ICAP Summer School, Seoul, S. Korea, July 19, 2016

2. Feshbach Resonances

Paul S. Julienne

Joint Quantum Institute
NIST and The University of Maryland

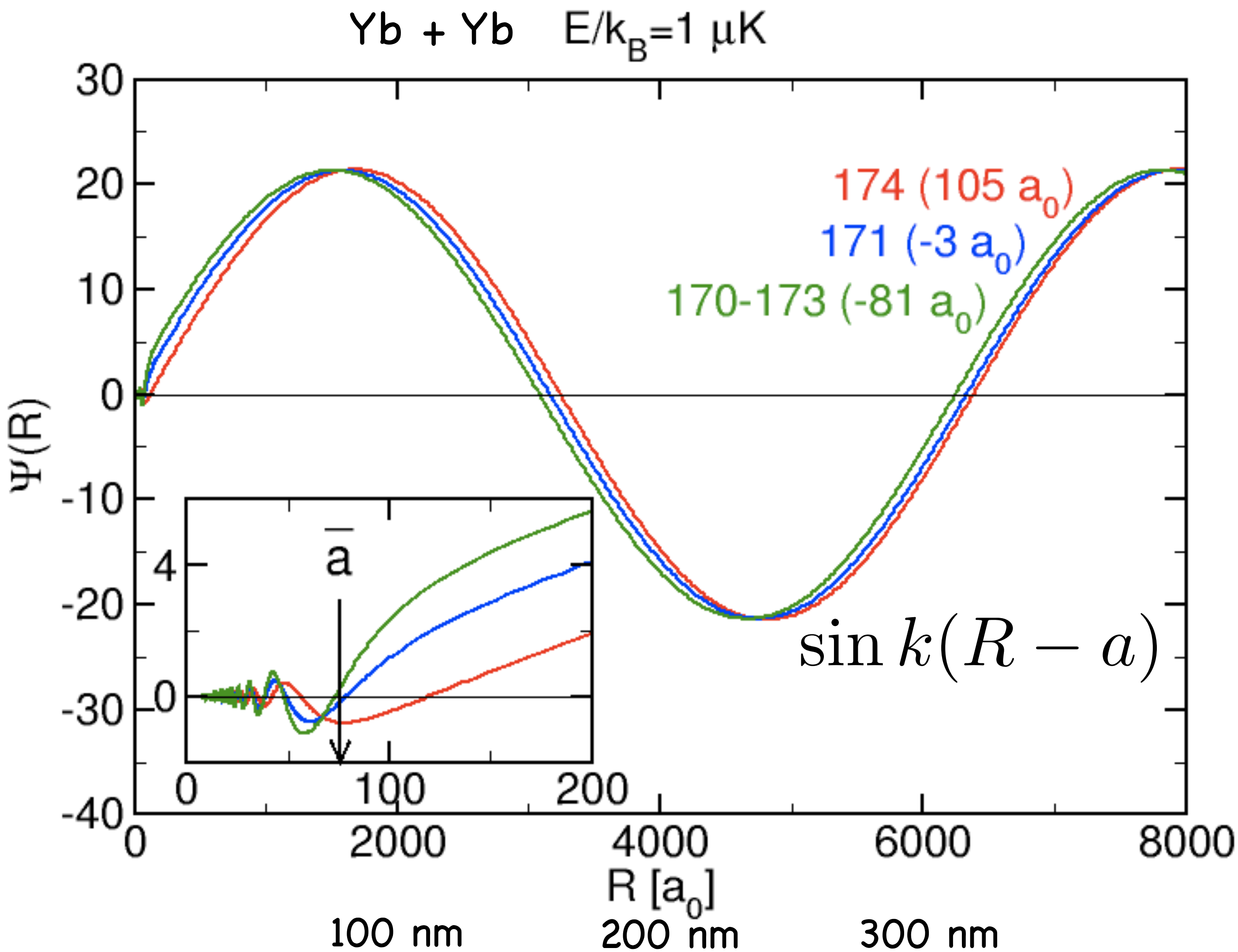
Thanks to many colleagues in theory and experiment who have contributed to this work

http://www.jqi.umd.edu/

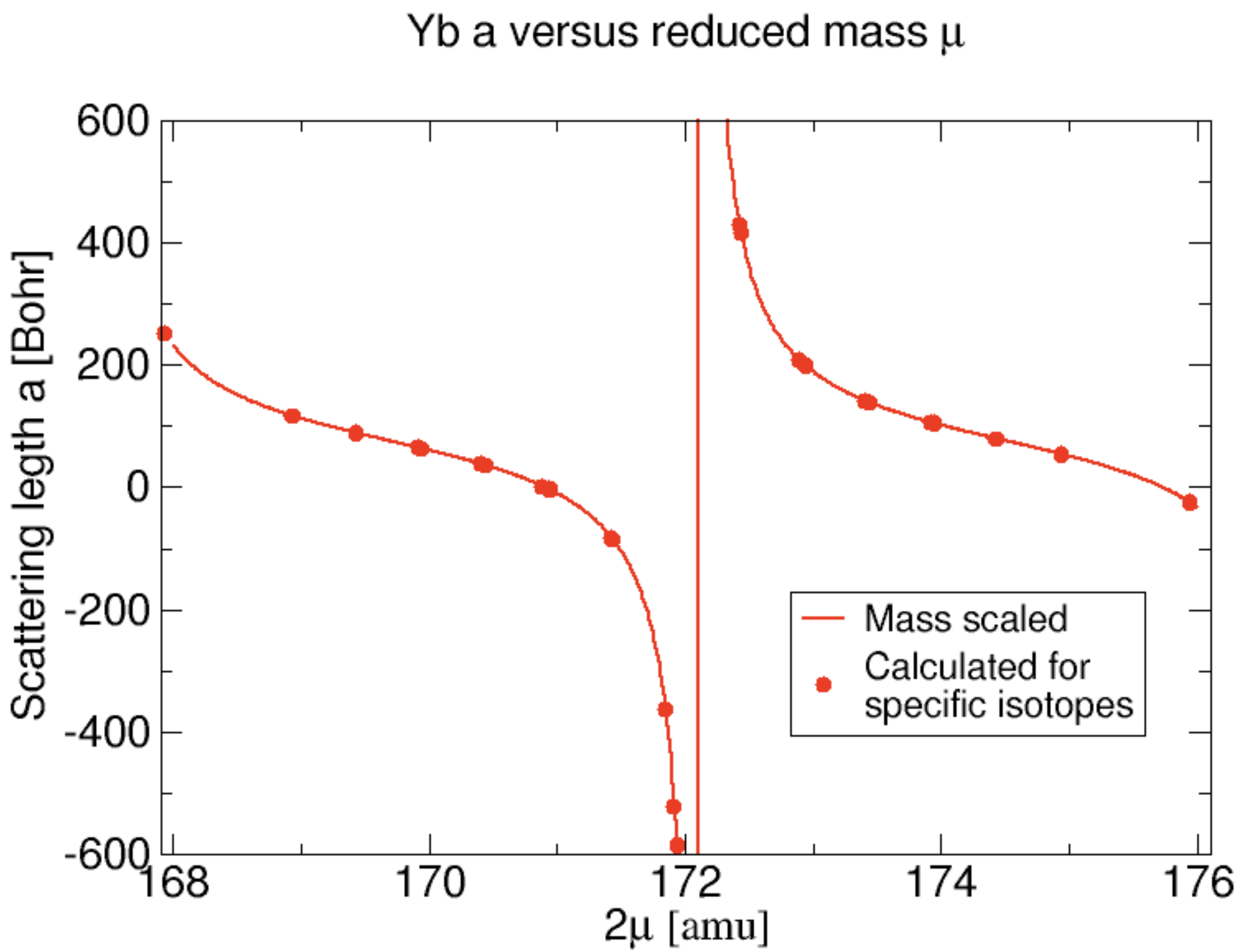
NIST

Joint Quantum Institute

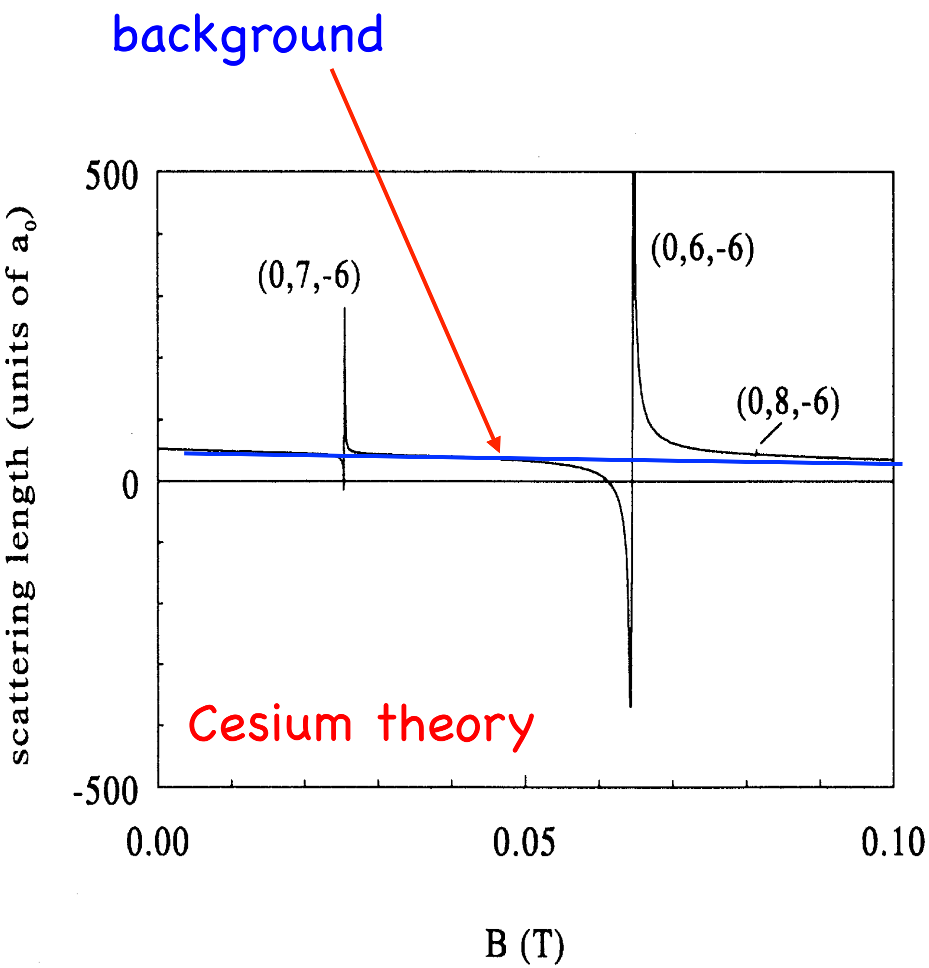
Review: s-wave scattering length + long range potential determine the scattering and bound state properties of all partial waves near threshold.



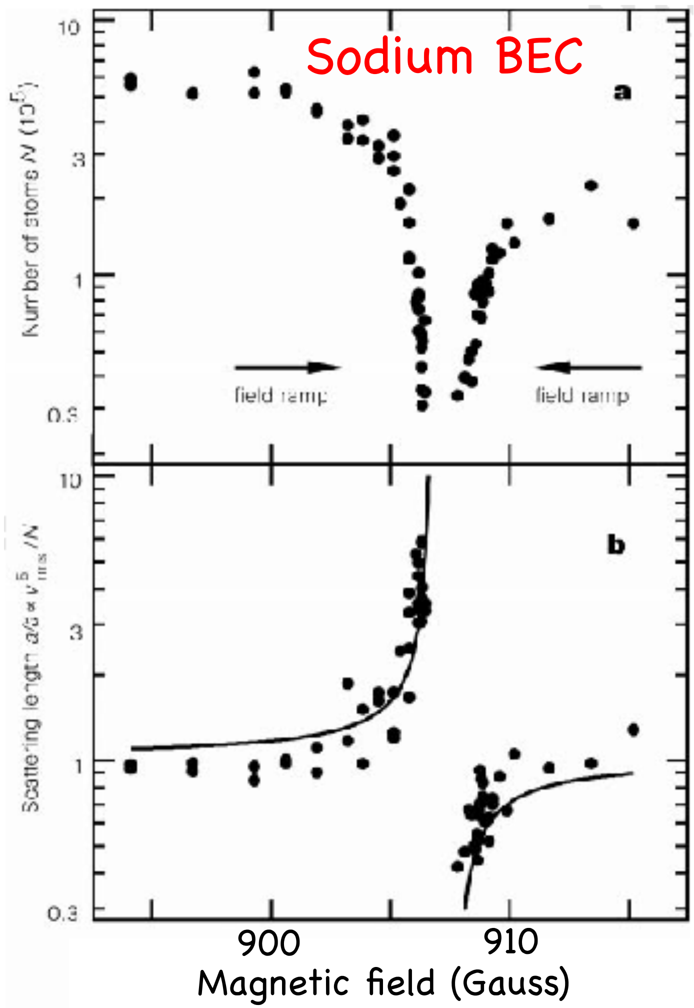
From Krems et al, Cold Molecules, Chapter 6, arXiv:0902.1727


Yb a versus reduced mass μ
Scattering legth a [Bohr]
600
400
200
0
-200
-400
-600
168
170
172
174
176
2μ [amu]
Mass scaled
Calculated for specific isotopes

Some examples of Feshbach resonances



E. Tiesinga *et al.*, Phys. Rev. A **47**, 4114 (1993)

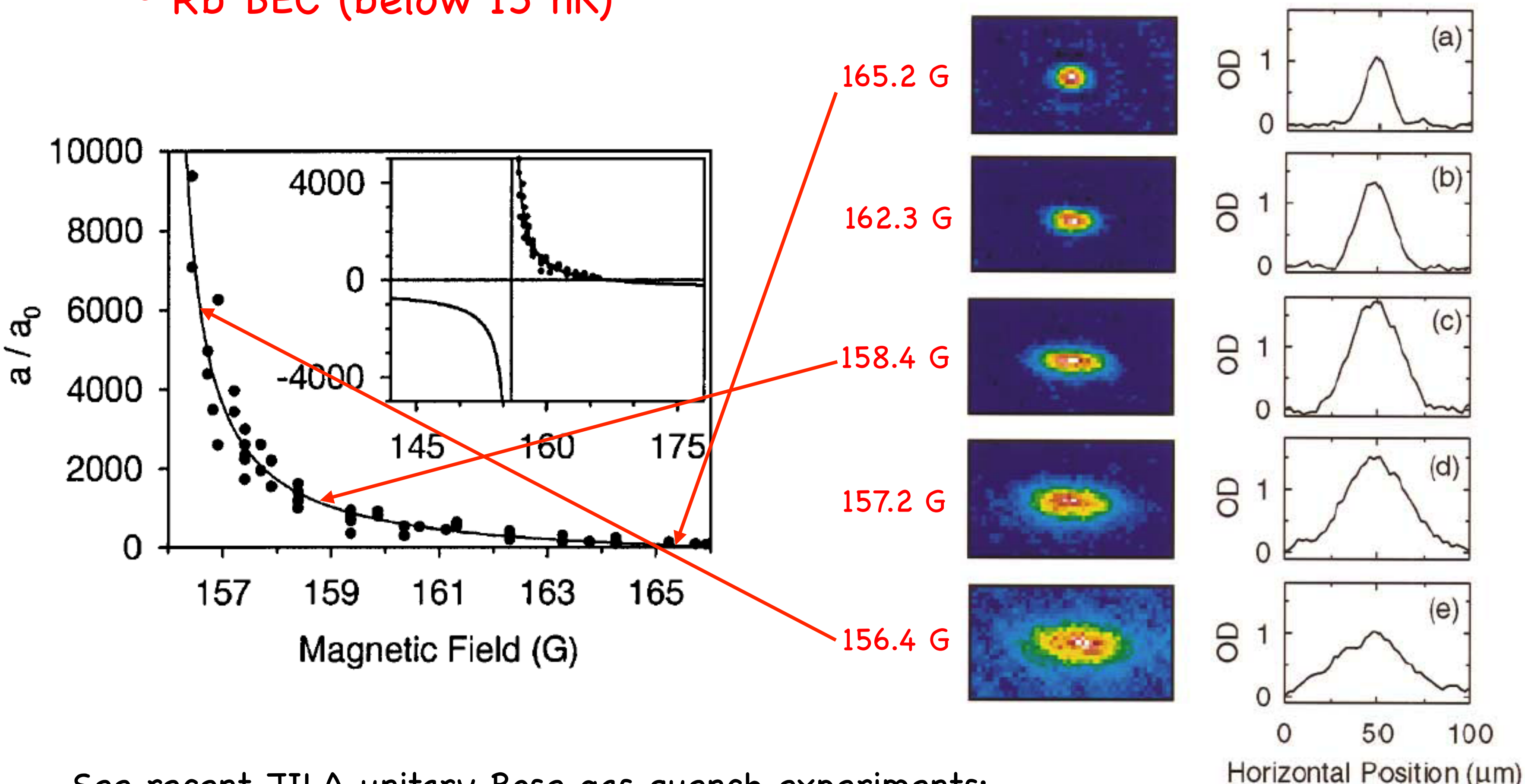


S. Inouye *et al*
Nature **392**, 141 (1998)

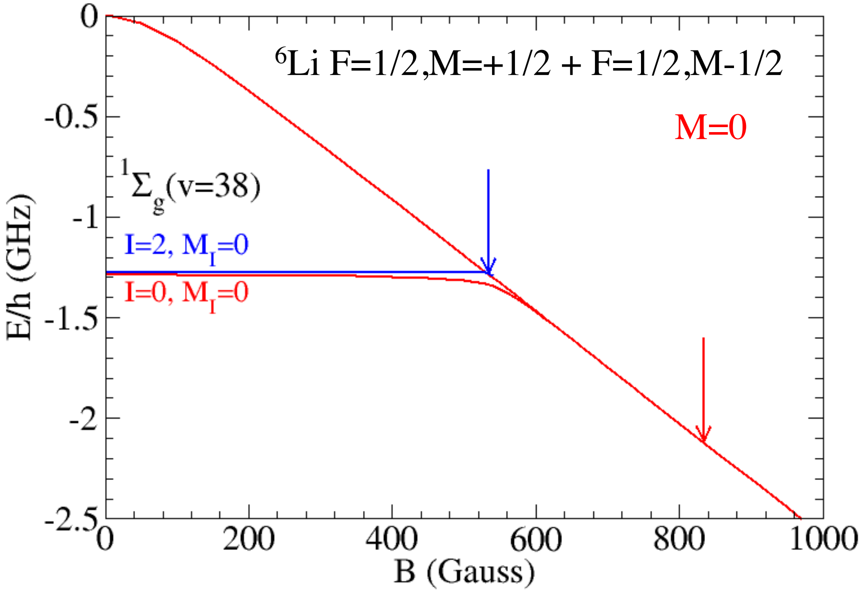
An example for E→ 0

Cornish, Claussen, Roberts, Cornell, Wieman,
Phys. Rev. Lett. 85, 1795 (2000)

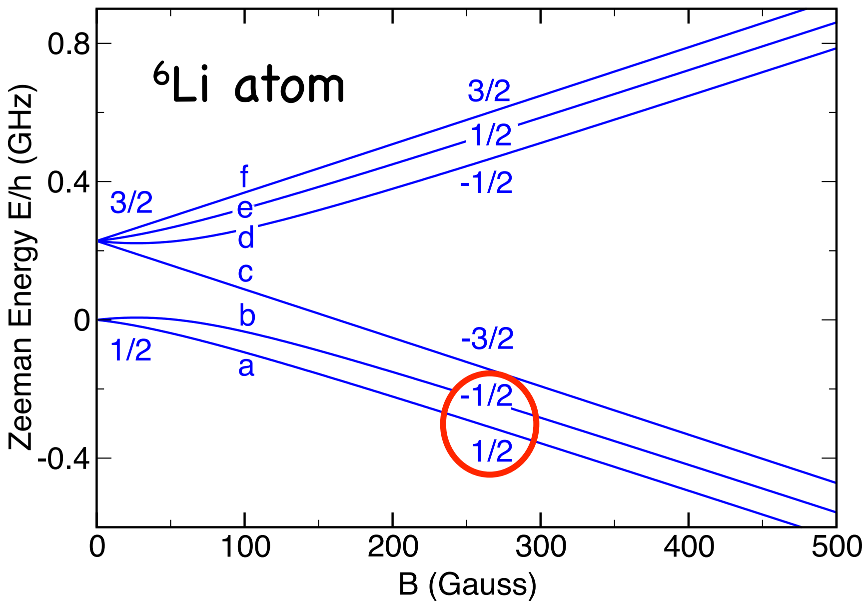
^{85}Rb BEC (below 15 nK)



See recent JILA unitary Bose gas quench experiments:
Makotyn et al, Nat. Phys. 10, 115 (2014)
Sykes, et al, Phys. Rev. A 89, 021601(R) (2014)

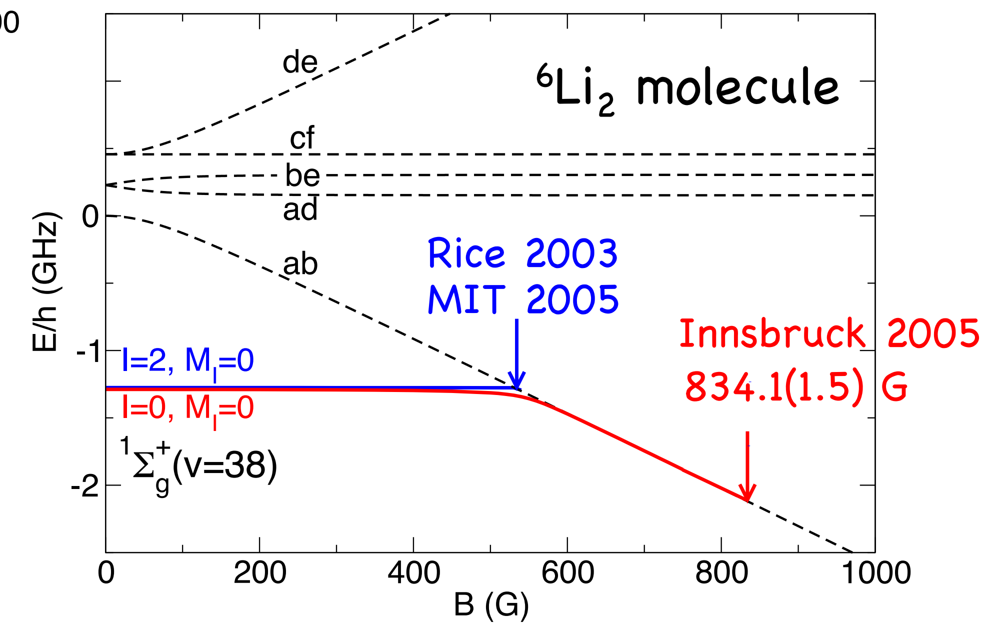


6Li F=1/2,M=+1/2 + F=1/2,M-1/2
M=0
1Σg(v=38)
I=2, MI=0
I=0, MI=0
E/h (GHz)
B (Gauss)
0
-0.5
-1
-1.5
-2
-2.5
0
200
400
600
800
1000



Quantum degenerate Fermi mixtures
BEC-BCS crossover
Strongly interacting gas
Equation of state

a+b s-wave

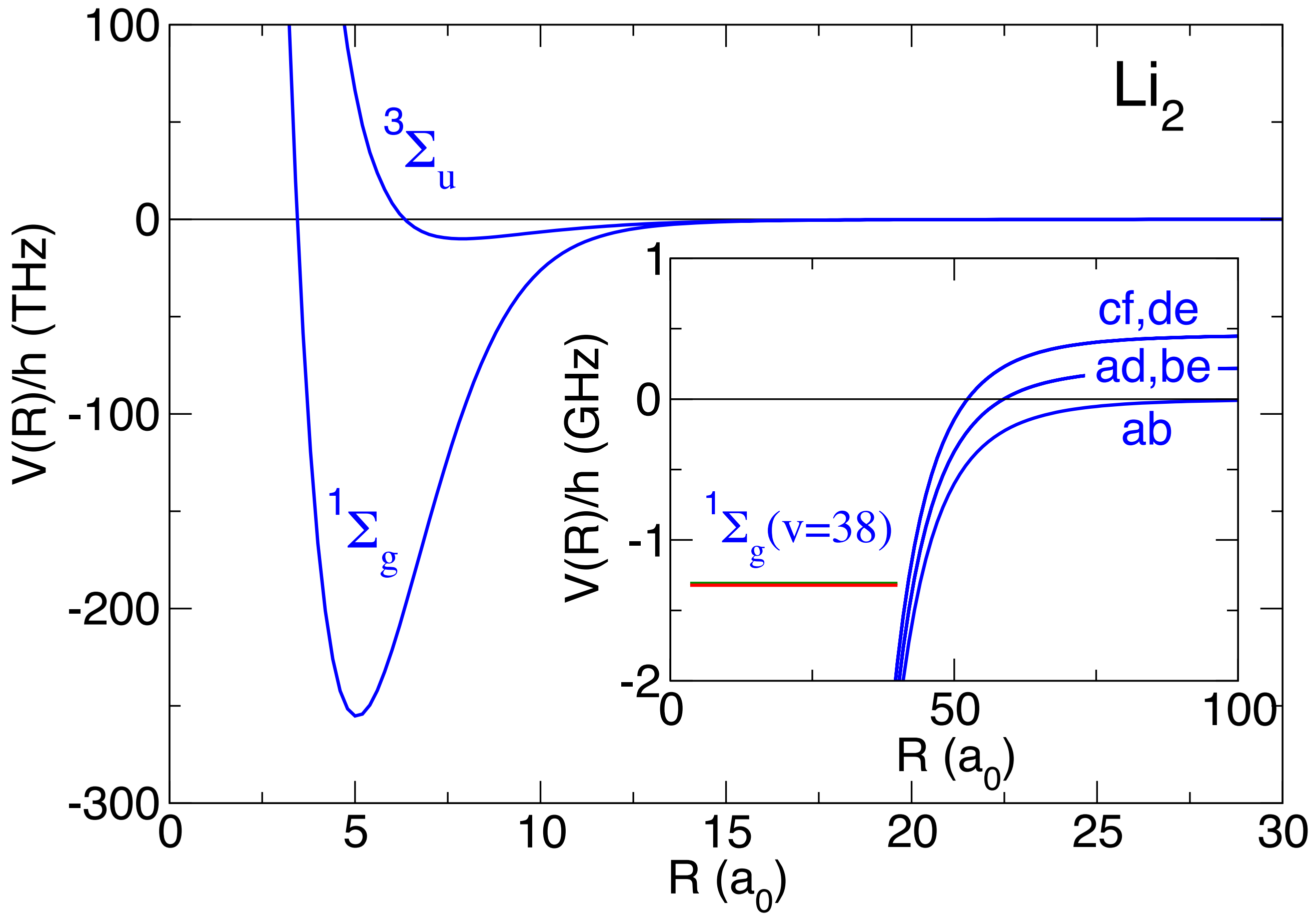


Molecular physics of Li+Li is well-known.

See Simonucci, Pieri, Strinati, Europhys. Lett. **69**, 713-718 (2005)

M. Bartenstein, *et al.*, PRL 94, 103201(2005)
Zürn *et al*, PRL 110, 135301(2013), 832.18(8)G

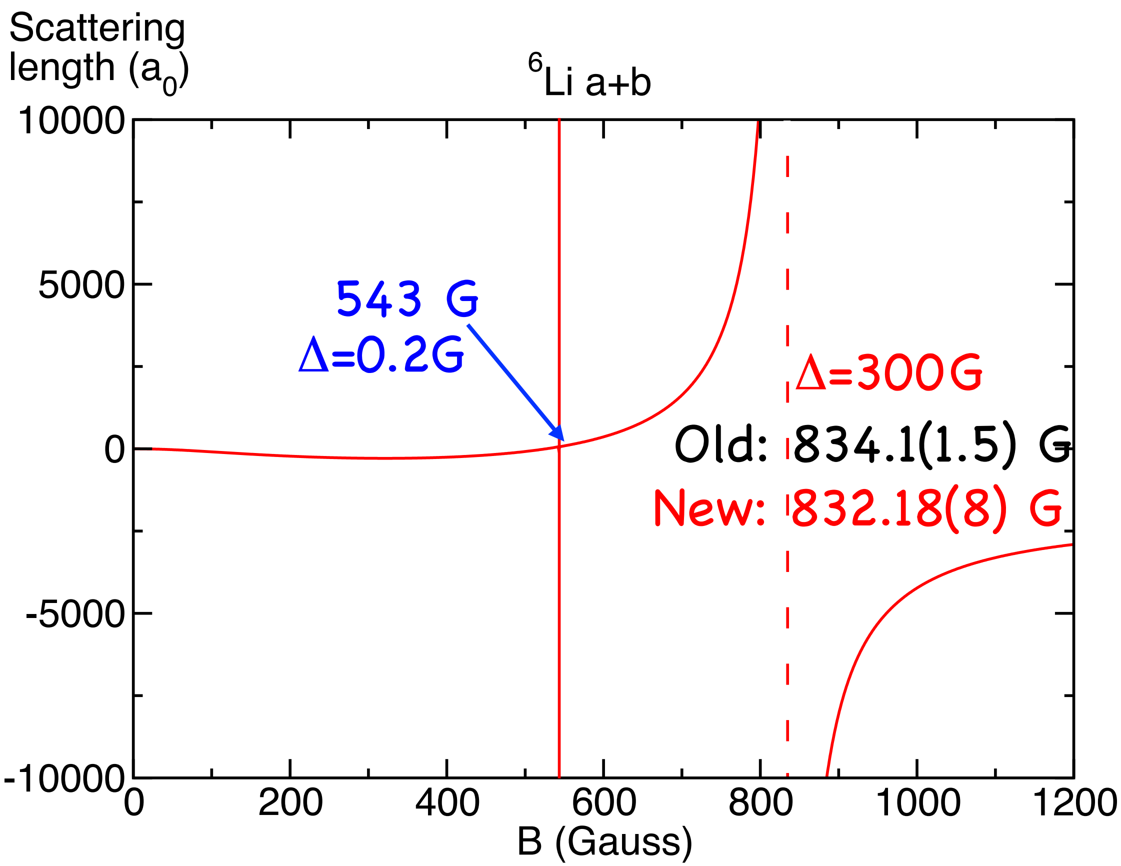
Coupled channels calculations based on accurately known potentials
All spin-dependent interactions treated in Hamiltonian
2 free parameters: S and T scattering lengths



PRL 110, 135301 (2013) PHYSICAL REVIEW LETTERS week ending 29 MARCH 2013

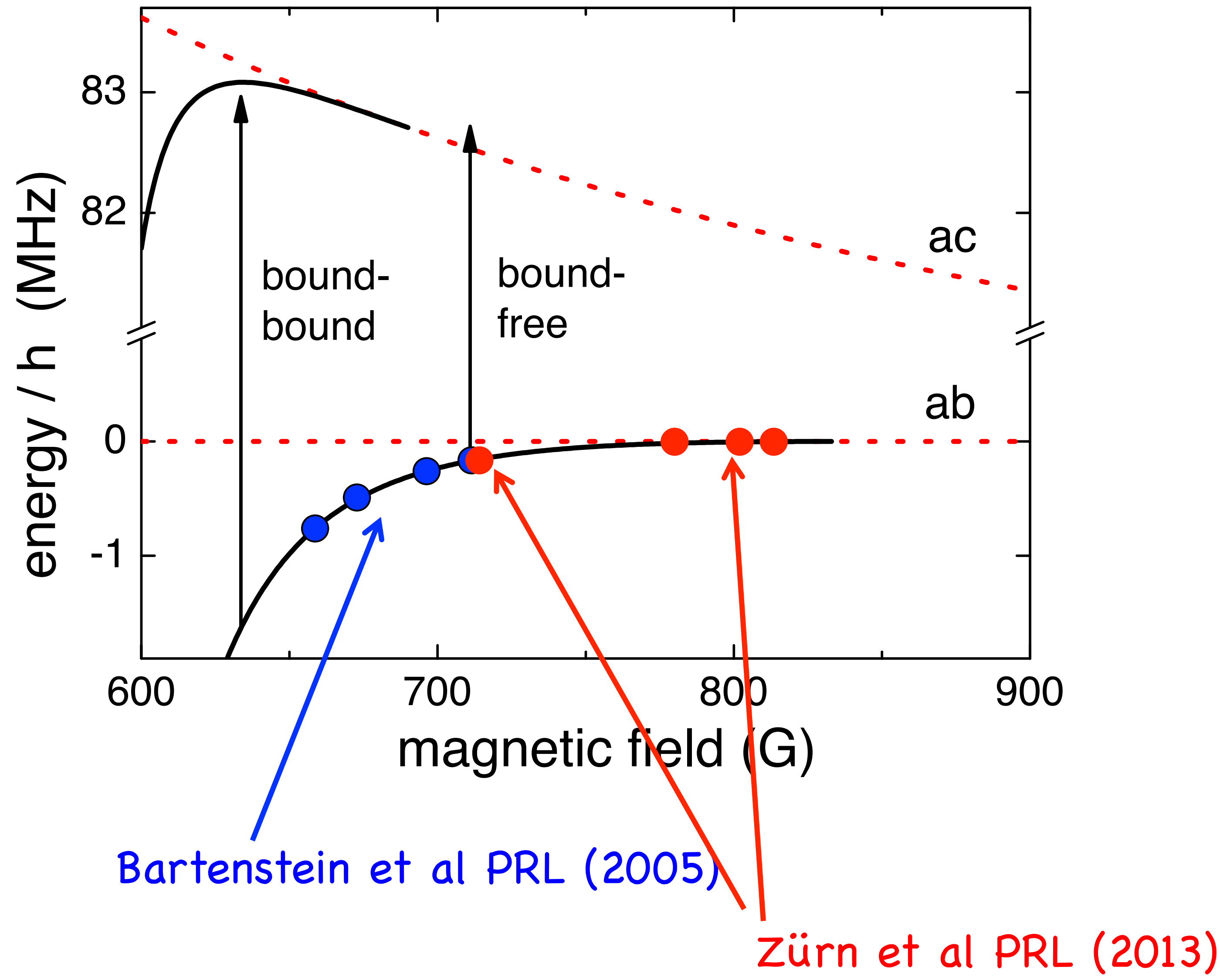
Precise Characterization of ^{6}Li Feshbach Resonances Using Trap-Sideband-Resolved RF Spectroscopy of Weakly Bound Molecules

G. Zürn,[1,2] T. Lompe,[1,2,3,*] A. N. Wenz,[1,2] S. Jochim,[1,2,3] P. S. Julienne,[4] and J. M. Hutson[5,†]

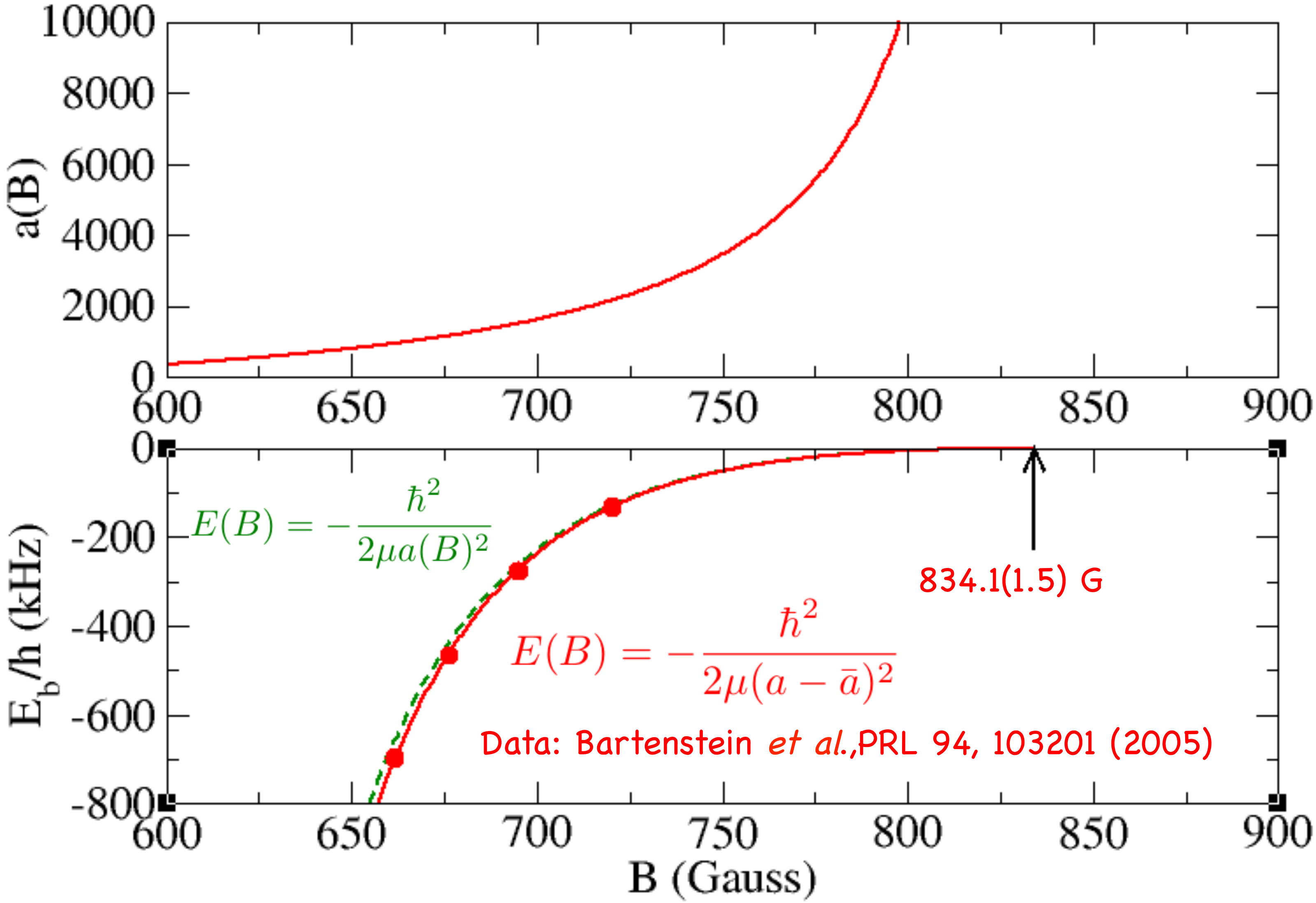


4 new binding energy measurements for B closer to resonance.

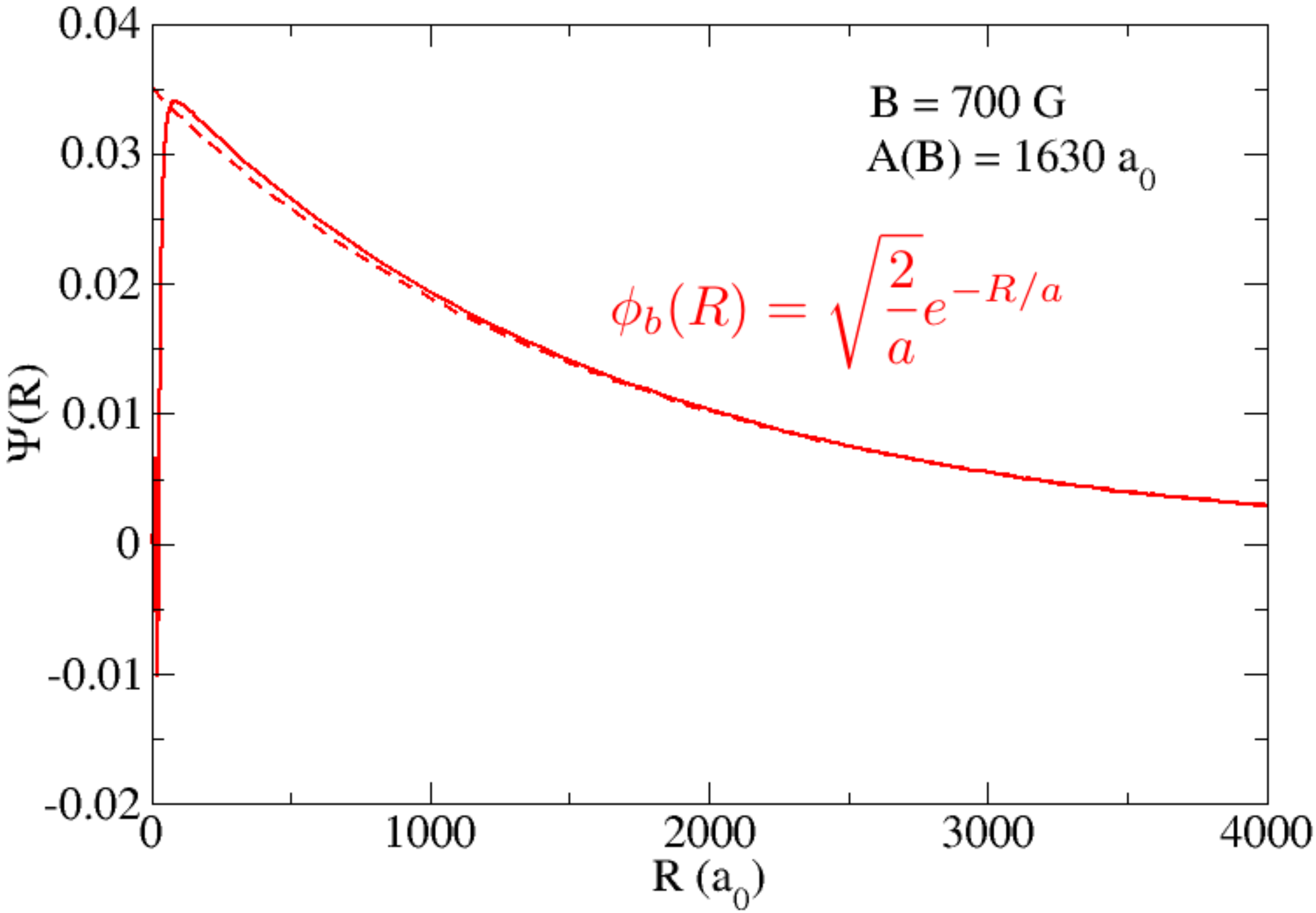
Old: Bartenstein, *et al.*, *PRL* 94, 103201(2005)



83
82
0
-1
energy / h (MHz)
600
700
800
900
magnetic field (G)
ac
ab
bound-
bound
bound-
free
Bartenstein et al PRL (2005)
Zürn et al PRL (2013)



10000
8000
6000
4000
2000
0
a(B)
600
650
700
750
800
850
900
0
-200
-400
-600
-800
E_b/h (kHz)
$E(B) = -\frac{\hbar^2}{2\mu a(B)^2}$
834.1(1.5) G
$E(B) = -\frac{\hbar^2}{2\mu(a-\bar{a})^2}$
Data: Bartenstein *et al.*,PRL 94, 103201 (2005)
B (Gauss)



0.04
0.03
0.02
0.01
0
-0.01
-0.02
Ψ(R)
0
1000
2000
3000
4000
R (a0)
B = 700 G
A(B) = 1630 a0
$\phi_b(R) = \sqrt{\frac{2}{a}} e^{-R/a}$

Long history of resonance scattering

O. K. Rice, J. Chem. Phys. 1, 375 (1933)

U. Fano, Nuovo Cimento 12, 154 (1935)

J. M. Blatt and V. F. Weisskopf, Theoretical Nuclear Physics (1952)

H. Feshbach, Ann. Phys. (NY) 5, 357 (1958); 19, 287 (1962)

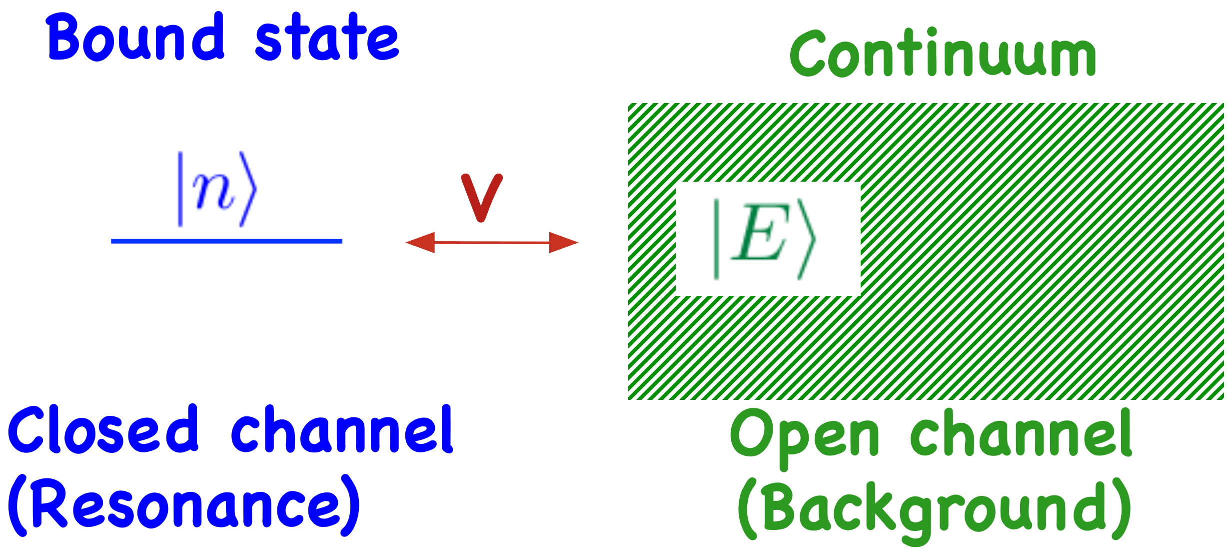
U. Fano, Phys. Rev. 124, 1866 (1961)

Separation of system into:

An (approximate) bound state
A scattering continuum
with some coupling between them

Resonant Scattering Picture

following U. Fano, Phys. Rev. 124, 1866 (1961); see Chin et al, RMP (2010)



$$\eta(E) = \eta_{\rm bg} + \eta_{\rm res}(E)$$

$$\eta_{\rm res} = -\tan^{-1} \frac{\frac{1}{2}\Gamma_n}{E - E_n - \delta E_n}$$

width

$$\Gamma_n = 2\pi |\langle n|V|E\rangle|^2$$

shift

$$\delta E_n = \int \frac{|\langle n|V|E'\rangle|^2}{E_n - E'} dE'$$

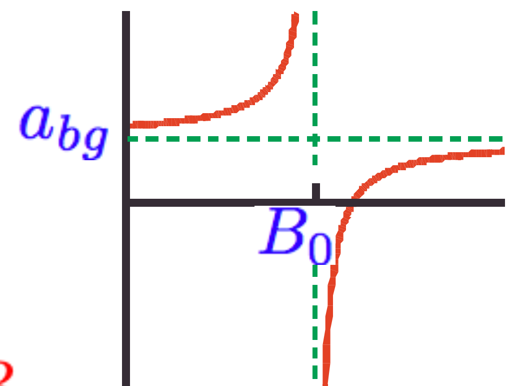
Threshold Resonant Scattering

$$E_n = \delta\mu(B - B_n)$$

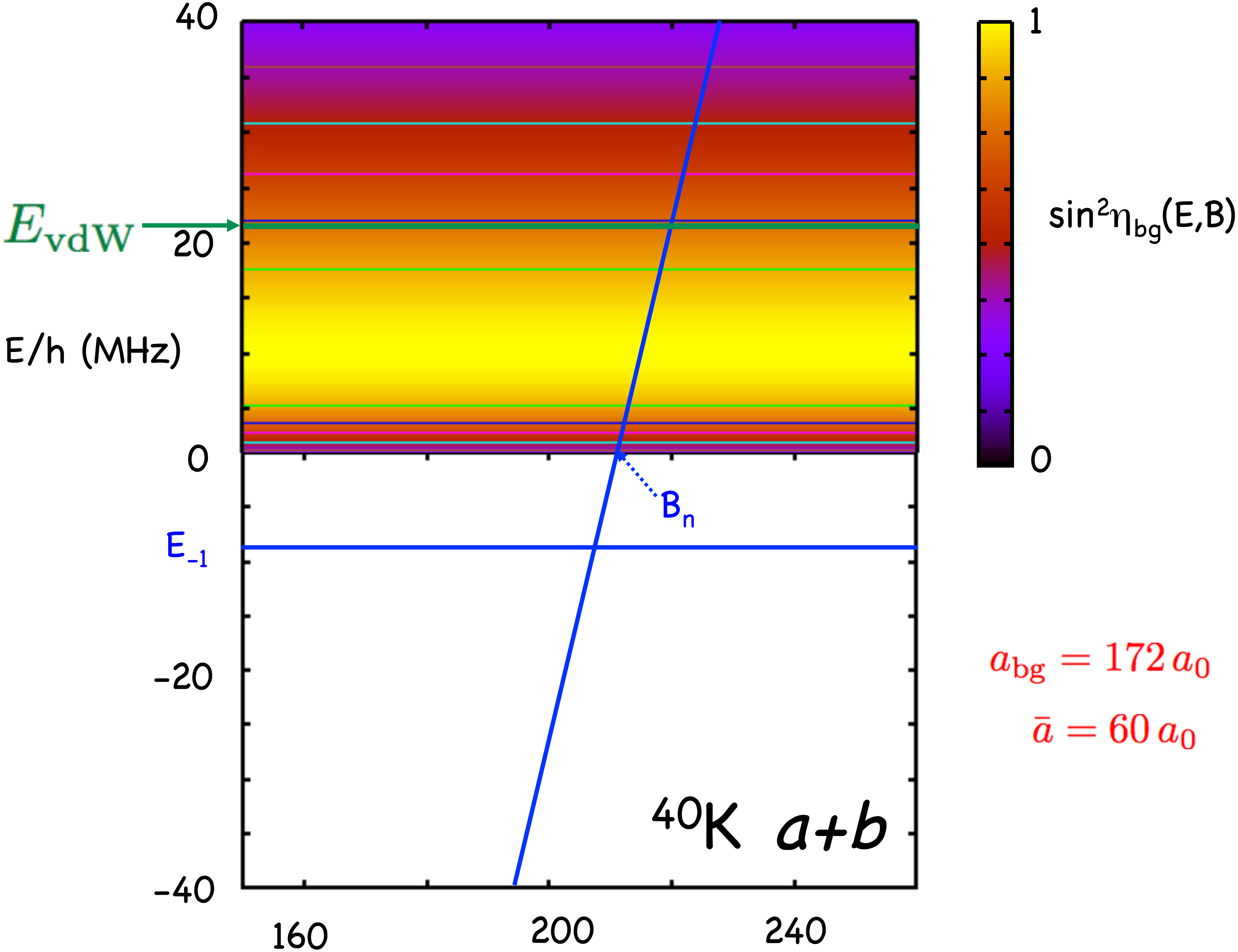
$$\eta(E,B) = \eta_{\mathrm{bg}}(E) - \tan^{-1}\frac{\frac{1}{2}\Gamma(E)}{E - E_n - \delta E_n(E)}$$

As E → 0 $\quad \eta_{\mathrm{bg}} = -k a_{\mathrm{bg}} \qquad \frac{1}{2}\Gamma_n(E) = (k a_{\mathrm{bg}})\, \delta\mu\, \Delta_n$

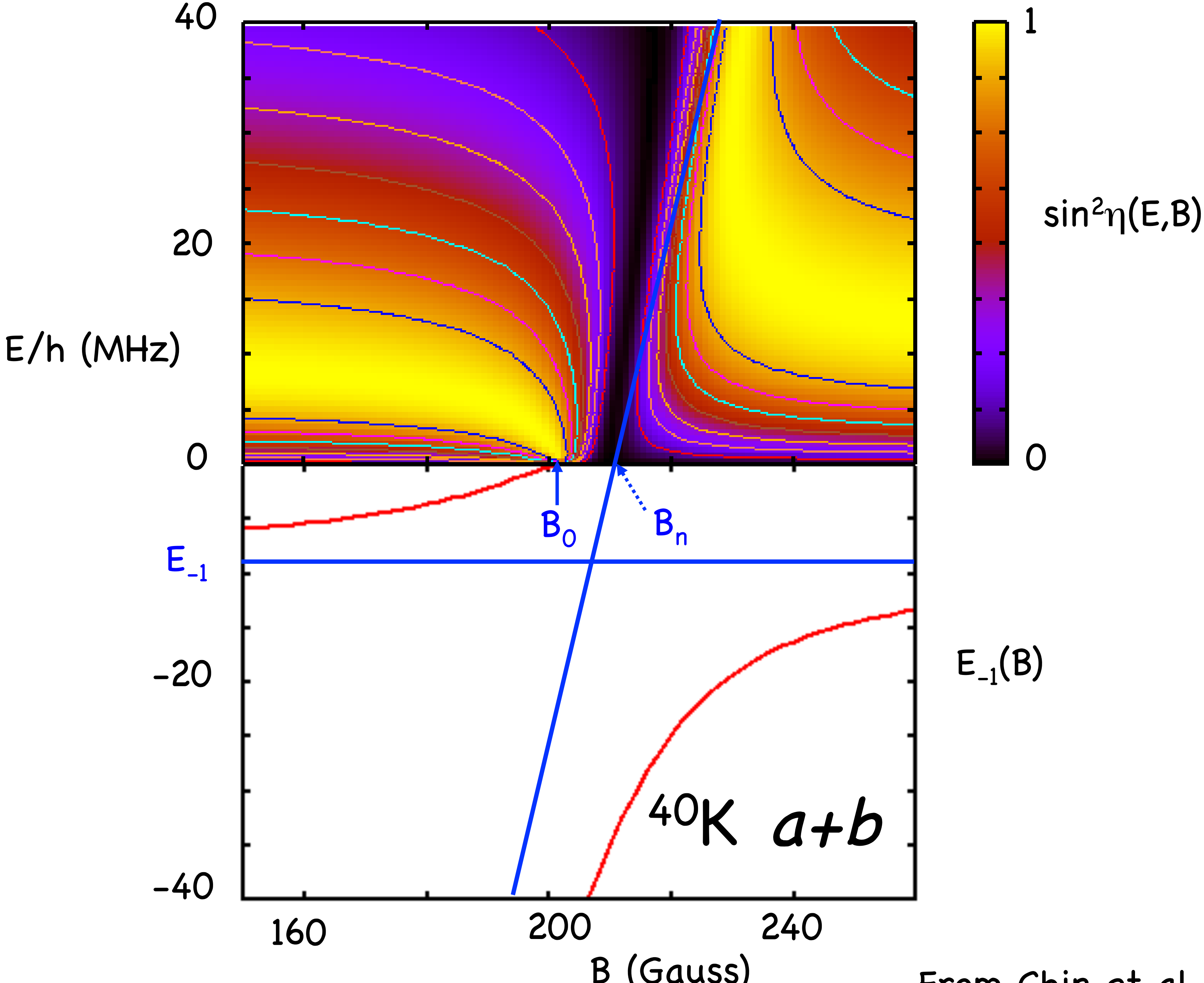
$$a(B) = a_{bg}\left(1 - \frac{\Delta_n}{B - B_0}\right)$$



Shifted $B_0 = B_n + \delta B_n$

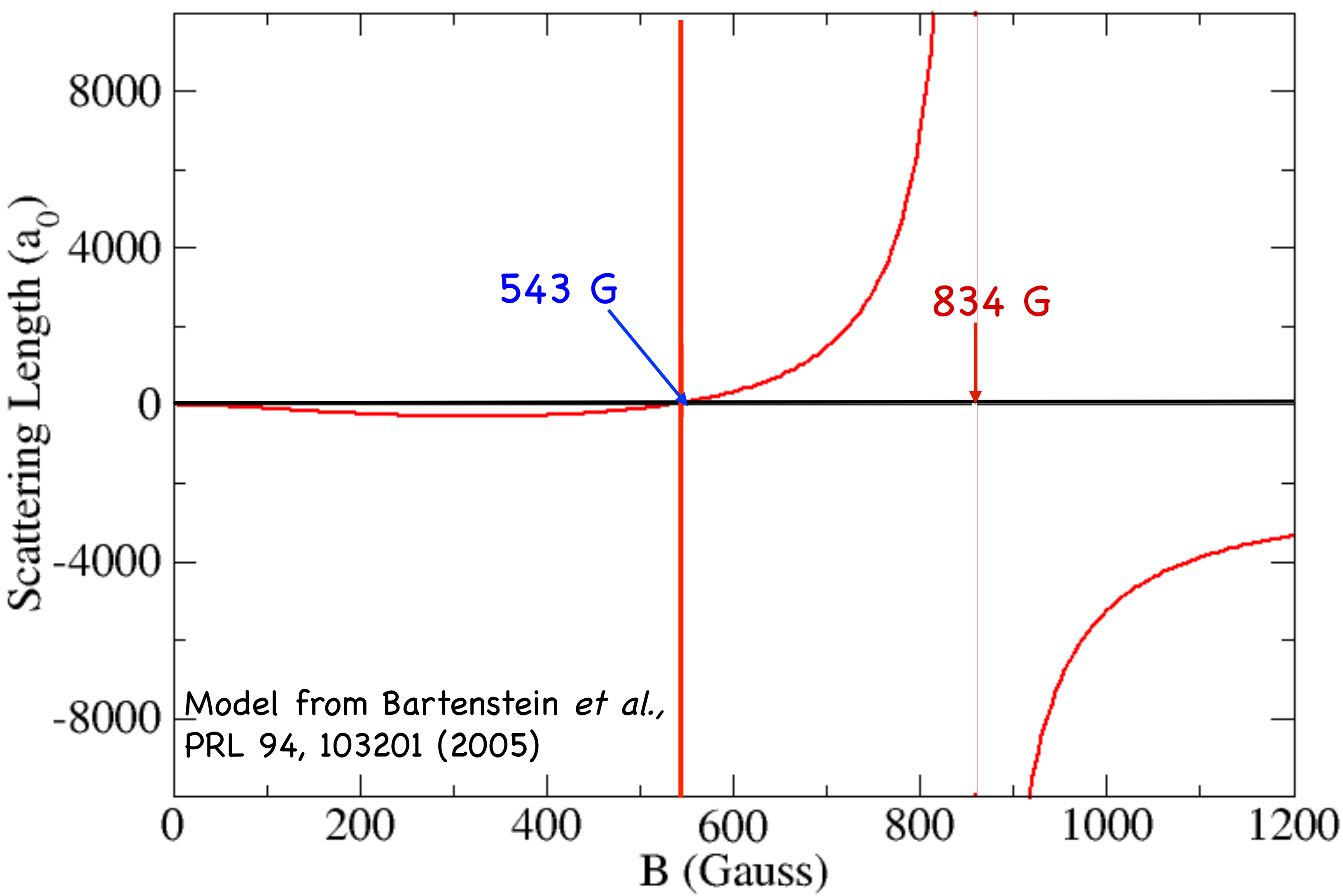


From Chin et al (2010)



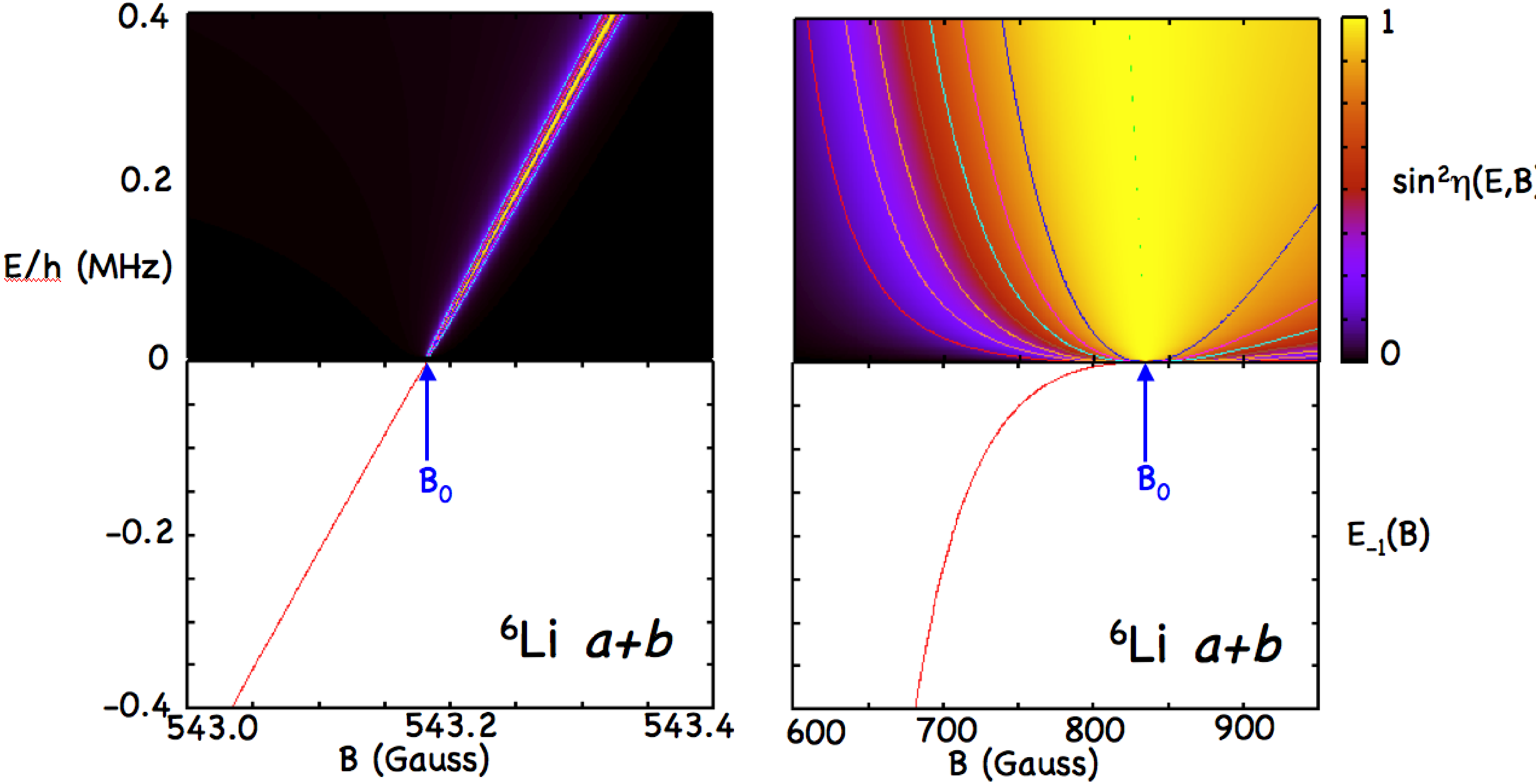
From Chin et al (2010)

^{6}Li *a+b* Scattering Length vs. B



Closed channel dominated

Open channel dominated



Chin, Grimm, PSJ, Tiesinga, Rev. Mod. Phys. 82, 1225 (2010)

Classification of resonances by strength

Resonance strength $s_{\mathrm{res}} = \frac{a_{bg}}{\bar{a}} \frac{\delta\mu\Delta}{\bar{E}}$

See Kohler et al, Rev. Mod. Phys. 78, 1311 (2006)
And Chin, et al, Rev. Mod. Phys. 82, 1225 (2010)

$$a(B) = a_{\mathrm{bg}} \left(1 - \frac{\Delta}{B - B_0}\right)$$

For magnetically tunable resonances: $E_c = \delta\mu(B - B_c)$

Bound state norm Z as E → 0
and B → B_0

$$Z = \zeta^{-1} \left| \frac{B - B_0}{\Delta} \right|$$

$$\zeta = \frac{1}{2} s_{\mathrm{res}} \frac{a_{bg}}{\bar{a}}$$

Universal van der Waals for an isolated Feshbach resonance

Reduced (dimensionless) variables for a van der Waals potential:

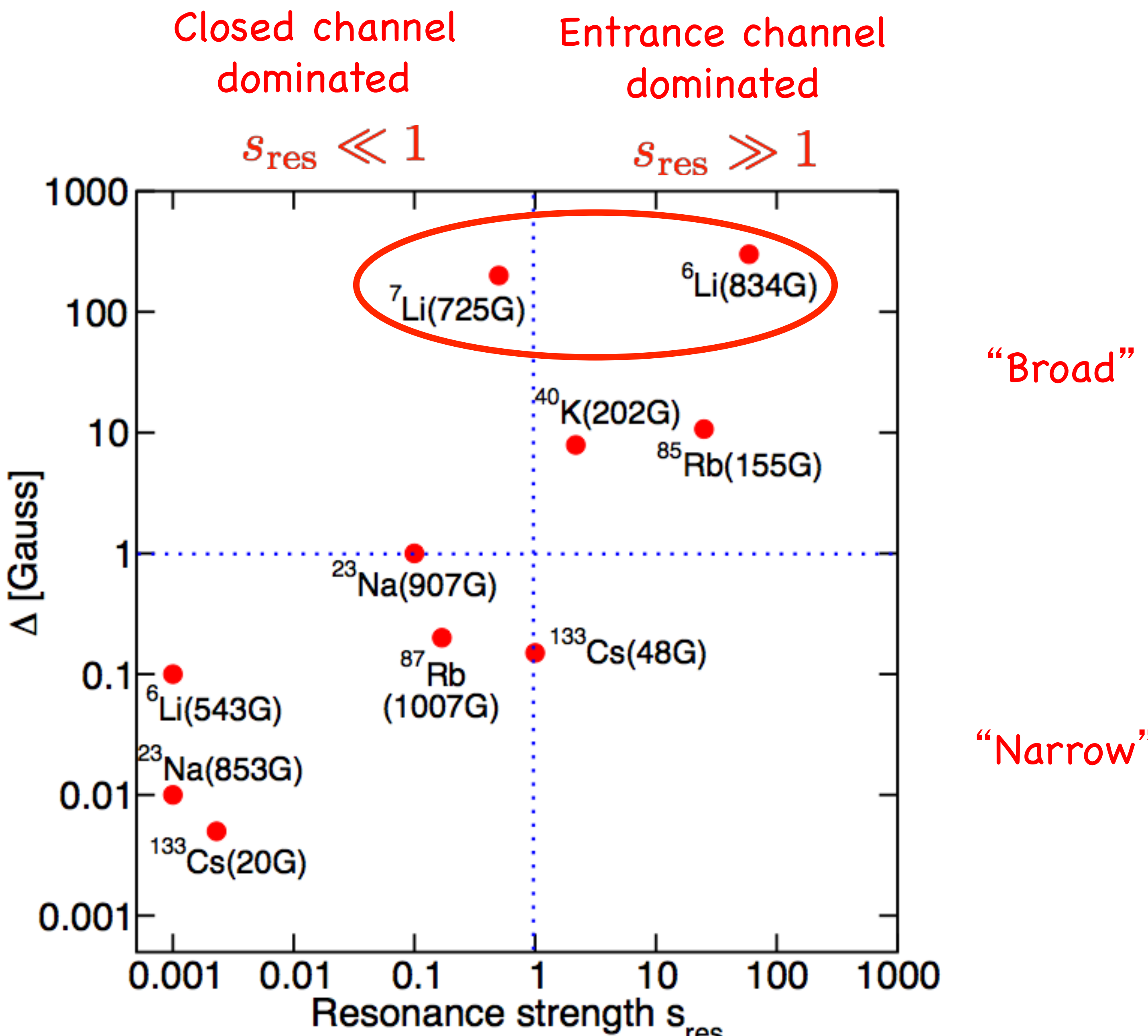
$$\varepsilon = E/\bar{E} \qquad \kappa = k\bar{a} \qquad r = a/\bar{a}$$

$$\beta = B/\Delta \qquad m_{\rm res} = \mu_{\rm diff}\Delta/\bar{E}$$

Universal van der Waals expression in the limit E → 0

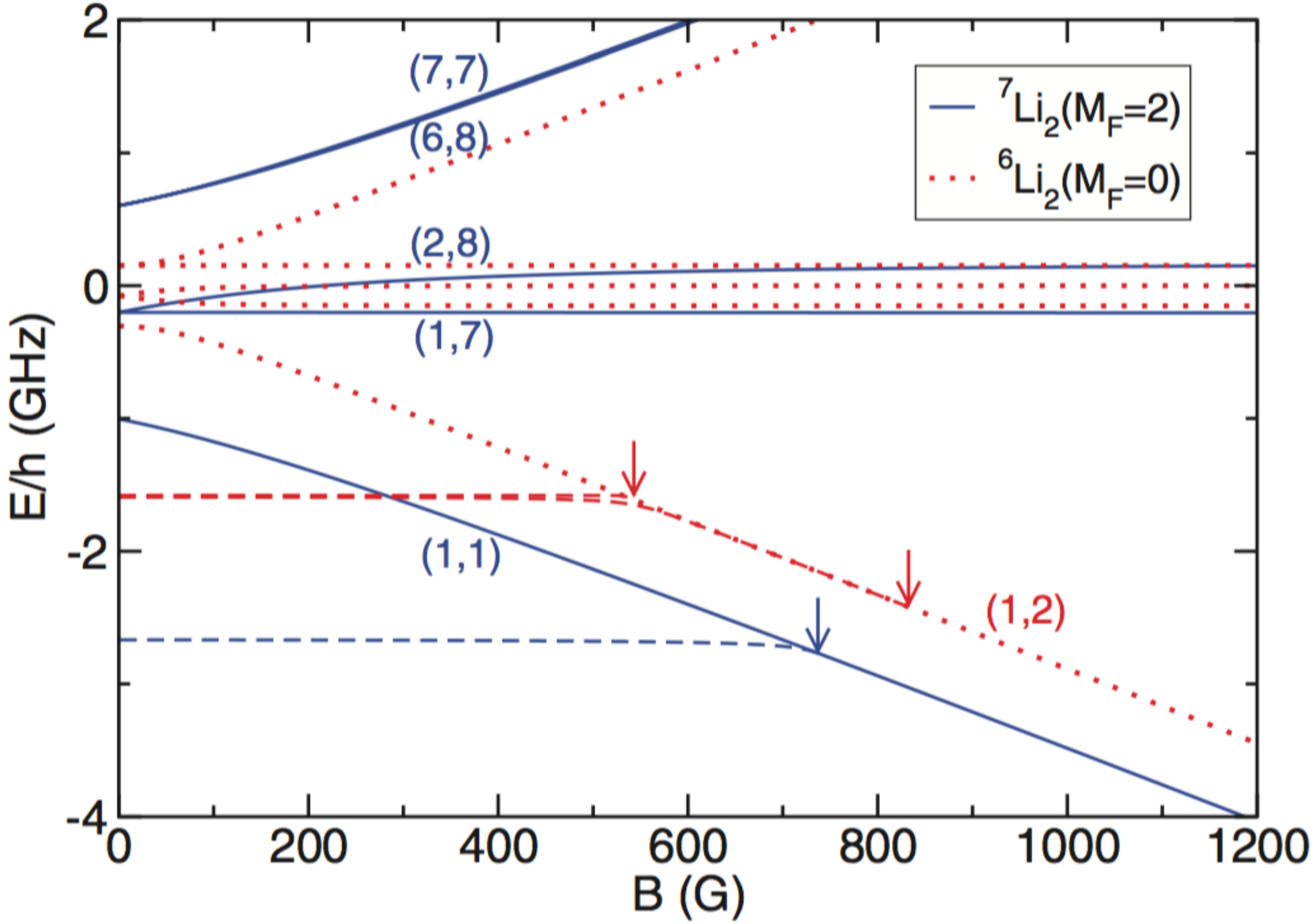
$$\eta_{\rm res}(\varepsilon,\beta) = -\tan^{-1}\frac{\kappa s_{\rm res}}{\varepsilon - m_{\rm res}\left(\beta - \beta_{\rm c} - \frac{r_{\rm bg}(1-r_{\rm bg})}{1+(r_{\rm bg}-1)^2}\right)}$$

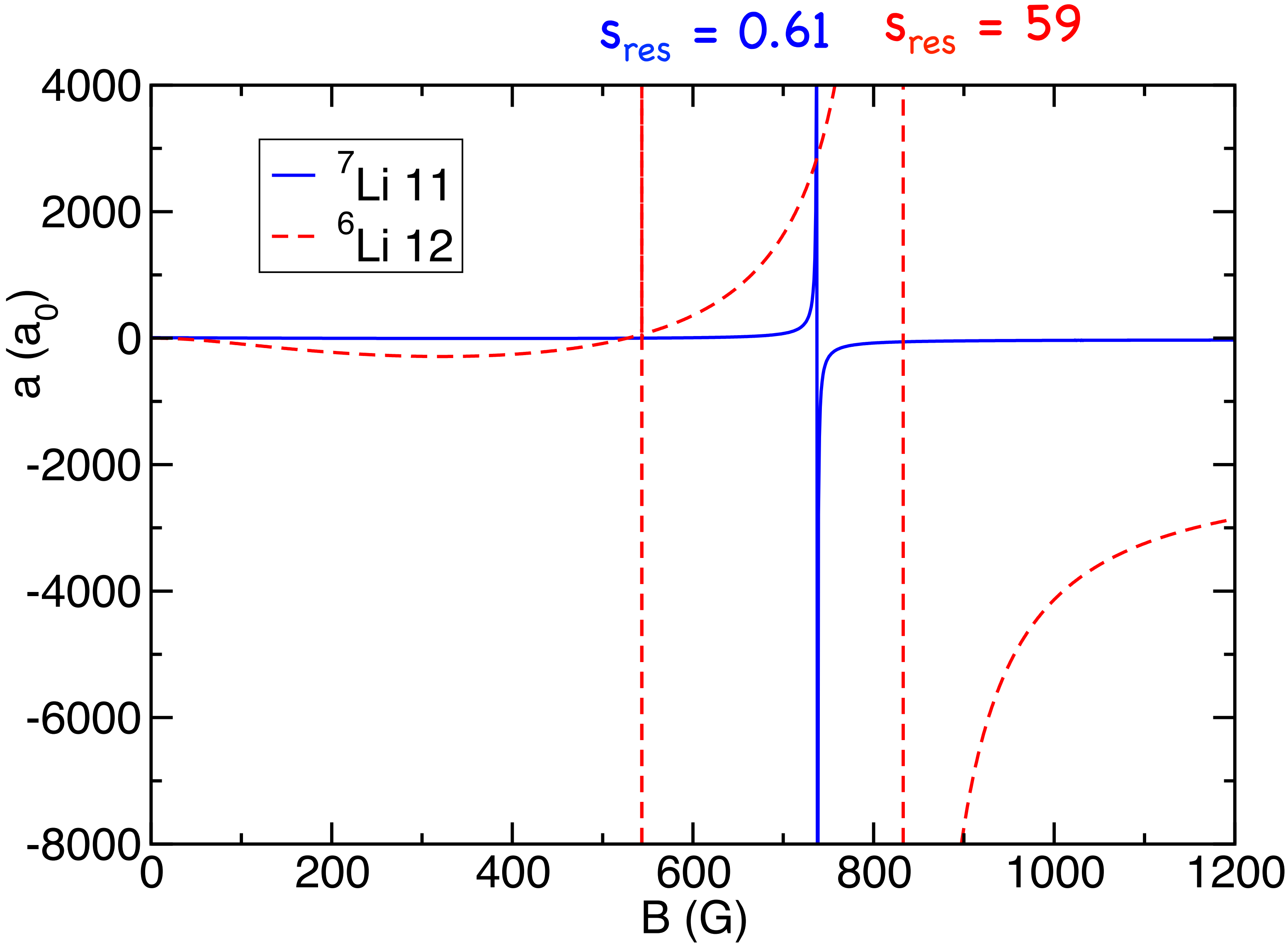
(Analytic vdW functions can be used away from E = 0)



From Chin et al (2010)

^7Li (blue) and ^6Li (red)





s_res = 0.61
s_res = 59
4000
2000
0
-2000
-4000
-6000
-8000
a (a_0)
0
200
400
600
800
1000
1200
B (G)
7Li 11
6Li 12

$s_{\mathrm{res}} = 59$

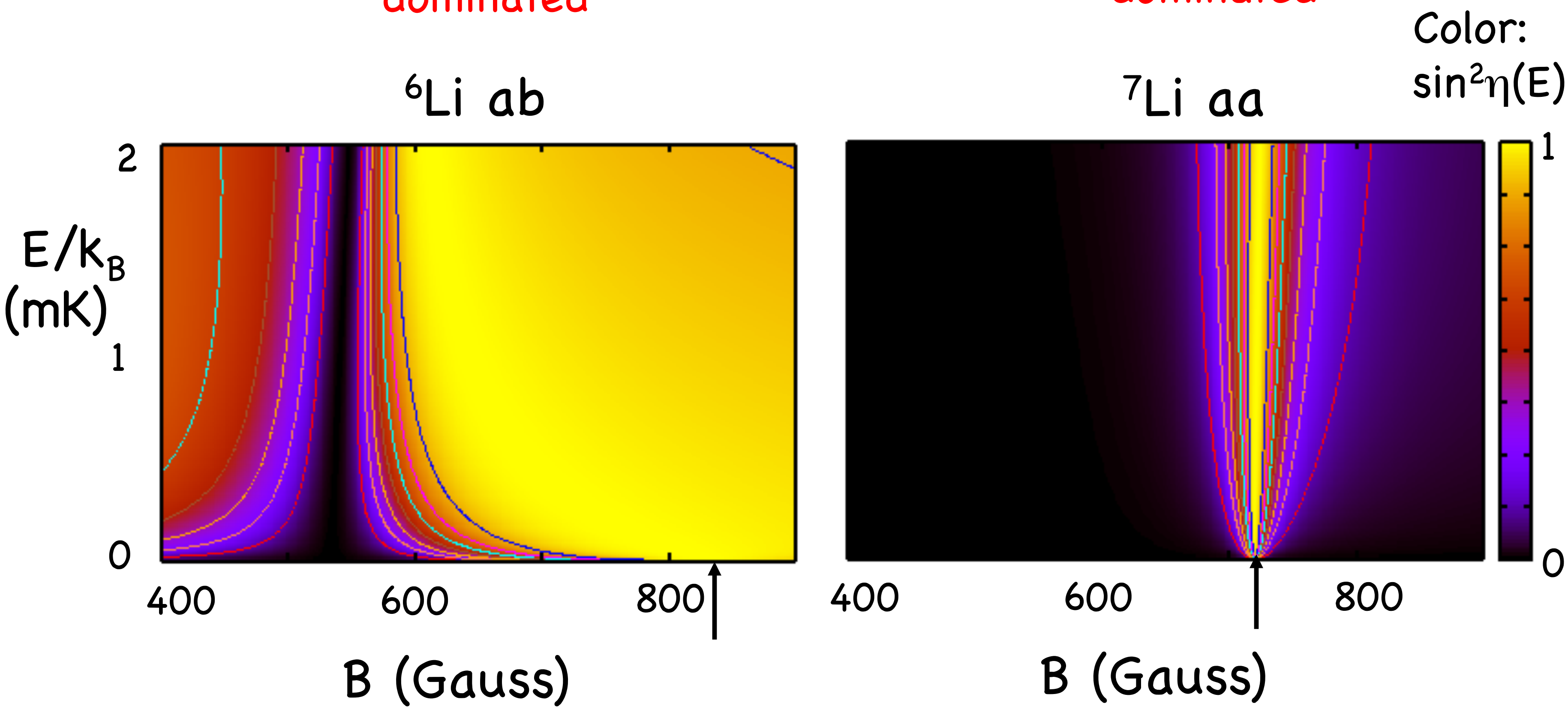
$\Delta = 300\,\mathrm{G}$

Entrance channel dominated

$s_{\mathrm{res}} = 0.61$

$\Delta = 180\,\mathrm{G}$

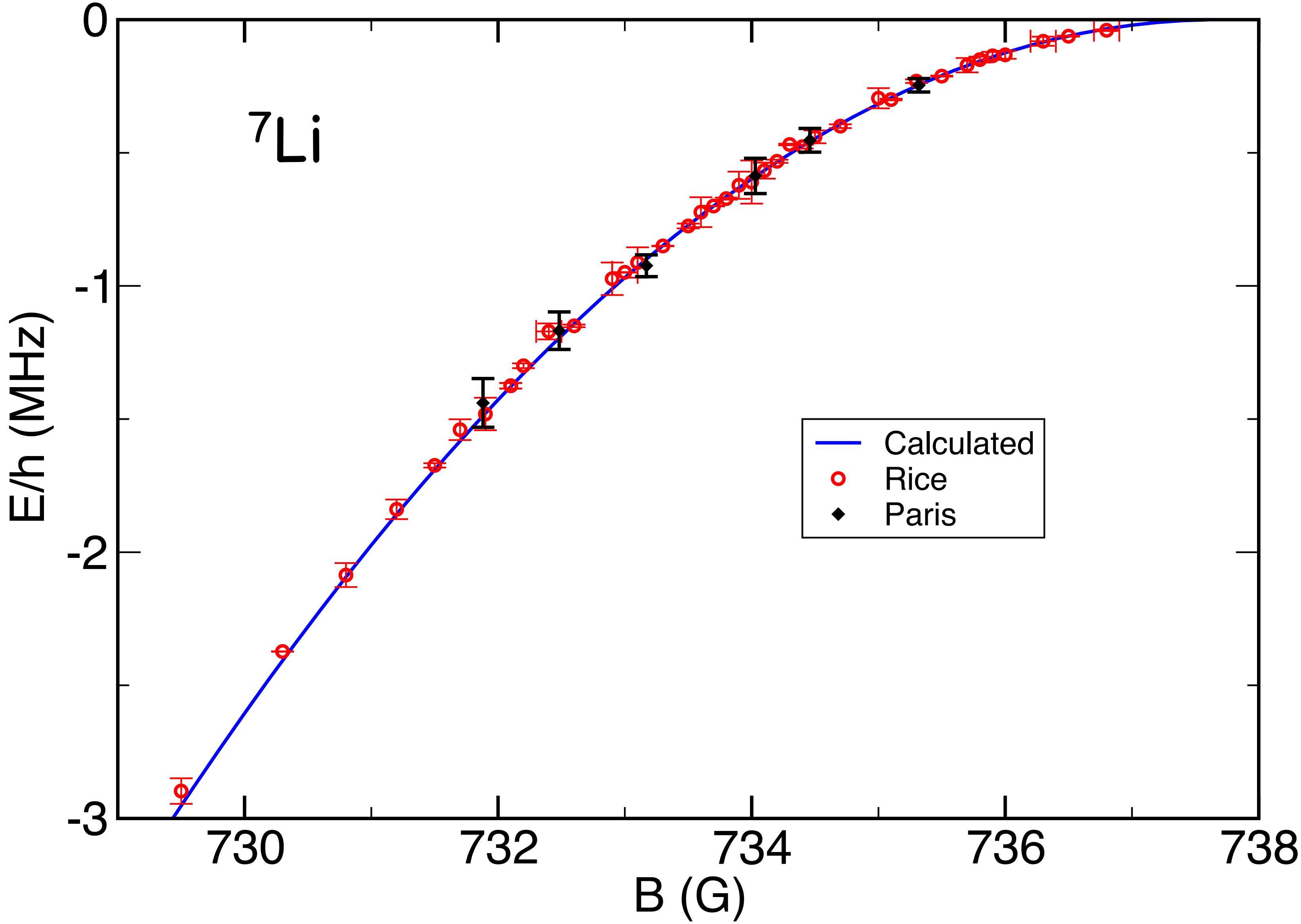
Closed channel dominated



$a_{bg} = -1400\ a_0$

$a_{bg} = -27\ a_0$

Coupled channels fit, PSJ & Hutson, Phys. Rev. A89, 052715(2014)



Universal energy: $$E^{\mathrm{U}} = -\frac{\hbar^2}{2\mu a^2}$$

Reduced E and length: $$\epsilon = E/\bar{E} \text{ and } r = a/\bar{a}$$

$$\epsilon^{\mathrm{U}} = -\frac{1}{r^2}$$

$$\epsilon^{\mathrm{U}} r^2 = -1$$

$$\epsilon^{\mathrm{U}} = -\frac{1}{r^2}$$

Universal

$$\epsilon^{\mathrm{GF}} = -\frac{1}{(r-1)^2}$$

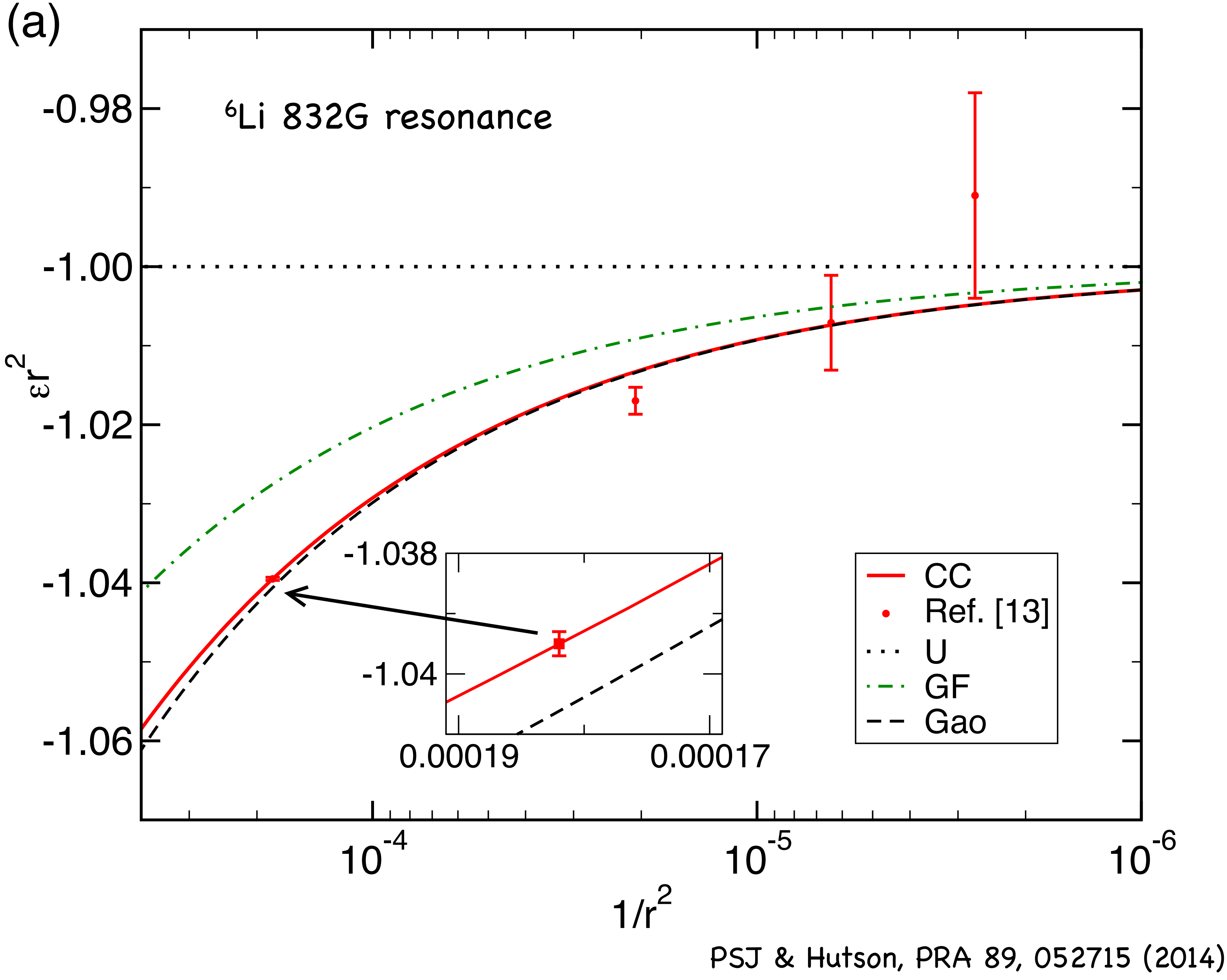
Gribakin and Flambaum, PRA 48, 546 (1993)

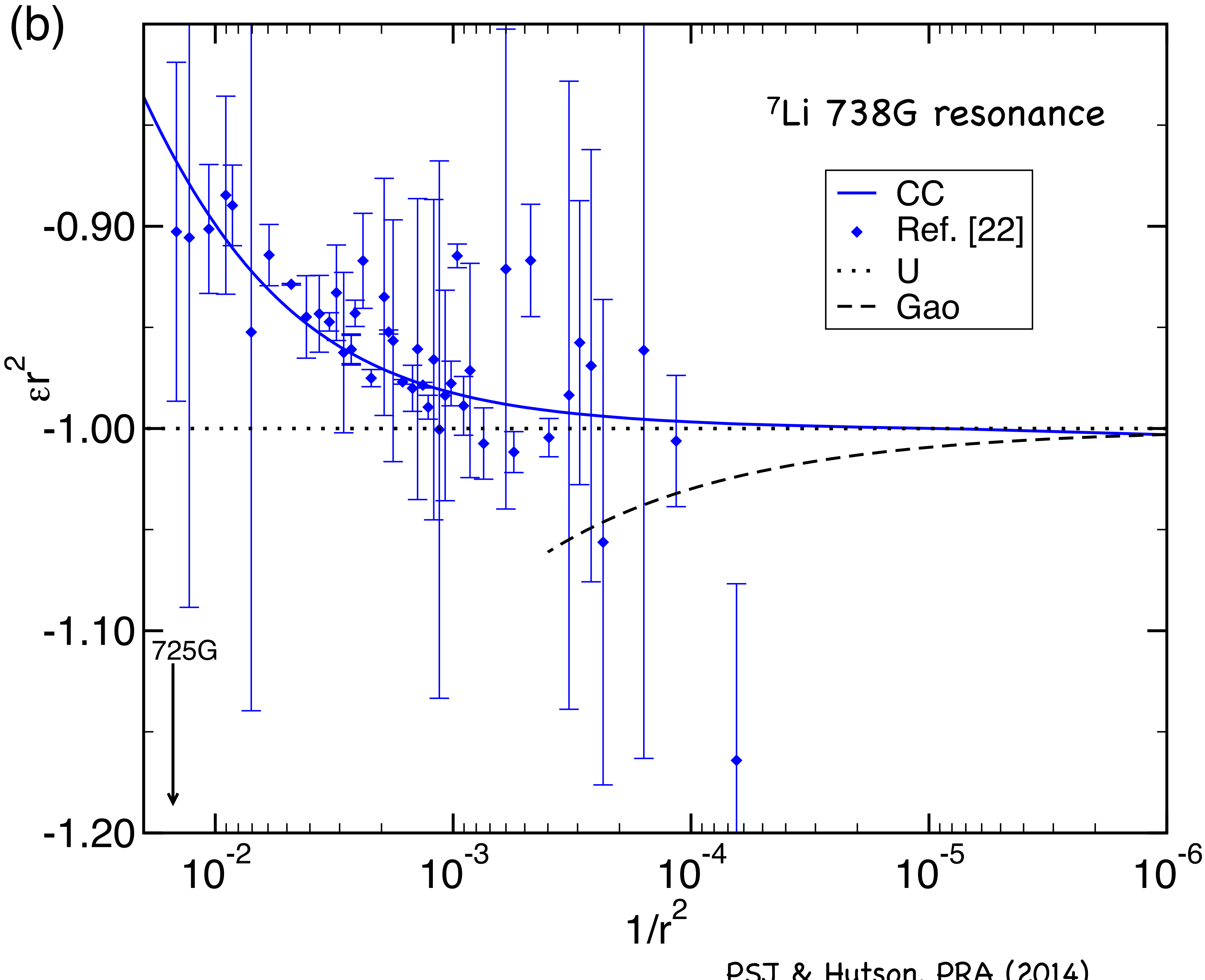
$$\epsilon^{\mathrm{G}} = \epsilon^{\mathrm{GF}} \left(1 + \frac{g_1}{r-1} + \frac{g_2}{(r-1)^2} \right)$$

$$g_1 = \Gamma(\frac{1}{4})^4/(6\pi^2) - 2 \approx 0.9179$$

$$g_2 = (5/4)g_1^2 - 2 \approx -0.9468$$

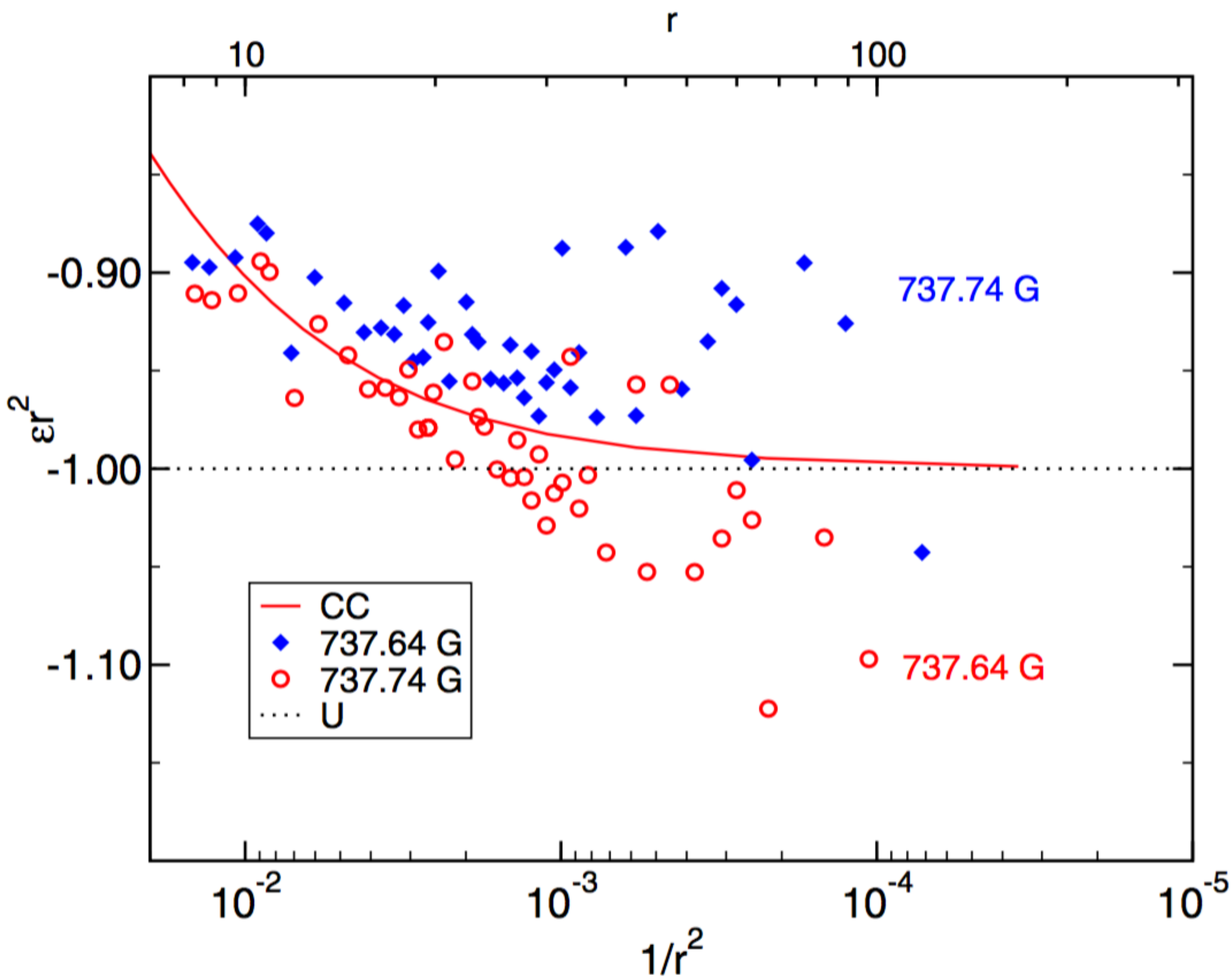
Gao, J. Phys. B 37, 4273 (2004)


(a)
6Li 832G resonance
-0.98
-1.00
-1.02
-1.04
-1.06
εr2
10-4
10-5
10-6
1/r2
-1.038
-1.04
0.00019
0.00017
CC
Ref. [13]
U
GF
Gao
PSJ & Hutson, PRA 89, 052715 (2014)



(b)
^{7}Li 738G resonance
CC
Ref. [22]
U
Gao
-0.90
-1.00
-1.10
-1.20
εr^2
725G
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
1/r^2
PSJ & Hutson, PRA (2014)

If a different B_0 is used in a(B) expression



PSJ & Hutson, PRA (2014)

“Quantum Defect” Theory
(QDT)

Quantum defect theory (QDT)

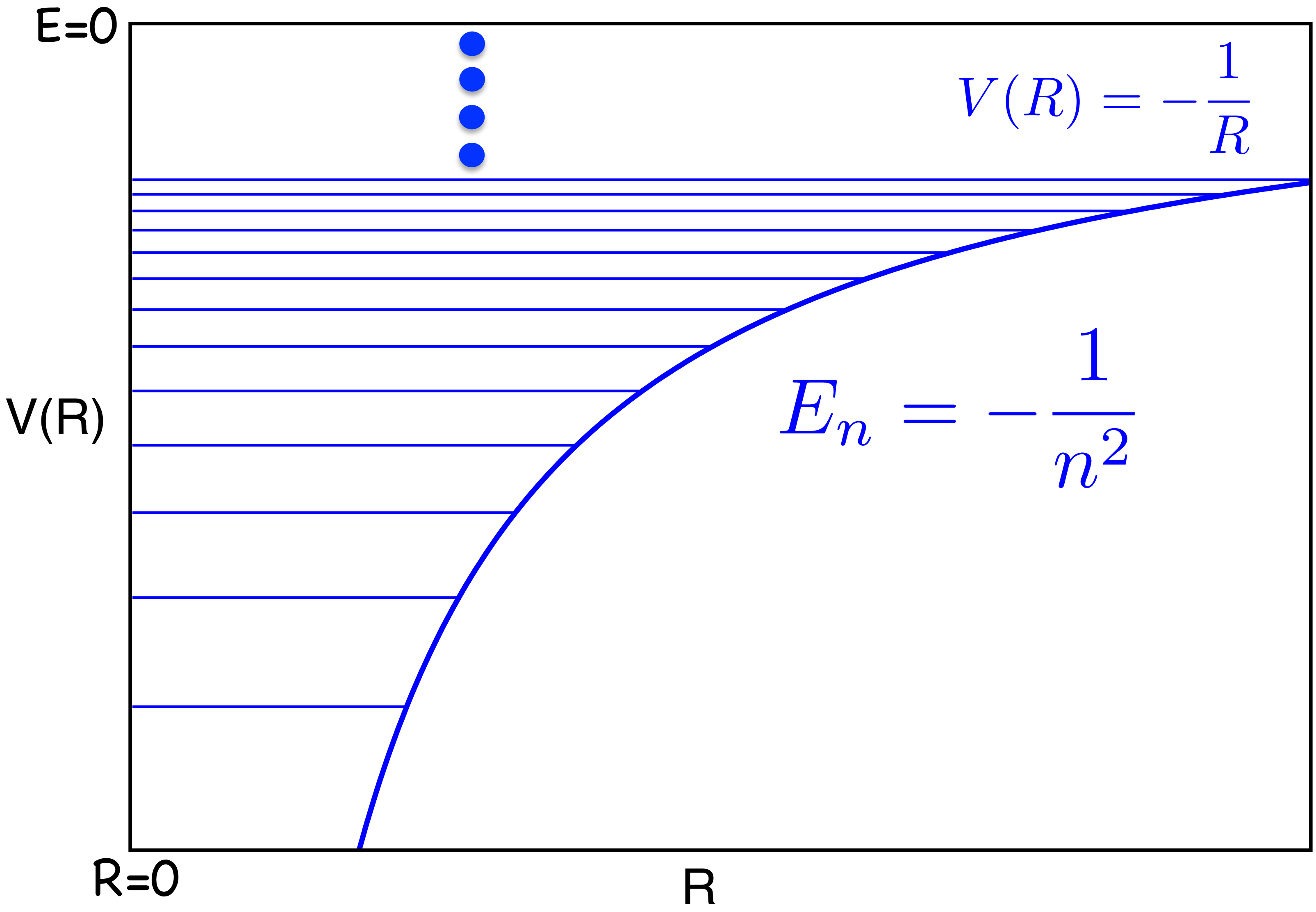
1. Pick a reference problem we can solve
   Classic example: Coulomb potential, H-like atom
   Here van der Waals potential, B. Gao, 1998-2009

2. Parameterize dynamics by a few "physical" parameters
   subject to experimental fitting
   and theoretical interpretation
      scattering length
      resonance strength
      reaction probability

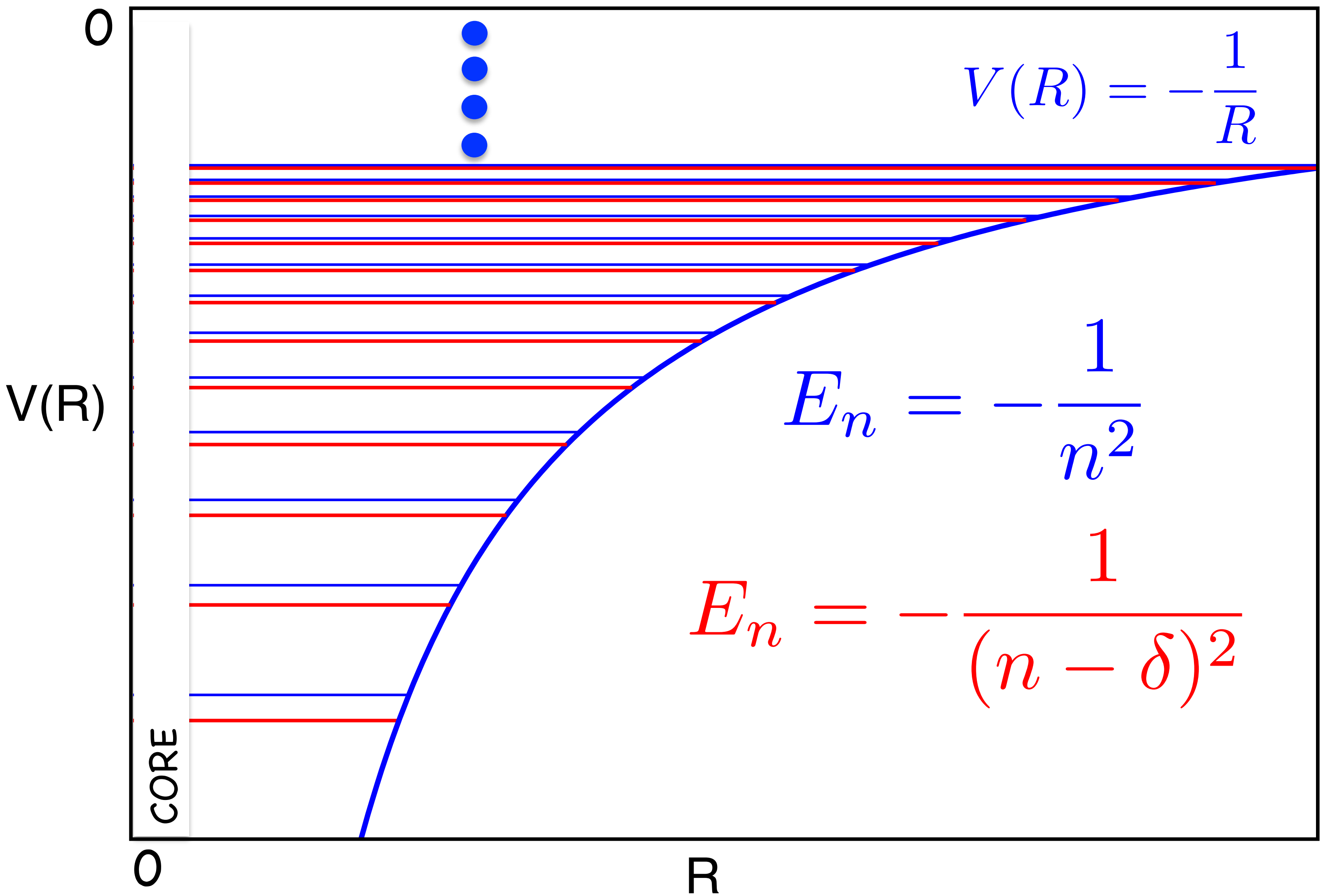
3. Take advantage of separation of energy, length scales
   Preparation, control: $E/h \approx$ kHz
   Short range (chemical): > THz

4. Use methods of QDT for
   bound and scattering states, resonances, cross sections, etc.

H atom



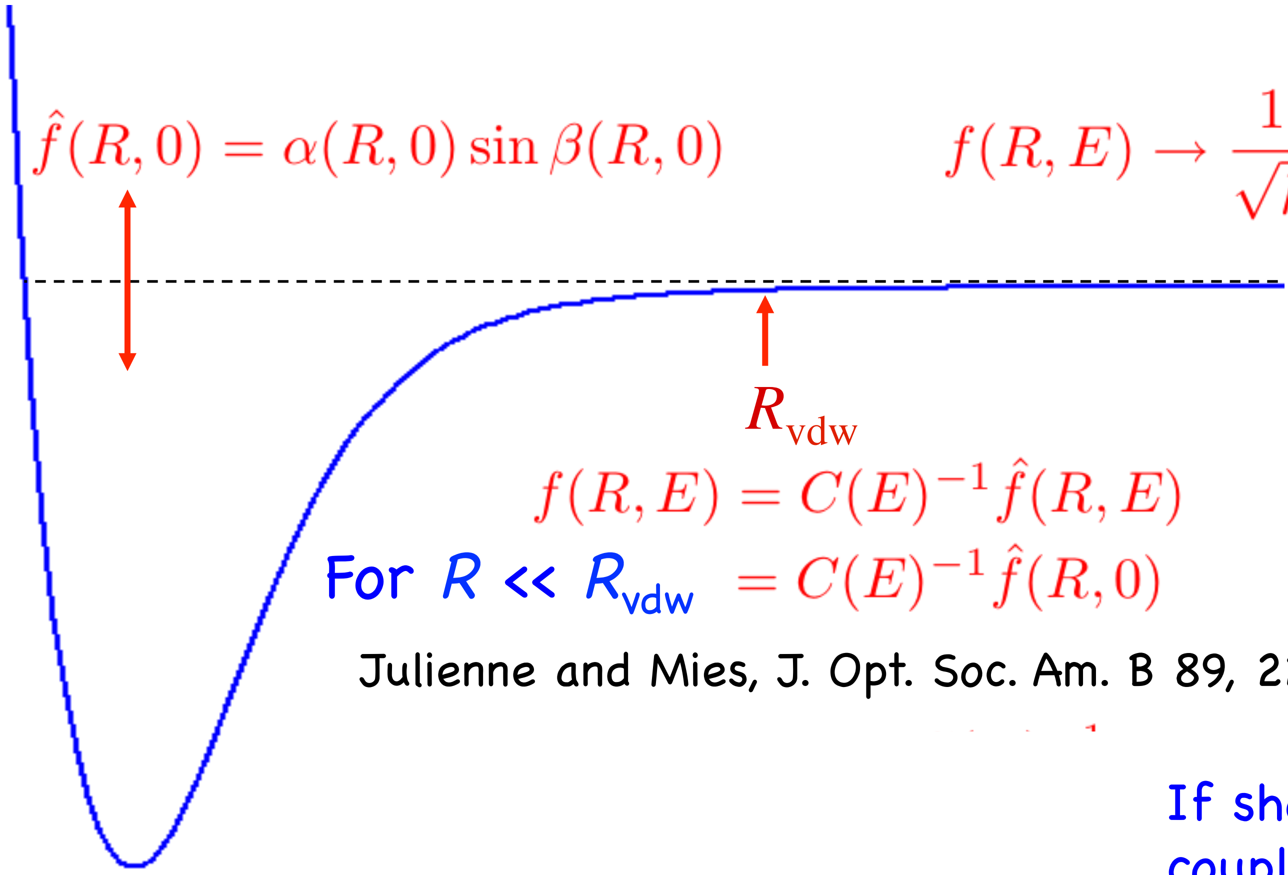
Multi-electron atom



Semiclassical considerations continued

$$\hat{f}(R,0) = \alpha(R,0)\sin\beta(R,0)$$

$$f(R,E) \to \frac{1}{\sqrt{k}}\sin(kR + \eta(E))$$



$$f(R,E) = C(E)^{-1}\hat{f}(R,E)$$

For R << R_{vdw} $= C(E)^{-1}\hat{f}(R,0)$

Julienne and Mies, J. Opt. Soc. Am. B 89, 2257 (1989)

If short-range coupling << R_{vdw}

$\alpha(R,E) =$

Consequently: $V_n(E) = \langle n|H|E\rangle = C(E)^{-1}\hat{V}_n$

$\beta(R,E) =$

Van der Waals Quantum Defect Theory (QDT)

PSJ and B. Gao, in Vol 869 AIP Conf. Proc., 261–268 (2006), arXiv:physics/0609013v1
Chin et al, RMP (2010)

$$\eta(E,B) = \eta_{\mathrm{bg}}(E) + \eta_{\mathrm{res}}(E,B)$$

$$\eta_{\mathrm{res}}(E,B) = -\tan^{-1}\frac{\frac{1}{2}\Gamma(E)}{E - \mu_{\mathrm{diff}}(B - B_{\mathrm{c}}) + \delta E_{\mathrm{c}}(E)}$$

$$\Gamma(E) = \frac{1}{2}\bar{\Gamma}C(E)^{-2}$$

$$\delta E_{\mathrm{c}}(E) = \frac{1}{2}\bar{\Gamma}\tan\lambda(E)$$

$\bar{\Gamma}$ = short range strength independent of E, B

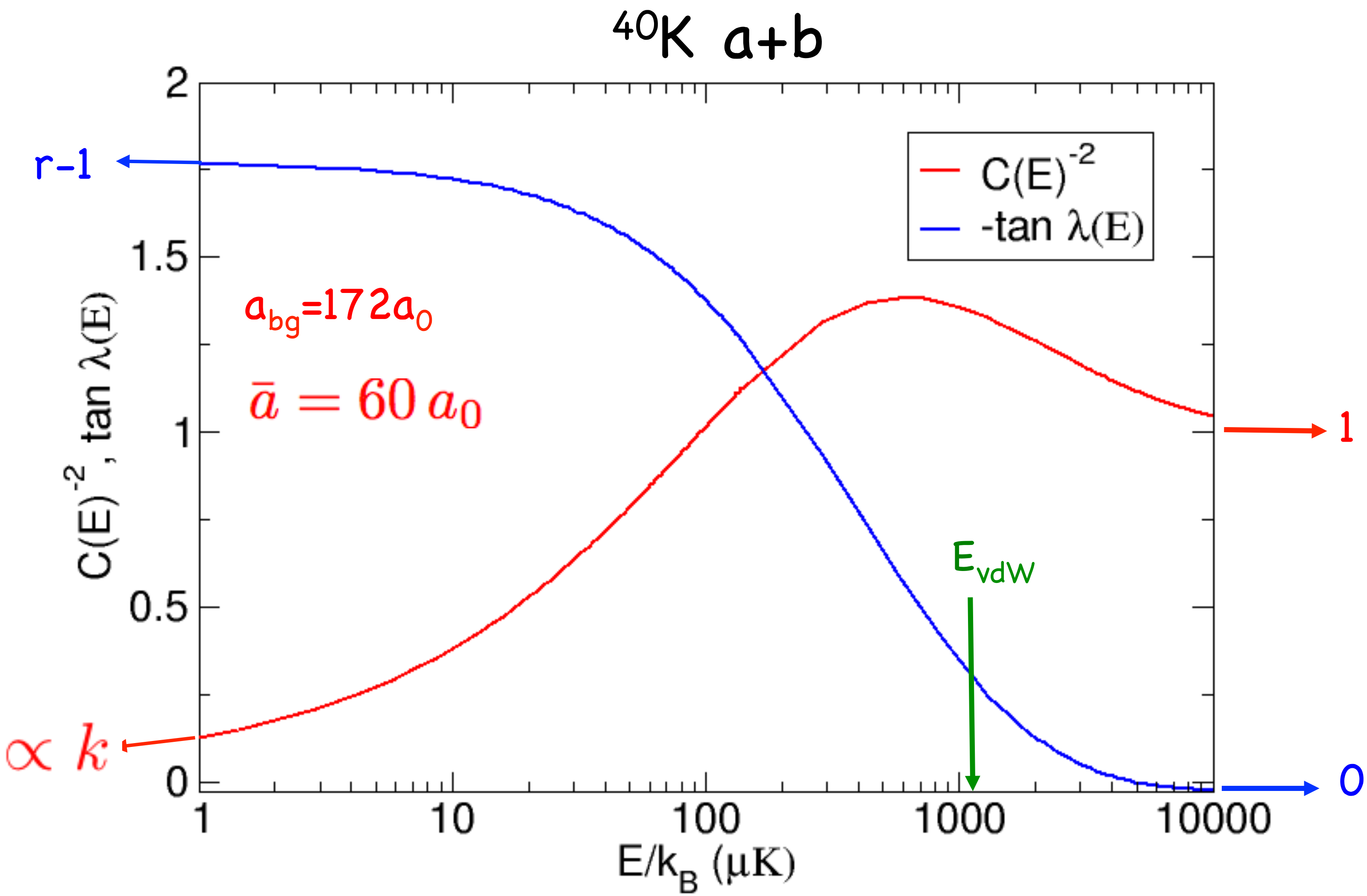
$\eta_{bg}(E)$, $C(E)^{-2}$, $\tan\lambda(E)$ are analytic QDT functions of the background channel, given C_6 and a_{bg}

$$\lim_{E\to 0}\eta(E) = -ka_{\mathrm{bg}}$$

$$\lim_{E\to 0} C(E)^{-2} = k\bar{a}\left(1 + \left(1 - \frac{a_{\mathrm{bg}}}{\bar{a}}\right)^2\right)$$

$$\lim_{E\to 0}\tan\lambda(E) = 1 - \frac{a_{\mathrm{bg}}}{\bar{a}}$$

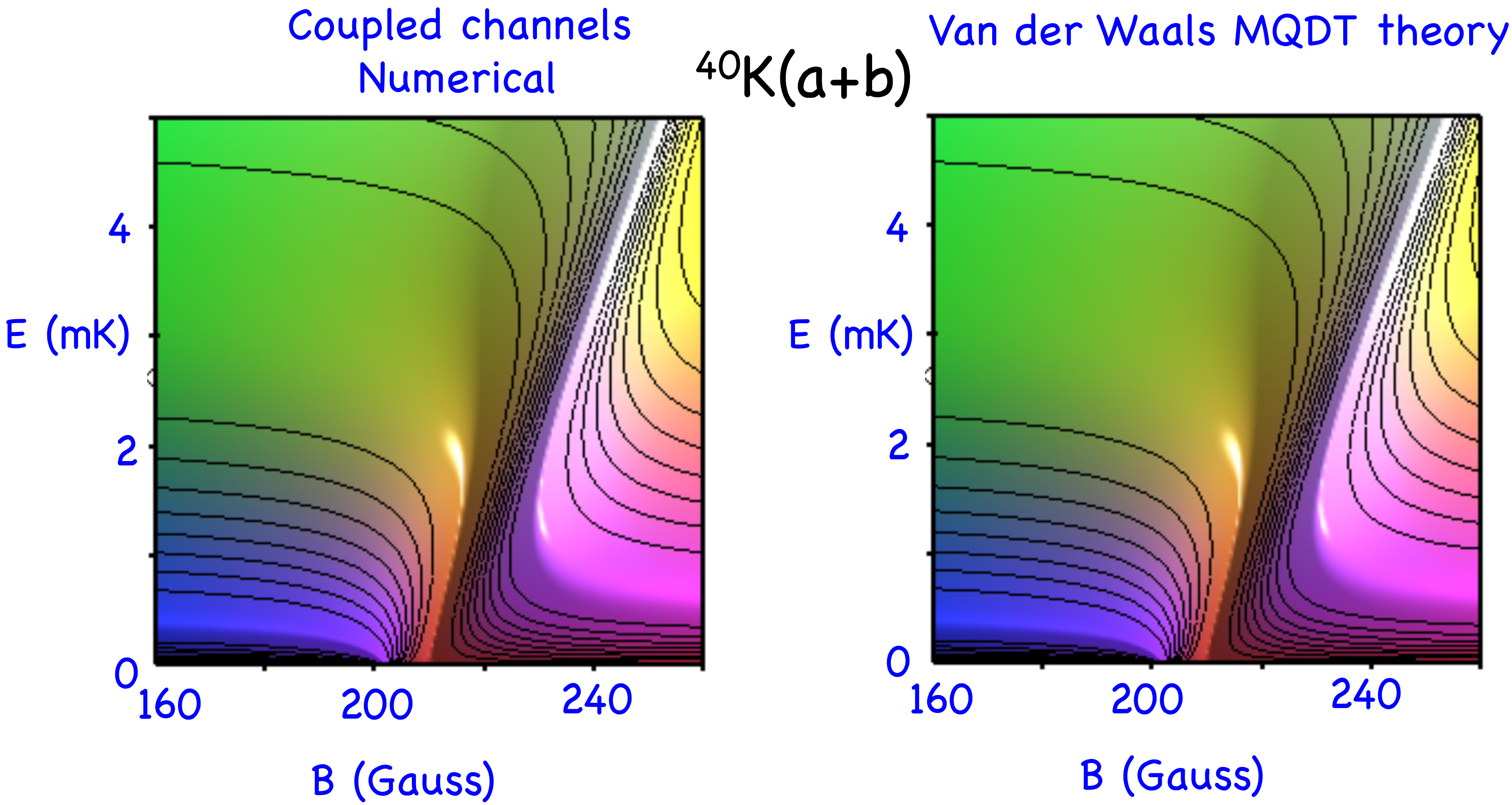
MQDT Functions

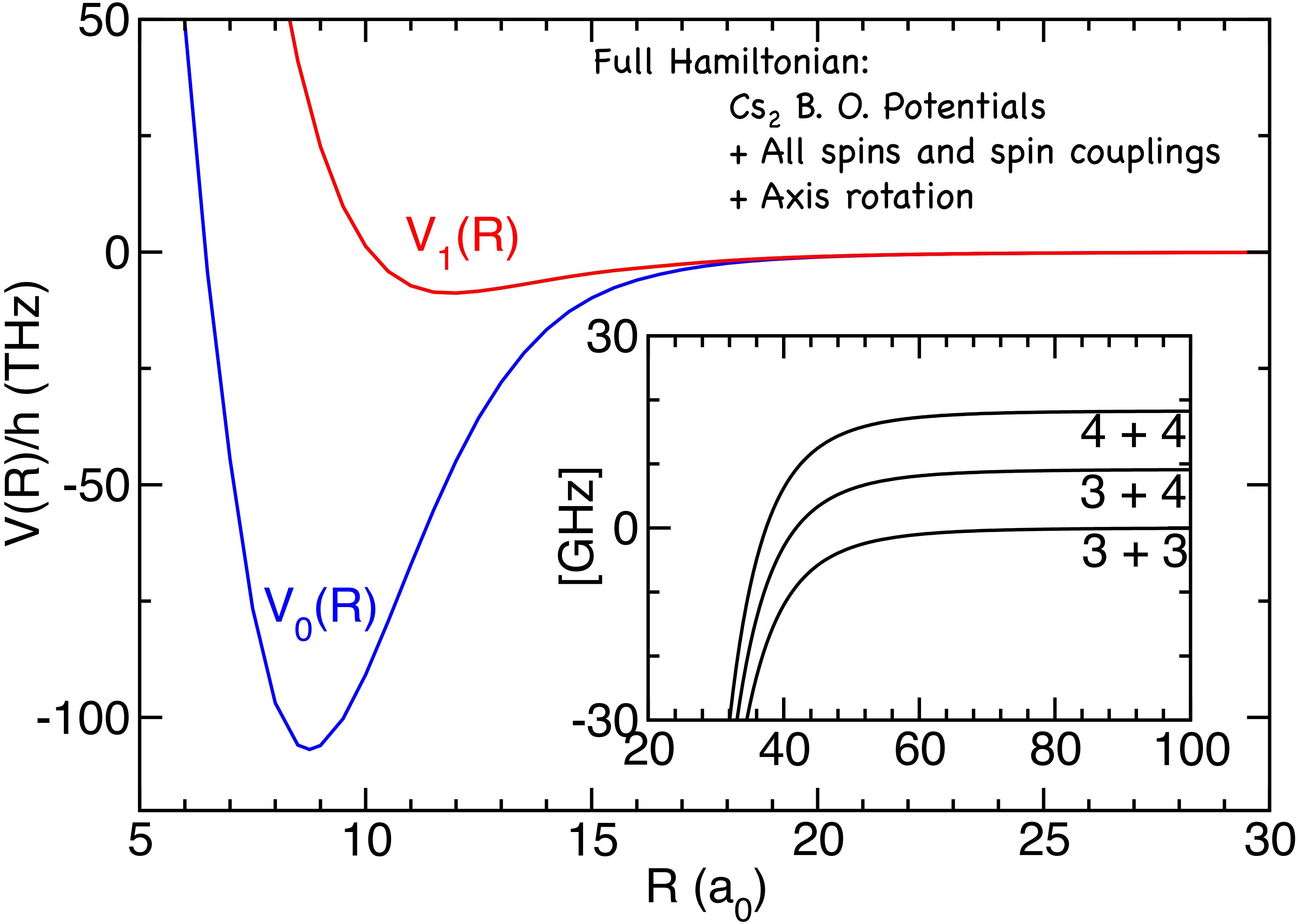


PSJ and B. Gao, in Vol 869 AIP Conf. Proc., 261–268 (2006), arXiv:physics/0609013v1

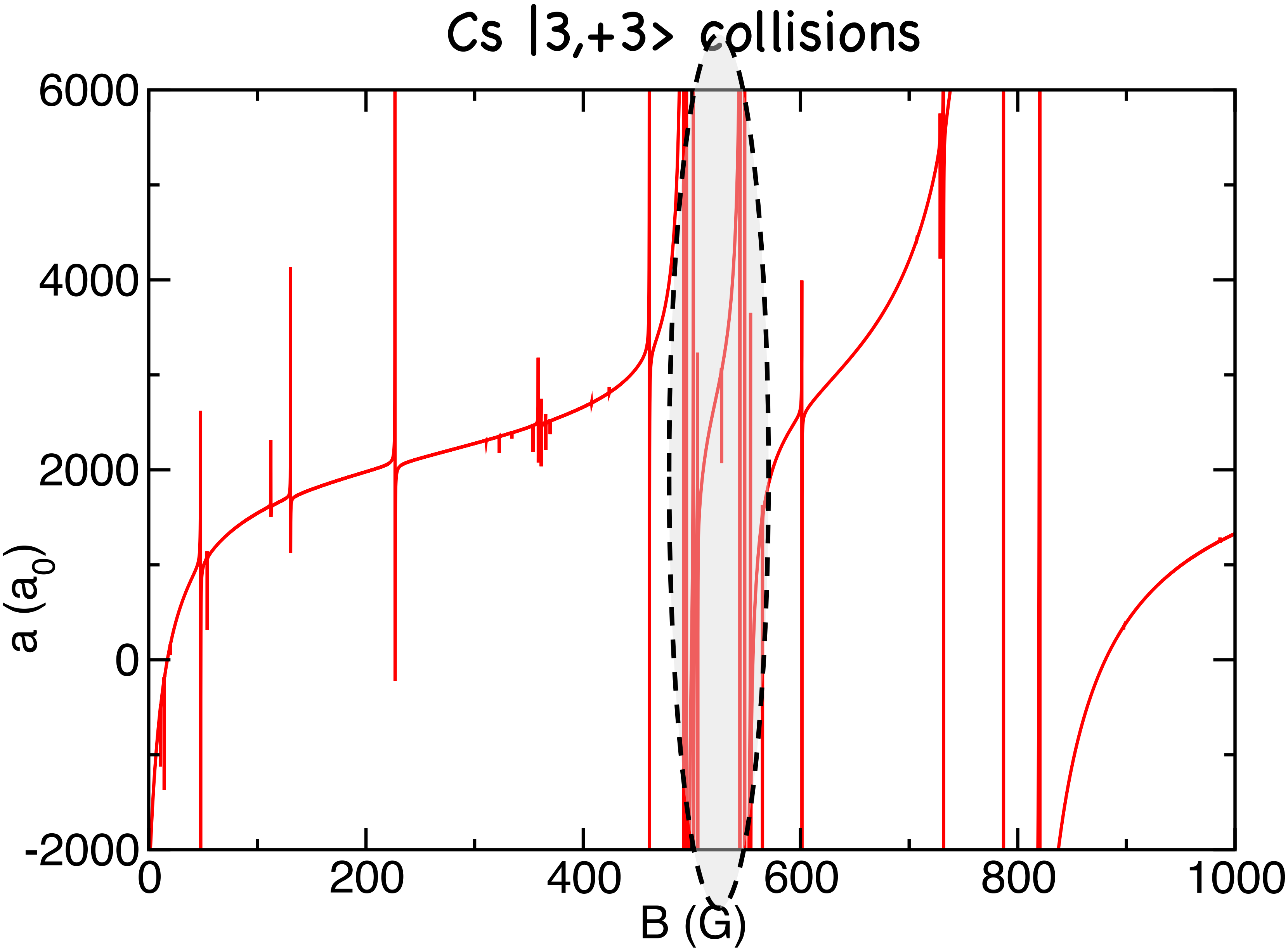
$$\eta(E,B) = \eta_{\rm bg}(E) + \eta_{\rm res}(E,B)$$

$$\sin^2 \eta(E,B)$$



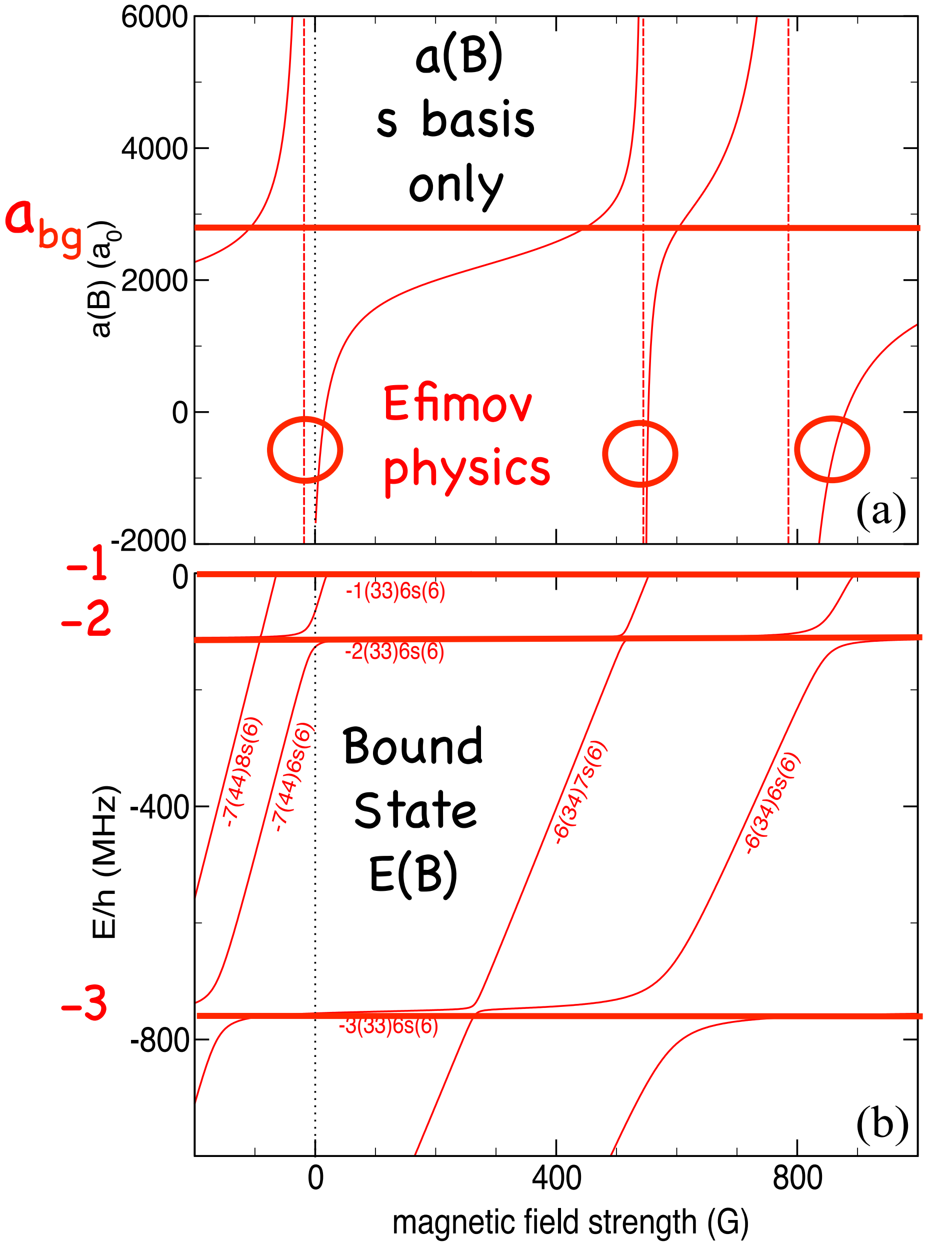


Full Hamiltonian:
Cs2 B. O. Potentials
+ All spins and spin couplings
+ Axis rotation
V1(R)
V0(R)
V(R)/h (THz)
R (a0)
[GHz]
4 + 4
3 + 4
3 + 3



Berninger et al, PRA 87, 032517 (2013)

Cs+Cs Innsbruck/Hutson/PSJ Phys. Rev. A 87, 032517 (2013)

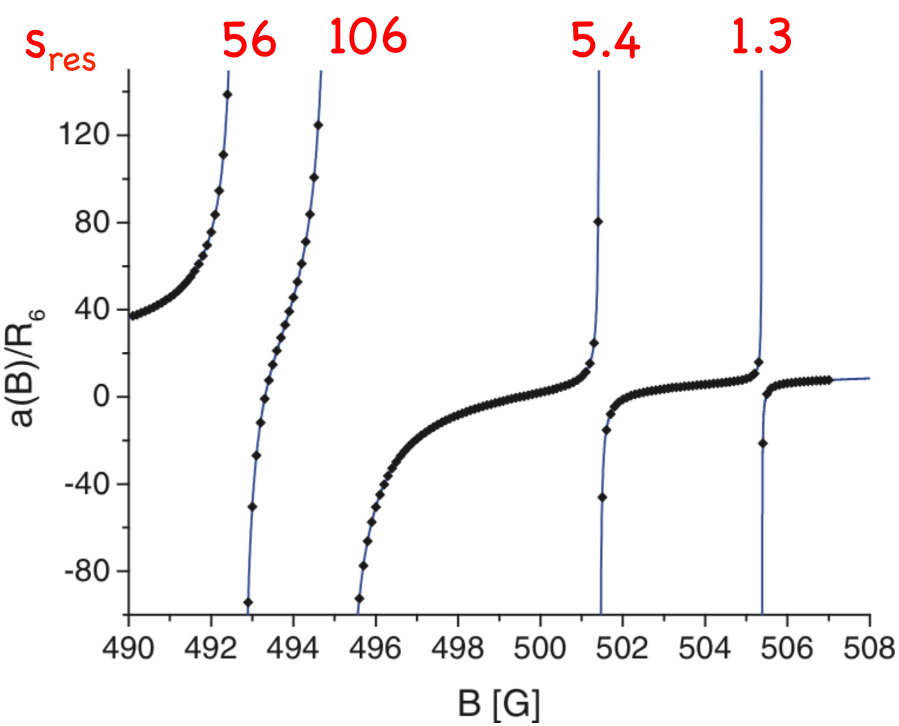


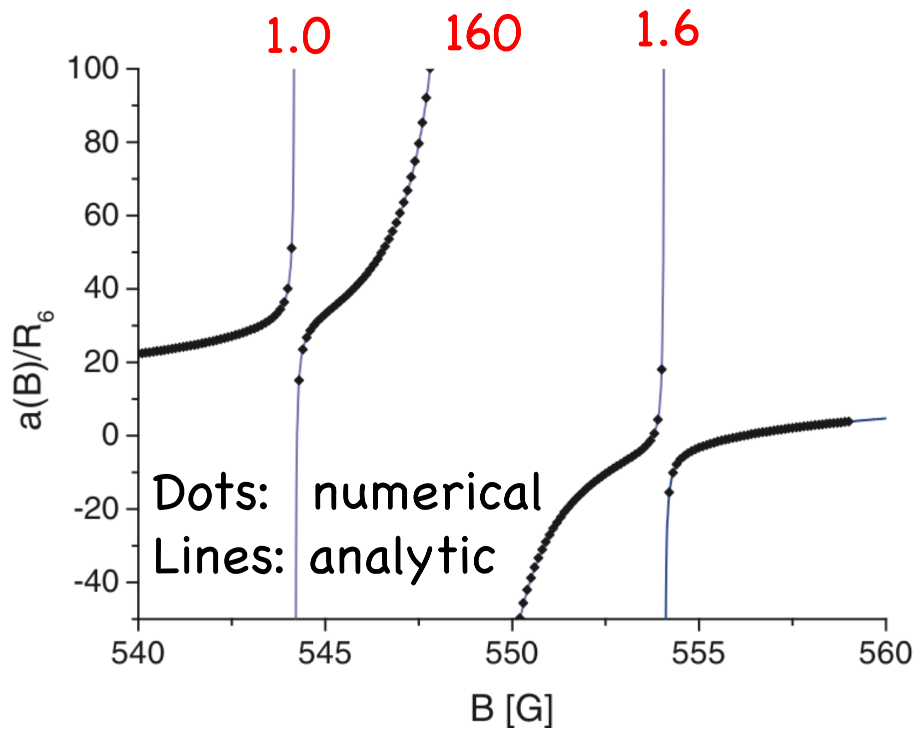
PHYSICAL REVIEW A **88**, 052701 (2013)

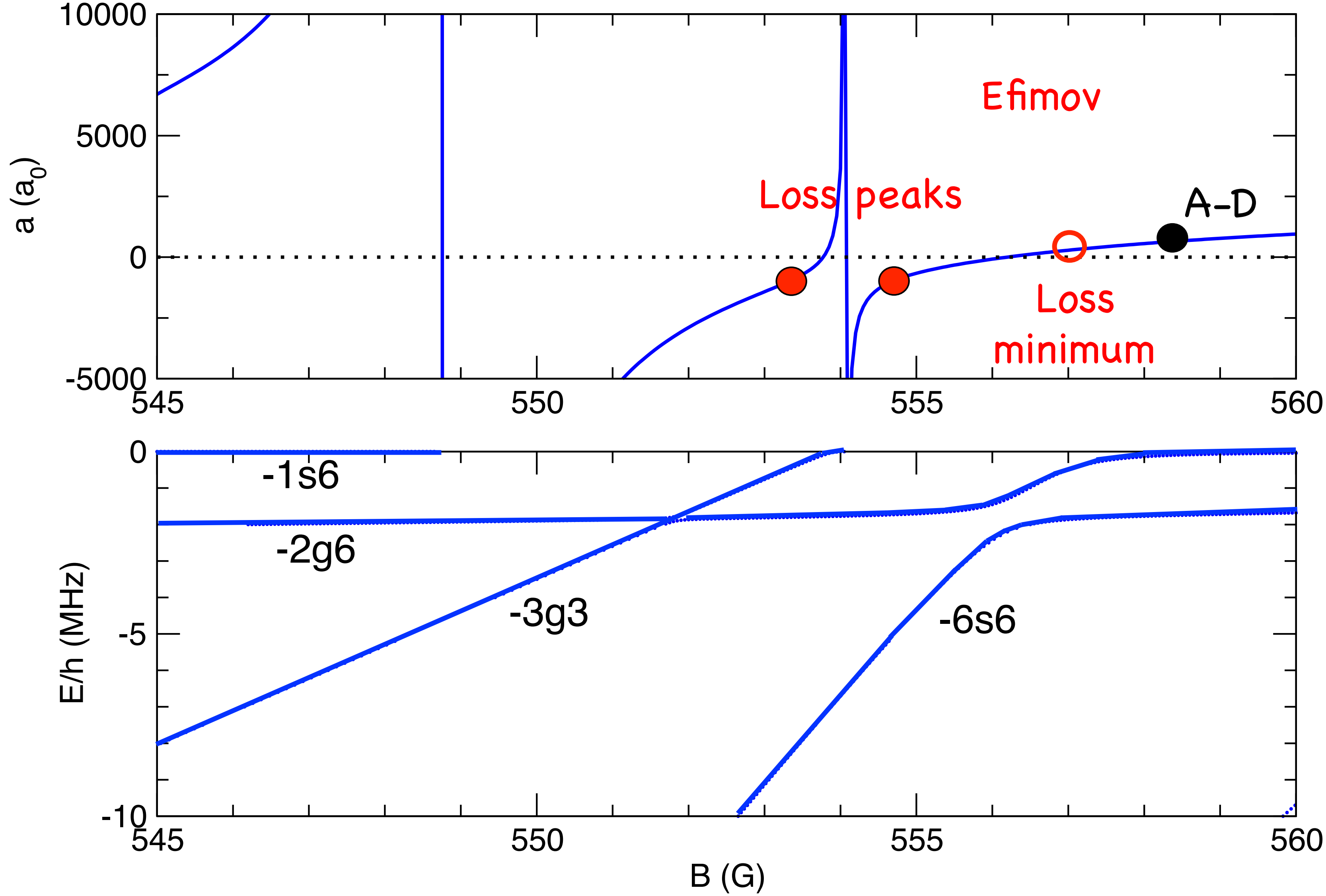
Analytical model of overlapping Feshbach resonances

Krzysztof Jachymski[1] and Paul S. Julienne[2]

$$a(B) = a_{\mathrm{bg}} - \sum_{i=1}^{N} P_i(B) \qquad a(B) = a_{bg} \prod_{i=1}^{N} \left(1 - \frac{\tilde{\Delta}_i}{B - B_i^{\mathrm{res}}}\right)$$







Coupled channels model from Berninger, et al, Phys. Rev. A 87, 032517 (2013)
Overlapping resonance treatment: Jachymski, PSJ, PRA 88, 052701(2013)
3-body and Efimov: Wang and PSJ, Nat. Phys. 10, 768 (2014)

$$a(B) = a_{\rm bg} - \sum_{i=1}^{N} P_i(B)$$

$$P_i(B) = \frac{\frac{1}{2}\frac{\hat{\Gamma}_i}{\delta\mu_i} C^{-2}(E)/k}{B - B_i - \frac{1}{2}\tan\lambda(E)\left(\frac{\hat{\Gamma}_i}{\delta\mu_i} - \sum_{j\neq i}\frac{B-B_i}{B-B_j}\frac{\hat{\Gamma}_j}{\delta\mu_j}\right)}$$

"global" background

Short-range coupling

$$a(B) = a_{bg}\left(1 - \sum_i \frac{\Delta_i}{B - B_i - \delta B_i - \sum_{j\neq i}\frac{B-B_i}{B-B_j}\delta B_j}\right)$$

Interaction shift

$$a(B) = a_{bg}\prod_{i=1}^{N}\left(1 - \frac{\tilde{\Delta}_i}{B - B_i^{\rm res}}\right)$$

$$a(\mathrm{near}\ B_i) = \tilde{a}_{bg,i}\left(1 - \frac{\tilde{\Delta}_i}{B - B_i^{\rm res}}\right)$$

$$\tilde{a}_{bg,i} = a_{bg}\prod_{j\neq i}^{N}\left(1 - \frac{\tilde{\Delta}_j}{B_i^{\rm res} - B_j^{\rm res}}\right)$$

"local" background

The End

Effective range expansion

$$k \cot \eta(E) = -\frac{1}{a} + \frac{1}{2} r_0 k^2$$

$$r_0 = 2.918\bar{a}\frac{\bar{a}^2 + (a - \bar{a})^2}{a^2}$$

Flambaum, Gribakin, and Harabati, Phys. Rev. A 59, 1998 (1999)

Gao, Phys. Rev. A 62, 050702 (2000)

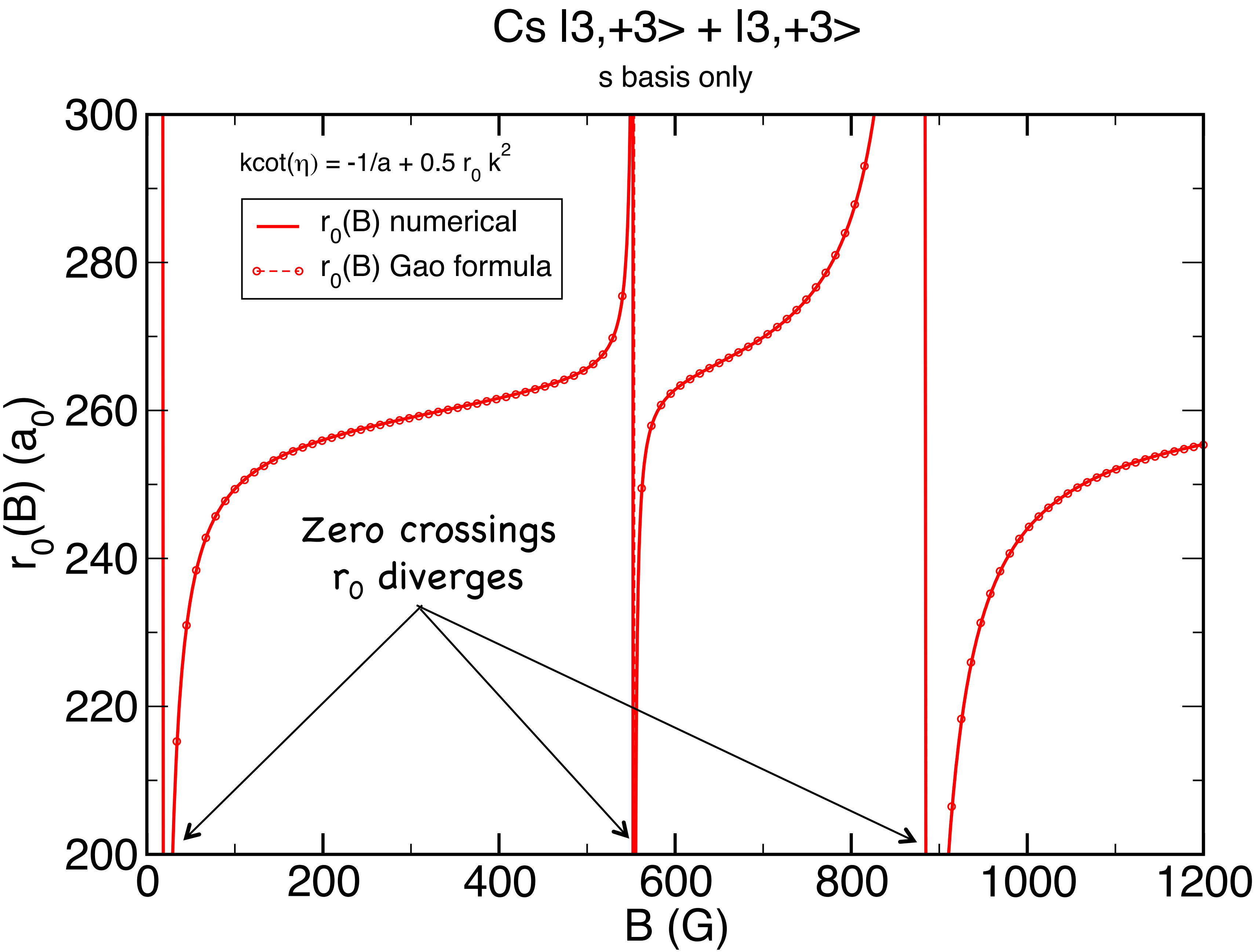
$$\bar{a} = \frac{2\pi}{\Gamma(\frac{1}{4})^2}\left(\frac{2\mu C_6}{\hbar^2}\right)^{\frac{1}{4}}$$

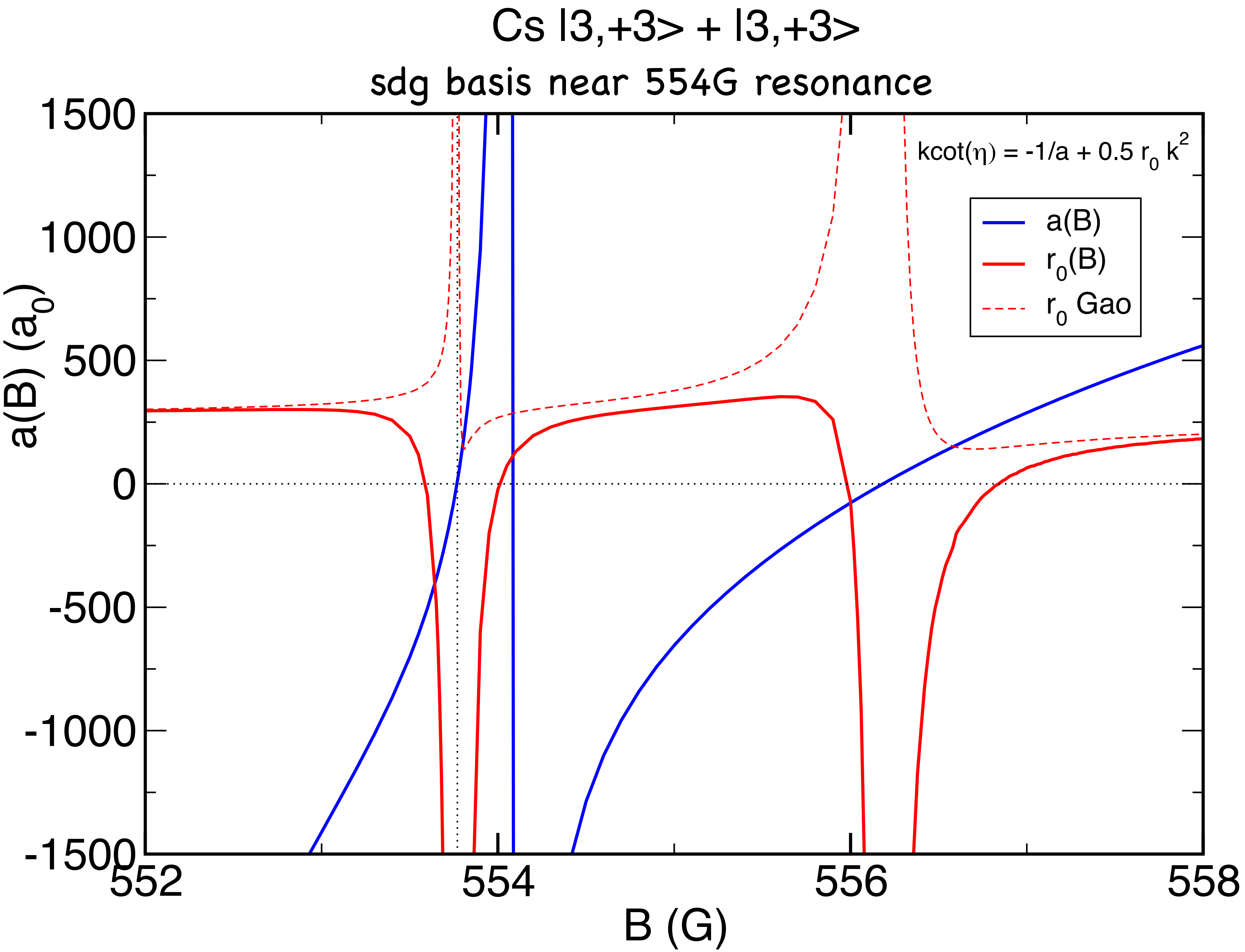
See also:

Blackley, Hutson, PSJ, Phys. Rev. A 89, 042701 (2014)

Shotan, Machtey, Kokkelmans, Khaykovich, Phys. Rev. Lett. 113, 053202 (2014)

Werner and Castin, Phys. Rev. A 86, 013626 (2012), Eq. 185

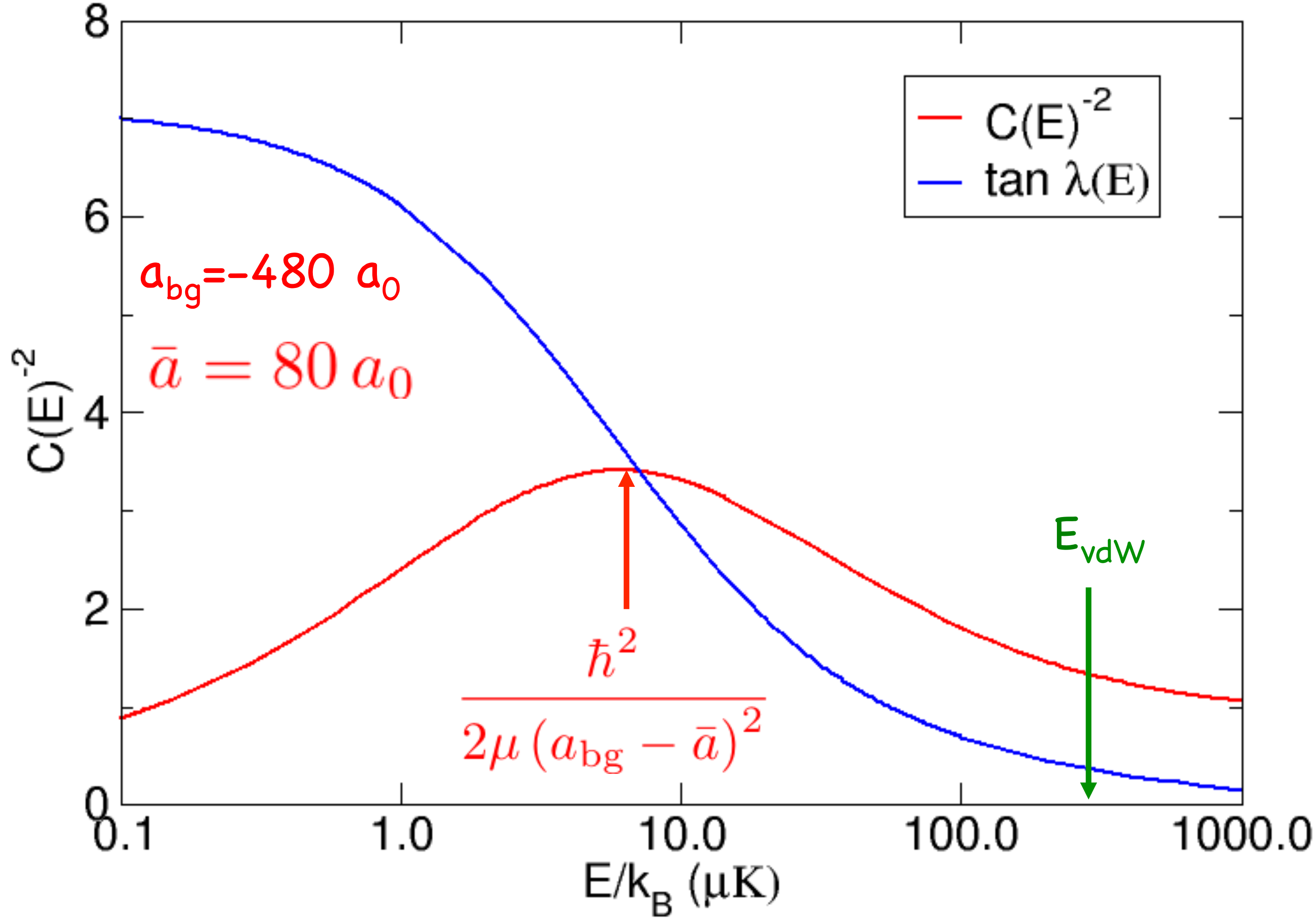

Cs |3,+3> + |3,+3>
s basis only
kcot(η) = -1/a + 0.5 r0 k2
r0(B) numerical
r0(B) Gao formula
Zero crossings
r0 diverges
r0(B) (a0)
B (G)
300
280
260
240
220
200
0
200
400
600
800
1000
1200



Cs |3,+3> + |3,+3>
sdg basis near 554G resonance
kcot(η) = -1/a + 0.5 r0 k2
a(B)
r0(B)
r0 Gao
a(B) (a0)
1500
1000
500
0
-500
-1000
-1500
552
554
556
558
B (G)

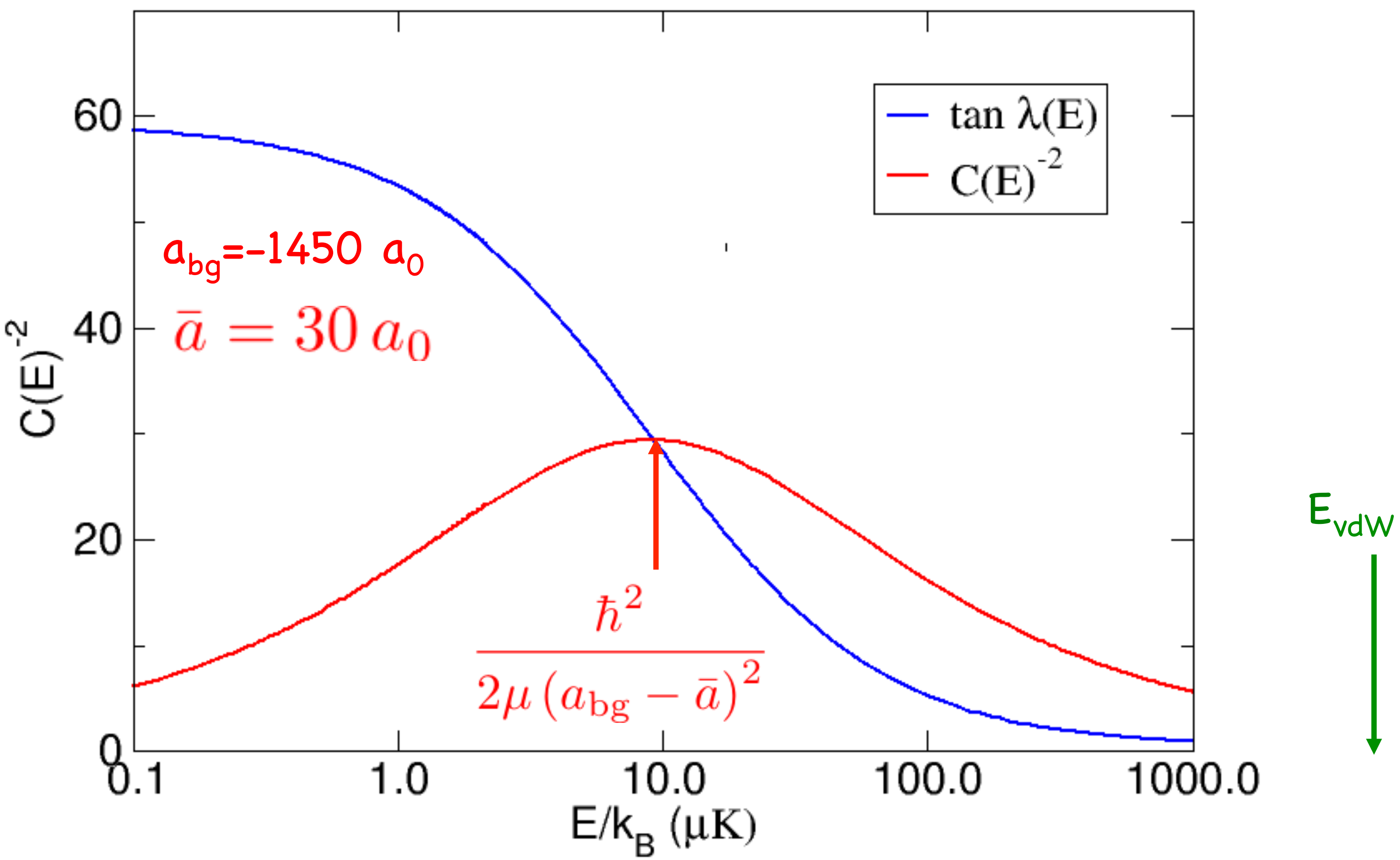
MQDT Functions

^{85}Rb e+e



MQDT Functions

^{6}Li a+b



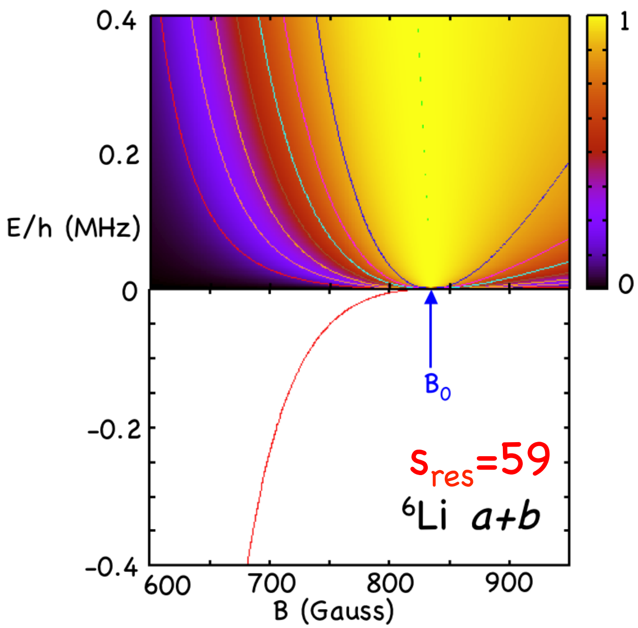
Open and closed channel dominated resonances

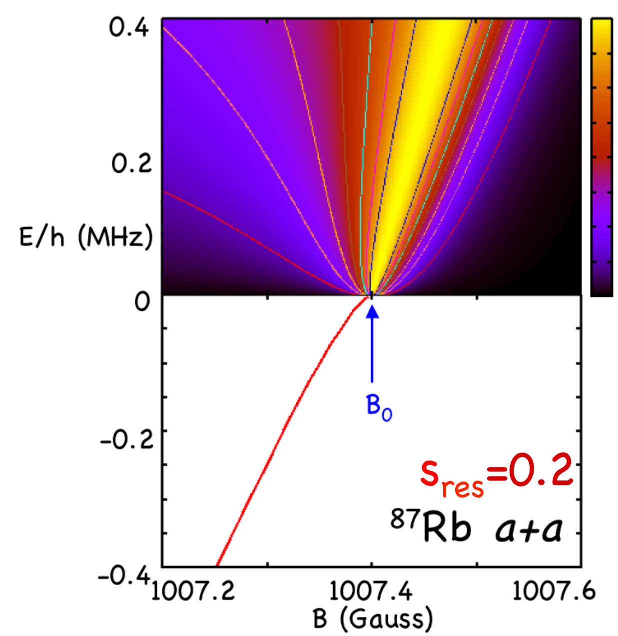
$$s_{\mathrm{res}} = \frac{a_{\mathrm{bg}}}{\bar{a}} \frac{\mu_{\mathrm{diff}} \Delta}{\bar{E}}$$

Köhler, et al, Rev. Mod. Phys. (2006)
Chin, et al, Rev. Mod. Phys. (2010)

Open: $s_{res} \gg 1$
universal, $Z \ll 1$
over $\approx \Delta$
$\Gamma(E) \gg E$ when $E < E_{vdw}$

Closed: $s_{res} \ll 1$
Not universal, $Z \rightarrow 1$
over most of Δ
$\Gamma(E) \ll E$ when $E < E_{vdw}$





^{6}Li: Simonucci, Pieri and Strinati, Europhys. Lett. 69, 713 (2005)