(0, 4) dualities

KIAS-YITP Workshop 2015
Geometry in gauge theories and string theory

Jaewon Song (UCSD)

in collaboration with
Pavel Putrov (IAS), Wenbin Yan (Harvard)

arXiv:1505.07110

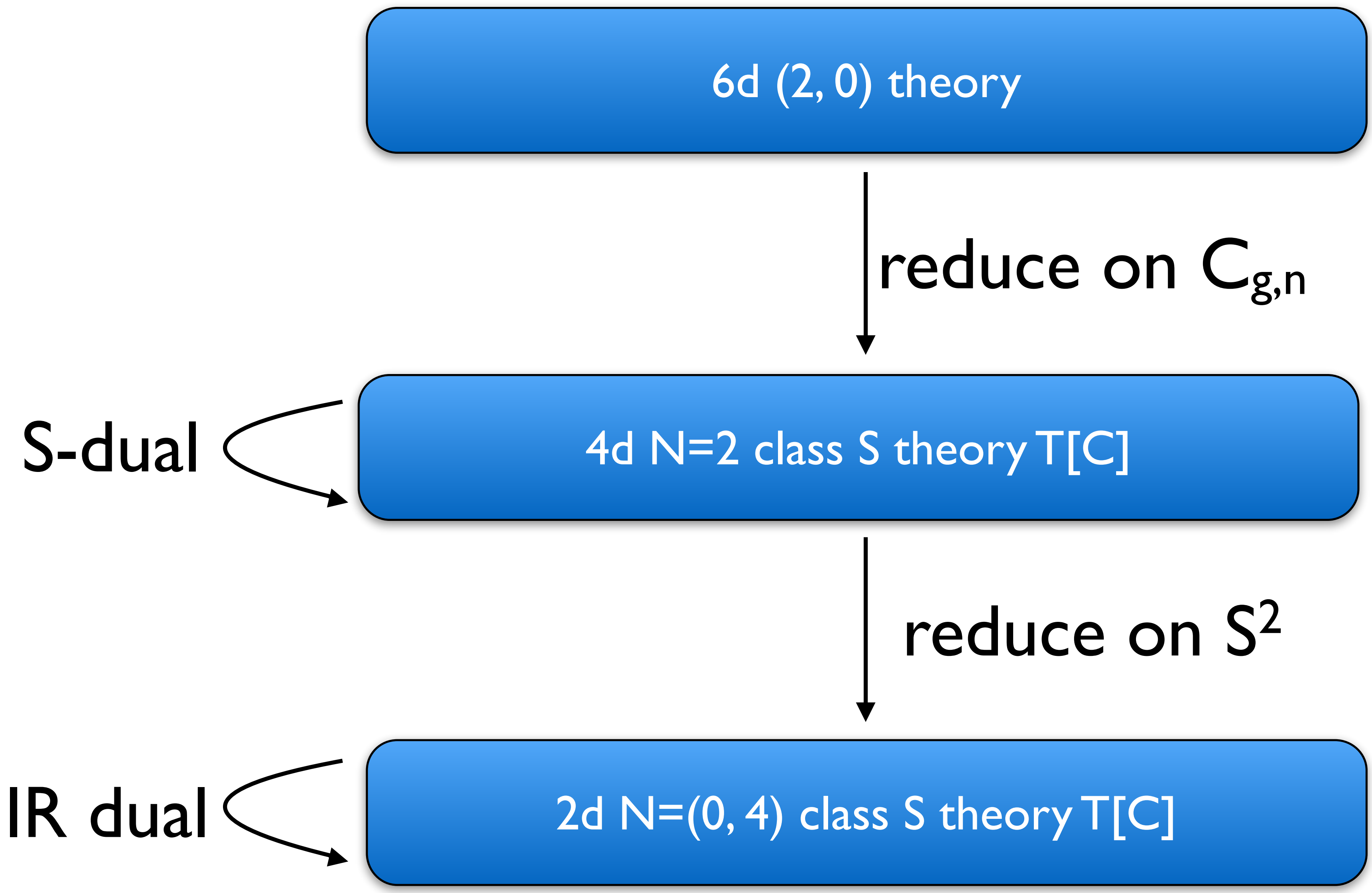
Motivation

- N=(0, 4) gauge theories flow to SCFT
  - Recent developments on N=(0, 2) theory - IR dualities [Gadde-Gukov-Putrov][Jia-Sharpe-Wu][Kutasov-Lin] See E.Sharpe's talk
  - Self-dual strings in 6d (1, 0) (2, 0) theories. "M-string" [Haghighat-Iqbal-Kozcaz-Lockart-Vafa] [Kim-Kim-Lee-Park-Vafa]….
  - Not so many literatures on N=(0,4) non-abelian gauge theory. [Tong]
- M5-branes wrapped on a 4-manifold M_4
  - d=2 (0, 2) SCFT $T[M_4]$ <=> TQFT on M_4 [Gadde-Gukov-Putrov]
  - Special class: $C_{g1,n1} \times C_{g2,n2}$ [Benini-Bobev]

Summary

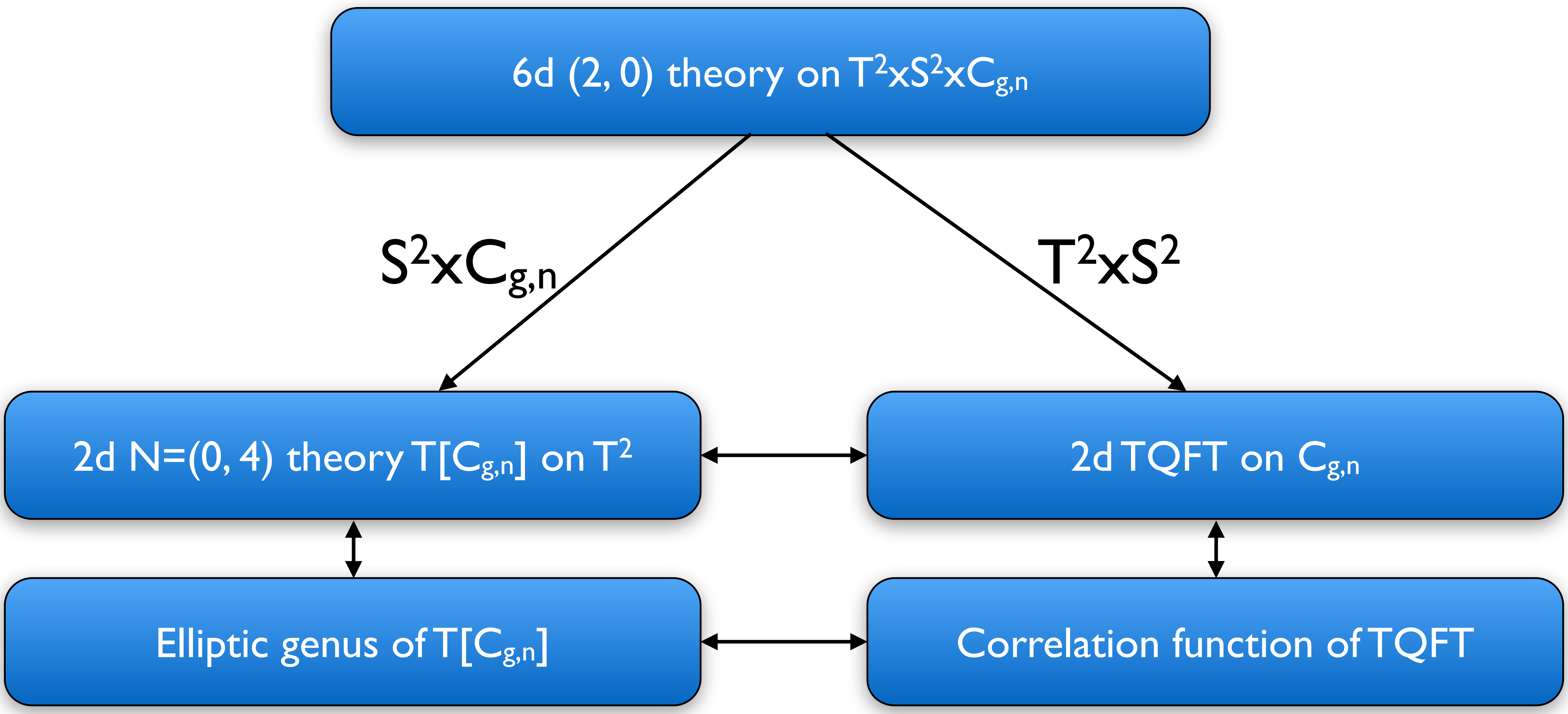
Summary (1)

[Gaiotto]
[Gaiotto-Moore-Neitzke]



Summary (2)

[Alday-Gaiotto-Tachikawa]
[Gadde-Pomoni-Rastelli-Razamat-Yan]
[Dimfte-Gaiotto-Gukov]
[Gadde-Gukov-Putrov]

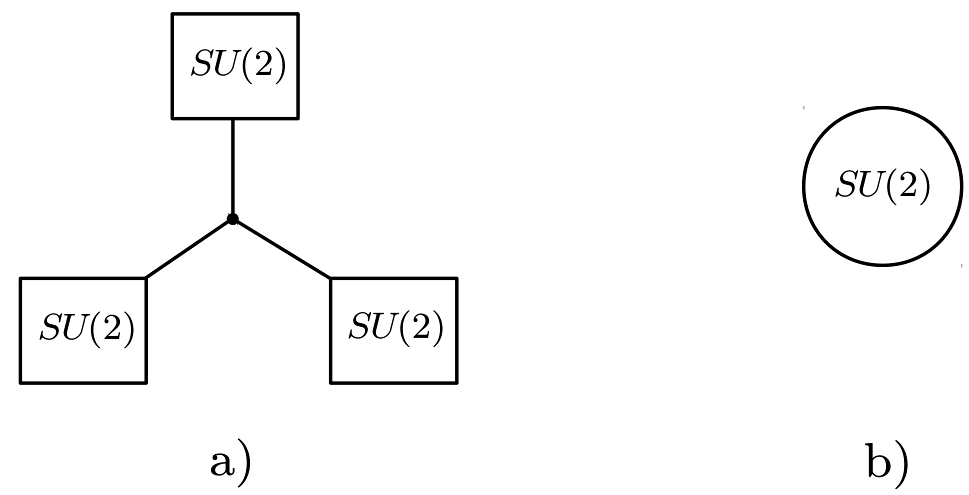


SU(2) theories

(0, 4) gauge theory

- (0, 4) multiplets
  - vector multiplet = (0, 2) vector + (0, 2) Fermi
    (A_μ, λ_-) (ψ_-)
  - (twisted) hypermultiplet = (0, 2) chiral + (0, 2) chiral
    (ψ_+, ϕ)
  - Fermi multiplet = (0, 2) Fermi
- Lagrangian for a given matter content fixed by SUSY?
- Gauge theories generally realize non-compact CFTs with holomorphic bundle over a hyperkahler target space

SU(2) building blocks



- a) Sphere with 3-punctures: $SU(2)^3$-trifundamental half-hypermultiplet
  - R-charge for the hypermultiplets are zero.
  - CFT on the Higgs branch [Witten]
- b) Cylinder: SU(2) gauge multiplet

Superconformal Index

[Gadde-Gukov]
[Benini-Eager-Hori-Tachikawa]

- We verify duality by computing the elliptic genus or superconformal index.

$$\mathcal{I}^{(0,2),NS}(\mathbf{a};q) = \mathrm{Tr}_{NS}(-1)^F q^{H_L}\bar{q}^{H_R-\frac{1}{2}J_R}\prod_i a_i^{f_i}$$

- Vector multiplet $\mathcal{I}^{(0,4),NS}_{\Lambda,G}(\mathbf{z};q) = (\theta(q^{\frac{1+\alpha}{2}}v^{-2};q))^{\mathrm{rk}\,G}\prod_{\substack{\alpha\in\mathrm{adj}_G\\ \alpha\neq 0}}\theta(q^{\frac{1+\alpha}{2}}v^{-2}\mathbf{z}^{\alpha};q)\theta(\mathbf{z}^{\alpha};q)$
- Hypermultiplet $\mathcal{I}^{(0,4),NS}_{\Phi',\mathcal{R}}(\mathbf{x};q) = \prod_{\rho\in\mathcal{R}}\frac{1}{\theta(q^{\frac{1+\alpha}{4}}v^{-1}\mathbf{x}^{\rho};q)}$
- Fermi multiplet $\mathcal{I}^{(0,4),NS}_{\Psi,\mathcal{R}}(\mathbf{x};q) = \prod_{\rho\in\mathcal{R}}\theta(q^{\frac{1}{2}}\mathbf{x}^{\rho};q)$
- α denotes a choice of R-charge. We choose $\alpha=1$, which sets R charge for the hypermultiplets to be zero.

SU(2) N_f=4 theory

- Can be obtained from a 4-punctured sphere
- We get a CFT on the Higgs branch = SO(8) 1-instanton moduli space
- Elliptic genus can be written in terms of SO(8) characters

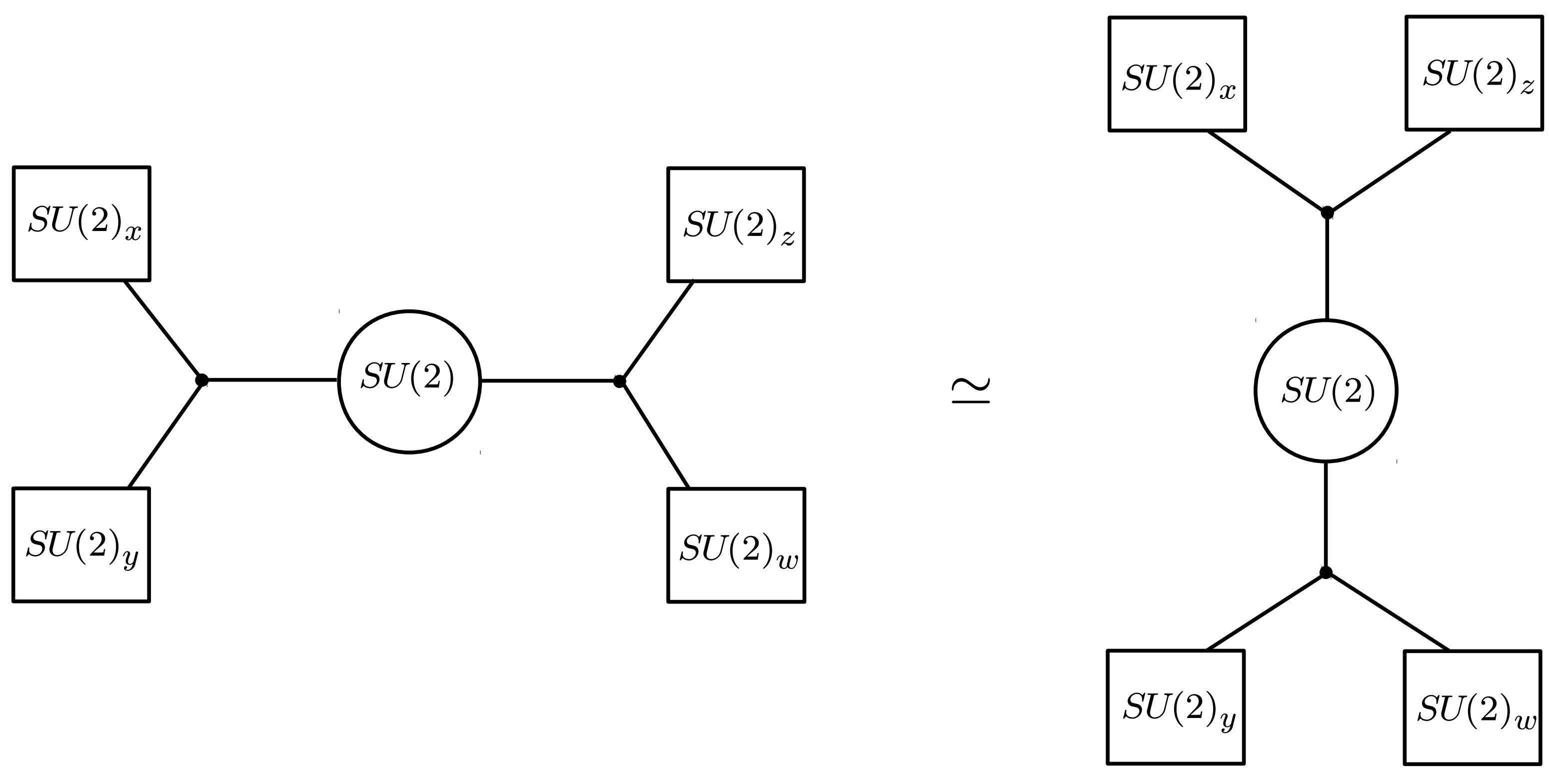
$$\mathcal{I}^{(0,4)}_{\succ\!\prec}(x,y,z,w;v;q) = \int_{\text{JK}} \frac{d\xi}{2\pi i \xi}\, \mathcal{I}^{(0,4)}_{T_2}(x,y,\xi;v;q)\, \mathcal{I}^{(0,4)}_{V,SU(2)}(\xi;v,q)\, \mathcal{I}^{(0,4)}_{T_2}(1/\xi,z,w;v,q)$$

$$\mathcal{I}^{(0,4)}_{\succ\!\prec}(\mathbf{x};v;q) = \left(\mathbf{1} + \mathbf{28}\, v^2 + \mathbf{300}\, v^4 + \mathbf{1925}\, v^6 + \ldots\right) + \left((\mathbf{1}+\mathbf{28}) + (2\cdot\mathbf{28} + \mathbf{300} + \mathbf{350})v^2 + \ldots\right)\, q + \ldots$$

- EG can be written in terms of F_4-characters as in 4d [Gadde-Pomoni-Rastelli-Razamat], so that Weyl group can act. NO conserved current of F_4

$$\mathcal{I}^{(0,4)}_{\succ\!\prec}(\mathbf{x};v;q) = \left(\mathbf{1} + (\mathbf{52} - \mathbf{26} + 2\cdot\mathbf{1})\, v^2 + \mathbf{300}\, v^4 + \ldots\right) + \left((\mathbf{52} - \mathbf{26} + 3\cdot\mathbf{1}) + \ldots\right) q + \ldots$$

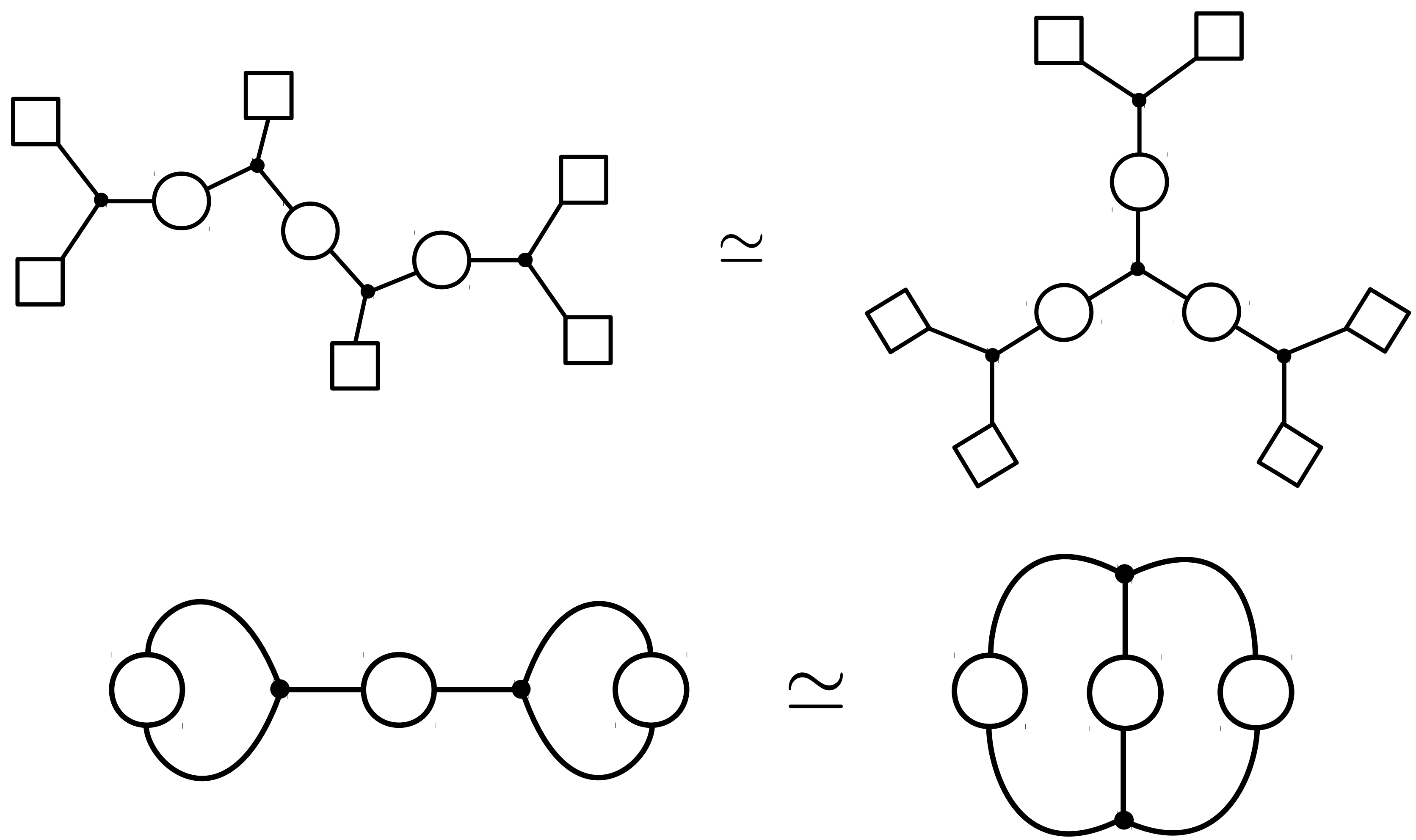
Crossing-symmetry



$$\mathcal{I}^{(0,4)}_{\succ\!\!-\!\!\prec}(x,y,z,w;v;q) - \mathcal{I}^{(0,4)}_{\succ\!\!-\!\!\prec}(x,z,y,w;v;q) = 0$$

- The "dual theory" has the same Lagrangian. Constraints on the spectrum.

Generalized quivers



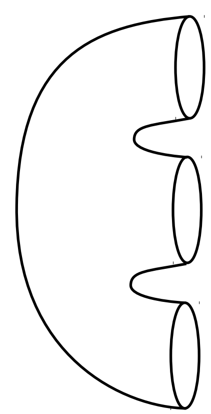
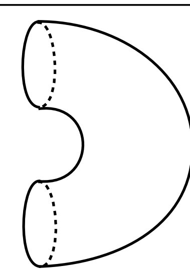
Elliptic genus and 2d TQFT

$$\mathcal{H}^{(0,4)}_{S^1} = \{ f : \mathbb{C}^* \to \mathbb{C} \,|\, f(x) = f(1/x),\ f(qx) = q^4 x^8 f(x) \}$$

Building blocks

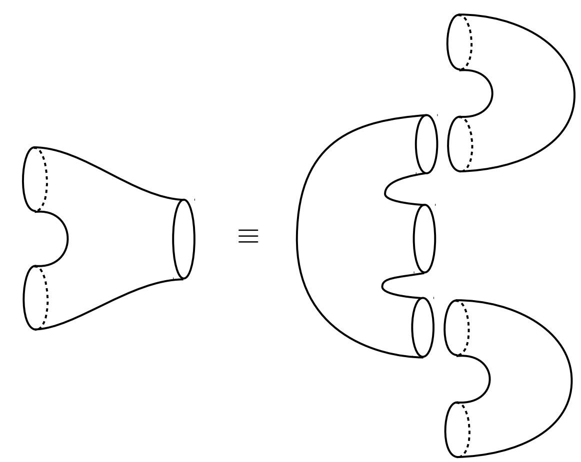
1) 3-point function

2) propagator

	$C:\ \mathbb{C} \longrightarrow \mathcal{H}^{(0,4)}_{S^1} \otimes \mathcal{H}^{(0,4)}_{S^1} \otimes \mathcal{H}^{(0,4)}_{S^1}$ $1 \longmapsto \mathcal{I}^{(0,4)}_{T_2}(x,y,z;v;q)$
	$\eta:\ \mathcal{H}^{(0,4)}_{S^1} \otimes \mathcal{H}^{(0,4)}_{S^1} \longrightarrow \mathbb{C}$ $f(x,y) \longmapsto \int_{\mathrm{JK}} \frac{d\xi}{2\pi i \xi}\, \mathcal{I}^{(0,4)}_{V,SU(2)}(\xi;v;q) f(\xi,\xi)$

Elliptic genus and 2d TQFT

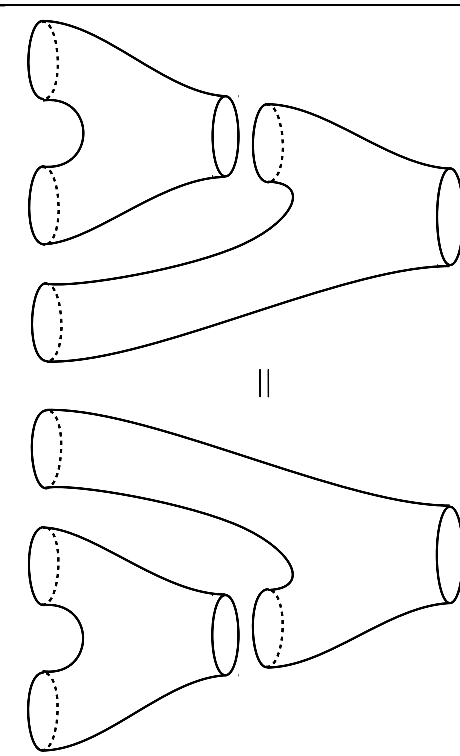
commutative product



$$\mu \equiv (\eta \otimes \mathrm{id} \otimes \eta) \circ (\mathrm{id} \otimes C \otimes \mathrm{id})$$

$$\mu : \mathcal{H}_{S^1}^{(0,4)} \otimes \mathcal{H}_{S^1}^{(0,4)} \longrightarrow \mathcal{H}_{S^1}^{(0,4)}$$

associativity



$$\mu \circ (\mu \otimes \mathrm{id}) = \mu \circ (\mathrm{id} \otimes \mu)$$

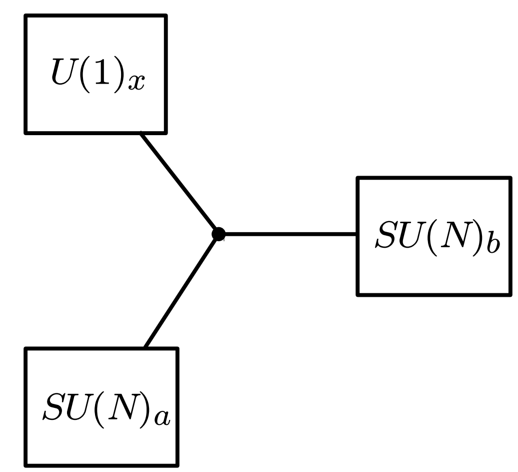
What TQFT is it?

- We have a complete formal definition, but no Lagrangian
- When q=0, it reduces to the Hilbert series of the Higgs branch (when g=0) or more precisely the Hall-Littlewood index of 4d N=2 theory [Gadde-Rastelli-Razamat-Yan]
- HL index is a limit of Macdonald index, where the TQFT is given by the (q, t)-deformed 2d Yang-Mills theory
- For non-zero q, we need to find an elliptic version of the Hall-Littlewood polynomial.

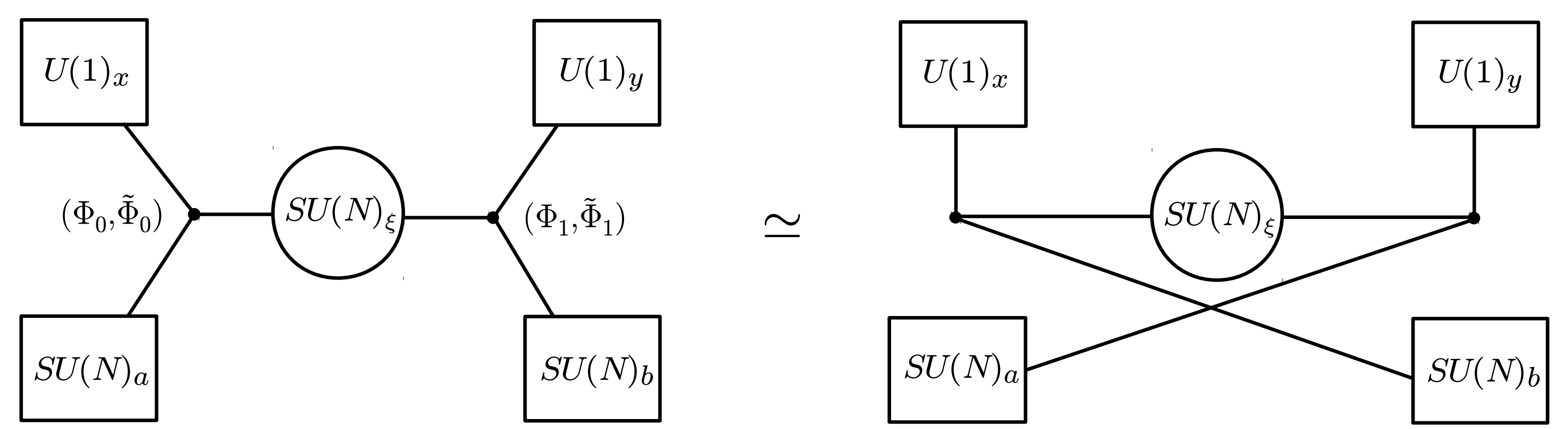
SU(N) theories

SU(N) N_f=2N

- Basic building block U_N - 2 maximal, 1 minimal punctures



- A pair of U_N realizes SU(N) theory



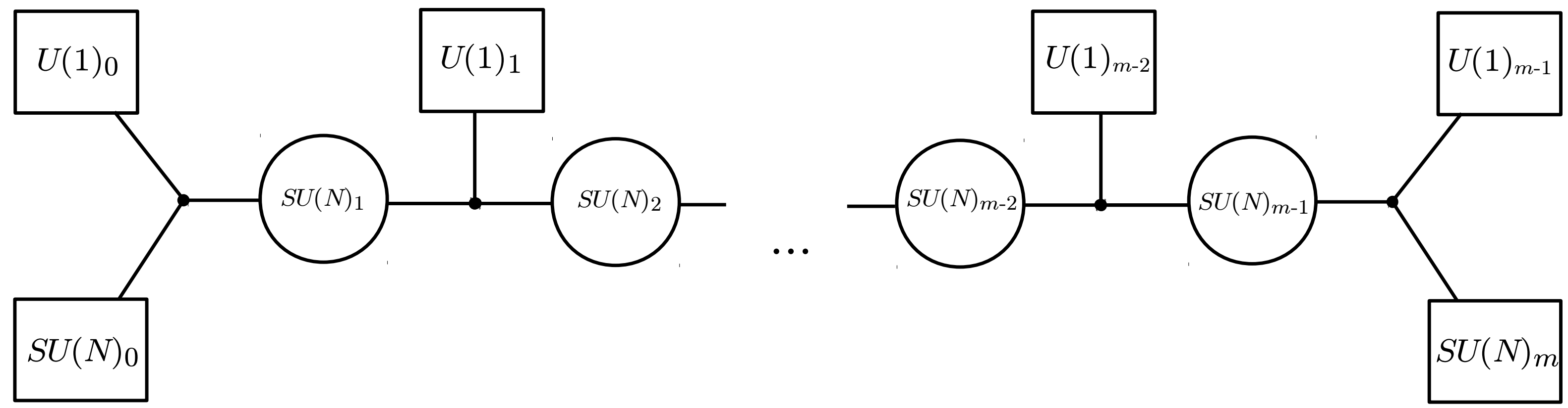
- Crossing-symmetry

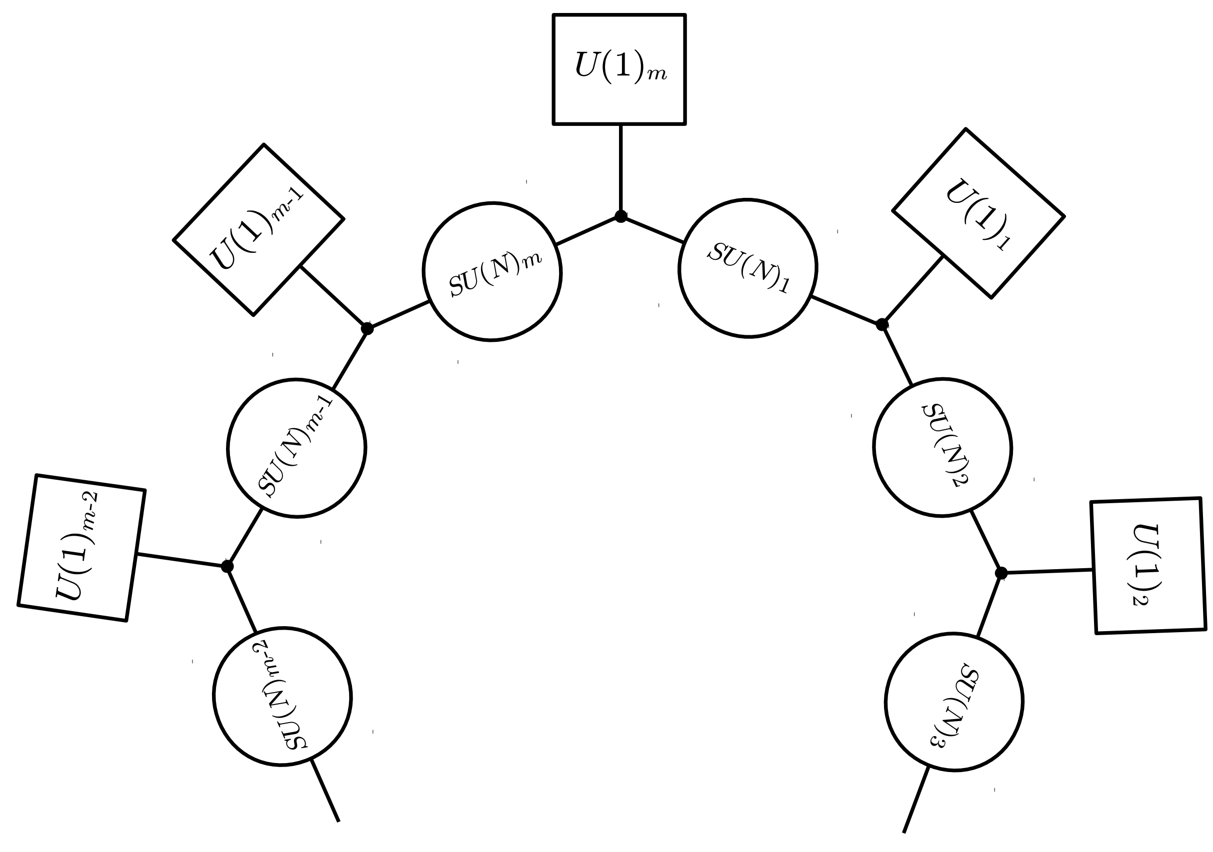
$$\mathcal{I}^{(0,4)}_{\succ\!-\!\prec}(\mathbf{a},\mathbf{b},x,y) = \int_{\mathrm{JK}} \left(\prod_{i=1}^{N-1} \frac{d\xi_i}{2\pi i \xi_i} \right) \mathcal{I}^{(0,4)}_{U_N}(\mathbf{a},\boldsymbol{\xi},x) \mathcal{I}^{(0,4)}_{V,SU(N)}(\boldsymbol{\xi}) \mathcal{I}^{(0,4)}_{U_N}(\boldsymbol{\xi}^{-1},\mathbf{b},y)$$

$$\mathcal{I}^{(0,4)}_{\succ\!-\!\prec}(\mathbf{a},\mathbf{b},x,y) = \mathcal{I}^{(0,4)}_{\succ\!-\!\prec}(\mathbf{b},\mathbf{a},x,y) = \mathcal{I}^{(0,4)}_{\succ\!-\!\prec}(\mathbf{a},\mathbf{b},y,x)$$

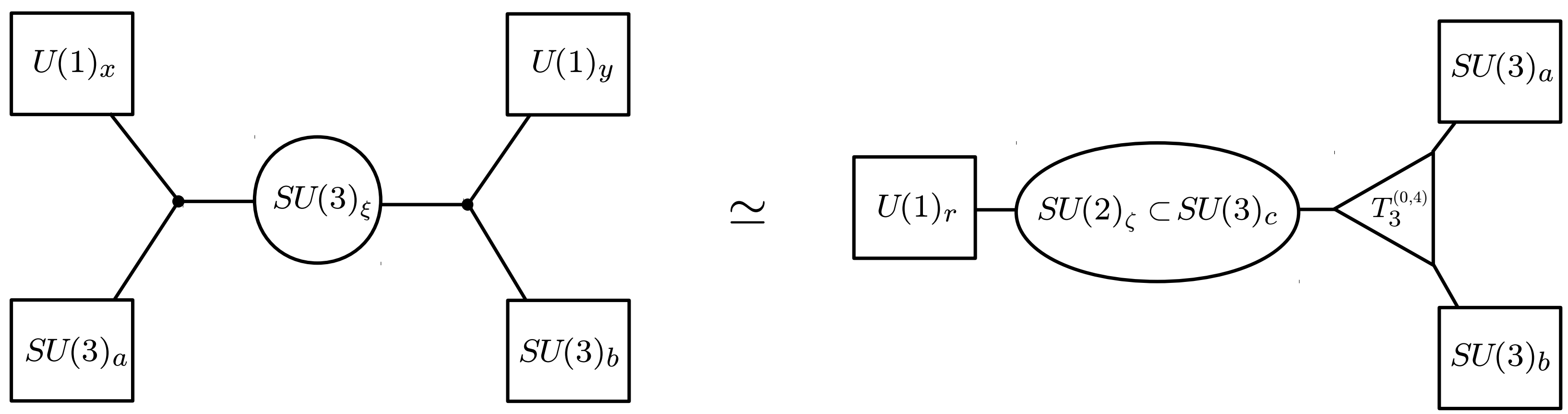
SU(N) quiver theories

- Symmetric under the permutations of $U(1)_i$'s





Argyres-Seiberg duality



$$I^{(0,4)}_{\succ\!\prec}(\mathbf{a},\mathbf{b};x,y) = \frac{1}{2}\int_{\mathrm{JK}} \frac{d\zeta}{2\pi i\zeta}\,\frac{I^{(0,4)}_{V,SU(2)}(\zeta)}{\theta(vs^{\pm1}\zeta^{\pm1})}\,I^{(0,4)}_{T_3}(\mathbf{a},\mathbf{b},\mathbf{c})\,,$$

$$(c_1,c_2,c_3)\equiv(r\zeta,r/\zeta,1/r^2),\qquad x\equiv s^{1/3}/r,\qquad y\equiv s^{-1/3}/r$$

(0,4) E_6 SCFT

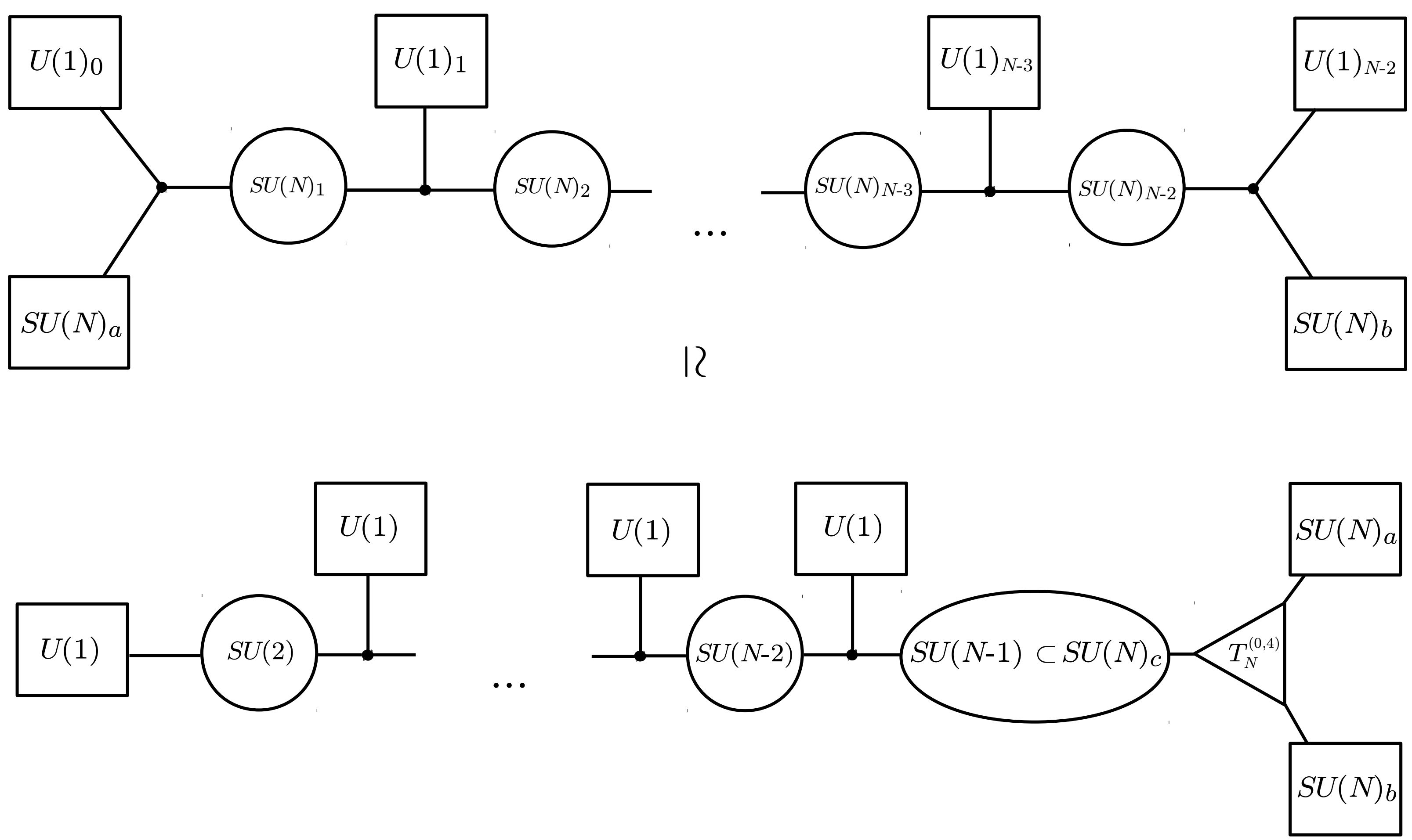
- We find that it is possible to invert the integral equations as in 4d N=2 [Gadde-Rastelli-Razamat-Yan]
- We prove an inversion formula analogous to [Spridonov-Warnaar] for the theta function integral.

$$I^{(0,4)}_{T_3}(\mathbf{a},\mathbf{b},\mathbf{c}) = \frac{1}{2\,\theta(v^2\zeta^{\pm 2})}\int_{\mathrm{JK}} \frac{ds}{2\pi i\, s}\,\frac{\theta(s^{\pm 2})\theta(v^{-2})}{\theta(v^{-1}s^{\pm 1}\zeta^{\pm 1})}\, I^{(0,4)}_{\rightarrowtail\!\!\prec}(\mathbf{a},\mathbf{b};x,y)$$

- This can be also understood as the index for an N=(0, 2) theory with SUSY enhancement in the IR. [Gadde-Razamat-Willett]
- It gives the elliptic genus of E_6 1-instanton string.

$$\begin{aligned} I^{(0,4)}_{T_3} &= \left(\mathbf{1} + \mathbf{78}\, v^2 + \mathbf{2430}\, v^4 + \ldots\right) \\ &\quad + \left((\mathbf{1}+\mathbf{78}) + (\mathbf{1} + 2\cdot\mathbf{78} + \mathbf{2430} + \mathbf{2925})v^2 + \ldots\right) q + \ldots \end{aligned}$$

(0, 4) T$_N$ SCFT and duality



(2, 2) and (0, 2) duality

New N=(2, 2), (0, 2) duality

- Consider a (2, 2) or (0, 2) theory of the form

  $$[SU(N)]_{(a,\,x)} \times SU(N) \times [SU(N)]_{(b,\,y)}$$

  where the bifundamentals are given by chiral multiplets.

- We find that the following index is symmetric under $\mathbf{a} \leftrightarrow \mathbf{b}$ and $x \leftrightarrow y$

$$\mathcal{I}^{\mathcal{N}}_{\rangle\!-\!\langle}(\mathbf{a}, \mathbf{b}, x, y) = \frac{1}{N!}\int_{\mathrm{JK}} \prod_{i=1}^{N-1} \frac{d\xi_i}{2\pi i\, \xi_i}\, \mathcal{I}^{\mathcal{N}}_{U_N}(\mathbf{a}, \boldsymbol{\xi}, x)\, \mathcal{I}^{\mathcal{N}}_{V,SU(N)}(\boldsymbol{\xi})\, \mathcal{I}^{\mathcal{N}}_{U_N}(\boldsymbol{\xi}^{-1}, \mathbf{b}, y)$$

- IR fixed point is invariant under $U(1)_x \leftrightarrow U(1)_y$

(0, 2) duality and 1d TQFT

- Let us define a Landau-Ginzburg model $K_N^{(0,2)}$ with N^2 chiral and a Fermi with superpotential $W = \Gamma\,\mathrm{det}\Phi$

- We have an identity

$$\frac{1}{N!}\int_{\mathrm{JK}} \frac{d\boldsymbol{\xi}}{2\pi i\boldsymbol{\xi}}\,\mathcal{I}^{(0,2)}_{K_N}(\mathbf{a},\boldsymbol{\xi}^{-1},x)\,\mathcal{I}^{(0,2)}_{V,SU(N)}(\boldsymbol{\xi})\,\mathcal{I}^{(0,2)}_{K_N}(\boldsymbol{\xi},\mathbf{b}^{-1},1/y) = \mathcal{I}^{(0,2)}_{K_N}(\mathbf{a},\mathbf{b}^{-1},x/y)$$

- The quiver theory of arbitrary copies collapses to single copy.

- It can be written as a 1d TQFT on a line with two endpoints.

Summary

Summary

- We find dualities of "class S" N=(0, 4) theories
- Elliptic genus of (0, 4) theory T[C] = TQFT on C
    - Elliptic genus of E_6 1-instanton string
- New dualities of N=(2, 2) and (0, 2) quiver theories.

Thank you!