The Higher Spin Square

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ICTS-TIFR
Bangalore, India

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Based on

- M. R. Gaberdiel and R. G.
  "Higher Spins and Strings", arXiv:1406.6103
- M. R. Gaberdiel and R. G.
  "Stringy Symmetries and the Higher Spin Square", arXiv:1501.07236
- M. R. Gaberdiel and R. G.
  In Progress/To Appear

Plan of the Talk

- Brief Motivation
  The search for a maximally unbroken symmetric phase of string theory.
- The Symmetric Product CFT
  The partition function, conserved currents and their single particle generators.
- Simpler Case: Single Boson Symmetric Product
  Its horizontal and vertical algebras.
- The Higher Spin Square
  Its minimal representation; the Clifford Algebra Square; Twisted Sectors
- Outlook

Unbroken Symmetries of String Theory

- Why is the structure of String Theory so rigid? Unique?
- Perhaps due to a large underlying invariance which is mostly unmanifest (Gross, Witten, Moore, Sagnotti etc.).
- In flat space, only YM gauge symmetry and diffeos manifest.
- But, if we consider $\alpha' E^2 \to \infty$ limit, there seem to be relations between amplitudes of different string levels.
- Thus suggests tensionless limit of string theory is a good place to look for unbroken symmetry.
- In flat space, this limit cannot be taken uniformly on the whole theory because of absence of any other dimensionless parameter.
- What about other backgrounds?

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Unbroken Symmetries of String Theory (Contd.)

Why AdS_3 is Special

- Holography tells us that the tensionless limit of Strings on $AdS \leftrightarrow$ free CFTs ($\lambda \propto \frac{R_{AdS}^2}{\alpha'} \to 0$). (Sundborg, Witten)
- Free Yang-mills theory has a single tower of higher spin conserved currents (bilinear in the fields) dual to Vasiliev H-spin gauge fields.
- In fact, Vasiliev theory an example of structure quite determined by underlying gauge invariance.
- Can there be more unbroken symmetries (amongst AdS vacua)?
- Yes. In AdS_3.
- The dual free CFTs have a large number of conserved currents - not just bilinears. Exponentially larger unbroken symmetry algebra.
- Can these constrain the theory away from the symmetric point?

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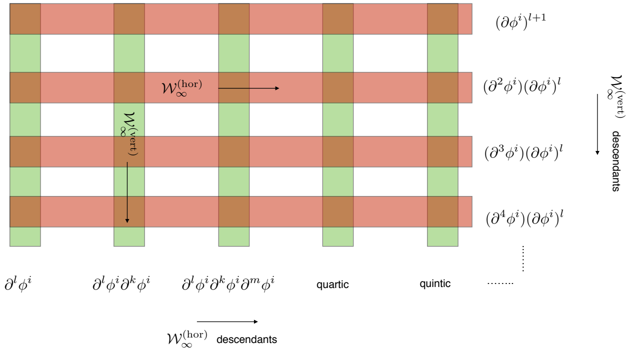
The Punchline

- Vasiliev HS symmetry organises the unbroken symmetry algebra of $AdS_3 \times S^3 \times T^4$ in terms of a Higher Spin Square. The square is effectively generated by two $\mathcal{W}_\infty$ symmetries in that all commutators are expressed in terms of these two higher spin commutators.



The Symmetric Product CFT

- String theory on $AdS_3 \times S^3 \times T^4$ believed to be on the moduli space of the orbifold CFT $(T^4)^N/S_N$ (at large N).
- Partition function known explicitly (DMVV). If $Z(X) = \sum_{h,\bar{h}} c(h,\bar{h}) q^h \bar{q}^{\bar{h}}$, then

$$\sum_{N\geq 0} p^N Z(X^N/S_N) = \prod_{n>0} \prod_{h,\bar{h}} (1 - p^n q^h \bar{q}^{\bar{h}})^{-c(h,\bar{h})}.$$

- This is essentially a formula of taking multiparticles. In particular, applies to the chiral sector ($\bar{h} = 0$) of conserved currents.
- Thus single particle conserved current generators of X^N/S_N are in 1-1 association with the chiral algebra of X.
- The correspondence is $J^{(s)} \leftrightarrow \frac{1}{\sqrt{N}} \sum_{i=1}^{N} J_i^{(s)}$.

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The $X = T^4$ CFT

- For $(T^4)^N/S_N$ there is an 1-1 correspondence with the chiral algebra of (supersymmetric) T^4 i.e. four free bosons and four free fermions.
- Note that at large N the generators are all independent. Thus $\sum_{i=1}^N (\partial\phi_i)^2$ is independent generator compared to $\sum_{i=1}^N (\partial\phi_i)^4$.
- Generating function of single particle chiral algebra of the symmetric product thus

$$\prod_{n=1} \frac{(1+yq^{n-\frac{1}{2}})^2(1+y^{-1}q^{n-\frac{1}{2}})^2}{(1-q^n)^4}.$$

- Thus the number of currents at any given spin (or dimension) and thus massless gauge fields in the dual AdS_3 grows exponentially (Cardy growth).
- Can we usefully view these stringy symmetries through the lens of higher spin symmetry?

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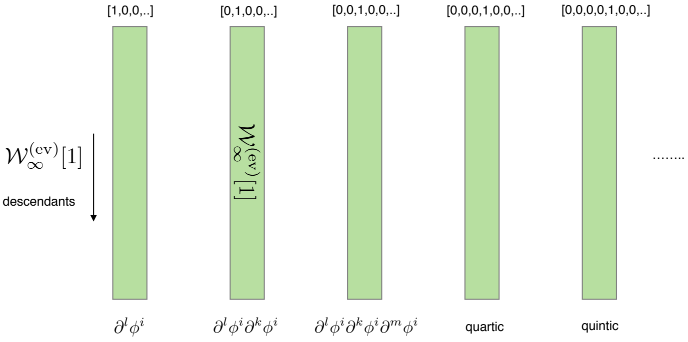
Simplifying the Algebra

- To strip off all the decorations of $\mathcal{N} = 4$ SUSY and see the bare bones of the symmetry algebra, we restrict to the case of the symmetric product of a single real boson.
- Thus single particle generators in correspondence with chiral algebra of a single boson with generating function $\prod_{n=1}^{\infty} \frac{1}{(1-q^n)}$.
- Built from monomials $\prod_j (\partial^j \phi)^{k_j}$.
- The bilinears $\sum_{j=1}^{s-1} c_j (\partial^j \phi)(\partial^{s-j} \phi)$ correspond to $\mathcal{W}_\infty[\lambda = 1]$ symmetry generators.

Vertical $\mathcal{W}_\infty$ Algebra

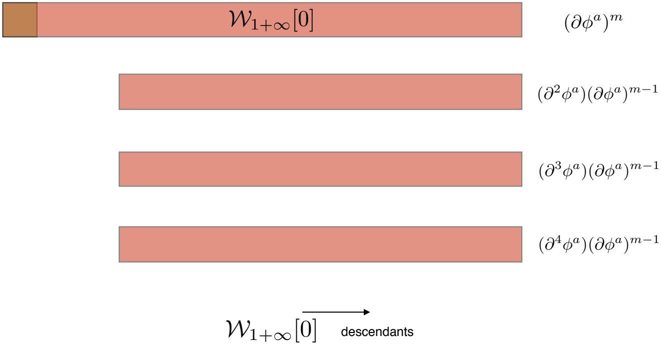
- The others fall into representations of this $\mathcal{W}_\infty[\lambda = 1]$. The terms with n ϕ's transform in the representation $\Lambda_+ = [0^{n-1}, 1, 0 \ldots 0]$.

$$\prod_{k=1}^{\infty} \frac{1}{(1-q^k)} = 1 + \sum_{n=1}^{\infty} \frac{q^n}{\prod_{j=1}^{n}(1-q^j)}.$$



Horizontal $\mathcal{W}_\infty$ Algebra

- The novel observation is that we can alternatively organise the generators in a horizontal way starting with the top row.
- By fermionisation, the top row are bilinears of fermions which generates a different higher spin symmetry - $\mathcal{W}_{1+\infty}[\lambda = 0]$.



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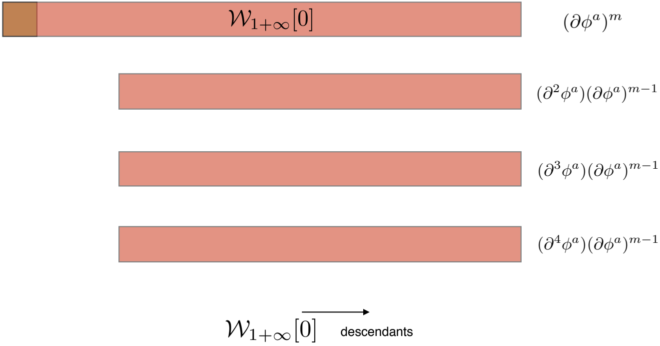

Horizontal $\mathcal{W}_\infty$ Algebra (Contd.)

Another Decomposition

- The other rows are different representations of the horizontal $\mathcal{W}_{1+\infty}[0]$ algebra.
- They are labelled as $\Lambda_+ = 0$ and $\Lambda_- = [m, 0 \ldots 0, m]$.
- This is a different decomposition

$$\prod_{k=1}^{\infty} \frac{1}{(1-q^k)} = 1 + \sum_{m=1}^{\infty} \frac{q^{\frac{m^2}{2}}}{\prod_{j=1}^{m}(1-q^j)^2}.$$

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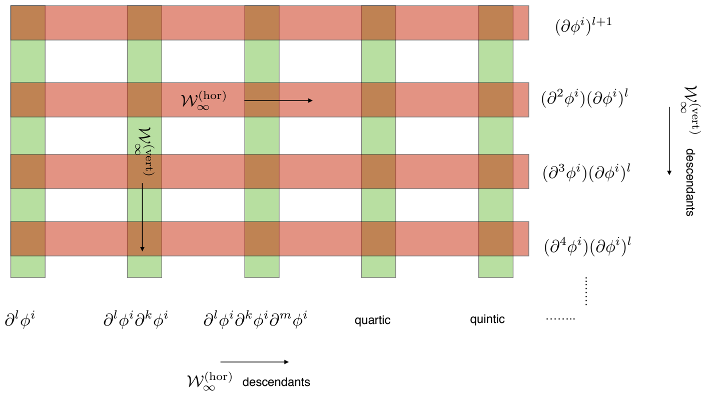
The Higher Spin Square

- The vertical and horizontal algebra together generate the structure of a higher spin square. Characterises all commutators of the stringy algebra in terms of higher spin commutators.



The Clifford Square

- The stringy algebra is, however, not a tensor product in any sense of the underlying higher spin algebras.
- It is exponentially larger in size than either the horizontal or vertical $\mathcal{W}_\infty$ algebras.
- This can be illustrated in a very similar toy example - a Clifford square. Consider the $SO(2d)$ clifford algebra $\{\gamma^i, \gamma^j\} = 2\delta^{ij}$.
- Bilinears $\gamma^{[i}\gamma^{j]}$ are generators of $SO(2d)$.
- Other products with $n \leq 2d$ γ's i.e $\gamma^{[i_i} \ldots \gamma^{i_n]}$ transform in the nth antisymmetric representation of the $SO(2d)$. Each forms a column.
- But because of the clifford multiplication, there is an algebra across columns as well. In fact, the set of all these gamma matrices and their products generates $SU(2^d)$.
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- Other products with $n \leq 2d$ γ's i.e $\gamma^{[i_i} \ldots \gamma^{i_n]}$ transform in the nth antisymmetric representation of the $SO(2d)$. Each forms a column.
- But because of the clifford multiplication, there is an algebra across columns as well. In fact, the set of all these gamma matrices and their products generates $SU(2^d)$.
- Exponentially larger rank than $SO(2d)$.

The Clifford Square

- The stringy algebra is, however, not a tensor product in any sense of the underlying higher spin algebras.
- It is exponentially larger in size than either the horizontal or vertical $\mathcal{W}_\infty$ algebras.
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Matter and the Higher Spin Square

- How can we exploit this structure of the Higher Spin Square?
- View the (non-chiral) spectrum through the lens of higher spin.
- The full untwisted sector given by

$$q^{\frac{N}{4}}\,\bar{q}^{\frac{N}{4}}\;Z_{\mathrm{U}}(q,y,\bar{q},\bar{y})=\prod_{r,\bar{r}=0}^{\prime}\prod_{l,\bar{l}\in\mathbb{Z}}\left(1-(-1)^{2r+2\bar{r}}\,q^{r}y^{l}\bar{q}^{\bar{r}}\bar{y}^{\bar{l}}\right)^{-d(r,l)d(\bar{r},\bar{l})}$$

where $\prod_{n=1}^{\infty}\frac{\left(1-yq^{n-1/2}\right)^{2}\left(1-y^{-1}q^{n-1/2}\right)^{2}}{(1-q^{n})^{4}}\;=\;\sum_{r,l}d(r,l)\,q^{r}y^{l}.$

- This can be expressed as

$$Z_{\mathrm{U}}(q,y,\bar{q},\bar{y})=|Z_{vac}|^{2}[1+\sum_{R}|\chi_{R}|^{2}]$$

where χ_R is the multiparticling of the minimal irrep χ_1.

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Matter and the Higher Spin Square (Contd.)

- This minimal irrep is built from the basic representation of the vertical higher spin algebra.
- In Vasiliev theory, this is the minimally coupled matter field.
- Thus the untwisted sector behaves like the "perturbative" sector of coset holography - the states $(\Lambda; 0)$ (Gaberdiel-R.G.-Hartman-Raju).
- The twisted sectors behave, instead, like the "non-perturbative" sector $(0; \Lambda)$ (Perlmutter, Prochazka, Raeymakers).
- Thus, in the two-cycle twisted sector

$$Z^{(2)}(q, y, \bar{q}, \bar{y}) = |Z_{vac}|^2(|\chi_+|^2 + |\chi_-|^2)[1 + \sum_R |\chi_R|^2].$$

  Here $\chi_\pm$ are "near minimal" representations.
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Looking Ahead

- Stringy symmetries can be usefully viewed through the lens of Vasiliev theory - Higher Spin Square.
- The entire untwisted sector of the string theory is one representation - the minimal one of the higher spin square.
- Also have a reasonable understanding of the representations corresponding to the twisted sector.
- Need to use this to understand how the stringy symmetries are broken in going away from the orbifold point - symmetry is "higgsed" since one is giving a vev to a charged field (two cycle twisted sector).
- Hope: Use Wigner-Eckart philosophy to constrain matrix elements even away from the unbroken point.