Magnetic Monopoles

Since Maxwell discovered the unified theory, Maxwell equations, of electric and magnetic forces, people have studied its implications. Maxwell conjectured the electromagnetic wave in his equations to be the light, based on the speed of electromagnetic wave being close to the speed of light. Since Herz and Heaviside, independently, have written the current form of Maxwell's equations, 1905 Poincare and Einstein independently found the Lorentz transformation which is the property of Maxwell equation. The study of cathode ray has led to the discovery of electrons. The electron orbits are bending around when a magnet was brought near it. By imagining the motion of electron near a tip of a very long solenoid, Poincare introduced a magnetic monopole around which the magnetic field comes out radially.

$$\mathbf{B} = g\mathbf{r}/r^3$$

1. Consider the motion of a charged particle with equation of motion,

$$m\, d^2\mathbf{r}/dt^2 = e\, \mathbf{v} \times \mathbf{B}$$

Find the conserved energy E and the explicit form of the conserved angular momentum $\mathbf{J}=m\mathbf{r} \times d\mathbf{r}/dt+\ldots$.

2. Find the orbit of the charged particle.

3. Consider the motion of two magnetic monopoles appearing in the 4-dim field theory. Magnetic monopoles are characterized by their positions and internal phase angle. The Lagrangian for the relative position **r** and phase ψ is given as below.
where $\psi \sim \psi+2\pi$ and $\nabla\times\mathbf{w}(\mathbf{r}) = -\mathbf{r}/r^3$, and r_0 and a are constant positive parameters..

$$L = \frac{\mu}{2}\left\{\left(1+\frac{r_0}{r}\right)\dot{\mathbf{r}}^2 + r_0^2\left(1+\frac{r_0}{r}\right)^{-1}(\dot{\psi} + \mathbf{w}(\mathbf{r})\cdot\dot{\mathbf{r}})^2\right\} + \frac{1}{2\mu r_0^2}\frac{a^2}{1+\frac{r_0}{r}}$$

Calling the conserved charge q under the shift symmetry of ψ and the conjugate momentum **π** of the coordinate **r**, find the expression for the conserved energy and angular momentum **J**. (There is no such vector potential w, without Dirac string attached. Dirac string is a long and very thin solenoid which brings magnetic flux to the tip of the monopole.)

4. There is a conserved Runge-Lenz vector $\mathbf{K}=\boldsymbol{\pi} \times \mathbf{J}+\ldots$, similar to the hydrogen atom case. Find its explicit form.

5. Analyze the orbital motion of the relative motion of monopoles for a given relative charge q and the particle as we have done for the hydrogen atom, like unbounded and bounded orbits.

6. Dirac found the electromagnetic charge quantization rule $eg= 2\pi\hbar n$ where n is an integer by requiring the solenoid being not detectable. Show that it is consistent with the angular momentum quantization rule. Dirac charge quantizationn rule tells us that all electric charge observed in the Universe should be integer-multiplet of a unit charge if there exists at least a single magnetic monopole.