Generalising Calabi–Yau Geometries

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Introduction

Supersymmetric background with no flux

$$\nabla_m \epsilon = 0 \qquad \Longrightarrow \qquad \text{special holonomy}$$

Classic case: Type II on Calabi–Yau

Geometry encoded in pair of integrable objects

$$\mathrm{d}\omega = 0 \qquad\qquad \text{symplectic structure}$$
$$\mathrm{d}\Omega = 0 \qquad\qquad \text{complex structure}$$

Supersymmetric background with flux (eg type II)?

$$\left(\nabla_m \mp \tfrac{1}{8} H_{mnp}\gamma^{np}\right)\varepsilon^{\pm} + \tfrac{1}{16}\mathrm{e}^{\phi}\sum_i \not{F}_{(i)}\gamma_m\varepsilon^{\mp} = 0$$

$$\gamma^m\left(\nabla_m \mp \tfrac{1}{24} H_{mnp}\gamma^{np} - \partial_m\phi\right)\varepsilon^{\pm} = 0$$

What is the geometry?

- special holonomy? analogues of ω and Ω?
- integrability?
- deformations? moduli spaces? ...

(n.b. no-go means non-compact/bdry for Minkowski)

G structures

Killing spinors $\varepsilon_i^{\pm}$ invariant under

$$G = \mathrm{Stab}(\{\varepsilon_i^{\pm}\}) \subset \mathrm{SO}(6) \subset \mathrm{GL}(6)$$

define G-structure and flux gives lack of integrability, eg

$$G = \mathrm{SU}(3) \qquad \begin{cases} \mathrm{d}\omega \simeq \text{flux} & \mathrm{Sp}(6,\mathbb{R}) \text{ structure} \\ \mathrm{d}\Omega \simeq \text{flux} & \mathrm{SL}(3,\mathbb{C}) \text{ structure} \end{cases}$$

- classification, new solutions, ...
- global questions: G can change, ..., moduli hard, ...

[*Gauntlett, Martelli, Pakis & DW; Gauntlett & Pakis; ...,*]

Is there some integrable geometry?

supersymmetry $\Leftrightarrow$ *integrable G-structure in generalised geometry*

Generalised Calabi–Yau structures

Generalised tangent space $E \simeq TM \oplus T^*M$ with

$$\mathsf{Stab}(\{\varepsilon_i^\pm\}) = \mathsf{SU}(3) \times \mathsf{SU}(3) \subset \mathsf{SO}(6) \times \mathsf{SO}(6) \subset \mathsf{O}(6,6) \times \mathbb{R}^+$$

gives class of pure NS-NS backgrounds

$$G = \mathsf{SU}(3) \times \mathsf{SU}(3) \qquad \begin{cases} \mathrm{d}\Phi^+ = 0 & \mathsf{SU}(3,3)_+ \text{ structure} \\ \mathrm{d}\Phi^- = 0 & \mathsf{SU}(3,3)_- \text{ structure} \end{cases}$$

for generalised spinor $\Phi^\pm \in S^\pm(E) \simeq \Lambda^\pm T^*M$

$$\Phi^+ = \mathrm{e}^{-\phi}\mathrm{e}^{-B-\mathrm{i}\omega} \qquad \Phi^- = \mathrm{e}^{-\phi}\mathrm{e}^{-B}\left(\Omega_1 + \Omega_3 + \Omega_5\right)$$

[*Hitchin, Gualtieri; Graña, Minasian, Petrini and Tomasiello*]

Generic $\mathcal{N} = 2$ backgrounds

Warped compactification

$$\mathrm{d}s^2 = \mathrm{e}^{2\Delta}\mathrm{d}s^2(\mathbb{R}^{3,1}) + \mathrm{d}s^2(M) \qquad \begin{cases} M_6 & \text{type II} \\ M_7 & \text{M-theory} \end{cases}$$

- "exceptional generalised geometry" with $E_{7(7)} \times \mathbb{R}^+$
- spinors in SU(8) vector rep $\epsilon = (\varepsilon^+, \varepsilon^-)$
- so for $\mathcal{N} = 2$ we have

$$\mathrm{Stab}(\epsilon_1, \epsilon_2) = \mathrm{SU}(6) \subset \mathrm{SU}(8) \subset E_{7(7)} \times \mathbb{R}^+$$

The problem

Generalised ω and Ω

$$G = \mathrm{SU}(6) \qquad \begin{cases} ??? & \text{“H structure”} \\ ??? & \text{“V structure”} \end{cases}$$

- how do we define structures?
- what are integrability conditions?

[*cf. Graña, Louis, Sim & DW; Graña & Orsi; Graña & Triendl*]

Generalised geometry

$\mathrm{O}(d,d) \times \mathbb{R}^+$ generalised geometry

Unify symmetries of NS-NS sector $\delta g = \mathcal{L}_v g$, $\delta B = \mathcal{L}_v B + \mathrm{d}\lambda$

- generalised tangent space $E \simeq TM \oplus T^*M$
- natural $\mathrm{O}(d,d)$ metric given $V^M = v + \lambda \in \Gamma(E)$

$$\eta(V,V) = v^m \lambda_m$$

- infinitesimal symmetries as "generalised Lie derivative "

$$L_V = \text{diffeo by } v + \text{gauge by } \lambda$$

[*Hitchin, Gualtieri*]

$E_{d(d)} \times \mathbb{R}^+$ generalised geometry ($d \leq 7$)

Unify all symmetries of fields restricted to M_{d-1} in type II

$$\delta g = \mathcal{L}_v g \qquad \delta C_\pm = \mathcal{L}_v C_\pm + \mathrm{d}\lambda^\mp + \dots$$
$$\delta B = \mathcal{L}_v B + \mathrm{d}\lambda \qquad \delta \tilde{B} = \mathcal{L}_v \tilde{B} + \mathrm{d}\tilde{\lambda} + \dots$$

gives generalised tangent space

$$E \simeq TM \oplus T^*M \oplus \Lambda^5 T^*M \oplus \Lambda^\pm T^*M \oplus (T^*M \otimes \Lambda^6 T^*M)$$
$$V^M = (v^m, \lambda_m, \tilde{\lambda}_{m_1\cdots 5}, \lambda^\pm, \dots)$$

Transforms under $E_{d(d)} \times \mathbb{R}^+$ rep with $\mathbb{R}^+$ weight $(\det T^*M)^{1/(9-d)}$

[*Hull; Pacheco & DW*]

In M-theory

$$E \simeq TM \oplus \Lambda^2 T^*M \oplus \Lambda^5 T^*M \oplus (T^*M \otimes \Lambda^7 T^*M)$$
$$V^M = (v^m, \lambda_m, \tilde{\lambda}_{m_1\cdots 5}, \dots)$$

Generalised Lie derivative

$$L_V = \text{diffeo} + \text{gauge transf} = V \cdot \partial - (\partial \times_{\mathrm{ad}} V)$$

where type IIA, IIB and M-theory distinguished by

$$\partial_M f = (\partial_m f, 0, 0, \dots) \in E^*$$

[*Coimbra, S-Constable & DW*]

Generalised tensors: $E_{d(d)} \times \mathbb{R}^+$ representations

For example, adjoint includes potentials

$$\begin{aligned} \text{ad}\,\tilde{F} \simeq \mathbb{R} \oplus (TM \otimes T^*M) \oplus \Lambda^2 T^*M \oplus \Lambda^2 TM \\ \oplus \Lambda^6 TM \oplus \Lambda^6 T^*M \oplus \Lambda^\pm TM \oplus \Lambda^\pm T^*M, \end{aligned}$$

$$A^M{}_N = (\dots, B_{mn}, \dots, \tilde{B}_{m_1 \dots m_6}, \dots, C^\pm)$$

Gives "twisting" of generalised vector and adjoint

$$V = \mathrm{e}^{B+\tilde{B}+C^\pm}\,\tilde{V} \qquad R = \mathrm{e}^{B+\tilde{B}+C^\pm}\,\tilde{R}\,\mathrm{e}^{-B-\tilde{B}-C^\pm}$$

Generalised geometry and supergravity

Unified description of supergravity on M

- Generalised metric

$$G_{MN} \qquad \text{invariant under max compact } H_d \subset \mathsf{E}_{\mathsf{d(d)}} \times \mathbb{R}^+$$

  equivalent to $\{g, \phi, B, \tilde{B}, C^\pm, \Delta\}$

- Generalised Levi–Civita connection $D_M V^N = \partial_M V^N + \Omega_M{}^N{}_P V^P$

  exists gen. torsion-free connection D with $DG = 0$

  but not unique

- Analogue of Ricci tensor is unique gives bosonic action

$$S_{\mathrm{B}} = \int_M |\mathrm{vol}_G|\, R \qquad\qquad \text{eom} = \text{gen. Ricci flat}$$

where $|\mathrm{vol}_G| = \sqrt{g}\, \mathrm{e}^{2\Delta}$

- Leading-order fermions and supersymmetry

$$\delta\psi = D \curlyvee \epsilon \qquad\qquad \delta\rho = \not{D}\epsilon \qquad\qquad \text{etc}$$

unique operators, full theory has local SU(8) invariance

[*CSW*] (c.f [*Berman & Perry,...*] and [*Siegel; Hohm, Kwak & Zweibach; Jeon, Lee & Park*] for $\mathrm{O}(d, d)$)

H and V structures

H and V structures

Generalised structures in $E_{7(7)} \times \mathbb{R}^+$

H structure	$G = \mathrm{Spin}^*(12)$	"hypermultiplets"
V structure	$G = E_{6(2)}$	"vector-multiplets"

[*Graña, Louis, Sim & DW*]

Invariant tensor for V structure

Generalised vector in $\mathbf{56_1}$

$$K \in \Gamma(E) \qquad \text{such that } q(K) > 0$$

where q is $E_{7(7)}$ quartic invariant, determines second vector $\hat{K}$

Invariant tensor for H structure

Weighted tensors in $\mathbf{133_1}$

$$J_\alpha(x) \in \Gamma(\operatorname{ad}\tilde{F} \otimes (\det T^*M)^{1/2})$$

forming highest weight $\mathfrak{su}_2$ algebra

$$[J_\alpha, J_\beta] = 2\kappa\epsilon_{\alpha\beta\gamma}J_\gamma$$
$$\operatorname{tr} J_\alpha J_\beta = -\kappa^2\delta_{\alpha\beta}$$

where $\kappa^2 \in \Gamma(\det T^*M)$

Compatible structures and SU(6)

The H and V structures are compatible if

$$J_\alpha \cdot K = 0 \qquad\qquad \sqrt{q(K)} = \tfrac{1}{2}\kappa^2$$

analogues of $\omega \wedge \Omega = 0$ and $\frac{1}{6}\omega^3 = \frac{1}{8}\mathrm{i}\Omega \wedge \bar{\Omega}$

the compatible pair $\{J_\alpha, K\}$ define an SU(6) structure

J_α and K come from spinor bilinears.

Example: CY in IIA

$$J_+ = \tfrac{1}{2}\kappa\,\Omega - \tfrac{1}{2}\kappa\,\Omega^\sharp$$

$$J_3 = \tfrac{1}{2}\kappa\, I + \tfrac{1}{2}\kappa\,\mathrm{vol}_6 - \tfrac{1}{2}\kappa\,\mathrm{vol}_6^\sharp$$

$$\begin{aligned}\operatorname{ad}\tilde{F} \simeq {} & (TM \otimes T^*M) \\ & \oplus \Lambda^2 T^*M \oplus \Lambda^2 TM \\ & \oplus \mathbb{R} \oplus \Lambda^6 TM \oplus \Lambda^6 T^*M \\ & \oplus \Lambda^- TM \oplus \Lambda^- T^*M,\end{aligned}$$

where $\kappa^2 = \mathrm{vol}_6 = \frac{1}{8}\mathrm{i}\Omega \wedge \bar{\Omega}$ and I is complex structure

$$K + \mathrm{i}\hat{K} = \mathrm{e}^{-\mathrm{i}\omega}$$

$$\begin{aligned}E \simeq {} & TM \oplus T^*M \oplus \Lambda^+ T^*M \\ & \oplus \Lambda^5 T^*M \oplus (T^*M \otimes \Lambda^6 T^*M)\end{aligned}$$

Example: CY in IIB

$$J_+ = \tfrac{1}{2}\kappa\, \mathrm{e}^{-\mathrm{i}\omega} - \tfrac{1}{2}\kappa\, \mathrm{e}^{-\mathrm{i}\omega^\sharp}$$

$$J_3 = \tfrac{1}{2}\kappa\,\omega + \tfrac{1}{2}\kappa\,\omega^\sharp - \tfrac{1}{2}\kappa\,\mathrm{vol}_6 - \tfrac{1}{2}\kappa\,\mathrm{vol}_6^\sharp$$

where $\kappa^2 = \mathrm{vol}_6 = \frac{1}{6}\omega^3$

$$K + \mathrm{i}\hat{K} = \Omega$$

$$\begin{aligned} \operatorname{ad}\tilde{F} \simeq\ & (TM \otimes T^*M) \\ & \oplus \Lambda^2 T^*M \oplus \Lambda^2 TM \\ & \oplus \mathbb{R} \oplus \Lambda^6 TM \oplus \Lambda^6 T^*M \\ & \oplus \Lambda^+ TM \oplus \Lambda^+ T^*M, \end{aligned}$$

$$\begin{aligned} E \simeq\ & TM \oplus T^*M \oplus \Lambda^- T^*M \\ & \oplus \Lambda^5 T^*M \oplus (T^*M \otimes \Lambda^6 T^*M) \end{aligned}$$

Example: D3-branes in IIB

Smeared branes on $M_{\mathsf{SU}(2)} \times \mathbb{R}^2$

$$\mathrm{d}s^2 = \mathrm{e}^{2\Delta}\mathrm{d}s^2(\mathbb{R}^{3,1}) + \mathrm{d}\tilde{s}^2(M_{\mathsf{SU}(2)}) + \zeta_1^2 + \zeta_2^2,$$

with integrability

$$\mathrm{d}(\mathrm{e}^{\Delta}\zeta_i) = 0, \qquad \mathrm{d}(\mathrm{e}^{2\Delta}\omega_\alpha) = 0, \qquad \mathrm{d}\Delta = -\tfrac{1}{4} \star F,$$

where ω_α triplet of two-forms defining SU(2) structure. (Can also add anti-self-dual three-form flux.)

The H and V structures are

$$\tilde{J}_\alpha = -\tfrac{1}{2}\kappa\, I_\alpha - \tfrac{1}{2}\kappa\,\omega_\alpha \wedge \zeta_1 \wedge \zeta_2 + \tfrac{1}{2}\kappa\,\omega^\sharp_\alpha \wedge \zeta^\sharp_1 \wedge \zeta^\sharp_2,$$

$$\begin{aligned} \operatorname{ad}\tilde{F} \simeq (TM \otimes T^*M) &\oplus \Lambda^2 T^*M \oplus \Lambda^2 TM \\ &\oplus \mathbb{R} \oplus \Lambda^6 TM \oplus \Lambda^6 T^*M \\ &\oplus \Lambda^+ TM \oplus \Lambda^+ T^*M, \end{aligned}$$

where $\kappa^2 = \mathrm{e}^{2\Delta}\,\mathrm{vol}_6$ and I_α are complex structures

$$\tilde{K} + \mathrm{i}\hat{\tilde{K}} = n^i \mathrm{e}^{\Delta}(\zeta_1 - \mathrm{i}\zeta_2) + \mathrm{i}n^i \mathrm{e}^{\Delta}(\zeta_1 - \mathrm{i}\zeta_2) \wedge \mathrm{vol}_4$$

$$\begin{aligned} E \simeq TM \oplus T^*M \oplus \Lambda^- T^*M \\ \oplus \Lambda^5 T^*M \oplus (T^*M \otimes \Lambda^6 T^*M) \end{aligned}$$

where $n^i = (\mathrm{i}, 1)$ is S-duality doublet, then twist by C_4

$$K = \mathrm{e}^{C_4}\tilde{K} \qquad J_\alpha = \mathrm{e}^{C_4}\,\tilde{J}_\alpha\,\mathrm{e}^{C_4}$$

Generic form?

Complicated but

- interpolates between symplectic, complex, product and hyper-Kähler structures
- can construct from bilinears and twisting

Integrability

GDiff and moment maps

Symmetries of supergravity give generalised diffeomorphisms

$$\mathrm{GDiff} = \mathrm{Diff} \ltimes \text{gauge transf.}$$

acts on the spaces of H and V structures

$$\text{integrability} \quad \Leftrightarrow \quad \text{vanishing moment map}$$

Ubiquitous in supersymmetry equations

- flat connections on Riemann surface (Atiyah–Bott)
- Hermitian Yang–Mills (Donaldson-Uhlenbeck-Yau)
- Hitchin equations, Kähler–Einstein, ...

Space of H structures, $\mathcal{A}_\mathrm{H}$

Consider infinite-dimensional space of structures, $J_\alpha(x)$ give coordinates

$$\mathcal{A}_\mathrm{H} \text{ has hyper-Kähler metric}$$

inherited fibrewise since at each $x \in M$

$$J_\alpha(x) \in W = \frac{\mathrm{E}_{7(7)} \times \mathbb{R}^+}{\mathrm{Spin}^*(12)}$$

and W is HK cone over homogenous quaternionic-Kähler (Wolf) space (n.b. $\mathcal{A}_\mathrm{H}$ itself is HK cone by *global* $\mathbb{H}^+ = \mathrm{SU}(2) \times \mathbb{R}^+$)

Triplet of moment maps

Infinitesimally parametrised by $V \in \Gamma(E) \simeq \mathfrak{gdiff}$ and acts by

$$\delta J_\alpha = L_V J_\alpha \qquad \in \Gamma(T\mathcal{A}_\mathsf{H})$$

preserves HK structure giving maps $\mu_\alpha : \mathcal{A}_\mathsf{H} \to \mathfrak{gdiff}^* \otimes \mathbb{R}^3$

$$\mu_\alpha(V) = -\tfrac{1}{2}\epsilon_{\alpha\beta\gamma} \int_M \operatorname{tr} J_\beta(L_V J_\gamma)$$

functions of coordinates $J_\alpha(x) \in \mathcal{A}_\mathsf{H}$

Integrability

$$\text{integrable H structure} \quad \Leftrightarrow \quad \mu_\alpha(V) = 0, \quad \forall V$$

for CY gives $\mathrm{d}\omega = 0$ or $\mathrm{d}\Omega = 0$

Moduli space

Since structures related by GDiff are equivalent

$$\mathcal{M}_\mathsf{H} = \mathcal{A}_\mathsf{H} /\!\!/\!\!/ \mathrm{GDiff} = \mu_1^{-1}(0) \cap \mu_2^{-1}(0) \cap \mu_3^{-1}(0) / \mathrm{GDiff}.$$

moduli space is HK quotient, actually HK cone over QK space of hypermultiplets (as for CY $4h_{1,1} + 4$ or $4h_{2,1} + 4$)

Space of V structures, $\mathcal{A}_{\mathsf{V}}$

Consider infinite-dimensional space of structures, $K(x)$ give coordinates

$\mathcal{A}_{\mathsf{H}}$ has (affine) special-Kähler metric

(explains $\hat{K}$) inherited fibrewise since at each $x \in M$

$$K(x) \in P = \frac{\mathsf{E}_{7(7)} \times \mathbb{R}^+}{\mathsf{E}_{6(2)}}$$

and P is homogenous special-Kähler space

Moment map

Infinitesimally $\mathfrak{gdiff}$ and acts as

$$\delta K = L_V K \qquad \in \Gamma(T\mathcal{A}_{\mathsf{V}})$$

preserves SK structure giving maps $\mu : \mathcal{A}_{\mathsf{V}} \to \mathfrak{gdiff}^*$

$$\mu(V) = -\tfrac{1}{2} \int_M \operatorname{tr} s(K, L_V K)$$

where $s(\cdot, \cdot)$ is $\mathsf{E}_{7(7)}$ symplectic invariant

Integrability

$$\text{integrable V structure} \quad \Leftrightarrow \quad \mu(V) = 0, \quad \forall V$$

for CY gives $\omega \wedge \mathrm{d}\omega = 0$ or $(\mathrm{d}\Omega)_{3,1} = 0$ Weak!

Moduli space

Since structures related by GDiff are equivalent

$$\mathcal{M}_{\mathsf{H}} = \mathcal{A}_{\mathsf{H}} /\!\!/ \mathsf{GDiff} = \mu^{-1}(0)/\mathsf{GDiff}.$$

moduli space is symplectic quotient, giving SK space

SU(6) structure integrability

Integrable H and V structures not sufficient: need extra condition

$$\mu_\alpha(V) = \mu(V) = 0 \qquad \text{and} \qquad L_K J_\alpha = L_{\hat{K}} J_\alpha = 0$$

for CY give $\mathrm{d}\omega = \mathrm{d}\Omega = 0$

Integrability and generalised intrinsic torsion

- in all cases integrability implies exists torsion-free, compatible D
- integrable SU(6) is equivalent to KS equations [*CSW*]

Why moment maps?

Reformulate IIA or M-theory as $D = 4$, $\mathcal{N} = 2$ but keep all KK modes

- ∞-number of hyper- and vector multiplets: $\mathcal{A}_{\mathrm{H}}$ and $\mathcal{A}_{\mathrm{V}}$
- gauged 4d supergravity with $G = \mathrm{GDiff}$
- integrability is just $\mathcal{N} = 2$ vacuum conditions of [*Hristov, Looyestijn, & Vandoren; Louis, Smyth & Triendl*]

[*Graña, Louis & DW; GLSW*]

Other cases

- In M-theory, same H and V strutures, generalises notion of $\mathrm{CY} \times S^1$
- For type II or M-theory in $D = 5, 6$ with eight supercharges

$$
\mathrm{E}_{6(6)} \qquad \begin{cases} J_\alpha : \mathrm{SU}^*(6) & \text{hyper-Kähler} \\ K : \mathrm{F}_{4(4)} & \text{very special real} \end{cases}
$$

$$
\mathrm{E}_{5(5)} \simeq \mathrm{Spin}(5,5) \qquad \begin{cases} J_\alpha : \mathrm{SU}(2) \times \mathrm{Spin}(1,5) & \text{hyper-Kähler} \\ Q : \mathrm{Spin}(4,5) & \text{flat (tensor)} \end{cases}
$$

- Extend to AdS backgrounds, generalises Sasaki–Einstein

Application : marginal deformations

with Ashmore, Gabella, Graña, Petrini

$\mathcal{N} = 1$ marginal deformations for $\mathcal{N} = 4$ [*Leigh & Strassler*]

Superpotential deformation

$$W = \epsilon_{ijk} \operatorname{tr} Z^i Z^j Z^k + f_{ijk} \operatorname{tr} Z^i Z^j Z^k$$

- f_{ijk} symmetric giving 10 complex marginal deformations
- but beta-function constrains moment map for SU(3) symmetry

$$f_{ikl} \bar{f}^{jkl} - \tfrac{1}{3} \delta_i^j f_{klm} \bar{f}^{klm} = 0$$

- only 2 exactly marginal deformation as symplectic quotient

$$\widetilde{\mathcal{M}} = \{f_{ijk}\} /\!\!/ \mathrm{SU}(3)$$

long calculation in supergravity [*Aharony, Kol & Yankielowicz*]

General analysis [*Green, Komargodski, Seiberg, Tachikawa & Wecht*]

1. "Kähler deformation" dual to bulk vector multiplets
2. "superpotential deformation" dual to bulk hypermultiplets

Field theory analysis

- no Kähler deformations
- every marginal superpotential deform. is exactly marginal unless ...
- if global symmetry G (other then $U(1)_R$) then

$$\text{exactly marginal} = \text{marginal} /\!\!/ G$$

H and V structures in $E_{6(6)} \times \mathbb{R}^+$ generalised geometry

Bulk is $D = 5$, $\mathcal{N} = 1$ supergravity

$$\left.\begin{array}{rl} \text{H structure, } J_\alpha & G = \mathrm{SU}^*(6) \\ \text{V structure, } K & G = \mathrm{F}_{4(4)} \end{array}\right\} \quad G = \mathrm{USp}(6)$$

with integrability to AdS

$$\mu_\alpha(V) = \lambda_\alpha \int_M c(K, K, V)$$

$$L_K J_\alpha = \epsilon_{\alpha\beta\gamma} \lambda_\beta J_\gamma \qquad\qquad L_K K = 0$$

where $c(\cdot, \cdot, \cdot)$ is $E_{6(6)}$ cubic invariant

Kähler deformations: $\delta K \neq 0$, $\delta J_\alpha = 0$

No solution to moment map equation ...

Superpotential deformations: $\delta K = 0$, $\delta J_\alpha \neq 0$

For moment maps no obstruction to linearised solution unless fixed point

- fixed point of GDiff is a gen. Killing vector ie. global symmetry G
- obstruction is moment map of G on linearised problem

Summary

- supersymmetry is special holonomy in generalised geometry
- natural extension of CY geometry

Questions/Extensions

- $\mathcal{N} = 1$ backgrounds ?✓
- deformation theory: underlying DGLA, cohomology, ...?✓
- topological string?: J_α is generalisation of Kähler and Kodaira/Spencer gravity even to M-theory ...
- algebraic geometry?: CFT gives (non-commuatative) algebraic description ...