Can Emergent Spacetime Kill the Multiverse Hypothesis?

Hyun Seok Yang

KIAS

Joint Winter Conference on Particle, String and Cosmology
High1, January 25-31, 2015

In this talk

- I want to emphasize that NC spacetime is much more radical and mysterious than we thought.

- I will give you an overall picture why NC spacetime implies emergent spacetime. The emergent spacetime opens a new prospect which may cripple all the rationales to introduce the multiverse hypothesis.

- Every mathematical details will be addressed in my coming paper "Emergent Spacetime and Cosmic Inflation" which will appear pretty soon.

Who understands quantum mechanics?

1. Quantum mechanics: mechanics on NC phase space $[x, p] = i\hbar$.
   The concept of (phase) space is doomed. The (phase) space is replaced by a state in $\mathcal{H}$ and dynamical variables become operators acting on $\mathcal{H}$.

2. Every point in $\mathbb{R}^3$ is unitarily equivalent because translation in $\mathbb{R}^3$ is generated by an inner automorphism of $\mathcal{A}_\hbar$, i.e.,
   $f(x+a) = U(a)f(x)U(a)^\dagger$ where $U(a) = e^{ipa/\hbar}$.
   Thus the NC *space* is a misnomer. There is no space but an algebra $\mathcal{A}_\hbar$ only.

3. The Hilbert space $\mathcal{H}$ in most cases is separable and so has a countable basis.
   Thus dynamical variables are represented by $N \times N$ matrices in $End(\mathcal{H})$ where $N = \dim(\mathcal{H}) \to \infty$. The dynamics is generated by inner derivations and any derivation of the matrix algebra is inner.

4. Since the Hilbert space $\mathcal{H}$ is a *complex linear vector space*, the superposition of two states can be allowed which brings about the interference of states.
   We are easily puzzled, e.g., by two-slit experiment or EPR experiment.

We understood NC spacetime too easily!

What about NC spacetime? : $[y^\mu, y^\nu] = i\theta^{\mu\nu}$.
Note that the mathematical structure of NC spacetime is essentially equivalent to NC phase space in quantum mechanics.

But NC spacetime is much more radical and mysterious than quantum mechanics.
I think next revolution after quantum mechanics may come from the NC spacetime.
Let me explain why.

1. NC spacetime admits a (dynamical) diffeomorphism symmetry. So gravity emerges from the NC spacetime: *Emergent gravity*.

2. NC spacetime implies *emergent spacetime*. A classical spacetime must be derived from a NC algebra $\mathcal{A}_\theta$.

3. Large N duality or gauge/gravity duality such as the AdS/CFT correspondence is an inevitable consequence of the NC spacetime.

Consider D=$(d+2n)$-dimensional NC $U(1)$ gauge theory on $\mathbb{R}_C^d \times \mathbb{R}_{NC}^{2n}$ whose coordinates are $X^M = (x^\mu, y^a)$, $M = 0,1,\cdots,D-1$, $\mu = 0,1,\cdots,d-1$, $a = 1,\cdots,2n$ where

$$[y^a, y^b] = i\theta^{ab}.$$

The D-dimensional $U(1)$ connections are split as

$$D_M(X) = \partial_M - iA_M(x,y) = \left(D_\mu, D_a\right)(x,y)$$

where

$$D_a\,(x,y) = -i\left(B_{ab}y^b + A_a(x,y)\right) \equiv -i\Phi_a(x,y). \qquad (1)$$

Using the matrix representation $\mathcal{A}_\theta \to \mathcal{A}_N$ by

$$\Xi(x,y) \mapsto \Xi(x) \in U(N),$$

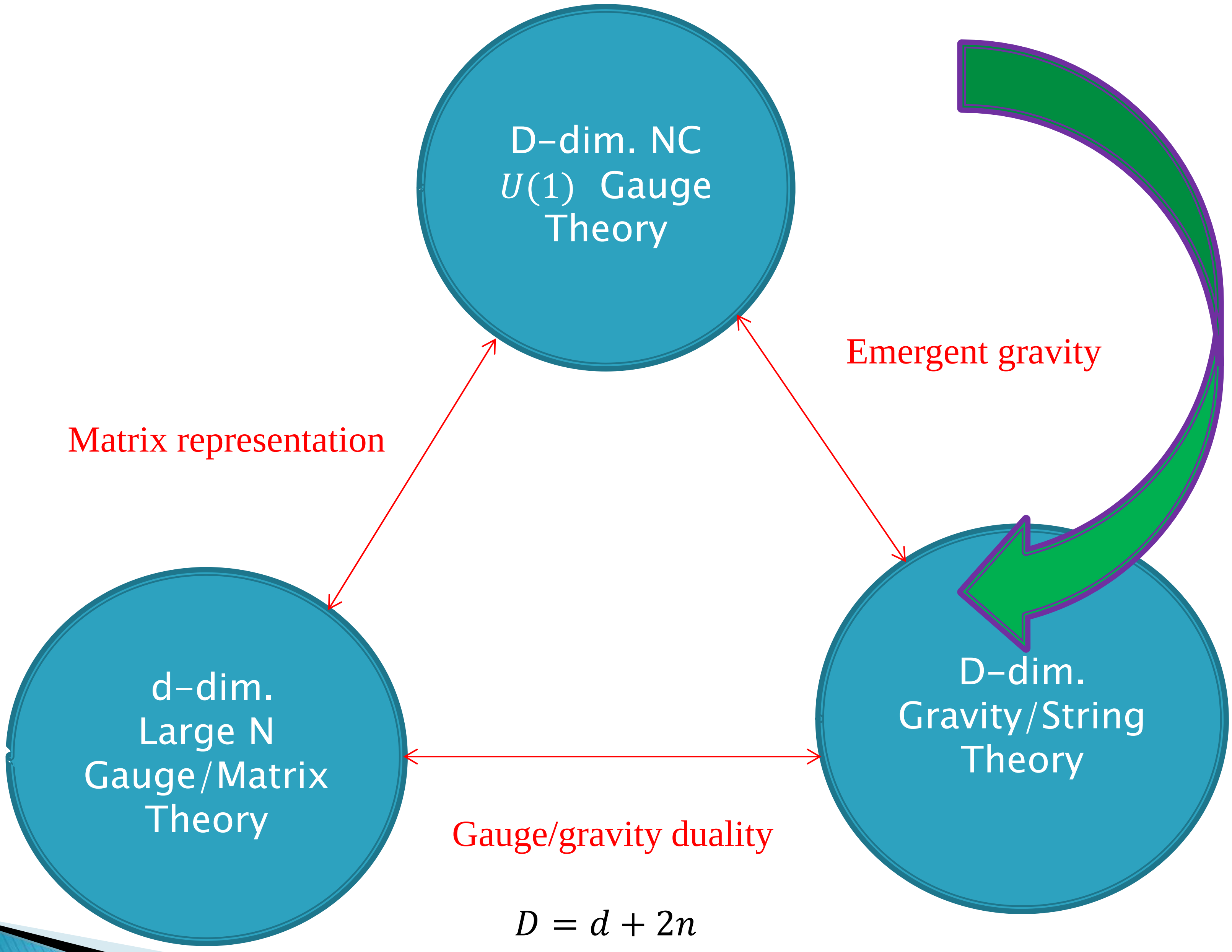
the D-dimensional NC $U(1)$ gauge theory is exactly mapped to the d-dimenisonal $U(N)$ Yang-Mills theory

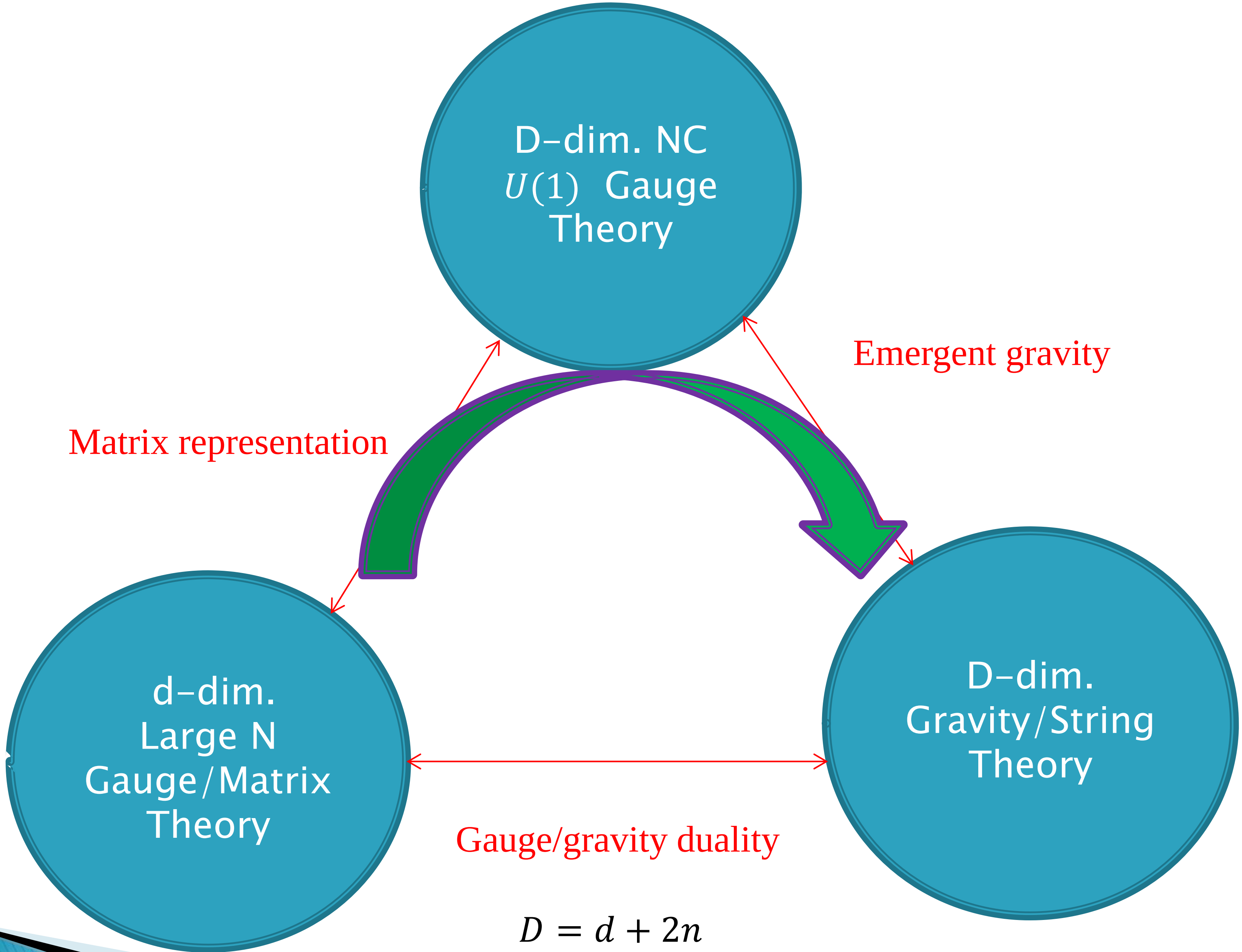
$$S = -\frac{1}{4G_{YM}^2}\int d^D X (F_{MN} - B_{MN})^2 \qquad (2)$$

$$= -\frac{1}{g_{YM}^2}\int d^d\mathrm{x}\,\mathrm{Tr}\left(\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}D_\mu\Phi_a D^\mu\Phi^a - \frac{1}{4}[\Phi_a,\Phi_b]^2\right)$$

where $B_{MN} = \begin{pmatrix} 0 & 0 \\ 0 & B_{ab} \end{pmatrix}$.

d=0: IKKT, d=1: BFSS, d=2: DVV, $\cdots$, d=4: AdS/CFT, $\cdots$.


D-dim. NC
$U(1)$ Gauge
Theory
Emergent gravity
Matrix representation
d-dim.
Large N
Gauge/Matrix
Theory
D-dim.
Gravity/String
Theory
Gauge/gravity duality
$D = d + 2n$



D-dim. NC
$U(1)$ Gauge
Theory
Matrix representation
Emergent gravity
d-dim.
Large N
Gauge/Matrix
Theory
D-dim.
Gravity/String
Theory
Gauge/gravity duality

$$D = d + 2n$$

Emergent spacetime from BFSS matrix model

Let us consider the BFSS matrix model- matrix quantum mechanics, whose action is given by

$$S_{BFSS} = \frac{1}{g^2}\int dt\, Tr(-\frac{1}{2}(D_0\Phi_a)^2 + \frac{1}{4}[\Phi_a, \Phi_b]^2) \qquad (6)$$

where $D_0\Phi_a = \frac{\partial \Phi_a}{\partial t} - i[A_0, \Phi_a]$. Using the relationship between large N matrices and NC fields under the Moyal vacuum with $\langle A_0 \rangle_{vac} = 0$, one can show that the MQM (6) is equivalent to $(2n+1)$-dimensional NC U(1) gauge theory. Then it is straightforward to show that the emergent geometry dual to the BFSS matrix model is precisely given by the $(2n+1)$-dimensional Lorentzian metric

$$ds^2 = \lambda^2(-dt^2 + V_\mu^a V_\nu^a\, (dy^\mu - \mathbf{A}^\mu)\, (dy^\nu - \mathbf{A}^\nu)) \qquad (7)$$

where $\mathbf{A}^\mu = -\theta^{\mu\nu}\frac{\partial A_0}{\partial y^\nu} dt$ and $\lambda^2 = \nu(V_0, \cdots, V_{2n})$ with volume form $\tilde{\nu} = dt \wedge \nu$.

Interestingly, the $(2n+1)$-dimensional metric (7) appears as an emergent geometry of the BFSS matrix model. If all fluctuations are turned off, the vacuum geometry reduces to flat Minkowski space $\mathbb{R}^{1,2n}$ and the global Lorentz symmetry should be emergent too as an isometry of the vacuum geometry $\mathbb{R}^{1,2n}$.

Dynamical origin of spacetime

We have considered a translation invariant vacuum defined by

$$\langle \Phi_a \rangle_{vac} = B_{ab}\, y^b \in \mathcal{A}_N, \qquad (8)$$

where $[y^a, y^b] = i\theta^{ab} \mathbb{1}_{N\times N}$ and $B_{ab} = (\theta^{-1})_{ab}$. The Heisenberg-Moyal algebra is a consistent vacuum of the BFSS matrix model. Fluctuations were introduced around the vacuum (8)

$$\Phi_a = B_{ab} y^b + \hat{A}_a(y).$$

Then we see that the emergent geometry for the fluctuations is described by the Lorentzian metic (7) and flat space is emergent from a uniform condensate of gauge fields in vacuum:

$$\langle \Phi_a \rangle_{vac} = B_{ab}\, y^b \Rightarrow E_a^{(0)} = \delta_a^\mu \partial_\mu \Rightarrow \langle g_{\mu\nu} \rangle_{vac} = \delta_{\mu\nu}.$$

We can calculate the energy density for the vacuum condensate which is responsible for the generation of flat spacetime:

$\rho_{vac} \sim \frac{1}{g_{YM}^2} |B_{ab}|^2 \sim g_{YM}^2 M_P^4 \sim 10^{-2} M_P^4$ where $G\hbar^2/c^2 \sim g_{YM}^2 |\theta|$ and

$M_P = (8\pi G)^{-\frac{1}{2}} \sim 10^{18}$ GeV.

A striking fact is that the vacuum responsible for the generation of flat spacetime is not empty. Rather the flat spacetime is originated from the uniform vacuum energy known as the cosmological constant in general relativity.

But the spacetime was not existent at the beginning. Thus the vacuum condensate must be regarded as a dynamical process. Since the dynamical scale for the condensate is about of the Planck energy, the typical time scale for the condensate will be roughly of the Planck time $\sim 10^{-44}$ sec. Thus it is natural to consider the instantaneous condensate of vacuum energy enormously spreading out spacetime as the cosmic inflation of our universe in which the metric (7) corresponds to a final state of the inflation.

So the question is how to describe the dynamical process for the vacuum condensate. Now I will argue that a natural phase space describing the cosmic inflation of our universe is a locally conformal cosymplectic manifold.

Picture of emergent inflation

- Inflation is simply an (exponential) expansion of a preexisting spacetime triggered by the potential energy carried by inflaton(s):

$$H^2 = \frac{8\pi G}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right)$$

- Inflation is a time-dependent dynamical system and corresponds to a non-Hamiltonian system

$\ddot{\phi} + 3H\dot{\phi} + \frac{\delta V}{\delta \phi} = 0 \quad \rightarrow$ frictional force $\approx$ external force during inflation.

NC spacetime is a phase space with a symplectic structure $B = \frac{1}{2} B_{\mu\nu} dy^\mu \wedge dy^\nu$.

In a conservative dynamical system described by a Hamiltonian vector field, time coordinate t is not a phase space coordinate but an affine parameter on particle trajectories.

But, for a general time-dependent system, it is necessary to include the time coordinate as an extra phase space coordinate. The corresponding $(2n + 1)$-dimensional symplectic manifold is known as a contact manifold which is a triple (M, ω, η) where ω and η are a 2-form and a 1-form on M such that $\eta \wedge \omega^n \neq 0$. If ω and η are closed, i.e., $d\omega = d\eta = 0$, then M is said to be a cosymplectic manifold.

In a conservative Hamiltonian system, we have the Liouville theorem stating the invariance of phase space volume under Hamiltonian time evolution.

But, the cosmic inflation means that the volume of spacetime phase space has to exponentially expand. Thus we need a generalized Liouville theorem describing the exponential expansion of spacetime.

The corresponding almost symplectic manifold is known as a locally conformal symplectic (LCS) manifold which is a triple (M, ω, b) where b is a closed 1-form and ω is a nondegenerate (but not closed) 2-form η satisfying $d\omega - b \wedge \omega = 0$. Locally by choosing $b = d\lambda^{(i)}$ using the Poincar$\acute{e}$ lemma for a local function $\lambda^{(i)}: U_i \to \mathbb{R}$ on an open neighborhood U_i, we have the relation $d\left(e^{-\lambda^{(i)}}\omega\right) = 0$, so local geometry of LCS manifolds is exactly the same as that of symplectic manifolds. In this case, a Hamiltonian vector field is defined by $\iota_X \omega = df - f\omega$ where $f \in C^\infty(M)$ is a globally defined function. This implies an important condition $\mathcal{L}_X \omega = b(X)\omega$. A Hamiltonian vector field X corresponds to $b(X) = 0$, which leads to $\mathcal{L}_X \frac{\omega^n}{n!} = 0$, the Liouville theorem.

Conformal Hamiltonian dynamics

Example: Mechanical systems with friction.

(I) $\omega = dq^i \wedge dp_i = da$ where $a = -p_i dq^i$. Solve $\iota_X \omega = a + dH$.

$\Rightarrow X = \kappa p_i \frac{\partial}{\partial p_i} + X_H$ where $\iota_{X_H}\omega = dH$.

(II) $\omega = db$ where $b = \frac{1}{2}\left(p_i dq^i - q^i dp_i\right) = a - d\lambda$ and $\lambda = \frac{1}{2} q^i p_i$.

$\Rightarrow \ X = \frac{\kappa}{2}(q^i \frac{\partial}{\partial q^i} + p_i \frac{\partial}{\partial p_i}) + X_H.$

Hamilton's equation for the case (II) with $H = \frac{1}{2} p_i^2 + U(q)$

$$\frac{dq^i}{dt} = X\left(q^i\right) = \frac{\kappa}{2} q^i + \frac{\partial U}{\partial p_i}, \qquad \frac{dp^i}{dt} = X\left(p^i\right) = \frac{\kappa}{2} p^i - \frac{\partial U}{\partial q^i}.$$

$\Longrightarrow \ \ddot{q}^i - \kappa \dot{q}^i + \frac{\partial V}{\partial q^i} = 0$ where $V(q) = U(q) + \frac{\kappa^2}{8} q_i^2$.

In this case, the flow generated by the conformal vector field X has the property

$$\phi^* \omega = \ e^{\kappa t}\, \omega.$$

Inflation corresponds to $\kappa = -3H$.

Predictions of emergent inflation (cont'd)

- Inflation metric described by the conformal Hamiltonian vector field X_I is given by

  $$ds^2 = -dt^2 + e^{\kappa t} dx^\mu dx^\mu \qquad (\kappa = H),$$

  after ignoring all fluctuations around the inflation background.

  This geometry precisely describes the FRW universe for the cosmic inflation.

- Emergent gravity predicts the existence of dark energy $\rho_{DE} \sim M_H^4$ which put the size $L_H = M_H^{-1} \sim (10^{-3} eV)^{-1}$ of the cosmic horizon (observable Universe).

  This implies that the size of entire universe is extremely large, $L_U \sim e^{60} L_H$, compared to our visible universe L_H.

- When including fluctuations, the inflationary metric is generalized to

  $$ds^2 = -(1+\phi)dt^2 + e^{\kappa t}(\delta_{\mu\nu} + h_{\mu\nu}) dx^\mu dx^\nu \qquad (\kappa = H).$$

  ∃ reasonable size of density and tensor perturbations.

- Inflation is a single event due to the exclusion principle of NC spacetime.

  Our universe is (almost) stable. Chaotic, eternal inflations and cyclic universe seem to be inconsistent with our picture. Our visible universe is an extremely small part $\sim 10^{-27}$, but the emergent spacetime picture strongly suggests the orthodox universe (not multiverse) picture.

Cosmic inflation: orthodox vs. emergent spacetime

- Existence of spacetime is a priori assumed.
- Effective field theory: (modified) gravity $\oplus$ (quantum) field theory
- Inflation is triggered by a potential energy carried by inflaton.
- Inflation is an expansion of a preexisting spacetime.
- $E \leq 10^{16}$ GeV (BICEP 2)
- Inflation is (almost) eternal. E.g., chaotic and eternal inflations, cyclic universe, etc.
- Slow roll inflation with enough e-folding $\geq 50 \sim 60$. Multiverse is almost inevitable.

Inflation is an expansion of a preexisting spacetime

- Existence of spacetime is not assumed but defined by the theory.
- Background independent theory: matrix models
- Inflation is triggered by a condensate of Planck energy into vacuum.
- Inflation is a dynamical process generating spacetime.
- 10^{15} GeV $\sim E \sim 10^{17}$ GeV
- Inflation is a single event due to the exclusion principle of NC spacetime. Our universe is (almost) stable.
- Neither inflaton nor inflation potential. Universe with our visible part $\sim 10^{-27}$ after 60 e−folding.

Inflation is a dynamical process generating spacetime

Conclusion and open issues

In emergent gravity, the cosmic inflation can be triggered by the vacuum condensate of Planck energy that is responsible for the generation of space and time.

There is no initial value problem (cf. no boundary proposal by Hawking and Hartle) and predicts a spatially flat universe, $k = 0$.

Any dilaton field and its inflation potential need not be introduced.

Then an urgent question is how to end the inflation?
Some nonlinear damping mechanism (e.g., Landau damping) through the interaction with density fluctuations ???

How to generalize the inflation scenario to (3+1)-dimensions ???
Simple minded recipe: Wick rotation from $\mathbb{R}^4$ to $\mathbb{R}^{3,1}$ or
KK compactification $\mathbb{R}^{4,1} \to \mathbb{R}^{3,1} \times \mathbb{S}^1$.
I am not happy with these recipes. I am thinking of alternatives:
Nambu structure (3-algebra of M-theory) or symplectic groupoids ?