler, Lang, Schmid, Grimm, Hecker Denschlag, "Long-lived Feshbach molecules in a 3D optical lattice," PRL 96, 050402(2006)

Complexity and Chaos

Physics

B. Laburthe-Tolra, Physics 5, 58 (2012)
"Quantum Dipolar Gases in Boson or Fermion Flavor"

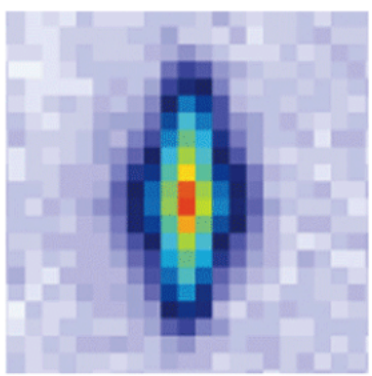


(a)

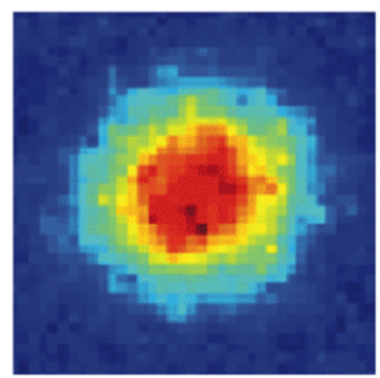
$4f^{12}6s^2\ ^3H_6$

$7\mu_B$

Ferlaino group
Innsbruck
^{168}Er
30000 atoms
order 100nK



(b)

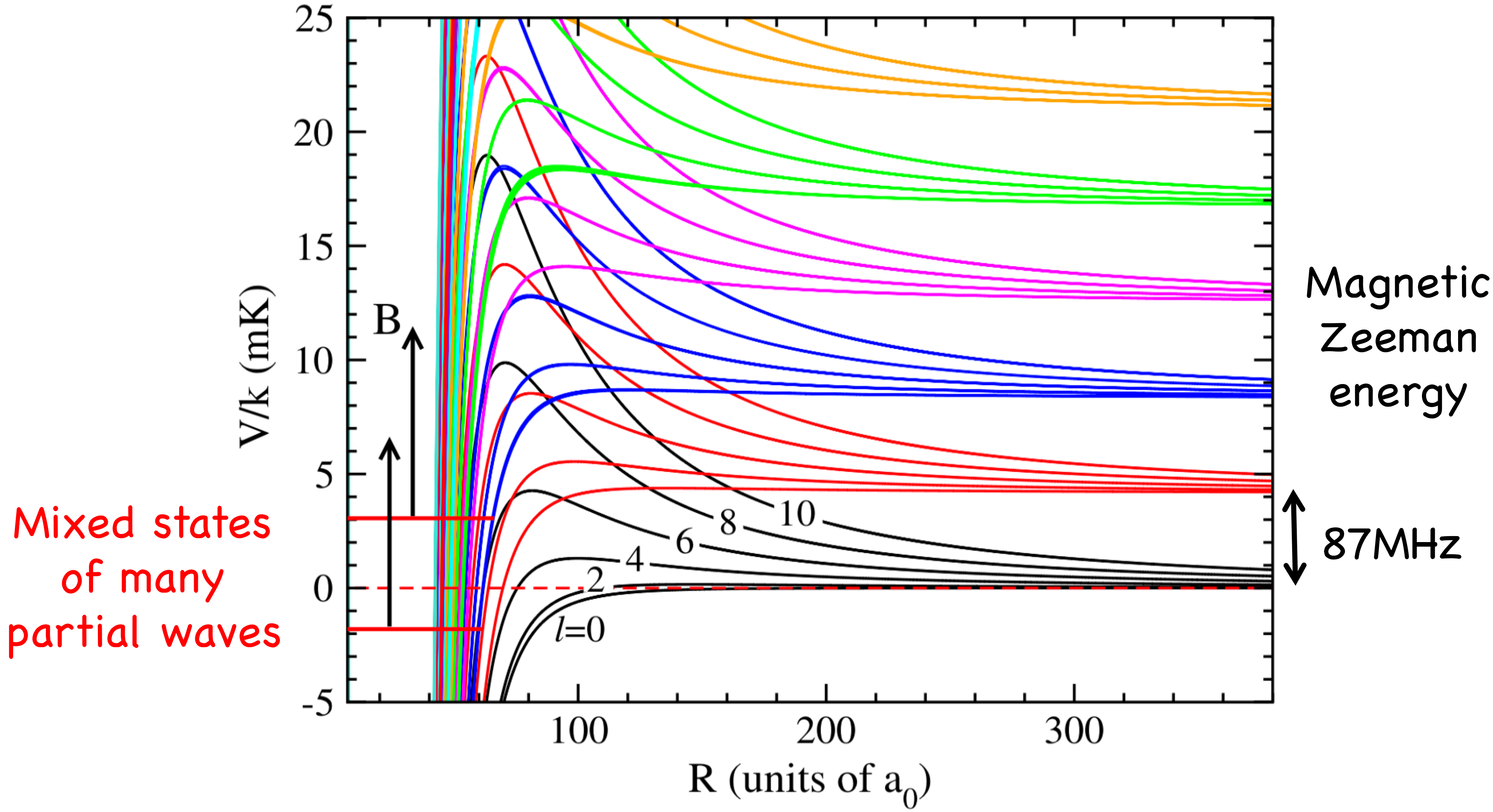


(c)

$4f^{10}6s^2\ ^5I_8$

$10\mu_B$

Lev group
Stanford
^{161}Dy
6000 atoms
64nK T/T_F=0.2



Diagonal potential energy curves for ^{164}Dy + ^{164}Dy at B = 50G

Asymptotic $|(j_1j_2)jm_j,lm_l>$ channels, $m_j+m_l = -16$, $0 \le l \le 10$

From Petrov, Tiesinga, Kotochigova, PRL 109, 103002 (2012)

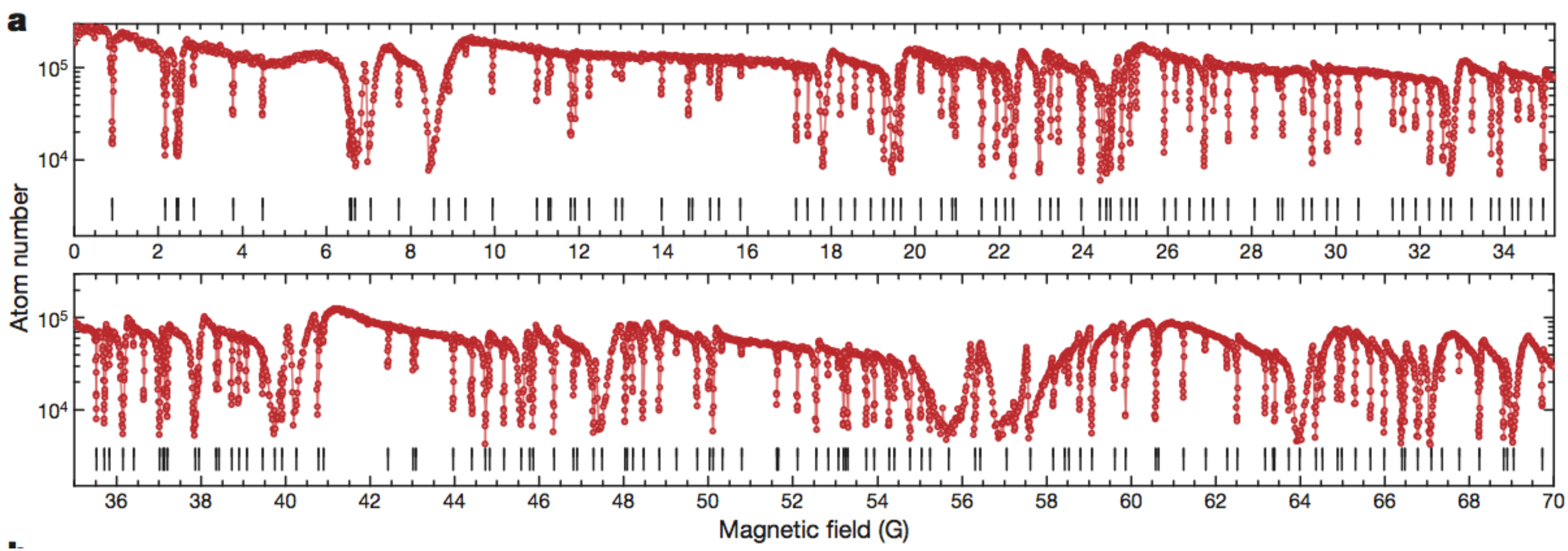
LETTER

Nature online, March 12, 2014

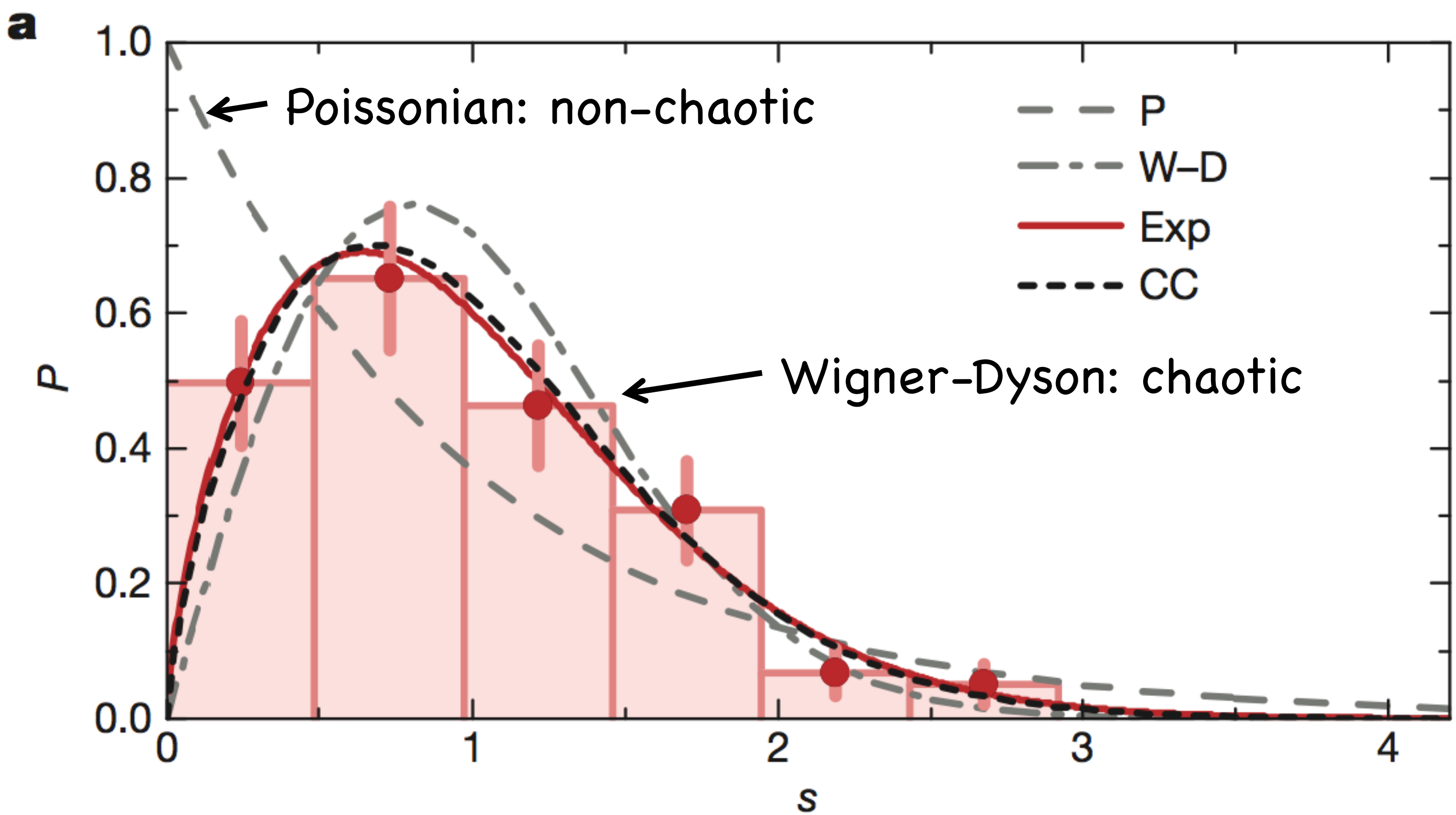
doi:10.1038/nature13137

Quantum chaos in ultracold collisions of gas-phase erbium atoms

Albert Frisch[1], Michael Mark[1], Kiyotaka Aikawa[1], Francesca Ferlaino[1], John L. Bohn[2], Constantinos Makrides[3], Alexander Petrov[3,4,5] & Svetlana Kotochigova[3]



^{168}Er in ground state $^3H_6(m=-6)$



Dense set of overlapping (interacting) resonances:
mixed eigenstates of "random" character

From Frisch et al, Nature 507, 475 (2014)

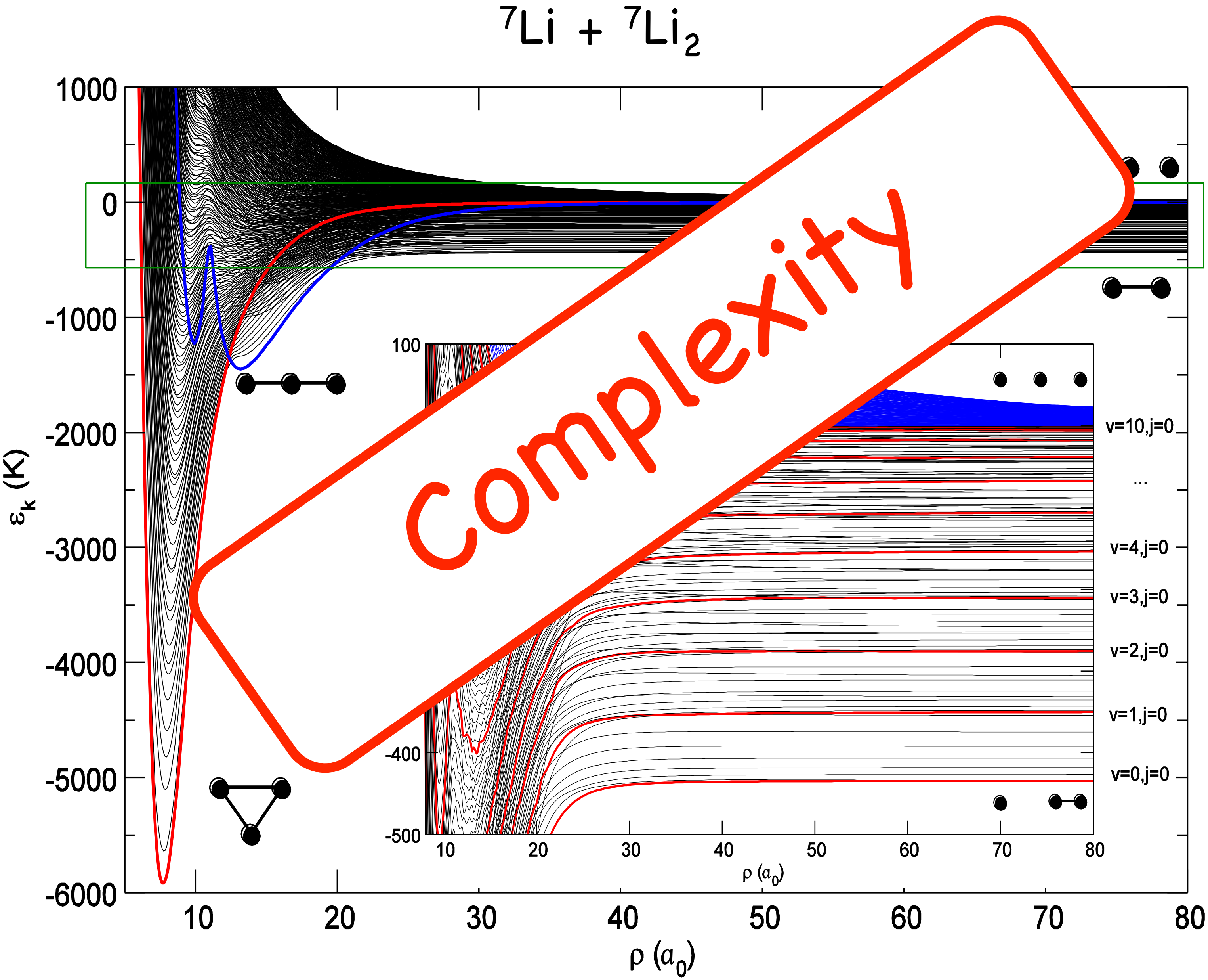
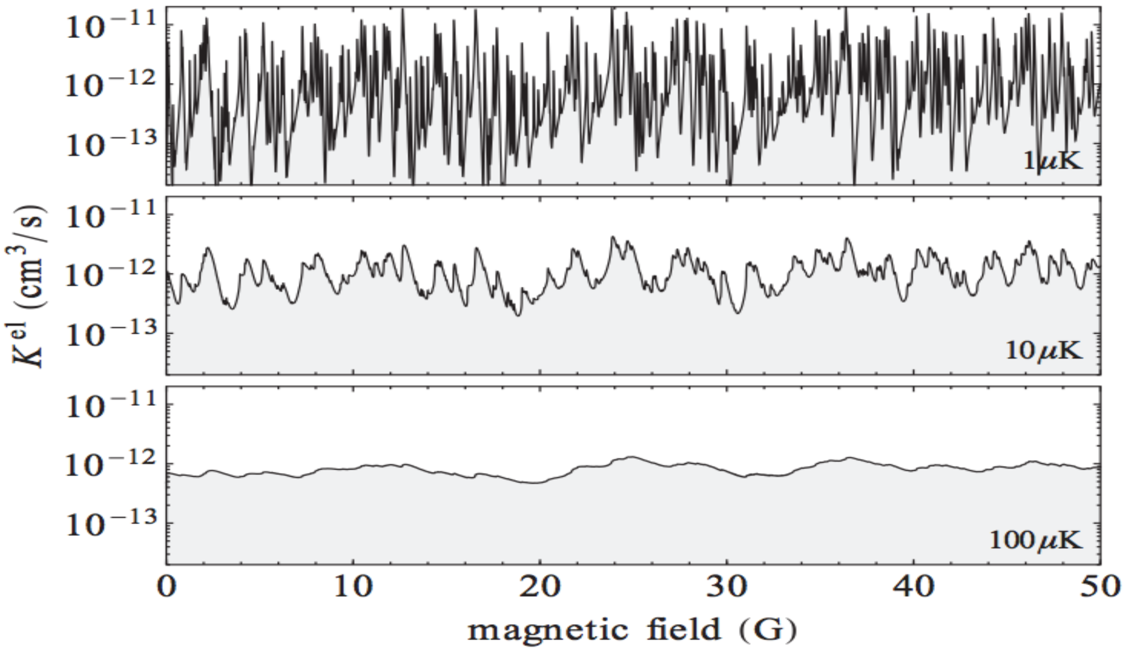


Fig. 40 of Quéméner and PSJ, Chem. Rev. 112, 4949 (2012)

From Mayle, Ruzic, Bohn, Phys. Rev. A 85, 062712 (2012)

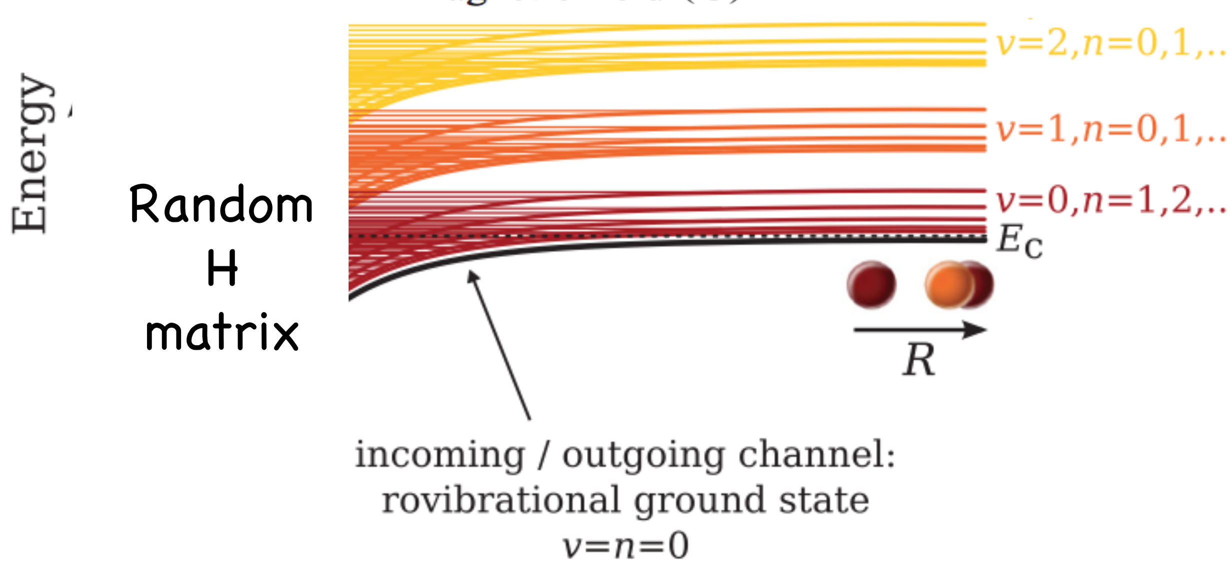


Toy Statistical model

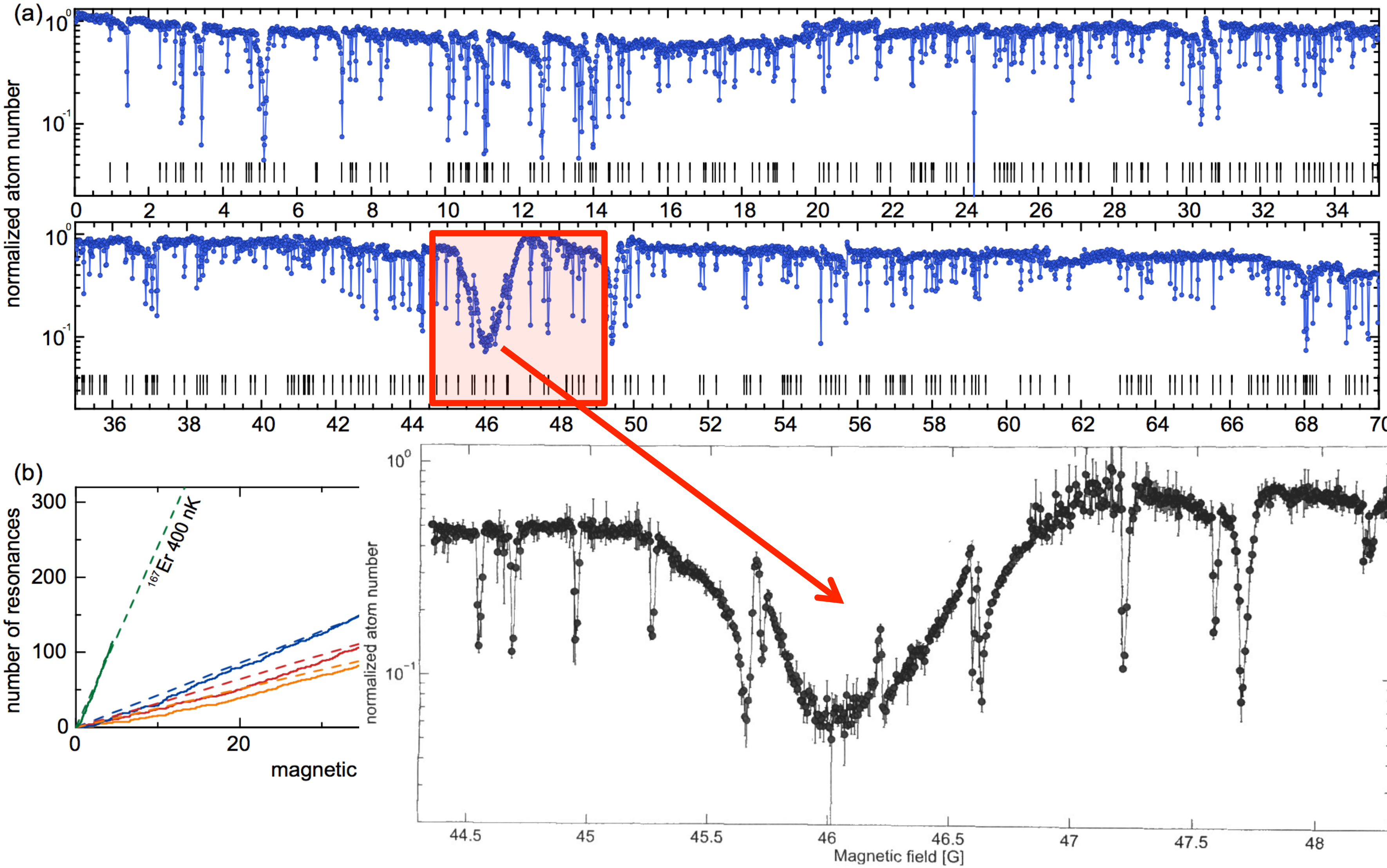
Rb + KRb

Random matrix Theory in a QDT framework

Random H matrix



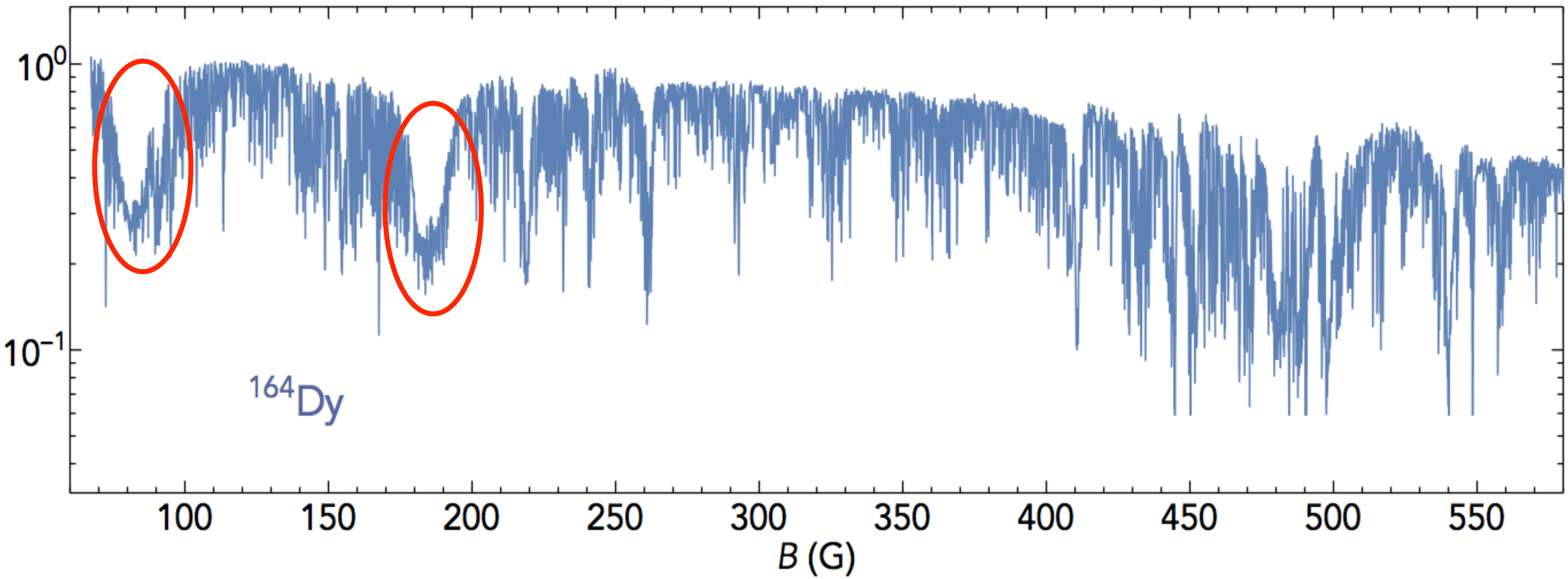
^{164}Dy M=-8



Pfau/Ferlaino/Kotochigova collaboration, Phys. Rev. X 5, 041029 (2015)

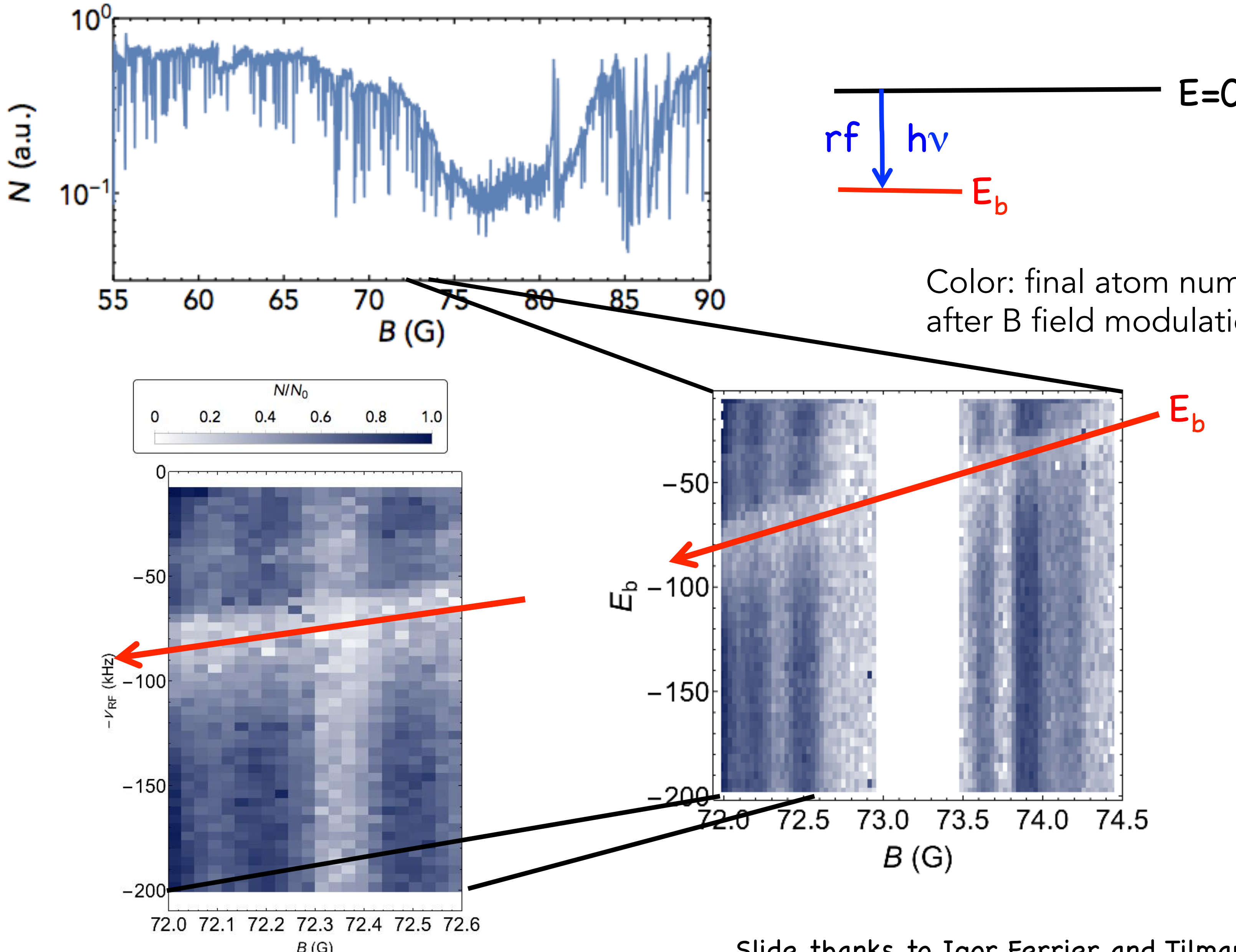
Feshbach resonances in Dysprosium

Atom-loss spectroscopy of ^{164}Dy at high field, observation of several broad features



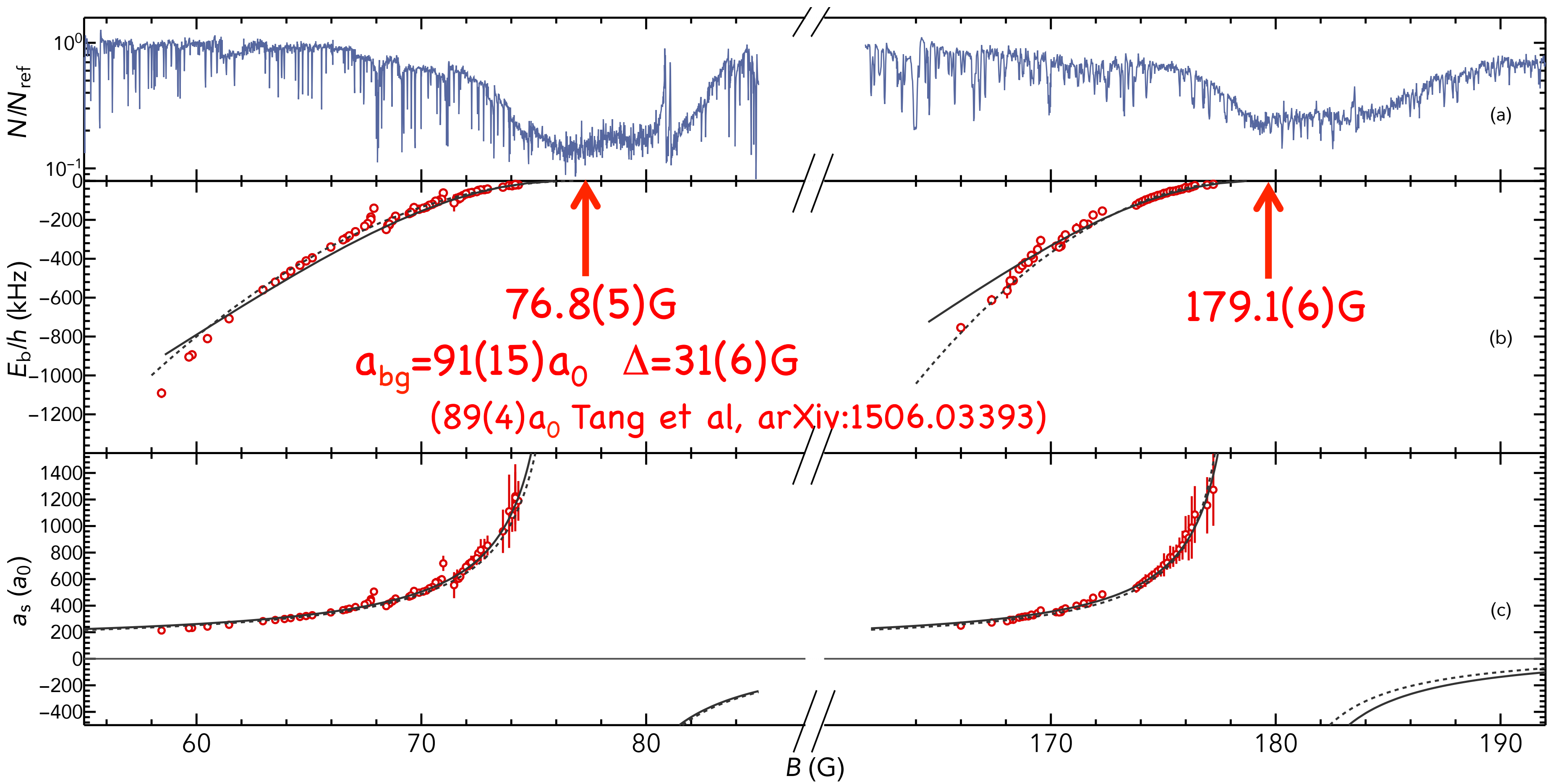
"Patterned" complexity → underlying simplicity

T. Maier, I. Ferrier-Barbut, H. Kadau, M. Schmitt, M. Wenzel, C. Wink, T. Pfau, K. Jachymski, and PSJ, Phys. Rev. A, 92, 060702 (2015)
Plus Supplemental Material (data & theory)



Color: final atom number after B field modulation.

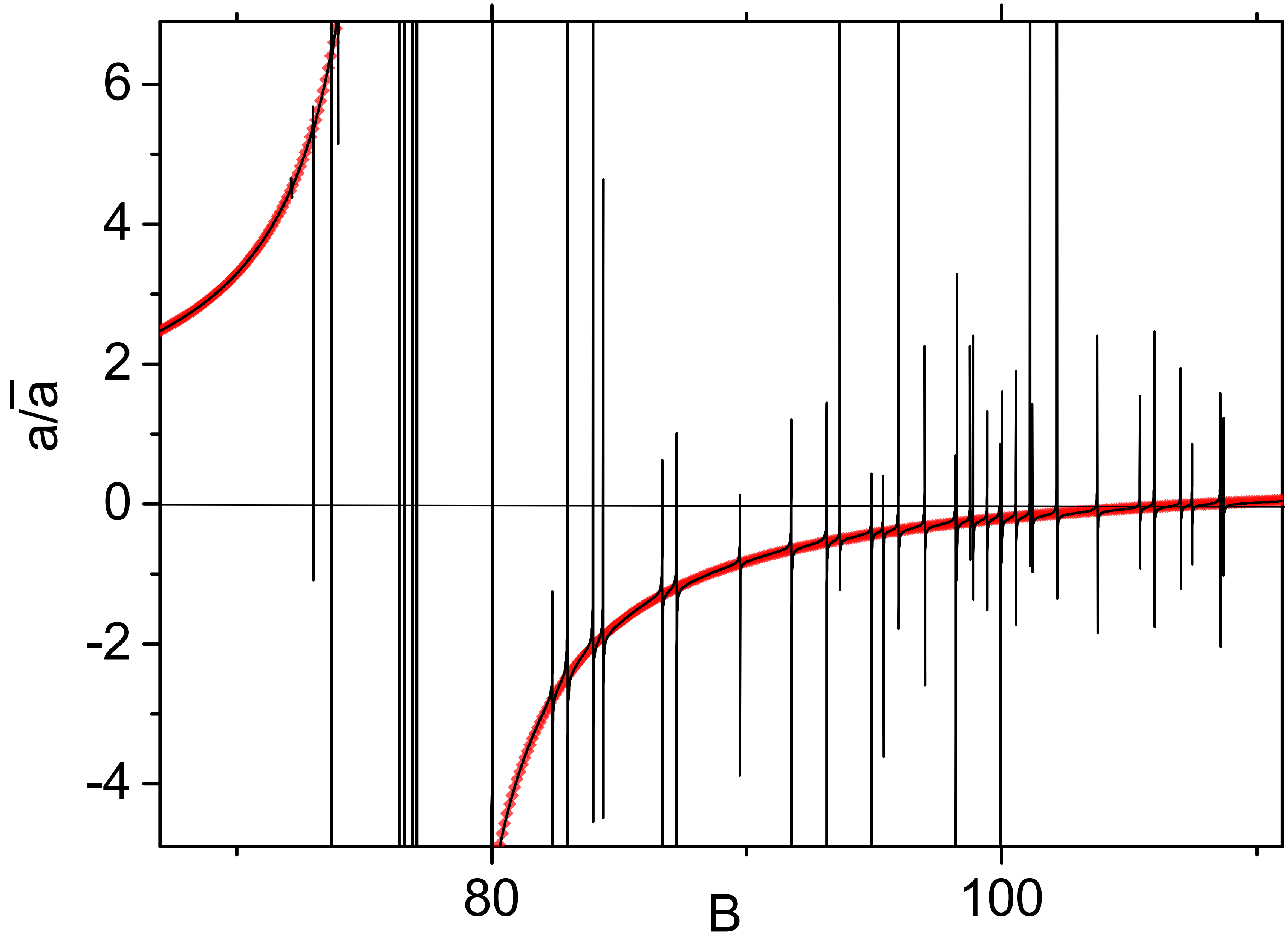
Slide thanks to Igor Ferrier and Tilman Pfau



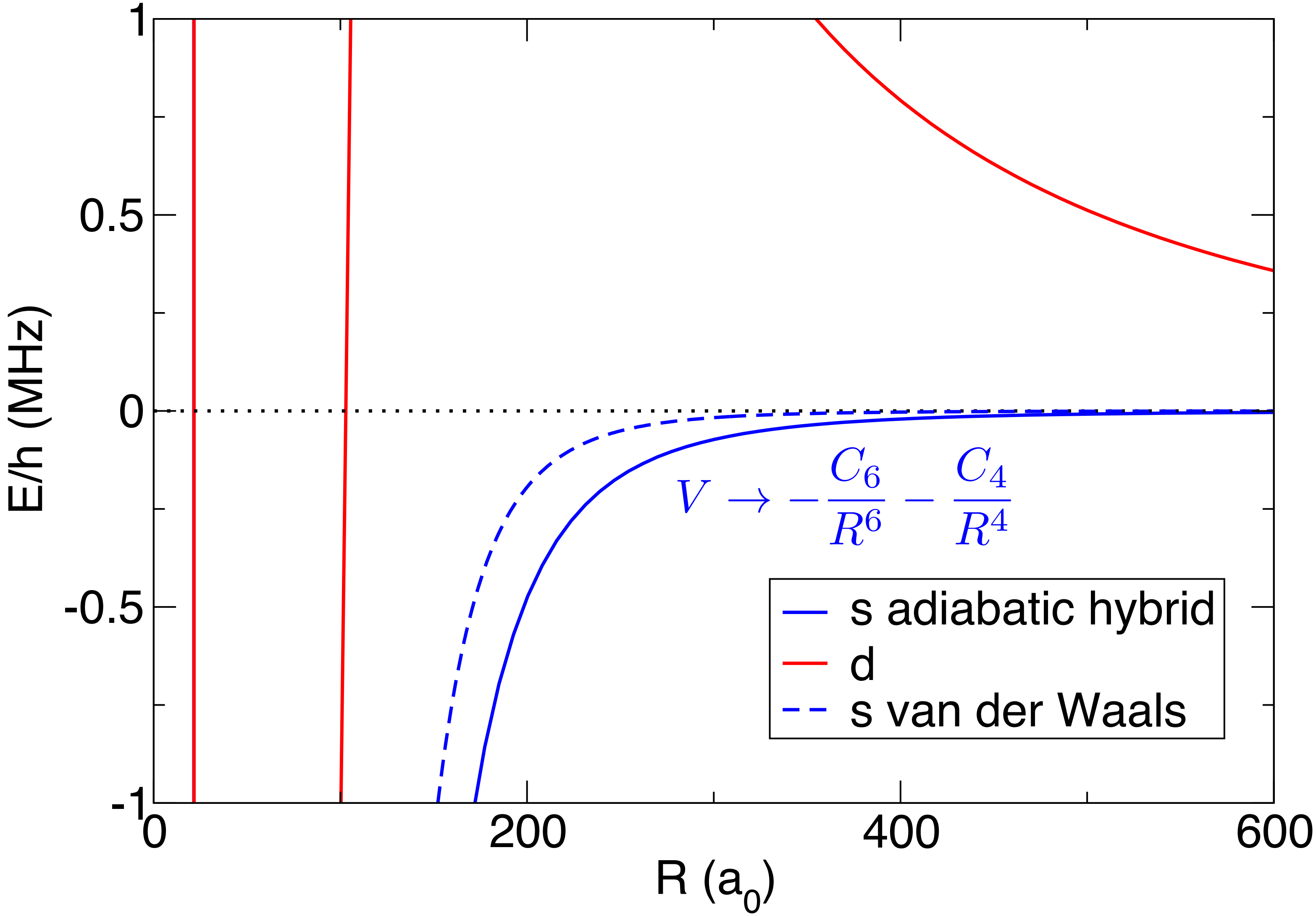
$$a = a_{\rm bg} - a_{\rm bg}\frac{\Delta}{B - B_0}$$

T. Maier, I. Ferrier-Barbut, H. Kadau, M. Schmitt, M. Wenzel, C. Wink, T. Pfau, K. Jachymski, and PSJ, Phys. Rev. A, 92, 060702 (2015) + Supplemental material

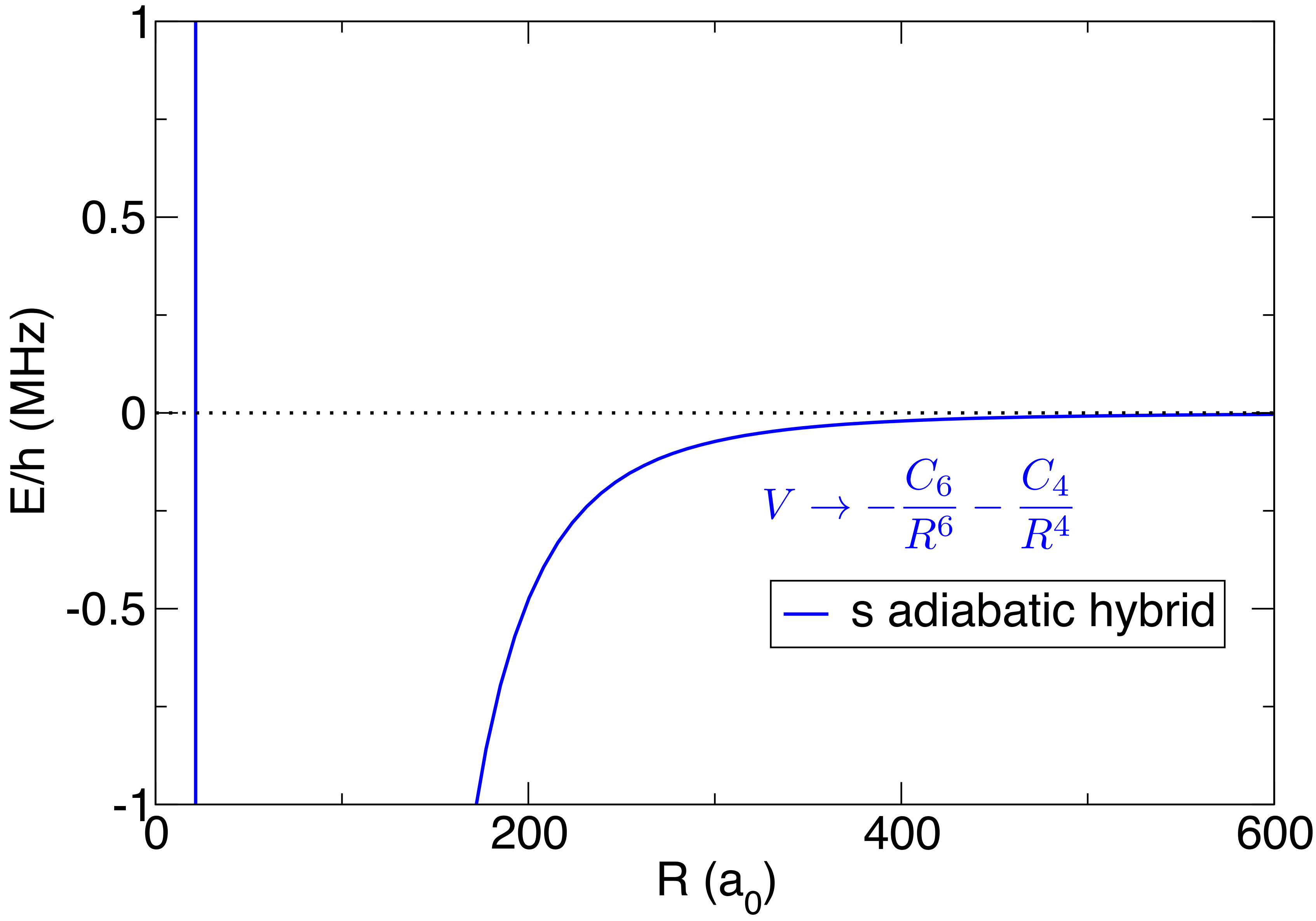
Dy "Toy" QDT model: 1 strong (broad), many narrow (weak) resonances



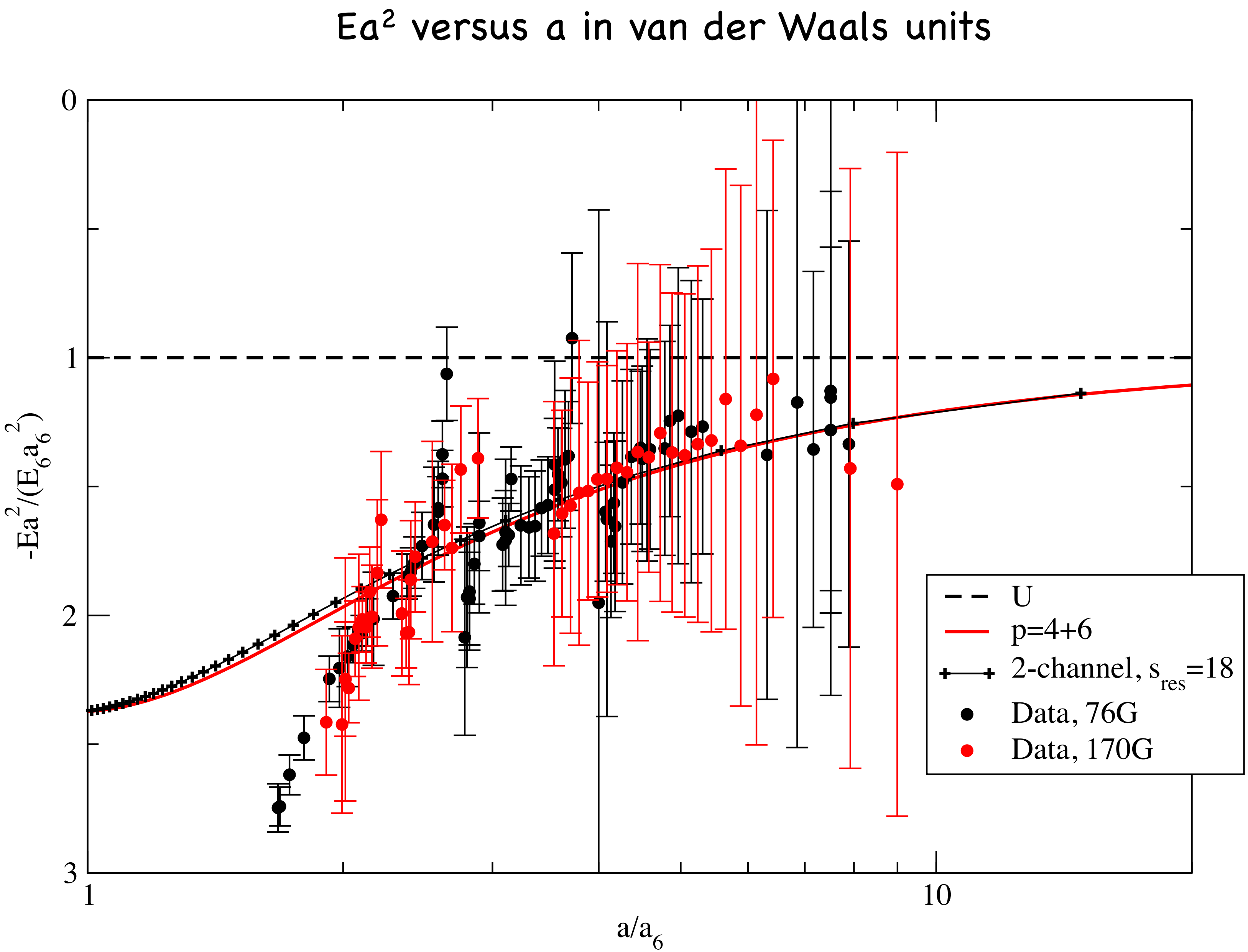
Bohn–Cavagnero–Ticknor model



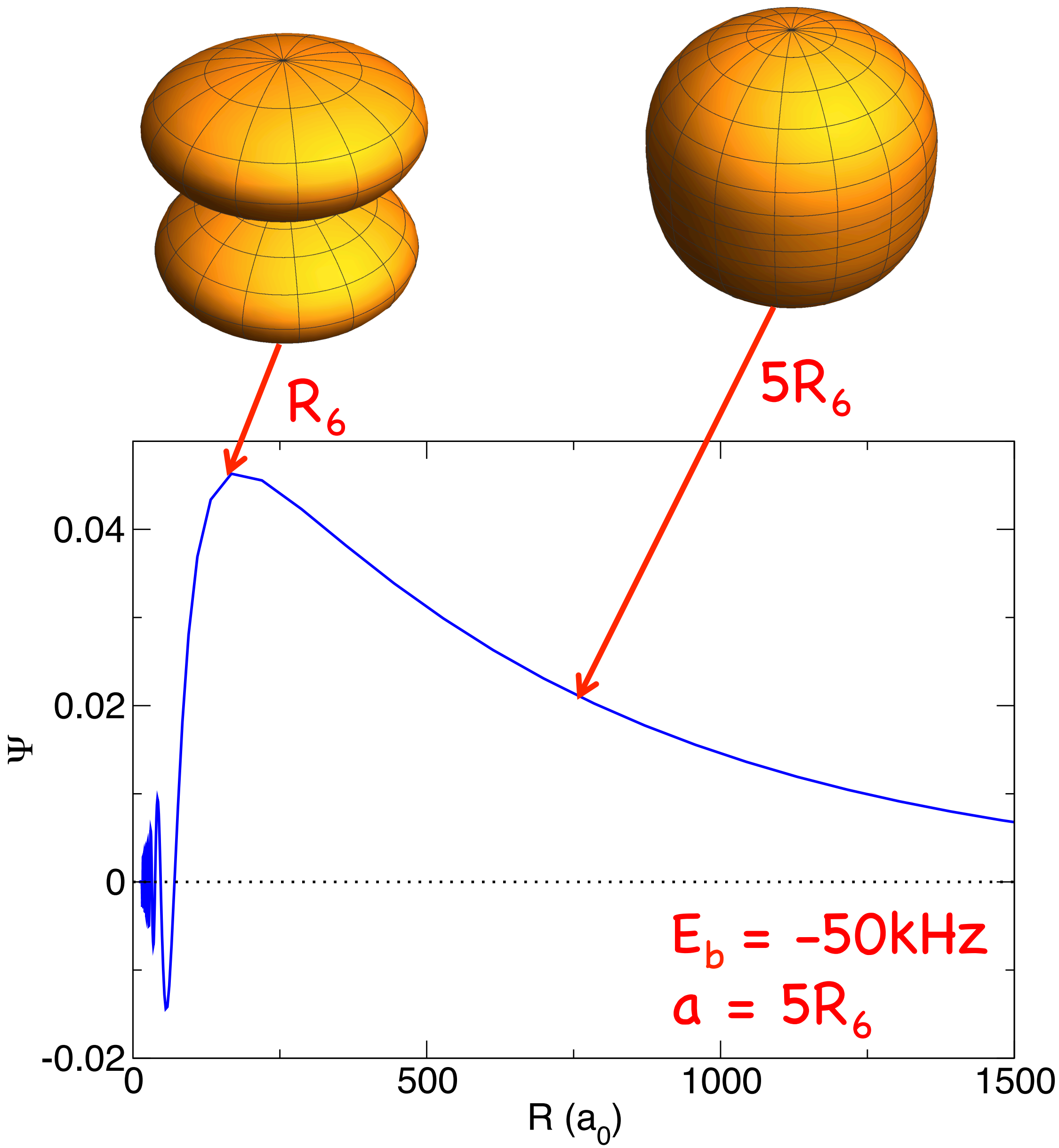
Bohn, Čavagnero, Ticknor, New. J. Phys. *11*, 055039 (2009)



1
0.5
0
-0.5
-1
0
200
400
600
E/h (MHz)
R (a_0)
$V \to -\frac{C_6}{R^6} - \frac{C_4}{R^4}$
s adiabatic hybrid



Ea² versus a in van der Waals units
0
1
2
3
-Ea²/(E₆a₆²)
1
10
a/a₆
U
p=4+6
2-channel, s_res=18
Data, 76G
Data, 170G



R6
5R6
0.04
0.02
0
-0.02
Ψ
0
500
1000
1500
R (a0)
Eb = -50kHz
a = 5R6

Non-reactive cold molecular collisions
e.g., NaK, RbCs, NaRb, KCs

Many open research questions:

- Are their collisions "chaotic" due to dense resonance set?
- Do their collisions exhibit "patterned complexity" when tuned?
- Are hyperfine spin-relaxation collisions fast?
- Do they exhibit "sticky loss collisions" at universal rates?
- Is collision control possible in reduced dimensional structures?

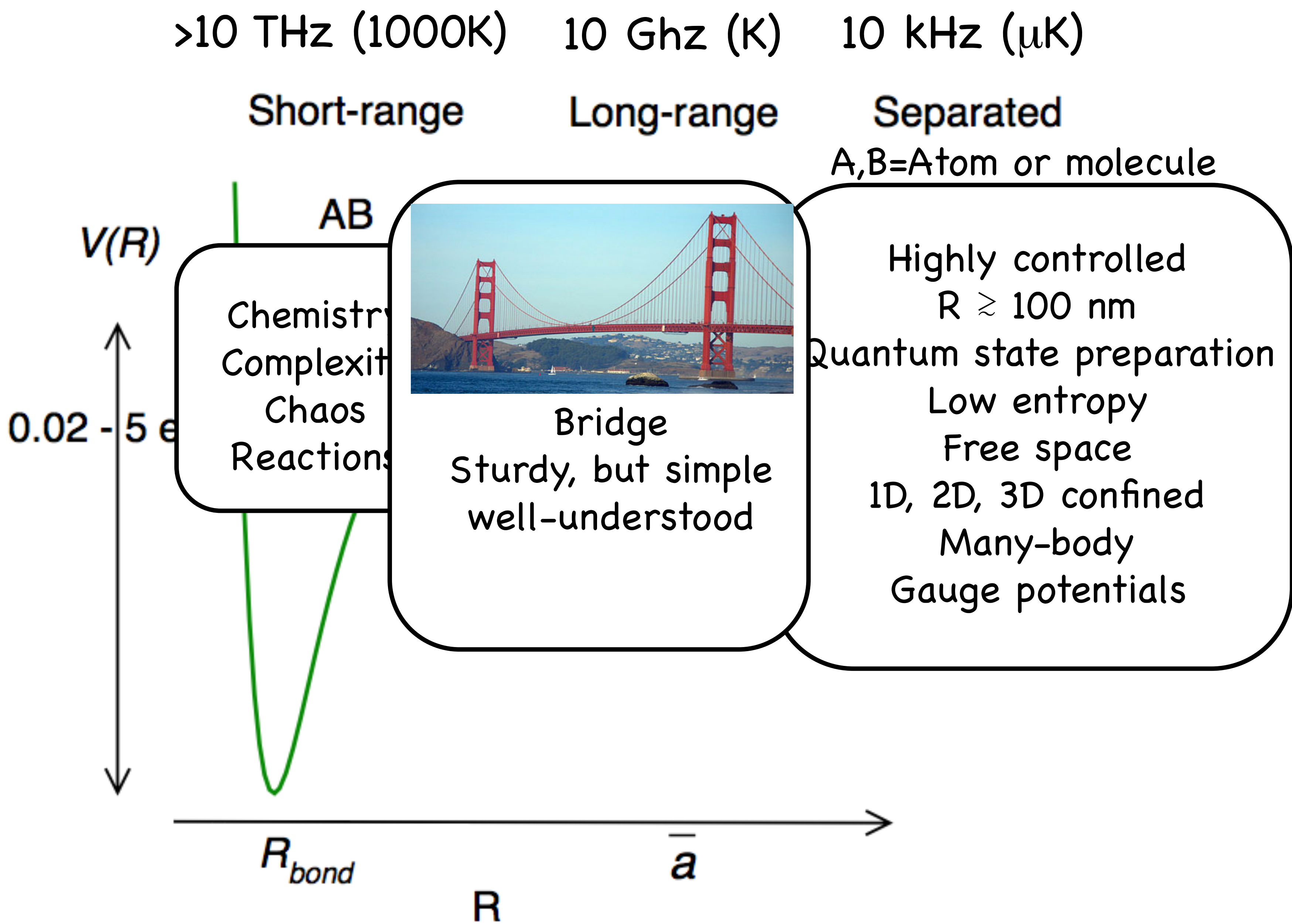


Fig. 2, PSJ Faraday Disc 142, 361 (2009)

>10 THz (1000K) | 10 Ghz (K) | 10 kHz (μK)

Short-range | Long-range | Separated

A,B=Atom or molecule

A few
parameters

Phase (a_{bg})
Feshbach strength (s_{res})
Reactivity (y)

Statistical, chaotic
Patterned complexity
"chemical" modes
Semiclassical
???

Not unique
More than one way
To build a bridge
(Mies/PSJ/Hutson
Greene/Bohn, Gao)
Analytic
Numerical

Highly controlled
R $\gtrsim$ 100 nm
Quantum state preparation
Low entropy
Free space
1D, 2D, 3D confined
Many-body
Gauge potentials

R_{bond} — R — $\bar{a}$

Fig. 2, PSJ Faraday Disc 142, 361 (2009)

The End