Self-dual strings in 6d and factorization of 3d superconformal index with an adjoint matter

Jaemo Park

[1]Postech

KIAS-YITP joint workshop 2015
KIAS, 2015. 09. 17

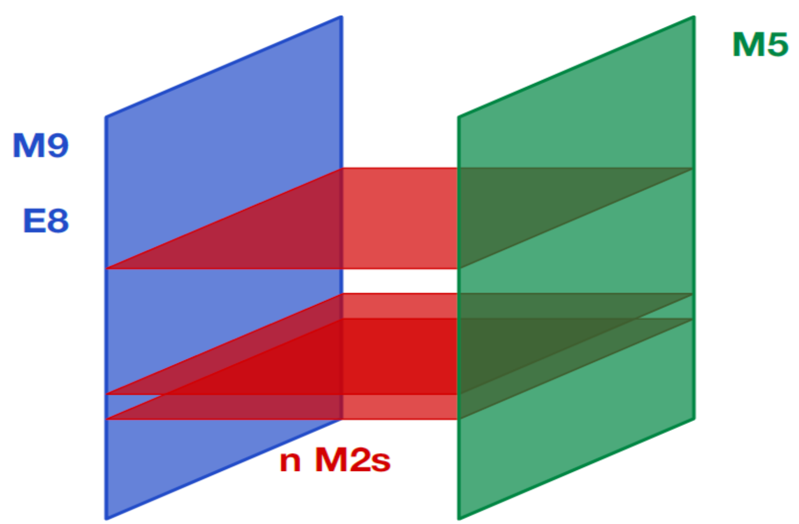
- Based on two papers and two topics (and previous works)
- Elliptic genus of E-strings
  by Joonho Kim, Seok Kim, Kimyeong Lee, JP, Cumrun Vafa
  1411.2324 [arXiv[hep-th]]
- Exploring aspects of 6d SCFTs using 2d field theory
- Factorization of the 3d superconformal index with an adjoint matter
  by Chiung Hwang and JP
  1506.03951 [arXiv[hep-th]]
- Exploring dualities with 3d index

Outline

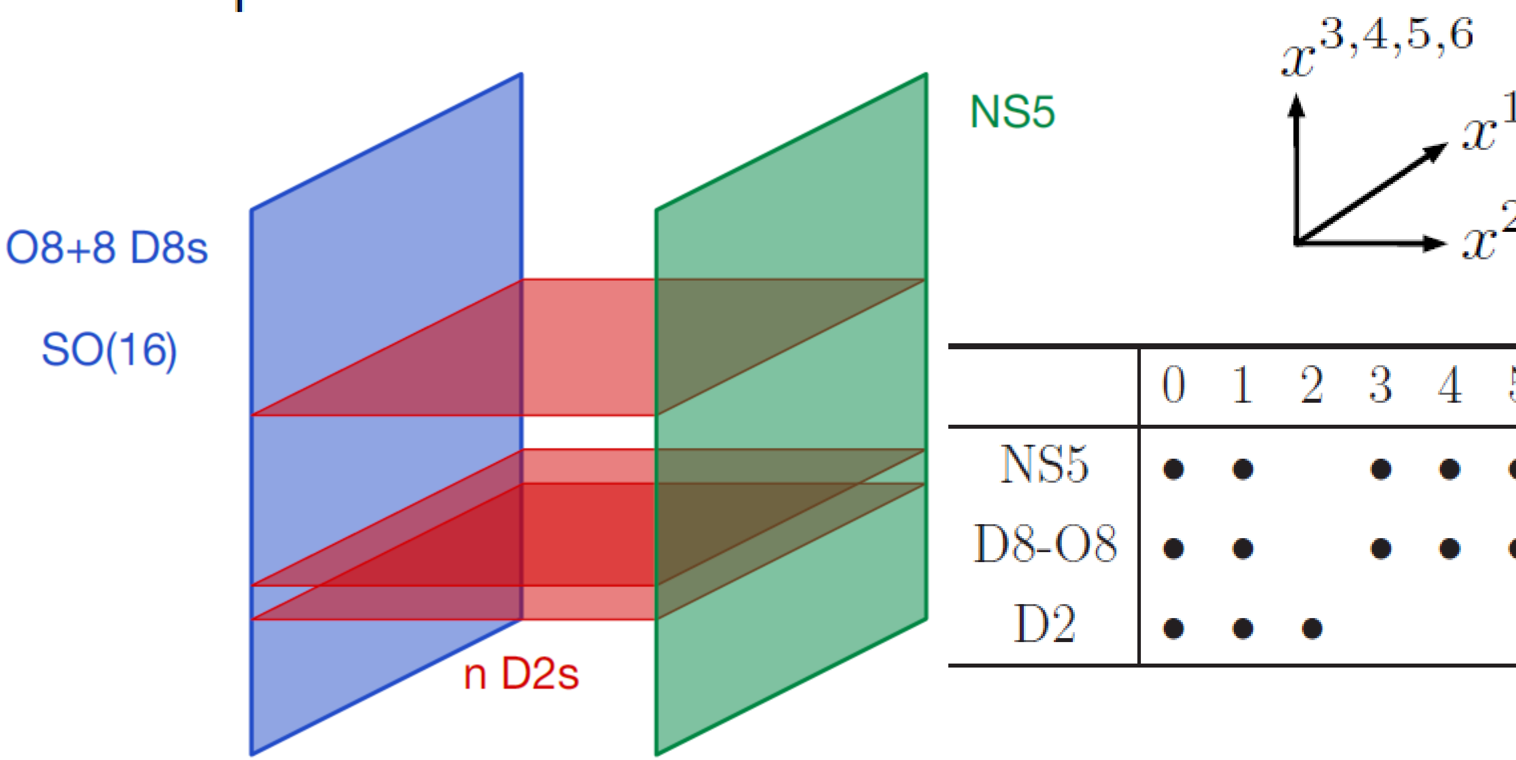
- Self-dual strings in 6d SCFT
- 3d N=2 gauge theories
- Conclusions I
- Factorization of the 3d superconformal index
- Applications to duality
  a. N=4 Seiberg-like duality
  b. N=2 Seiberg-like duality with an adjoint
- Conclusions II

Self-dual string

- Many interesting (0,1) SCFTs arise in the M theory or F-theory setting
- Focus on (0,1) E_8 theory
  (Witten; Ganor, Hanany; Seiberg, Witten; Klemm, Mayr, Vafa; circa mid 90s + many more ....)
- Coincidence limit of M5 and M9 produces the tensionless self-dual string
- which is an interacting 2D SCFT
- Long standing puzzle: How can we describe such self-dual string theories?



- Use the related gauge theory configuration
- and take $g_{2YM} \to \infty$ limit to obtain the 2d SCFT
- which corresponds to the decompactifying limit in Type IIA setup



	0	1	2	3	4	5	6	7	8	9
NS5	•	•		•	•	•	•			
D8-O8	•	•		•	•	•	•	•	•	•
D2	•	•	•							

- The resulting 2D theory has (0,4) world-sheet SUSY
- The symmetry is
  $SO(4)_{3456} \times SO(3)_{789} \sim SU(2)_L \times SU(2)_R \times SU(2)_I$
  doublet indices $\alpha, \beta \quad \dot{\alpha}, \dot{\beta} \quad A, B$

- matter content

$$\begin{aligned}
\text{vector} &: \quad O(n) \text{ antisymmetric} \quad (A_\mu,\ \lambda_+^{\dot{\alpha} A}) \\
\text{hyper} &: \quad O(n) \text{ symmetric} \quad (\varphi_{\alpha\dot{\beta}},\ \lambda_-^{\alpha A}) \\
\text{Fermi} &: \quad O(n) \times SO(16) \text{ bifundamental} \quad \Psi_l
\end{aligned}$$

- interactions determined by (0, 4) SUSY

- We work out the elliptic genus as an evidence for the proposal
  (For the (0,2) 2D gauge theory, the elliptic genus was worked out by Gadde, Gukov and Benini, Eager, Hori, Tachikawa)
- With a choice of (0,2) worldsheet SUSY, the elliptic genus is given by

$$Z_n(q, \epsilon_{1,2}, m_l) = \mathrm{Tr}_{\mathrm{RR}}\left[(-1)^F q^{H_L} \bar{q}^{H_R} e^{2\pi i \epsilon_1 (J_1 + J_I)} e^{2\pi i \epsilon_2 (J_2 + J_I)} \prod_{l=1}^{8} e^{2\pi i m_l F_l}\right]$$

- J_1, J_2 Cartans of $SO(4)_{3456}$, J_I Cartans of $SU(2)_{789}$, F_l are the Cartans of $SO(16)$
- elliptic genus gives the BPS spectrum of 6d theory and all-genus topological string amplitudes on related CY_3 (F-theory, M-theory setup)
- important check: E_8 symmetry in IR

single string

- $O(1) \sim Z_2$ and we have four discrete holonomies on T^2

$$Z_1 = \sum_{i=1}^{4} \frac{Z_{1(i)}}{2} = -\frac{\Theta(q, m_l)}{\eta^6 \theta_1(\epsilon_1)\theta_1(\epsilon_2)}$$

- where $\Theta(q, m_l)$ is the E_8 theta function
- the sum over discrete holonomies is the same as that of R, NS sectors with GSO projections to obtain $E_8(\times E_8)$ heterotic string out of the free fermion formalism

- We call it E-string

Higher E-string

- Two E-string: $O(2)$ gauge theory has 7 holonomy sectors

$$Z_2(\tau, \epsilon_{1,2}, m_l) = \frac{1}{2} Z_{2(0)} + \frac{1}{4} \sum_{a=1}^{6} Z_{2(a)}$$

$$Z_{2(0)} = \frac{1}{2\eta^{12}\theta_1(\epsilon_1)\theta_1(\epsilon_2)} \sum_{i=1}^{4} \left[\frac{\prod_{l=1}^{8} \theta_i(m_l \pm \frac{\epsilon_1}{2})}{\theta_1(2\epsilon_1)\theta_1(\epsilon_2 - \epsilon_1)} + \frac{\prod_{l=1}^{8} \theta_i(m_l \pm \frac{\epsilon_2}{2})}{\theta_1(2\epsilon_2)\theta_1(\epsilon_1 - \epsilon_2)} \right]$$

$$Z_{2(1)} = \frac{\theta_2(0)\theta_2(2\epsilon_+) \prod_{l=1}^{8} \theta_1(m_l)\theta_2(m_l)}{\eta^{12}\theta_1(\epsilon_1)^2\theta_1(\epsilon_2)^2\theta_2(\epsilon_1)\theta_2(\epsilon_2)} \ , \quad Z_{2(2)} = \frac{\theta_2(0)\theta_2(2\epsilon_+) \prod_{l=1}^{8} \theta_3(m_l)\theta_4(m_l)}{\eta^{12}\theta_1(\epsilon_1)^2\theta_1(\epsilon_2)^2\theta_2(\epsilon_1)\theta_2(\epsilon_2)}$$

$$Z_{2(3)} = \frac{\theta_4(0)\theta_4(2\epsilon_+) \prod_{l=1}^{8} \theta_1(m_l)\theta_4(m_l)}{\eta^{12}\theta_1(\epsilon_1)^2\theta_1(\epsilon_2)^2\theta_4(\epsilon_1)\theta_4(\epsilon_2)} \ , \quad Z_{2(4)} = \frac{\theta_4(0)\theta_4(2\epsilon_+) \prod_{l=1}^{8} \theta_2(m_l)\theta_3(m_l)}{\eta^{12}\theta_1(\epsilon_1)^2\theta_1(\epsilon_2)^2\theta_4(\epsilon_1)\theta_4(\epsilon_2)}$$

$$Z_{2(5)} = \frac{\theta_3(0)\theta_3(2\epsilon_+) \prod_{l=1}^{8} \theta_1(m_l)\theta_3(m_l)}{\eta^{12}\theta_1(\epsilon_1)^2\theta_1(\epsilon_2)^2\theta_3(\epsilon_1)\theta_3(\epsilon_2)} \ , \quad Z_{2(6)} = \frac{\theta_3(0)\theta_3(2\epsilon_+) \prod_{l=1}^{8} \theta_2(m_l)\theta_4(m_l)}{\eta^{12}\theta_1(\epsilon_1)^2\theta_1(\epsilon_2)^2\theta_3(\epsilon_1)\theta_3(\epsilon_2)}$$

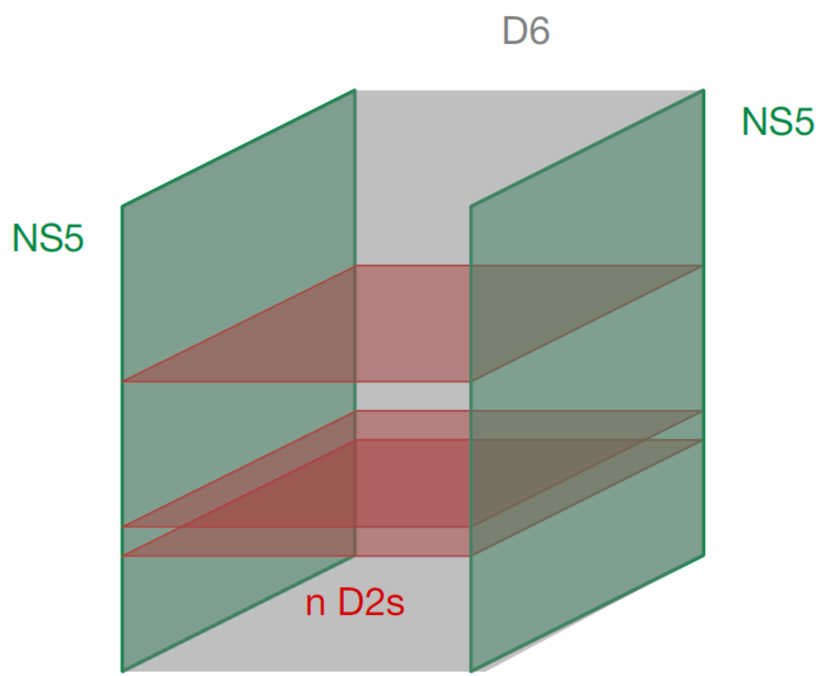
- Coincides with the previous result of Haghighat, Lockhart, Vafa obtained using the E_8 symmetry with low genus expansion, where E_8 symmetry is manifest

$$
\begin{aligned}
Z_2 \;=\; & \frac{1}{576\eta^{12}\theta_1(\epsilon_1)\theta_1(\epsilon_2)\theta_1(\epsilon_2-\epsilon_1)\theta_1(2\epsilon_1)}\Big[4A_1^2(\phi_{0,1}(\epsilon_1)^2-E_4\theta_{-2,1}(\epsilon_1)^2) \\
& + 3A_2(E_4^2\phi_{-2,1}(\epsilon_1)^2-E_6\phi_{-2,1}(\epsilon_1)\phi_{0,1}(\epsilon_1)) + 5B_2(E_6\phi_{-2,1}(\epsilon_1)^2 - E_4\phi_{-2,1}(\epsilon_1)\phi_{0,1}(\epsilon_1))\Big] \\
& + (\epsilon_1 \leftrightarrow \epsilon_2)
\end{aligned}
$$

- Higher string results coincide with the known results of topological string amplitudes

More self-dual strings

- M strings: M2 branes between two M5 branes worked out by Haghighat, Kozcaz, Lockhart, Vafa before E string
- Type IIA description



- 2D (0,4) theory with U(n) vector multiplet, one adjoint hyper, one fundamental hyper, one fundamental Fermi

More self-dual strings

- F theory on ellptic CY_3 with the base $O(-n) \to P^1$
- Self-dual string is D3 wrapping on a shrinking 2-cycle
- $n = 1$ E-string $n = 2$ M-string (Witten 1996)
- $n = 4$ admits perturbative IIB orientifold description (Bershadsky, Vafa 1997)
  on C^2/Z_2 with $\Omega\Pi$ where $\Pi : z_1 \to z_1 \;\; z_2 \to -z_2$
- The resulting (0,4) 2D theory is $Sp(k)$ gauge theory with $Sp(k)$ vector multiplets, $Sp(k) \times SO(8+2p)$ bifundamental hyper, $Sp(k) \times Sp(p)$ bifundamental Fermi
  (Haghighat, Klemm, Lockhart, Vafa after E-string)

Conclusions for the part I

- For (0,1) E_8 theory, we propose the tensionless string as IR limit of the related gauge theory
- Obviously this method should work for many other cases
- Good starting point for exploring details beyond elliptic genus
- Precise description of the CFT of the self-dual string?
  cf) 3d N=8 U(N) SYM in the strong couling limit: ABJM theory

3d N=2 SUSY gauge theories

- dimensional reduction of N=1 4d gauge theories
- vector multiplet $(A_\mu, \lambda) \to (A_i, \sigma, \lambda)$
- chiral multiplet (ϕ, ψ)
- vector is dual to scalar $\partial_\mu a = \epsilon_{\mu\nu\lambda} F^{\nu\lambda}$ a is periodic
- Coulomb branch of 3d theory is parametrized by the complex scalar $Y = exp(\sigma + ia)$.
- Valid for generic point of moduli space of Coulomb branch where all charged fields are massive
- Most of Coulomb branches are lifted due to (monopole)-instanton effects.
  e.g. $U(N), \;\; \sigma = (\sigma_1, \sigma_2, \cdots \sigma_N)$
  unlifted Coulomb branch are parametrized by $V_+ = exp(\sigma_1 + ia_1), V_- = exp(-\sigma_N - ia_N)$ in the convention $\sigma_1 \geq \sigma_2 \geq \cdots \sigma_N$.
- One can define the monopole operator which creates the unit monopole flux around x. This flows in IR to the above Coulomb branch coordinate.

3d N=2 SUSY gauge theories

- For general matters it's not clear how many independent monopole operators and what type of monopole operators are allowed
- For later purposes 3d N=2 can also have Chern-Simons terms
  $S_{cs} = k \int Tr(A \wedge dA + \frac{2}{3}A^3 - \bar{\lambda}\lambda + 2D\sigma)$

3d superconformal index

- $I = Tr(-1)^F e^{\beta' \{Q,S\}} x^{\epsilon + j_3} y_j^{F_j}$
- No dependence on β'
- defined on $S^2 \times S^1$
- Evaluation can be done via localization (S. Kim: Imamura and Yokoyama, arbitrary R-charge)
- It has holomorphic dependence on $m + ir$, combination of the mass and R-charge. Dual theories have the same index with arbitrary R charge consistent with symmetry.

3d superconformal index

$$I(x) = \sum_m \int da\, e^{-S^{(0)}_{CS}} e^{ib_0(a)} y_j^{q_{0j}} x^{\epsilon_0} \exp\left[\sum_{n=1}^{\infty} \frac{1}{n} f_{tot}(e^{ina}, y_j^n, x^n)\right]$$

$$S^{(0)}_{CS} = i \sum_{\rho \in R_\Phi} k\rho(m)\rho(a)$$

$$b_0(a) = -\frac{1}{2} \sum_\Phi \sum_{\rho \in R_\Phi} |\rho(m)| \rho(a)$$

$$y_j^{q_{0j}} = y_i^{\frac{1}{2} \sum_\Phi \sum_{\rho \in R_\Phi} |\rho(m)| F_i(\Phi)}$$

$$\epsilon_0 = \frac{1}{2} \sum_\Phi (1 - \Delta_\Phi) \sum_{\rho \in R_\Phi} |\rho(m)| - \frac{1}{2} \sum_{\alpha \in G} |\alpha(m)|$$

$$f_{chiral}(e^{ia}, y_j, x) = \sum_\Phi \sum_{\rho \in R_\Phi} \left[e^{i\rho(a)} y_j^{F_j} \frac{x^{|\rho(m)| + \Delta_\Phi}}{1 - x^2} - e^{-i\rho(a)} y_j^{-F_j} \frac{x^{|\rho(m)| + 2 - \Delta_\Phi}}{1 - x^2} \right]$$

$$f_{vector}(e^{ia}, x) = \sum_{\alpha \in G} \left(-e^{i\alpha(a)} x^{|\alpha(m)|} \right)$$

Factorization of the superconformal index

- 3d superconformal index has the factorization property. This was discussed in the context of the conformal block of 3d SCFTs. (Beem, Dimofte, Pasquetti 2012)
- $S^1 \times S^2$ partition function can be thought as the overlap of the wave function

$$Z = \langle 0_q | 0_{\tilde{q}} \rangle$$
$$q = e^{2\pi i}, \;\; \tilde{\tau} = -\tau$$

- Previously we explicitly worked out the factorization of the index by the residue evaluation for $U(N)$ with fundamental and antifundamental matters. (C. Hwang, C Kim, J.P. 2012) Similar factorization was worked out for the partition function on the squashed sphere. (M. Taki 2013)

Factorization of the superconformal index

- The factorized part has the interpretation as the vortex partition function on $R^2 \times S^1$. This was evaluated by H. Kim, J. Kim, S. Kim, K. Lee (2012).
- For $\mathcal{N} = 2$ $U(N)$ Chern-Simons theory with N_f fundamentals and $\tilde{N}_f$ antifundamentals, the factorized index is given by

$$I(x,t,w,\kappa) = \sum_{\vec{m}\in\mathbb{Z}^N/S_N} \oint \prod_j \frac{dz_j}{2\pi i z_j} \frac{1}{|\mathcal{W}_m|} w^{\sum_j m_j} e^{-S_{CS}(a,m)} Z_{gauge}(x,z,m) \prod_\Phi Z_\Phi(x,t,z,m)$$

w is the fugacity for $U(1)_T$ with $J^\mu = \epsilon^{\mu\nu\lambda} F_{\nu\lambda}$

$$I^{N_f,\tilde{N}_f}(x,t,\tilde{t},w,\kappa)$$
$$=\frac{1}{N!(N_f-N)!}\sum_{\sigma(t)} I^{pert}(x,\sigma(t),\tilde{t},\tau)\left[\sum_{\vec{n}=0}^{\infty}(-w)^n I_{\{n_j\}}(x,\sigma(t),\tilde{t},\tau,\kappa)\right]\left[\sum_{\vec{\bar{n}}=0}^{\infty}(-w)^{-\bar{n}} I_{\{\bar{n}_j\}}(x,\sigma(t),\tilde{t},\tau,-\kappa)\right]$$

$$I^{pert}(x,t,\tilde{t},\tau)=\prod_{i\neq j}^{N} 2\sinh\frac{iM_i-iM_j}{2}\prod_{j=1}^{N}\prod_{k=0}^{\infty}\left[\prod_{a=1(\neq j)}^{N_f}\frac{1-t_j t_a^{-1}x^{2k+2}}{1-t_j^{-1}t_a x^{2k}}\right]\left[\prod_{a=1}^{\tilde{N}_f}\frac{1-t_j^{-1}\tilde{t}_a^{-1}\tau^{-2}x^{2k+2}}{1-t_j\tilde{t}_a\tau^2x^{2k}}\right]$$

$$I_{\{n_j\}}(x,t,\tilde{t},\tau,\kappa)=(-1)^{-\kappa n-(N_f-\tilde{N}_f)n/2}e^{i\kappa\sum_j(M_jn_j+\mu n_j+i\gamma n_j^2)}$$
$$\prod_{j=1}^{N}\prod_{k=1}^{n_j}\frac{\prod_{a=1}^{\tilde{N}_f}2\sinh\frac{-i\tilde{M}_a-iM_j-2i\mu+2\gamma(k-1)}{2}}{\prod_{i=1}^{N}2\sinh\frac{iM_i-iM_j+2\gamma(k-1-n_i)}{2}\prod_{a=N+1}^{N_f}2\sinh\frac{iM_a-iM_j+2\gamma k}{2}}$$

$$x=e^{-\gamma},t=e^{iM},\tilde{t}=e^{i\tilde{M}},\tau=e^{i\mu}$$

Factorization of the superconformal index

- Since we know the explicit expressions for the index, the index agreement for various dual pair is equivalent to nontrivial identities for the vortex partition function expression.
- Recently we generalize the factorization including one adjoint chiral matter multiplet
- For $N = 2 U(N)$ Chern-Simons theory with N_f fundamentals and N_a antifundamentals and one adjoint matter, the factorized index is given by

$$I(x,t,\tilde{t},\tau,\upsilon,w) = \sum_{\substack{p_a \geq 0, \\ \sum_a p_a = N_c}} I^{(p_a)}_{\text{pert}}(x,t,\tilde{t},\tau,\upsilon) Z^{(p_a)}_{\text{vortext}}(x,t,\tilde{t},\tau,\upsilon,\mathfrak{w}) Z^{(p_a)}_{\text{antivortex}}(x,t,\tilde{t},\tau,\upsilon,\mathfrak{w})$$

$$\mathfrak{w} = (-1)^{-\kappa - \frac{N_f - N_a}{2}} w \qquad \sum_{a=1}^{N_f} p_a = N_c$$

$$I^{(p_a)}_{\text{pert}}(x, t = e^{iM}, \tilde{t}, \tau, \upsilon = e^{i\nu})$$

$$= \left(\prod_{a,b=1}^{N_f} \prod_{q=1}^{p_a} \prod_{\substack{r=1 \\ (\neq q \text{ if } a=b)}}^{p_b} 2\sinh \frac{iM_a - iM_b + i\nu(q-r)}{2} \right) \left(\prod_{a,b=1}^{N_f} \prod_{q=1}^{p_a} \prod_{r=1}^{p_b} \frac{\left(t_a t_b^{-1} \upsilon^{q-r-1} x^2; x^2\right)_\infty}{\left(t_a^{-1} t_b \upsilon^{-q+r+1}; x^2\right)'_\infty} \right)$$

$$\times \left(\prod_{a=1}^{N_f} \prod_{q=1}^{p_a} \frac{\prod_{b=1}^{N_f} \left(t_a t_b^{-1} \upsilon^{q-1} x^2; x^2\right)_\infty}{\prod_{b=1}^{N_a} \left(t_a \tilde{t}_b \tau^2 \upsilon^{q-1}; x^2\right)_\infty} \frac{\prod_{b=1}^{N_a} \left(t_a^{-1} \tilde{t}_b^{-1} \tau^{-2} \upsilon^{-q+1} x^2; x^2\right)_\infty}{\prod_{b=1}^{N_f} \left(t_a^{-1} t_b \upsilon^{-q+1}; x^2\right)'_\infty} \right)$$

$$Z^{(p_a)}_{\text{vortex}}(x, t, \tilde{t}, \tau, \upsilon, \mathfrak{w}) = \sum_{\mathfrak{n}_j \geq 0} \mathfrak{w}^{\sum_{j=1}^{N_c} \sum_{n=0}^{l_j-1} \mathfrak{n}_j^n} \mathfrak{Z}^{(p_a)}_{(\mathfrak{n}_j)}(x, t, \tilde{t}, \tau, \upsilon),$$

$$Z^{(p_a)}_{\text{antivortex}}(x, t, \tilde{t}, \tau, \upsilon, \mathfrak{w}) = \sum_{\bar{\mathfrak{n}}_j \geq 0} \mathfrak{w}^{-\sum_{j=1}^{N_c} \sum_{n=0}^{l_j-1} \bar{\mathfrak{n}}_j^n} \mathfrak{Z}^{(p_a)}_{(\bar{\mathfrak{n}}_j)}(x^{-1}, t^{-1}, \tilde{t}^{-1}, \tau^{-1}, \upsilon^{-1})$$

$$\mathfrak{Z}^{(p_a)}_{(\mathfrak{n}_j)}(x=e^{-\gamma},t=e^{iM},\tilde{t}=e^{i\tilde{M}},\tau=e^{i\mu},\upsilon=e^{i\nu})$$

$$= e^{-S^{(p_a)}_{(\mathfrak{n}_j)}(x,t,\tau,\upsilon)}\left(\prod_{a,b=1}^{N_f}\prod_{q=1}^{p_a}\prod_{\substack{r=1\\(\neq q \text{ if } a=b)}}^{p_b}\prod_{k=1}^{\sum_{n=1}^{r}\mathfrak{n}_{(b,n)}}\frac{\sinh\frac{iM_a-iM_b+i\nu(q-r)+2\gamma k}{2}}{\sinh\frac{iM_a-iM_b+i\nu(q-r)+2\gamma(k-1-\sum_{n=1}^{q}\mathfrak{n}_{(a,n)})}{2}}\right)$$

$$\times\left(\prod_{a,b=1}^{N_f}\prod_{q=1}^{p_a}\prod_{\substack{r=1\\(\neq q \text{ if } a=b)}}^{p_b}\prod_{k=1}^{\sum_{n=1}^{r}\mathfrak{n}_{(b,n)}}\frac{\sinh\frac{iM_a-iM_b+i\nu(q-r-1)+2\gamma(k-1-\sum_{n=1}^{q}\mathfrak{n}_{(a,n)})}{2}}{\sinh\frac{iM_a-iM_b+i\nu(q-r+1)+2\gamma k}{2}}\right)$$

$$\times\left(\prod_{b=1}^{N_f}\prod_{r=1}^{p_b}\prod_{k=1}^{\sum_{n=1}^{r}\mathfrak{n}_{(b,n)}}\frac{\prod_{a=1}^{N_a}\sinh\frac{-i\tilde{M}_a-iM_b-2i\mu-i\nu(r-1)+2\gamma(k-1)}{2}}{\prod_{a=1}^{N_f}\sinh\frac{iM_a-iM_b-i\nu(r-1)+2\gamma k}{2}}\right),$$

$$e^{-S^{(p_a)}_{(\mathfrak{n}_j)}(x,t,\tau,\upsilon)}=\prod_{b=1}^{N_f}\prod_{r=1}^{p_b}\left(t_b\tau\upsilon^{r-1}x^{\sum_{n=1}^{r}\mathfrak{n}_{(b,n)}}\right)^{\kappa\sum_{n=1}^{r}\mathfrak{n}_{(b,n)}}$$

$$|\kappa|\leq\frac{N_f-N_a}{2}$$

Seiberg-like duality for 3d N=4 $U(N)$ with N_f hypermultiplets

	$U(1)_R$	$SU(N_f)$	$U(1)_A$	$U(1)_T$
Q	$1/2$	$\overline{\mathbf{N}}_f$	1	0
$\tilde{Q}$	$1/2$	$\mathbf{N}_f$	1	0
X	1	$\mathbf{1}$	-2	0
$V_{i,\pm}$	$\frac{1}{2}N_f - N_c + 1 + i$	$\mathbf{1}$	$-N_f + 2N_c - 2 - 2i$	± 1

- good theory $N_f > 2N_c - 1$ monopole operator R charge greater than 1/2 satisfying unitarity bound
  bad theory $N_f = 2N_c - 1$ monopole operator R charge can be 1/2 saturating unitarity bound
  ugly theory $N_f < 2N_c - 1$ monopole operator R charge can be less than 1/2 violating unitarity bound
- The fate of monopole operators of the ugly theory? Conjectured to be decoupled in some cases (Yaakov)

Seiberg-like duality for 3d N=4 $U(N)$ with N_f hypermultiplets

- N=4 Seibeg-like duality conjeture
  $U(N_c)$ with N_f hypermultiplets with $N_c \leq N_f \leq 2N_c$ is dual to
  $U(N_f - N_c)$ with N_f hypers with $2N_c - N_f$ decoupled free hypers. (Razamat, Willett)

- If we use usual 3d index with the expansion in conformal dimension, the claim is difficult to check
- Using the factorized form of the index, we can attack this problem so we show analyticaly
  the index of the above dual pair coincides with each other

$$
\begin{aligned}
& I^{N_c,N_f}(x,t,\tau,w) \\
&= I^{N_f-N_c,N_f}(x,t^{-1},\tau,w) \times \prod_{i=1}^{2N_c-N_f} I_{\text{hyper}}(x,\tau,w\tau^{2N_c-N_f-2i+1}x^{-(2N_c-N_f-2i+1)/2})
\end{aligned}
$$

Seiberg-like duality for 3d N=4 $U(N)$ with N_f hypermultiplets

- The peculiar thing of the free hyper is that they have the nonstandard R charges $-(2N_c - N_f - 2i)/2$, corresponding to that of monopole operators $V_{i-1\ \pm}$.
- This is due to the fact that the accidental symmetry in IR is not accessible to UV theory

$U(N)$ with adjoint, no CS term

- Consider the $U(N)$ theory with N_f flavors, one adjoint X, $W = \mathrm{Tr}X^{n+1}$.
- One important question is how many independent monopole operators parametrizing Coulomb branch
- Considering the deformation

$$W = \sum_{j=0}^{n} \frac{s_j}{n+1-j} \mathrm{Tr}X^{n+1-j}$$

$$W'(x) = \sum_{j=0}^{n} s_j x^{n-j} \equiv s_0 \prod_{j=1}^{n} (x - a_j)$$

- If all a_j are different, the adjoints get massive and the gauge group is broken to

$$U(N_c) \to U(r_1) \times U(r_2) \times \cdots \times U(r_n)$$

- We have n decoupled $N = 2U(N)$ theory with N_f flavors
- Thus we need at leat n pairs of monopole operators. It turns out that we have just n pairs of monopole operators

$U(N)$ with adjoint, no CS term

- Independent n pairs of monopole operators can be constructed as follows. Consider the radially quantized theory on $R \times S^2$. Turning on the monopole of unit flux $| \pm 1, 0, \cdots, 0 >$. The guage group is broken to $U(1) \times U(N-1)$ and

$$X = \begin{pmatrix} X_{11} & 0 \\ 0 & X' \end{pmatrix}$$

- Define $v_{i,\pm} = X_1^i | \pm 1, 0, \cdots, 0 >, \ \ i = 0 \cdots n-1$
- Due to the characteristic equation for X, $v_{j,\pm}, j \geq n$ is not independent

$U(N)$ with adjoint, no CS term

- Electric theory: $U(N_c)$ gauge theory(without Chern-Simons term), N_f pairs of fundamental/anti-fundamental chiral superfields Q^a, $\tilde{Q}_b$(where a, b denote flavor indices), an adjoint superfield X, and the superpotential $W_e = \mathrm{Tr}\, X^{n+1}$.
- Magnetic theory: $U(nN_f - N_c)$ gauge theory(without Chern-Simons term), N_f pairs of fundamental/anti-fundamental chiral superfields q_a, $\tilde{q}^a$, $N_f \times N_f$ singlet superfields $(M_j)^a_b$, $j = 0, \ldots, n-1$, $2n$ singlet superfields $v_{0,\pm}, \ldots, v_{n-1,\pm}$, an adjoint superfield Y, and a superpotential $W_m = \mathrm{Tr}\, Y^{n+1} + \sum_{j=0}^{n-1} M_j \tilde{q} Y^{n-1-j} q + \sum_{i=0}^{n-1} (v_{i,+} \tilde{v}_{n-1-i,-} + v_{i,-} \tilde{v}_{n-1-i,+})$.

- This was first worked out by H. Kim and JP (2013)
- This is the 3d analogue of the duality of 4d Kutasov-Schwimmer
- Using the factorized expression of the index, one can generate nontrivial identities, which can be checked numerically

Conclusions for the part II

- We derived the factorized expression for the 3d index with one adjoint chiral multiplet
- Using this we prove the index identities analytically arising from 3d N=4 Seiberg-like dual pair. The identities are the strong indication that monopole opertors violating the unitarity bound are decoupled
- Fate of such monopole operators in general N=2 theories?
- Factorization for other matters e.g. more adjoints or bifundamentals
- Structure of holomorphic blocks in 3-d?