

# 5d/6d SCFTs & new dualities

**Kim, Sung-Soo**

Korea Institute for Advanced Study

KIAS-YTTP Joint Workshop 2015

Geometry in Gauge Theories and String Theory

September 15-18, 2015

1114 International Conference Hall, 1fl., KIAS, Seoul, Korea

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This work is based on various collaborations with

Hiroataka Hayashi (Instituto de Fisica Teorica),  
Kimyeong Lee (KIAS),  
Masato Taki (RIKEN),  
Futoshi Yagi (KIAS)

[arXiv:1504.03672](https://arxiv.org/abs/1504.03672)

[arXiv:1505.04439](https://arxiv.org/abs/1505.04439)

[arXiv:1509.03300](https://arxiv.org/abs/1509.03300)

Also See Futoshi Yagi's poster presentation tomorrow

[6d  \$N=\(1,0\)\$  CFT via 5-brane web](#)

**We report  
a new understanding of 5d SCFTs  
in connection with 6d SCFTs**

# 6d SCFTs

**(2,0) theory: worldvolume theory of M5 branes  
but mysterious..**

**Quite recently, F-theory classification of (1,0) theories**

[Heckman-Morrison-Vafa '13]  
[Del Zotto-Heckman-Tomasiello-Vafa '14]  
[Heckman-Morrison-Rudelius-Vafa '15]  
[Bhardway '15]  
.....

**M-theory perspective:**

[Yuji Tachikawa's talk]

One of the simplest (1,0) theory : E-string theory

Reduction to low dimensional theories  $\longrightarrow$  Rich physics

Dualities

# 5d/6d relation

5d **Maximally supersymmetric Yang-Mills** theory  
= a circle compactification of **6d** (2,0) theory

[Lambert, Papageorgakis, Schmidt-Sommerfeld '10]  
[Douglas '10]

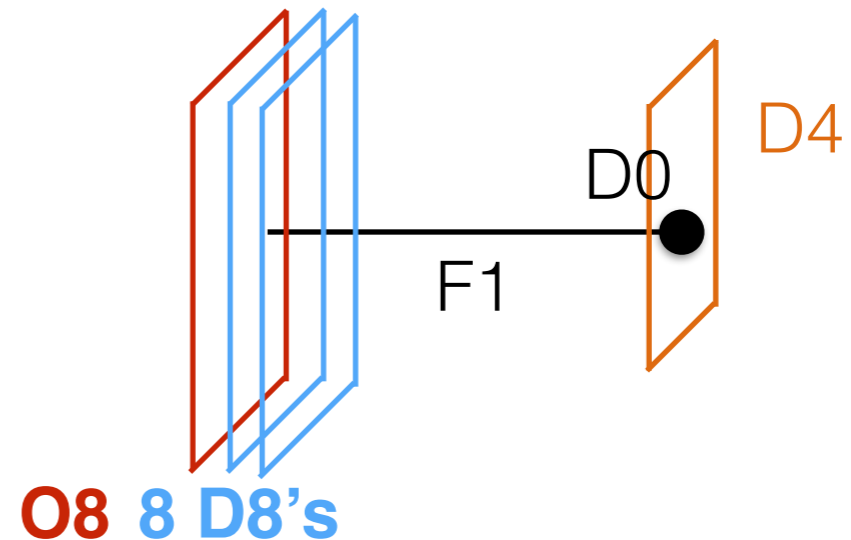
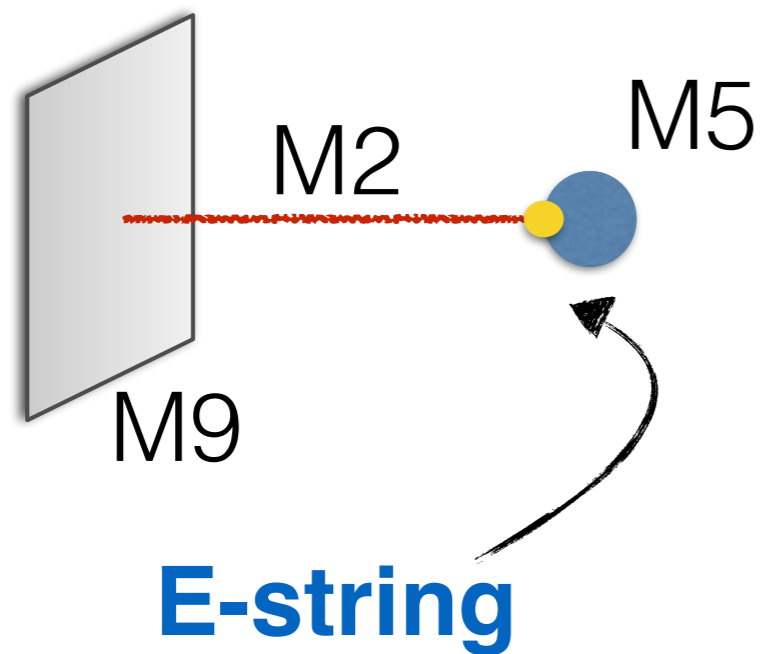
$$g_{YM}^2 = R_6$$



circle radius

# UV completion as 6d SCFT

E-string theory



**E-string theory on a circle = 5d SU(2) theory with Nf=8**  
**KK modes = Instantons**

5d SU(2) Nf=8 Nekrasov partition function

[ '14 Chiung Hwang, Joonho Kim, Seok Kim, Jaemo Park ]

E-string partition function  
(elliptic genus)

[ '14 Seok Kim, Joonho Kim, Kimyeong Lee, Jaemo Park, Vafa ]

# 5d SCFTs

Flavor decoupling of 5d SU(2) theory of  $N_f=8$



SU(2) gauge theory with  $N_f = 0, 1, \dots, 7$   
flavors has non-trivial **5d UV fixed point**  
(Superconformal theory)

[Seiberg '96]



**Q1: How do we determine what 5d theories have UV completion as 6d SCFTs?**

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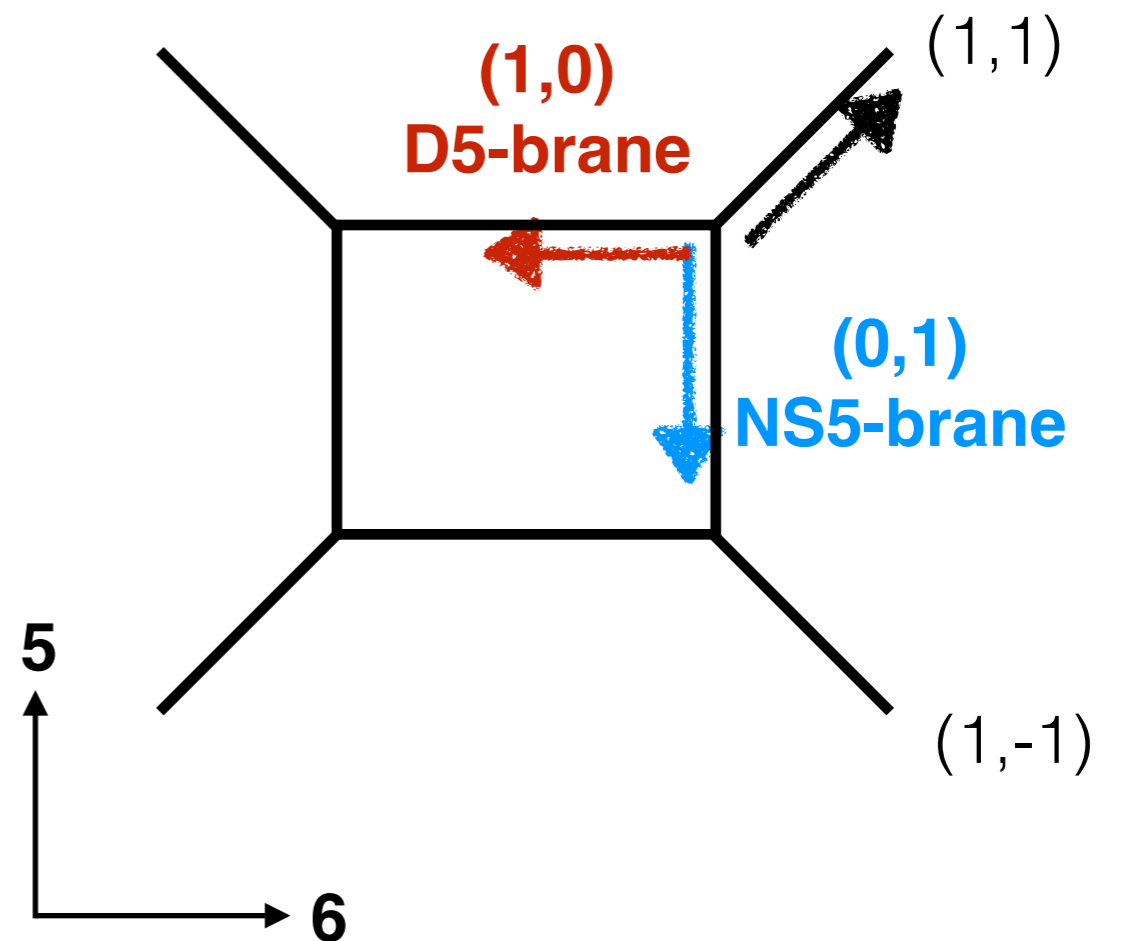
**A1: We developed a diagrammatic way that distinguishes 6d UV completions from 5d UV completions → Tao diagram**

# $(p,q)$ web diagram and 5d $SU(2)$ theory

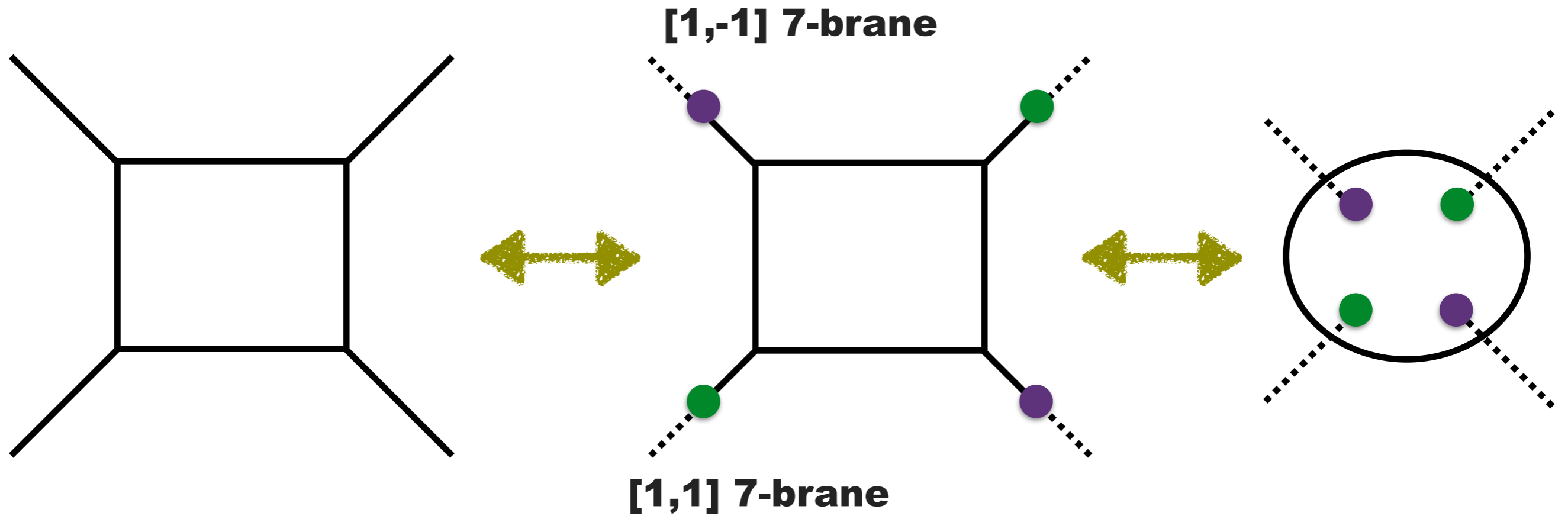
[Aharony-Hanany, '97]

Configuration with **charge conservation, tension balance**  
: the  $(p,q)$  web diagram

	0	1	2	3	4	5	6	7	8	9
NS5	-	-	-	-	-	-	-	-	-	-
D5	-	-	-	-	-	-	-	-	-	-
(1,1)	-	-	-	-	-	-	-	-	-	-



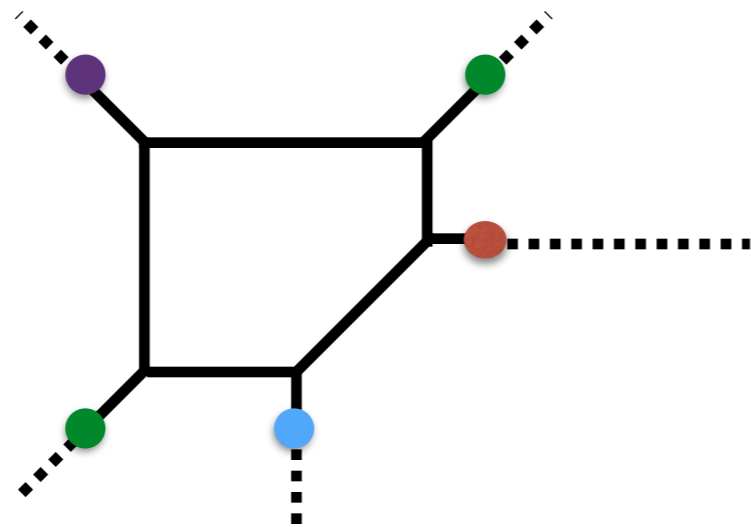
# 5d SU(2) theory via 5-branes and 7-branes



	0	1	2	3	4	5	6	7	8	9
NS5	-	-	-	-	-	-	.			
D5	-	-	-	-	-	.	-			
D7	-	-	-	-	-	.	.	-	-	-

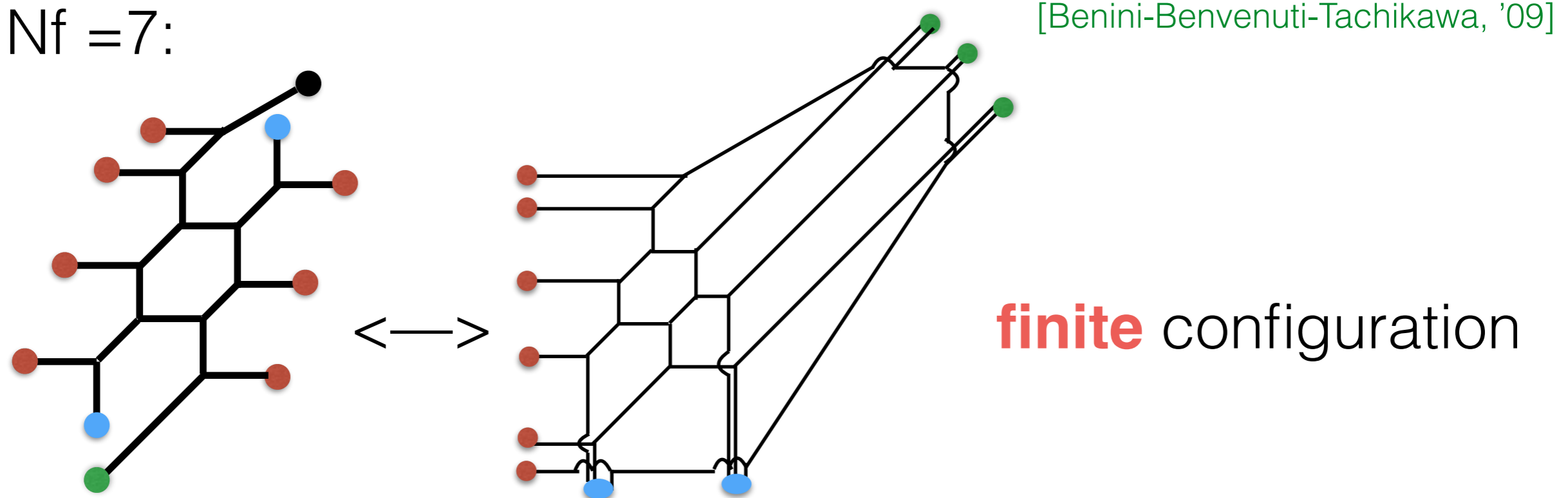
# Nf Flavours : adding Nf D7branes ●

Nf = 1:



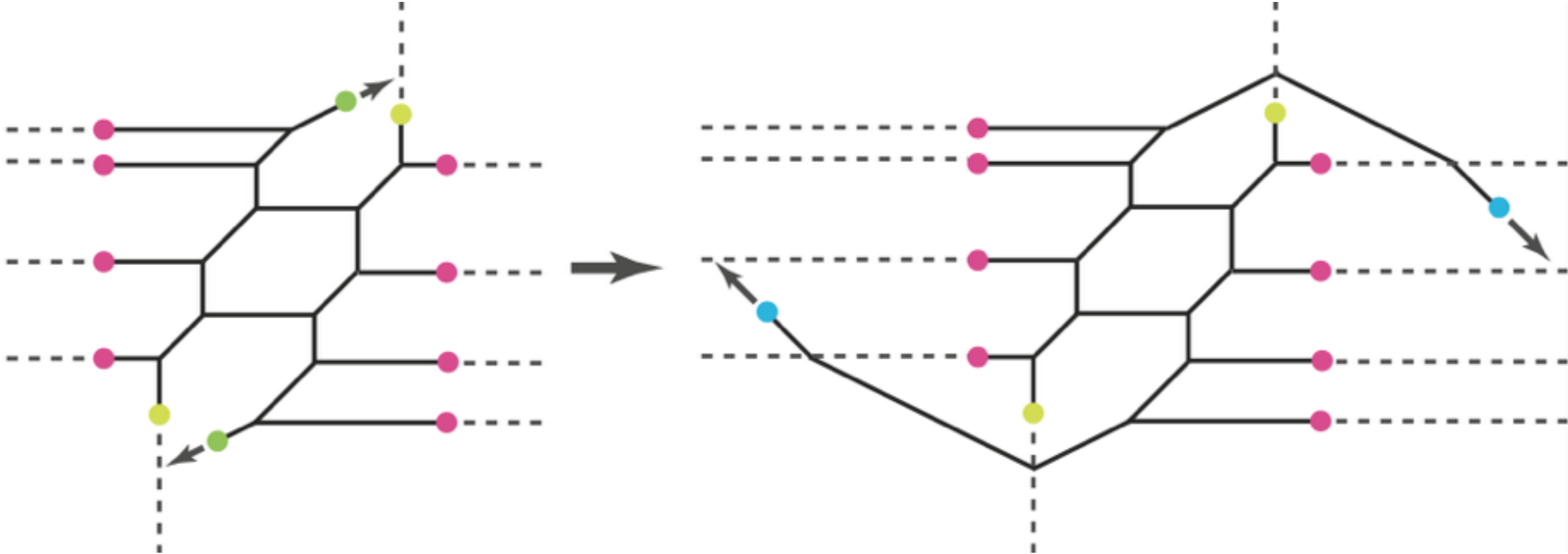
Adding flavors up to Nf=7 can be done systematically

Nf = 7:

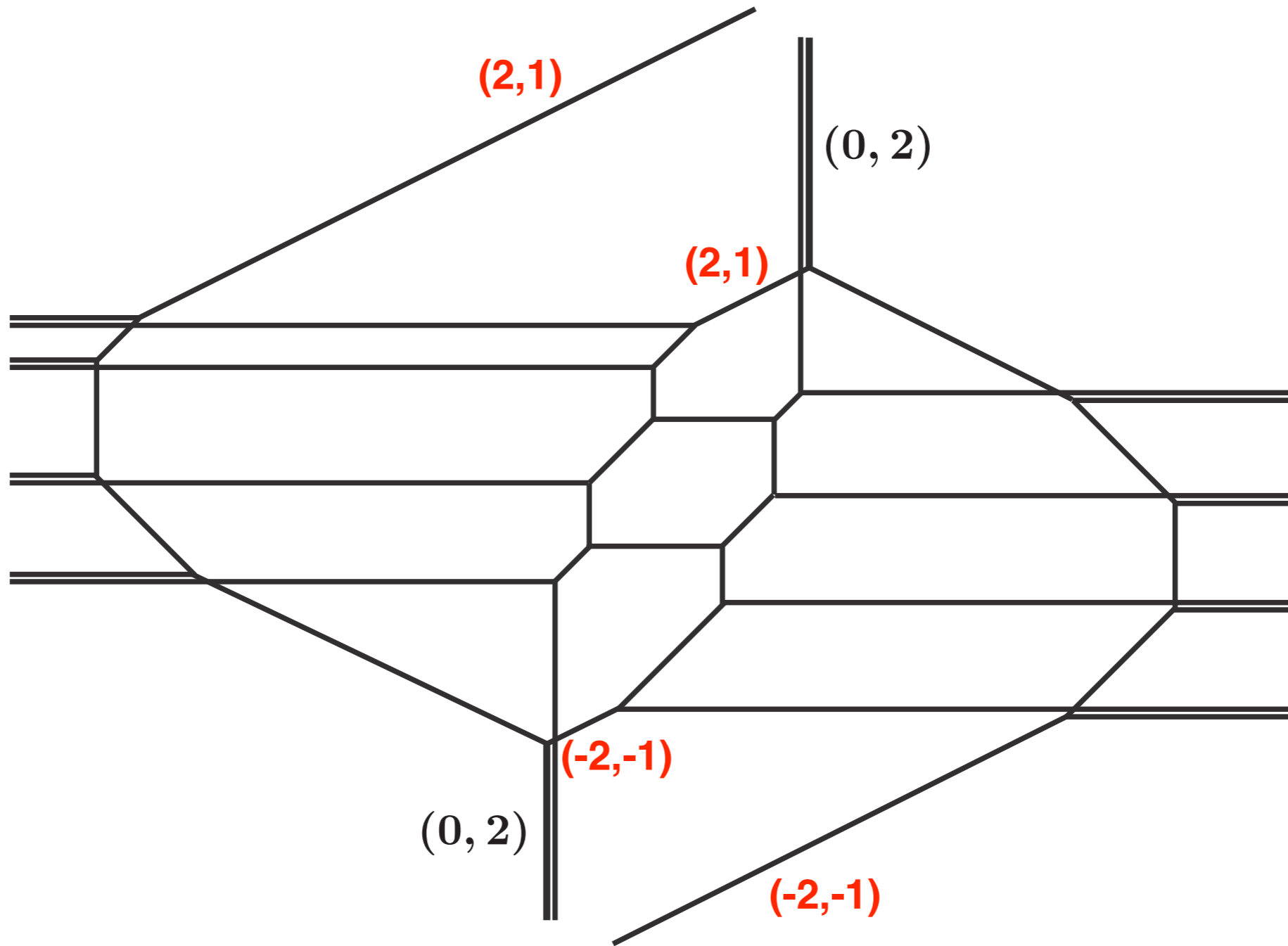


**Nf=8 brane web diagram?**

Naively,



By pulling out 7-branes to infinity



Spirally rotating! One revolution, charges remain the same.

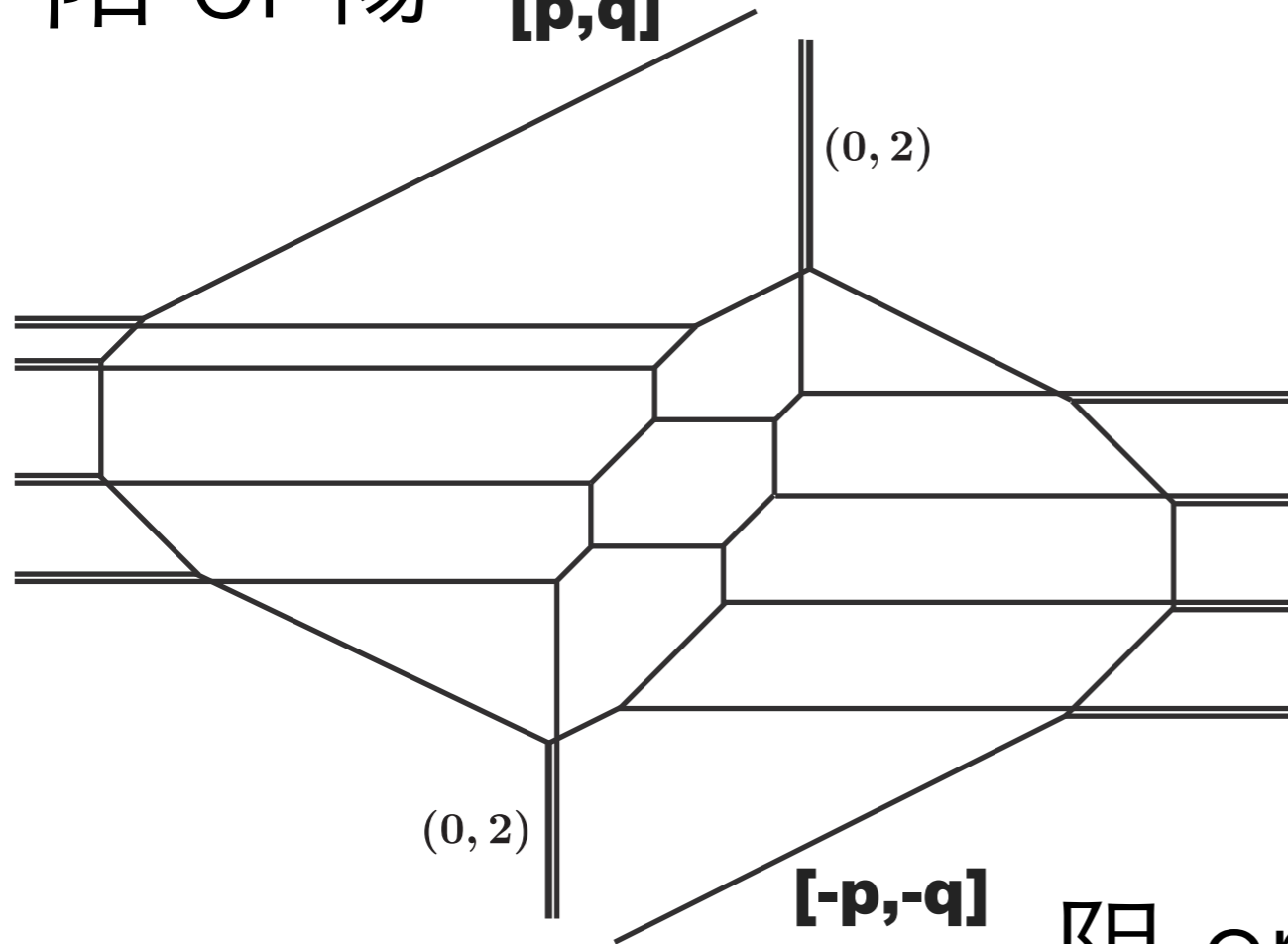
**Infinitely rotating spiral diagram**



# The shape looks like

阳 or 陽

$[p, q]$



$(0, 2)$

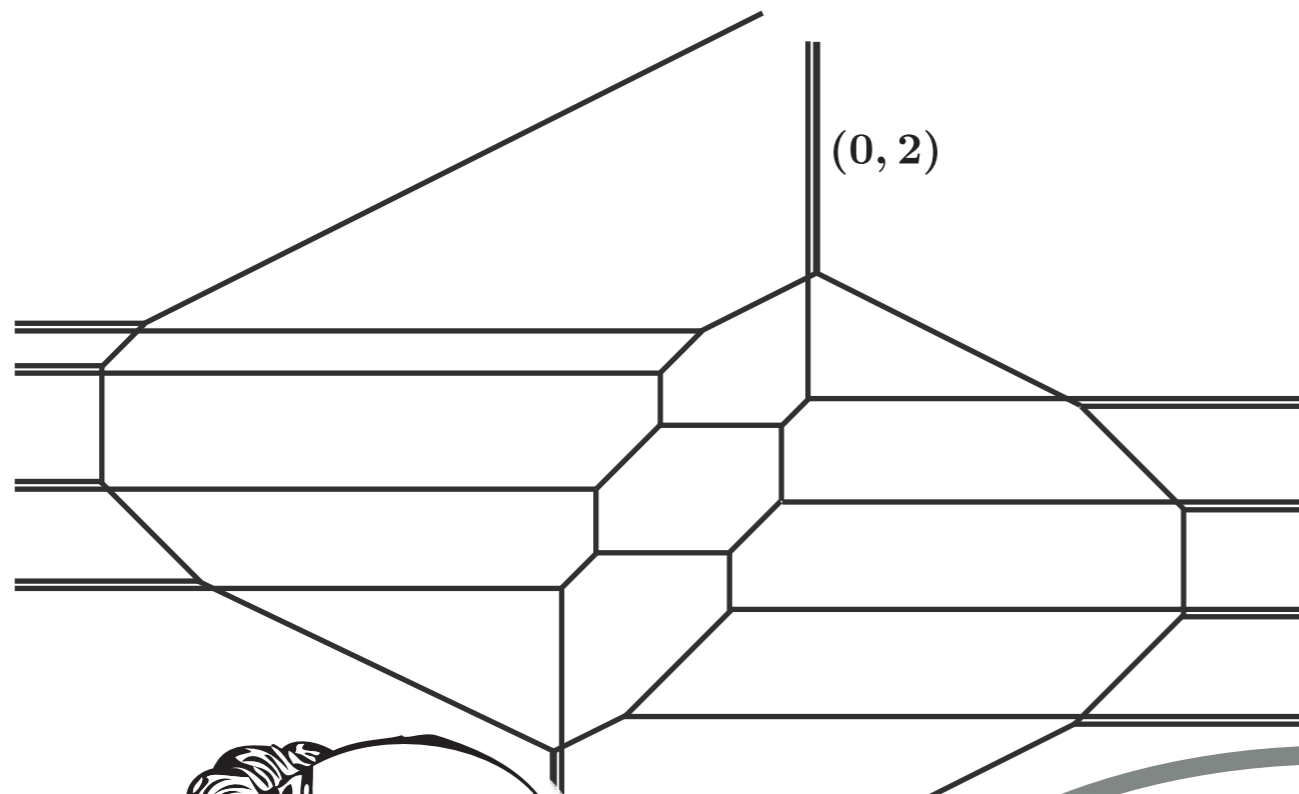
$[-p, -q]$

阴 or 陰



# We call it **Tao diagram**...

[SSK-Taki-Yagi, '15]



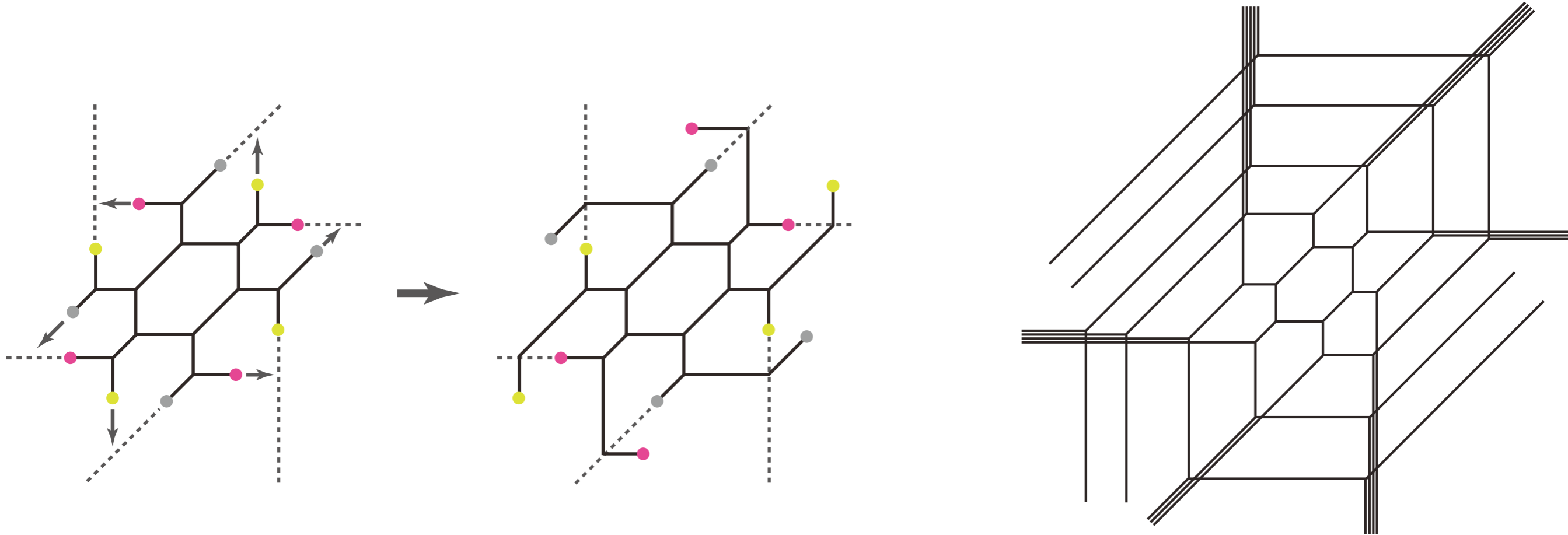
**Tao probes  
the end of the  
world (M9)...**



**Laozi 老子**

There are **various equivalent** forms of Tao diagram.

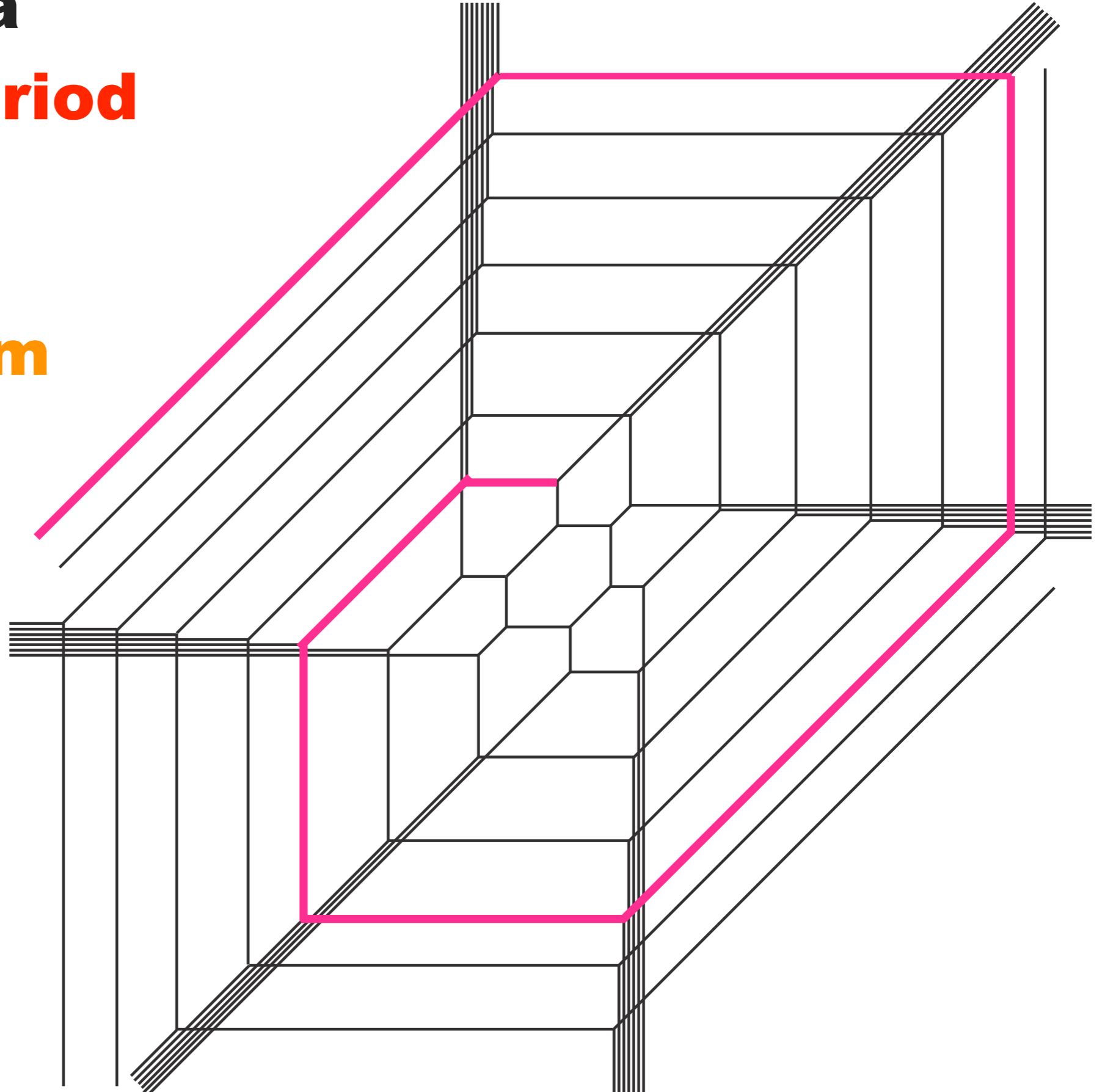
Another **Tao diagram for SU(2) gauge theory with 8 flavors**



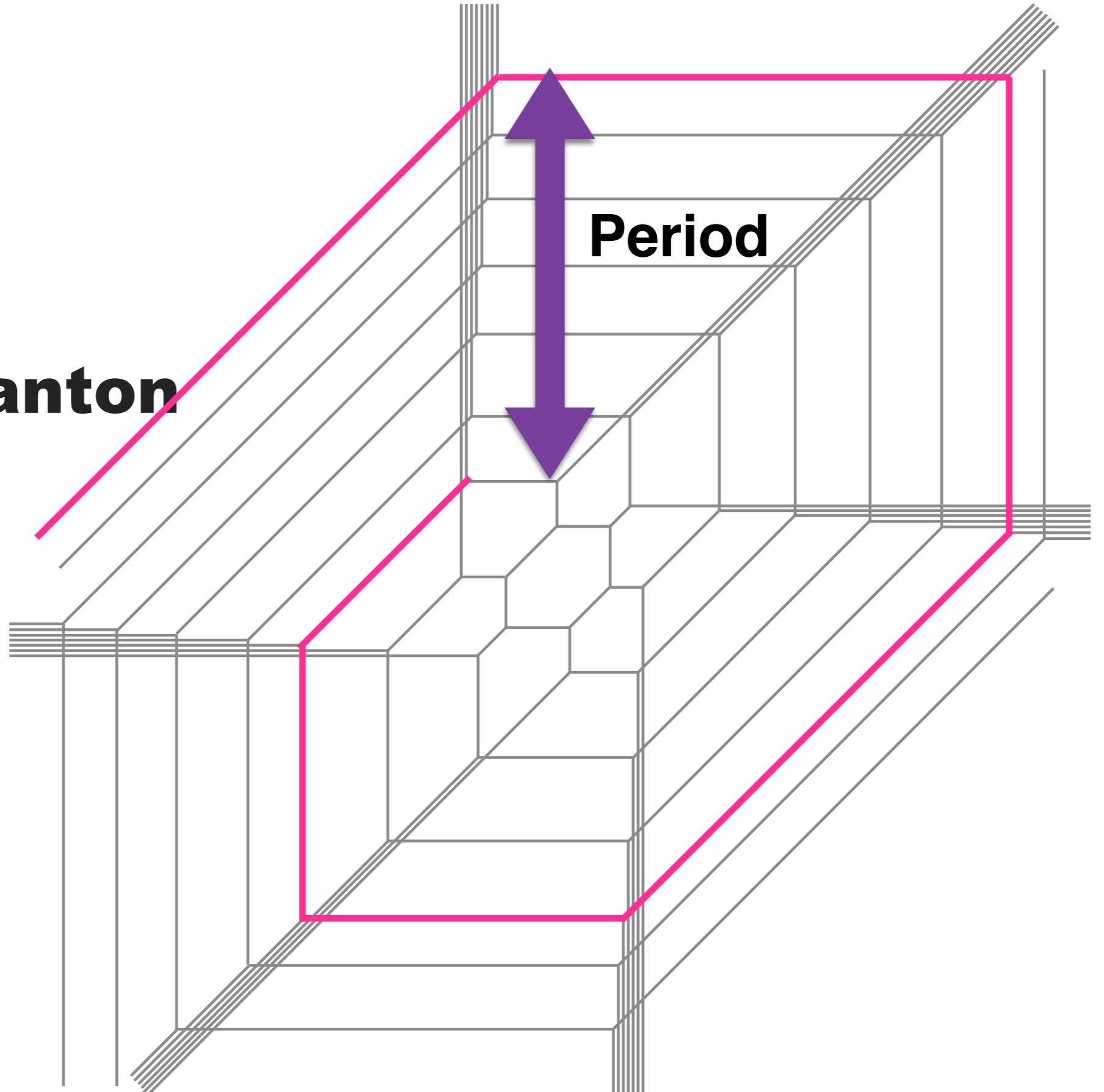
This is more practical and **useful for computations**

**spiral with a  
constant period**

**KK spectrum**



**period = instanton**  
 $\sim R^{-1}$



# Tao diagrams give new perspective on 6d SCFTs

**Tao diagram:** spiral, constant period

- Naturally identified as **a 6d theory on a circle**  
(a compact dimension arises)
- Computational tool:  
**Partition function**

**(Difficult) 6d SCFTs may be tractable!**

# Partition function from Tao diagram

$$Z_{E-string} = \text{PE} \left[ \sum_{m=0}^{\infty} \mathcal{F}_m(y, A, q) \mathfrak{q}^m \right] = \text{PE} \left[ \frac{1}{(1-q)(1-q^{-1})} \sum_{n=1}^{\infty} \tilde{f}_n A^n \right]$$

$$\begin{aligned} \tilde{f}_1 &= \chi^{(1)} + \chi_c \mathfrak{q} + \left( 2\chi_2(q)\chi^{(1)} + \chi^{(3)} + \chi^{(1)} \right) \mathfrak{q}^2 + \left( \chi^{(1)}\chi_s + 2\chi_2(q)\chi_c \right) \mathfrak{q}^3 \\ &\quad + \left( 3\chi_3(q) + 4\chi_2(q) + 2 \right) \chi^{(1)} + 2\chi_2(q)\chi^{(3)} + \chi^{(5)} + \chi^{(1)}\chi^{(2)} \mathfrak{q}^4 + \mathcal{O}(\mathfrak{q}^5), \\ \tilde{f}_2 &= -2 - 2\chi_s \mathfrak{q} - \left( 2\chi^{(4)} + (3\chi_2(q) + 2)\chi^{(2)} + 4(\chi_3(q) + \chi_2(q) + 1) \right) \mathfrak{q}^2 \\ &\quad - \left( 2\chi^{(2)}\chi_s + 3\chi_2(q)\chi^{(1)}\chi_c + 4(\chi_3(q) + \chi_2(q) + 1)\chi_s \right) \mathfrak{q}^3 \\ &\quad + \left( (5\chi_4(q) + 6\chi_3(q) + 11\chi_2(q) + 8)\chi^{(2)} + (4\chi_3(q) + 4\chi_2(q))\chi^{(4)} + (3\chi_2(q) - 2)\chi^{(6)} \right. \\ &\quad \left. + (4\chi_3(q) + 3\chi_2(q) + 2)(\chi^{(1)})^2 + 3\chi_2(q)\chi^{(1)}\chi^{(3)} + 2\chi^{(1)}\chi^{(5)} + 2(\chi^{(2)})^2 + 2(\chi_s)^2 \right. \\ &\quad \left. + (6\chi_5(q) + 8\chi_4(q) + 16\chi_3(q) + 20\chi_2(q) + 10) \right) \mathfrak{q}^4 + \mathcal{O}(\mathfrak{q}^5). \end{aligned}$$

**reproduces the E-string partition function (elliptic genus)**

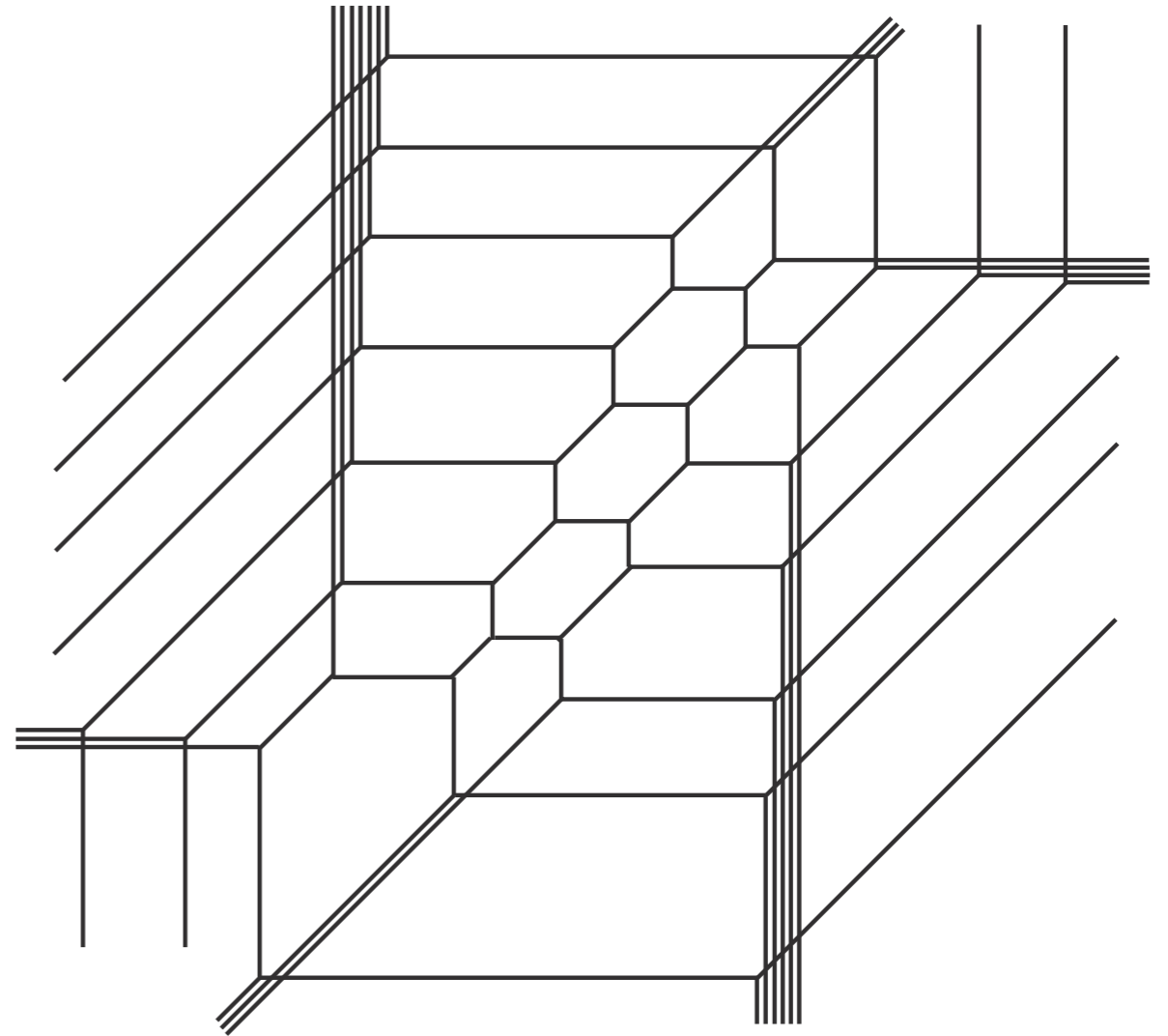
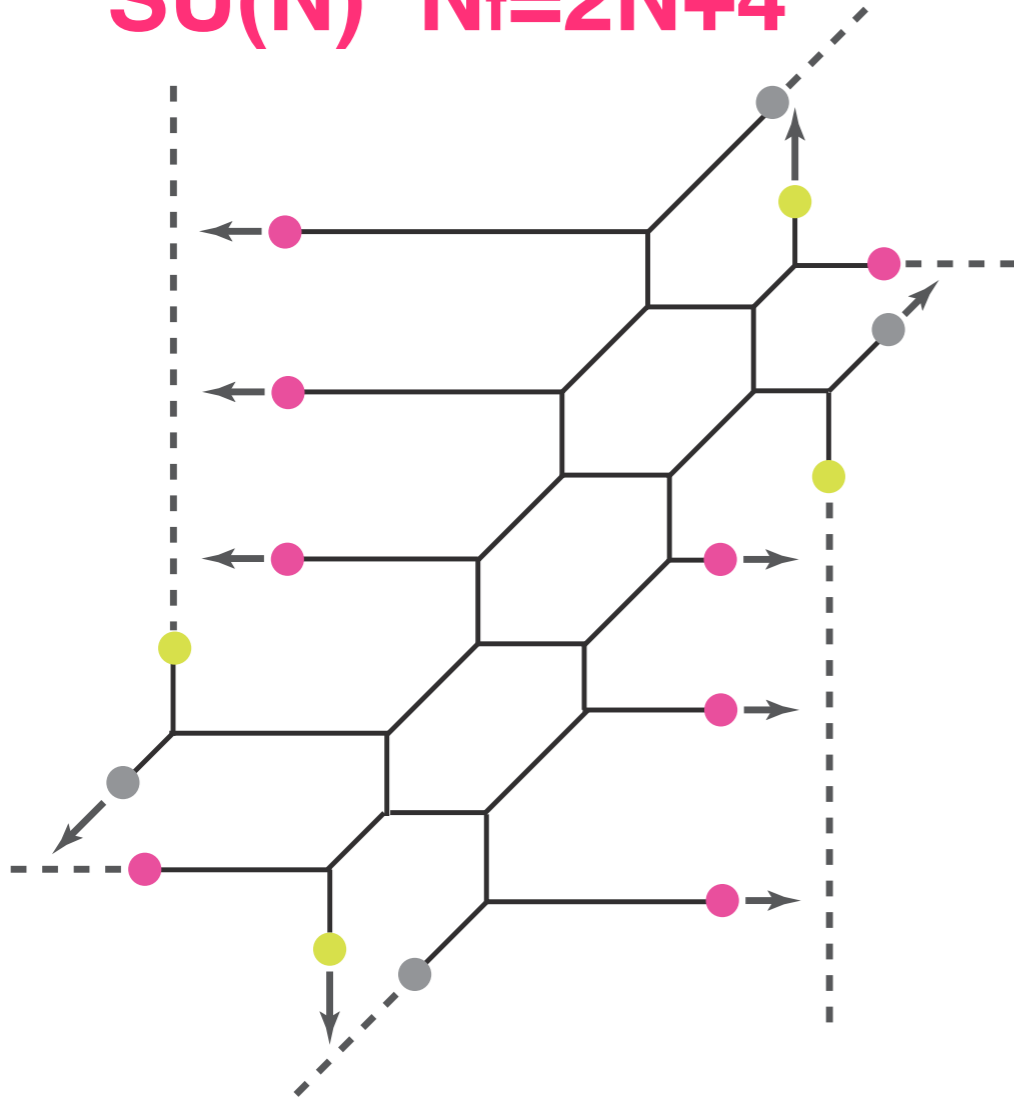
by (up to 4 instantons)

[ '14 Kim, Kim, Lee, Park, Vafa ]

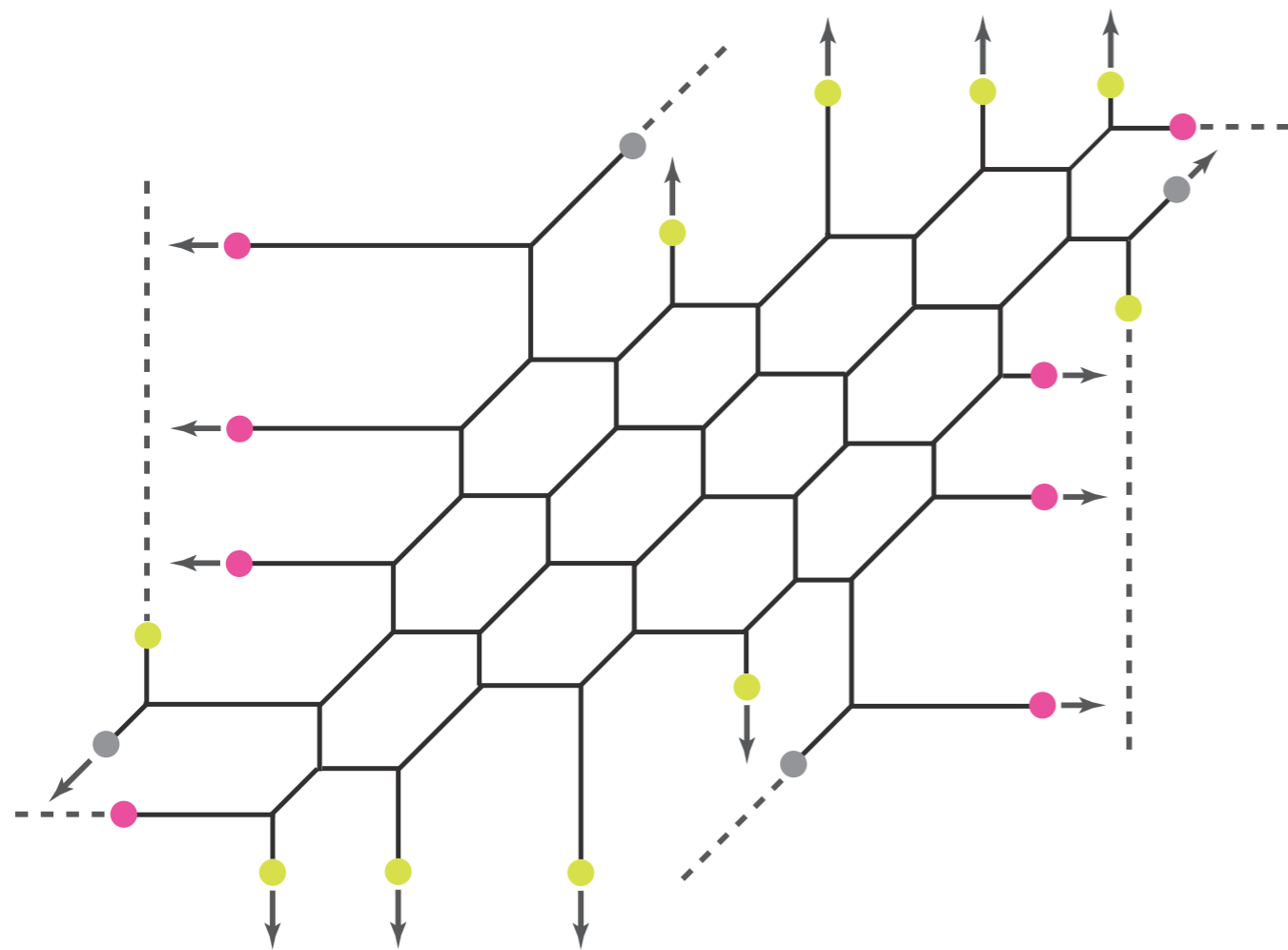
**Tao diagram indeed sees the E-string theory on a circle**

# Many more Tao web diagrams

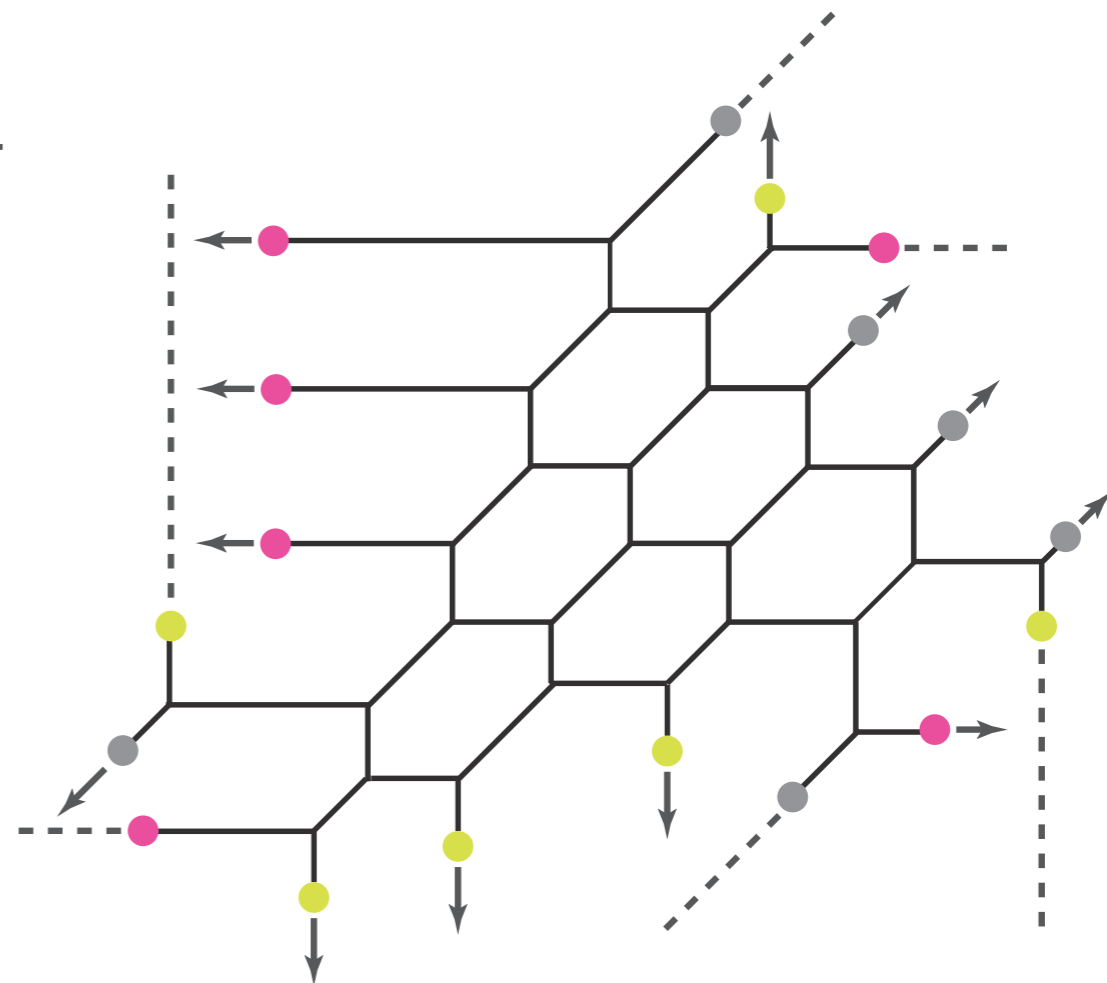
$SU(N)$   $N_f=2N+4$







quiver type



T<sub>N</sub> type

**Claim:**

**Tao web diagrams imply  
that a 5d theory has UV  
completion as a 6d SCFT**

**Q2: What are 6d SCFTs that  
Tao web diagrams correspond to?**

**6d SCFT  $\rightarrow$  5d Tao web diagram**

Previous Tao web diagrams' are reproduced

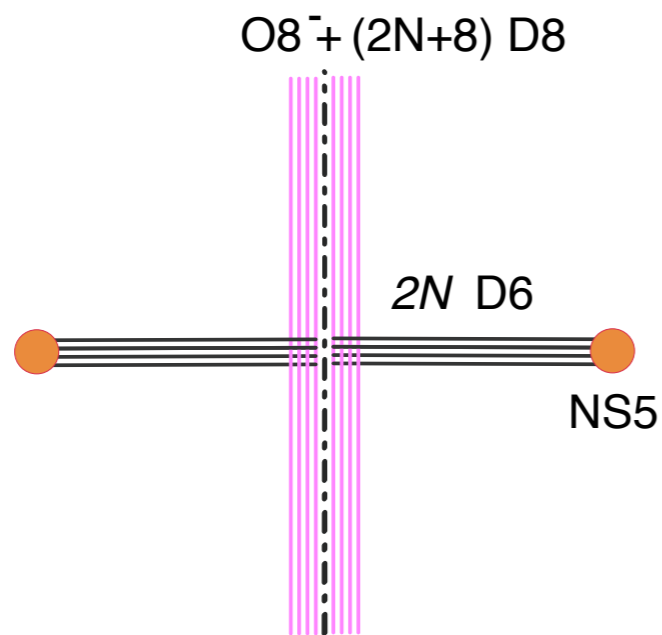
**Along the way, new dual picture arises...**

6d ( $D_{N+4}, D_{N+4}$ ) minimal conformal matter:

M5 probing the  $D_{N+4}$  Singularity [Del Zotto-Heckman-Tomasiello-Vafa '14]

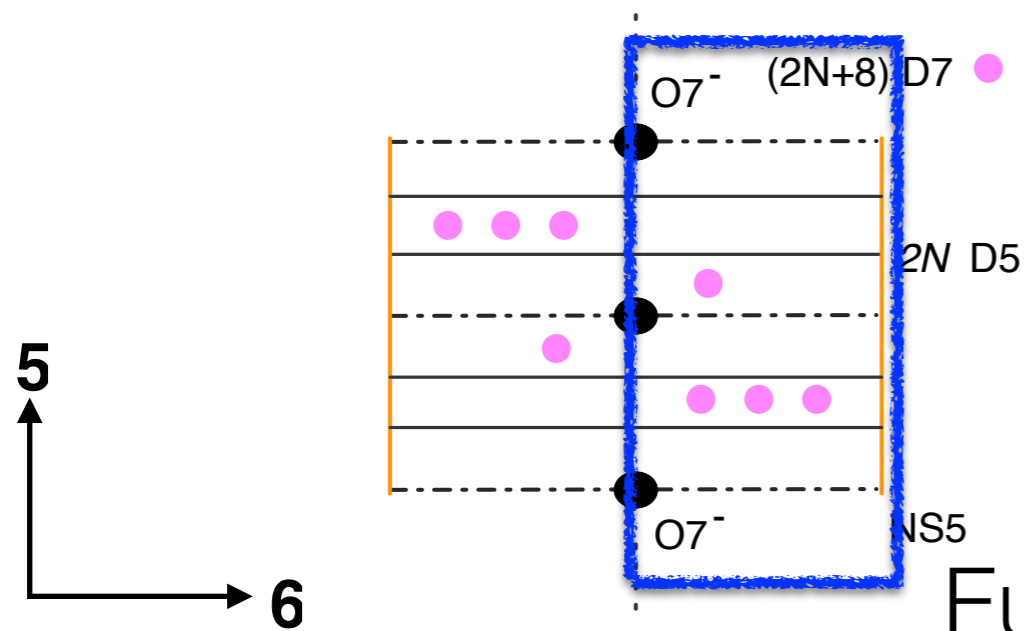
## 6d $Sp(N)$ theory with $N_f = 2N+8$ , one tensor (NS5)

[Hanany-Zaffaroni '97][Brunner-Karch '97]



	0	1	2	3	4	5	6	7	8	9
NS5	-	-	-	-	-	-	-			
D6	-	-	-	-	-	-	-	-		
D8/O8	-	-	-	-	-	-	-	-	-	-

$x^5$  : compactify & T-dual:  $O8 \rightarrow 2 O7$



	0	1	2	3	4	5	6	7	8	9
NS5	-	-	-	-	-	-				
D5	-	-	-	-	-		-			
D7/O7	-	-	-	-	-	.	.	-	-	-

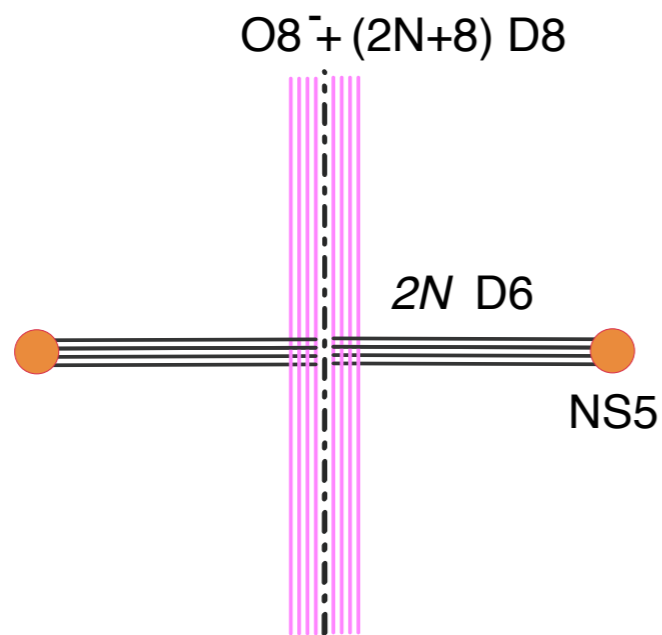
Fundamental region

6d ( $D_{N+4}, D_{N+4}$ ) minimal conformal matter:

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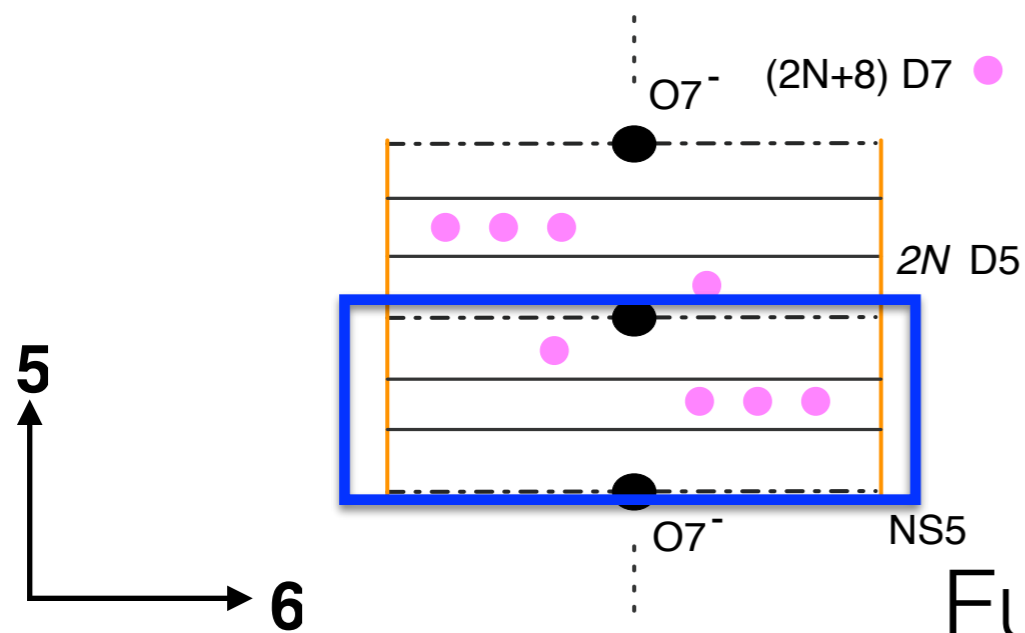
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	0	1	2	3	4	5	6	7	8	9
NS5	-	-	-	-	-	-	-	-	-	-
D6	-	-	-	-	-	-	-	-	-	-
D8/O8	-	-	-	-	-	-	-	-	-	-

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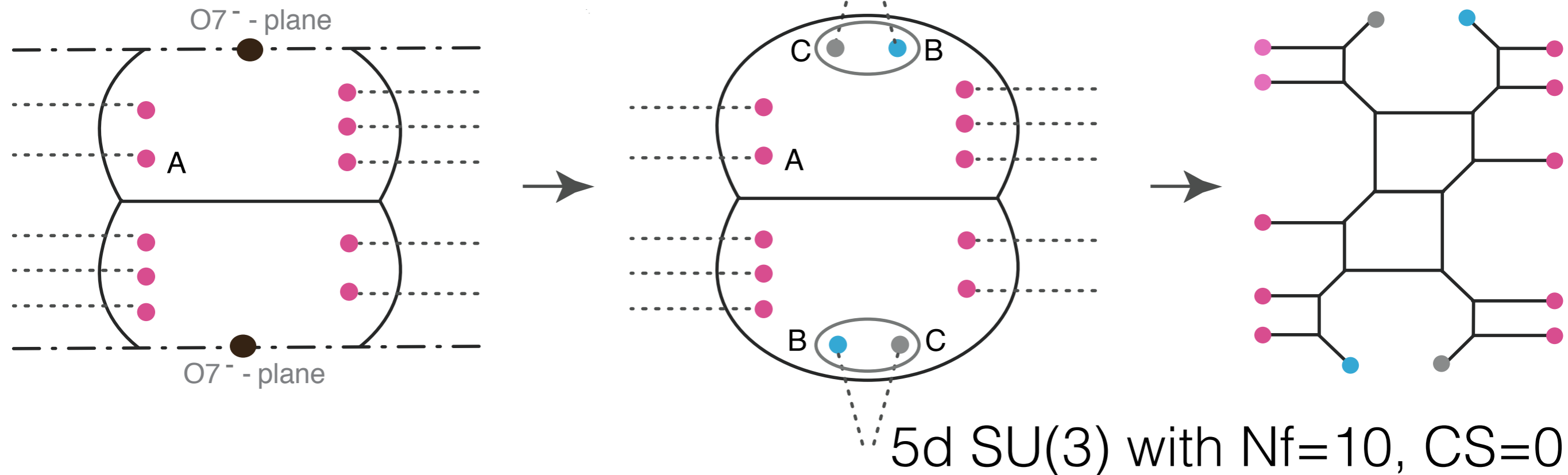
	0	1	2	3	4	5	6	7	8	9
NS5	-	-	-	-	-	-	-	-	-	-
D5	-	-	-	-	-	-	-	-	-	-
D7/O7	-	-	-	-	-	.	.	-	-	-

Fundamental region

# 5d $SU(N+2)$ theory with $N_f = 2N+8$

ex:  $N=1 \rightarrow$  6d  $Sp(1)$  with  $N_f=10$

$$A = [1, 0], B = [1, -1], C = [1, 1]$$



Following Ashoke Sen,  $O7$  can be resolved non-perturbatively into a pair of two 7-branes

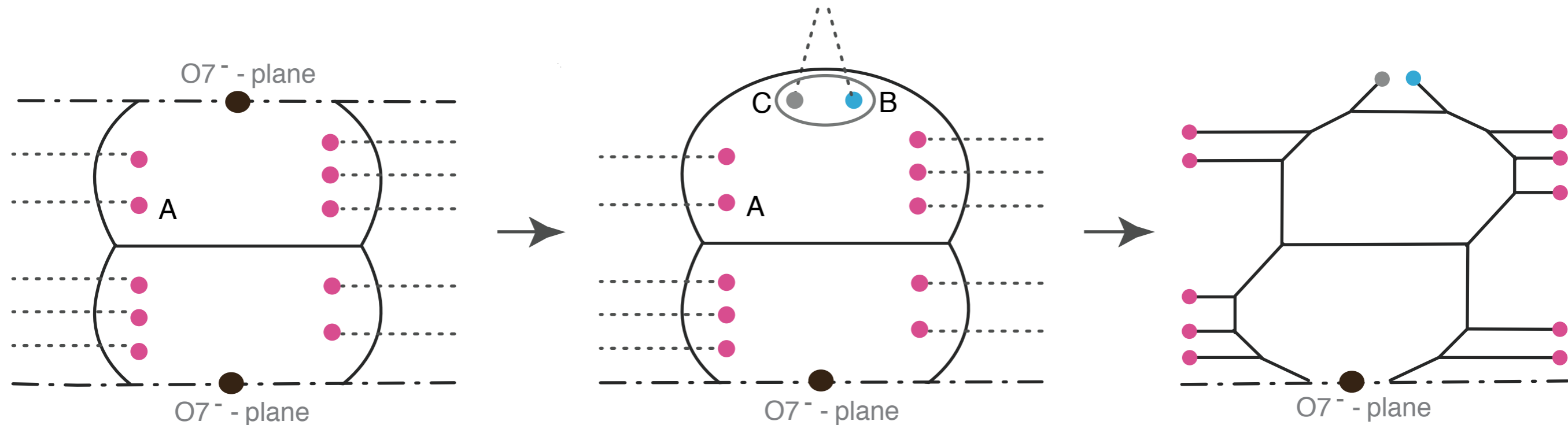
[Hayashi-**SSK**-Lee-Taki-Yagi '15]

[Yonekura '15]

# 5d $Sp(N+1)$ theory with $N_f=2N+8$

Resolving **only one**  $O7^-$ :

[Hayashi-SSK-Lee-Yagi '15]



We thus have

5d  $Sp(2)$  with  $N_f=10$

**5d  $SU(N+2)$  theory with  $N_f=2N+8$**



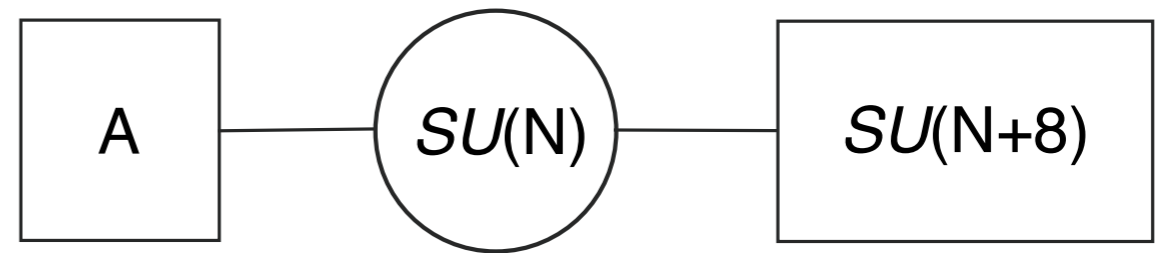
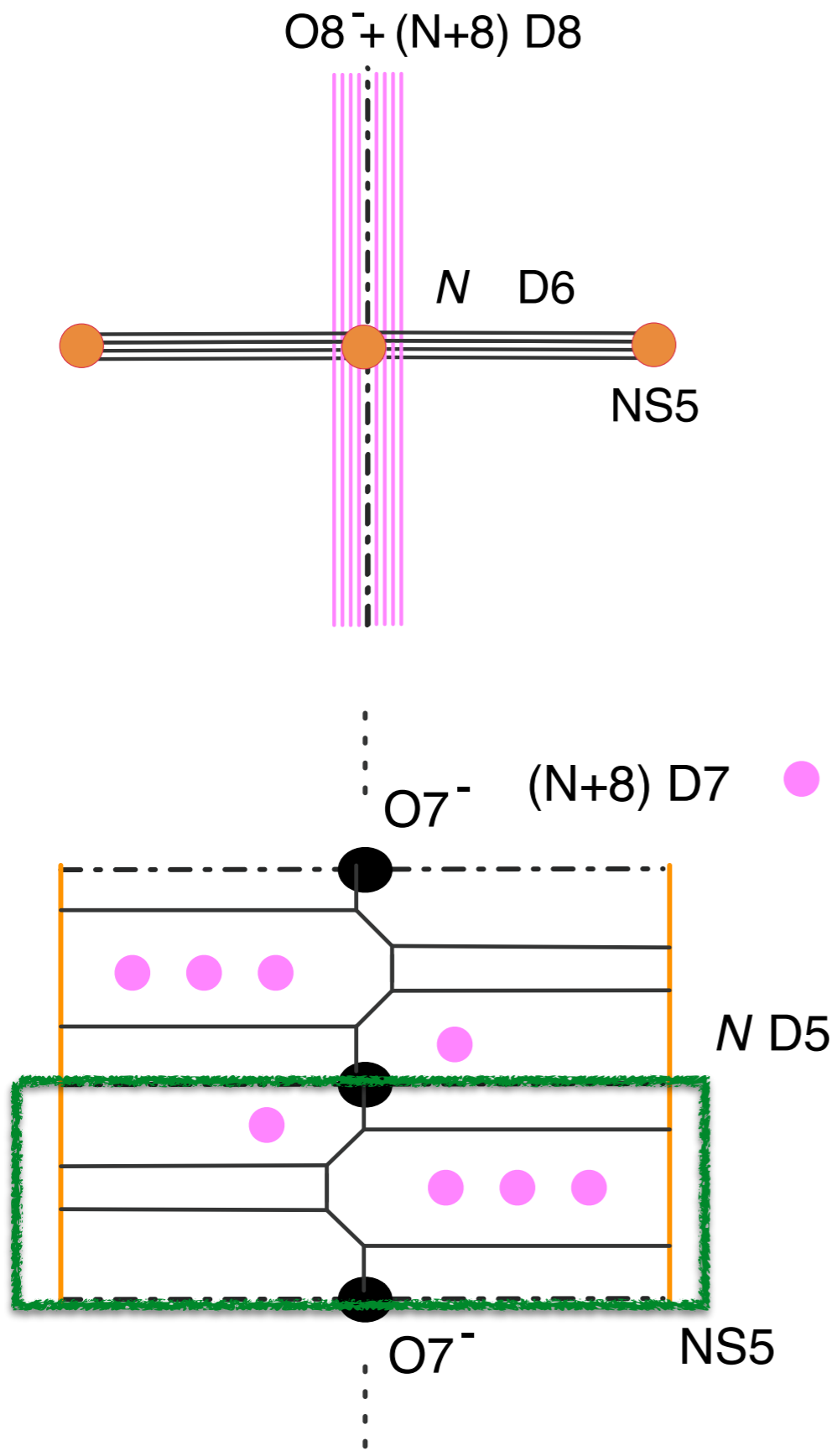
**5d  $Sp(N+1)$  theory with  $N_f=2N+8$**

[Hee-Cheol Kim's talk]

Flavor decoupling  $\rightarrow$  5d dualities

[Gaiotto-Kim '15]

# Distribution duality

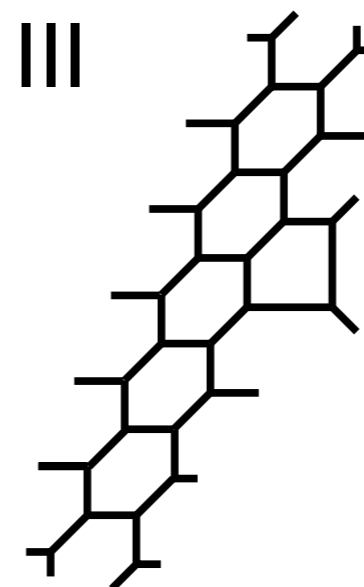
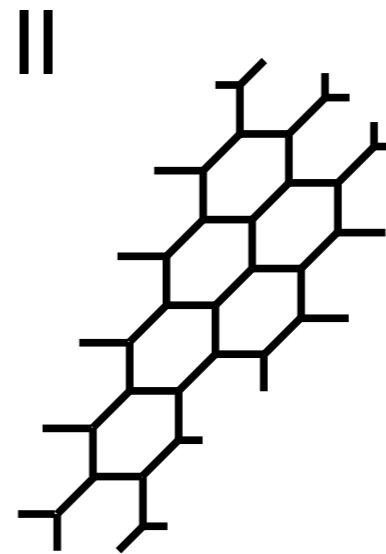
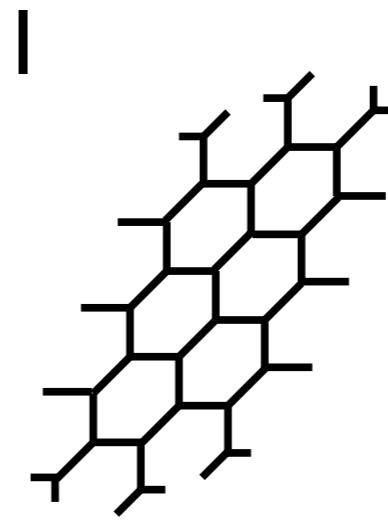




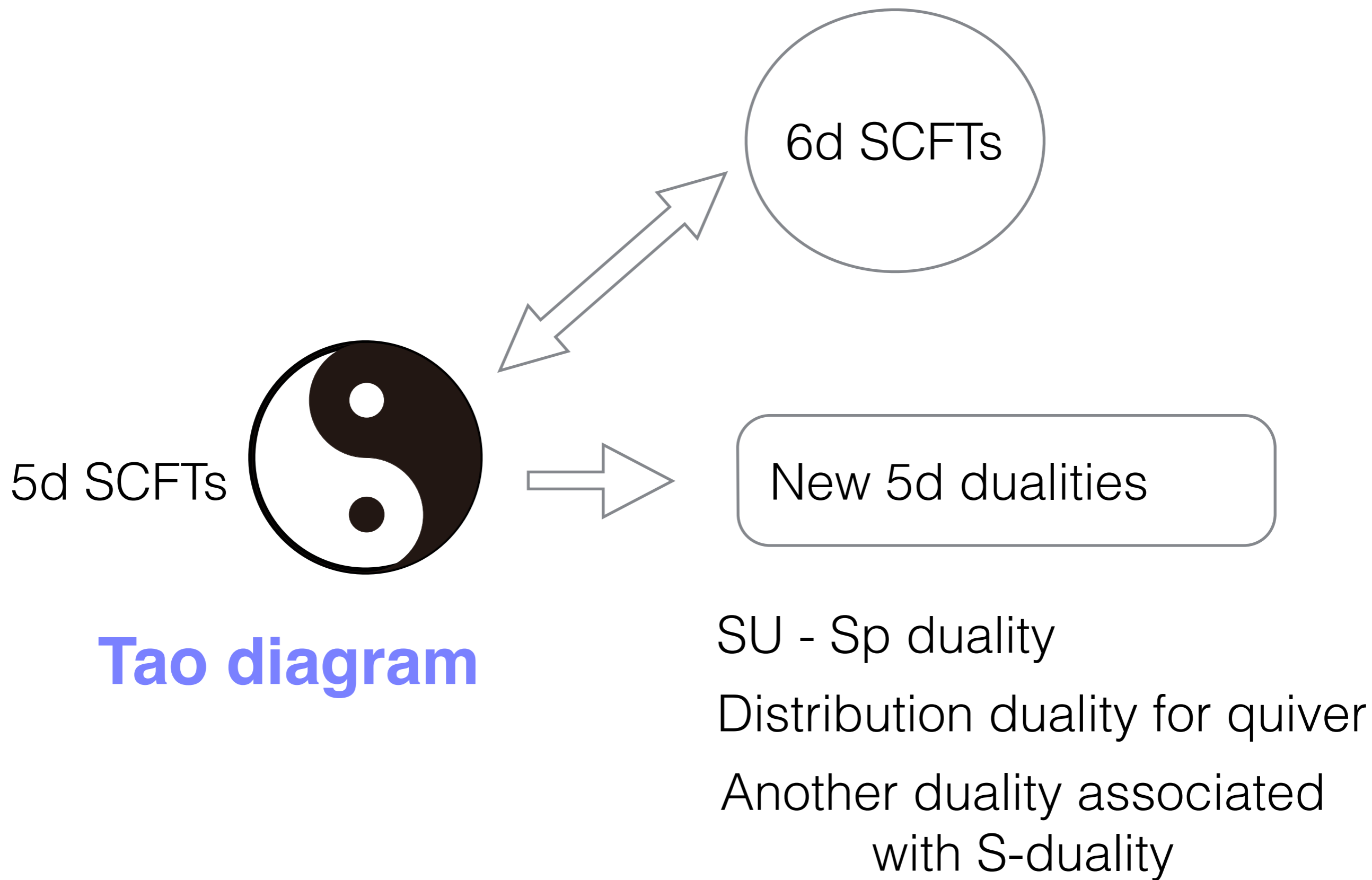
6d  $SU(6)$  theory  
with  $N_f=14$ ,  $N_a=1$



various 5d quivers  
(depending on  
D5, D7 distributions)



# New understandings of 5d SCFTs : Summary



# Conclusion

From Tao diagrams and 6d brane configuration

We claim

- Any 5d configurations of the critical number of flavors that make **Tao web diagram**, have UV completion as 6d SCFTs
- 6d  $Sp(N)$  gauge theory with  $N_f=2N+8$  and a tensor multiplet
  - 5d  **$Sp(N+1)$**  gauge theory with  $N_f=2N+8$  **one 07**
  - 5d  **$SU(N+2)$**  gauge theory with  $N_f=2N+8$  **two 07**

dual to each other;

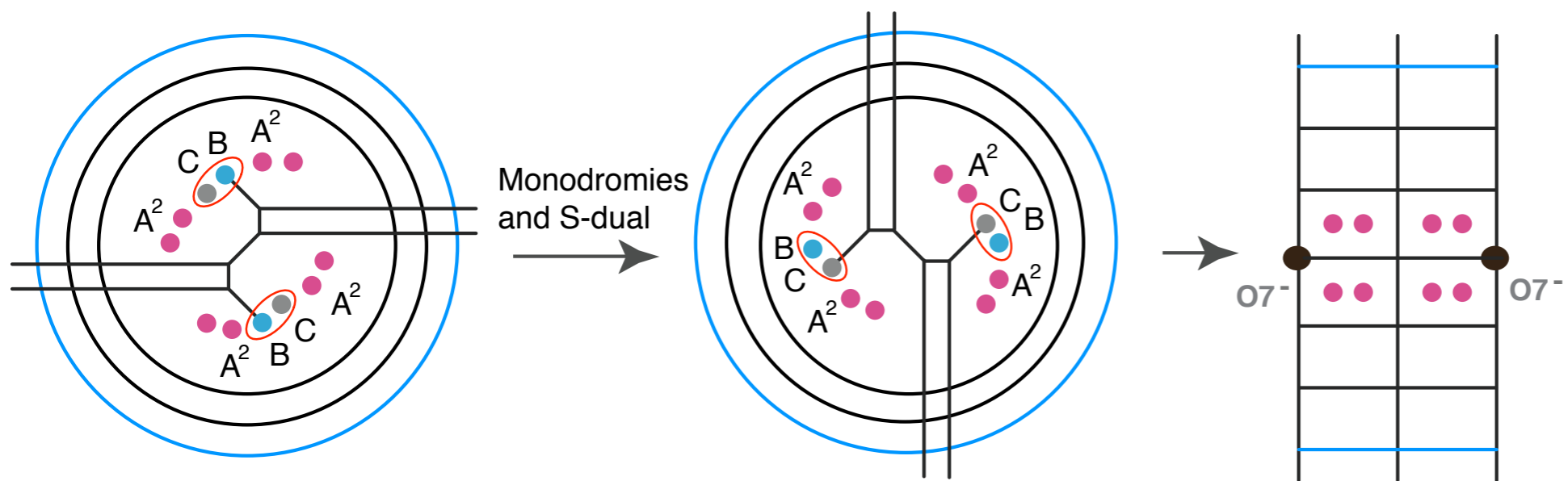
as they have the same UV fixed point.

Flavor decoupling limit

extra slides

6d  $[1_A, 8] - \text{SU}(4) - \text{SU}(4) - \text{SU}(4) - [4]$

5d  $[1_A, 4] - \text{SU}(7) - \text{SU}(7) - [1_A, 4]$



5d  $[3] - \text{Sp}(3) - (\text{SU}(7) - [2]) - \text{Sp}(3) - [3]$

