A Membrane Paradigm at Large D

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References

- Talk based on ArXiv 1504.06613 (S. Bhattacharya, A. De, S.M, R. Mohan, A. Saha)
- And ongoing work with above plus Y. Dandekar, K. Inbasekar, S. Mazumdar, M. Mandlik, S. Thakur, U. Mehta, U. Sharma
- Builds on observations and earlier work by Emparan, Tanabe, Suzuki (EST)
- Other related work: 4 recent papers (one paper simultaneous with ours three papers since ours by EST and collaborators)

Introduction

- In this talk we study the classical vacuum Einstein Equations.

$$R_{\mu\nu} = 0$$

- Deceptively simple looking equations. Extremely complicated dynamics, in particular involving black holes.
- E.g. consider the collision of two black holes. Analytically intractable. Phenonenon seems too complicated to ever admit an exact solution. Progress in numerics, but also very difficult.
- Natural instinct of a theorist: search for a parameter and do perturbation theory. However Einstein's equations do not have a parameter. Can we invent one?

Introduction

- Similar lack of parameters in quantum $SU(3)$ Yang Mills theory in $D = 4$. t' Hooft introduced an effective parameter by generalizing to the study of $SU(N)$ Yang Mills theory. New parameter $= \frac{1}{N}$.
- In the early 1980s Witten studied the problem of quantum bound states with the $\frac{1}{r}$ potential in a large number of dimensions. Emergent semiclassical picture of (e.g.) the Helium atom, with definite inter nuclear separation and 'bond angle'. While Witten's motivations were pedegogical, some chemists today find this approximation in the study of complicated real world molecules.
- This talk. Buiding on suggestions by Emparan, Suzuki, Tanabe and collaborators, we adopt a similar strategy for the analysis of black hole dynamics in classical gravity. Our parameter is $\frac{1}{D}$. D is the number of spacetime dimensions.

Introduction: the membrane region

- Schwarschild Black hole in D spacetime dimenstions.

$$ds^2 = -\left(1 - \left(\frac{r_0}{r}\right)^{D-3}\right) dt^2 + \frac{dr^2}{\left(1 - \left(\frac{r_0}{r}\right)^{D-3}\right)} + r^2 d\Omega_{D-2}^2 \qquad (1)$$

- EST made the following important observation. If r is held fixed at any value greater than r_0 as $D \to \infty$ then metric reduces to flat space.
- On the other hand set $r = r_0(1 + \frac{R}{D-3})$ and keep R fixed as D is sent to ∞ then

$$\lim_{D\to\infty} \left(\frac{r_0}{r}\right)^{D-3} = \lim_{D\to\infty} (1 + \frac{R}{D-3})^{-(D-3)} = e^{-R}.$$

Thus 'tail' of the black hole extends only over the distance $\delta r \sim \frac{r_0}{D}$. We refer to this thin layer as the 'membrane' region.

Introduction: Light Qasinormal Modes

- EST proceeded to compute the quasinormal modes of the Schwarzschild Black hole in an expansion in $\frac{1}{D}$. As usual there are an infinite number of qasinormal modes at every angular momentum. At any angular momentum EST find:
- 1.A finite number of modes with frequecies of order $1/r_0$. Light modes.
- 2. All remaining infinite number of modes have frequencies of order $\sim 1/\delta r$. Heavy modes.
- All quasinormal modes have imaginary components of frequency atleast as large as the real parts. Heavy modes decay over time scale δr. Light modes decay over time scale r_0.

Introduction: Intuition for scales

- We can understand the qualitative features of EST's results as follows. Qasinormal mode problem, similar to the computation of the eigenmodes of ∇^2 in a leaky spherical shell of radius r_0 and thickness δr. Modes expanded in spherical and radial harmonics. All modes with nonzero radial harmonics have freqency $\sim 1\delta r$. Howevere modes with zero radial harmonic have frequency $\sim 1/r_0$.
- Light modes are static over time scales δr. Interesting way to understand these modes as we now explain.
- Infinitesimal translations, boosts and scale transformations produce static linearized solutions to the equations of motion. If we zoom onto a patch of size δr about any point on the horizon, it turns out that the light modes well approximate one of the zero modes. However the zero mode parameter varies as a function of the location of the patch. As a consequqnce these configurations evolve on time scale r_0.

Introduction: Effective low energy theory?

- In other words EST's light quasi normal modes describe the linearized collective coordinate dynamics of black holes. Obvious question: can we find the nonlinear 'chiral lagrangian' of these modes.
- As dynamics will turn out to be dissipative we should really search for an equation of motion not a Lagrangain.
- In this talk we will determine the nonlinear equations of motion of the effective theory (to leading order) by a direct analysis of Einstein's equations. The method we employ is reminicent of the Fluid Gravity correspondence. However we work in flat space; our approximations are justified by the expansion in $\frac{1}{D}$ rather than the long wavelength limit; we do not require derivatives to be small in units of the Schwarschild radius.

Einstein's equations with $SO(d+1)$ Isometry

- The large D limit is simplest when observables are kept fixed as D is taken to infinity.
- We divide up the D dimensions into two sets of $p+2$ and $d+1$ respectively ($D = p+d+3$). We then study only those spacetimes that enjoy an $SO(d+1) = SO(D-p-2)$ invariance.
- In other words we require the metric to take the form

$$ds^2_{full} = g_{\mu\nu} dx^\mu dx^\nu + e^\phi d\Omega_d^2 \tag{2}$$

E.g. flat space

$$ds^2 = dw_a dw^a + ds^2 + s^2 d\Omega_d^2$$

- In this talk we take the limit $D \to \infty$ with p held fixed. In particular this means $d \to \infty$.

The large d limit

- Einstein's equations become

$$\begin{aligned} e^{-\phi}(d-1) - \frac{d}{4}(\partial\phi)^2 - \frac{1}{2}\nabla^2\phi = 0 \\ R_{\mu\nu} = \frac{d}{2}\nabla_\mu\nabla_\nu\phi + \frac{d}{4}\nabla_\mu\phi\nabla_\nu\phi \end{aligned} \tag{3}$$

- Note factors of d behind $\partial\phi$. Intuitive reason: ϕ controls the size of a d sphere. Wiggles of ϕ much more expensive than those of $g_{\mu\nu}$. Sensible large d limit requires $g_{\mu\nu}$ and ϕ to be treated asymmetrically. ϕ varies on length scale unity. $g_{\mu\nu}$ varies on length scale $\frac{1}{d}$.
- Solutions of interest are nontrivial over length scales of order unity. However metric varies over length scale $\frac{1}{d}$. In order to write metric we think of spacetime collection of approximately d^{p+2} patches, each of size $\frac{1}{d}$. Solve Einsten's equations in each region and smoothly match.

Coordinates in a patch

- Consider a particular patch centred around x_0^μ. We use rescaled patch coordinates and metric (also gauge field when we study generalizations ahead)
-

$$
\begin{aligned}
x^\mu &= x_0^\mu + \frac{\alpha^\mu(y^a)}{d} \\
G_{ab} &= d^2 \times \partial_a \alpha^\mu \, \partial_b \alpha^\nu g_{\mu\nu} \\
A_a &= d \times \partial_a \alpha^\mu A_\mu
\end{aligned}
\tag{4}
$$

Equations in the patch

- Adapted to patch coordinates the equations of motion become

$$\begin{aligned}\nabla_a \chi^a &= e^{-\phi}\frac{d-1}{d} - \frac{1}{4}\chi^2 \\ R_{ab} &= \frac{1}{2}\nabla_a \chi_b + \frac{1}{4d}\chi_a\chi_b\end{aligned} \qquad (5)$$

  Here R_{ab} is the curvature with G_{ab} regarded as the metric. Similarly for covariant derivatives
- We can now easily take the large d limit of the equations. At leading order in $\frac{1}{d}$ ϕ and χ are constants in this equation. In actual computations it is usually convenient to partially fix coordinate freedom to ensure $e^{\phi} = s^2$. s is the radius of the sphere in the big space R^{d+1}.

Patches of Black Holes

- Can find exact large d solutions by studying patches of known exact solutions.
- Consider the Schwarschild BH in the Kerr Schild form. To get this form we move to 'radial' Eddington Finklestein coordinates in which

$$ds^2 = 2dvd\tilde{r} - dv^2\left(1 - (r_0/\tilde{r})^{D-3}\right) + r^2 d\Omega_{D-2}^2.$$

- Then define a new 'time' coordinate t by $v = t + r$. We have

$$ds^2 = ds^2_{flat} + (dt + d\tilde{r})^2 (r_0/\tilde{r})^{D-3}$$

- Finally divide up flat space coordinates into $p + 2$ and $d + 1$. Let

$$\tilde{r}^2 = r^2 + s^2$$

.

Patches of black holes

- The 'reduced' $p+3$ dimensional metric is given by

$$ds^2 = dw_a dw^a + ds^2 + \frac{(u+n)_\mu (u+n)_\nu dx^\mu dx^\nu}{\psi^{p+d}}$$

  with

$$\psi = \frac{\sqrt{r^2+s^2}}{r_0}, \quad u_\mu dx^\mu = dt, \quad n_\mu dx^\mu = \frac{d\psi}{|d\psi|}$$

- We can obtain more general black hole solutions by translating and boosting the function ψ and the oneform field n in the $p+2$ directions (boosts or translations in the $d+1$ dimensions violate $SO(d+1)$ symmetry and so are not considered).

Wigly membranes

- While boosts change u and n they do not alter the fact that

$$u^2 = -1, \quad u.n = 0, \quad n^2 = 1, \quad |d\psi|^2 = r_0^2 = \left(\frac{s}{n_s}\right)^2$$

- This suggests a strategy to find a much larger class of solutions at large d. We could simply try to replace ψ by an arbitrary function and n by an arbitrary oneform field, subject only to local constraints above and a few other simple constraints.
- At leading order in large d this produces a class of spacetimes, whose even horizon may be shown to lie at $\psi = 1$. Moreover $\psi - 1 \gg \frac{1}{d}$ the metric is exponentially close to flat space.

The wiggly moving membrane

- Thus from the point of view of the outside observer, the class of metrics described above are nontrivial only in a shell of thickness $\frac{1}{d}$ around the surface $\psi = 1$. We call this the membrane region. It follows that, at leading order in large d, the spacetimes obtained from this procedure are characterized only by the location of the geometrical surface $\psi = 1$ and the value of u on this surface. The constraints listed above also have to be obeyed only on this surface.
- It is easy to construct ψ functions that satisfy the 'norm' equation above. We start with an unconstrained function B whose zeroes will define the 'membrane' $\psi = 1$
- In terms of B define an auxilliary function ψ by

$$\psi = 1 + \frac{n.ds}{sn.n} B, \quad n = dB \tag{6}$$

ψ is less than one inside and greater than one outside the membrane.

Wiggly Moving Membrane

- We then use ψ to construct the spacetime metric

$$ds^2 = ds^2_{flat} + \frac{(u+n)_\mu (u+n)_\nu dx^\mu dx^\nu}{\psi^{d+p}} \tag{7}$$

- $u =$ any oneform field with $u_s = 0$, $u.n = 1$ and $u^2 = -1$.
- Because the metric at leading order in d cares only about surface data, it is convenient to write (7) entirely in terms of this data. This may be achieved quite naturally. Given a codimension one timelike surface swept out by a membrane in flat space we construct a congruence of spacelike geodesics that intersect this surface normal to it. the function B at any point is defined as the proper distance from that point to the membrane, along the geodesic that connects the two. Let $n = dB$. It follows that $n.n = 1$ $n.\nabla n = 0$. Similarly we define u^μ away from the membrane by parallel transport along these geodesics so that $n.\nabla u = 0$.

Membrane spacetimes solve the leading large d equations

- At leading order in d, a patch centred about the membrane region of (7) is identical to the patch about the event horizon of a Schwarschild black hole of radius $r_0 = \frac{s}{n_s}$ and boost velocity u. We refer to u as the velocity of the membrane. recall that u is tangent to the membrane, and so is a velocity that does not modify the membrane surface.
- It follows that (7) solves the leading order large d equations in the membrane region and so everywhere outside its event horizon. Inside the membrane region all hell breaks loose in the spacetime (7), but the chaos is causally disconnected from the outside so we dont care. Outside the membrane region we simply have flat space.

The first $\frac{1}{d}$ correction to the membrane metric

- In order to correct the membrane metric above, we zoom in on a patch about a membrane point. We then evaluate Einstein's equations to first subleading order in $\frac{1}{d}$. The equations are not satisfied at this order. A particular component of Einstein's equations may, for instance, evaluate to $\frac{1}{d}u.K.uF(\psi^{p+d})$ where K is the extrinsic curvature of the membrane surface $B = 0$ and F is an arbitrary function. Note that at first order in $\frac{1}{d}$ the 'source' terms listed above cannot depend on more than one derivative of u or n.
- In order to obtain a solution to Einstein's equations to this order we allow our spacetime metric to be corrected at first order in $\frac{1}{d}$. The form of this correction metric is constrained by symmetry. Our original metric has three special oneforms: u, n and ds. Let these oneforms be c_m ($m = 1 \ldots 3$). Let dy^i denote a basis of the remaining directions.

First order correction

- First order corrections to the metric are of three sorts: scalar corrections proportional to $c_m c_n$ or $\sum_i dy^i dy^i$ 'vector' corrections proportional to $dc_m dy^i$ and 'tensor' corrections proportional to $dy^i dy^j$. Tensor terms must be proportional to tensor first order data like $\nabla_i u_j$ or $\nabla_i n_j$. Vector terms must be proportional to vector first order data like $u.\nabla n_s$ or $c_m.\nabla n_i$. Scalar terms are either proportional to scalar first order data or to constants. Each of these terms are further multiplied by unkonwn functions of ψ^{p+d}.
- At first order Einstein's equations yield ODEs for all these unknown functions. Luckily these coupled ODEs all turn out to be easily solvable.
- It turns out that the most general solution generically has physically unacceptable singularities at $\psi = 1$. Nonsingular solutions that also asymptote to flat space exist if and only if the membrane location and velocities obey the following equations of motion.

Membrane Equations of Motion

- $$V_\perp.K.V_\perp = \frac{1-n_s^2}{s n_s}$$
  $$\mathcal{P}\left(V_\perp.\nabla(u-n)_i\right) = 0 \qquad (8)$$
  $$V_\perp = \frac{ds}{n_s} - (n+u)$$

  Here $\mathcal{P}$ is the projector orthogonal to the three dimensional subspace spanned by n, u, ds
- Total number of equations $1+p$. As many equations as variables. Well defined initial value problem for the shape of the membrane and the velocity field.

Rewriting the equation of motion

- It turns out that two equations of motion above may be combined into a single vector equation

$$V_{\perp} \cdot \nabla \left(\frac{S}{n_s}(u-n) \right) + \frac{dS}{n_s} - n = 0 \tag{9}$$

- This looks like $p+3$ equations, but the dot product of this equation with $n-u$ and ds turns out to vanish identically, so its actually $p+1$ equations.
- The equations may be rewritten in explicitly geometric form as

$$-\frac{1}{d}\nabla K + \frac{1}{K}\nabla^2 u - \frac{d}{K} u \cdot \nabla u + n \left[\frac{1}{d\,K^2}\nabla^2 K - \frac{d}{K^2} u \cdot \nabla K - 1 \right] + \frac{d}{K^2}(u \cdot \nabla K) u = 0 \tag{10}$$

Metric correction at first order

- Provided the equations above are obeyed We find the following explicit expression for the first order corrected metric.
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$$
\begin{aligned}
ds^2 = \; & ds^2_{\text{flat}} + \psi^{-d+p}(O\cdot dx)^2 \\
& + \left(\frac{\psi^{-d}}{d}\right)(O\cdot dx)\Bigg[\left(\frac{S\,\mathfrak{S}^{(4)}}{1-n_S^2}+\frac{n_S S\,\mathfrak{S}^{(2)}}{(1-n_S^2)^2}\right)X_\mu \\
& - d(\rho-1)\frac{S\left(2n_S\mathfrak{V}^{(1)}_\mu+\mathfrak{V}^{(4)}_\mu\right)}{n_S^2} \\
& + \left(2-\frac{S\,\mathfrak{S}^{(3)}}{n_S}-\frac{2S\,\mathfrak{S}^{(2)}}{n_S^2(1-n_S^2)}-\frac{(2-n_S^2)S\,\mathfrak{S}^{(1)}}{n_S^2(1-n_S^2)}\right)O_\mu\Bigg]dx^\mu
\end{aligned}
\tag{11}
$$

Redistribution Invariance

- The equations above must pass a simple consistency check. Any solution of the large d equations with the split $D = (p + 2) + (d + 1)$ may also be regarded as a solution of the equations with the split $D = (p + 2 + k) + (d + 1 - k)$ where k is a finite positive number held fixed in the large d limit. This is because an $SO(d + 1)$ invariant solution is ofcourse also $SO(d - k + 1)$ invariant.
- This 'redistribution invariance' is not manifest in the original form of our membrane equations. However it is manifest in the geometrical form of these equations. The geometrical form of the equations applies to any metric situation that preserves $SO(D - p - 2)$ isometry for any fixed p.
- It is conceivable that the geometrical form of the equations has a larger domain of validity.

Simple Solutions

- The equations above are quite unfamiliar. In order to build intuition it is useful to search for simple solutions.
- Lets search for solutions that undergo 'rigid rotations'. Let p be odd. We group the spatial $p+1$ dimensional coordinates into $\frac{p+1}{2}$. two planes and work in polar coordinates in each two plane. A membrane undergoes rigid rotations if its velocity vector in these coordinates is given by

$$u^\mu = \gamma(1, 0, \omega_1, 0, \omega_2, \ldots), \quad \gamma = \frac{1}{\sqrt{1 - \omega_i^2 r_i^2}}$$

  where our coordinates are $(t, r_1, \theta_1, r_2, \theta_2 \ldots)$
- We also assume that angular rotations are symmetries of the membrane world volume so that the time independent membrane shape can be described by the equation

$$s^2 = 2g(r_i)$$

Stationary Black Holes

- Plugging into the equations above it is possible to show that they are solved if and only if the shape function g obeys

$$2g + (\partial g)^2 = K(1 - \omega_a^2 r_a^2)$$

- This equation admits the following simple exact solution:

$$g = \frac{K}{2} + \sum_i \frac{a_i r_i^2}{2}, \quad where \quad a_i + a_i^2 + K\omega_i^2 = .0$$

We have verified that the spacetime dual to this solution of the membrane equations agrees with the large d limit of the Myers Perry black hole solutions.

Stationary solutions

- The equation above also admits solutions of different topologies. These solutions are dual to black ring solutions of general relativity. A thorough classification of the solutions to this equation would be an interesting - and I believe tractable - exercise.
- A version of the equation above first appeared in a paper by Tanabe and Suzuki a few weeks after our paper. Their procedure was to constrain the shape of stationary membrane spacetimes at large d using a method that crucially relies on the the existence of a killing time translation vector.
- The connection between the Suzuki-Tanabe equation and our general membrane equation was initially not clear. We now see the relationship; our membrane equations reduce to the Suzuki-Tanabe equation under the assumption of stationary rigid rotation. This satisfying observation may be regarded as a consistency check of both our equations.

Perturbation around Schwarzschild

- The simplest of the stationary solution has no rotations and is given by
$$s^2 = 2g = K - r_i r^i .$$
This is a spherical $D - 2$ dimensional membarane of radius $\sqrt{K}$ dual to the Schwarzschild B H of the same radius.
- Linearized the membrane equations around this simple solution we find the following spectrum (for scalar and vector modes)
$$w_s = -i(l-1) \pm \sqrt{l-1}, \quad w_V = i(l-1) \tag{12}$$
where l is a positive integer related to the angular momentum of the corresponding modes. Our results are in perfect agreement with the leading order spectrum of light quasinormal modes obtained by ESR from direct analysis of the linearized equations around black holes.
- Note that this spectrum is highly dissipative, even at leading order in $\frac{1}{d}$.

Addition of Charge

- In work to appear we have repeated this computation for the Eisntein Maxwell System. Story very similiar, except we also need to specify a charge distribution function Q on the membrane. Corresponding to this new degree of freedom we have a new equation. The membrane equations with charge are
- $$\begin{aligned} &V_\perp.K.V_\perp - \left(\frac{2Q^2}{(1-Q^2)} V_\perp.K.u\right) = \frac{1-n_s^2}{s n_s} \\ &V_\perp.\nabla Q = Q V_\perp.K.u \\ &V_\perp.\nabla(u-n)_i + Q^2\left(V_\perp.\nabla n_i - u.\nabla(u-n)_i\right) \\ &V_\perp = \frac{ds}{n_s} - (n+u) \end{aligned} \tag{13}$$

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&V_{\perp}.\nabla Q = Q V_{\perp}.K.u \\
&V_{\perp}.\nabla(u-n)_i + Q^2 \left(V_{\perp}.\nabla n_i - u.\nabla(u-n)_i\right) \\
&V_{\perp} = \frac{ds}{n_s} - (n+u)
\end{aligned}
\tag{14}
$$

Once again the equations may be combined and geometrized

Charged stationary solutions

- It is natural to ask if the charged membrane equations also admit ‘rigidly rotating’ stationary solutions like their uncharged counterparts. We have investigated this question over the last two days, and find the answer is yes.
- The solutions we have found have u given by rigid rotation,

$$
\begin{aligned}
Q &= \alpha\gamma \\
s^2 &= 2g(r_i) \\
2g + (\partial g)^2 &= \beta \frac{\left(1 - \alpha - \sum_j \omega_j^2 r_j^2\right)^2}{1 - \sum_j \omega_j^2 r_j^2}
\end{aligned}
\tag{15}
$$

- It is easily argued that these equations admit solutions dual to charged rotating black holes. We have not managed to find the corresponding membrane solutions solutions in closed analytic form for nonzero ω, but have

Entropy Current

- The dissipative nature of black hole physics is quantified by the Hawking area increase theorem, which states that the area form on the event horizon always ‘increases’ along the future directed generators of the event horizon
- The area form may be used to construct an entropy current on our membrane which is guaranteed to be of positive divergence. Using our first order metric solution, we have computed the entropy current to first order in the $\frac{1}{d}$ expansion. We find

$$J^{\mu} = s^{d}\left(u^{\mu} - \frac{s}{d}\nabla.u\delta^{\mu s} + \frac{J_1^a}{d}\delta^{\mu a}\right)$$

where a runs over the $p+2$ coordinates (all membrane coordinates excluding s) and J_1^a are known functions). Can check that $\nabla.J$ vanishes at leading order. Expect first entropy production at order $1/d$. Need 2nd order corrected metric to compute that.

Charge and stress tensor

- Presumably our membrane is equiped with a charge current and stress tensor. We have not yet determined the formula for these currents valid to arbitrary orders in the $1/D$ expansion. At leading order, however, the expressions are easily guessed just from the fact that each little patch of our membrane is a little bit of a Schwarzschild (or RN) black hole.
- Using these formulas we have verified that we correctly reproduce the known thermodynamical formulas for uncharged rotating black holes at leading order in large D, and also have predictions for the thermodynamics of the (new) charged rotating black holes in high dimensions.

Radiation

- The stress tensor and charge furrent also presumably act as sources for radiation to infinity. It would be very interesting - and should not be too hard - to determine a formula for the charge and energy radiation fields (that result from any given membrane motion) at leading order the large d expansion. We hope to study this issue shortly.

‘Duality’ between black holes and membranes

- The ‘membrane’ equations of motion presented above are the main result of this talk. Let us recap their significance.
- Given any solution to the membrane equations, we have constructed a corresponding solution to large d gravity to first subleading order in $\frac{1}{d}$. The gravity solution reduces to flat space outside the membrane region. We expect that every large d solution of gravity that reduces to flat space outside the world volume of a compact world tube is dual to some membrane solution by our construction.
- A solution of gravity that vanishes outside the membrane clearly describes the intrinsic dynamics *of* the black hole in flat space. It follows the intrinsic dynamics of black holes in large dimensions is governed by our membrane equations (well defined initial value problem). It is tempting to use the name ‘membrane paradigm’ for this phenomenon.

Application to four dimensions

- I started this talk asking for a parameter for general relativity. We have found a parameter but is it of any use at $D = 4$?
- Probably unlikely worth testing. The following strategy suggests itself. Take a tough problem in $D = 4$ (like the collision of two black holes). Solve the corresponding membrane equations. Then compute the resultant radiation field and boldly set $D = 4$. Compare with the results of a full simulation. If there are even qualitative similarities between the answers, our expansion might prove useful for physicists calliberating gravity wave detectors to measure black hole mergers.

Conclusions

- We have reduced the equations that govern intrinsic black hole dynamics in the limit of a large number of dimensions to a well defined initial value problem for wiggly membrane membrane. The degrees of freedom on this membrane are its shape and a velocity field.
- Our construction should be generalized in many ways: to understand radiation, stress energy, entropy, charge and higher orders, ...
- In my opinion the construction presented in this paper diserves the name 'the membrane paradigm of black hole physics': we see it emerges at large D.
- It will be interesting to see how well our large D solution compares with results from numerical simulations in $d = 4$.

Things to do

- Determine the membrane charge current and stress tensor at subleading orders in $1/D$
- Find the radiation formula
- Uncharged and charged computation to 2nd order
- Compute divergence of entropy current to 1st nontrivial order
- Study potentially new stationary solutions
- Numerically investigate collision of two membranes and compare with GTR simulations of collision of two black holes.
- ...