

Neutrino Physics

(Neutrino Masses, Mixing, Oscillations, the Nature of Massive Neutrinos and Leptonic CP Violation)

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Plan of the Lectures

1. Introduction.
2. Massive Neutrinos, Neutrino Mixing and Oscillations: Overview.
3. Three Neutrino Mixing. Massive Majorana versus Massive Dirac Neutrinos I. Dirac and Majorana CP Violation.
4. Neutrino Oscillations in Vacuum: Theory and Experimental Evidences.
5. Matter Effects in Neutrino Oscillations: Theory.
Neutrino Oscillations in the Earth.
CP Violation in Neutrino Oscillations.
Flavour Conversions of Solar Neutrinos.
6. Three Neutrino Mixing: the Angle θ_{13} and Indications for Dirac CP Violation.
7. Open Questions in the Physics of Massive Neutrinos.
8. Understanding the Pattern of Neutrino Mixing.
9. The Absolute Scale of Neutrino Masses.
10. The Nature of Massive Neutrinos.
I. Massive Majorana versus Massive Dirac Neutrinos.
II. Origins of Dirac and Majorana Massive Neutrinos.
III. The Seesaw Mechanisms of Neutrino Mass Generation.
11. Determining the Nature of Massive Neutrinos.
12. Future LBL Neutrino Oscillation Experiments on $\text{sgn}(\Delta m_{31}^2)$ and CP Violation.
13. Conclusions.

3 Families of Fundamental Particles

$$\begin{pmatrix} \nu_e & u \\ e & d \end{pmatrix} \quad \begin{pmatrix} \nu_\mu & c \\ \mu & s \end{pmatrix} \quad \begin{pmatrix} \nu_\tau & t \\ \tau & b \end{pmatrix} \quad + \text{ their antiparticles}$$

- 3 types (flavours) of active ν 's and $\tilde{\nu}$'s
- The notion of “type” (“flavour”) - dynamical;

$$\nu_e: \nu_e + n \rightarrow e^- + p; \quad \nu_\mu: \pi^+ \rightarrow \mu^+ + \nu_\mu; \text{ etc.}$$

- The flavour of a given neutrino is Lorentz invariant.
- $\nu_l \neq \nu_{l'}, \tilde{\nu}_l \neq \tilde{\nu}_{l'}, l \neq l' = e, \mu, \tau; \nu_l \neq \tilde{\nu}_{l'}, l, l' = e, \mu, \tau.$

The states must be orthogonal (within the precision of the corresponding data): $\langle \nu_{l'} | \nu_l \rangle = \delta_{l'l}, \langle \tilde{\nu}_{l'} | \tilde{\nu}_l \rangle = \delta_{l'l}, \langle \tilde{\nu}_{l'} | \nu_l \rangle = 0.$

- **Data (relativistic ν 's):** ν_l ($\tilde{\nu}_l$) - predominantly LH (RH).
Standard Theory: $\nu_l, \tilde{\nu}_l - \nu_{lL}(x)$;

$\nu_{lL}(x)$ form $SU(2)_L$ doublets with $l_L(x)$, $l = e, \mu, \tau$:

$$\begin{pmatrix} \nu_{lL}(x) \\ l_L(x) \end{pmatrix}, \quad l = e, \mu, \tau.$$

- **No (compelling) evidence for existence of (relativistic) ν 's ($\tilde{\nu}$'s) which are predominantly RH (LH):** ν_R ($\tilde{\nu}_L$.)

If $\nu_R, \tilde{\nu}_L$ exist, must have much weaker interaction than $\nu_l, \tilde{\nu}_l$: $\nu_R, \tilde{\nu}_L$ - “sterile”, “inert” .

B. Pontecorvo, 1967

In the formalism of the ST, ν_R and $\tilde{\nu}_L$ - RH ν fields $\nu_R(x)$; can be introduced in the ST as $SU(2)_L$ singlets.

No experimental indications exist at present whether the SM should be minimally extended to include $\nu_R(x)$, and if it should, how many $\nu_R(x)$ should be introduced.

$\nu_R(x)$ appear in many extensions of the ST, notably in $SO(10)$ GUT's.

The RH ν 's can play crucial role

- i) in the generation of $m(\nu) \neq 0$,
- ii) in understanding why $m(\nu) \ll m_l, m_q$,
- iii) in the generation of the observed matter-antimatter asymmetry of the Universe (via leptogenesis).

The simplest hypothesis is that to each $\nu_{lL}(x)$ there corresponds a $\nu_{lR}(x)$, $l = e, \mu, \tau$.

ST + $m(\nu) = 0$: $L_l = \text{const.}$, $l = e, \mu, \tau$;
 $L \equiv L_e + L_\mu + L_\tau = \text{const.}$

There have been remarkable discoveries in neutrino physics in the last ~ 17 years.

Compelling Evidence for ν -Oscillations

– ν_{atm} : **SK UP-DOWN ASYMMETRY**

θ_{z-} , L/E – dependences of μ -like events

Dominant $\nu_{\mu} \rightarrow \nu_{\tau}$ K2K, MINOS, T2K; CNGS (OPERA)

– ν_{\odot} : **Homestake, Kamiokande, SAGE, GALLEX/GNO**

Super-Kamiokande, SNO, BOREXINO; KamLAND

Dominant $\nu_e \rightarrow \nu_{\mu,\tau}$ **BOREXINO**

– $\bar{\nu}_e$ (from reactors): **Daya Bay, RENO, Double Chooz**

Dominant $\bar{\nu}_e \rightarrow \bar{\nu}_{\mu,\tau}$

T2K, MINOS (ν_{μ} from accelerators): $\nu_{\mu} \rightarrow \nu_e$

Compelling Evidences for ν -Oscillations: ν mixing

$$|\nu_l\rangle = \sum_{j=1}^n U_{lj}^* |\nu_j\rangle, \quad \nu_j : m_j \neq 0; \quad l = e, \mu, \tau; \quad n \geq 3;$$

$$\nu_{lL}(x) = \sum_{j=1}^n U_{lj} \nu_{jL}(x), \quad \nu_{jL}(x) : m_j \neq 0; \quad l = e, \mu, \tau.$$

B. Pontecorvo, 1957; 1958; 1967;

Z. Maki, M. Nakagawa, S. Sakata, 1962;

U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix.

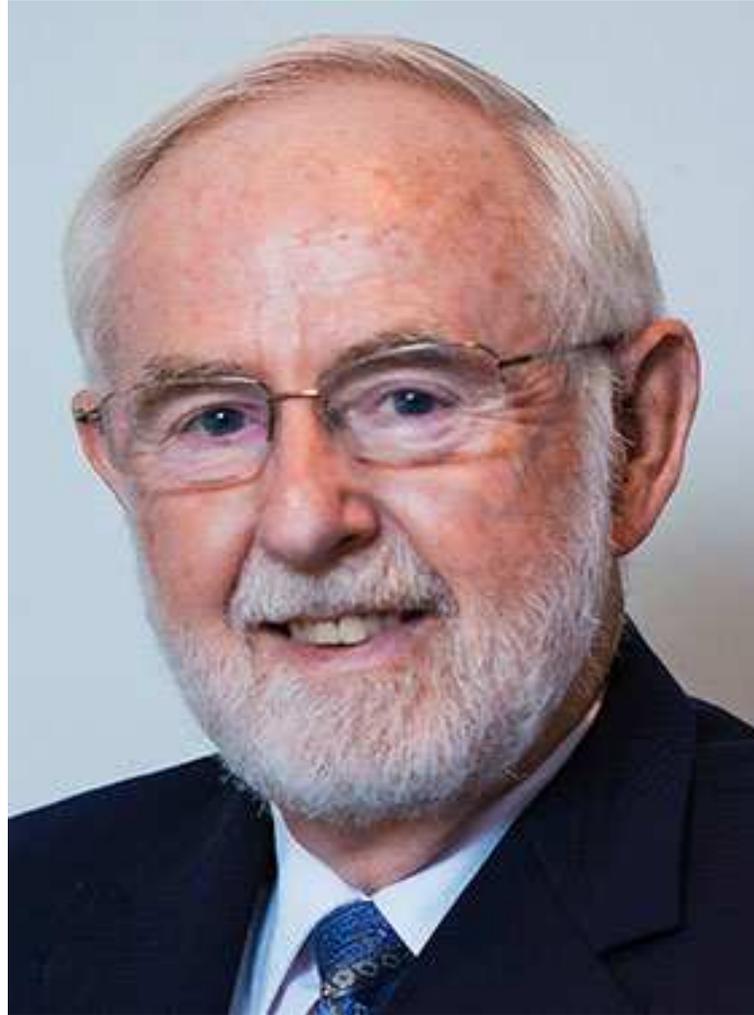
$\nu_j, m_j \neq 0$: Dirac or Majorana particles.

Data: at least 3 ν s are light: $\nu_{1,2,3}, m_{1,2,3} \lesssim 1$ eV.

The Charged Current Weak Interaction Lagrangian:

$$\mathcal{L}^{CC}(x) = -\frac{g}{2\sqrt{2}} \sum_{l=e,\mu,\tau} \bar{l}(x) \gamma_\alpha (1 - \gamma_5) \nu_{lL}(x) W^\alpha(x) + \text{h.c.},$$

$$\nu_{lL}(x) = \sum_{j=1}^n U_{lj} \nu_{jL}(x), \quad \nu_{jL}(x) : m_j \neq 0; \quad l = e, \mu, \tau.$$



Dr. T. Kajita, Prof. A. McDonald, Nobel Prize for Physics winners, 2015



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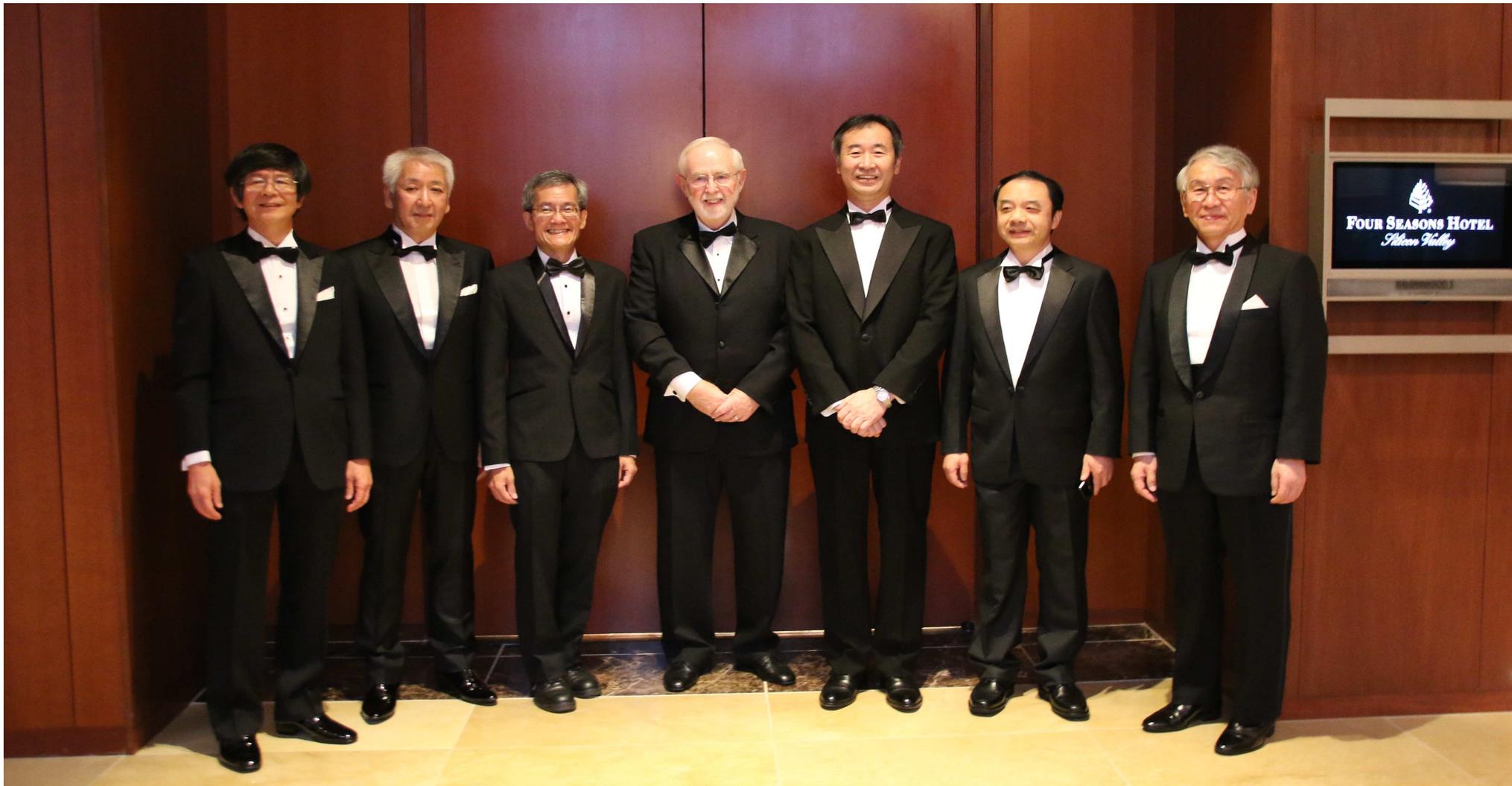
[Arthur B. McDonald and
the SNO Collaboration](#)



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S.T. Petcov, KIAS/KNRC School on Neutrino Physics, Seoul, 15/02/2016

These data imply that

$$m_{\nu_j} \lll m_{e,\mu,\tau}, m_q, \quad q = u, c, t, d, s, b$$

For $m_{\nu_j} \lesssim 1$ eV: $m_{\nu_j}/m_{l,q} \lesssim 10^{-6}$

For a given family: $10^{-2} \lesssim m_{l,q}/m_{q'} \lesssim 10^2$

**These discoveries suggest the existence of
New Physics beyond that of the ST.**

The New Physics can manifest itself (can have a variety of different “flavours”):

- In the existence of more than 3 massive neutrinos: $n > 3$ ($n = 4$, or $n = 5$, or $n = 6, \dots$).
- In the observed pattern of neutrino mixing and in the values of the CPV phases in the PMNS matrix.
- In the Majorana nature of massive neutrinos.
- In the existence of LFV processes: $\mu \rightarrow e + \gamma$, $\mu \rightarrow 3e$, $\mu - e$ conversion, etc., which proceed with rates close to the existing upper limits.
- In the existence of new particles, e.g., at the TeV scale: heavy Majorana Neutrinos N_j , doubly charged scalars,...
- In the existence of new (FChNC, FCFNSNC) neutrino interactions.
- In the existence of “unknown unknowns” ...

We can have $n > 3$ ($n = 4$, or $n = 5$, or $n = 6, \dots$) if, e.g., sterile ν_R , $\tilde{\nu}_L$ exist and they mix with the active flavour neutrinos ν_l ($\tilde{\nu}_l$), $l = e, \mu, \tau$.

Two (extreme) possibilities:

i) $m_{4,5,\dots} \sim 1$ eV;

in this case $\nu_{e(\mu)} \rightarrow \nu_S$ oscillations are possible (hints from LSND and MiniBooNE experiments, re-analyses of short baseline (SBL) reactor neutrino oscillation data (“reactor neutrino anomaly”, data of radioactive source calibration of the solar neutrino SAGE and GALLEX experiments (“Gallium anomaly”));

ii) $M_{4,5,\dots} \sim (10^2 - 10^3)$ GeV, TeV scale seesaw models;
 $M_{4,5,\dots} \sim (10^9 - 10^{13})$ GeV, “classical” seesaw models.

We can also have, in principle:

$m_4 \sim 1$ eV ($\nu_{e(\mu)} \rightarrow \nu_S$), $m_5 \sim 5$ keV (DM), $M_6 \sim (10 - 10^3)$ GeV (seesaw).

All compelling data compatible with 3- ν mixing:

$$\nu_{lL} = \sum_{j=1}^3 U_{lj} \nu_{jL} \quad l = e, \mu, \tau.$$

The PMNS matrix U - 3×3 unitary to a good approximation (at least: $|U_{l,n}| \lesssim (\ll) 0.1$, $l = e, \mu$, $n = 4, 5, \dots$).

ν_j , $m_j \neq 0$: Dirac or Majorana particles.

3- ν mixing: 3-flavour neutrino oscillations possible.

ν_μ , E ; at distance L : $P(\nu_\mu \rightarrow \nu_{\tau(e)}) \neq 0$, $P(\nu_\mu \rightarrow \nu_\mu) < 1$

$$P(\nu_l \rightarrow \nu_{l'}) = P(\nu_l \rightarrow \nu_{l'}; E, L; U; m_2^2 - m_1^2, m_3^2 - m_1^2)$$

Three Neutrino Mixing

$$\nu_{lL} = \sum_{j=1}^3 U_{lj} \nu_{jL} .$$

U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix,

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix}$$

- U - $n \times n$ unitary:

	n	2	3	4
mixing angles:	$\frac{1}{2}n(n-1)$	1	3	6

CP-violating phases:

- ν_j - Dirac: $\frac{1}{2}(n-1)(n-2)$ 0 1 3
- ν_j - Majorana: $\frac{1}{2}n(n-1)$ 1 3 6

$n = 3$: 1 Dirac and
2 additional CP-violating phases, Majorana phases

S.M. Bilenky, J. Hosek, S.T.P., 1980

Majorana Neutrinos

Can be defined in QFT using fields or states.

Fields: $\chi_k(x)$ - 4 component (spin 1/2), complex, m_k

Majorana condition:

$$C (\bar{\chi}_k(x))^T = \xi_k \chi_k(x), \quad |\xi_k|^2 = 1, \quad C^{-1} \gamma_\mu C = - \gamma_\mu^T$$

- Invariant under proper Lorentz transformations.
- Reduces by 2 the number of components in $\chi_k(x)$.

Implications:

$$U(1) : \chi_k(x) \rightarrow e^{i\alpha} \chi_k(x) - \text{impossible}$$

- $\chi_k(x)$ cannot absorb phases.
- $Q_{U(1)} = 0 : Q_{el} = 0, L_l = 0, L = 0, \dots$
- $\chi_k(x)$: 2 spin states of a spin 1/2 absolutely neutral particle
- $\chi_k \equiv \bar{\chi}_k$

Propagators: $\Psi(x)$ –Dirac, $\chi(x)$ –Majorana

$$\langle 0|T(\Psi_\alpha(x)\bar{\Psi}_\beta(y))|0 \rangle = S_{\alpha\beta}^F(x-y) ,$$

$$\langle 0|T(\Psi_\alpha(x)\Psi_\beta(y))|0 \rangle = 0 , \quad \langle 0|T(\bar{\Psi}_\alpha(x)\bar{\Psi}_\beta(y))|0 \rangle = 0 .$$

$$\langle 0|T(\chi_\alpha(x)\bar{\chi}_\beta(y))|0 \rangle = S_{\alpha\beta}^F(x-y) ,$$

$$\langle 0|T(\chi_\alpha(x)\chi_\beta(y))|0 \rangle = -\xi^* S_{\alpha\kappa}^F(x-y) C_{\kappa\beta} ,$$

$$\langle 0|T(\bar{\chi}_\alpha(x)\bar{\chi}_\beta(y))|0 \rangle = \xi C_{\alpha\kappa}^{-1} S_{\kappa\beta}^F(x-y)$$

$$U_{CP} \chi(x) U_{CP}^{-1} = \eta_{CP} \gamma_0 \chi(x'), \quad \eta_{CP} = \pm i .$$

PMNS Matrix: Standard Parametrization

$$U = V P, \quad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix},$$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- $s_{ij} \equiv \sin \theta_{ij}$, $c_{ij} \equiv \cos \theta_{ij}$, $\theta_{ij} = [0, \frac{\pi}{2}]$,
- δ - Dirac CPV phase, $\delta = [0, 2\pi]$; CP inv.: $\delta = 0, \pi, 2\pi$;
- α_{21}, α_{31} - Majorana CPV phases; CP inv.: $\alpha_{21(31)} = k(k')\pi$, $k(k') = 0, 1, 2, \dots$
S.M. Bilenky, J. Hosek, S.T.P., 1980
- $\Delta m_{\odot}^2 \equiv \Delta m_{21}^2 \cong 7.54 \times 10^{-5} \text{ eV}^2 > 0$, $\sin^2 \theta_{12} \cong 0.308$, $\cos 2\theta_{12} \gtrsim 0.28$ (3σ),
- $|\Delta m_{31(32)}^2| \cong 2.47$ (2.42) $\times 10^{-3} \text{ eV}^2$, $\sin^2 \theta_{23} \cong 0.437$ (0.455), NO (IO),
- θ_{13} - the CHOOZ angle: $\sin^2 \theta_{13} = 0.0234$ (0.0240), Capozzi et al. NO (IO).
F. Capozzi et al. (Bari Group), arXiv:1312.2878v2 (May 5, 2014)

- Fogli et al., Phys. Rev. D86 (2012) 013012, global analysis, b.f.v.: $\sin^2 \theta_{13} = 0.0241$ (0.0244), NO (IO).

- $\Delta m_{\odot}^2 \equiv \Delta m_{21}^2 \cong 7.54 \times 10^{-5} \text{ eV}^2 > 0$, $\sin^2 \theta_{12} \cong 0.308$, $\cos 2\theta_{12} \gtrsim 0.28$ (3σ),

- $|\Delta m_{31(32)}^2| \cong 2.47$ (2.42) $\times 10^{-3} \text{ eV}^2$, $\sin^2 \theta_{23} \cong 0.437$ (0.455), NO (IO),

- θ_{13} - the CHOOZ angle: $\sin^2 \theta_{13} = 0.0234$ (0.0240), NH (IH).

- $1\sigma(\Delta m_{21}^2) = 2.6\%$, $1\sigma(\sin^2 \theta_{12}) = 5.4\%$;

- $1\sigma(|\Delta m_{31(23)}^2|) = 2.6\%$, $1\sigma(\sin^2 \theta_{23}) = 9.6\%$;

- $1\sigma(\sin^2 \theta_{13}) = 8.5\%$;

- $3\sigma(\Delta m_{21}^2) : (6.99 - 8.18) \times 10^{-5} \text{ eV}^2$; $3\sigma(\sin^2 \theta_{12}) : (0.259 - 0.359)$;

- $3\sigma(|\Delta m_{31(23)}^2|) : 2.27(2.23) - 2.65(2.60) \times 10^{-3} \text{ eV}^2$;

- $3\sigma(\sin^2 \theta_{23}) : 0.374(0.380) - 0.628(0.641)$;

- $3\sigma(\sin^2 \theta_{13}) : 0.0176(0.0178) - 0.0295(0.0298)$.

F. Capozzi et al. (Bari Group), arXiv:1312.2878v2 (May 5, 2014)

- Dirac phase δ : $\nu_l \leftrightarrow \nu_{l'}, \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}, l \neq l'$; $A_{CP}^{(l,l')} \propto J_{CP} \propto \sin \theta_{13} \sin \delta$:

P.I. Krastev, S.T.P., 1988

$$J_{CP} = \text{Im} \{ U_{e1} U_{\mu 2} U_{e2}^* U_{\mu 1}^* \} = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta$$

Current data: $|J_{CP}| \lesssim 0.040 |\sin \delta|$ (can be relatively large!); b.f.v. with $\delta = 3\pi/2$:
 $J_{CP} \cong -0.035$.

- Majorana phases α_{21}, α_{31} :

– $\nu_l \leftrightarrow \nu_{l'}, \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$ not sensitive;

S.M. Bilenky, J. Hosek, S.T.P., 1980;

P. Langacker, S.T.P., G. Steigman, S. Toshev, 1987

– $|\langle m \rangle|$ in $(\beta\beta)_{0\nu}$ -decay depends on α_{21}, α_{31} ;

– $\Gamma(\mu \rightarrow e + \gamma)$ etc. in SUSY theories depend on $\alpha_{21,31}$;

– BAU, leptogenesis scenario: $\delta, \alpha_{21,31}$!

Solar Neutrinos ν_e , $E \sim 1$ MeV: B. Pontecorvo 1946



R. Davis et al., 1967 - 1996: 615 t C_2Cl_4 ; 0.5 Ar atoms/day, exposure 60 days.



Kamiokande (1986-1994), Super-Kamiokande (1996 -), SNO (2000 - 2006), BOREXINO (2007 -);



Super-Kamiokande: 50000t ultra-pure water;

SNO: 1000t heavy water (D_2O)



SAGE (60t), 1990-; GALLEX/GNO (30t, LNGS), 1991-2003

Atmospheric Neutrinos $\nu_\mu, \bar{\nu}_\mu, \nu_e, \bar{\nu}_e, E \sim 1$ GeV (0.20 - 100 GeV)

$$\nu_\mu + N \rightarrow \mu^- + X, \quad \bar{\nu}_\mu + N \rightarrow \mu^+ + X$$

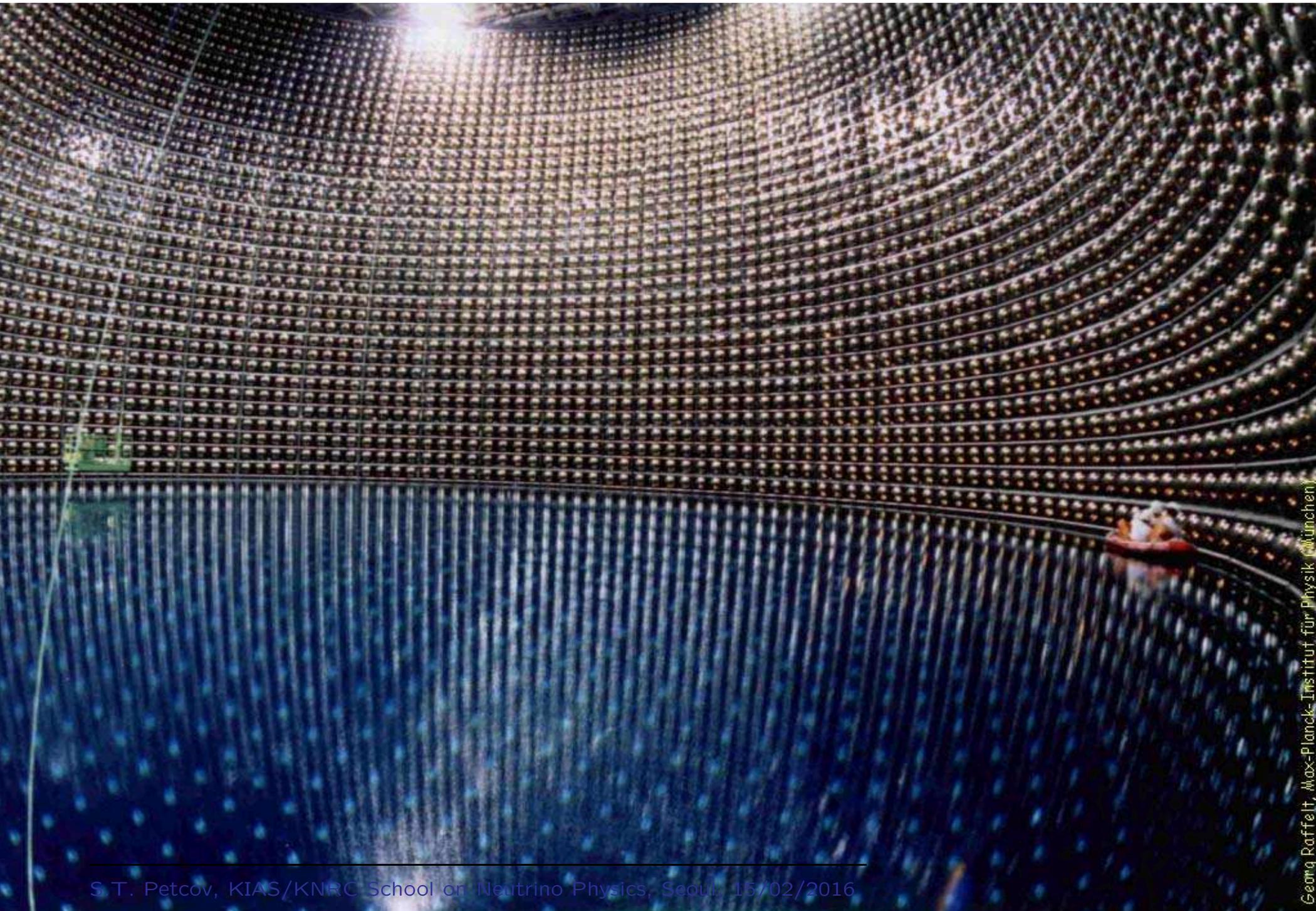
$$\nu_e + N \rightarrow e^- + X, \quad \bar{\nu}_e + N \rightarrow e^+ + X$$

K2K, MINOS, T2K, ν_μ ($\bar{\nu}_\mu$), $E \sim 1$ GeV

$$\nu_\mu + N \rightarrow \mu^- + X \quad (\nu_e + N \rightarrow e^- + X)$$

Reactor $\bar{\nu}_e$: CHOOZ, KamLAND, Double Chooz, RENO, Daya Bay ($E \cong 2 - 8$ MeV)

$$\bar{\nu}_e + p \rightarrow e^+ + n$$



S.T. Petcov, KIAS/KNRC School on Neutrino Physics, Seoul, 15/02/2016

Georg Raffelt, Max-Planck-Institut für Physik (München)



Neutrino Oscillations in Vacuum

Suppose at $t = 0$ in vacuum

$$|\nu_e\rangle = |\nu_1\rangle \cos\theta + |\nu_2\rangle \sin\theta,$$

$$|\nu_{\mu(\tau)}\rangle = -|\nu_1\rangle \sin\theta + |\nu_2\rangle \cos\theta; \quad \nu_{1,2} : m_{1,2} \neq 0$$

After time t in vacuum

$$|\nu_e\rangle_t = e^{-iE_1 t} |\nu_1\rangle \cos\theta + e^{-iE_2 t} |\nu_2\rangle \sin\theta, \quad E_{1,2} = \sqrt{p^2 + m_{1,2}^2}$$

$$A(\nu_e \rightarrow \nu_\mu; t) = \langle \nu_\mu | \nu_e \rangle_t = \frac{1}{2} \sin 2\theta (e^{-iE_2 t} - e^{-iE_1 t})$$

$$P(\nu_e \rightarrow \nu_\mu; t) = \frac{1}{2} \sin^2 2\theta (1 - \cos(E_2 - E_1)t)$$

$$P(\nu_e \rightarrow \nu_e; t) \equiv P_{ee} = 1 - P(\nu_e \rightarrow \nu_\mu; t)$$

V. Gribov, B. Pontecorvo, 1969

Neutrinos are relativistic: $t \cong L$, $E_2 - E_1 \cong (m_2^2 - m_1^2)/(2p)$
 $(E_2 - E_1)t \cong (m_2^2 - m_1^2)L/(2p) = 2\pi \frac{L}{L_{osc}^{vac}}$, $L_{osc}^{vac} \equiv \frac{4\pi E}{\Delta m^2}$

$$P(\nu_e \rightarrow \nu_\mu; t) = \frac{1}{2} \sin^2 2\theta \left(1 - \cos 2\pi \frac{L}{L_{osc}^{vac}}\right), \quad L_{osc}^{vac} \equiv \frac{4\pi E}{\Delta m^2}$$

$$L_{osc}^{vac} \cong 2.5 \text{ m} \frac{E[\text{MeV}]}{\Delta m^2[\text{eV}^2]}$$

$$E \cong 3 \text{ MeV}, \quad \Delta m^2[\text{eV}^2] \cong 8 \times 10^{-5} : \quad L_{osc}^{vac} \cong 100 \text{ km}$$

$$E \cong 1 \text{ GeV}, \quad \Delta m^2[\text{eV}^2] \cong 2.5 \times 10^{-3} : \quad L_{osc}^{vac} \cong 1000 \text{ km}$$

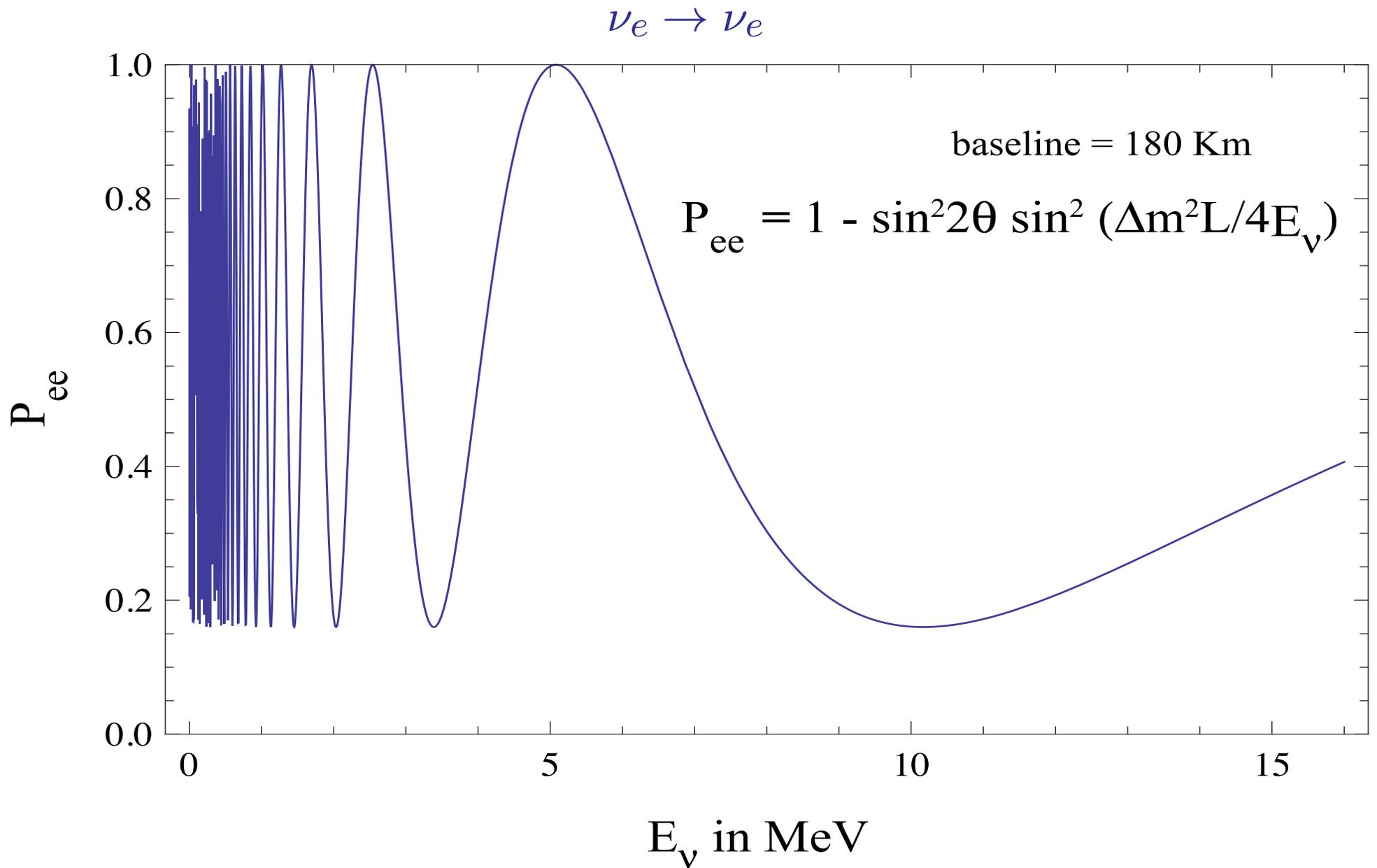
Effects of oscillations observable if

$$\sin^2 2\theta - \text{sufficiently large}, \quad L \gtrsim L_{osc}^{vac}$$

Two basic parameters: $\sin^2 2\theta$, Δm^2

SK, K2K, MINOS; CNGS (OPERA): dominant $\nu_\mu \rightarrow \nu_\tau$

KamLAND: $\bar{\nu}_e \rightarrow \bar{\nu}_e$; $\bar{\nu}_e \rightarrow (\bar{\nu}_\mu + \bar{\nu}_\tau)/\sqrt{2}$



Source	Type of ν	\bar{E} [MeV]	L [km]	$\min(\Delta m^2)$ [eV ²]
Reactor	$\tilde{\nu}_e$	~ 1	1	$\sim 10^{-3}$
Reactor	$\tilde{\nu}_e$	~ 1	100	$\sim 10^{-5}$
Accelerator	$\nu_\mu, \tilde{\nu}_\mu$	$\sim 10^3$	1	~ 1
Accelerator	$\nu_\mu, \tilde{\nu}_\mu$	$\sim 10^3$	1000	$\sim 10^{-3}$
Atmospheric ν 's	$\nu_{\mu,e}, \tilde{\nu}_{\mu,e}$	$\sim 10^3$	10^4	$\sim 10^{-4}$
Sun	ν_e	~ 1	1.5×10^8	$\sim 10^{-11}$

Correspond to: CHOOZ, Double Chooz, RENO, Daya Bay ($L \sim 1$ km), KamLAND ($L \sim 100$ km);

$\tilde{\nu}_e$ disappearance; $E = (1.8 \div 8.0)$ MeV;

to accelerator experiments - past ($L \sim 1$ km);

past, current: K2K ($L \sim 250$ km), MINOS ($L \sim 730$ km), ν_μ disappearance; OPERA ($L \sim 730$ km), $\nu_\mu \rightarrow \nu_\tau$;

T2K ($L \sim 295$ km), NO ν A ($L \sim 800$ km), ν_μ disappearance, $\nu_\mu \rightarrow \nu_e$; $E \sim 1$ GeV;

SK experiment studying atmospheric $\nu_\mu, \tilde{\nu}_\mu, \nu_e, \tilde{\nu}_e$ ($E \cong 0.1 \div 100$ GeV), and solar ν_e ($E \cong 5 \div 14$ MeV) oscillations, and to the solar ν experiments ($E \cong 0.29 \div 14$ MeV).

$$|\nu_l\rangle = \sum_j U_{lj}^* |\nu_j; \tilde{p}_j\rangle, \quad l = e, \mu, \tau$$

$\pi^+ \rightarrow \mu^+ + \nu_\mu$ decay at rest:

$$E_j = E + m_j^2/(2m_\pi), \quad p_j = E - \xi m_j^2/(2E), \quad E = (m_\pi/2)(1 - m_\mu^2/m_\pi^2) \cong 30 \text{ MeV}, \quad \xi = (1 + m_\mu^2/m_\pi^2)/2 \cong 0.8.$$

Taking $m_j = 1 \text{ eV}$: $E_j \cong E (1 + 1.2 \times 10^{-16})$,
 $p_j \cong E (1 - 4.4 \times 10^{-16})$.

Problem avoided if one uses the fact that the ν_j state is entangled with the μ^+ state.

$$A(\nu_l \rightarrow \nu_{l'}) = \sum_j U_{l'j} D_j U_{jl}^\dagger, \quad l, l' = e, \mu, \tau,$$

$$D_j = e^{-i\tilde{p}_j(x_f - x_0)} = e^{-i(E_j T - p_j L)}, \quad p_j \equiv |\mathbf{p}_j|.$$

$$\begin{aligned} \delta\varphi_{jk} &= (E_j - E_k)T - (p_j - p_k)L \\ &= (E_j - E_k) \left[T - \frac{E_j + E_k}{p_j + p_k} L \right] + \frac{m_j^2 - m_k^2}{p_j + p_k} L; \end{aligned}$$

First term - negligible:

- L and T related: $T = (E_j + E_k) L / (p_j + p_k) = L / \bar{v}$,
 $\bar{v} = (E_j / (E_j + E_k)) v_j + (E_k / (E_j + E_k)) v_k$ - the “average” velocity of ν_j and ν_k ,
 $v_{j,k} = p_{j,k} / E_{j,k}$;
- $E_j = E_k = E_0$;
- $p_j = p_k = p$
 (additionally suppressed by $(m_j^2 + m_k^2) / p^2$: $L = T$ up to $\sim m_{j,k}^2 / p^2$);
- $E_j \neq E_k$, $p_j \neq p_k$, $j \neq k$: the same conclusion
 (neutrinos are relativistic, $L \cong T$ up to corrections $\sim m_{j,k}^2 / E_{j,k}^2$).

$$\delta\varphi_{jk} \simeq \frac{m_j^2 - m_k^2}{2p} L = 2\pi \frac{L}{L_{jk}^{\nu}} \text{sgn}(m_j^2 - m_k^2), \quad p = (p_j + p_k)/2,$$

$$L_{jk}^{\nu} = 4\pi \frac{p}{|\Delta m_{jk}^2|} \simeq 2.5 \text{ m} \frac{p[\text{MeV}]}{|\Delta m_{jk}^2|[\text{eV}^2]}$$

is the neutrino oscillation length associated with Δm_{jk}^2 .

- One can safely neglect the dependence of p_j and p_k on the masses m_j and m_k and consider p to be the zero neutrino mass momentum, $p = E$.
- The phase $\delta\varphi_{jk}$ is Lorentz invariant.

$$\sigma_{m^2} = \sqrt{(2E\sigma_E)^2 + (2p\sigma_p)^2}$$

Condition for producing coherently ν_1, ν_2, \dots :

$$\sigma_{m^2} > |\Delta m_{jk}^2|$$

The equation used above corresponds to a plane wave description of the propagation of neutrinos ν_j . It accounts only for the movement of the center of the wave packet describing ν_j . In the wave packet treatment of the problem, the interference between the states of ν_j and ν_k is subject to a number of conditions, the localisation condition (in space and time) and the condition of overlapping of the wave packets of ν_j and ν_k at the detection point being the most important. For relativistic neutrinos, the localisation condition in space reads: $\sigma_{xP}, \sigma_{xD} < L_{jk}^v / (2\pi)$, $\sigma_{xP(D)}$ being the spatial width of the production (detection) wave packet. Thus, the interference will not be suppressed if the spatial width of the neutrino wave packets determined by the neutrino production and detection processes is smaller than the corresponding oscillation length in vacuum. In order for the interference to be nonzero, the wave packets describing ν_j and ν_k should also overlap in the point of neutrino detection. This requires that the spatial separation between the two wave packets at the point of neutrinos detection, caused by the two wave packets having different group velocities $v_j \neq v_k$, satisfies $|(v_j - v_k)T| \ll \max(\sigma_{xP}, \sigma_{xD})$. If the interval of time T is not measured, T in the preceding condition must be replaced by the distance L between the neutrino source and the detector.

Examples

- Spatial localisation condition

ΔL - dimensions of the ν - source (and/or detector):

$$2\pi\Delta L/L_{jk}^{\nu} \lesssim 1.$$

- Time localisation condition

ΔE - detector's energy resolution:

$$2\pi(L/L_{jk}^{\nu})(\Delta E/E) \lesssim 1.$$

If $2\pi\Delta L/L_{jk}^{\nu} \gg 1$, and/or $2\pi(L/L_{jk}^{\nu})(\Delta E/E) \gg 1$,

$$\bar{P}(\nu_l \rightarrow \nu_{l'}) = \bar{P}(\bar{\nu}_l \rightarrow \bar{\nu}_{l'}) \cong \sum_j |U_{l'j}|^2 |U_{lj}|^2$$

Two-Neutrino Oscillations in Vacuum

SK ((100-12742) km), K2K (250 km); CNGS (OPERA), MINOS (730 km); T2K (295 km); dominant $\nu_\mu \rightarrow \nu_\tau$;

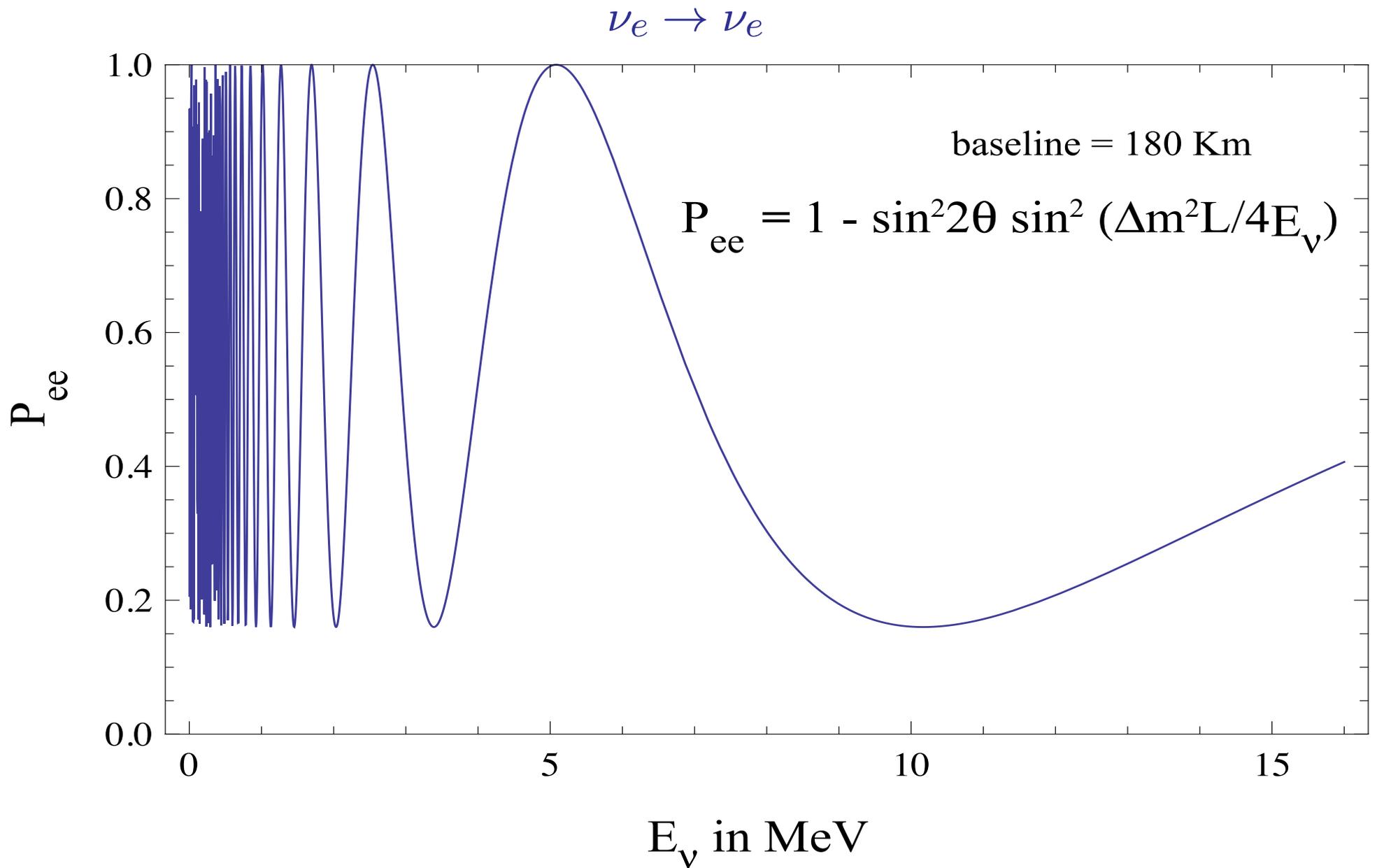
$$P(\nu_\mu \rightarrow \nu_\tau; L) = P(\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau; L) \cong \sin^2 2\theta_{23} \sin^2 \frac{\Delta m_{31}^2 L}{4E};$$
$$P(\nu_\mu \rightarrow \nu_\mu; L) = P(\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu; L) = 1 - P(\nu_\mu \rightarrow \nu_\tau; L).$$

KamLAND (~ 180 km): $\bar{\nu}_e \rightarrow \bar{\nu}_e$

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e; L) \cong 1 - \frac{1}{2} \sin^2 2\theta_{12} \left(1 - \cos \frac{\Delta m_{21}^2 L}{2E}\right).$$

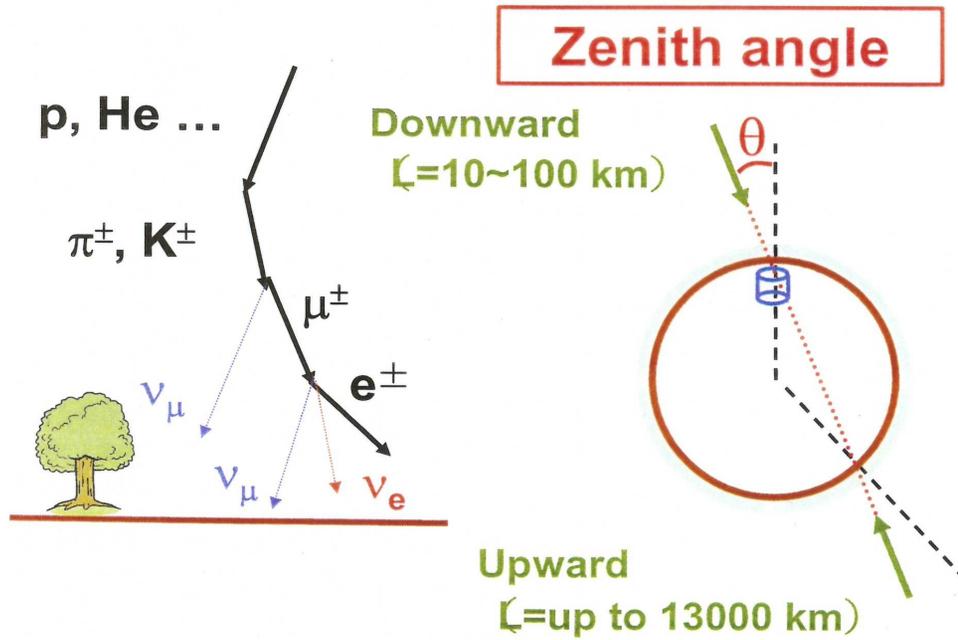
CHOOZ, Double Chooz, Daya Bay, RENO (~ 1 km):
 $\bar{\nu}_e \rightarrow \bar{\nu}_e$

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e; L) \cong 1 - \frac{1}{2} \sin^2 2\theta_{13} \left(1 - \cos \frac{\Delta m_{31}^2 L}{2E}\right).$$

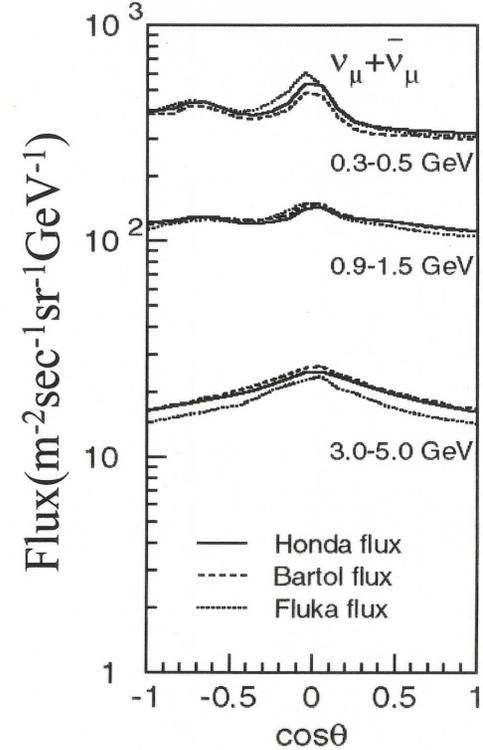


Observing the Oscillations of Neutrinos

Atmospheric neutrinos

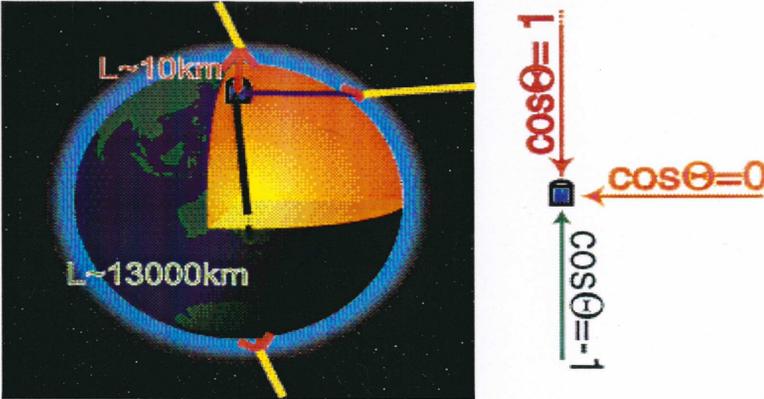


Zenith angle dist. of Atmospheric ν flux

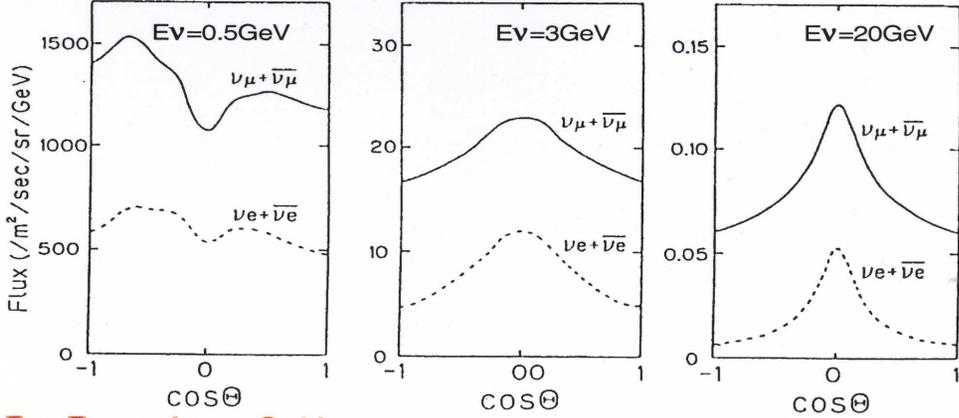


$E_\nu > \text{a few GeV}$
Up/Down Symmetry

Zenith angle distribution(1D)

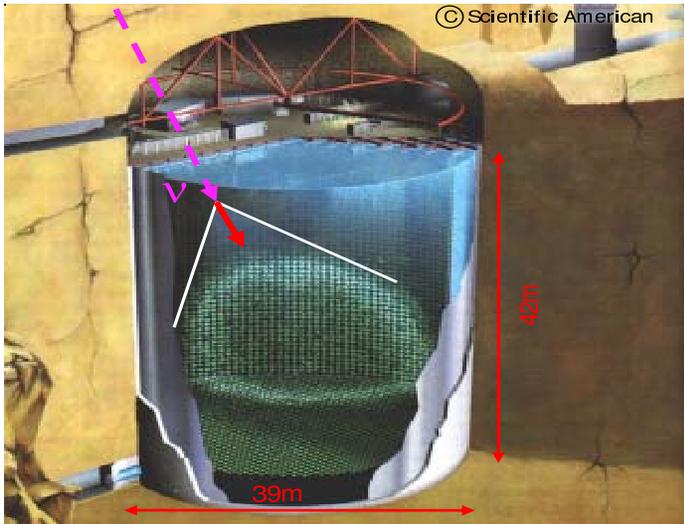


Calculated zenith angle distribution



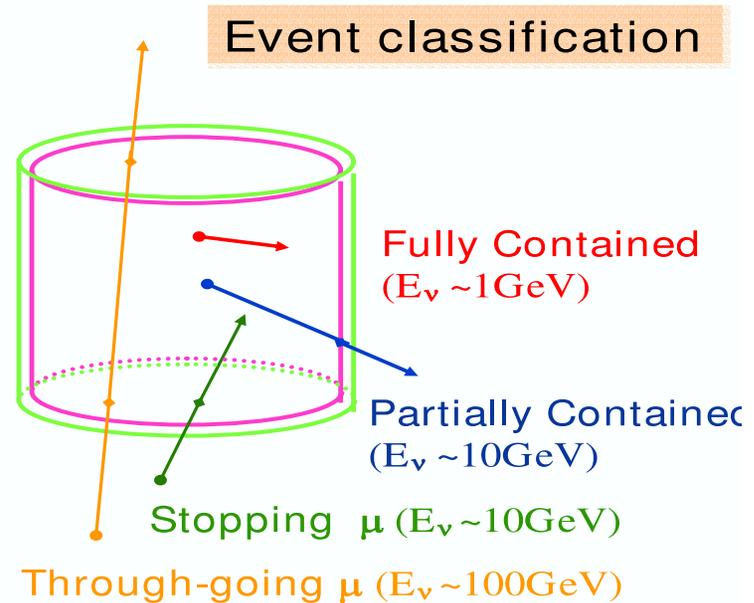
For $E_\nu >$ a few GeV,
Upward / downward = 1 (within a few %)

↳ Up/Down asymmetry for neutrino oscillations



Water Cherenkov detector

1000 m underground
 50,000 ton (22,500 ton fid.)
 inner-detector(ID): 11,146
 outer-detector(OD): 1,885

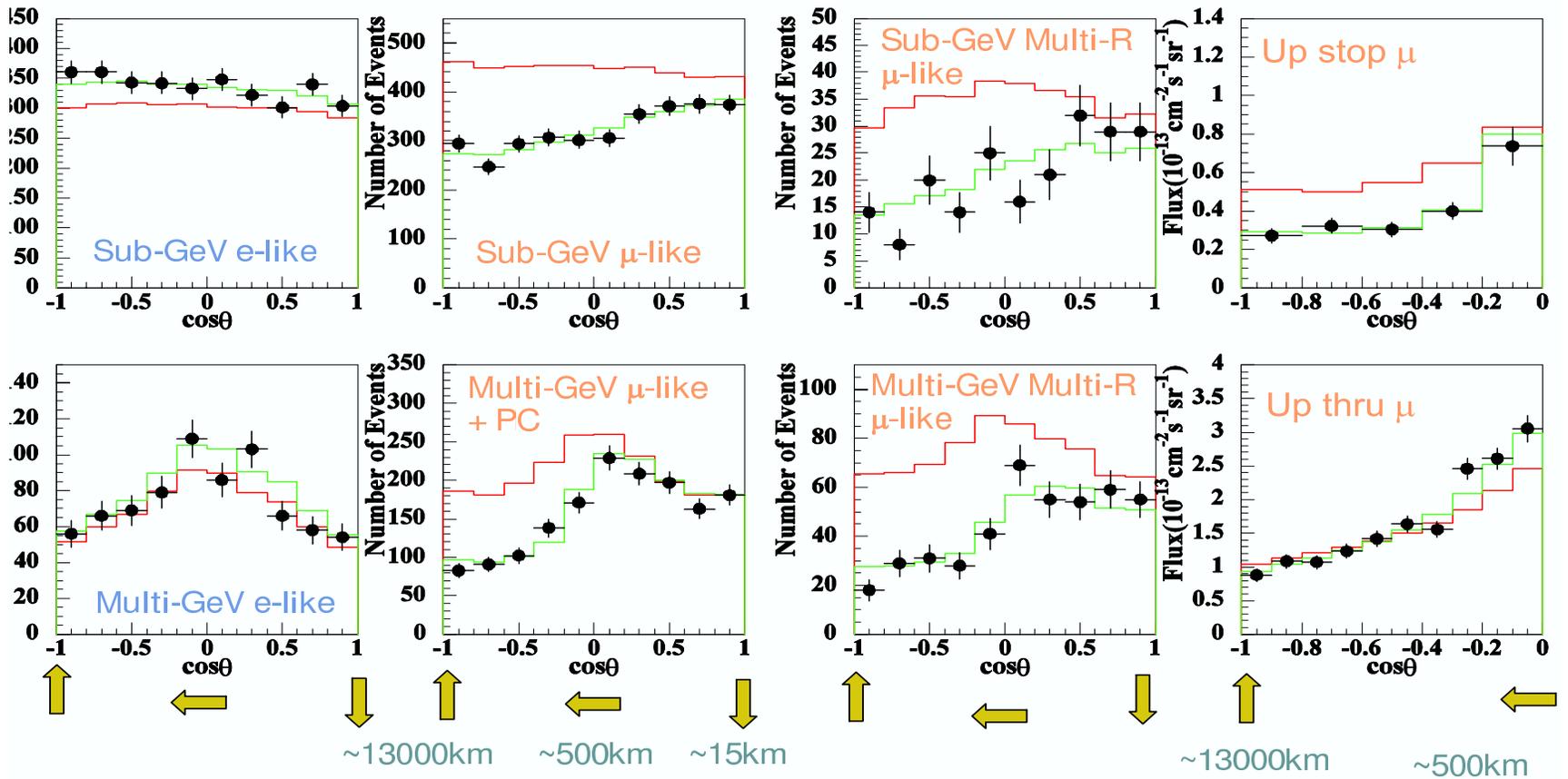


20 inch PMTs(SK-I)
 8 inch PMTs

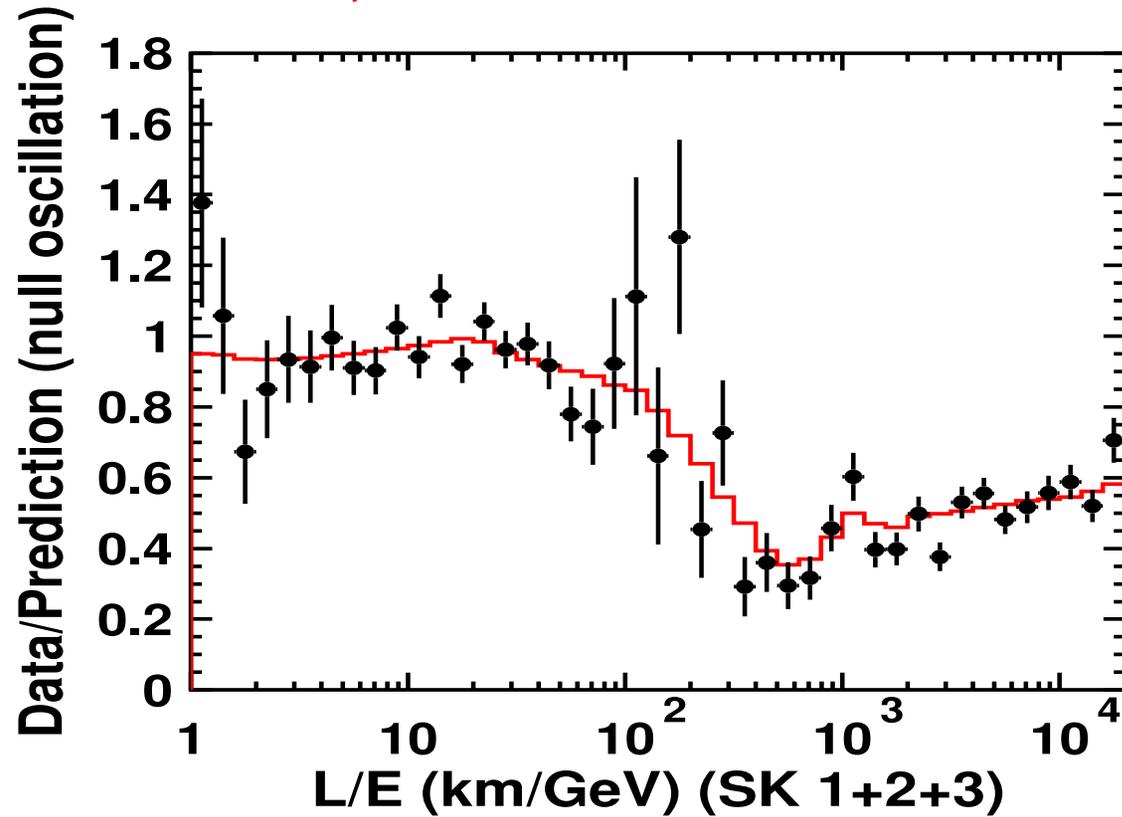
Zenith angle distributions

$\nu_\mu \leftrightarrow \nu_\tau$
2-flavor oscillations

Best fit
 $\sin^2 2\theta = 1.0, \Delta m^2 = 2.0 \times 10^{-3} \text{ eV}^2$
Null oscillation



SK: L/E Dependence, μ -Like Events

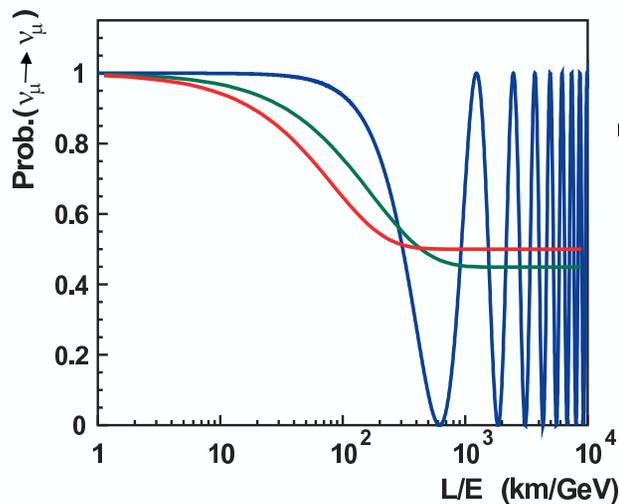


L/E analysis

Neutrino oscillation : $P_{\mu\mu} = 1 - \sin^2 2\theta \sin^2(1.27 \frac{\Delta m^2 L}{E})$

Neutrino decay : $P_{\mu\mu} = (\cos^2 \theta + \sin^2 \theta \times \exp(-\frac{m}{2\tau} \frac{L}{E}))^2$

Neutrino decoherence : $P_{\mu\mu} = 1 - \frac{1}{2} \sin^2 2\theta \times (1 - \exp(-\gamma_0 \frac{L}{E}))$



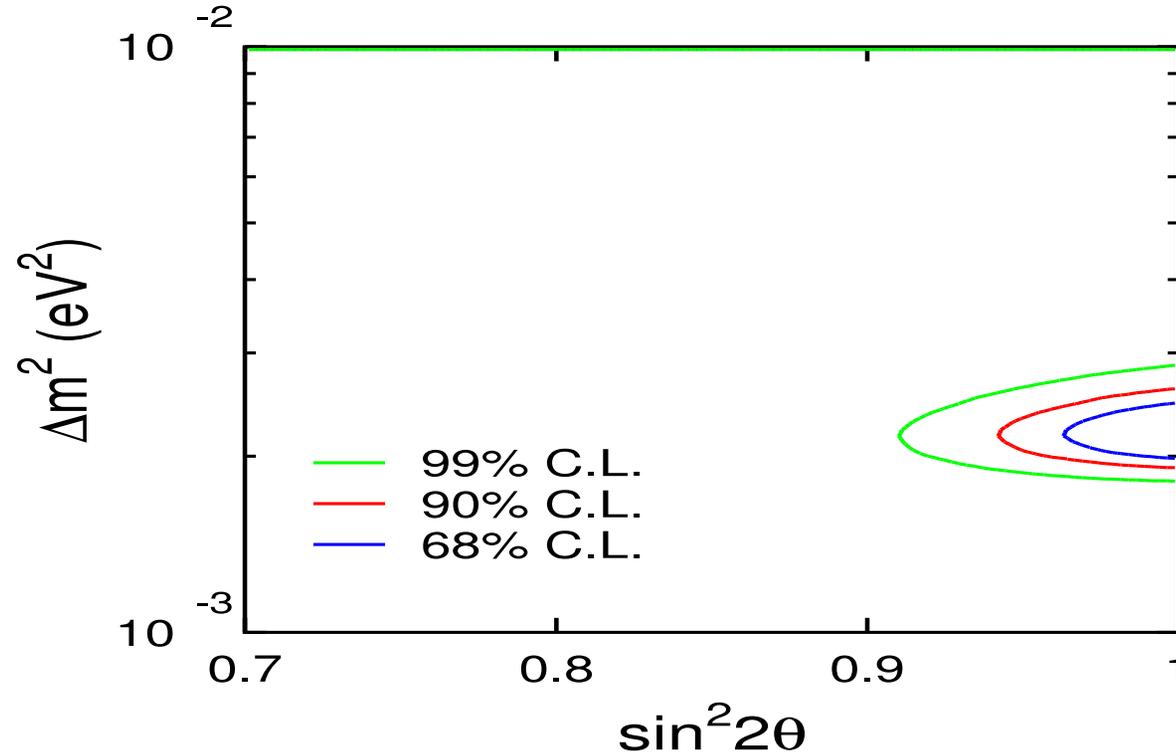
Use events with high resolution in L/E



The first dip can be observed

- Direct evidence for oscillations
- Strong constraint to oscillation parameters, especially Δm^2 value

SK: Atmospheric ν Data



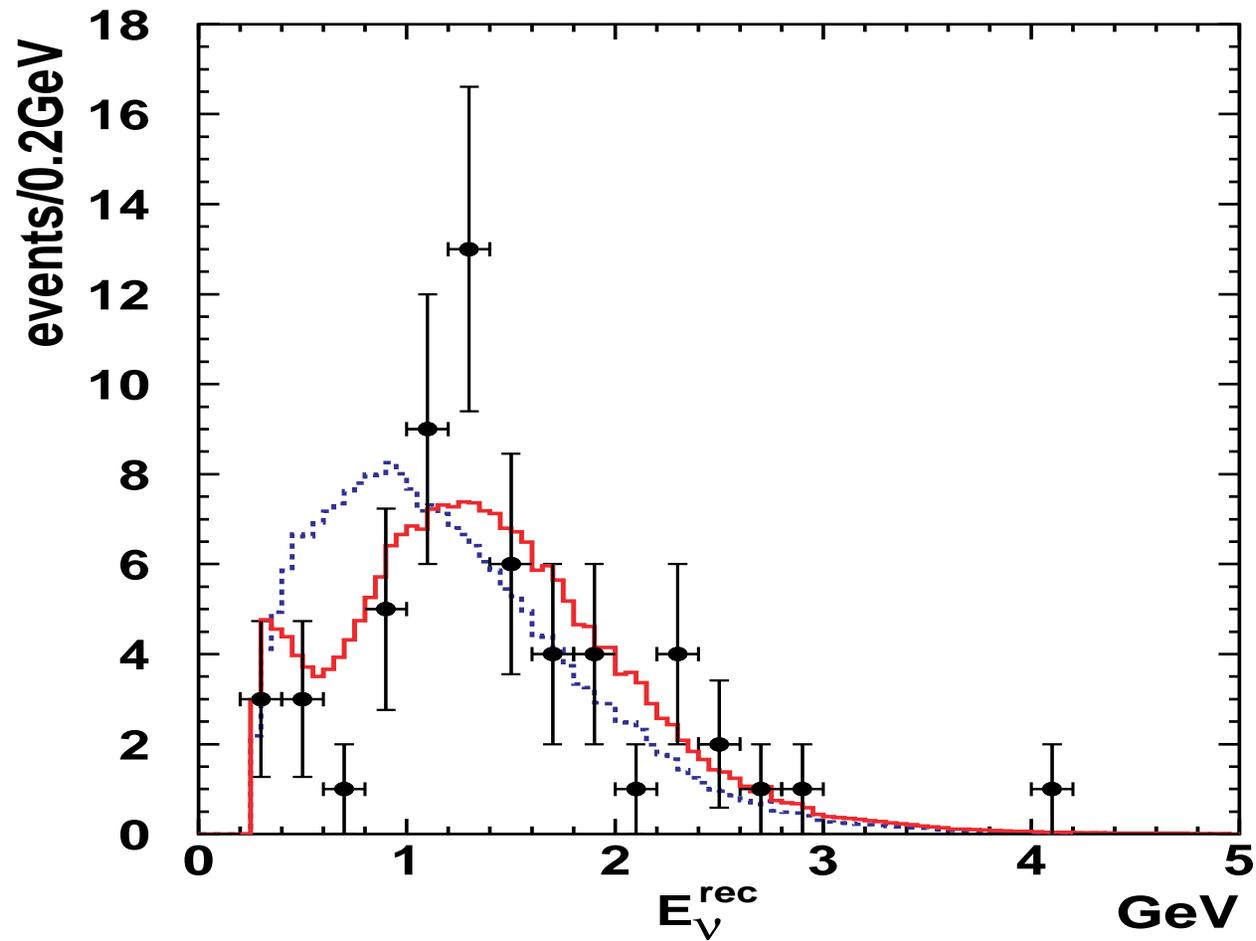
$$\Delta m_{\text{atm}}^2 \equiv \Delta m_{31}^2 = 2.4 \times 10^{-3} \text{ eV}^2, \quad \sin^2 2\theta_{\text{atm}} \equiv \sin^2 2\theta_{23} = 1.0 ;$$

$$\Delta m_{31}^2 = (1.9 - 2.9) \times 10^{-3} \text{ eV}^2, \quad \sin^2 2\theta_{23} \geq 0.92, \quad 99\% \text{ C.L.}$$

- sign of Δm_{atm}^2 not determined. If $\theta_{23} \neq \frac{\pi}{4}$: $\theta_{23}, (\frac{\pi}{4} - \theta_{23})$ ambiguity.

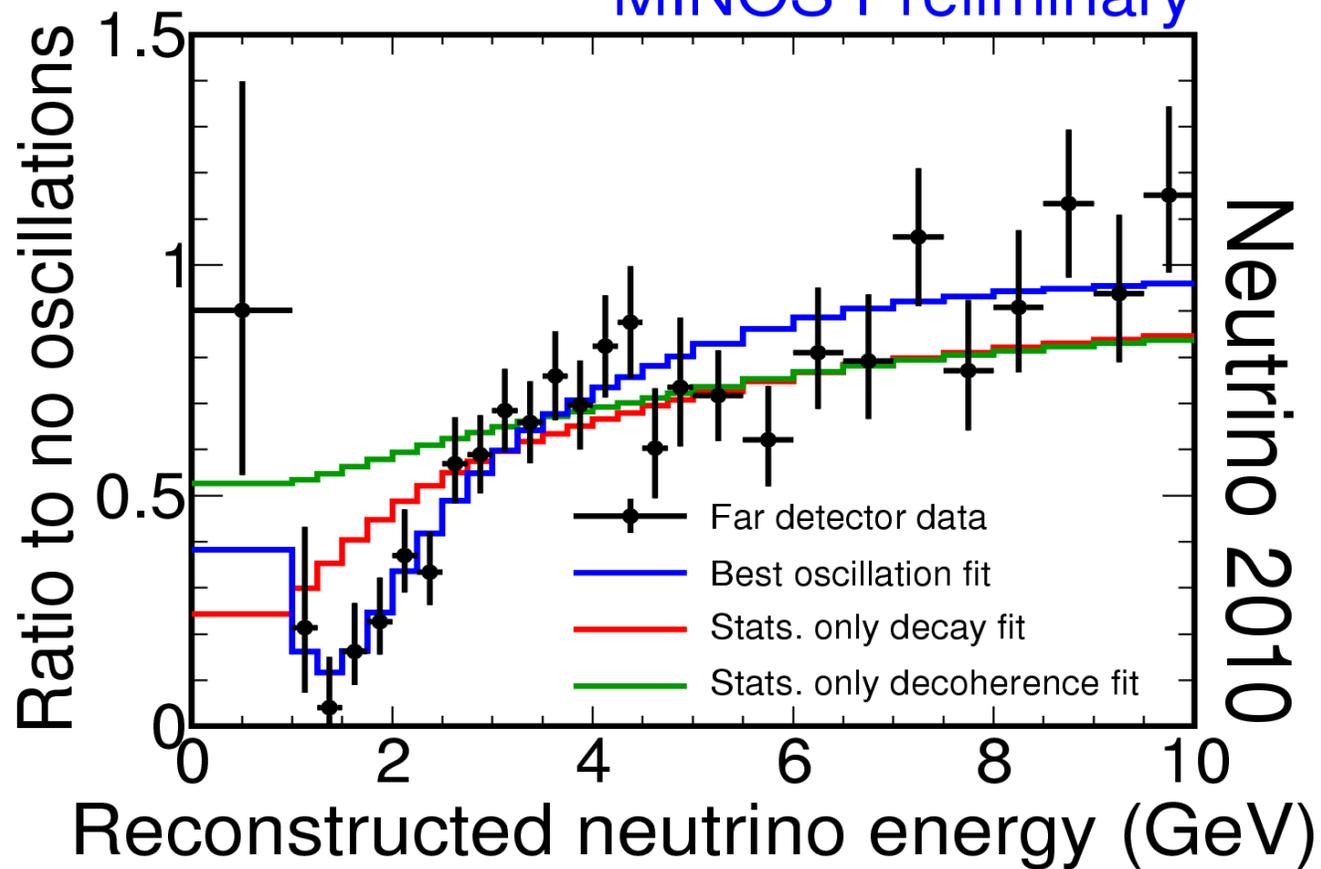
3- ν mixing: $\Delta m_{31}^2 > 0, m_1 < m_2 < m_3$ (NH); $\Delta m_{31}^2 < 0, m_3 < m_1 < m_2$ (IH).

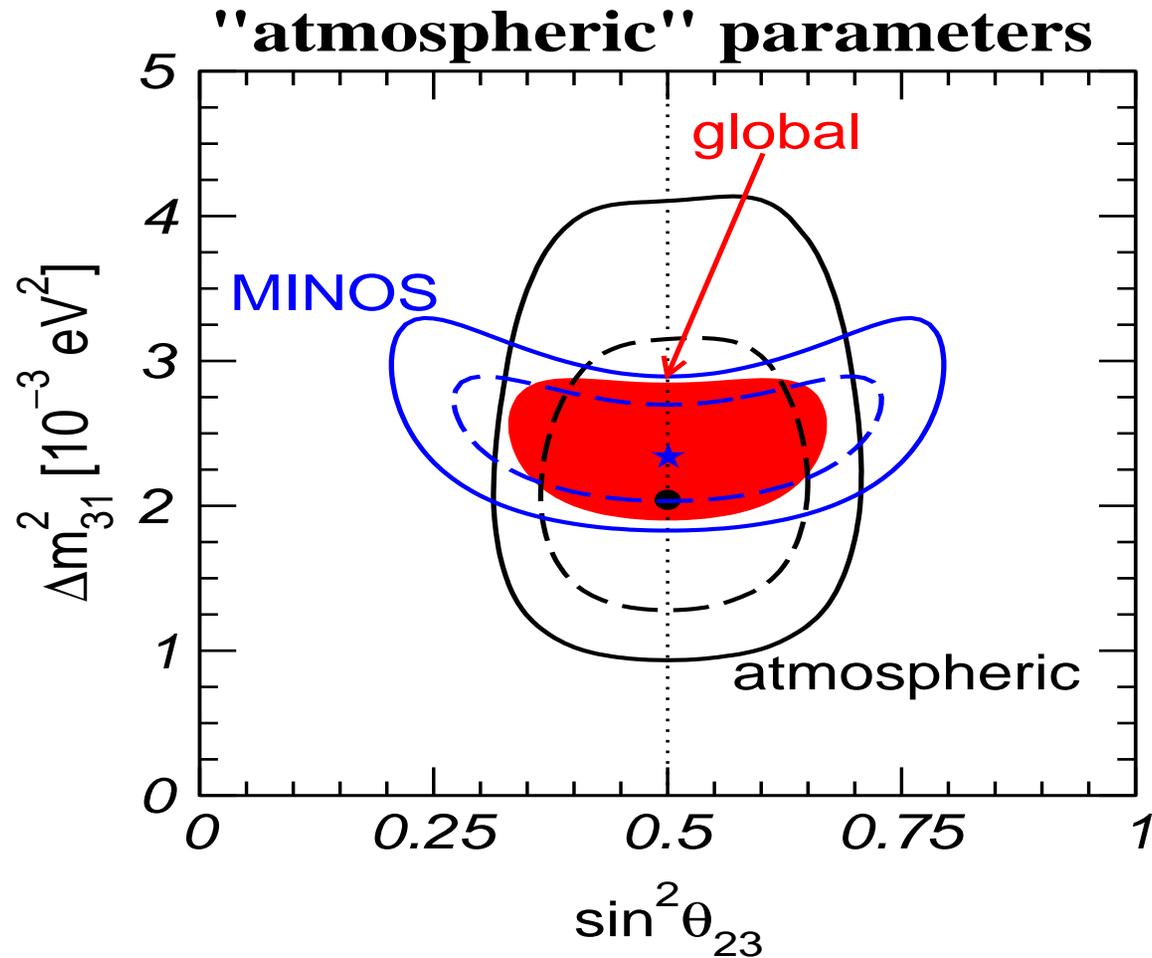
K2K: ν_μ Spectrum (ν_μ "disappearance")



MINOS: ν_μ Spectrum (ν_μ "disappearance")

MINOS Preliminary





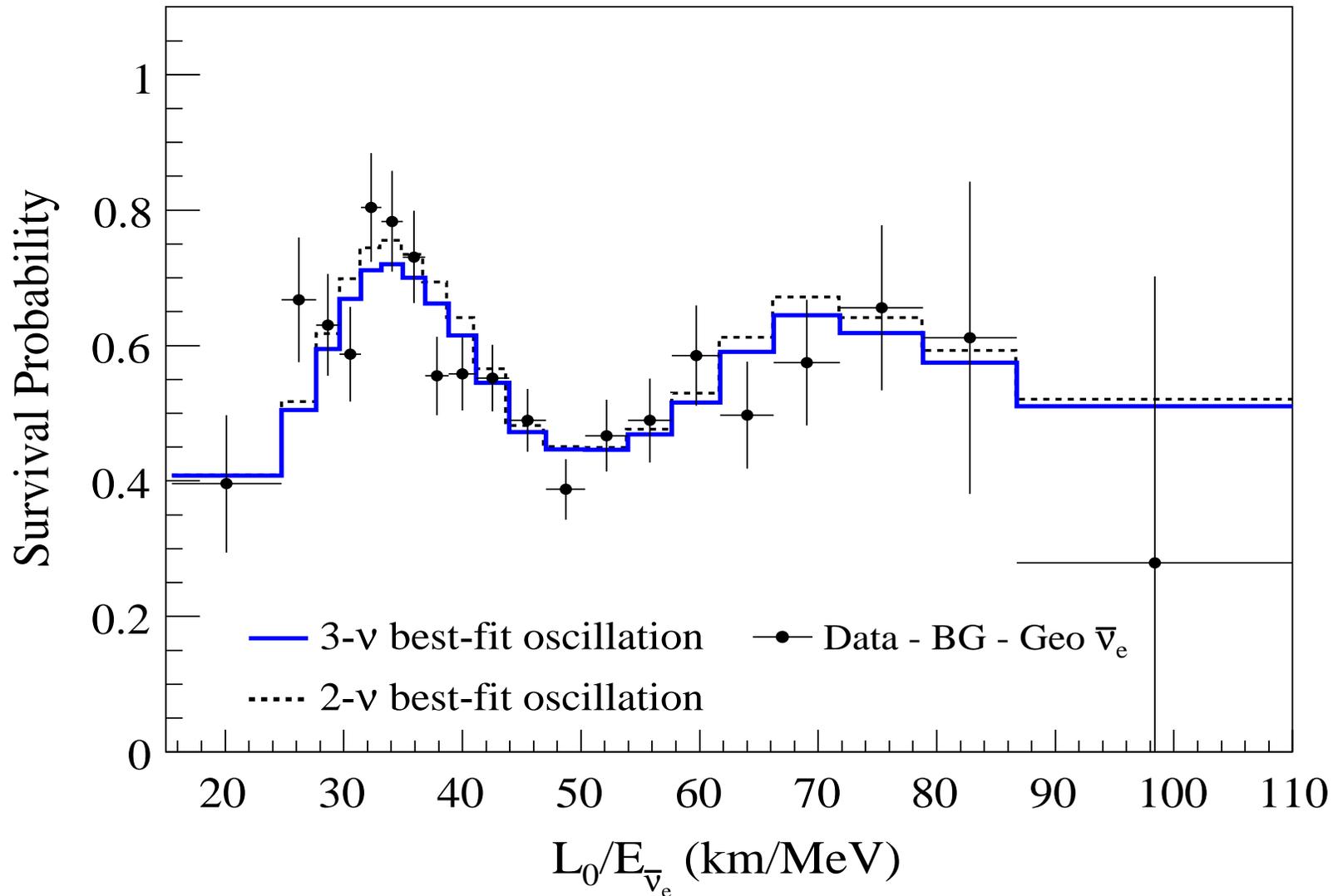
- sign of Δm_{atm}^2 not determined;

T. Schwetz, arXiv:0710.5027[hep-ph]

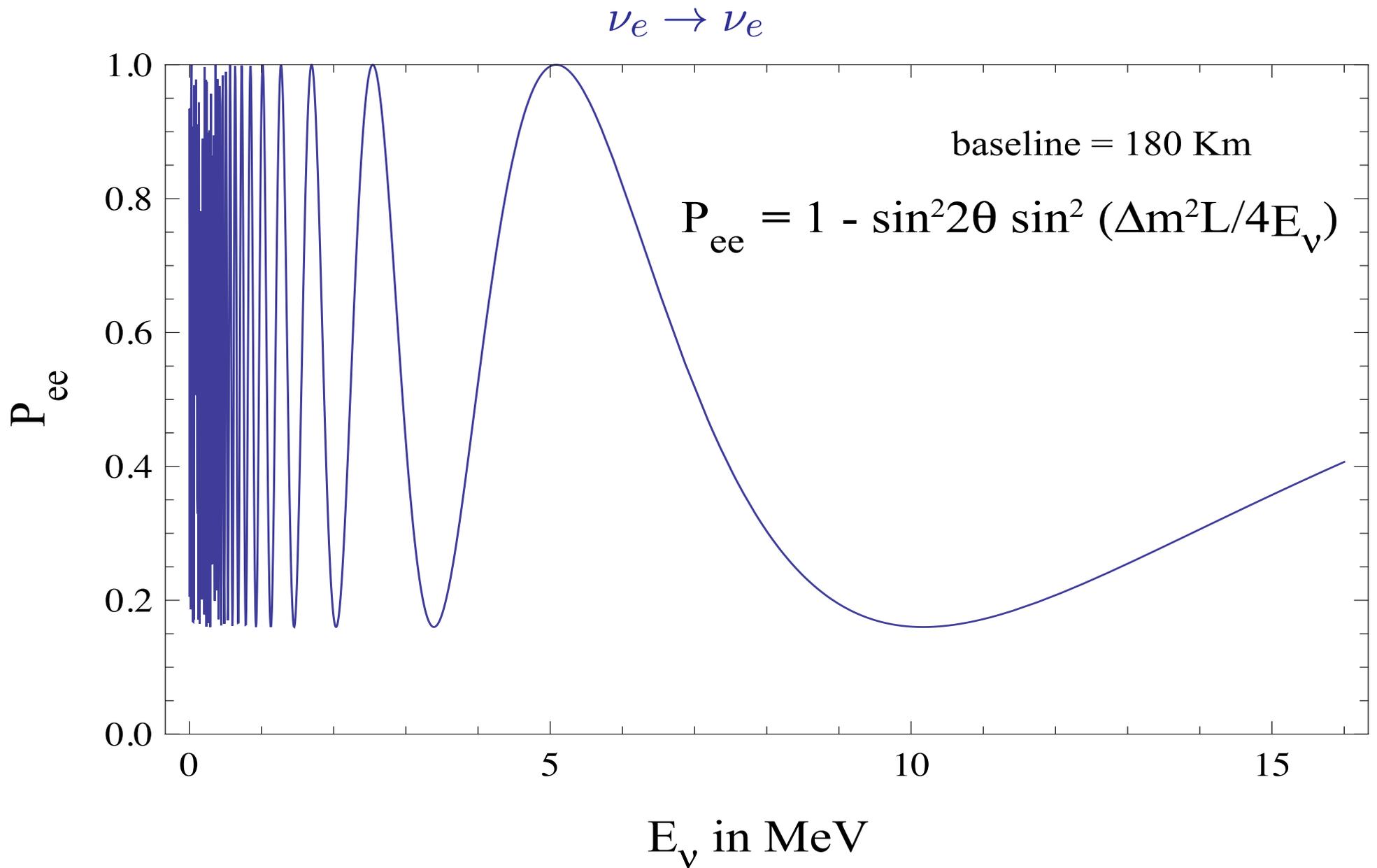
3- ν mixing: $\Delta m_{31}^2 > 0$, $m_1 < m_2 < m_3$ (normal ordering (NO));

$\Delta m_{31}^2 < 0$, $m_3 < m_1 < m_2$ (inverted ordering (IO)).

- If $\theta_{23} \neq \frac{\pi}{4}$: θ_{23} , $(\frac{\pi}{4} - \theta_{23})$ ambiguity.



KamLAND: L/E -Dependence (reactor $\bar{\nu}_e$, $\bar{L} = 180$ km, $E = (1.8 - 10)$ MeV)



Solar Neutrinos: ν_e , $E \sim (0.26 - 14.4)$ MeV

Super-Kamiokande, $E \cong (5.0 - 14.4)$ MeV

$$\begin{aligned} R(SK) &\propto \Phi_E^0(\nu_e) \sum_{l=e,\mu,\tau} P(\nu_e \rightarrow \nu_l) \sigma(\nu_l e^- \rightarrow \nu_l e^-) \\ &= \sigma(\nu_e e^- \rightarrow \nu_e e^-) [\Phi_E^0(\nu_e) P(\nu_e \rightarrow \nu_e) \\ &\quad + \Phi_E^0(\nu_e) (1 - P(\nu_e \rightarrow \nu_e)) \frac{\sigma(\nu_{\mu(\tau)} e^- \rightarrow \nu_{\mu(\tau)} e^-)}{\sigma(\nu_e e^- \rightarrow \nu_e e^-)}] \\ &= \sigma(\nu_e e^- \rightarrow \nu_e e^-) [\Phi_E(\nu_e) + 0.16(\Phi_E(\nu_\mu) + \Phi_E(\nu_\tau))] \end{aligned}$$

$$\begin{aligned} P(\nu_e \rightarrow \nu_e) + P(\nu_e \rightarrow \nu_\mu) + P(\nu_e \rightarrow \nu_\tau) &= 1, \\ \sigma(\nu_\mu e^- \rightarrow \nu_\mu e^-) &= \sigma(\nu_\tau e^- \rightarrow \nu_\tau e^-). \end{aligned}$$

SNO, CC: $E \cong (5.0 - 14.4)$ MeV



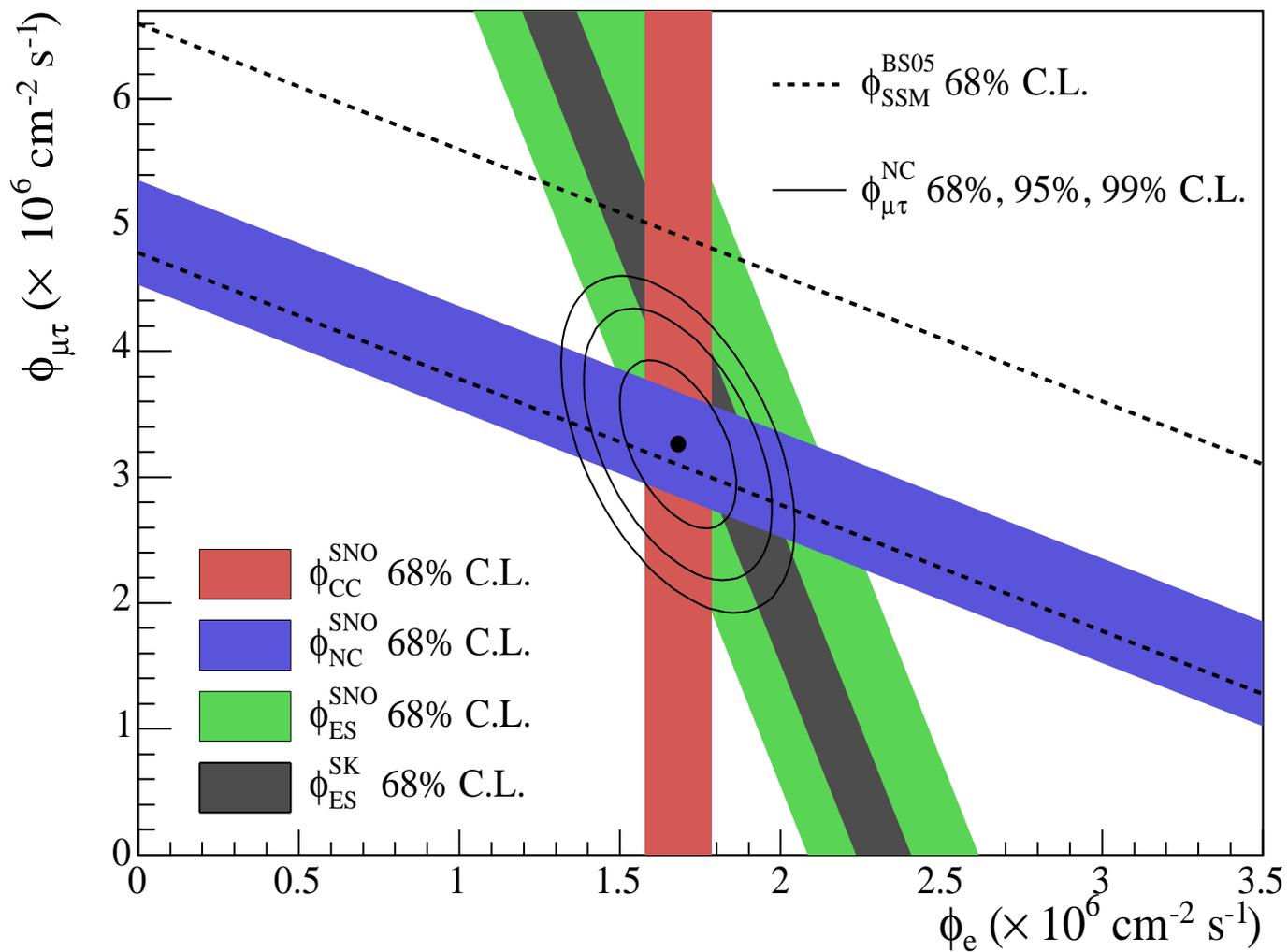
$$R(SNO) \propto \sigma(\nu_e + D \rightarrow e^- + p + p) \Phi_E^0(\nu_e) P(\nu_e \rightarrow \nu_e)$$

$$= \sigma(\nu_e + D \rightarrow e^- + p + p) \Phi_E(\nu_e)$$

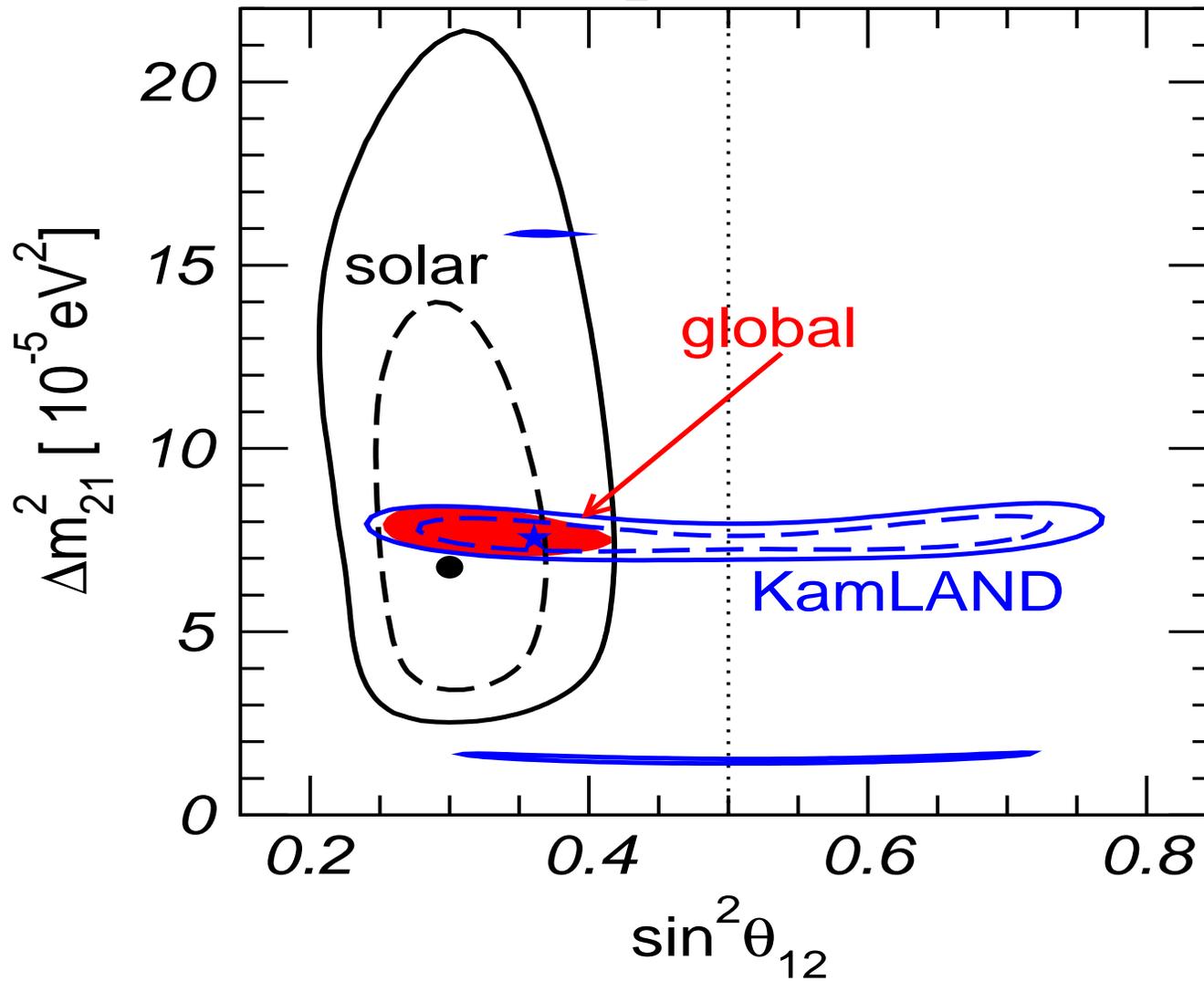
$$SK: \Phi^{SK}(\nu_\odot) = \Phi_E(\nu_e) + 0.16(\Phi_E(\nu_\mu) + \Phi_E(\nu_\tau))$$

$$SNO \text{ CC: } \Phi^{SNO}(\nu_\odot) = \Phi_E(\nu_e)$$

$$\text{No oscillations: } \Phi_E(\nu_\mu) + \Phi_E(\nu_\tau) = 0, \quad \Phi^{SK}(\nu_\odot) = \Phi^{SNO}(\nu_\odot)$$



"solar" parameters



T. Schwetz, arXiv:0710.5027[hep-ph]

Dirac CP-Nonconservation: δ in U_{PMNS}

Observable manifestations in

$$\nu_l \leftrightarrow \nu_{l'} , \quad \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'} , \quad l, l' = e, \mu, \tau$$

- not sensitive to Majorana CPVP α_{21}, α_{31}

CP-invariance:

$$P(\nu_l \rightarrow \nu_{l'}) = P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'}) , \quad l \neq l' = e, \mu, \tau$$

N. Cabibbo, 1978
S.M. Bilenky, J. Hosek, S.T.P., 1980;
V. Barger, S. Pakvasa et al., 1980.

CPT-invariance:

$$P(\nu_l \rightarrow \nu_{l'}) = P(\bar{\nu}_{l'} \rightarrow \bar{\nu}_l)$$
$$l = l' : \quad P(\nu_l \rightarrow \nu_l) = P(\bar{\nu}_l \rightarrow \bar{\nu}_l)$$

T-invariance:

$$P(\nu_l \rightarrow \nu_{l'}) = P(\nu_{l'} \rightarrow \nu_l), \quad l \neq l'$$

3ν —mixing:

$$A_{\text{CP}}^{(l,l')} \equiv P(\nu_l \rightarrow \nu_{l'}) - P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'}) , \quad l \neq l' = e, \mu, \tau$$

$$A_{\text{T}}^{(l,l')} \equiv P(\nu_l \rightarrow \nu_{l'}) - P(\nu_{l'} \rightarrow \nu_l), \quad l \neq l'$$

$$A_{\text{T}(\text{CP})}^{(e,\mu)} = A_{\text{T}(\text{CP})}^{(\mu,\tau)} = -A_{\text{T}(\text{CP})}^{(e,\tau)}$$

P.I. Krastev, S.T.P., 1988; V. Barger, S. Pakvasa et al., 1980

In vacuum:

$$A_{\text{CP(T)}}^{(e,\mu)} = J_{\text{CP}} F_{\text{osc}}^{\text{vac}}$$

$$J_{\text{CP}} = \text{Im} \{ U_{e1} U_{\mu 2} U_{e2}^* U_{\mu 1}^* \} = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta$$

$$F_{\text{osc}}^{\text{vac}} = \sin\left(\frac{\Delta m_{21}^2 L}{2E}\right) + \sin\left(\frac{\Delta m_{32}^2 L}{2E}\right) + \sin\left(\frac{\Delta m_{13}^2 L}{2E}\right)$$

P.I. Krastev, S.T.P., 1988

In matter: Matter effects violate

CP : $P(\nu_l \rightarrow \nu_{l'}) \neq P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'})$

CPT : $P(\nu_l \rightarrow \nu_{l'}) \neq P(\bar{\nu}_{l'} \rightarrow \bar{\nu}_l)$

P. Langacker et al., 1987

Can conserve the T-invariance (**Earth**)

$$P(\nu_l \rightarrow \nu_{l'}) = P(\nu_{l'} \rightarrow \nu_l), \quad l \neq l'$$

In matter with constant density: $A_{\text{T}}^{(e,\mu)} = J_{\text{CP}}^{\text{mat}} F_{\text{osc}}^{\text{mat}}$

$$J_{\text{CP}}^{\text{mat}} = J_{\text{CP}}^{\text{vac}} R_{\text{CP}}$$

R_{CP} does not depend on θ_{23} and δ ; $|R_{\text{CP}}| \lesssim 2.5$

P.I. Krastev, S.T.P., 1988

Rephasing Invariants Associated with CPVP

Dirac phase δ :

$$J_{CP} = \text{Im} \{ U_{e1} U_{\mu 2} U_{e2}^* U_{\mu 1}^* \} .$$

C. Jarlskog, 1985 (for quarks)

CP-, T- violation effects in neutrino oscillations

P. Krastev, S.T.P., 1988

Majorana phases α_{21}, α_{31} :

$$S_1 = \text{Im} \{ U_{e1} U_{e3}^* \}, \quad S_2 = \text{Im} \{ U_{e2} U_{e3}^* \} \quad (\text{not unique}); \quad \text{or}$$
$$S'_1 = \text{Im} \{ U_{\tau 1} U_{\tau 2}^* \}, \quad S'_2 = \text{Im} \{ U_{\tau 2} U_{\tau 3}^* \}$$

J.F. Nieves and P. Pal, 1987, 2001

G.C. Branco et al., 1986

J.A. Aguilar-Saavedra and G.C. Branco, 2000

CP-violation: both $\text{Im} \{ U_{e1} U_{e3}^* \} \neq 0$ and $\text{Re} \{ U_{e1} U_{e3}^* \} \neq 0$.

S_1, S_2 appear in $|\langle m \rangle|$ in $(\beta\beta)_{0\nu}$ -decay.

In general, J_{CP}, S_1 and S_2 are independent.

Matter Effects in Neutrino Oscillations

Matter can affect strongly ν -oscillations:

Mean free path in matter with $\bar{\rho} = \bar{\rho}(\text{Earth})$: $N \cong 4N_A \text{cm}^{-3}$,

$E \sim 1 \text{ MeV}$, $L_f \sim 2.5 \times 10^{14} \text{ km}$; $R_E = 6371 \text{ km}$

$E \sim 1 \text{ GeV}$, $L_f \sim 2.5 \times 10^8 \text{ km}$

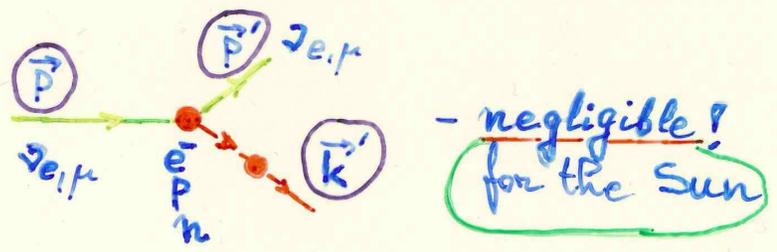
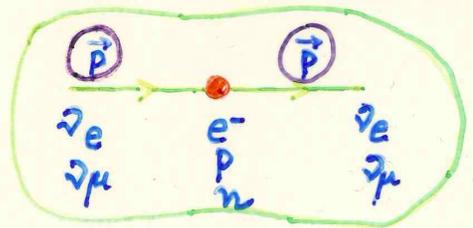
For $\bar{\rho} = \rho(\text{center of the Sun})$: $N \cong 100 N_A \text{cm}^{-3}$,

$E \sim 1 \text{ MeV}$, $L_f \sim 10^{13} \text{ km}$; $R_{Sun} = 6.96 \times 10^5 \text{ km}$

ν **coherent** scattering on e^- , p , n - effective potential (index of refraction)

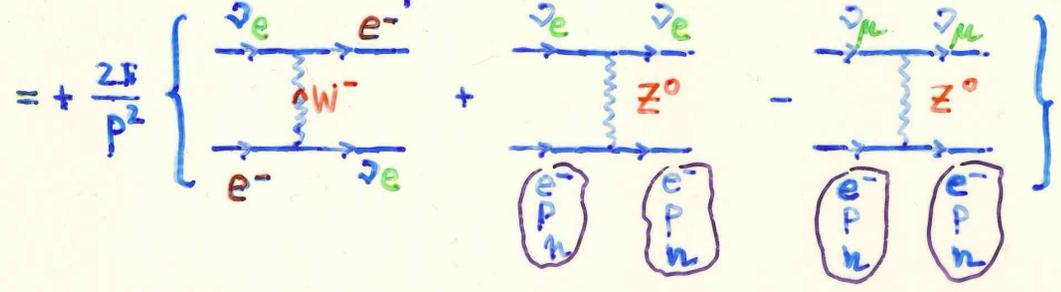
The presence of matter can change drastically the pattern of ν -oscillations

$$H_m = H_{vac} + H_{int}$$



$$n(\nu_e) \neq 1, n(\nu_\mu) \neq 1$$

$$n(\nu_e) - n(\nu_\mu) = \frac{2\pi}{p^2} [F_{\nu_e}^{\nu_e}(0) - F_{\nu_e}^{\nu_\mu}(0)] =$$



$$= -\frac{1}{p} \sqrt{2} G_F N_e$$

ν coherent scattering on e^- , p , n - effective potential (index of refraction)

$$V_{e\mu} = V(\nu_e) - V(\nu_\mu) = \sqrt{2}G_F N_e$$

$$\bar{V}_{e\mu} = V(\bar{\nu}_e) - V(\bar{\nu}_\mu) = -\sqrt{2}G_F N_e$$

$$V_{\mu\tau} = V(\nu_\mu) - V(\nu_\tau) = 0 \text{ (leading order)}$$

L. Wolfenstein, 1978; V. Barger et al., 1980; P. Langacker et al., 1983

$$V_{es} = V(\nu_e) - V(\nu_s) = \sqrt{2}G_F(N_e - \frac{1}{2}N_n)$$

$$\bar{V}_{es} = V(\bar{\nu}_e) - V(\bar{\nu}_s) = -\sqrt{2}G_F(N_e - \frac{1}{2}N_n) = -V_{es}$$

$$V_{\mu s} = V(\nu_\mu) - V(\nu_s) = \sqrt{2}G_F(-\frac{1}{2}N_n)$$

$$\bar{V}_{\mu s} = V(\bar{\nu}_\mu) - V(\bar{\nu}_s) = -\sqrt{2}G_F(-\frac{1}{2}N_n) = -V_{\mu s}$$

$V_{e\mu} \neq \bar{V}_{e\mu}$: CP, CPT violated

P. Langacker, S.T.P. et al., 1987

Neutrino Oscillations in Matter

When neutrinos propagate in matter, they interact with the background of electrons, protons and neutrons, which generates an effective potential in the neutrino Hamiltonian: $H = H_{vac} + V_{eff}$.

This modifies the neutrino mixing since the eigenstates and the eigenvalues of H_{vac} and of $H = H_{vac} + V_{eff}$ are different, leading to a different oscillation probability w.r.t to that in vacuum.

Typically the matter background is not CP and CPT symmetric, e.g., the Earth and the Sun contain only electrons, protons and neutrons, and the resulting oscillations violate CP and CPT symmetries.

$$P(\nu_\mu \rightarrow \nu_e) \cong \sin^2 \theta_{23} \sin^2 2\theta_{13}^m \sin^2 \frac{\Delta M_{31}^2 L}{4E}$$

$$i \frac{d}{dt} \begin{pmatrix} A_\alpha(t, t_0) \\ A_\beta(t, t_0) \end{pmatrix} = \begin{pmatrix} -\epsilon(t) & \epsilon'(t) \\ \epsilon'(t) & \epsilon(t) \end{pmatrix} \begin{pmatrix} A_\alpha(t, t_0) \\ A_\beta(t, t_0) \end{pmatrix}$$

where $\alpha = \nu_e$, $\beta = \nu_{\mu(\tau)}$,

$$\epsilon(t) = \frac{1}{2} \left[-\frac{\Delta m^2}{2E} \cos 2\theta - \sqrt{2} G_F N_e(t) \right],$$

$$\epsilon'(t) = \frac{\Delta m^2}{4E} \sin 2\theta, \text{ with } \Delta m^2 = m_2^2 - m_1^2.$$

In matter, $H_m = H_0 + H_{int}$.

$H_0 |\nu_{1,2}\rangle = E_{1,2} |\nu_{1,2}\rangle$, not eigenstates of H_m .

Consider first $N_e = \text{const.}$

$$H_m |\nu_{1,2}^m\rangle = E_{1,2}^m |\nu_{1,2}^m\rangle.$$

Then at $t = 0$ in matter

$$|\nu_e\rangle = |\nu_1^m\rangle \cos \theta_m + |\nu_2^m\rangle \sin \theta_m,$$

$$|\nu_{\mu(\tau)}\rangle = -|\nu_1^m\rangle \sin \theta_m + |\nu_2^m\rangle \cos \theta_m;$$

$$\sin 2\theta_m = \frac{\epsilon'}{\sqrt{\epsilon^2 + \epsilon'^2}} = \frac{\tan 2\theta}{\sqrt{(1 - \frac{N_e}{N_e^{\text{res}}})^2 + \tan^2 2\theta}},$$

$$\cos 2\theta_m = \frac{1 - N_e/N_e^{\text{res}}}{\sqrt{(1 - \frac{N_e}{N_e^{\text{res}}})^2 + \tan^2 2\theta}},$$

$$N_e^{\text{res}} = \frac{\Delta m^2 \cos 2\theta}{2E\sqrt{2}G_F} \simeq 6.56 \times 10^6 \frac{\Delta m^2 [\text{eV}^2]}{E [\text{MeV}]} \cos 2\theta \text{ cm}^{-3} N_A,$$

$$E_2^m - E_1^m = \frac{\Delta m^2}{2E} \left(\left(1 - \frac{N_e}{N_e^{\text{res}}}\right)^2 \cos^2 2\theta + \sin^2 2\theta \right)^{\frac{1}{2}}$$

$$P_m^{2\nu}(\nu_e \rightarrow \nu_\mu) = |A_\mu(t)|^2 = \frac{1}{2} \sin^2 2\theta_m \left[1 - \cos 2\pi \frac{L}{L_m} \right],$$

$$L_m = \frac{E_2^m - E_1^m}{2\pi} = L^v \left(\left(1 - \frac{N_e}{N_e^{res}} \right)^2 \cos^2 2\theta + \sin^2 2\theta \right)^{-\frac{1}{2}}.$$

The resonance condition: $N_e = N_e^{res} = \frac{\Delta m^2 \cos 2\theta}{2E\sqrt{2}G_F}$

At the resonance:

$$\sin^2 2\theta_m = 1, \quad \min(E_2^m - E_1^m), \quad L_m^{res} = L^v / \sin 2\theta.$$

Limiting cases:

$$N_e \ll N_e^{res}: \quad \theta_m \cong \theta, \quad E_{1,2}^m \cong E_{1,2}, \quad L_m \cong L^v.$$

$$N_e \gg N_e^{res}: \quad \theta_m \cong \frac{\pi}{2}, \quad \nu_e \rightarrow \nu_\mu \text{ suppressed.}$$

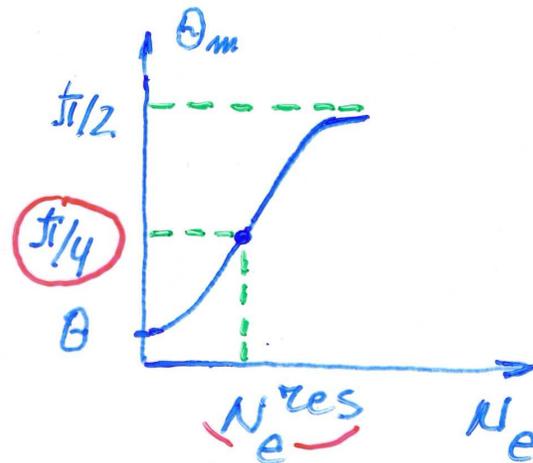
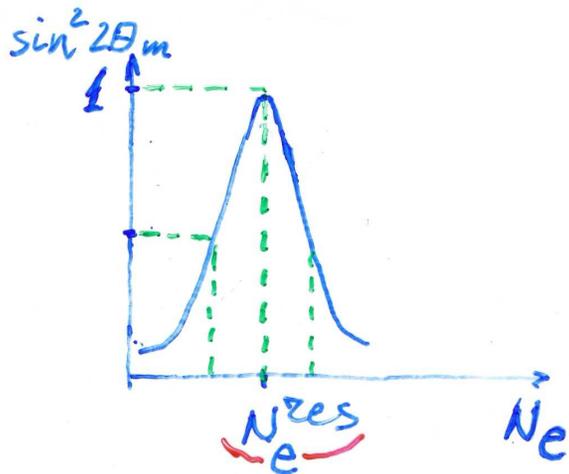
$$\text{In this case: } |\nu_e\rangle \cong |\nu_2^m\rangle, \quad |\nu_\mu\rangle = -|\nu_1^m\rangle.$$

$$N_e^{\text{res}} = \frac{\Delta m^2 \cos 2\theta}{2E\sqrt{2}G_F}$$

$$N_e \gg N_e^{\text{res}}, \theta_m \approx \pi/2$$

$$N_e \ll N_e^{\text{res}}, \theta_m \approx 0$$

$$N_e = N_e^{\text{res}}, \theta_m \approx \pi/4$$



$$\Delta N_e^{\text{res}} = 2N_e^{\text{res}} \tan 2\theta$$

$$E_2^m - E_1^m \Big|_{\text{res}} = \min (E_2^m - E_1^m)$$

Antineutrinos: $N_e \rightarrow (-N_e)$

$\Delta m^2 \cos 2\theta > 0$: $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$ suppressed by matter; $\nu_e \rightarrow \nu_\mu$ can be enhanced.

$\Delta m^2 \cos 2\theta < 0$: $\nu_e \rightarrow \nu_\mu$ suppressed by matter; $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$ can be enhanced.

V. Barger et al., 1980; S.P. Mikheyev, A.Yu. Smirnov, 1985

Oscillations in matter (Earth, Sun) are neither CP- nor CPT- invariant.

P. Langacker, S.T.P., S. Toshev, G. Steigman, 1987

Earth: $\bar{N}_e^{mant} \sim 2.3 N_A \text{ cm}^{-3}$, $\bar{N}_e^{core} \sim 6.0 N_A \text{ cm}^{-3}$

$$P^m(\nu_e \rightarrow \nu_\mu; t) = \frac{1}{2} \sin^2 2\theta_m \left(1 - \cos 2\pi \frac{L}{L_{osc}^m}\right), \quad L_{osc}^m \sim L_{osc}^{vac}$$

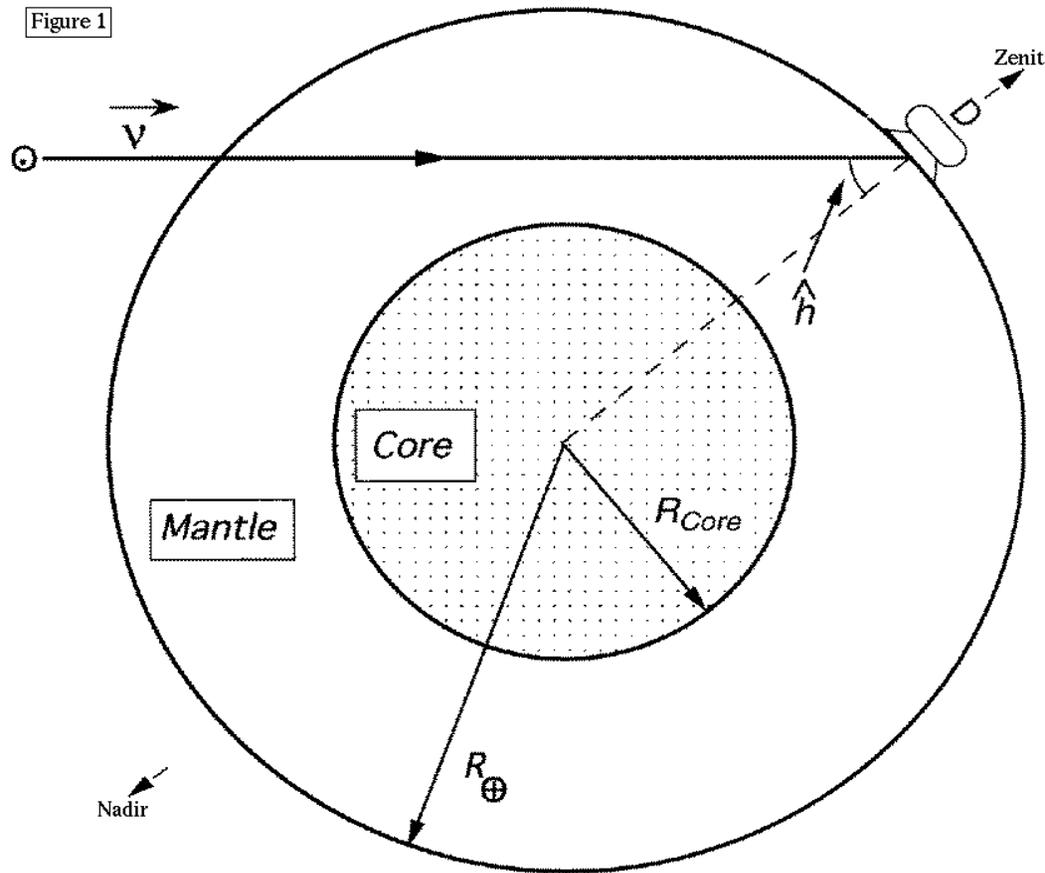
$$\sin^2 2\theta_m = \frac{\sin^2 2\theta}{\left(1 - \frac{N_e}{N_e^{res}}\right)^2 \cos^2 2\theta + \sin^2 2\theta}, \quad N_e^{res} \equiv \frac{\Delta m^2 \cos 2\theta}{2E\sqrt{2}G_F}$$

$N_e = N_e^{res}$: MSW (Mikheyev, Smirnov, Wolfenstein) resonance

$\Delta m^2 \cos 2\theta > 0$: $\nu_e \rightarrow \nu_\mu$

$\Delta m^2 \cos 2\theta < 0$: $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$

The Earth



Earth: $R_{core} = 3446 \text{ km}$, $R_{mant} = 2885 \text{ km}$

Earth: $\bar{N}_e^{mant} \sim 2.3 N_A \text{ cm}^{-3}$, $\bar{N}_e^{core} \sim 5.7 N_A \text{ cm}^{-3}$

The Earth

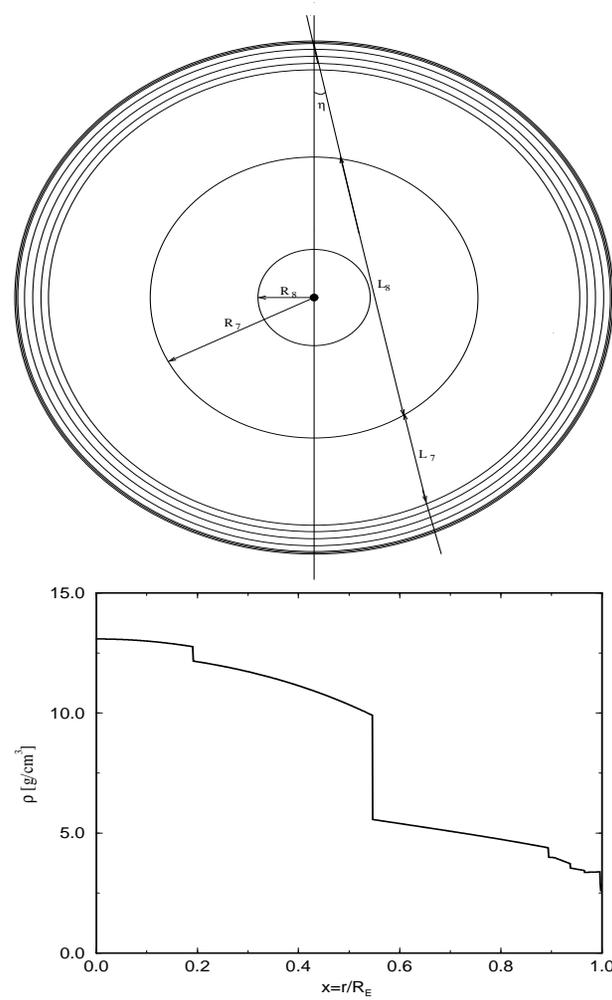
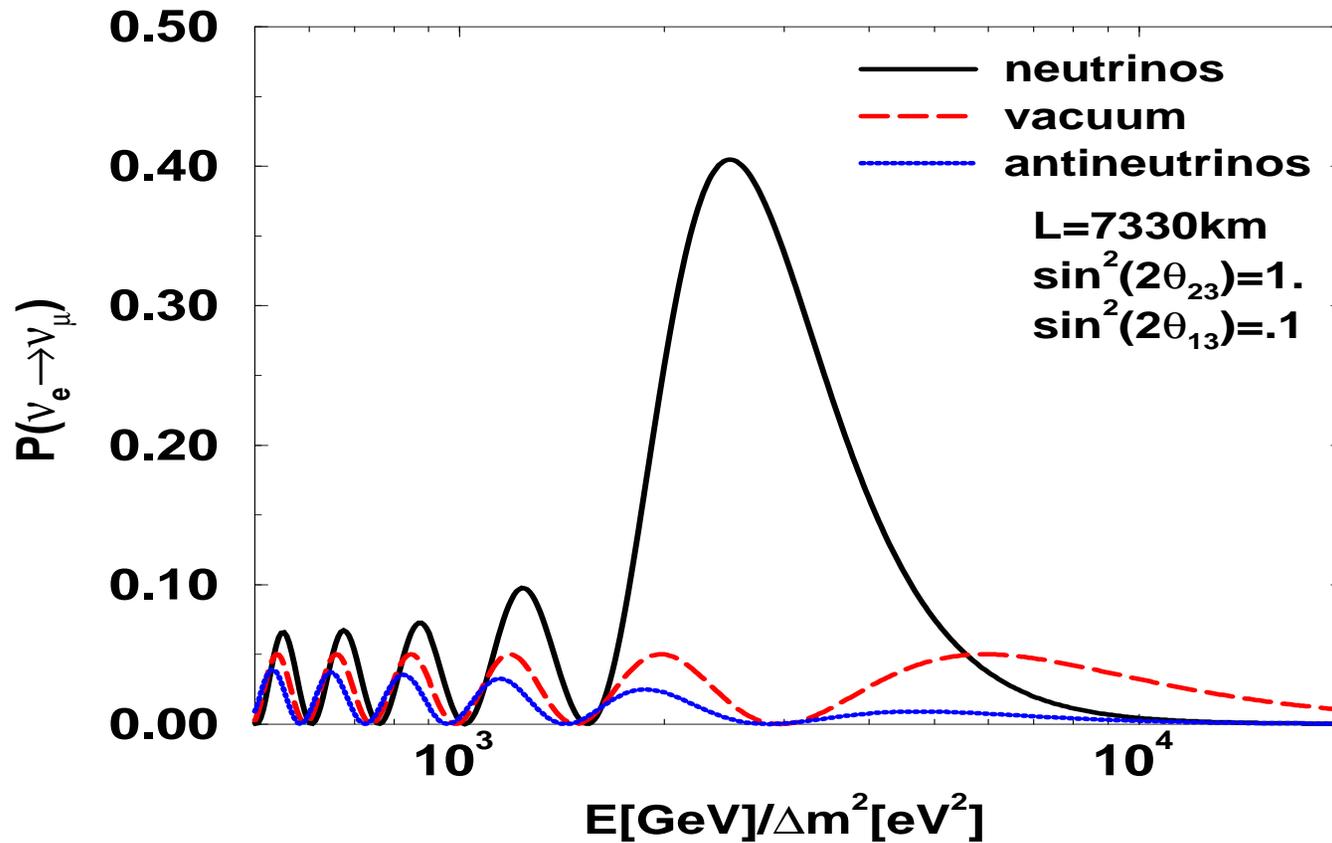


FIG. 1. Density profile of the Earth.

$$R_c = 3446 \text{ km}, R_m = 2885 \text{ km}; \bar{N}_e^{mant} \sim 2.3 N_A \text{ cm}^{-3}, \bar{N}_e^{core} \sim 5.7 N_A \text{ cm}^{-3}$$

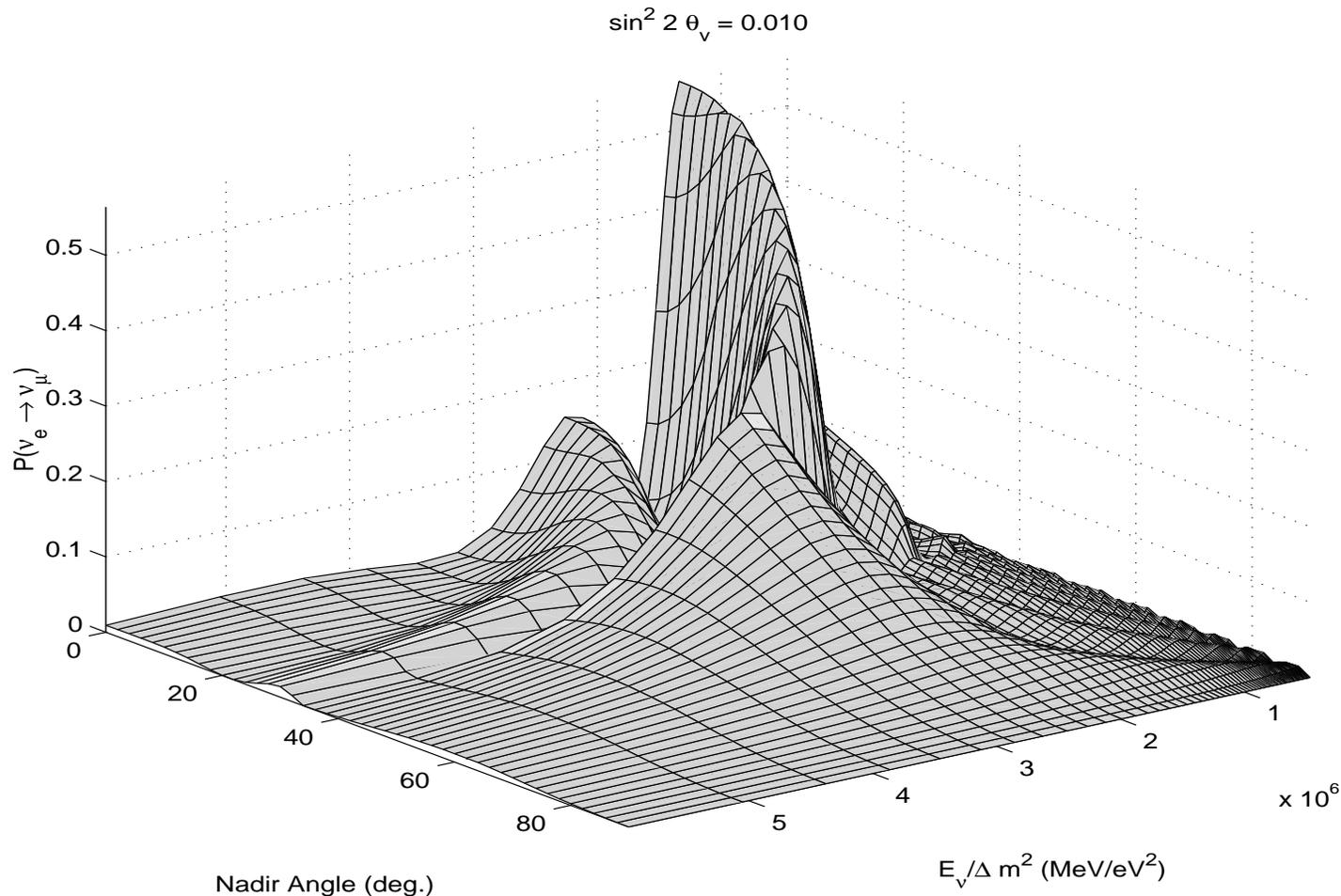
Earth matter effect in $\nu_\mu \rightarrow \nu_e, \bar{\nu}_\mu \rightarrow \bar{\nu}_e$ (MSW)



$$\Delta m^2 = 2.5 \times 10^{-3} \text{ eV}^2, E^{res} = 6.25 \text{ GeV}; P^{3\nu} = \sin^2 \theta_{23} P_m^{2\nu} = 0.5 P_m^{2\nu};$$

$$N_e^{res} \cong 2.3 \text{ cm}^{-3} N_A; L_m^{res} = L^v / \sin 2\theta_{13} \cong 6250 / 0.32 \text{ km}; 2\pi L / L_m \cong 0.75\pi (\neq \pi).$$

Earth matter effects in $\nu_\mu \rightarrow \nu_e, \bar{\nu}_\mu \rightarrow \bar{\nu}_e$ (NOLR)



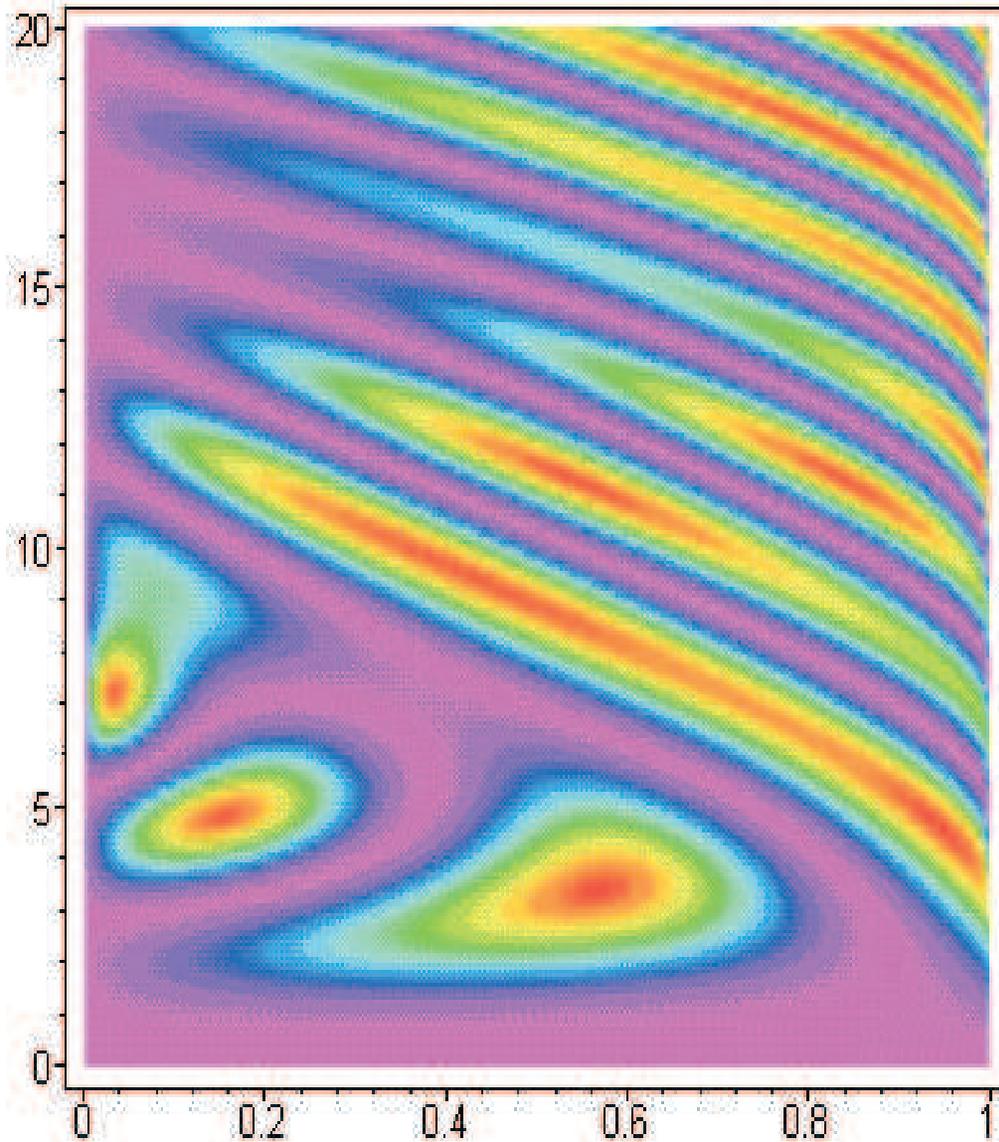
S.T.P., 1998;

M. Chizhov, M. Maris, S.T.P., 1998; M. Chizhov, S.T.P., 1999

$P(\nu_e \rightarrow \nu_\mu) \equiv P_{2\nu} \equiv (s_{23})^{-2} P_{3\nu}(\nu_{e(\mu)} \rightarrow \nu_{\mu(e)})$, $\theta_\nu \equiv \theta_{13}$, $\Delta m^2 \equiv \Delta m_{\text{atm}}^2$;

Absolute maximum: Neutrino Oscillation Length Resonance (NOLR);

Local maxima: MSW effect in the Earth mantle or core.



$(s_{23})^{-2} P_{3\nu}(\nu_{e(\mu)} \rightarrow \nu_{\mu(e)}) \equiv P_{2\nu}$; **NOLR: “Dark Red Spots”, $P_{2\nu} = 1$;**
Vertical axis: $\Delta m^2/E$ [$10^{-7} eV^2/MeV$]; horizontal axis: $\sin^2 2\theta_{13}$; $\theta_n = 0$

M. Chizhov, S.T.P., 1999 (hep-ph/9903399,9903424)

- For Earth center crossing ν 's ($\theta_n = 0$) and, e.g. $\sin^2 2\theta_{13} = 0.01$, **NOLR** occurs at $E \cong 4$ **GeV** ($\Delta m^2(atm) = 2.5 \times 10^{-3} \text{ eV}^2$).

S.T.P., hep-ph/9805262

- For the Earth core crossing ν 's: $P_{2\nu} = 1$ **due to NOLR** when

$$\tan \Phi^{\text{man}}/2 \equiv \tan \phi' = \pm \sqrt{\frac{-\cos 2\theta''_m}{\cos(2\theta''_m - 4\theta'_m)}},$$

$$\tan \Phi^{\text{core}}/2 \equiv \tan \phi'' = \pm \sqrt{\frac{\cos 2\theta'_m}{-\cos(2\theta''_m) \cos(2\theta''_m - 4\theta'_m)}}$$

Φ^{man} (Φ^{core}) - phase accumulated in the Earth mantle (core),
 θ'_m (θ''_m) - the mixing angle in the Earth mantle (core).

$P_{2\nu} = 1$ **due to NOLR** for $\theta_n = 0$ (Earth center crossing ν 's) at,
 e.g. $\sin^2 2\theta_{13} = 0.034; 0.154$, $E \cong 3.5; 5.2$ **GeV** ($\Delta m^2(atm) = 2.5 \times 10^{-3} \text{ eV}^2$).

M. Chizhov, S.T.P., Phys. Rev. Lett. 83 (1999) 1096 (hep-ph/9903399); Phys. Rev. Lett. 85 (2000) 3979 (hep-ph/0504247); Phys. Rev. D63 (2001) 073003 (hep-ph/9903424).

Dirac CP-Nonconservation: δ in U_{PMNS}

Observable manifestations in

$$\nu_l \leftrightarrow \nu_{l'} , \quad \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'} , \quad l, l' = e, \mu, \tau$$

- not sensitive to Majorana CPVP α_{21}, α_{31}

CP-invariance:

$$P(\nu_l \rightarrow \nu_{l'}) = P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'}) , \quad l \neq l' = e, \mu, \tau$$

N. Cabibbo, 1978
S.M. Bilenky, J. Hosek, S.T.P., 1980;
V. Barger, S. Pakvasa et al., 1980.

CPT-invariance:

$$P(\nu_l \rightarrow \nu_{l'}) = P(\bar{\nu}_{l'} \rightarrow \bar{\nu}_l)$$
$$l = l' : \quad P(\nu_l \rightarrow \nu_l) = P(\bar{\nu}_l \rightarrow \bar{\nu}_l)$$

T-invariance:

$$P(\nu_l \rightarrow \nu_{l'}) = P(\nu_{l'} \rightarrow \nu_l), \quad l \neq l'$$

3ν —mixing:

$$A_{\text{CP}}^{(l,l')} \equiv P(\nu_l \rightarrow \nu_{l'}) - P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'}) , \quad l \neq l' = e, \mu, \tau$$

$$A_{\text{T}}^{(l,l')} \equiv P(\nu_l \rightarrow \nu_{l'}) - P(\nu_{l'} \rightarrow \nu_l), \quad l \neq l'$$

$$A_{\text{T}(\text{CP})}^{(e,\mu)} = A_{\text{T}(\text{CP})}^{(\mu,\tau)} = -A_{\text{T}(\text{CP})}^{(e,\tau)}$$

P.I. Krastev, S.T.P., 1988; V. Barger, S. Pakvasa et al., 1980

In vacuum:

$$A_{\text{CP(T)}}^{(e,\mu)} = J_{\text{CP}} F_{\text{osc}}^{\text{vac}}$$

$$J_{\text{CP}} = \text{Im} \{ U_{e1} U_{\mu 2} U_{e2}^* U_{\mu 1}^* \} = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta$$

$$F_{\text{osc}}^{\text{vac}} = \sin\left(\frac{\Delta m_{21}^2 L}{2E}\right) + \sin\left(\frac{\Delta m_{32}^2 L}{2E}\right) + \sin\left(\frac{\Delta m_{13}^2 L}{2E}\right)$$

P.I. Krastev, S.T.P., 1988

In matter: Matter effects violate

CP : $P(\nu_l \rightarrow \nu_{l'}) \neq P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'})$

CPT : $P(\nu_l \rightarrow \nu_{l'}) \neq P(\bar{\nu}_{l'} \rightarrow \bar{\nu}_l)$

P. Langacker et al., 1987

Can conserve the T-invariance (**Earth**)

$$P(\nu_l \rightarrow \nu_{l'}) = P(\nu_{l'} \rightarrow \nu_l), \quad l \neq l'$$

In matter with constant density: $A_{\text{T}}^{(e,\mu)} = J_{\text{CP}}^{\text{mat}} F_{\text{osc}}^{\text{mat}}$

$$J_{\text{CP}}^{\text{mat}} = J_{\text{CP}}^{\text{vac}} R_{\text{CP}}$$

R_{CP} does not depend on θ_{23} and δ ; $|R_{\text{CP}}| \lesssim 2.5$

P.I. Krastev, S.T.P., 1988

Up to 2nd order in the two small parameters $|\alpha| \equiv |\Delta m_{21}^2|/|\Delta m_{31}^2| \ll 1$ and $\sin^2 \theta_{13} \ll 1$:

$$P_m^{3\nu \text{ man}}(\nu_\mu \rightarrow \nu_e) \cong P_0 + P_{\sin \delta} + P_{\cos \delta} + P_3,$$

$$P_0 = \sin^2 \theta_{23} \frac{\sin^2 2\theta_{13}}{(A-1)^2} \sin^2[(A-1)\Delta],$$

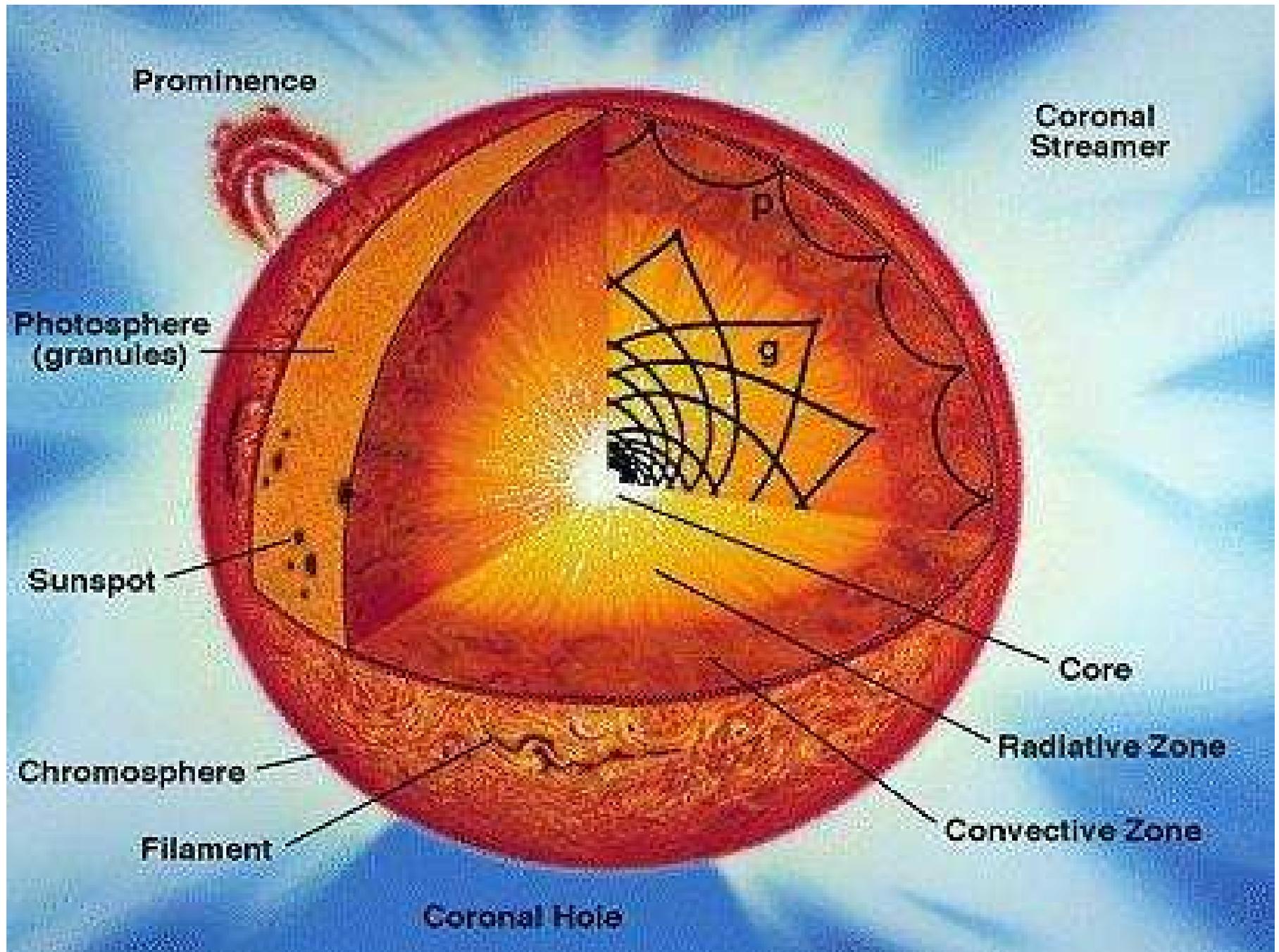
$$P_3 = \alpha^2 \cos^2 \theta_{23} \frac{\sin^2 2\theta_{12}}{A^2} \sin^2(A\Delta),$$

$$P_{\sin \delta} = -\alpha \frac{8 J_{CP}}{A(1-A)} (\sin \Delta)(\sin A\Delta) (\sin[(1-A)\Delta]),$$

$$P_{\cos \delta} = \alpha \frac{8 J_{CP} \cot \delta}{A(1-A)} (\cos \Delta)(\sin A\Delta) (\sin[(1-A)\Delta]),$$

$$\Delta = \frac{\Delta m_{31}^2 L}{4E}, \quad A = \sqrt{2} G_F N_e^{\text{man}} \frac{2E}{\Delta m_{31}^2}.$$

$$\bar{\nu}_\mu \rightarrow \bar{\nu}_e: \delta, \quad A \rightarrow (-\delta), \quad (-A)$$

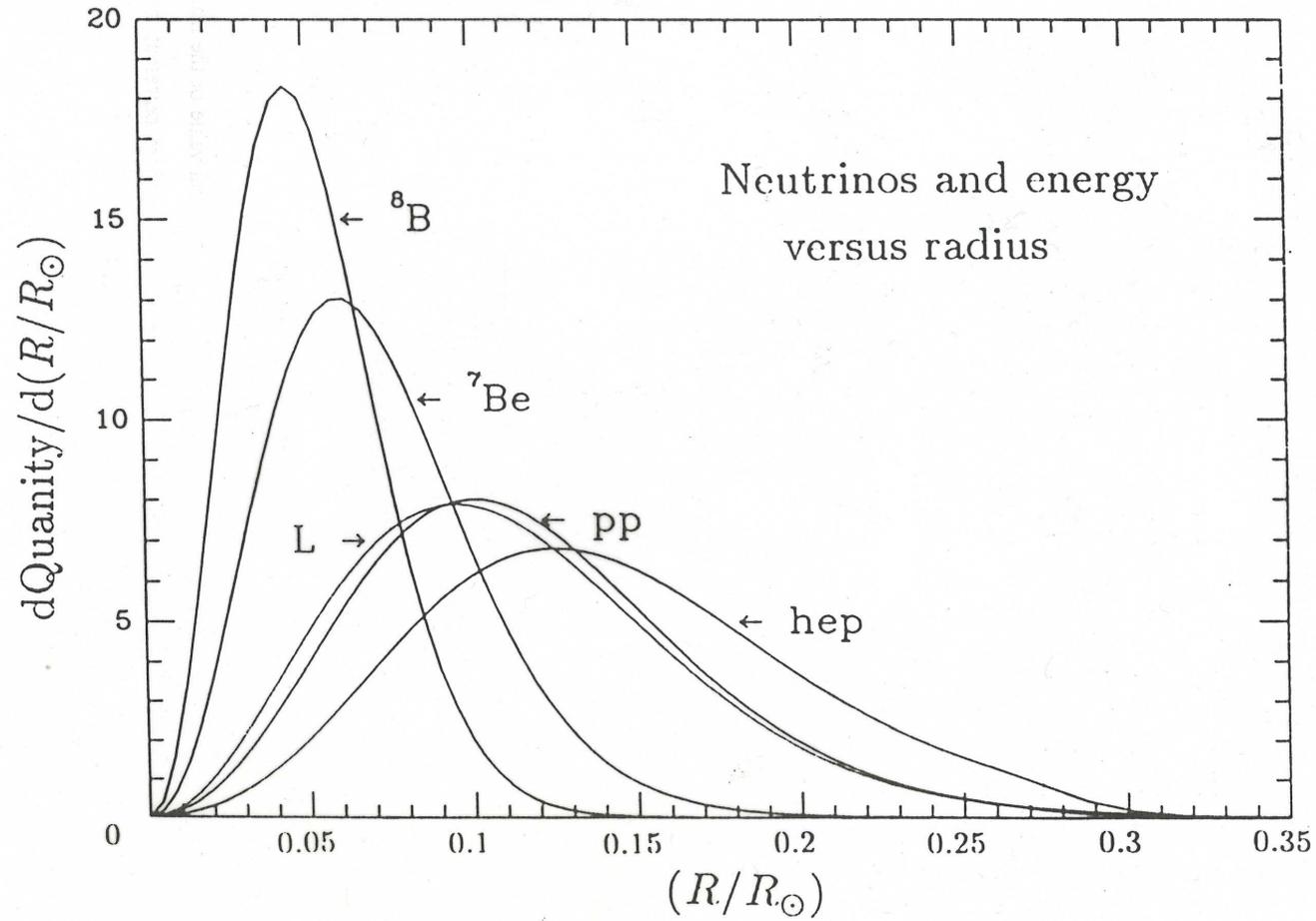


Solar Neutrino Production: pp Chain

REACTION	TERM. (%)	ν ENERGY (MeV)
$p + p \rightarrow {}^2\text{H} + e^+ + \nu_e$	(99.96)	≤ 0.420
or		
$p + e^- + p \rightarrow {}^2\text{H} + \nu_e$	(0.44)	1.442
${}^2\text{H} + p \rightarrow {}^3\text{He} + \gamma$	(100)	
${}^3\text{He} + {}^3\text{He} \rightarrow \alpha + 2 p$	(85)	
or		
${}^3\text{He} + {}^4\text{He} \rightarrow {}^7\text{Be} + \gamma$	(15)	
${}^7\text{Be} + e^- \rightarrow {}^7\text{Li} + \nu_e$	(15)	$\left\{ \begin{array}{l} 0.861 \text{ 90\%} \\ 0.383 \text{ 10\%} \end{array} \right.$
${}^7\text{Li} + p \rightarrow 2 \alpha$		
or		
${}^7\text{Be} + p \rightarrow {}^8\text{B} + \gamma$	(0.02)	
${}^8\text{B} \rightarrow {}^8\text{Be}^* + e^+ + \nu_e$		< 15
${}^8\text{Be}^* \rightarrow 2 \alpha$		
or		
${}^3\text{He} + p \rightarrow {}^4\text{He} + e^+ + \nu_e$	(0.000004)	18.8



- pp neutrinos, $E \leq 0.420$ MeV, $\bar{E} = 0.265$ MeV,
- ${}^7\text{Be}$ neutrinos, $E=0.862$ MeV (89.7% of the flux), 0.384 MeV (10.3%) ,
- ${}^8\text{B}$ neutrinos, $E \leq 14.40$ MeV, $\bar{E} = 6.71$ MeV,
- pep neutrinos, $E=1.442$ MeV,
- of ${}^{13}\text{N}$, $E \leq 1.199$ MeV, $\bar{E} = 0.707$ MeV,
- of ${}^{15}\text{O}$, $E \leq 1.732$ MeV, $\bar{E} = 0.997$ MeV.



Flux	BP'00	Cl-Ar	Ga-G
$\Phi_{pp} \times 10^{-10}$	5.95(1 $^{+0.01}_{-0.01}$)	0.00	69.
$\Phi_{pep} \times 10^{-8}$	1.40(1 $^{+0.01}_{-0.01}$)	0.22	2.
$\Phi_{Be} \times 10^{-9}$	4.77(1 $^{+0.09}_{-0.09}$)	1.15	34.
$\Phi_B \times 10^{-6}$	5.93(1 $^{+0.14}_{-0.15}$)	6.76	14.
$\Phi_N \times 10^{-8}$	5.48(1 $^{+0.19}_{-0.13}$)	0.09	3.
$\Phi_O \times 10^{-8}$	4.80(1 $^{+0.22}_{-0.15}$)	0.33	5.
Total		8.55 $^{+1.1}_{-1.2}$	129.8 $^{+1.1}_{-1.2}$

Solar Neutrinos ν_e , $E \sim 1$ MeV: B. Pontecorvo 1946



R. Davis et al., 1967 - 1996: 615 t C_2Cl_4 ; 0.5 Ar atoms/day, exposure 60 days.



Kamiokande (1986-1994), Super-Kamiokande (1996 -), SNO (2000 - 2006), BOREXINO (2007 -);



Super-Kamiokande: 50000t ultra-pure water;

SNO: 1000t heavy water (D_2O)



SAGE (60t), 1990-; GALLEX/GNO (30t, LNGS), 1991-2003

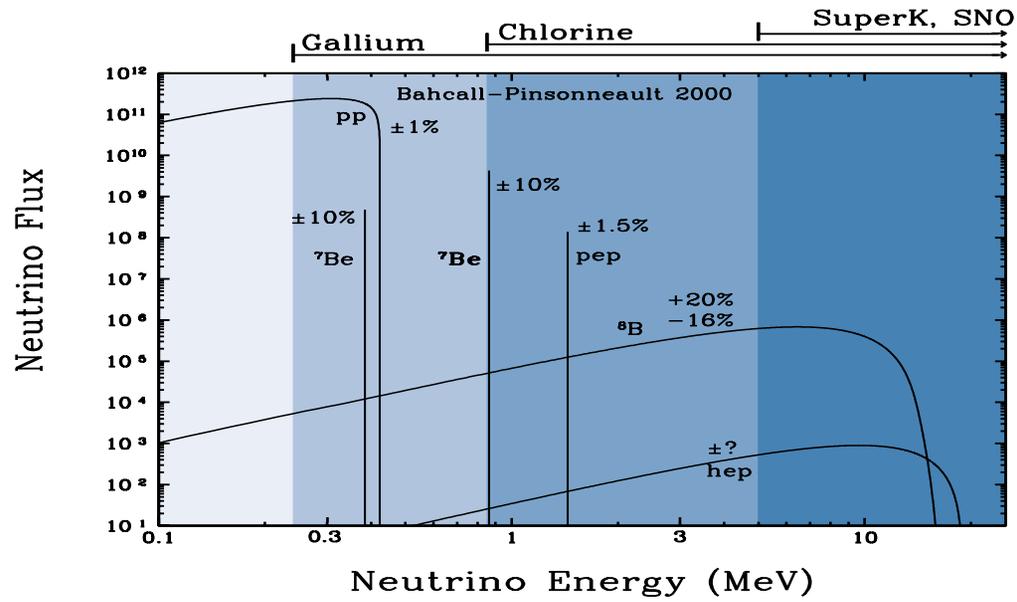


Figure 2: Differential Standard Solar Model neutrino fluxes [14].

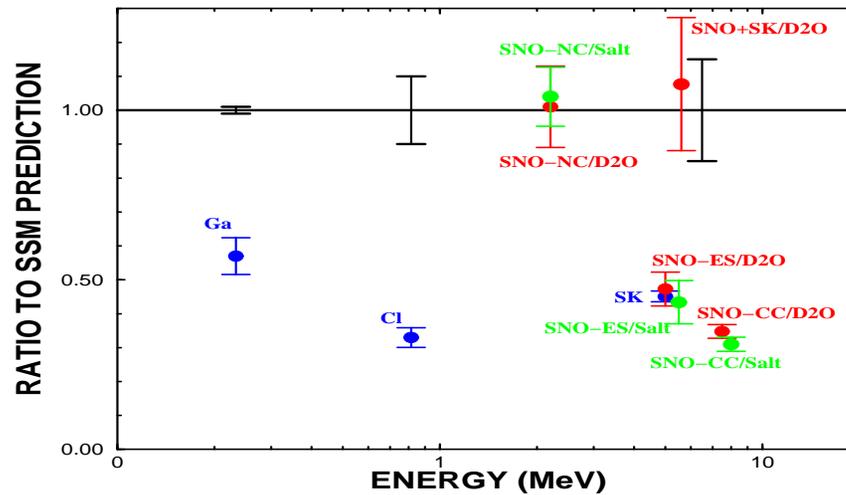
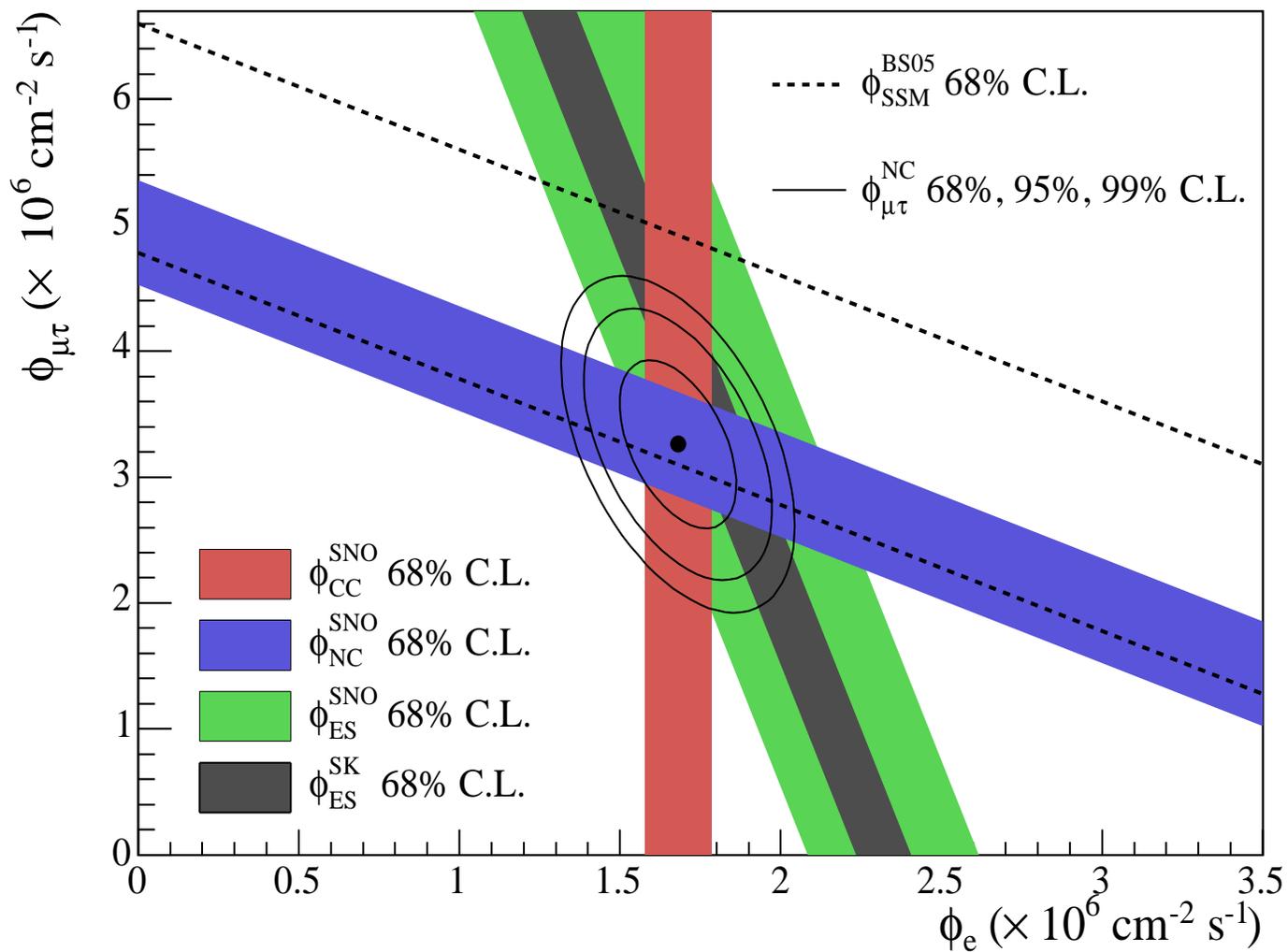


Figure 3: Comparison of measurements to Standard Solar Model predictions.

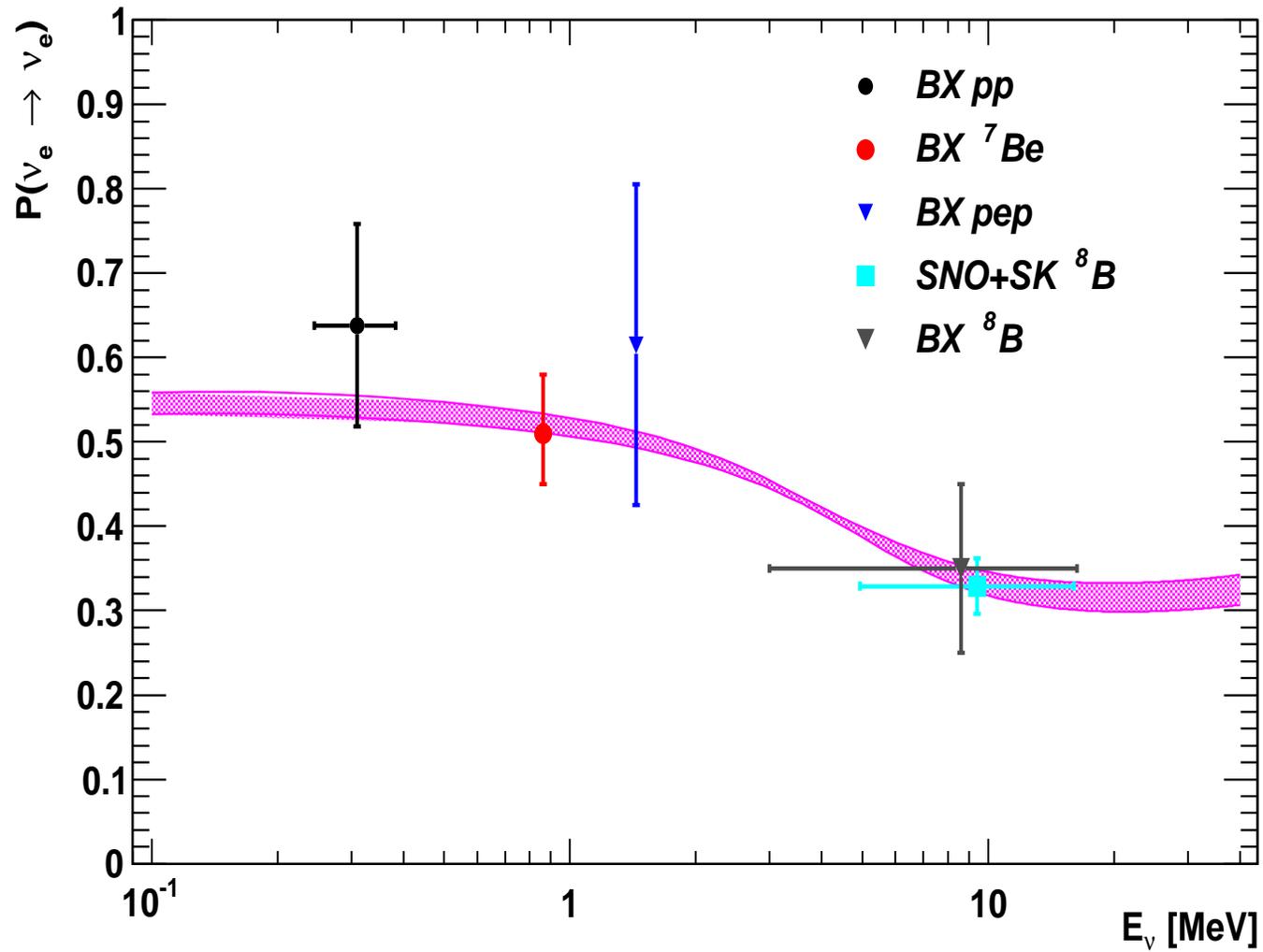
Flux	BP'00	Cl-Ar	Ga-Ge
$\Phi_{pp} \times 10^{-10}$	5.95(1 ^{+0.01} _{-0.01})	0.00	69.7
$\Phi_{pep} \times 10^{-8}$	1.40(1 ^{+0.01} _{-0.01})	0.22	2.8
$\Phi_{Be} \times 10^{-9}$	4.77(1 ^{+0.09} _{-0.09})	1.15	34.2
$\Phi_B \times 10^{-6}$	5.93(1 ^{+0.14} _{-0.15})	6.76	14.2
$\Phi_N \times 10^{-8}$	5.48(1 ^{+0.19} _{-0.13})	0.09	3.4
$\Phi_O \times 10^{-8}$	4.80(1 ^{+0.22} _{-0.15})	0.33	5.5
Total		8.55 ^{+1.1} _{-1.2}	129.8 ⁺⁹ ₋₇

Experiment	Observed rate/BP04 prediction	Predicted Rate at global best-fit	Predicted Rate at solar best-fit
Ga	0.52 ± 0.029	0.555	0.540
Cl	0.301 ± 0.027	0.356	0.345
SK(ES)	0.406 ± 0.014	0.394	0.395
SNO(CC)	0.274 ± 0.019	0.289	0.289
SNO(ES)	0.38 ± 0.052	0.386	0.386
SNO(NC)	0.895 ± 0.08	0.889	0.908

The observed rates w.r.t predictions from the latest Standard Solar Model BP04. Shown are also the predicted rates for the best fit values of Δm_{21}^2 and $\sin^2 \theta_{12}$, obtained in the analysis of the i) global solar neutrino data, and ii) global solar neutrino +KamLAND data.



Results from BOREXINO



MSW Transitions of Solar Neutrinos in the Sun and the Hydrogen Atom

$$i \frac{d}{dt} \begin{pmatrix} A_\alpha(t, t_0) \\ A_\beta(t, t_0) \end{pmatrix} = \begin{pmatrix} -\epsilon(t) & \epsilon'(t) \\ \epsilon'(t) & \epsilon(t) \end{pmatrix} \begin{pmatrix} A_\alpha(t, t_0) \\ A_\beta(t, t_0) \end{pmatrix}$$

where $\alpha = \nu_e$, $\beta = \nu_{\mu(\tau)}$,

$$\epsilon(t) = \frac{1}{2} \left[\frac{\Delta m^2}{2E} \cos 2\theta - \sqrt{2} G_F N_e(t) \right],$$

$$\epsilon'(t) = \frac{\Delta m^2}{4E} \sin 2\theta, \text{ with } \Delta m^2 = m_2^2 - m_1^2.$$

- Standard Solar Models

$$N_e(t) = N_e(t_0) \exp \left\{ -\frac{t-t_0}{r_0} \right\}, \quad r_0 \sim 0.1 R_\odot, \quad R_\odot = 6.96 \times 10^5 \text{ km}$$

The region of ν_\odot production: $r \lesssim 0.2R_\odot$

$$20 N_A \text{ cm}^{-3} \lesssim N_e(x_0) \lesssim 100 N_A \text{ cm}^{-3}$$

Suppose $N_e(x_0) \gg N_e^{res}$: $|\nu_e\rangle \cong |\nu_2^m\rangle$.

Possible evolution:

The system stays at this level; at the surface: $|\nu_2^m\rangle = |\nu_2\rangle$

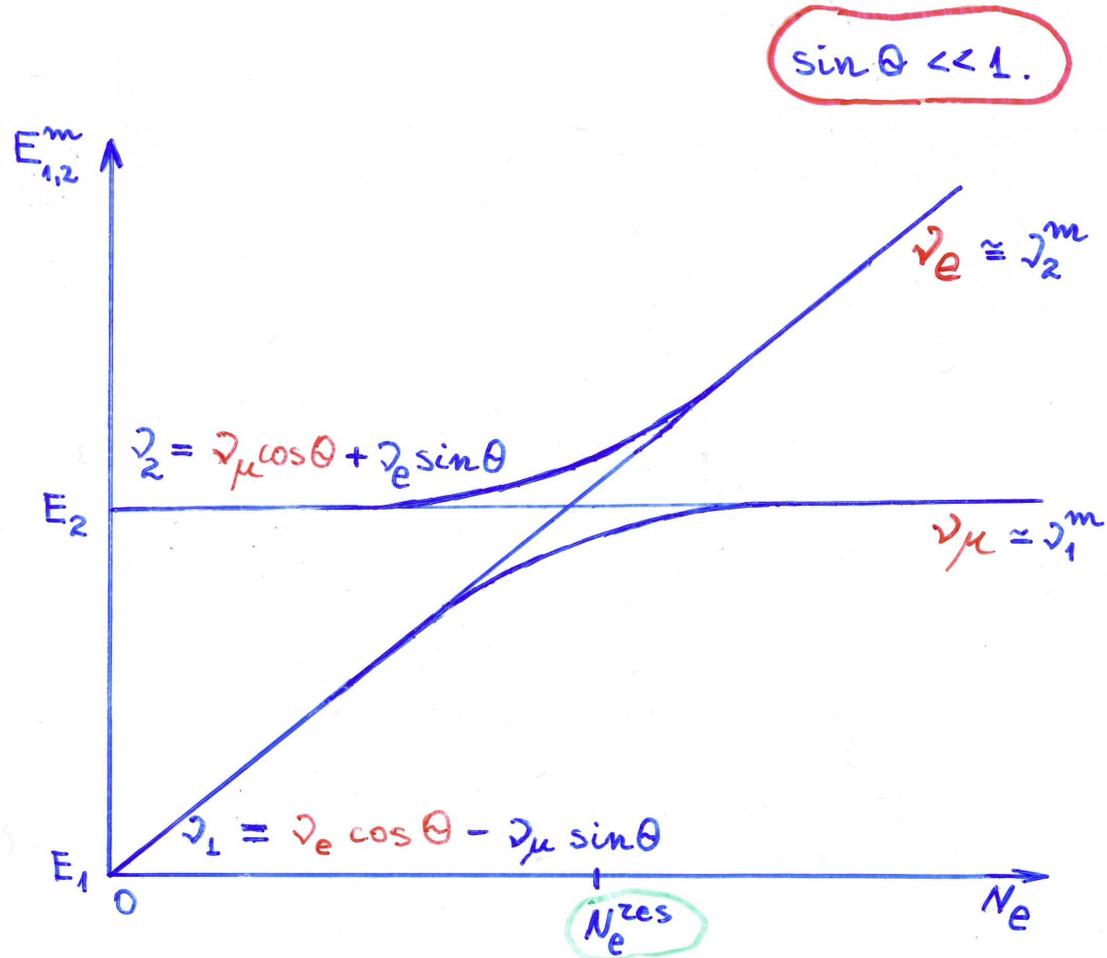
$$P(\nu_e \rightarrow \nu_e) \cong |\langle \nu_e | \nu_2 \rangle|^2 = \sin^2 \theta, \quad \text{Adiabatic}$$

At $N_e = N_e^{res}$, where $E_2^m - E_1^m$ is minimal, the system jumps to lower level $|\nu_1^m\rangle$; at the surface: $|\nu_1^m\rangle = |\nu_1\rangle$

$$P(\nu_e \rightarrow \nu_e) \cong |\langle \nu_e | \nu_1 \rangle|^2 = \cos^2 \theta, \quad \text{Nonadiabatic}$$

Type of transition: $P' \equiv P(\nu_2^m(t_0) \rightarrow \nu_1)$, **jump probability**

4.



- ①. $P(\nu_2^m \rightarrow \nu_1; t_0, t_0) \equiv P'$ - negligible: adiabatic transition
- ②. P' - nonnegligible: nonadiabatic

Introducing the dimensionless variable

$$Z = ir_0\sqrt{2}G_F N_e(t_0)e^{-\frac{t-t_0}{r_0}}, \quad Z_0 = Z(t = t_0),$$

and making the substitution

$$A_e(t, t_0) = (Z/Z_0)^{c-a} e^{-(Z-Z_0)+i\int_{t_0}^t \epsilon(t')dt'} A'_e(t, t_0),$$

$A'_e(t, t_0)$ satisfies the confluent hypergeometric equation (CHE):

$$\left\{ Z \frac{d^2}{dZ^2} + (c - Z) \frac{d}{dZ} - a \right\} A'_e(t, t_0) = 0,$$

where

$$a = 1 + ir_0 \frac{\Delta m^2}{2E} \sin^2 \theta, \quad c = 1 + ir_0 \frac{\Delta m^2}{2E}.$$

The confluent hypergeometric equation describing the ν_e oscillations in the Sun, coincides in form with the **Schroedinger (energy eigenvalue) equation obeyed by the radial part, $\psi_{kl}(r)$, of the non-relativistic wave function of the hydrogen atom,**

$$\Psi(\vec{r}) = \frac{1}{r} \psi_{kl}(r) Y_{lm}(\theta', \phi'),$$

r , θ' and ϕ' are the spherical coordinates of the electron in the proton's rest frame, l and m are the orbital momentum quantum numbers ($m = -l, \dots, l$), k is the quantum number labeling (together with l) the electron energy (the principal quantum number is equal to $(k+l)$), E_{kl} ($E_{kl} < 0$), and $Y_{lm}(\theta', \phi')$ are the spherical harmonics. The function

$$\psi'_{kl}(Z) = Z^{-c/2} e^{Z/2} \psi_{kl}(r)$$

satisfies the confluent hypergeometric equation in which the variable Z and the parameters a and c are in this case related to the physical quantities characterizing the hydrogen atom:

$$Z = 2 \frac{r}{a_0} \sqrt{-E_{kl}/E_I}, \quad a \equiv a_{kl} = l+1 - \sqrt{-E_I/E_{kl}}, \quad c \equiv c_l = 2(l+1)$$

$a_0 = \hbar/(m_e e^2)$ is the Bohr radius and $E_I = m_e e^4/(2\hbar^2) \cong 13.6 \text{ eV}$ is the ionization energy of the hydrogen atom.

Quite remarkably, the behavior of such different physical systems as solar neutrinos undergoing MSW transitions in the Sun and the non-relativistic hydrogen atom are governed by one and the same differential equation.

Any solution - linear combination of two linearly independent solutions:

$$\Phi(a, c; Z), Z^{1-c} \Phi(a - c + 1, 2 - c; Z); \Phi(a', c'; Z = 0) = 1, a', c' \neq 0, -1, -2, \dots$$

$$A(\nu_e \rightarrow \nu_{\mu(\tau)}) = \frac{1}{2} \sin 2\theta \left\{ \Phi(a - c, 2 - c; Z_0) - e^{i(t-t_0)\frac{\Delta m^2}{2E}} \Phi(a - 1, c; Z_0) \right\}.$$

$$\text{Sun: } N_e(x) \cong N_e(x_0)e^{-\frac{x}{r_0}}, r_0 \cong 0.1R_\odot, R_\odot \cong 7 \times 10^5 \text{ km}$$

The region of ν_\odot production:

$$20 N_A \text{ cm}^{-3} \lesssim N_e(x_0) \lesssim 100 N_A \text{ cm}^{-3}: |Z_0| > 500 (!)$$

The solar ν_e survival probability:

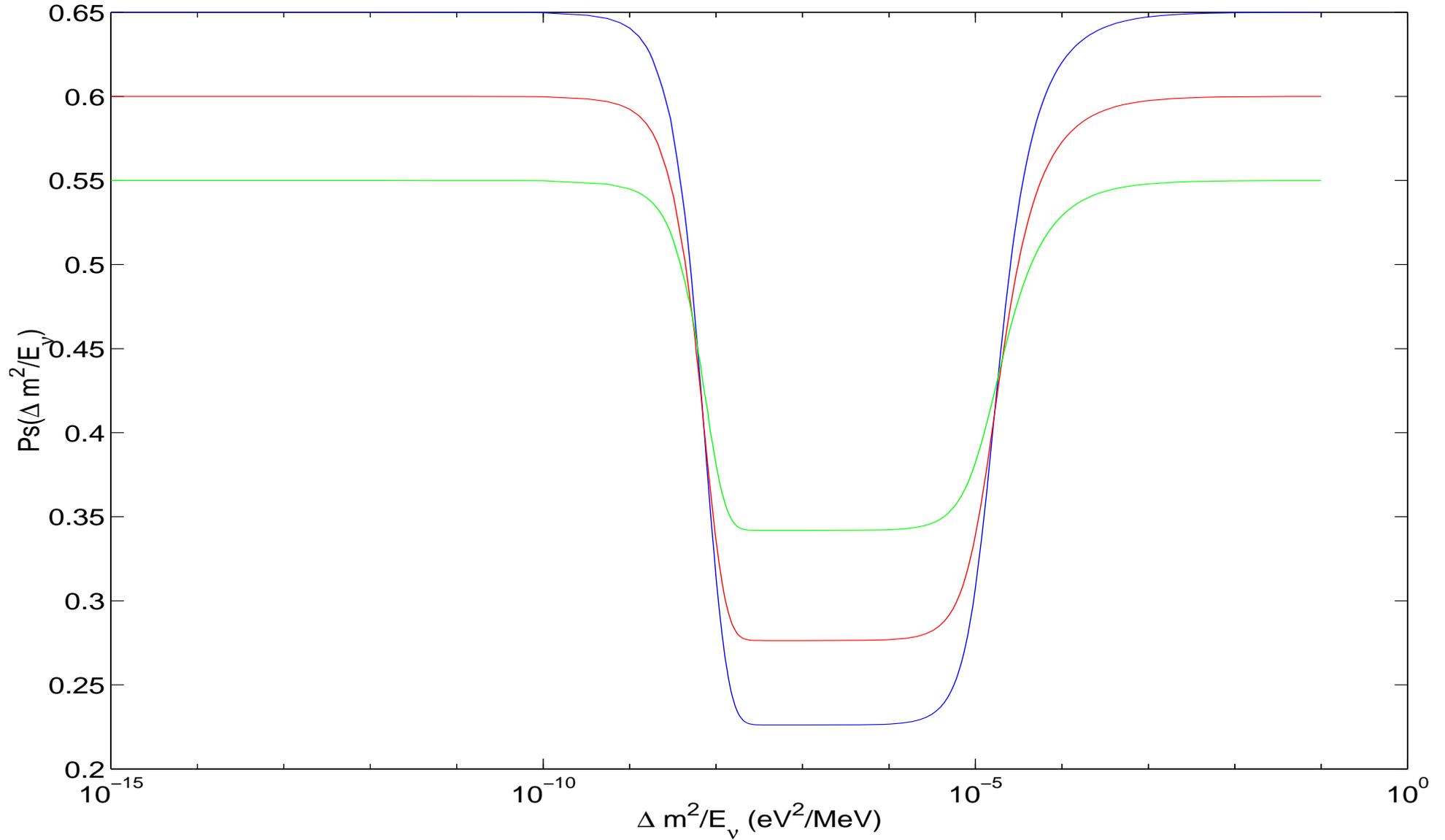
$$\bar{P}(\nu_e \rightarrow \nu_e) = \frac{1}{2} + \left(\frac{1}{2} - P'\right) \cos 2\theta_m^0 \cos 2\theta,$$

$$P' = \frac{e^{-2\pi r_0 \frac{\Delta m^2}{2E}} \sin^2 \theta - e^{-2\pi r_0 \frac{\Delta m^2}{2E}}}{1 - e^{-2\pi r_0 \frac{\Delta m^2}{2E}}}$$

S.T.P., 1988

$$\nu_e \rightarrow \nu_e$$

Averaged Survival Probability in the Sun



The solar ν_e survival probability:

$$\bar{P}(\nu_e \rightarrow \nu_e) = \frac{1}{2} + \left(\frac{1}{2} - P'\right) \cos 2\theta_m^0 \cos 2\theta,$$

$$P' = \frac{e^{-2\pi r_0 \frac{\Delta m^2}{2E}} \sin^2 \theta - e^{-2\pi r_0 \frac{\Delta m^2}{2E}}}{1 - e^{-2\pi r_0 \frac{\Delta m^2}{2E}}}$$

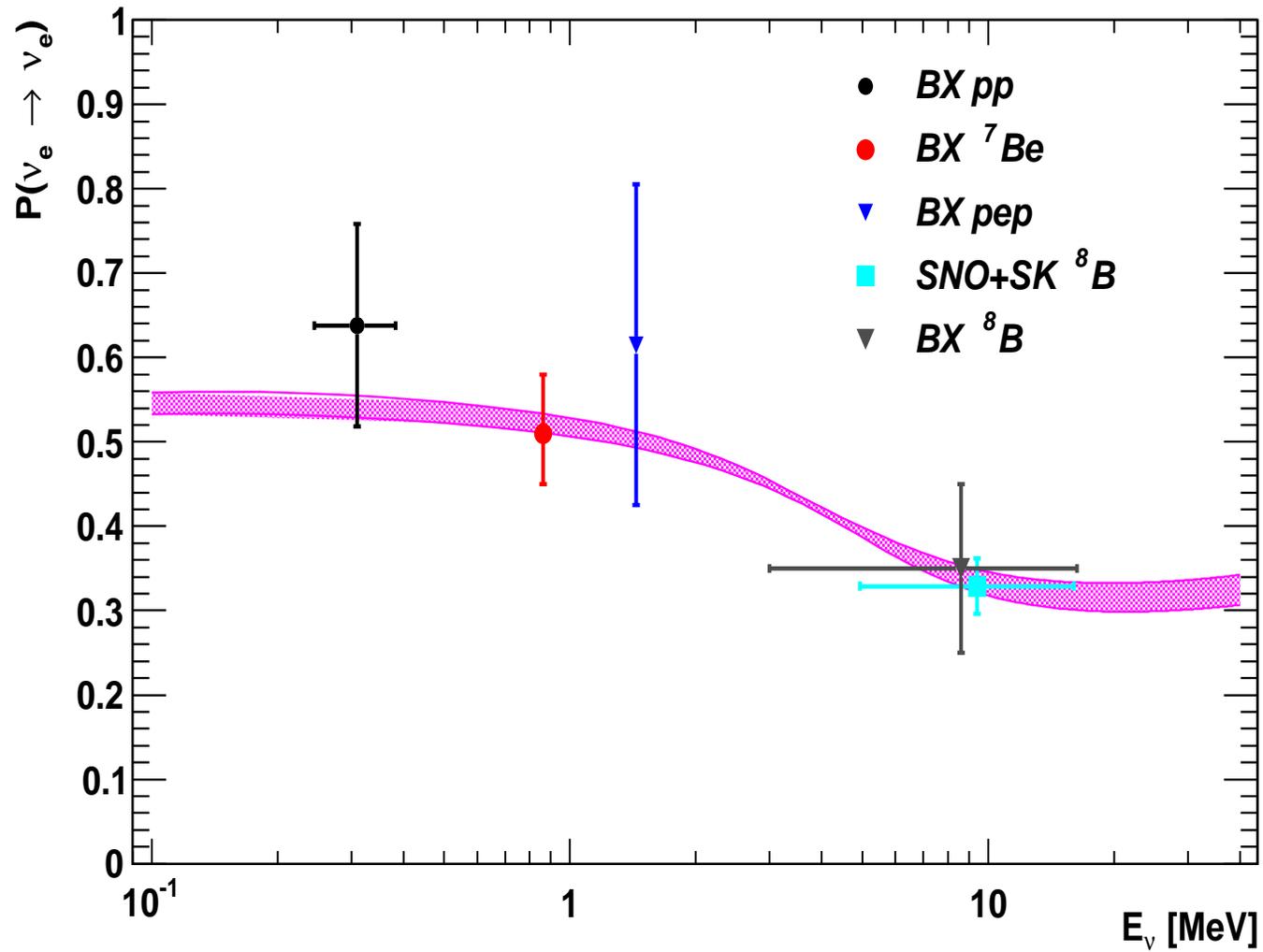
Case 1: $\cos 2\theta_m^0 = -1$, $P' = 0$, $\bar{P} = \frac{1}{2}(1 - \cos 2\theta)$.

Case 2: $\theta_m^0 = \theta$, $P' = 0$, $\bar{P}(\nu_e \rightarrow \nu_e) = 1 - \frac{1}{2} \sin^2 2\theta$

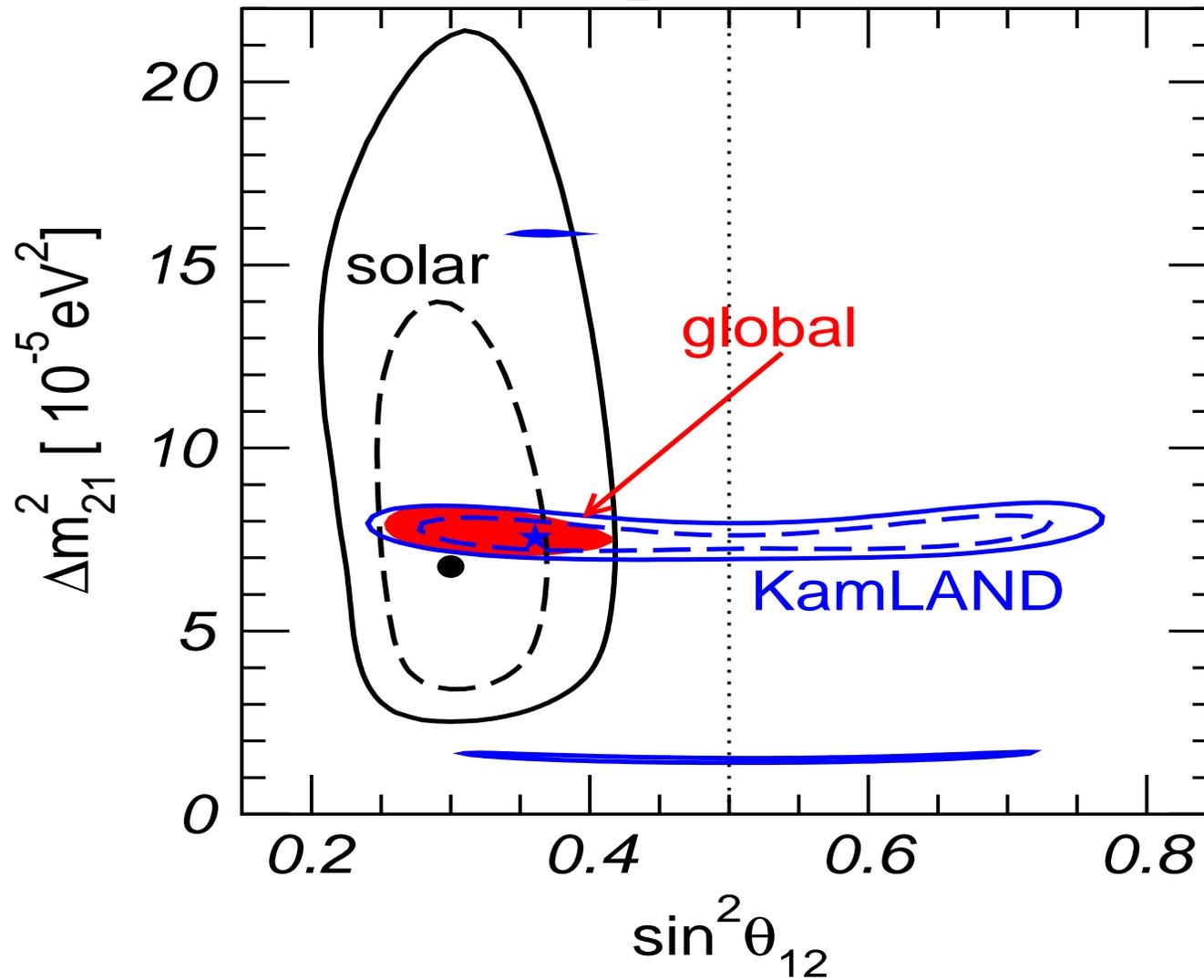
Case 1: SNO, Super Kamiokande; $\bar{P} \cong 0.3$: $\cos 2\theta > 0$!

Case 2: pp neutrinos.

Results from BOREXINO



"solar" parameters



T. Schwetz, arXiv:0710.5027[hep-ph]

J.N. BAHCALL, M.C. GONZALEZ-GARCIA, C. PEÑA GARAY '01

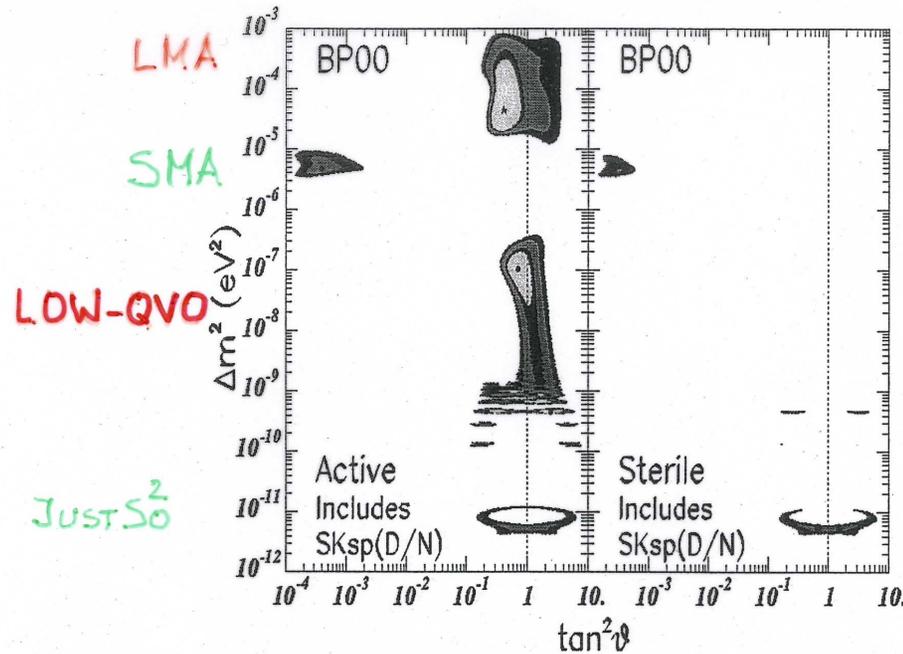


Figure 1: Global solutions including all available solar neutrino data. The input data include the total rates from the Chlorine [2], Gallium (averaged) [3, 5, 4], Super-Kamiokande [6], and SNO [1] experiments, as well as the recoil electron energy spectrum measured by Super-Kamiokande during the day and separately the energy spectrum measured at night. The C.L. contours shown in the figure are 90%, 95%, 99%, and 99.73% (3σ). The allowed regions are cutoff below 10^{-3}eV^2 by the Chooz reactor measurements [22]. The local best-fit points are marked by dark circles. The theoretical errors for the BP2000 neutrino fluxes are included in the analysis

The reference scheme: 3- ν mixing

$$\nu_{lL} = \sum_{j=1}^3 U_{lj} \nu_{jL} \quad l = e, \mu, \tau.$$

3-flavour neutrino oscillation probabilities

$$P(\nu_l \rightarrow \nu_{l'}) : m_2^2 - m_1^2 \equiv \Delta m_{21}^2, \quad m_3^2 - m_1^2 \equiv \Delta m_{31}^2$$

PMNS Matrix: Standard Parametrization

$$U = V P, \quad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix},$$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- $s_{ij} \equiv \sin \theta_{ij}$, $c_{ij} \equiv \cos \theta_{ij}$, $\theta_{ij} = [0, \frac{\pi}{2}]$,
- δ - Dirac CPV phase, $\delta = [0, 2\pi]$; CP inv.: $\delta = 0, \pi, 2\pi$;
- α_{21}, α_{31} - Majorana CPV phases; CP inv.: $\alpha_{21(31)} = k(k')\pi$, $k(k') = 0, 1, 2, \dots$
S.M. Bilenky, J. Hosek, S.T.P., 1980
- $\Delta m_{\odot}^2 \equiv \Delta m_{21}^2 \cong 7.54 \times 10^{-5} \text{ eV}^2 > 0$, $\sin^2 \theta_{12} \cong 0.308$, $\cos 2\theta_{12} \gtrsim 0.28$ (3σ),
- $|\Delta m_{31(32)}^2| \cong 2.47$ (2.42) $\times 10^{-3} \text{ eV}^2$, $\sin^2 \theta_{23} \cong 0.437$ (0.455), NO (IO),
- θ_{13} - the CHOOZ angle: $\sin^2 \theta_{13} = 0.0234$ (0.0240), Capozzi et al. NO (IO).
F. Capozzi et al. (Bari Group), arXiv:1312.2878v2 (May 5, 2014)

3- ν Mixing Analysis: $\Delta m_{\odot}^2 \ll |\Delta m_{\text{atm}}^2|$

$$P_{\odot}^{3\nu} \cong \sin^4 \theta_{13} + \cos^4 \theta_{13} P_{\odot}^{2\nu},$$

$$P_{\odot}^{2\nu} = \bar{P}_{\odot}^{2\nu} + P_{\odot}^{2\nu}{}_{\text{osc}},$$

$$\bar{P}_{\odot}^{2\nu} = \frac{1}{2} + \left(\frac{1}{2} - P'\right) \cos 2\theta_{12}^m(t_0) \cos 2\theta_{12} \quad (\theta_{12} \equiv \theta_{\odot}),$$

$P' = 0$: L. Wolfenstein, 1978; S. Mikheyev, A. Smirnov, 1985;

$P' \neq 0$ (general or LZ): S. Parke, W. Haxton, 1986;

P' -double exponential, $P_{\odot}^{2\nu}{}_{\text{osc}}$: S.T.P., 1988

$$N_e \rightarrow N_e \cos^2 \theta_{13},$$

$$P' = \frac{e^{-2\pi r_0 \frac{\Delta m^2}{2E}} \sin^2 \theta_{12} - e^{-2\pi r_0 \frac{\Delta m^2}{2E}}}{1 - e^{-2\pi r_0 \frac{\Delta m^2}{2E}}}, \quad r_0 \sim 0.1 R_{\odot}$$

S.T.P., 1988

LMA: $P' \ll 1, \quad \langle P_{\odot}^{2\nu}{}_{\text{osc}} \rangle \cong 0$

J. Rich, S.T.P., 1988

$$P_{\text{KL}}^{3\nu} \cong \sin^4 \theta_{13} + \cos^4 \theta_{13} \left[1 - \sin^2 2\theta_{12} \sin^2 \left(\frac{\Delta m_{\odot}^2}{4E} L \right) \right]$$

$$P_{\text{CHOOZ}}^{3\nu} \cong 1 - \sin^2 2\theta_{13} \sin^2 \left(\frac{\Delta m_{\text{atm}}^2}{4E} L \right)$$

- $\text{sgn}(\Delta m_{\text{atm}}^2) = \text{sgn}(\Delta m_{31(32)}^2)$ not determined

$$\Delta m_{\text{atm}}^2 \equiv \Delta m_{31}^2 > 0, \quad \text{normal mass ordering (NO)}$$

$$\Delta m_{\text{atm}}^2 \equiv \Delta m_{32}^2 < 0, \quad \text{inverted mass ordering (IO)}$$

Convention: $m_1 < m_2 < m_3$ - NO, $m_3 < m_1 < m_2$ - IO

$$\Delta m_{31}^2(\text{NO}) = -\Delta m_{32}^2(\text{IO})$$

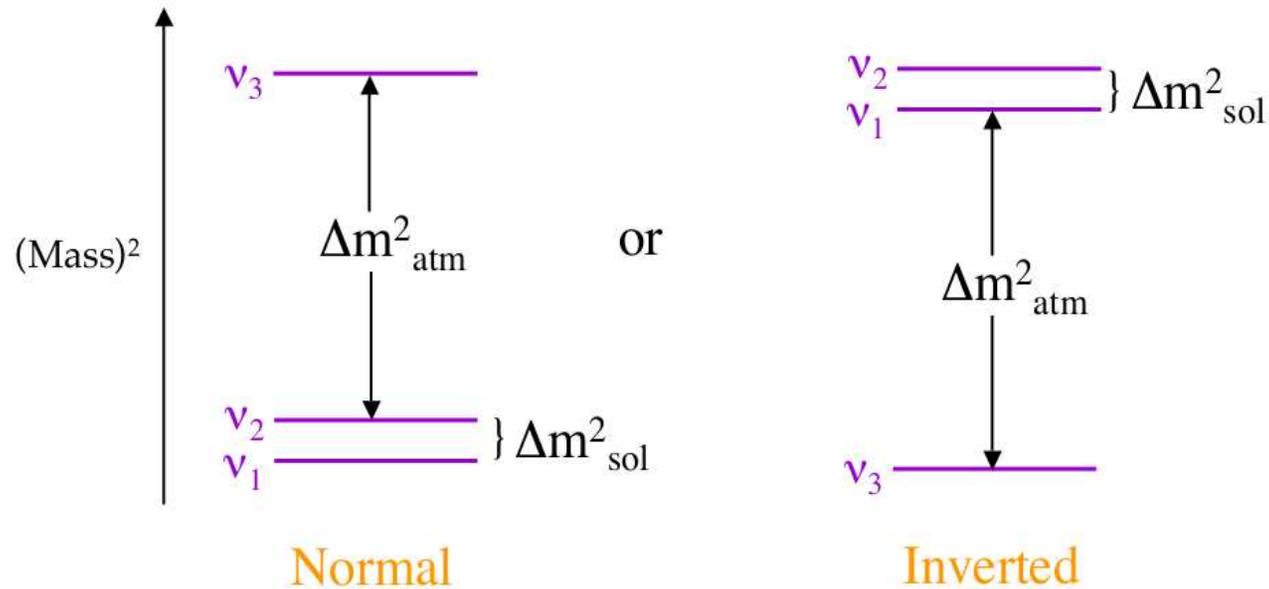
$$m_1 \ll m_2 < m_3, \quad \text{NH,}$$

$$m_3 \ll m_1 < m_2, \quad \text{IH,}$$

$$m_1 \cong m_2 \cong m_3, \quad m_{1,2,3}^2 \gg \Delta m_{\text{atm}}^2, \quad \text{QD; } m_j \gtrsim 0.10 \text{ eV.}$$

- $m_2 = \sqrt{m_1^2 + \Delta m_{21}^2}, \quad m_3 = \sqrt{m_1^2 + \Delta m_{31}^2}$ - NO;
- $m_1 = \sqrt{m_3^2 + \Delta m_{23}^2 - \Delta m_{21}^2}, \quad m_2 = \sqrt{m_3^2 + \Delta m_{23}^2}$ - IO;

The (Mass)² Spectrum

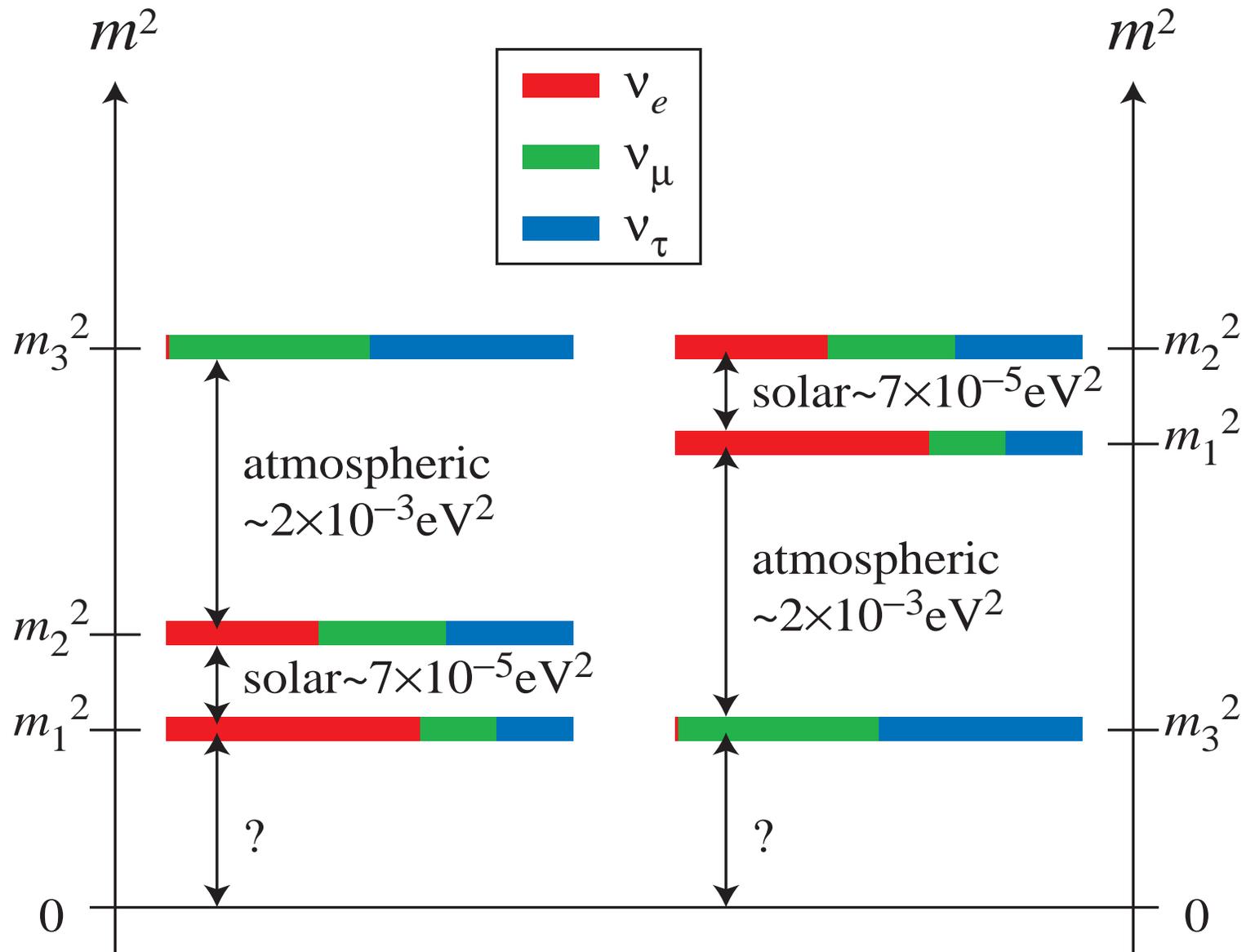


$$\Delta m^2_{\text{sol}} \cong 7.6 \times 10^{-5} \text{ eV}^2, \quad \Delta m^2_{\text{atm}} \cong 2.4 \times 10^{-3} \text{ eV}^2$$

Are there *more* mass eigenstates, as LSND suggests,
and MiniBooNE recently hints?

3

Due to B. Kayser



- Dirac phase δ : $\nu_l \leftrightarrow \nu_{l'}, \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}, l \neq l'$; $A_{CP}^{(l,l')} \propto J_{CP} \propto \sin \theta_{13} \sin \delta$:

P.I. Krastev, S.T.P., 1988

$$J_{CP} = \text{Im} \{ U_{e1} U_{\mu 2} U_{e2}^* U_{\mu 1}^* \} = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta$$

Current data: $|J_{CP}| \lesssim 0.035$ (can be relatively large!); b.f.v. with $\delta = 3\pi/2$:
 $J_{CP} \cong -0.035$.

- Majorana phases α_{21}, α_{31} :

– $\nu_l \leftrightarrow \nu_{l'}, \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$ not sensitive;

S.M. Bilenky, J. Hosek, S.T.P., 1980;

P. Langacker, S.T.P., G. Steigman, S. Toshev, 1987

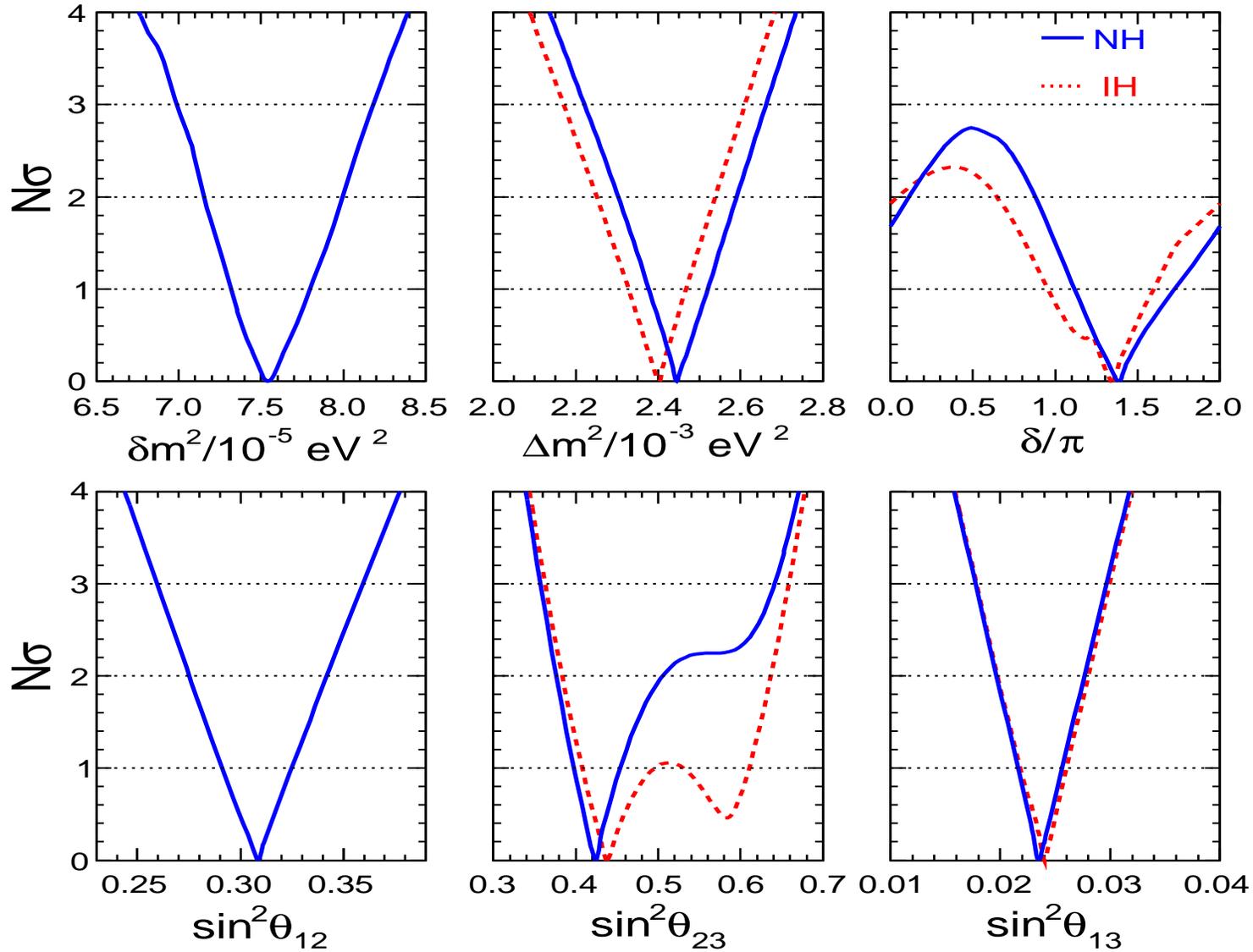
– $|\langle m \rangle|$ in $(\beta\beta)_{0\nu}$ -decay depends on α_{21}, α_{31} ;

– $\Gamma(\mu \rightarrow e + \gamma)$ etc. in SUSY theories depend on $\alpha_{21,31}$;

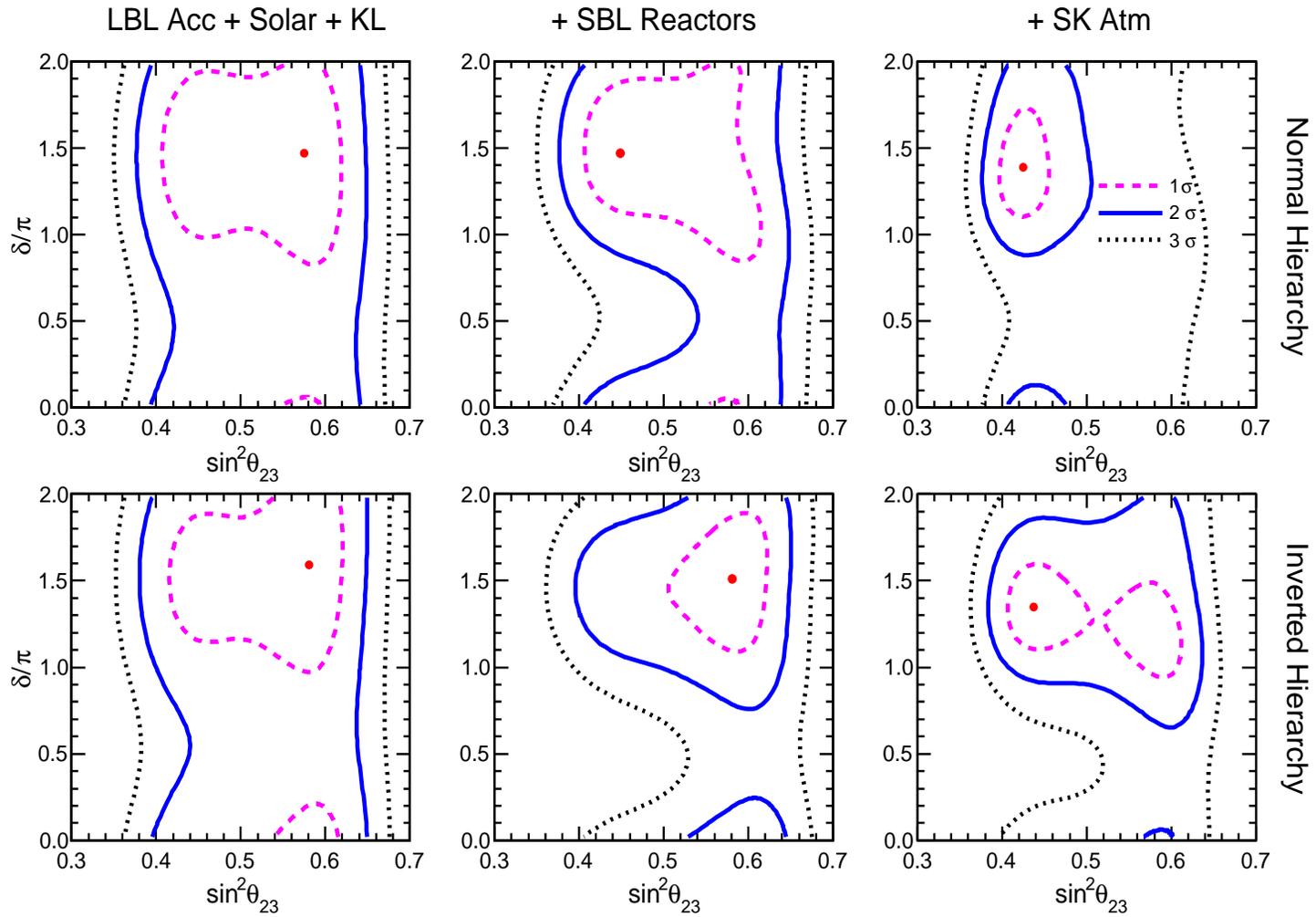
– BAU, leptogenesis scenario: $\delta, \alpha_{21,31}$!

$$\begin{aligned} J_{CP} &= \text{Im} \{ U_{e1} U_{\mu 2} U_{e2}^* U_{\mu 1}^* \} \\ &= \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta \end{aligned}$$

LBL Acc + Solar + KL + SBL Reactors + SK Atm



F. Capozzi, E. Lisi *et al.*, arXiv:1312.2878

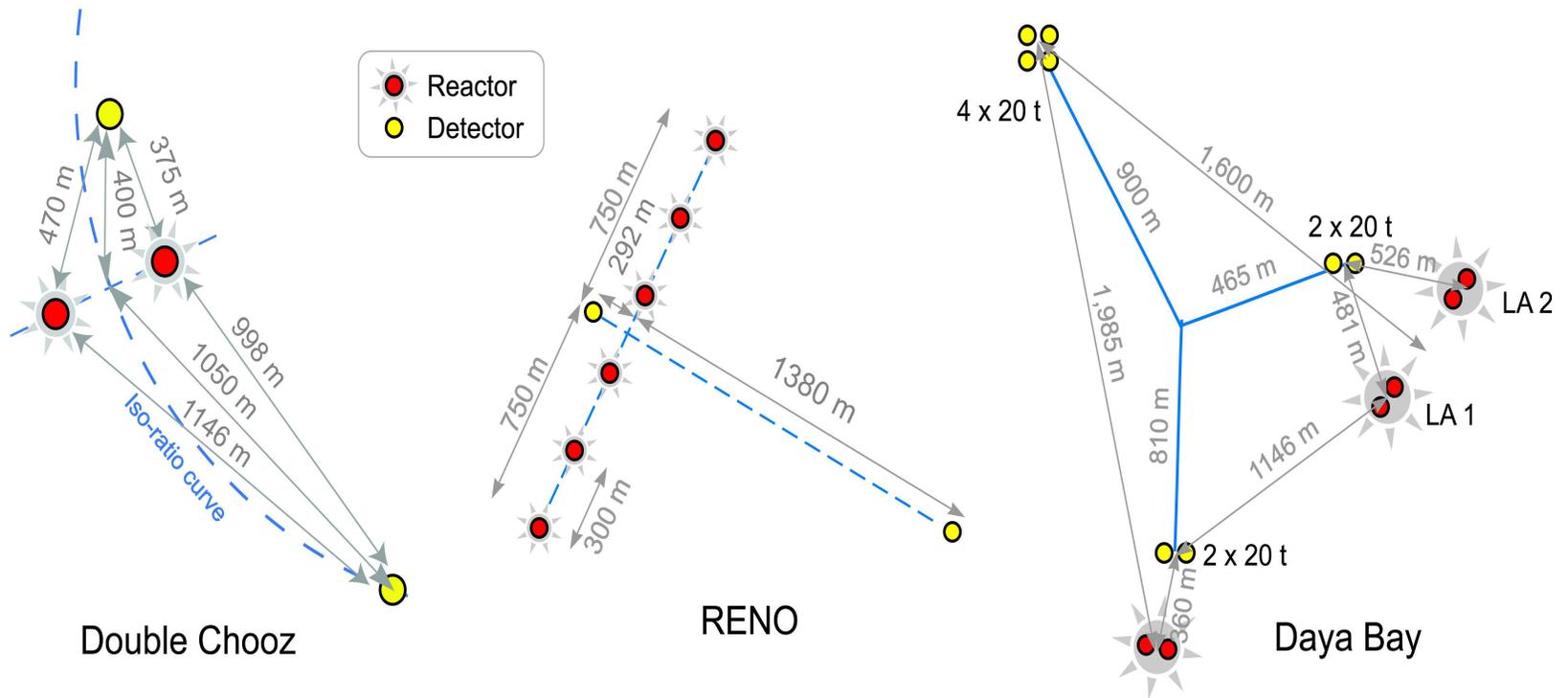


F. Capozzi, E. Lisi *et al.*, arXiv:1312.2878

- March 8, 2012, Daya Bay: 5.2σ evidence for $\theta_{13} \neq 0$, $\sin^2 2\theta_{13} = 0.092 \pm 0.016 \pm 0.005$.
- April 4, 2012, RENO: 4.9σ evidence for $\theta_{13} \neq 0$, $\sin^2 2\theta_{13} = 0.113 \pm 0.013 \pm 0.019$.
- Nu'2012 (June 4-9, 2012), T2K, Double Chooz: 3.2σ and 2.9σ evidence for $\theta_{13} \neq 0$.
- Daya Bay, 23/08/2013:
 $\sin^2 2\theta_{13} = 0.090 \pm 0.009$.
- RENO, 12/09/2013 (TAUP 2013):
 $\sin^2 2\theta_{13} = 0.100 \pm 0.010$ (*stat.*) ± 0.012 .

$$P^{3\nu}(\bar{\nu}_e \rightarrow \bar{\nu}_e) = P^{3\nu}(\theta_{13}, \Delta m_{31(32)}^2; \theta_{12}, \Delta m_{21}^2) \cong$$

$$1 - \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m_{31(32)}^2 L}{4E}\right), \text{ no dependence on } \theta_{23}, \delta.$$



M. Mezzetto, T. Schwetz, arXiv:1003.5800[hep-ph]

T2K: Search for $\nu_\mu \rightarrow \nu_e$ oscillations

T2K: first results March 2011 (2 events);
June 14, 2011 (6 events): evidence for $\theta_{13} \neq 0$ at 2.5σ ;
July, 2013 (28 events).

For $|\Delta m_{23}^2| = 2.4 \times 10^{-3} \text{ eV}^2$, $\sin^2 2\theta_{23} = 1$, $\delta = 0$, NO
(IO) spectrum:

$\sin^2 2\theta_{13} = 0.14$ (1.7), best fit.

This value is by a factor of ~ 1.6 (1.9) bigger than the value obtained in the Daya Bay and RENO experiments.

$$P_m^{3\nu}(\nu_\mu \rightarrow \nu_e) = P_m^{3\nu}(\theta_{13}, \Delta m_{31(32)}^2, \theta_{12}, \Delta m_{21}^2, \theta_{23}, \delta).$$



Up to 2nd order in the two small parameters $|\alpha| \equiv |\Delta m_{21}^2|/|\Delta m_{31}^2| \ll 1$ and $\sin^2 \theta_{13} \ll 1$:

$$P_m^{3\nu \text{ man}}(\nu_\mu \rightarrow \nu_e) \cong P_0 + P_{\sin \delta} + P_{\cos \delta} + P_3,$$

$$P_0 = \sin^2 \theta_{23} \frac{\sin^2 2\theta_{13}}{(A-1)^2} \sin^2[(A-1)\Delta],$$

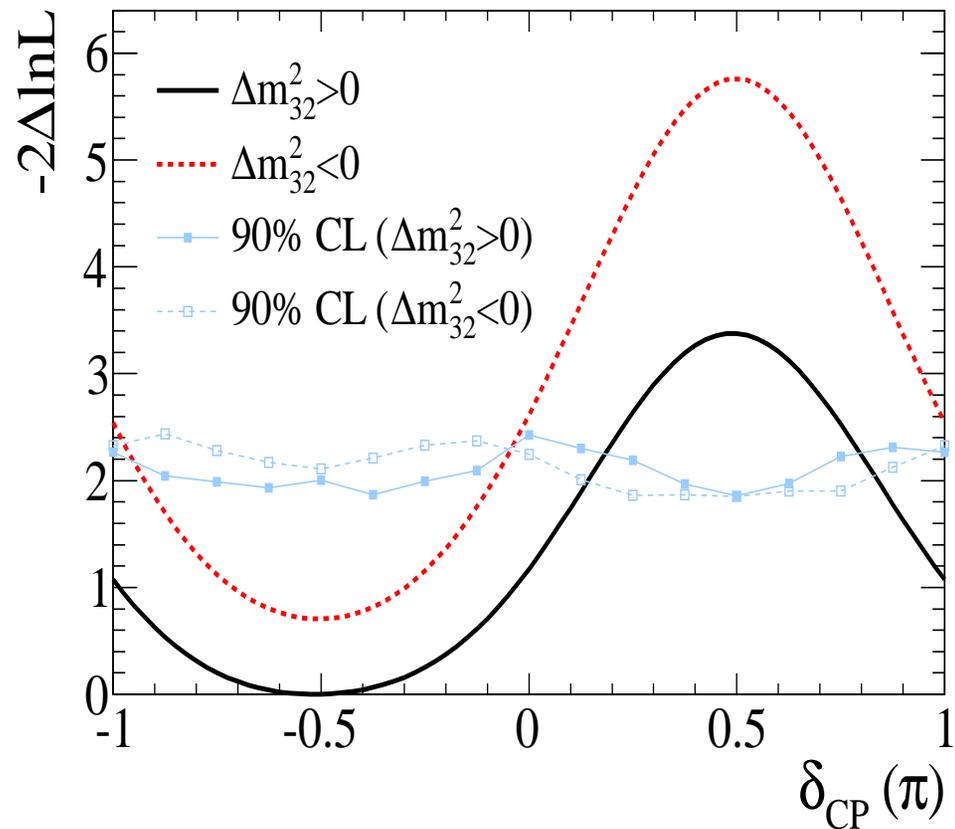
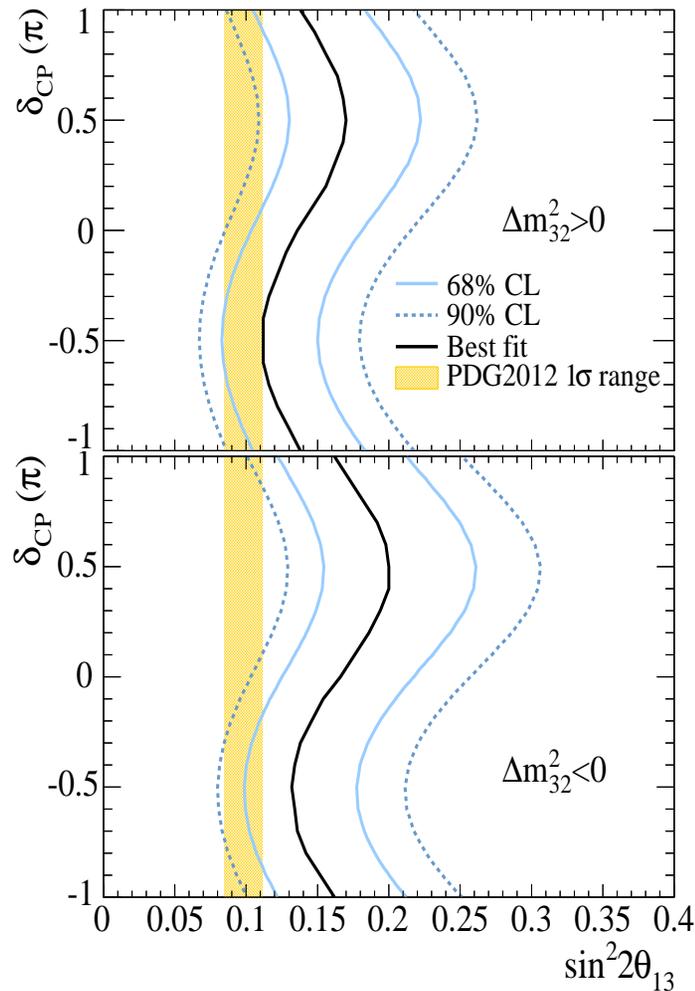
$$P_3 = \alpha^2 \cos^2 \theta_{23} \frac{\sin^2 2\theta_{12}}{A^2} \sin^2(A\Delta),$$

$$P_{\sin \delta} = -\alpha \frac{8 J_{CP}}{A(1-A)} (\sin \Delta)(\sin A\Delta) (\sin[(1-A)\Delta]),$$

$$P_{\cos \delta} = \alpha \frac{8 J_{CP} \cot \delta}{A(1-A)} (\cos \Delta)(\sin A\Delta) (\sin[(1-A)\Delta]),$$

$$\Delta = \frac{\Delta m_{31}^2 L}{4E}, \quad A = \sqrt{2} G_F N_e^{\text{man}} \frac{2E}{\Delta m_{31}^2}.$$

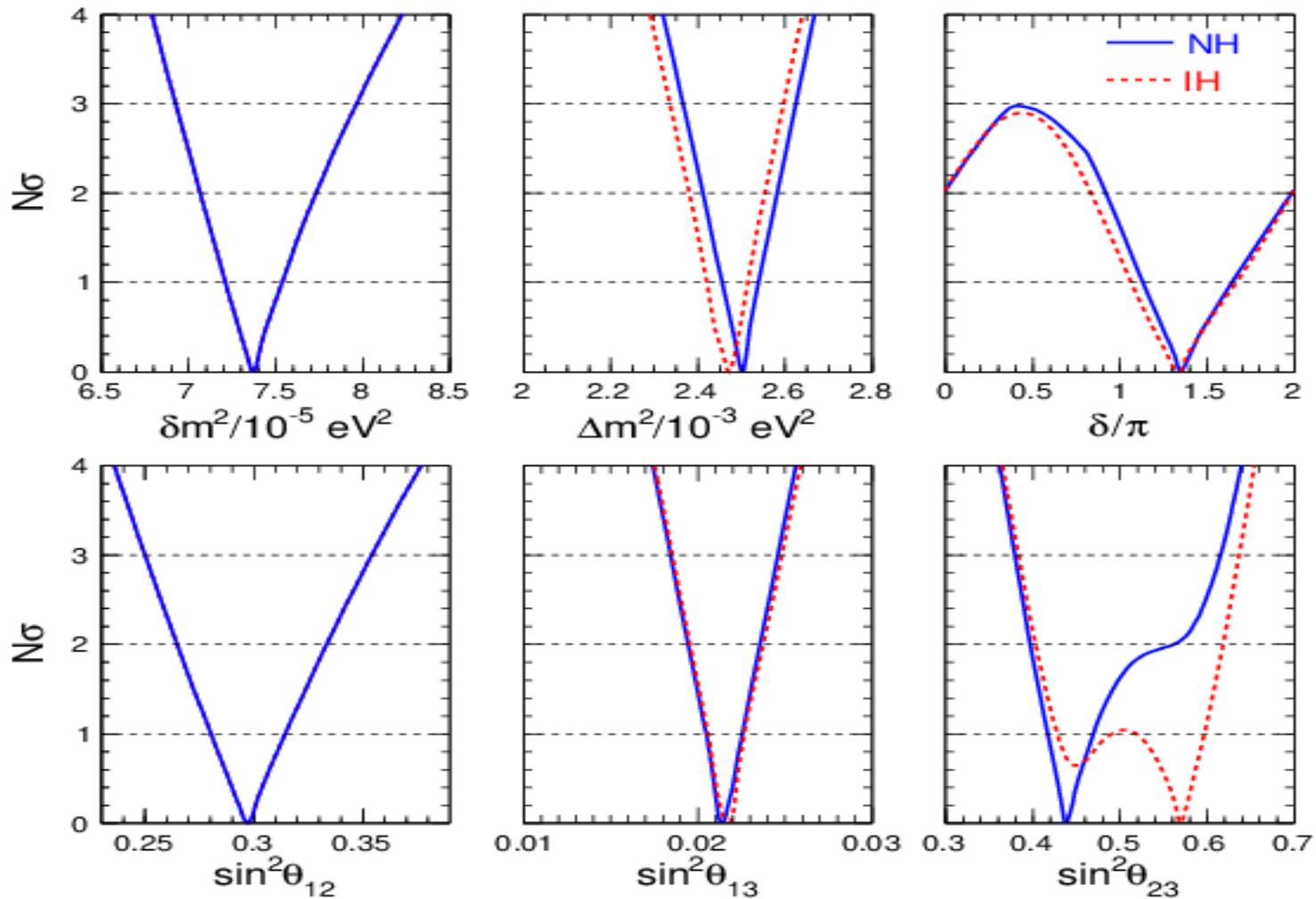
$$\bar{\nu}_\mu \rightarrow \bar{\nu}_e: \delta, \quad A \rightarrow (-\delta), \quad (-A)$$



K. Abe *et al.*, T2K Collaboration, arXiv:1311.4750

NO ν A first results (August, 2015) strengthened somewhat the hint that $\delta \cong 3\pi/2$.

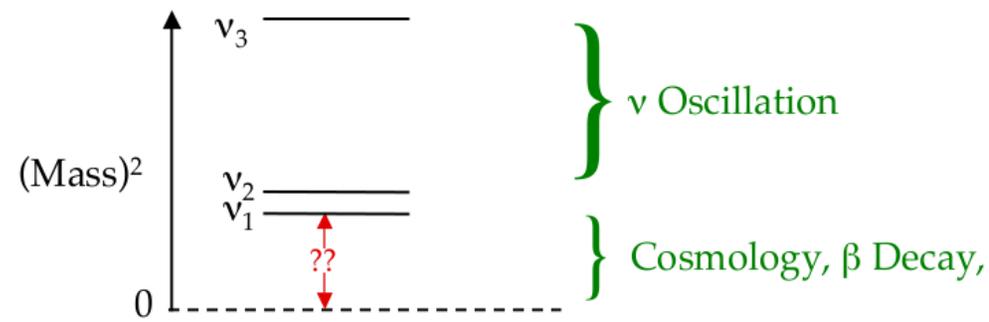
LBL Acc + Solar + KamLAND + SBL Reactors + Atmos



F. Capozzi, E. Lisi *et al.*, arXiv:1601.01777

Absolute Neutrino Mass Scale

The Absolute Scale of Neutrino Mass



How far above zero
is the whole pattern?

$$\text{Oscillation Data} \Rightarrow \sqrt{\Delta m_{\text{atm}}^2} < \text{Mass}[\text{Heaviest } \nu_i]$$

4

Due to B. Kayser

Absolute Neutrino Mass Measurements

Troitsk, Mainz experiments on ${}^3\text{H} \rightarrow {}^3\text{He} + e^- + \bar{\nu}_e$:
 $m_{\nu_e} < 2.2 \text{ eV}$ (95% C.L.)

We have $m_{\nu_e} \cong m_{1,2,3}$ in the case of QD spectrum. The upcoming **KATRIN** experiment is planned to reach sensitivity

KATRIN: $m_{\nu_e} \sim 0.2 \text{ eV}$

i.e., it will probe the region of the QD spectrum.

Improved β energy resolution requires a **BIG** β spectrometer.





Mass and Hierarchy from Cosmology

Cosmological and astrophysical data on $\sum_j m_j$: the Planck + WMAP (low $l \leq 25$) + ACT (large $l \geq 2500$) CMB data + Λ CDM (6 parameter) model + assuming 3 light massive neutrinos, implies

$$\sum_j m_j \equiv \Sigma < 0.66 \text{ eV} \quad (95\% \text{ C.L.})$$

Adding data on the baryon acoustic oscillations (BAO) leads to:

$$\sum_j m_j \equiv \Sigma < 0.23 \text{ eV} \quad (95\% \text{ C.L.})$$

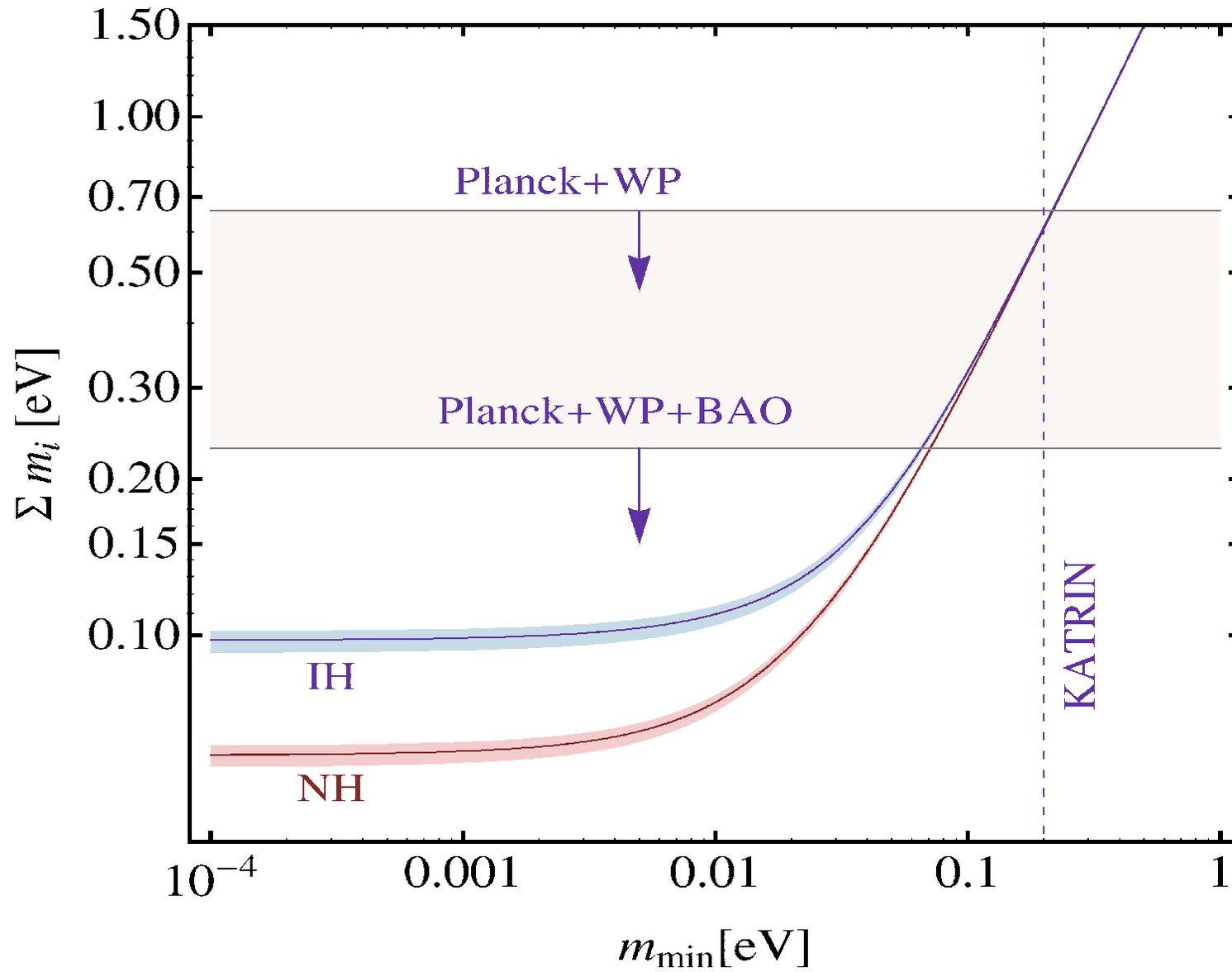
Data on weak lensing of galaxies by large scale structure, combined with data from the WMAP and Planck experiments might allow to determine

$$\sum_j m_j : \quad \delta \cong (0.01 - 0.04) \text{ eV.}$$

$$\text{NH: } \sum_j m_j \leq 0.05 \text{ eV} \quad (3\sigma);$$

$$\text{IH: } \sum_j m_j \geq 0.10 \text{ eV} \quad (3\sigma).$$

Mass and Hierarchy from Cosmology



These data imply that

$$m_{\nu_j} \lll m_{e,\mu,\tau}, m_q, \quad q = u, c, t, d, s, b$$

For $m_{\nu_j} \lesssim 1$ eV: $m_{\nu_j}/m_{l,q} \lesssim 10^{-6}$

For a given family: $10^{-2} \lesssim m_{l,q}/m_{q'} \lesssim 10^2$

Future Progress

- Determination of the nature - Dirac or Majorana, of ν_j .
- Determination of $\text{sgn}(\Delta m_{\text{atm}}^2)$, type of ν - mass spectrum

$$m_1 \ll m_2 \ll m_3, \quad \text{NH,}$$

$$m_3 \ll m_1 < m_2, \quad \text{IH,}$$

$$m_1 \cong m_2 \cong m_3, \quad m_{1,2,3}^2 \gg \Delta m_{\text{atm}}^2, \quad \text{QD; } m_j \gtrsim 0.10 \text{ eV.}$$

- Determining, or obtaining significant constraints on, the absolute scale of ν_j -masses, or $\min(m_j)$.
- Status of the CP-symmetry in the lepton sector: violated due to δ (Dirac), and/or due to α_{21}, α_{31} (Majorana)?
- High precision determination of $\Delta m_{\odot}^2, \theta_{12}, \Delta m_{\text{atm}}^2, \theta_{23}, \theta_{13}$
- Searching for possible manifestations, other than ν_l -oscillations, of the non-conservation of $L_l, l = e, \mu, \tau$, such as $\mu \rightarrow e + \gamma, \tau \rightarrow \mu + \gamma$, etc. decays.

- Understanding at fundamental level the mechanism giving rise to the ν – masses and mixing and to the L_l –non-conservation. Includes understanding
 - the origin of the observed patterns of ν -mixing and ν -masses ;
 - the physical origin of CPV phases in U_{PMNS} ;
 - Are the observed patterns of ν -mixing and of $\Delta m_{21,31}^2$ related to the existence of a new symmetry?
 - Is there any relations between q –mixing and ν – mixing? Is $\theta_{12} + \theta_c = \pi/4$?
 - Is $\theta_{23} = \pi/4$, or $\theta_{23} > \pi/4$ or else $\theta_{23} < \pi/4$?
 - Is there any correlation between the values of CPV phases and of mixing angles in U_{PMNS} ?
- Progress in the theory of ν -mixing might lead to a better understanding of the origin of the BAU.
 - Can the Majorana and/or Dirac $CPVP$ in U_{PMNS} be the leptogenesis CPV parameters at the origin of BAU?

The next most important steps are:

- determination of the nature - Dirac or Majorana, of massive neutrinos.
- determination of the neutrino mass hierarchy;
- determination of the absolute neutrino mass scale (or $\min(m_j)$);
- determination of the status of the CP symmetry in the lepton sector.

Large $\sin \theta_{13} \cong 0.16$ (Daya Bay, RENO) - far-reaching implications for the program of research in neutrino physics:

- For the determination of the type of ν - mass spectrum (or of $\text{sgn}(\Delta m_{\text{atm}}^2)$) in neutrino oscillation experiments.
- For understanding the pattern of the neutrino mixing and its origins (symmetry, etc.?).
- For the predictions for the $(\beta\beta)_{0\nu}$ -decay effective Majorana mass in the case of NH light ν mass spectrum (possibility of a strong suppression).

Large $\sin \theta_{13} \cong 0.16$ (Daya Bay, RENO) + $\delta = 3\pi/2$ - far-reaching implications:

- For the searches for CP violation in ν -oscillations; for the b.f.v. one has $J_{CP} \cong -0.030$;
- Important implications also for the “flavoured” leptogenesis scenario of generation of the baryon asymmetry of the Universe (BAU).

If all CPV, necessary for the generation of BAU is due to δ , a necessary condition for reproducing the observed BAU is

$$|\sin \theta_{13} \sin \delta| \gtrsim 0.09$$

S. Pascoli, S.T.P., A. Riotto, 2006.

The Nature of Massive Neutrinos I: Majorana versus Dirac Massive Neutrinos

Majorana Neutrinos

Can be defined in QFT using fields or states.

Fields: $\chi_k(x)$ - 4 component (spin 1/2), complex, m_k

Majorana condition:

$$C (\bar{\chi}_k(x))^T = \xi_k \chi_k(x), \quad |\xi_k|^2 = 1$$

- Invariant under proper Lorentz transformations.
- Reduces by 2 the number of components in $\chi_k(x)$.

Implications:

$$U(1) : \chi_k(x) \rightarrow e^{i\alpha} \chi_k(x) - \text{impossible}$$

- $\chi_k(x)$ cannot absorb phases.
- $Q_{U(1)} = 0 : Q_{el} = 0, L_l = 0, L = 0, \dots$
- $\chi_k(x)$: 2 spin states of a spin 1/2 absolutely neutral particle
- $\chi_k \equiv \bar{\chi}_k$

Propagators: $\Psi(x)$ –Dirac, $\chi(x)$ –Majorana

$$\langle 0|T(\Psi_\alpha(x)\bar{\Psi}_\beta(y))|0 \rangle = S_{\alpha\beta}^F(x-y) ,$$

$$\langle 0|T(\Psi_\alpha(x)\Psi_\beta(y))|0 \rangle = 0 , \quad \langle 0|T(\bar{\Psi}_\alpha(x)\bar{\Psi}_\beta(y))|0 \rangle = 0 .$$

$$\langle 0|T(\chi_\alpha(x)\bar{\chi}_\beta(y))|0 \rangle = S_{\alpha\beta}^F(x-y) ,$$

$$\langle 0|T(\chi_\alpha(x)\chi_\beta(y))|0 \rangle = -\xi^* S_{\alpha\kappa}^F(x-y) C_{\kappa\beta} ,$$

$$\langle 0|T(\bar{\chi}_\alpha(x)\bar{\chi}_\beta(y))|0 \rangle = \xi C_{\alpha\kappa}^{-1} S_{\kappa\beta}^F(x-y)$$

$$U_{CP} \chi(x) U_{CP}^{-1} = \eta_{CP} \gamma_0 \chi(x'), \quad \eta_{CP} = \pm i .$$

Special Properties of the Currents of $\chi(x)$ –Majorana:

$$\bar{\chi}(x)\gamma_\alpha\chi(x) = 0 : \quad Q_{U(1)} = 0 \quad (Q_{U(1)}(\Psi) \neq 0);$$

Has important implications, e.g. for SUSY DM (neutralino) abundance determination (calculation).

$$\bar{\chi}(x)\sigma_{\alpha\beta}\chi(x) = 0 : \quad \mu_\chi = 0 \quad (\mu_\Psi \neq 0)$$

$$\bar{\chi}(x)\sigma_{\alpha\beta}\gamma_5\chi(x) = 0 : \quad d_\chi = 0 \quad (d_\Psi \neq 0, \text{ if } CP \text{ is not conserved})$$

$\chi(x)$ cannot couple to a real photon (field).

$\chi(x)$ couples to a virtual photon through an anapole moment:

$$(g_{\alpha\beta} q^2 - q_\alpha q_\beta)\gamma_\beta\gamma_5 F_a(q^2).$$

Properties of Currents Formed by $\chi_1(x)$, $\chi_2(x)$: $\chi_2 \rightarrow \chi_1 + \gamma$, $\chi_2 \rightarrow \chi_1 \chi_1 \chi_1$, etc.

$$\bar{\chi}_1(x) \gamma_\alpha (v - a \gamma_5) \chi_2(x) \quad (\bar{\chi}_1(x) \gamma^\alpha (1 - \gamma_5) \chi_1(x), \dots) :$$

- CP is conserved: $v = 0$ ($a = 0$) if $\eta_{1CP} = \eta_{2CP}$ ($\eta_{1CP} = -\eta_{2CP}$)
- CP is not conserved: $v \neq 0$, $a \neq 0$

(Has important implications also, e.g. for SUSY neutralino phenomenology:
 $e^+ + e^- \rightarrow \chi_1 + \chi_2$, $\chi_2 \rightarrow \chi_1 + l^+ + l^-$, etc.)

$$\bar{\chi}_1(x) \sigma_{\alpha\beta} (\mu_{12} - d_{12} \gamma_5) \chi_2(x) \quad (F^{\alpha\beta}(x)) :$$

- CP is conserved: $\mu_{12} = 0$ ($d_{12} = 0$) if $\eta_{1CP} = \eta_{2CP}$ ($\eta_{1CP} = -\eta_{2CP}$)
- CP is not conserved: $\mu_{12} \neq 0$, $d_{12} \neq 0$

Pontecorvo, 1958:

$$\nu(x) = \frac{\chi_1 + \chi_2}{\sqrt{2}}, \quad m_1 \neq m_2 > 0, \eta_{1CP} = -\eta_{2CP}$$

$\chi_{1,2}$ - Majorana, maximal mixing .

Maki, Nakagawa, Sakata, 1962:

$$\nu_{eL}(x) = \Psi_{1L} \cos \theta_C + \Psi_{2L} \sin \theta_C ,$$

$$\nu_{\mu L}(x) = -\Psi_{1L} \sin \theta_C + \Psi_{2L} \cos \theta_C ,$$

$\Psi_{1,2}$ - Dirac (composite), θ_C - the Cabibbo angle .

Determining the Nature of Massive Neutrinos

Dirac CP-Nonconservation: δ in U_{PMNS}

Observable manifestations in

$$\nu_l \leftrightarrow \nu_{l'} , \quad \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'} , \quad l, l' = e, \mu, \tau$$

- not sensitive to Majorana CPVP α_{21}, α_{31}

S.M. Bilenky, J. Hosek, S.T.P., 1980;
P. Langacker et al., 1987

$$A(\nu_l \leftrightarrow \nu_{l'}) = \sum_j U_{l'j} e^{-i(E_j t - p_j x)} U_{jl}^\dagger$$

$$U = VP : P_j e^{-i(E_j t - p_j x)} P_j^* = e^{-i(E_j t - p_j x)}$$

P - diagonal matrix of Majorana phases.

The result is valid also in the case of oscillations in matter: ν_l oscillations are not sensitive to the nature of ν_j .

ν_j — Dirac or Majorana particles, **fundamental problem**

ν_j —Dirac: **conserved lepton charge exists**, $L = L_e + L_\mu + L_\tau$, $\nu_j \neq \bar{\nu}_j$

ν_j —Majorana: **no lepton charge is exactly conserved**, $\nu_j \equiv \bar{\nu}_j$

The observed patterns of ν —mixing and of Δm_{atm}^2 and Δm_{\odot}^2 can be related to Majorana ν_j and an **approximate** symmetry:

$$L' = L_e - L_\mu - L_\tau$$

S.T.P., 1982

See-saw mechanism: ν_j — Majorana

Establishing that ν_j are Majorana particles would be as important as the discovery of ν — oscillations.

If ν_j – Majorana particles, U_{PMNS} contains (3- ν mixing)

δ -Dirac, α_{21} , α_{31} - Majorana physical CPV phases

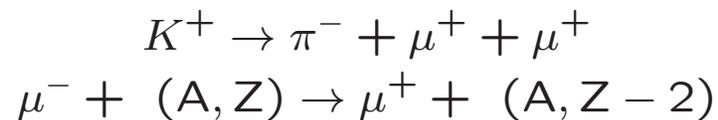
ν -oscillations $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$, $l, l' = e, \mu, \tau$,

- are not sensitive to the nature of ν_j ,

S.M. Bilenky et al., 1980;
P. Langacker et al., 1987

- provide information on $\Delta m_{jk}^2 = m_j^2 - m_k^2$, but not on the absolute values of ν_j masses.

The Majorana nature of ν_j can manifest itself in the existence of $\Delta L = \pm 2$ processes:



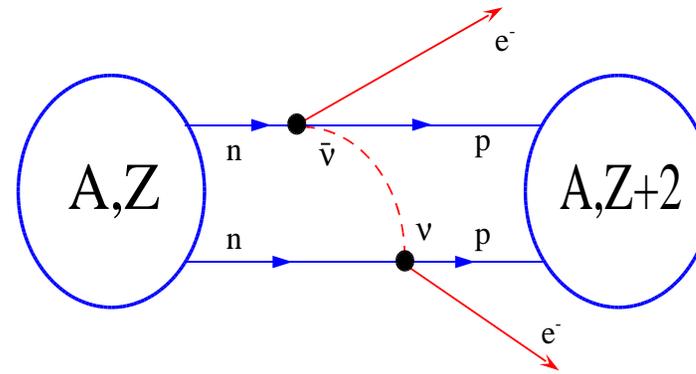
The process most sensitive to the possible Majorana nature of ν_j - $(\beta\beta)_{0\nu}$ -decay



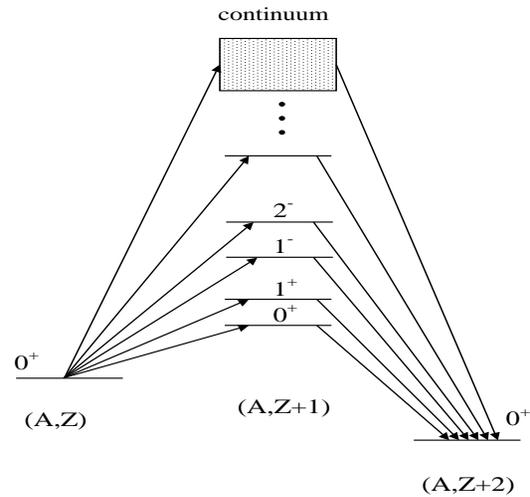
of even-even nuclei, ^{48}Ca , ^{76}Ge , ^{82}Se , ^{100}Mo , ^{116}Cd , ^{130}Te , ^{136}Xe , ^{150}Nd .

$2n$ from (A, Z) exchange a virtual Majorana ν_j (via the CC weak interaction) and transform into $2p$ of $(A, Z+2)$ and two free e^- .

Nuclear $0\nu\beta\beta$ -decay



strong in-medium modification of the basic process
 $dd \rightarrow uue^-e^-(\bar{\nu}_e\bar{\nu}_e)$



virtual excitation
of states of all multiplicities
in $(A, Z+1)$ nucleus

V. Rodin, talk at Gran Sasso, 2006

$(\beta\beta)_{0\nu}$ –Decay Experiments:

- Majorana nature of ν_j
- Type of ν –mass spectrum (NH, IH, QD)
- Absolute neutrino mass scale

${}^3\text{H}$ β -decay, cosmology: m_ν (QD, IH)

- CPV due to Majorana CPV phases

$$A(\beta\beta)_{0\nu} \sim \langle m \rangle M(A,Z), \quad M(A,Z) - \text{NME},$$

$$\begin{aligned} |\langle m \rangle| &= |m_1 |U_{e1}|^2 + m_2 |U_{e2}|^2 e^{i\alpha_{21}} + m_3 |U_{e3}|^2 e^{i\alpha_{31}}| \\ &= |m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 e^{i\alpha_{21}} + m_3 s_{13}^2 e^{i\alpha_{31}}|, \quad \theta_{12} \equiv \theta_{\odot}, \theta_{13} - \text{CHOOZ} \end{aligned}$$

α_{21}, α_{31} - the two Majorana CPVP of the PMNS matrix.

CP-invariance: $\alpha_{21} = 0, \pm\pi, \alpha_{31} = 0, \pm\pi;$

$$\eta_{21} \equiv e^{i\alpha_{21}} = \pm 1, \quad \eta_{31} \equiv e^{i\alpha_{31}} = \pm 1$$

relative CP-parities of ν_1 and ν_2 , and of ν_1 and ν_3 .

L. Wolfenstein, 1981;

S.M. Bilenky, N. Nedelcheva, S.T.P., 1984;

B. Kayser, 1984.

$$A(\beta\beta)_{0\nu} \sim \langle m \rangle M(A,Z), \quad M(A,Z) - \text{NME},$$

$$|\langle m \rangle| \cong \left| \sqrt{\Delta m_{\odot}^2} \sin^2 \theta_{12} e^{i\alpha} + \sqrt{\Delta m_{31}^2} \sin^2 \theta_{13} e^{i\beta_M} \right|, \quad m_1 \ll m_2 \ll m_3 \text{ (NH)},$$

$$|\langle m \rangle| \cong \sqrt{m_3^2 + \Delta m_{13}^2} |\cos^2 \theta_{12} + e^{i\alpha} \sin^2 \theta_{12}|, \quad m_3 < (\ll) m_1 < m_2 \text{ (IH)},$$

$$|\langle m \rangle| \cong m |\cos^2 \theta_{12} + e^{i\alpha} \sin^2 \theta_{12}|, \quad m_{1,2,3} \cong m \gtrsim 0.10 \text{ eV (QD)},$$

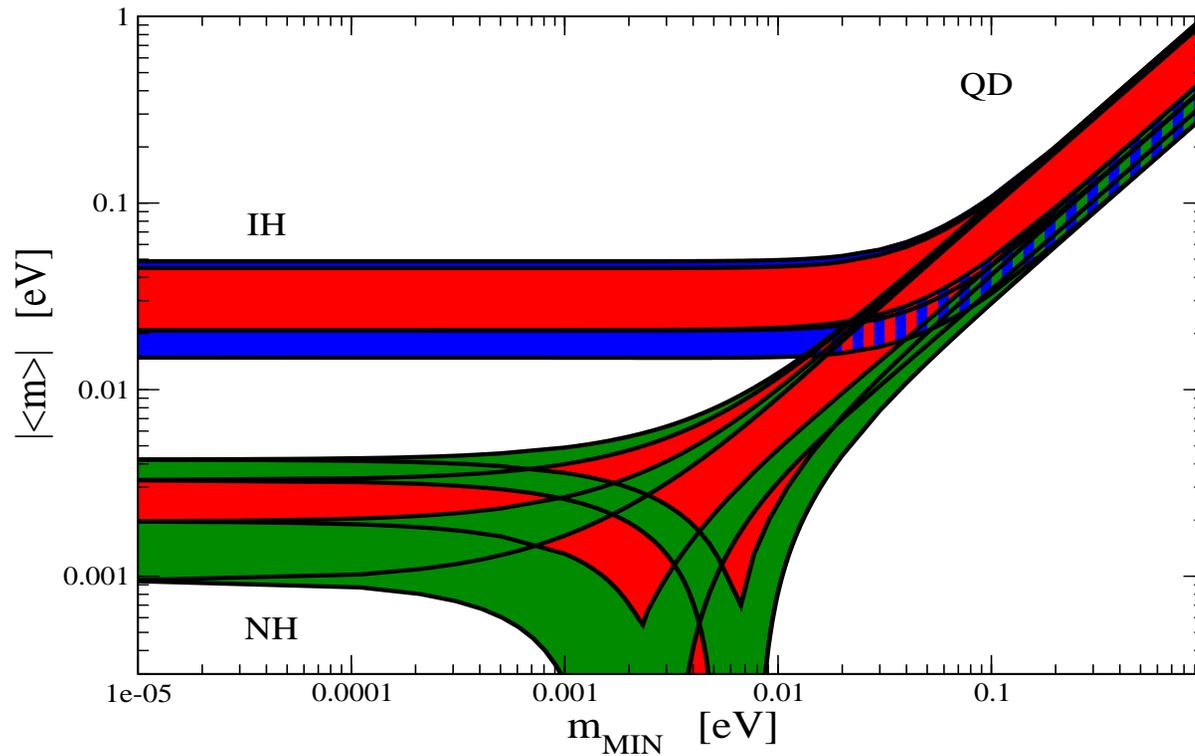
$$\theta_{12} \equiv \theta_{\odot}, \theta_{13} \text{-CHOOZ}; \quad \alpha \equiv \alpha_{21}, \beta_M \equiv \alpha_{31}.$$

CP-invariance: $\alpha = 0, \pm\pi, \beta_M = 0, \pm\pi;$

$$|\langle m \rangle| \lesssim 5 \times 10^{-3} \text{ eV, NH};$$

$$\sqrt{\Delta m_{13}^2} \cos 2\theta_{12} \cong 0.013 \text{ eV} \lesssim |\langle m \rangle| \lesssim \sqrt{\Delta m_{13}^2} \cong 0.055 \text{ eV, IH};$$

$$m \cos 2\theta_{12} \lesssim |\langle m \rangle| \lesssim m, \quad m \gtrsim 0.10 \text{ eV, QD}.$$



S. Pascoli, PDG, 2012

$$\sin^2 \theta_{13} = 0.0236 \pm 0.0042; \delta = 0.$$

$$1\sigma(\Delta m_{21}^2) = 2.6\%, \quad 1\sigma(\sin^2 \theta_{12}) = 5.4\%, \quad 1\sigma(|\Delta m_{31(23)}^2|) = 3\%.$$

From G.L. Fogli *et al.*, arXiv:1205.5254v3

$2\sigma(|\langle m \rangle|)$ used.

Best sensitivity: GERDA (^{76}Ge), EXO (^{136}Xe), KamLAND-ZEN (^{136}Xe).

Claim for a positive signal at $> 3\sigma$:

H. Klapdor-Kleingrothaus et al., PL B586 (2004),

$|\langle m \rangle| = (0.1 - 0.9) \text{ eV}$ (99.73% C.L.); b.f.v.: $|\langle m \rangle| = 0.33 \text{ eV}$.

IGEX ^{76}Ge : $|\langle m \rangle| < (0.33 - 1.35) \text{ eV}$ (90% C.L.).

Recent data - NEMO3 (^{100}Mo), CUORICINO (^{130}Te):

$|\langle m \rangle| < (0.45 - 0.96) \text{ eV}$, $|\langle m \rangle| < (0.18 - 0.64) \text{ eV}$ (90% C.L.).

H. Klapdor-Kleingrothaus et al., PL B586 (2004),

$$T(^{76}\text{Ge}) = 2.23_{0.31}^{+0.44} \times 10^{25} \text{ yr at 90\% C.L.}$$

Results from 2012-2013:

$$T(^{136}\text{Xe}) > 1.6 \times 10^{25} \text{ yr at 90\% C.L., EXO}$$

$$T(^{136}\text{Xe}) > 1.9 \times 10^{25} \text{ yr at 90\% C.L., KamLAND – Zen}$$

$$T(^{76}\text{Ge}) > 2.1 \times 10^{25} \text{ yr at 90\% C.L., GERDA.}$$

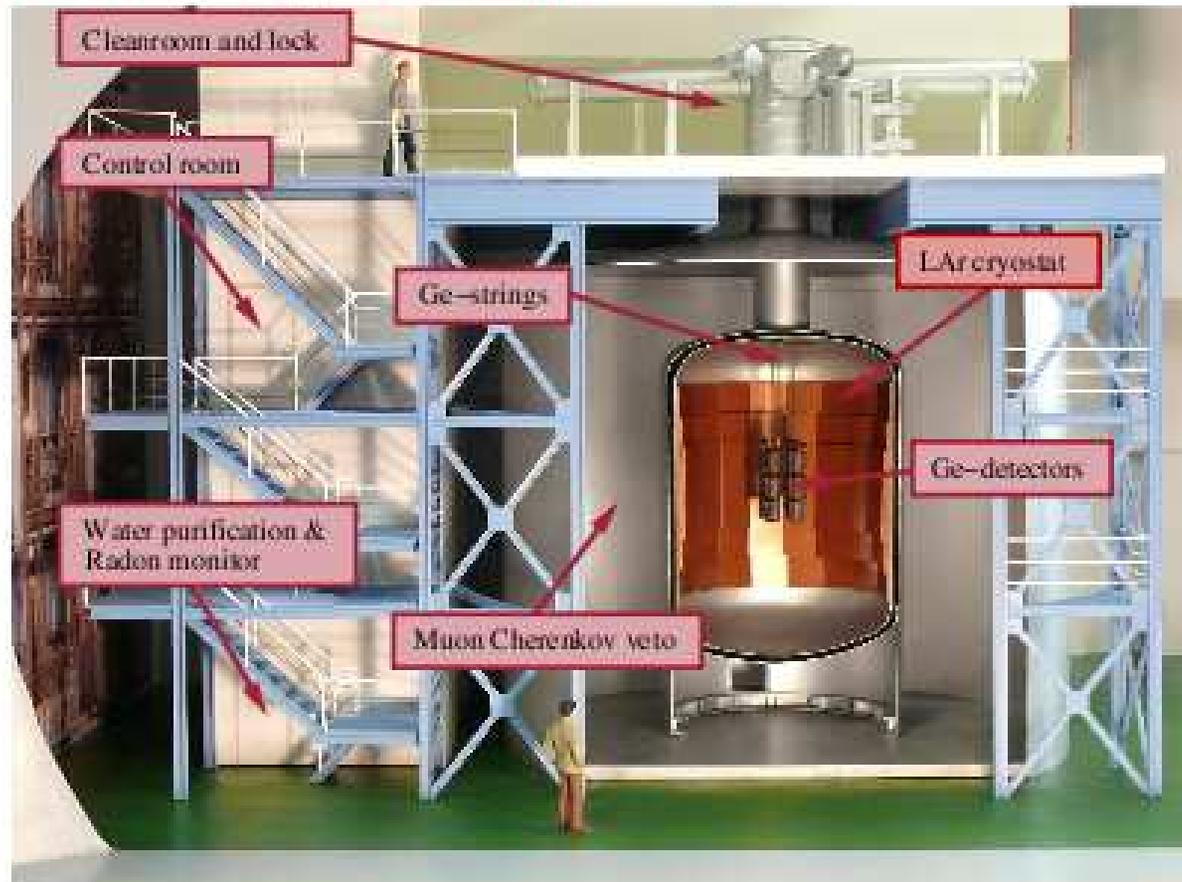
$$T(^{76}\text{Ge}) > 3.0 \times 10^{25} \text{ yr at 90\% C.L., GERDA + IGEX + HdM}$$

Large number of experiments: $|\langle m \rangle| \sim (0.01-0.05) \text{ eV}$

CUORE - ^{130}Te ,
GERDA - ^{76}Ge ,
KamLAND-ZEN - ^{136}Xe ;
EXO - ^{136}Xe ;
SNO+ - ^{130}Te ;
AMoRE - ^{100}Mo (S. Korea);
CANDLES - ^{48}Ca ;
SuperNEMO - $^{82}\text{Se}, \dots$;
MAJORANA - ^{76}Ge ;
COBRA - ^{116}Cd ;
MOON - ^{100}Mo .



GERDA: Experimental Setup



UNIVERSITÄT
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ESSEN



Majorana CPV Phases and $|\langle m \rangle|$

CPV can be established provided

- $|\langle m \rangle|$ measured with $\Delta \lesssim 15\%$;
- Δm_{atm}^2 (IH) or m_0 (QD) measured with $\delta \lesssim 10\%$;
- $\xi \lesssim 1.5$;
- α_{21} (QD): in the interval $\sim [\frac{\pi}{4} - \frac{3\pi}{4}]$, or $\sim [\frac{5\pi}{4} - \frac{3\pi}{2}]$;
- $\tan^2 \theta_{\odot} \gtrsim 0.40$.

S. Pascoli, S.T.P., W. Rodejohann, 2002

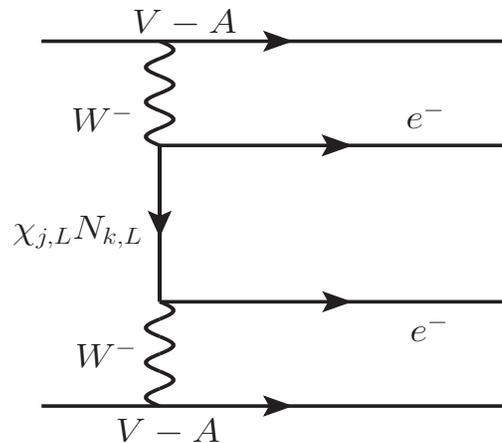
S. Pascoli, S.T.P., L. Wolfenstein, 2002

S. Pascoli, S.T.P., T. Schwetz, hep-ph/0505226

No “No-go for detecting CP-Violation via $(\beta\beta)_{0\nu}$ -decay”

V. Barger *et al.*, 2002

Different Mechanisms of $(\beta\beta)_{0\nu}$ -Decay



Light Majorana Neutrino Exchange

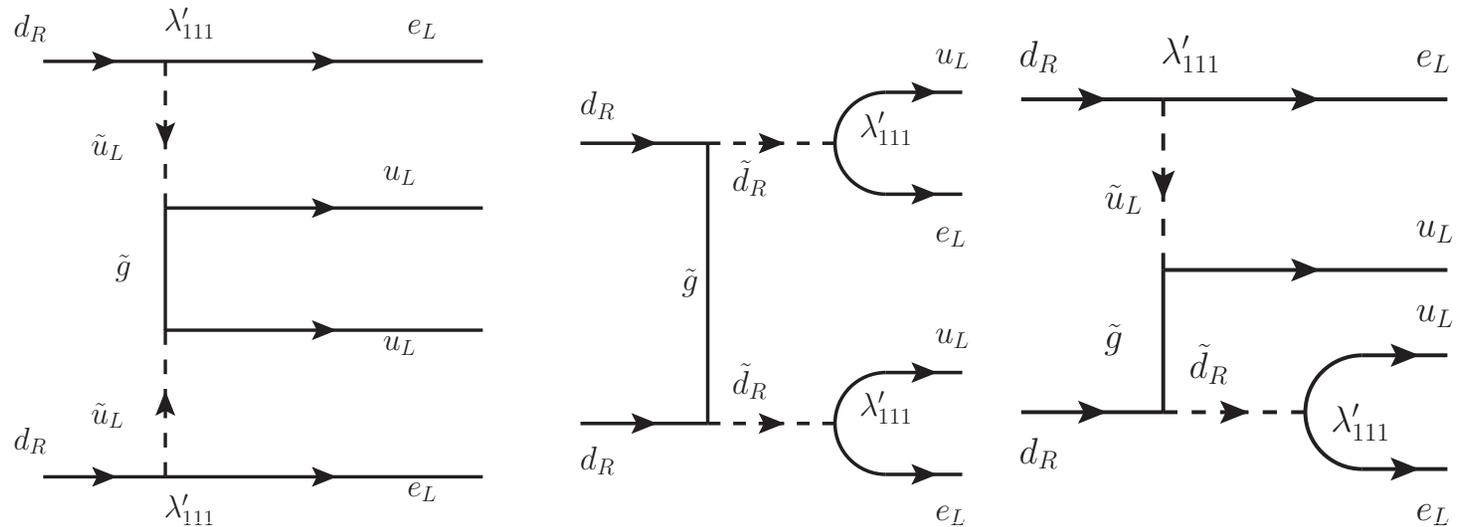
$$\eta_\nu = \frac{\langle m \rangle}{m_e}.$$

Heavy Majorana Neutrino Exchange Mechanisms

(V-A) Weak Interaction, LH N_k , $M_k \gtrsim 10$ GeV:

$$\eta_N^L = \sum_k^{\text{heavy}} U_{ek}^2 \frac{m_p}{M_k}, \quad m_p - \text{proton mass}, \quad U_{ek} - \text{CPV}.$$

SUSY Models with R-Parity Non-conservation



$$\begin{aligned}
 \mathcal{L}_{Rp} = & \lambda'_{111} \left[(\bar{u}_L \bar{d}_L) \begin{pmatrix} e_R^c \\ -\nu_{eR}^c \end{pmatrix} \tilde{d}_R + (\bar{e}_L \bar{\nu}_{eL}) d_R \begin{pmatrix} \tilde{u}_L^* \\ -\tilde{d}_L^* \end{pmatrix} \right. \\
 & \left. + (\bar{u}_L \bar{d}_L) d_R \begin{pmatrix} \tilde{e}_L^* \\ -\tilde{\nu}_{eL}^* \end{pmatrix} \right] + h.c.
 \end{aligned}$$

The problem of distinguishing between different sets of multiple (e.g., two) mechanisms being operative in $(\beta\beta)_{0\nu}$ -decay was studied in

1. A. Faessler, A. Meroni, S.T.P., F. Simkovic and J. Vergados, “Uncovering Multiple CP-Nonconserving Mechanisms of $(\beta\beta)_{0\nu}$ -Decay”, arXiv:1103.2434, Phys. Rev. D83 (2011) 113003.

2. A. Meroni, S.T.P. and F. Simkovic, “Multiple CP Non-conserving Mechanisms of $\beta\beta_{0\nu}$ -Decay and Nuclei with Largely Different Nuclear Matrix Elements”, (arXiv:1212.1331, JHEP **1302** (2013) 025).

Earlier studies include:

A. Halprin, S.T.P., S.P. Rosen, “Effects of Mixing of Light and Heavy Majorana Neutrinos in Neutrinoless Double Beta Decay”, Phys. Lett. 125B (1983) 335).

Determining the ν -Mass Hierarchy ($\text{sgn}(\Delta m_{\text{atm}}^2)$)

- **Reactor $\bar{\nu}_e$ Oscillations in vacuum (JUNO, RENO50).**
- **Atmospheric ν experiments: subdominant $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$ and $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ oscillations (matter effects) (HK, ORCA, PINGU (IceCube), INO).**
- **LBL ν -oscillation experiments (T2K, NO ν A; LBNO, LBNE, ν -factory); designed to search also for CP violation.**
- **^3H β -decay Experiments (sensitivity to 5×10^{-2} eV) (NH vs IH).**
- **$(\beta\beta)_{0\nu}$ -Decay Experiments; ν_j - Majorana particles (NH vs IH).**
- **Cosmology: $\sum_j m_j$ (NH vs IH).**
- **Atomic Physics Experiments: RENP.**

Reactor $\bar{\nu}_e$ Oscillations in vacuum

$$P_{\text{NO}}(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \frac{1}{2} \sin^2 2\theta_{13} \left(1 - \cos \frac{\Delta m_{\text{A}}^2 L}{2 E_\nu}\right) - \frac{1}{2} \cos^4 \theta_{13} \sin^2 2\theta_{\odot} \left(1 - \cos \frac{\Delta m_{\odot}^2 L}{2 E_\nu}\right) \\ + \sin^2 2\theta_{13} \sin^2 \theta_{\odot} \sin \frac{\Delta m_{\odot}^2 L}{4 E_\nu} \sin \left(\frac{\Delta m_{\text{A}}^2 L}{2 E_\nu} - \frac{\Delta m_{\odot}^2 L}{4 E_\nu}\right),$$

$$P_{\text{IO}}(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \frac{1}{2} \sin^2 2\theta_{13} \left(1 - \cos \frac{\Delta m_{\text{A}}^2 L}{2 E_\nu}\right) - \frac{1}{2} \cos^4 \theta_{13} \sin^2 2\theta_{\odot} \left(1 - \cos \frac{\Delta m_{\odot}^2 L}{2 E_\nu}\right) \\ + \sin^2 2\theta_{13} \cos^2 \theta_{\odot} \sin \frac{\Delta m_{\odot}^2 L}{4 E_\nu} \sin \left(\frac{\Delta m_{\text{A}}^2 L}{2 E_\nu} - \frac{\Delta m_{\odot}^2 L}{4 E_\nu}\right),$$

$\theta_{\odot} = \theta_{12}$, $\Delta m_{\odot}^2 = \Delta m_{21}^2 > 0$; $\sin^2 \theta_{12} \leq 0.36$ at 3σ ;

$\Delta m_{\text{A}}^2 = \Delta m_{31}^2 > 0$, NO spectrum,

$\Delta m_{\text{A}}^2 = \Delta m_{23}^2 > 0$, IO spectrum

M. Piai, S.T.P., hep-ph/0112074;

The reactor $\bar{\nu}_e$ detected via



The visible energy of the detected e^+ :

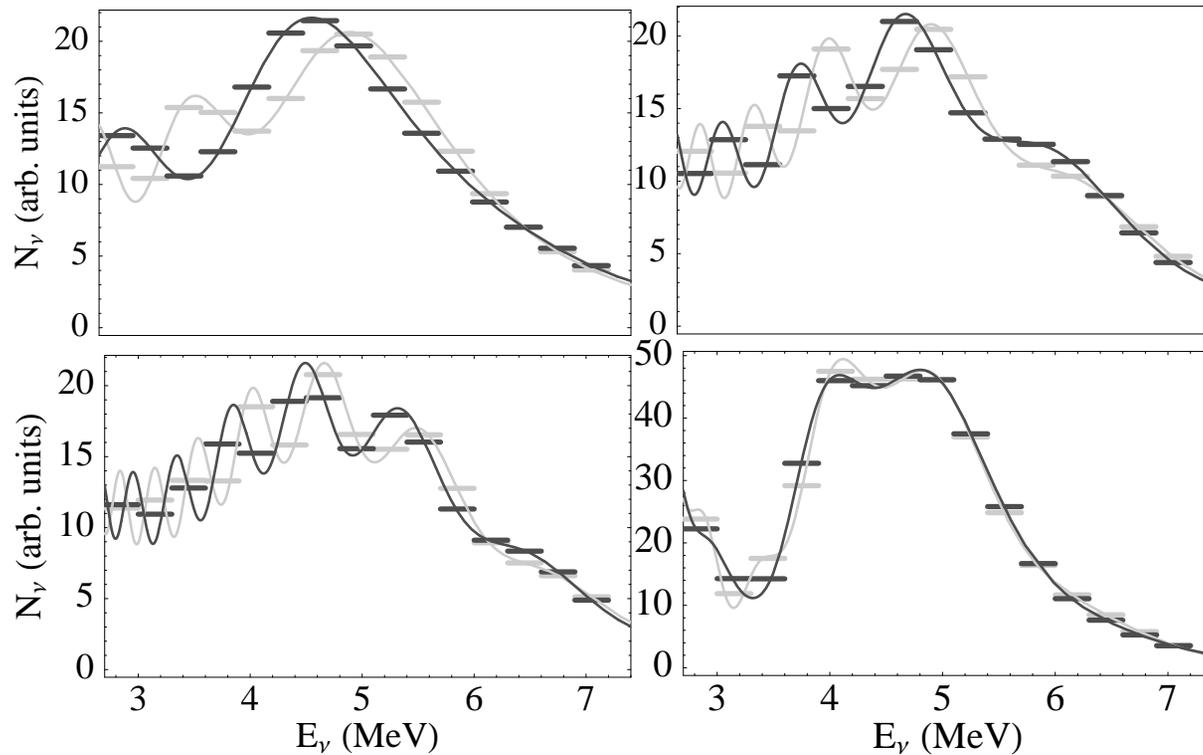
$$E_{vis} = E + m_e - (m_n - m_p) \simeq E - 0.8 \text{ MeV}.$$

The measured event rate spectrum vs. L/E_m :

$$N(L/E_m) = \int R(E, E_m) \Phi(E) \sigma(\bar{\nu}_e p \rightarrow e^+ n; E) P_{\bar{e}\bar{e}}^{NO(IO)} dE.$$

$$|P_{NO}(\bar{\nu}_e \rightarrow \bar{\nu}_e) - P_{IO}(\bar{\nu}_e \rightarrow \bar{\nu}_e)| \propto \sin^2 2\theta_{13} \cos 2\theta_{12}$$

$$\cos 2\theta_{12} \cong 0.38; \quad 3\sigma : \quad \cos 2\theta_{12} \geq 0.28; \quad \sin^2 2\theta_{13} \cong 0.09.$$



M. Piai, S.T.P., 2001

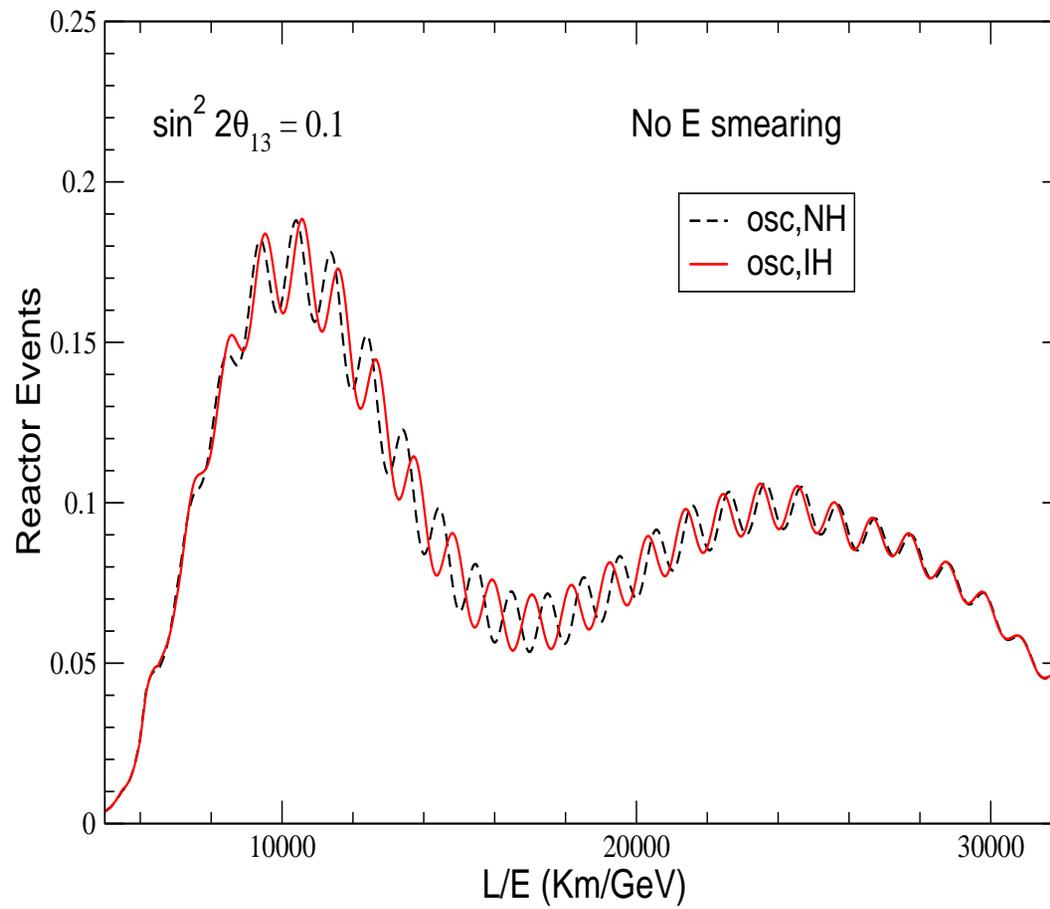
$$\sin^2 \theta_{13} = 0.05, \quad \Delta m_{21}^2 = 2 \times 10^{-4} \text{ eV}^2; \quad \Delta m_{\text{A}}^2 = 1.3; 2.5; 3.5 \times 10^{-3} \text{ eV}^2$$

$$L = 20 \text{ km}, \Delta E_\nu = 0.3 \text{ MeV}.$$

$$\Delta m_{21}^2 = 2 \times 10^{-4} \text{ eV}^2: \quad L = 20 \text{ km};$$

$$\Delta m_{21}^2 = 7.6 \times 10^{-5} \text{ eV}^2: \quad L \cong 53 \text{ km}.$$

NO – light grey; IO – dark grey



P. Ghoshal, S.T.P., arXiv:1011.1646

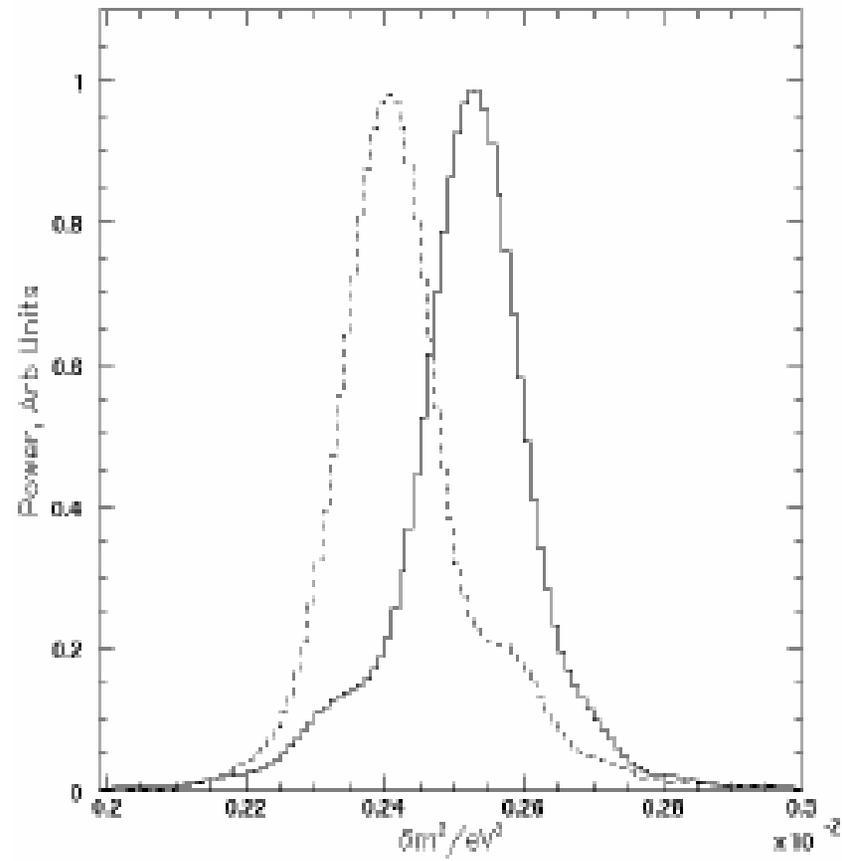
Fourier Analysis:

$$\text{NO} : \quad \cos^2 \theta_{12} \sin^2 \Delta + \sin^2 \theta_{12} \sin^2(\Delta - \Delta_{21}),$$

$$\text{IO} : \quad \sin^2 \theta_{12} \sin^2 \Delta + \cos^2 \theta_{12} \sin^2(\Delta - \Delta_{21}),$$

$$\Delta \equiv \Delta_{31}(\text{NO}) = |\Delta_{32}(\text{IO})|;$$

$$\sin^2 \theta_{12} \cong 0.31, \quad \cos^2 \theta_{12} \cong 0.69.$$



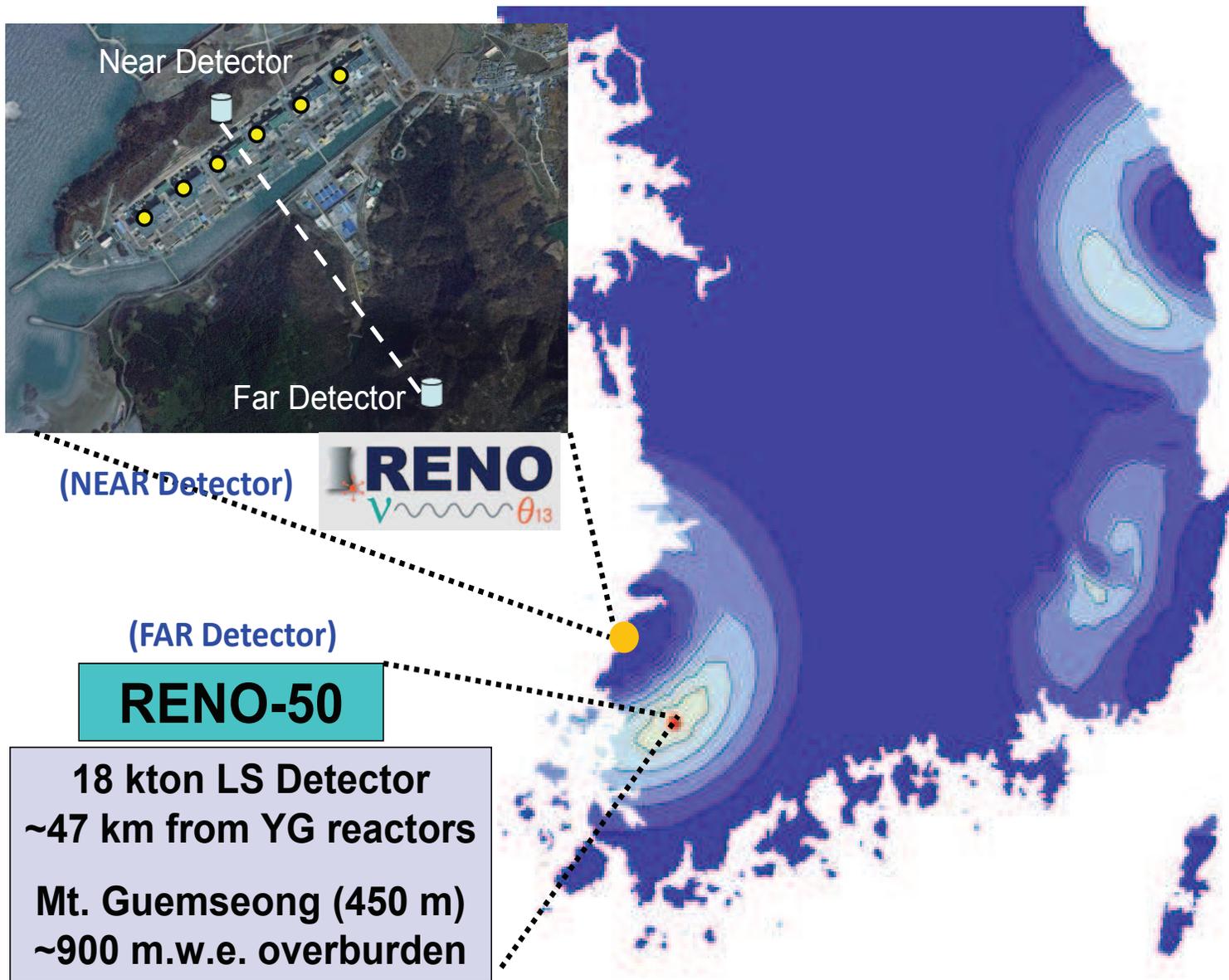
J.Learned et al., 2007

Very challenging; requires:

- energy resolution $\sigma/E_{\text{vis}} \lesssim 3\%/\sqrt{E_{\text{vis}}}$;
- relatively small energy scale uncertainty;
- relatively large statistics ($\sim (300 - 1000)$ kT GW yr);
- relatively small systematic errors;
- subtle optimisations (distance, number of bins, effects of “interfering distant” reactors).

Two experiments planned with $L \cong 50$ km: Juno (20 kT, approved), RENO50 (18 kT). Can measure also $\sin^2 \theta_{12}$, Δm_{21}^2 and $|\Delta m_{31}^2|$ with remarkably high precision. Can be used for detection of Geo, solar, SN neutrinos as well.





Atmospheric Neutrino Experiments on $\text{sgn}(\Delta m_{31}^2)$

Atmospheric ν experiments

Subdominant $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$ and $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ oscillations in the Earth.

$$P_{3\nu}(\nu_e \rightarrow \nu_\mu) \cong P_{3\nu}(\nu_\mu \rightarrow \nu_e) \cong s_{23}^2 P_{2\nu}, P_{3\nu}(\nu_e \rightarrow \nu_\tau) \cong c_{23}^2 P_{2\nu},$$

$$P_{3\nu}(\nu_\mu \rightarrow \nu_\mu) \cong 1 - s_{23}^4 P_{2\nu} - 2c_{23}^2 s_{23}^2 [1 - \text{Re}(e^{-i\kappa} A_{2\nu}(\nu_\tau \rightarrow \nu_\tau))],$$

$P_{2\nu} \equiv P_{2\nu}(\Delta m_{31}^2, \theta_{13}; E, \theta_n; N_e)$: 2- ν $\nu_e \rightarrow \nu'_\tau$ oscillations in the Earth,
 $\nu'_\tau = s_{23} \nu_\mu + c_{23} \nu_\tau$; $\Delta m_{21}^2 \ll |\Delta m_{31(32)}^2|$, $E_\nu \gtrsim 2$ GeV;

κ and $A_{2\nu}(\nu_\tau \rightarrow \nu_\tau) \equiv A_{2\nu}$ are known phase and 2- ν amplitude.

NO: $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$ **matter enhanced**, $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ - suppressed

IO: $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ **matter enhanced**, $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$ - suppressed

No charge identification (SK, HK, IceCube-PINGU, ANTARES-ORCA); event rate (DIS regime): $[2\sigma(\nu_l + N \rightarrow l^- + X) + \sigma(\bar{\nu}_l + N \rightarrow l^+ + X)]/3$

Neutrino Oscillations in Matter

When neutrinos propagate in matter, they interact with the background of electrons, protons and neutrons, which generates an effective potential in the neutrino Hamiltonian: $H = H_{vac} + V_{eff}$.

This modifies the neutrino mixing since the eigenstates and the eigenvalues of H_{vac} and of $H = H_{vac} + V_{eff}$ are different, leading to a different oscillation probability w.r.t to that in vacuum.

Typically the matter background is not CP and CPT symmetric, e.g., the Earth and the Sun contain only electrons, protons and neutrons, and the resulting oscillations violate CP and CPT symmetries.

$$P_{3\nu}(\nu_\mu \rightarrow \nu_e) \cong \sin^2 \theta_{23} \sin^2 2\theta_{13}^m \sin^2 \frac{\Delta M_{31}^2 L}{4E}$$

$\sin^2 2\theta_{13}^m$, ΔM_{31}^2 depend on the matter potential
 $V_{eff} = \sqrt{2} G_F N_e$,

For antineutrinos V_{eff} has the opposite sign:

$$V_{eff} = -\sqrt{2} G_F N_e.$$

$\Delta m_{31}^2 > 0$ (NO): $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$ matter enhanced,
 $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ - suppressed

$\Delta m_{31}^2 < 0$ (IO): $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ matter enhanced,
 $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$ - suppressed

$$\sin^2 2\theta_{13}^m = \frac{\tan^2 2\theta_{13}}{\left(1 - \frac{N_e}{N_e^{res}}\right)^2 + \tan^2 2\theta_{13}},$$

$$\cos 2\theta_{13}^m = \frac{1 - N_e/N_e^{res}}{\sqrt{\left(1 - \frac{N_e}{N_e^{res}}\right)^2 + \tan^2 2\theta_{13}}},$$

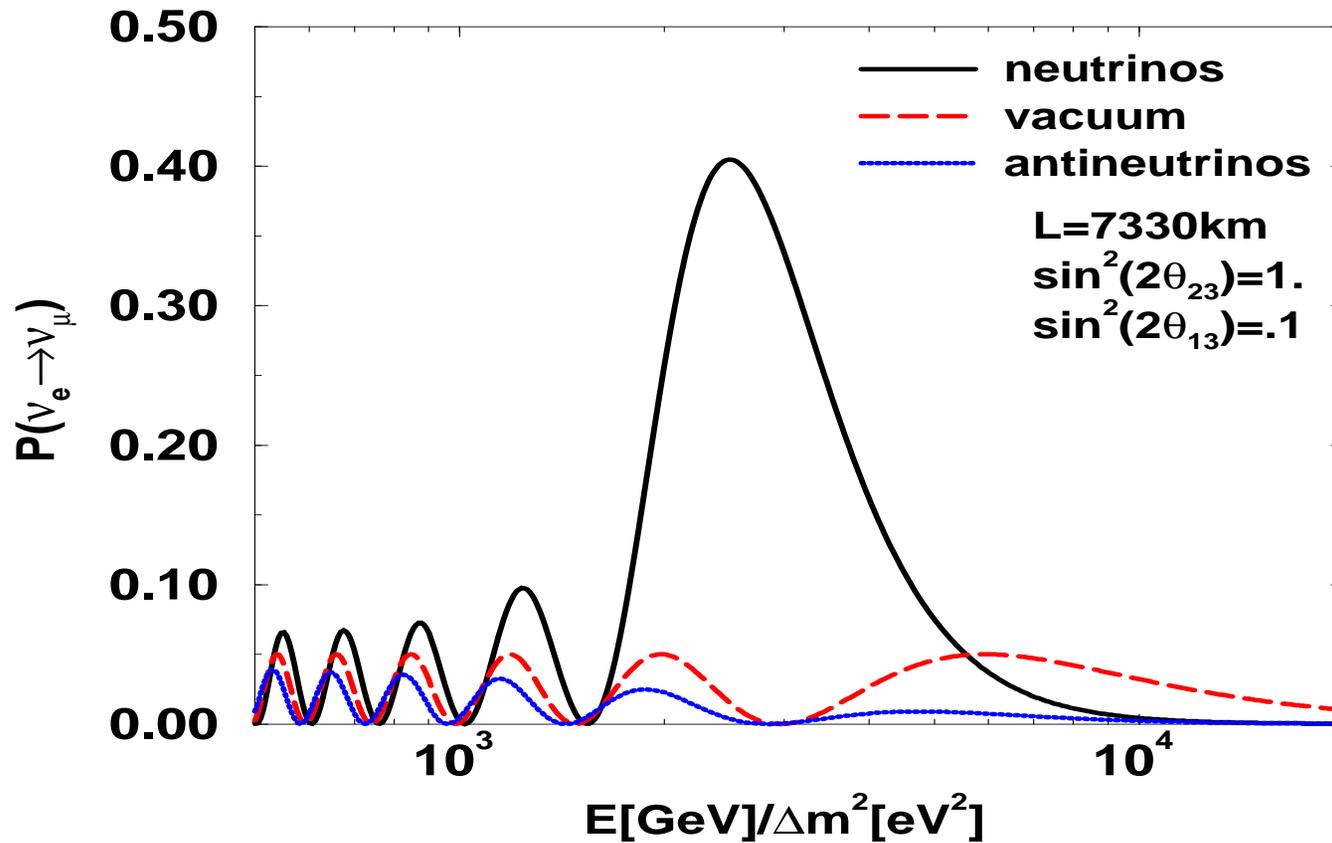
$$N_e^{res} = \frac{\Delta m_{31}^2 \cos 2\theta_{13}}{2E\sqrt{2}G_F} \quad |||$$

$$6.56 \times 10^6 \frac{\Delta m^2 [\text{eV}^2]}{E [\text{MeV}]} \cos 2\theta \text{ cm}^{-3} N_A,$$

$$\frac{\Delta M_{31}^2}{2E} \equiv \frac{\Delta m_{31}^2}{2E} \left(\left(1 - \frac{N_e}{N_e^{res}}\right)^2 \cos^2 2\theta_{13} + \sin^2 2\theta_{13} \right)^{\frac{1}{2}}$$

For $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$: $N_e \rightarrow (-N_e)$.

Earth matter effect in $\nu_\mu \rightarrow \nu_e, \bar{\nu}_\mu \rightarrow \bar{\nu}_e$ (MSW)



$$\Delta m^2 = 2.5 \times 10^{-3} \text{ eV}^2, E^{res} = 6.25 \text{ GeV}; P^{3\nu} = \sin^2 \theta_{23} P_m^{2\nu} = 0.5 P_m^{2\nu};$$

$$N_e^{res} \cong 2.3 \text{ cm}^{-3} N_A; L_m^{res} = L^v / \sin 2\theta_{13} \cong 6250 / 0.32 \text{ km}; 2\pi L / L_m \cong 0.75\pi (\neq \pi).$$

HyperKamiokande (10SK), IceCube-PINGU, ANTARES-ORCA;

Iron Magnetised detector: INO

INO: 50 or 100 kt (in India); ν_μ and $\bar{\nu}_\mu$ induced events detected (μ^+ and μ^-);
not designed to detect ν_e and $\bar{\nu}_e$ induced events.

IceCube at the South Pole: PINGU

PINGU: 50SK; ν_μ and $\bar{\nu}_\mu$ induced events detected (μ^+ and μ^- , no μ charge identification); Challenge: $E_\nu \gtrsim 2$ GeV (?)

ANTARES in Mediteranian sea: ORCA

Water-Cerenkov detector: Hyper Kamiokande (10SK)

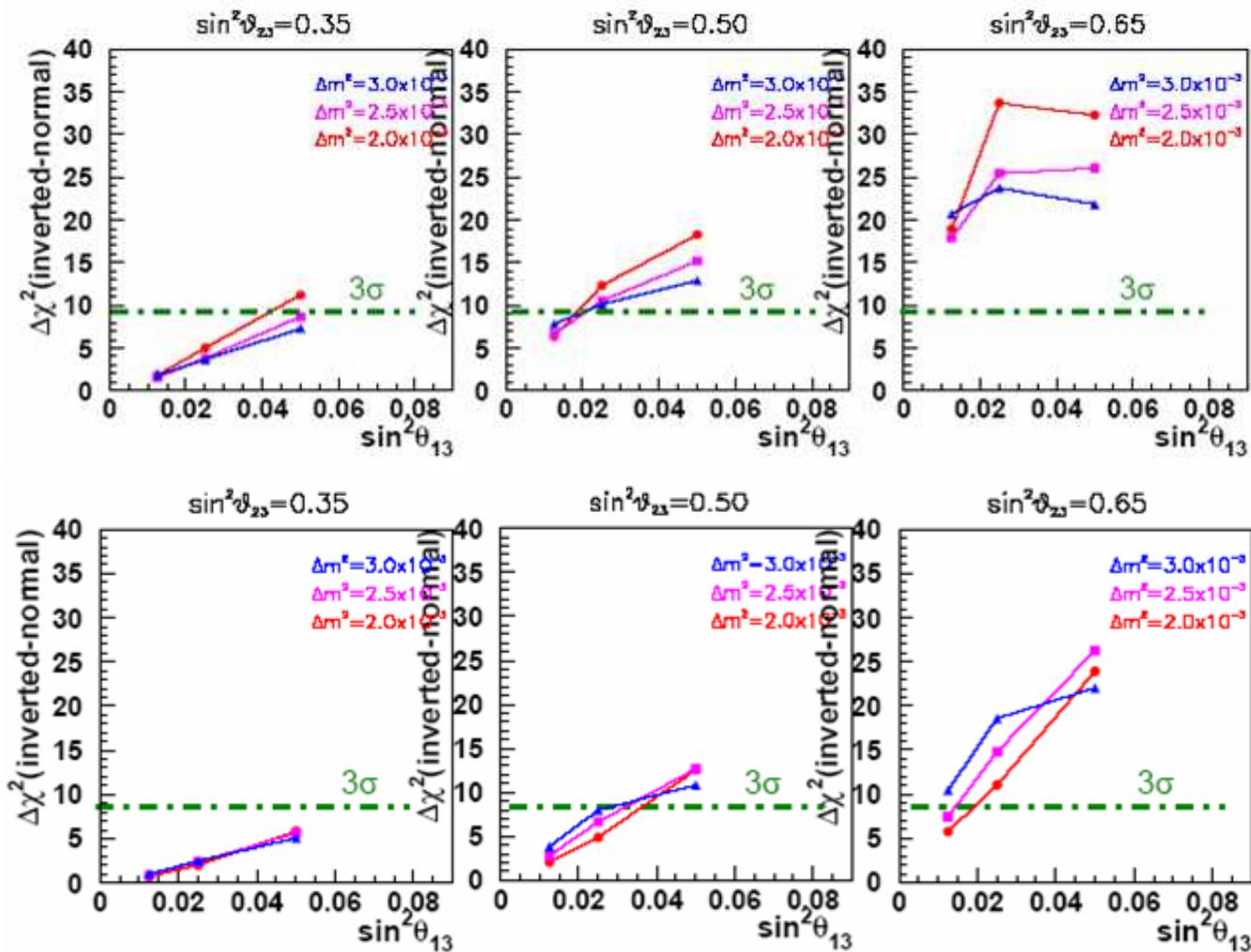
Sensitivity depends critically on θ_{23} , the “true” hierarchy.

J. Bernabeu, S. Palomares-Ruiz, S.T.P., 2003

$$P(\nu_\mu \rightarrow \nu_e) \cong \sin^2 \theta_{23} \sin^2 2\theta_{13}^m \sin^2 \frac{\Delta M_{31}^2 L}{4E}$$

No charge identification (SK, HK, PINGU, ORCA); event rate (DIS regime):

$$[2\sigma(\nu_l + N \rightarrow l^- + X) + \sigma(\bar{\nu}_l + N \rightarrow l^+ + X)]/3$$

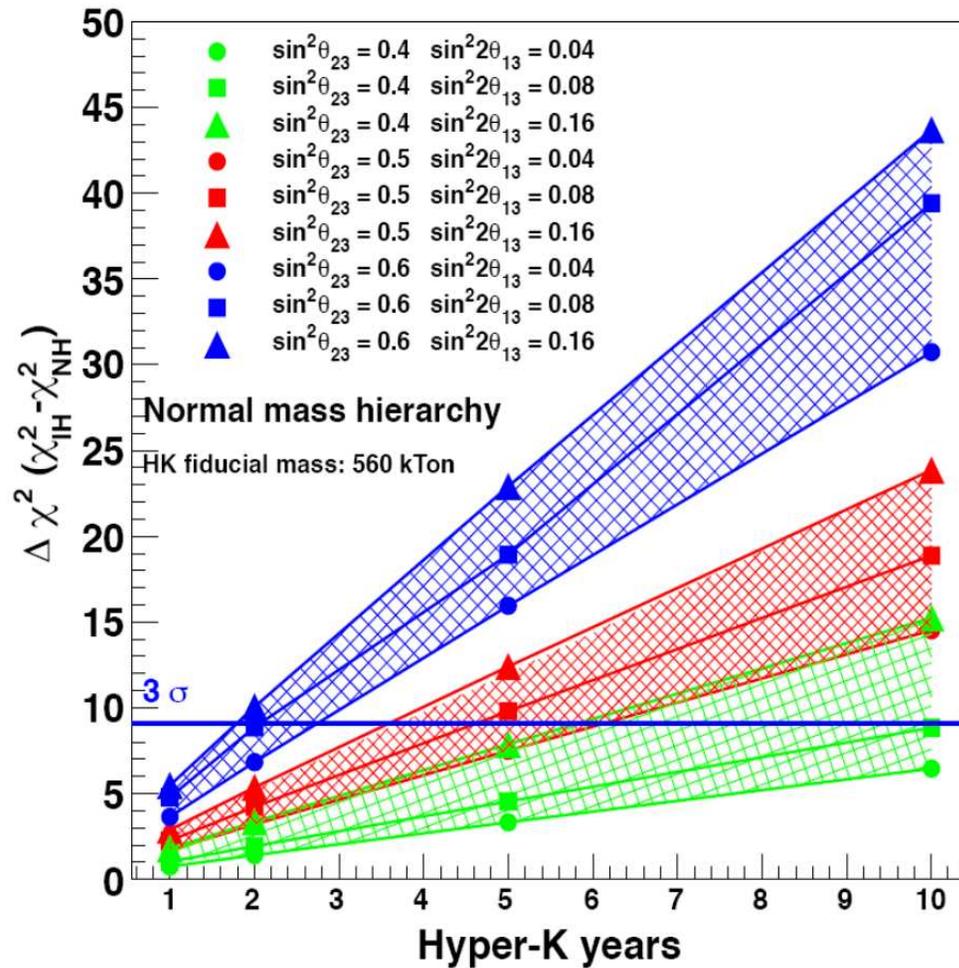


Water-Cerenkov detector, 1.8 MTy (HK = 10SK)

Critical dependence on θ_{23} , “true hierarchy”.

T. Kajita et al., 2004

J. Bernabeu, S. Palomares-Ruiz, S.T.P., 2003



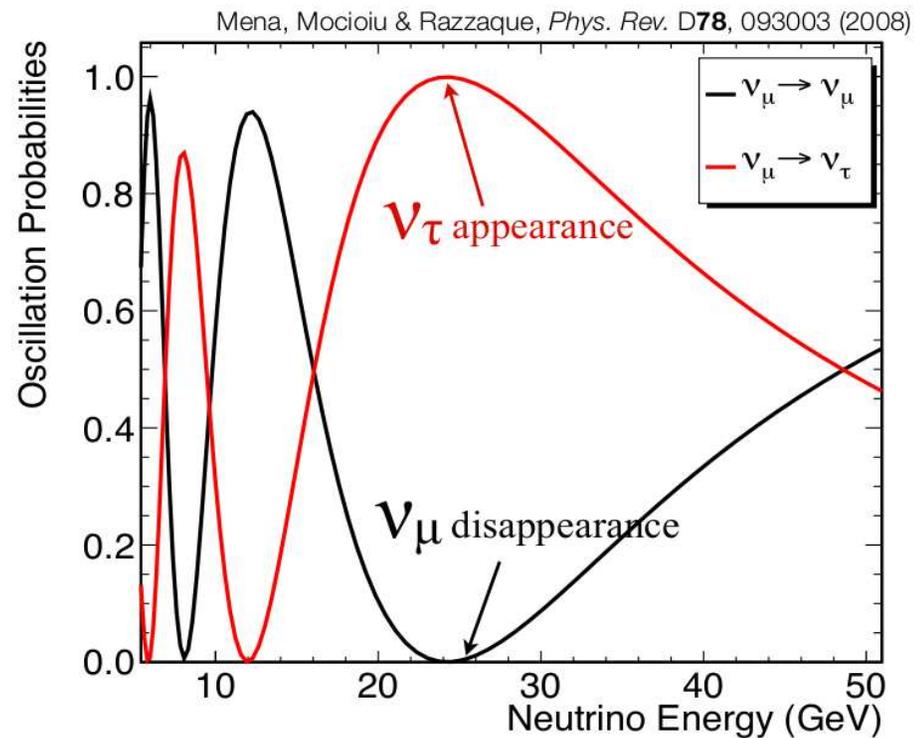
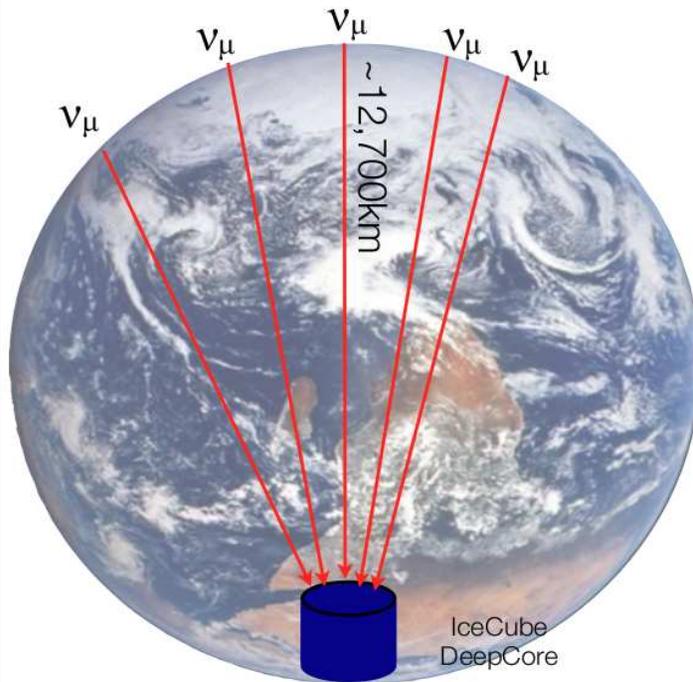
Sensitivity to the neutrino mass hierarchy from HK atmospheric neutrino data. θ_{23} and θ_{13} are assumed to be known as indicated in the figure.

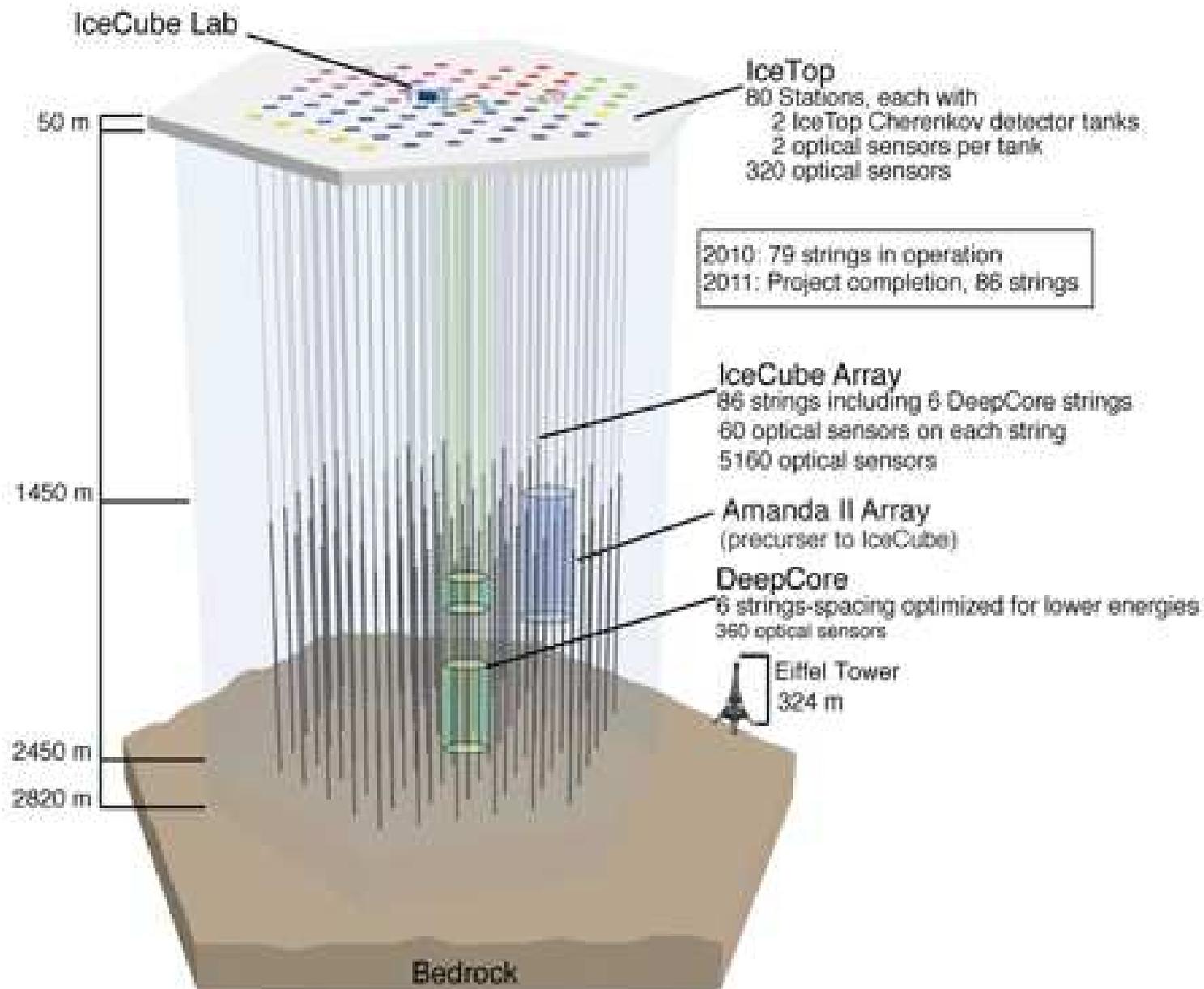
K. Abe et al. [Letter of intent: Hyper-Kamiokande Experiment], arXiv:1109.3262.

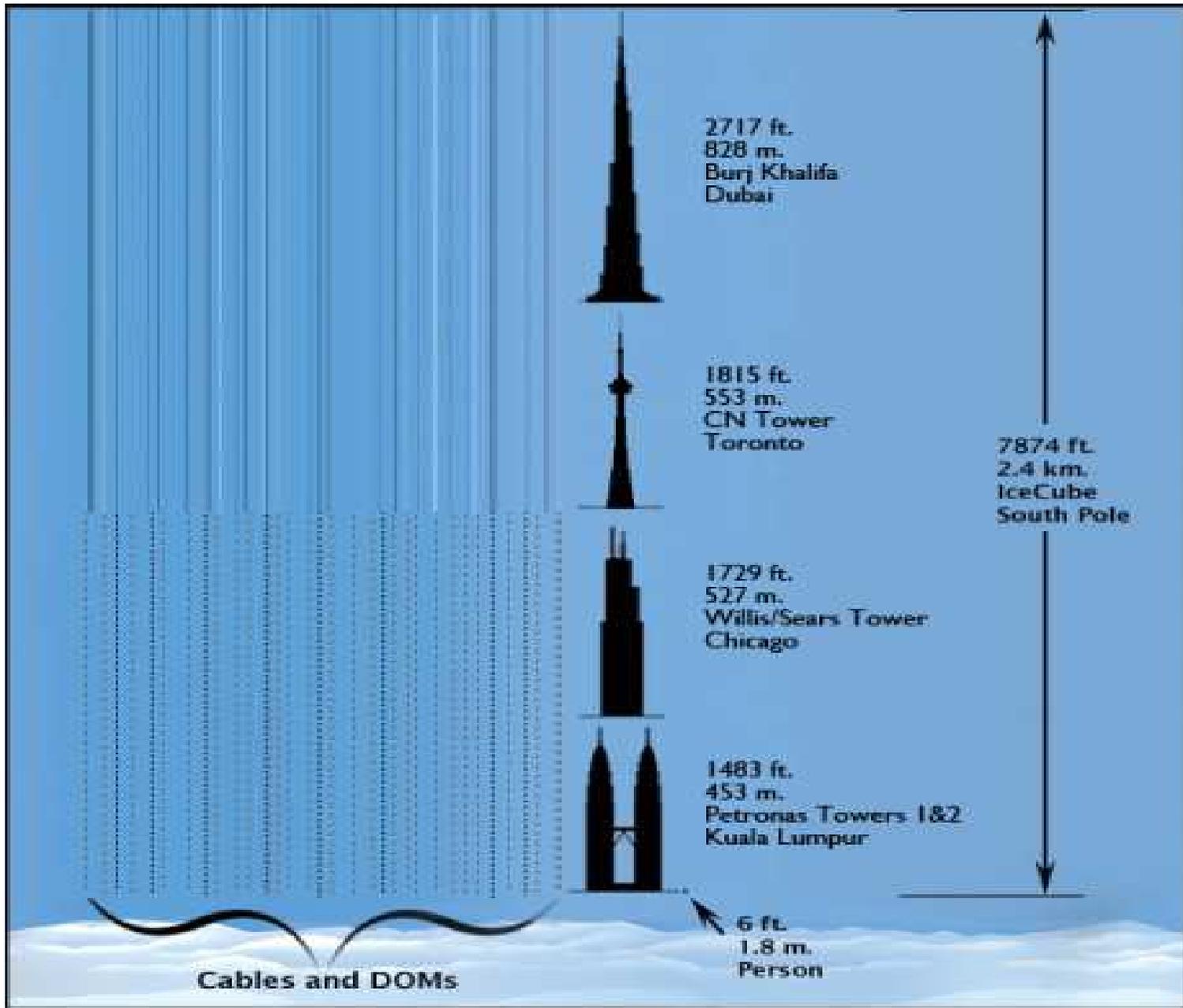
Neutrino Oscillation Source

- Oscillation
- IceCube-DeepCore Physics
- PINGU
- Beyond

- Northern Hemisphere ν_μ oscillating over one earth radii produces ν_μ (ν_τ) oscillation minimum(maximum) at ~ 25 GeV
 - Covers all possible terrestrial baselines
 - “Beam” is free and never turns off



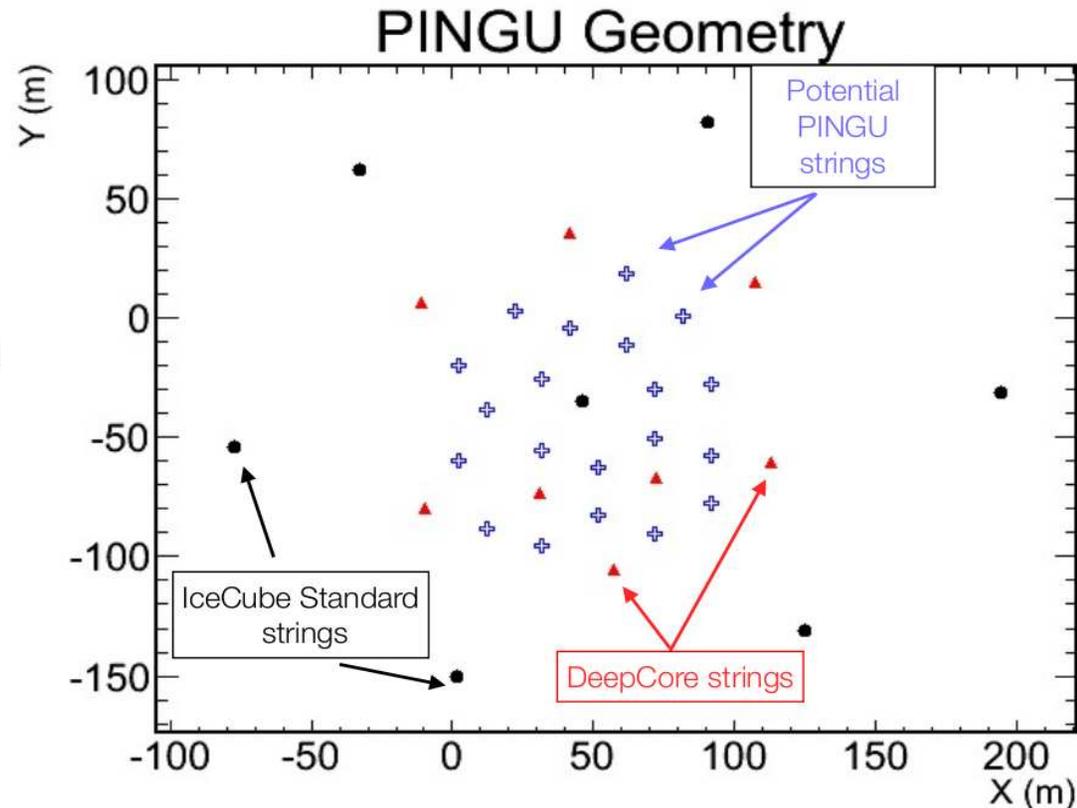




PINGU: Possible Geometry

- Oscillation
- IceCube-DeepCore Physics
- PINGU
- Beyond

- ~20 strings within DeepCore volume w/ short string-string spacing
 - IC-IC: 125m
 - DC-DC: ~80m
 - PINGU-PINGU: $\leq 26\text{m}$
- Shorter DOM-DOM spacing
 - IC-IC: 17m
 - DC-DC: 7m
 - PINGU-PINGU: $\leq 5\text{m}$
- R & D for future water/ice cerenkov

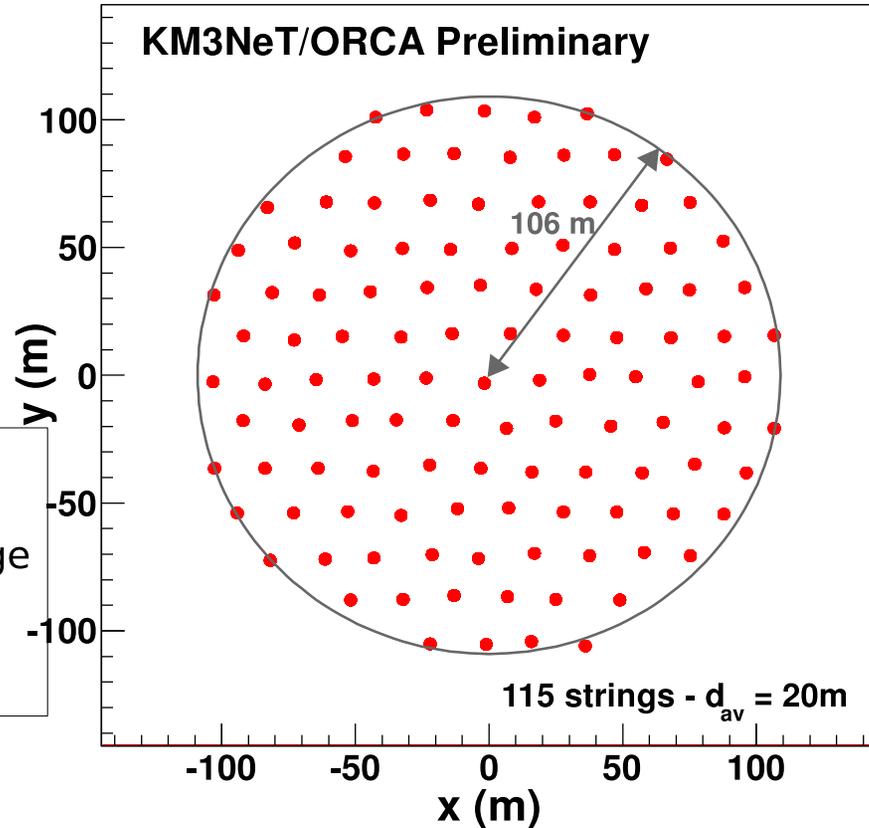


The ORCA proposed detector

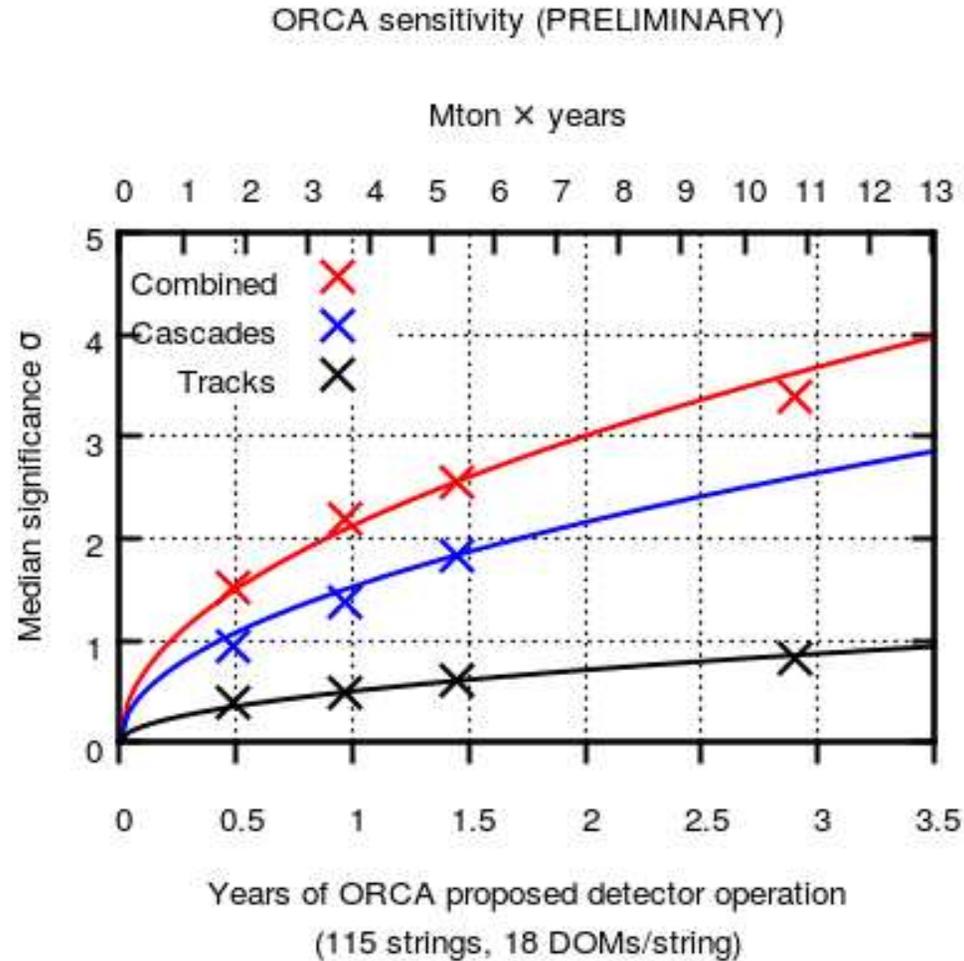


Multi-PMT DOM
 31 small PMTs
 Almost uniform coverage
 Photon counting
 Direction of photon
 All electronics inside

115 strings (building block) of 18 DOMs each
 Estimated cost 40M€



Sensitivity to the NMH



40 M€ investment

03/09/2014

Future LBL Neutrino Oscillation Experiments on $\text{sgn}(\Delta m_{31}^2)$ (the Hierarchy) and CP Violation

LBL Oscillation Experiments $\text{NO}\nu\text{A}$,
DUNE (LBNE+LBNO), T2HK (HK=HyperKamiokande
(10SK))

$\text{NO}\nu\text{A}$: Fermilab - site in Minnesota; off-axis ν beam,
 $E = 2$ GeV, $L \cong 810$ km, 14 kt liquid scintillator; 2014.

LBNE: Fermilab-DUSEL, $L = 1290$ km, 700 KW wide
band ν beam (first and second osc. maxima at $E = 2.4$
GeV and 0.8 GeV); 2 or 3 100 kt Water Cherenkov with
15% to 30% PMT coverage, or multiple 17 kt fiducial
volume LAr detectors; plans to run 5 years with ν_μ and 5
years with $\bar{\nu}_\mu$; 2025 (?)

LBNO: CERN-Pyhasalmi, $L = 2290$ km, wide band ν_μ
1.6 MW super beam (first and second osc. maxima at
 $E \cong 4$ GeV and 1.5 GeV); 440 kt Water Cherenkov, or
100 kt LAr, or 50 kt liquid scintillator detector; **aban-**
doned in 2014!

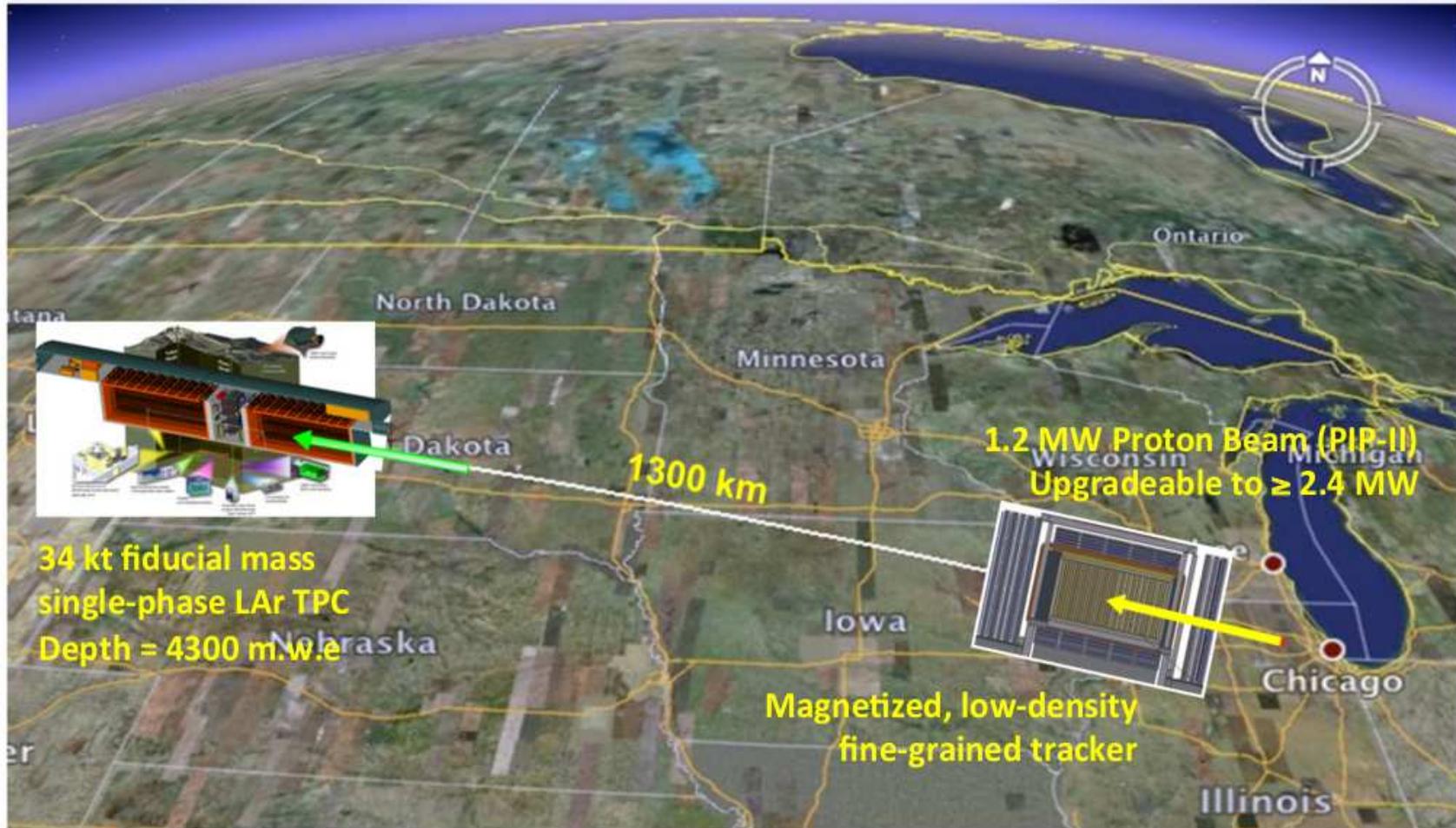
LBNE + LBNO \rightarrow ELBNF renamed recently to DUNE



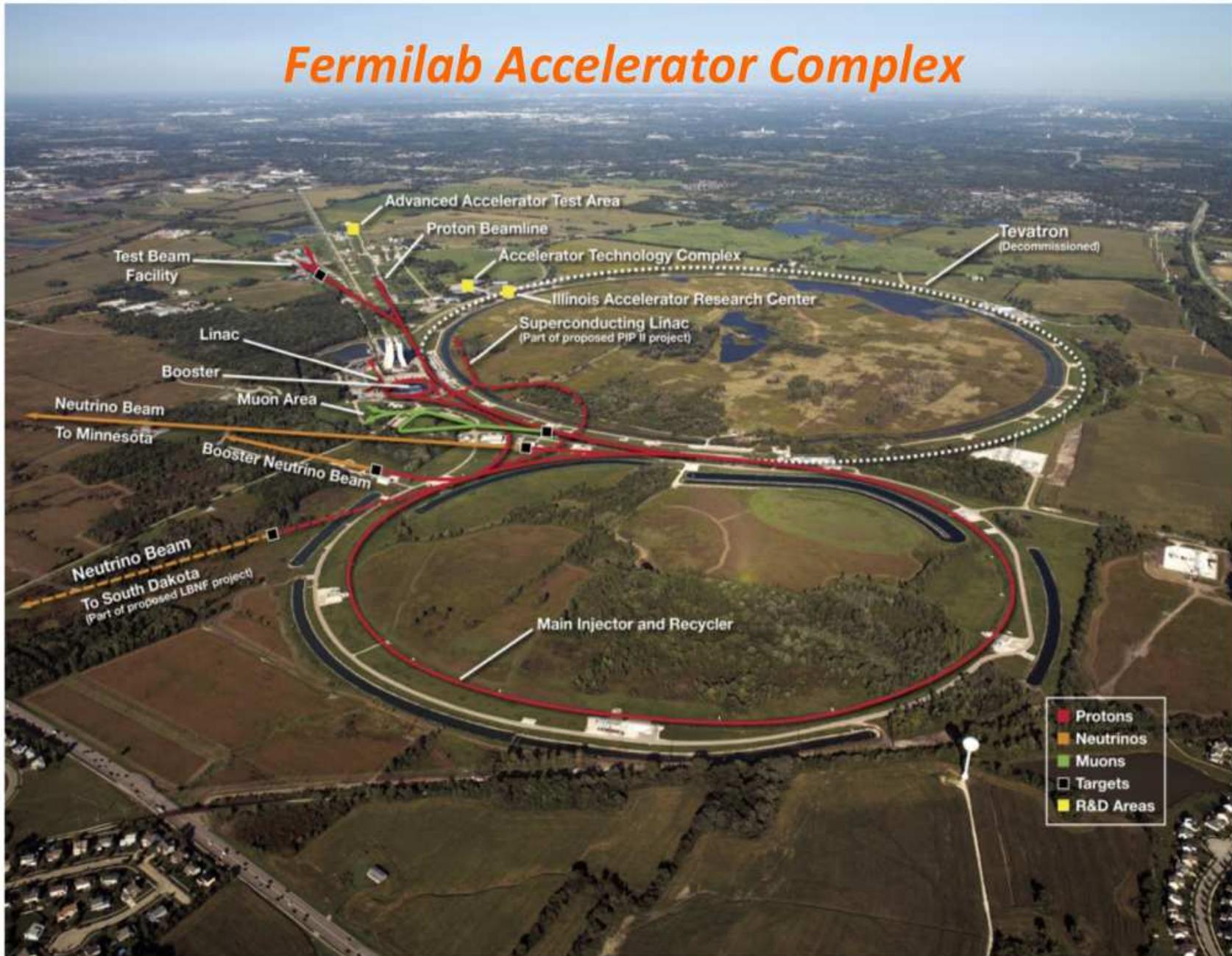
The slides on the LBNE (ELBNF \rightarrow DUNE) project are from the talk by B.C. Choudhary, given at the NuPhys (Prospects in Neutrino Physics) Workshop, December 15 - 17, 2014, London, UK.



LBNE Design



Fermilab Accelerator Complex





LBNE Parameters



- Wide band neutrino beam from FNAL
 - **protons: 60-120 GeV, 1.2 MW; upgradable to 2.3 MW**
 - 10 μ S pulses every 1.0 to 1.33 sec depending on P energy&power.
 - Neutrinos: sign selected, horn focused, 0.5 - 5 GeV
 - **1300 km** thru the Earth to Sanford Underground Research Facility.
- Liquid argon TPC parameters
 - **34 kt fiducial (50kt tot) at 4850 ft level. cosmics \sim 0.1Hz, beam \sim 9k CC/yr**
 - drift \sim 3.5 m, field: 500 V/cm, 2 mods = (14m(H)X 22m(W)X45m(L))
 - readout: x,u,v, pitch: 5 mm, wrapped wires, 2X108 APAs, 2X(275k ch)
 - Max Yield: \sim 9000 e/mm/MIP, 10000 ph/mm/MIP
- near detector parameters
 - distance \sim 450 m, \sim 3M events/ton/MW/yr
 - Magnetized Fine Grained Tracker (8 ton) with ECAL, and muon id.
 - Supplemented by a small LARTPC (few tons) or gas TPC.

Scale of project is dictated by physics. Beam and ND and FD detectors require high technology. Project can be done in phases with international partners.

Up to 2nd order in the two small parameters $|\alpha| \equiv |\Delta m_{21}^2|/|\Delta m_{31}^2| \ll 1$ and $\sin^2 \theta_{13} \ll 1$:

$$P_m^{3\nu \text{ man}}(\nu_\mu \rightarrow \nu_e) \cong P_0 + P_{\sin \delta} + P_{\cos \delta} + P_3,$$

$$P_0 = \sin^2 \theta_{23} \frac{\sin^2 2\theta_{13}}{(A-1)^2} \sin^2[(A-1)\Delta],$$

$$P_3 = \alpha^2 \cos^2 \theta_{23} \frac{\sin^2 2\theta_{12}}{A^2} \sin^2(A\Delta),$$

$$P_{\sin \delta} = -\alpha \frac{8 J_{CP}}{A(1-A)} (\sin \Delta)(\sin A\Delta) (\sin[(1-A)\Delta]),$$

$$P_{\cos \delta} = \alpha \frac{8 J_{CP} \cot \delta}{A(1-A)} (\cos \Delta)(\sin A\Delta) (\sin[(1-A)\Delta]),$$

$$\Delta = \frac{\Delta m_{31}^2 L}{4E}, \quad A = \sqrt{2} G_F N_e^{\text{man}} \frac{2E}{\Delta m_{31}^2}.$$

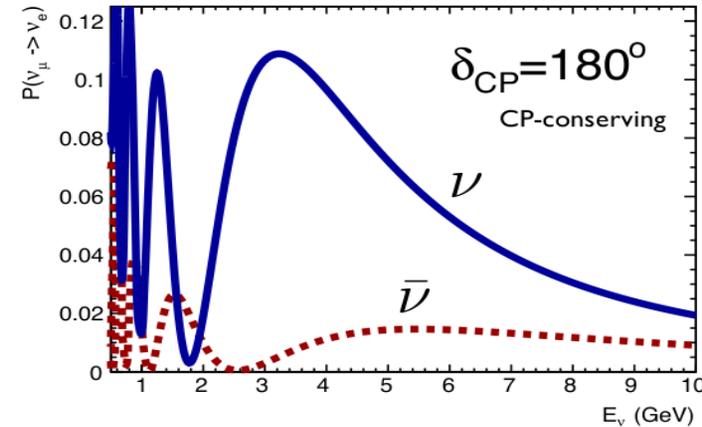
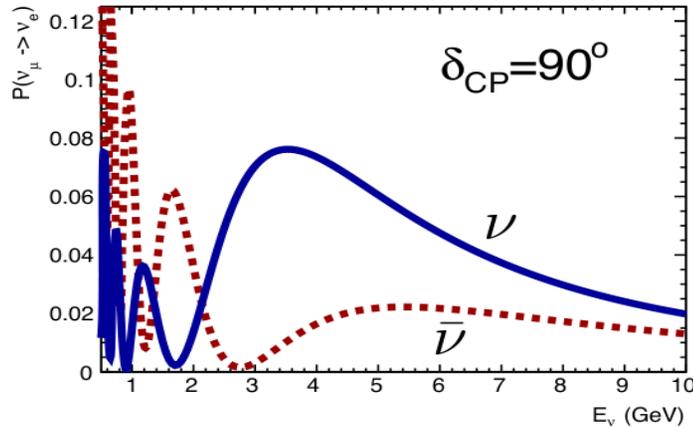
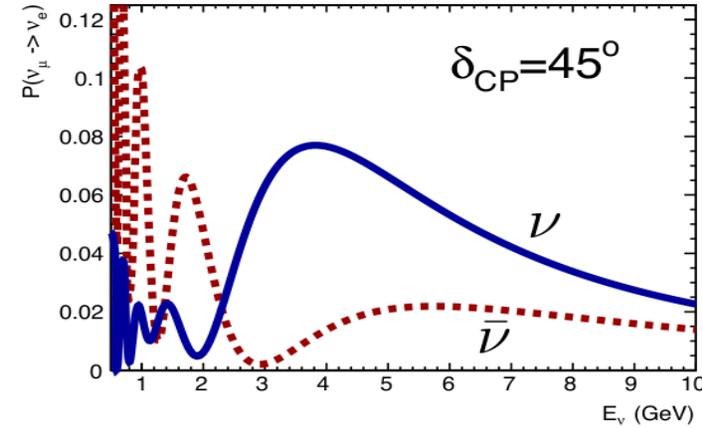
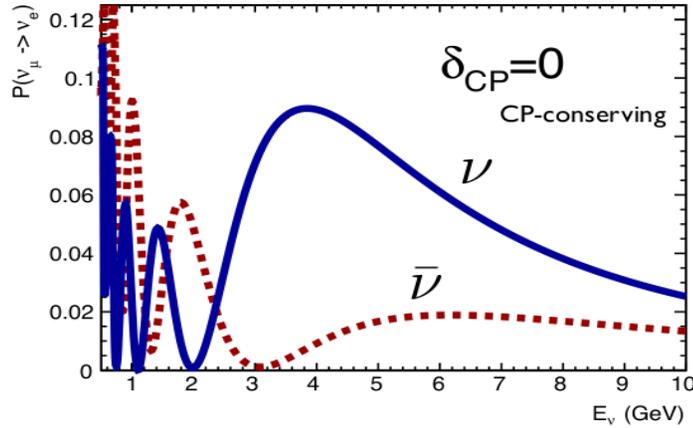
$$\bar{\nu}_\mu \rightarrow \bar{\nu}_e: \delta, \quad A \rightarrow (-\delta), \quad (-A)$$

CERN-Pyhäsalmi: CP-effect $\nu_\mu \rightarrow \nu_e$

★ Normal mass hierarchy

L=2300 km

$$\sin^2(2\theta_{13}) = 0.09$$



NPB2012

A. Rubbia – LAGUNA-LBNO

18

Wednesday, September 26, 12

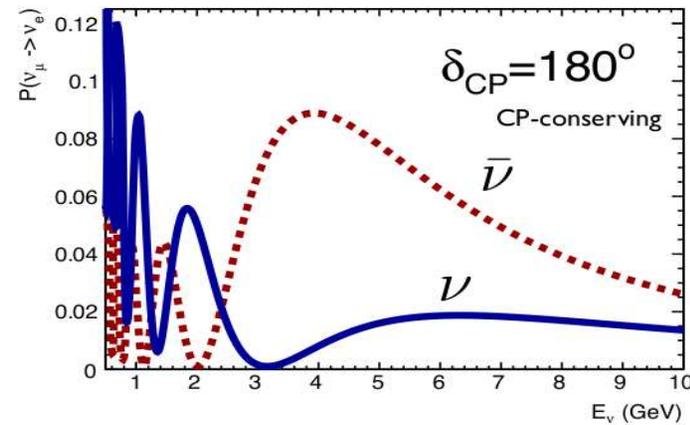
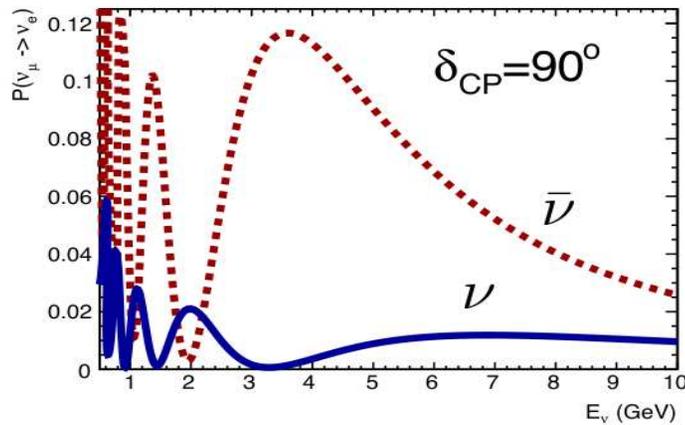
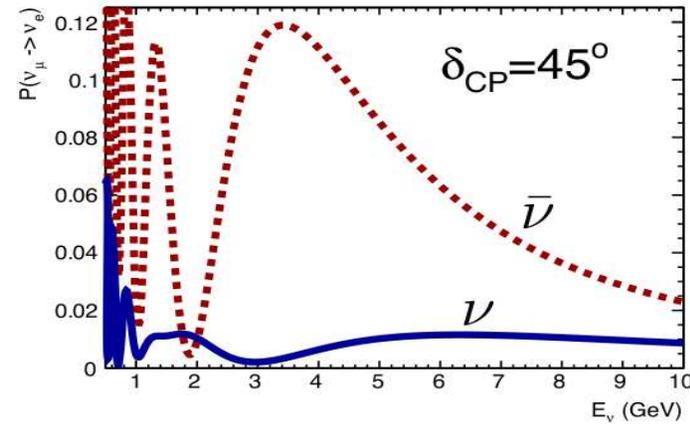
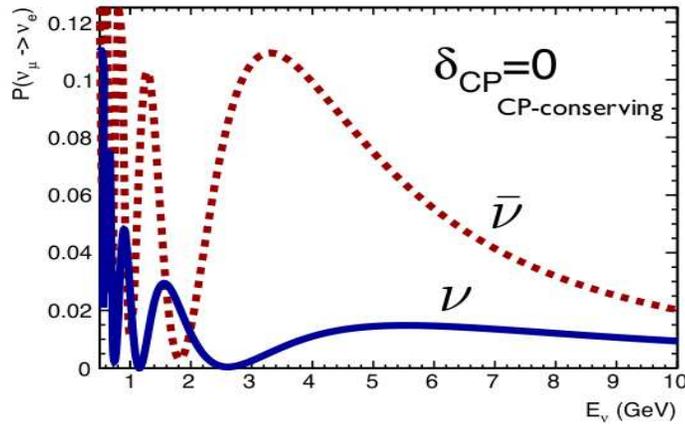
18

CERN-Pyhäsalmi: CP-effect $\nu_\mu \rightarrow \nu_e$

★ Inverted mass hierarchy

L=2300 km

$$\sin^2(2\theta_{13}) = 0.09$$



NPB2012

A. Rubbia – LAGUNA-LBNO

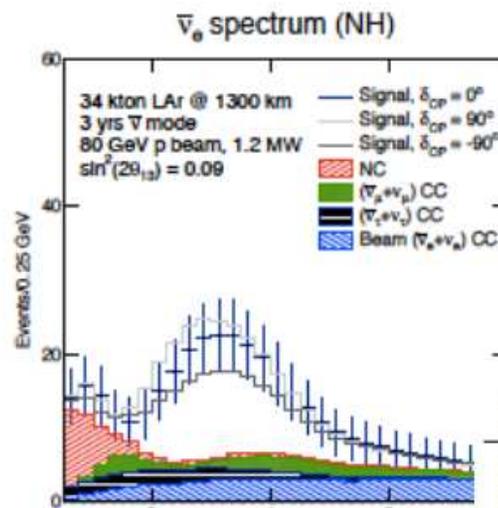
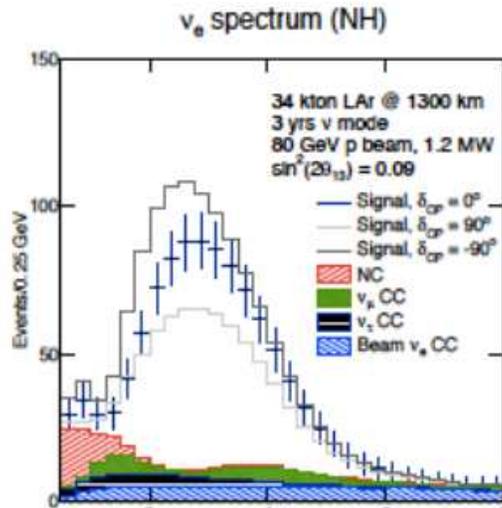
19

Wednesday, September 26, 12

19

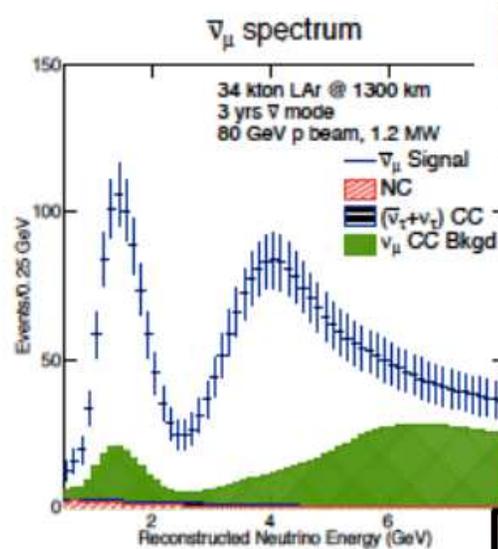
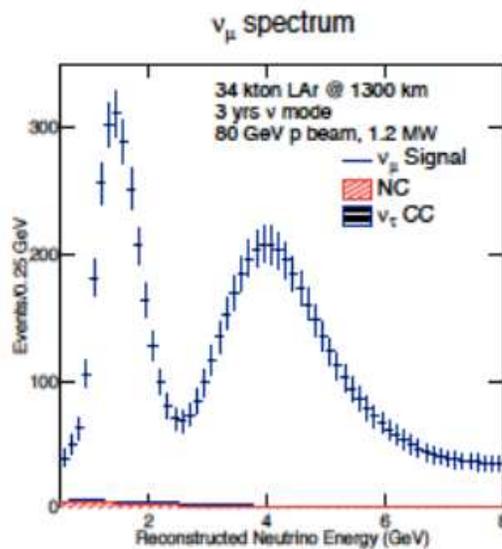


LBNE Event Rate



**34 kton Lar
1.2 MW beam
NH, $\delta_{CP} = 0$
Fit parameters
from 2012**

80 GeV Beam	ν mode	$\bar{\nu}$ mode
Signal: $\nu_e + \bar{\nu}_e$	777	189
BG: NC	67	39
BG: $\nu_\mu + \bar{\nu}_\mu$ CC	84	39
BG: Beam $\nu_e + \bar{\nu}_e$	147	81
BG: $\nu_\tau + \bar{\nu}_\tau$ CC	49	32

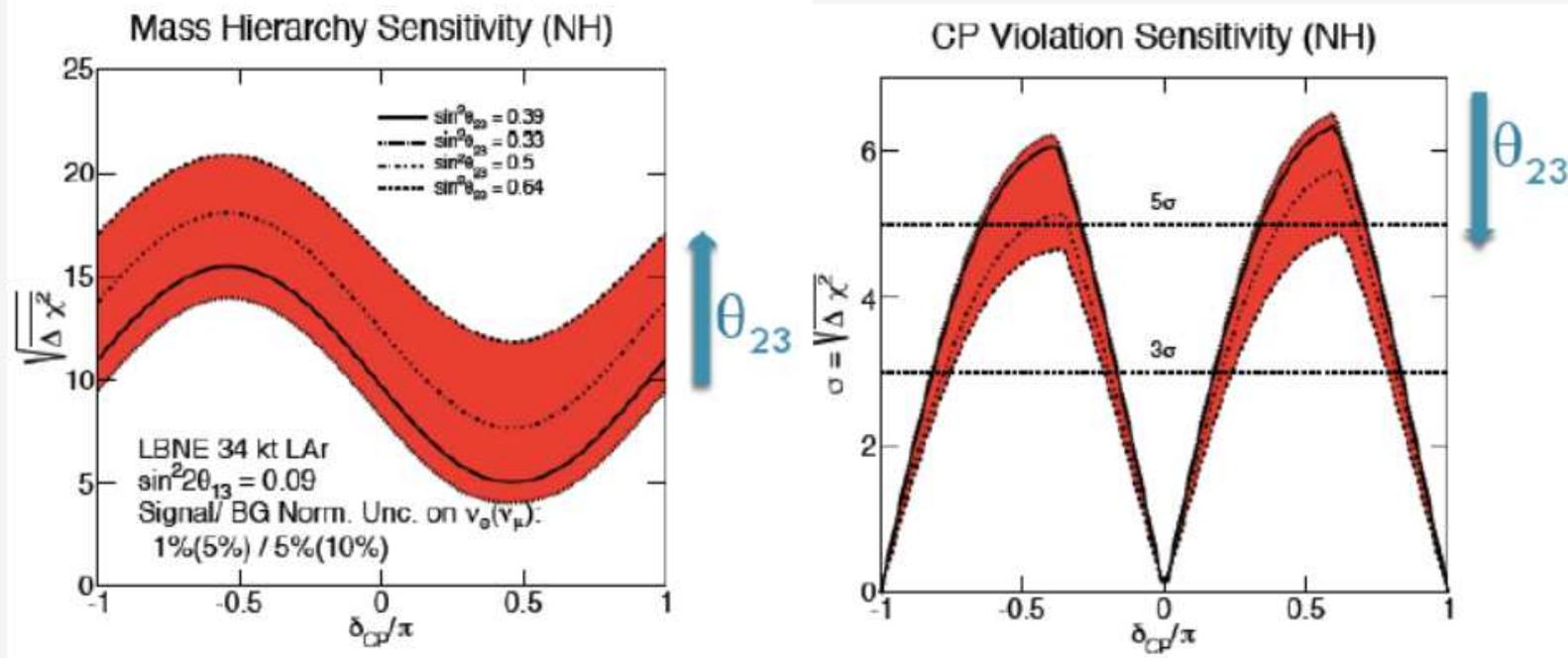


**Full oscillation structure
visible in energy spectrum. A
combined fit to the spectrum
to provide unambiguous
parameter sensitivity in one
experiment**

arXiv:1307.7335 – LBNE Document



LBNE Sensitivity to MH & CPV



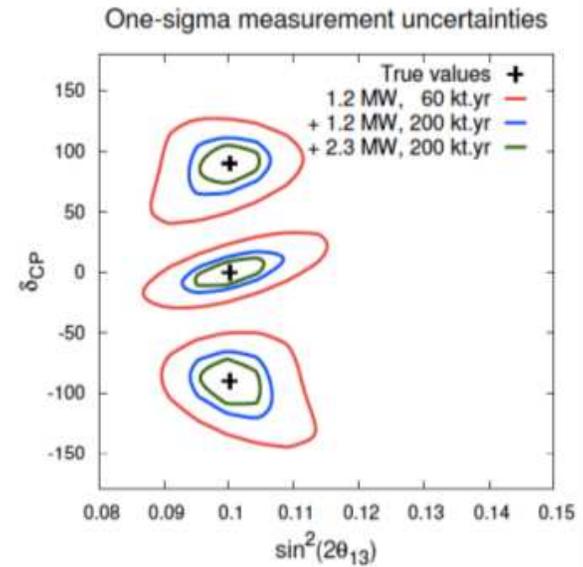
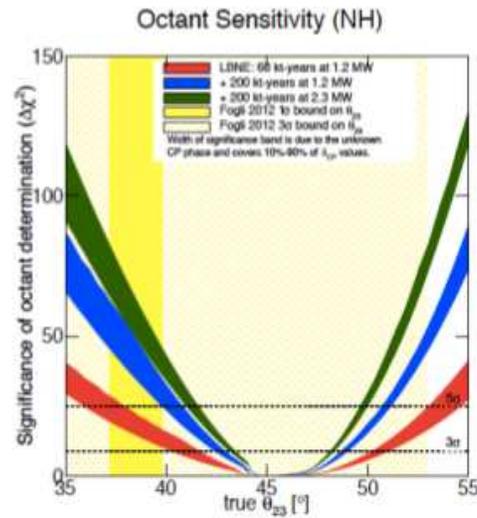
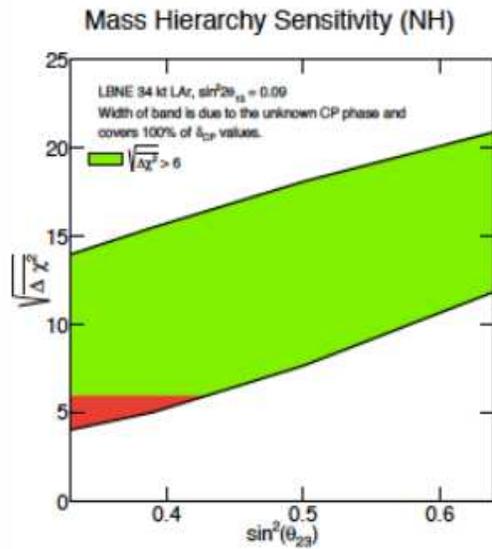
Width of the band indicates variation within the 2013 allowed range for θ_{23} .
Exposure $\sim 245\text{kTon} \sim 34\text{ kT} \times 1.2\text{ MW} \times (3\nu + 3\bar{\nu})$ years

Elizabeth Worchester – NOW 2014

arXiv:1307.7335 – LBNE Document



Other Measurements for a Comprehensive Program



34kton, 6 yrs, 1.2MW

LBNE, for example, could achieve the determination of the mass hierarchy at 3σ in less than a year.

LBNE could also have very good sensitivity to CP-violation with a 60% coverage at 3σ in the allowed range of values of $\sin^2 2\theta_{13}$, for a 200 kton Water Cherenkov or 34 kton LAr detectors (assuming it will run for 5 years in neutrinos and 5 years in antineutrinos).

The Nature of Massive Neutrinos II: Origins of Dirac and Majorana Massive Neutrinos

- Massive Dirac Neutrinos: $U(1)$, Conserved (Additive) Charge, e.g., L .
- Massive Majorana Neutrinos: No Conserved (Additive) Charge(s).

The type of massive neutrinos in a given theory is determined by the type of (effective) mass term $\mathcal{L}_m^\nu(x)$ neutrinos have, more precisely, by the symmetries $\mathcal{L}_m^\nu(x)$ and the total Lagrangian $\mathcal{L}(x)$ of the theory have.

Mass Term: any bi-linear in fermion (neutrino) fields invariant under the proper Lorentz transformations.

The type of massive neutrinos in a given theory is determined by the type of (effective) mass term $\mathcal{L}_m^\nu(x)$ neutrinos have, **more precisely**, by the symmetries $\mathcal{L}_m^\nu(x)$ and the total Lagrangian $\mathcal{L}(x)$ of the theory have.

- Dirac Neutrinos: Dirac Mass Term, requires $\nu_R(x)$ - $SU(2)_L$ singlet RH ν fields

$$\mathcal{L}_D^\nu(x) = - \overline{\nu_{lR}}(x) M_{Dl'l} \nu_{lL}(x) + h.c. , \quad M_D - \text{complex}$$

- $\mathcal{L}_D^\nu(x)$ conserves L : $L = \text{const.}$

$$M_D = V M_D^{\text{diag}} W^\dagger , \quad V, U - \text{unitary (bi - unitary transformation)} , \quad W \equiv U_{\text{PMNS}}$$

- ST + 3 $\nu_R(x)$ - RH ν fields: $n = 3$

$$\begin{aligned} \mathcal{L}_Y(x) &= Y_{l'l}^\nu \overline{\nu_{lR}}(x) \Phi^T(x) (i\tau_2) \psi_{lL}(x) + h.c. , \\ M_D &= \frac{v}{\sqrt{2}} Y^\nu , \quad v = 246 \text{ GeV} . \end{aligned}$$

No explanation why $m(\nu_j) \lll m_l, m_q$.

No DM candidate.

No mechanism for generation of the observed BAU.

The LFV processes $\mu^+ \rightarrow e^+ + \gamma$ decay, $\mu^- \rightarrow e^- + e^+ + e^-$ decay, $\tau^- \rightarrow e^- + \gamma$ decay, etc. are allowed.

However, they are predicted to proceed with unobservable rates:

$$BR(\mu \rightarrow e + \gamma) = \frac{3\alpha}{32\pi} \left| U_{ej} U_{\mu j}^* \frac{m_j^2}{M_W^2} \right|^2 \cong (2.5 - 3.9) \times 10^{-55},$$
$$M_W \cong 80 \text{ GeV, the } W^\pm \text{ - mass}$$

S.T.P., 1976

“New Physics”: $\nu_l \rightarrow \nu_{l'}, \bar{\nu}_l \rightarrow \bar{\nu}_{l'}, l, l' = e, \mu, \tau$ oscillations.

- Majorana ν_j : Majorana Mass Term of $\nu_{lL}(x)$, $l = e, \mu, \tau$

$$\mathcal{L}_M^\nu(x) = \frac{1}{2} \nu_{lL}^\top(x) C^{-1} M_{ll} \nu_{lL}(x) + h.c. , \quad C^{-1} \gamma_\alpha C = -\gamma_\alpha^\top$$

- If $M_{ll} \neq 0$, $L_l \neq \text{const.}$, $L \neq \text{const.}$, $n = 3$

- $\nu_{lL}(x)$ -fermions: $M = M^\top$, complex.

$$M^{diag} = U^\top M U , \quad U - \text{unitary (congruent transformation)}; \quad U \equiv U_{\text{PMNS}}$$

$$\nu_j \equiv \chi_j(x) = U_{jl}^\dagger \nu_{lL}(x) + U_{jl}^* \nu_{lR}^c = C (\overline{\chi_j}(x))^\top , \quad m_j \neq 0 , \quad j = 1, 2, 3$$

CP-invariance: $M^* = M$, M - real, symmetric.

$$M^{diag} = (m'_1, m'_2, m'_3): \quad m'_j = \rho_j m_j, \quad m_j \geq 0, \quad \rho_j = \pm 1$$

$$\chi_j: \quad m_j \geq 0: \quad \eta_{CP}(\chi_j) = i\rho_j$$

$\mathcal{L}_M^\nu(x)$ not possible in the ST: requires New Physics Beyond the ST

$(\beta\beta)_{0\nu}$ -decay is allowed; typically also $BR(\mu \rightarrow e + \gamma)$, $BR(\mu \rightarrow 3e)$, $CR(\mu^- + \mathcal{N} \rightarrow e^- + \mathcal{N})$ can be “large”, i.e., in the range of sensitivity of ongoing (MEG) and future planned experiments.

- Majorana ν_j : Dirac+Majorana Mass Term; requires both $\nu_{lL}(x)$ and $\nu_{lR}(x)$:

$$\mathcal{L}_{D+M}^\nu(x) = -\overline{\nu_{lR}}(x) M_{Dl} \nu_{lL}(x) + \frac{1}{2} \nu_{lL}^\top(x) C^{-1} M_{ll}^{LL} \nu_{lL}(x) + \frac{1}{2} \nu_{lR}^\top(x) C^{-1} (M^{RR})_{ll}^\dagger \nu_{lR}(x)$$

$$M = \begin{pmatrix} M^{LL} & M_D \\ M_D^T & M^{RR} \end{pmatrix} = M^T \quad \left((M^{LL})^T = M^{LL}, (M^{RR})^T = M^{RR} \right)$$

- If $M_{Dl} \neq 0$ and $M_{ll}^{LL} \neq 0$ and/or $M_{ll}^{RR} \neq 0$: $L_l \neq const.$, $L \neq const.$; $n = 6 (> 3)$
- $M = M^T$, complex.

$$M^{diag} = W^T M W, \quad W - \text{unitary}, \quad 6 \times 6; \quad W^T \equiv (U^T \quad V^T); \quad U \equiv U_{PMNS} : 3 \times 6.$$

$$\nu_{lL}(x) = \sum_{j=1}^6 U_{lj} \chi_j(x), \quad \chi_j(x) - \text{Majorana}, \quad m_j \neq 0, \quad l = e, \mu, \tau;$$

$$\nu_{lL}^C(x) \equiv C (\overline{\nu_{lR}}(x))^T = \sum_{j=1}^6 V_{lj} \chi_j(x), \quad \nu_{lL}^C(x) : \text{sterile antineutrino}$$

$\mathcal{L}_{D+M}^\nu(x)$ possible in the ST + ν_{lR} : $M^{LL} = 0$

$(\beta\beta)_{0\nu}$ -decay is allowed;

phenomenology depends on the relative magnitude of M_D and M^{RR} .

Dirac - Majorana Relation (if any...)

Majorana Mass Term of $\nu_{lL}(x)$, $l = e, \mu, \tau$, can lead to Dirac neutrinos with definite mass if it conserves some lepton charge:

$$\mathcal{L}_M^\nu(x) = -\frac{1}{2} \overline{\nu_{lR}^c}(x) M_{ll} \nu_{lL}(x) + h.c. , \nu_{lR}^c \equiv C (\overline{\nu_{lL}(x)})^\top$$

$\mathcal{L}_M^\nu(x)$ conserves, e.g. $L' = L_e - L_\mu - L_\tau$ if only $M_{e\mu} = M_{\mu e}, M_{e\tau} = M_{\tau e} \neq 0$

S.T.P., 1982

- Dirac ν , Ψ , is equivalent to two Majorana ν 's, $\chi_{1,2}$, having the same (positive) mass, opposite CP-parities, and which are “maximally mixed”:

$$\Psi(x) = \frac{\chi_1 + \chi_2}{\sqrt{2}}, \quad m_1 = m_2 = m_D > 0, \eta_{jCP} = i\rho_j, \rho_1 = -\rho_2 \quad (C (\overline{\chi_j})^\top = \rho_j \chi_j)$$

$$\text{Example ZKM } \nu : \nu_{eL}(x) = \Psi_L = \frac{\chi_{1L} + \chi_{2L}}{\sqrt{2}}, \quad \nu_{\mu L}(x) = \Psi_L^C = \frac{\chi_{1L} - \chi_{2L}}{\sqrt{2}}$$

- Pseudo-Dirac Neutrino: the symmetry of $\mathcal{L}_M^\nu(x)$ is not a symmetry of $\mathcal{L}_{tot}(x)$

Suppose: $\nu_{eL}(x) = \Psi_L = (\chi_{1L} + \chi_{2L})/\sqrt{2}$, and to “leading order” $m_1 = m_2$, but due to “higher order” corrections $m_1 \neq m_2$, $|m_2 - m_1| \equiv |\Delta m| \ll m_{1,2}$

All Majorana effects $\sim \Delta m$

- Suppose: $m_1 = m_2$, $\rho_1 = -\rho_2$, but $\chi_{1,2}$ are not maximally mixed:

$$\nu_{eL}(x) = \chi_{1L} \cos \phi + \chi_{2L} \sin \phi = \Psi_L \cos \phi' + \Psi_L^C \sin \phi'$$

All Majorana effects are $\sim m_D \cos \phi' \sin \phi'$

In the case of conserved $L' = L_e - L_\mu - L_\tau$:

$$M = \begin{pmatrix} 0 & M_{e\mu} & M_{e\tau} \\ M_{e\mu} & 0 & 0 \\ M_{e\tau} & 0 & 0 \end{pmatrix}$$

$$\theta_{12} = \pi/4, \theta_{13} = 0, \tan \theta_{23} = M_{e\tau}/M_{e\mu},$$

$m_3 = 0$ - spectrum with IH, $m_1 = m_2$, $\chi_{1,2}$ - equivalent to one Dirac ν , Ψ .

Adding L' -breaking term, e.g. M_{ee} , $|M_{ee}|/\sqrt{M_{e\mu}^2 + M_{e\tau}^2} \sim 0.01$, leads to $m_1 \neq m_2$ compatible with Δm_{\odot}^2 .

LECTURE III

Origin of Neutrino Masses and Mixing

The Seesaw Mechanisms of Neutrino Mass Generation

- Explain the smallness of ν -masses.
- Through **leptogenesis theory** link the ν -mass generation to the generation of baryon asymmetry of the Universe.

S. Fukugita, T. Yanagida, 1986.

Three Types of Seesaw Mechanisms

Require the existence of new degrees of freedom (particles) beyond those present in the SM

Type I seesaw mechanism: ν_{lR} - RH ν s' (heavy).

Type II seesaw mechanism: $\mathbf{H}(\mathbf{x})$ - a triplet of H^0, H^-, H^{--} Higgs fields (HTM).

Type III seesaw mechanism: $\mathbf{T}(\mathbf{x})$ - a triplet of fermion fields.

The scale of New Physics determined by the masses of the New Particles.

Massive neutrinos ν_j - Majorana particles.

All three types of seesaw mechanisms have TeV scale versions, predicting rich low-energy phenomenology ($(\beta\beta)_{0\nu}$ -decay, LFV processes, etc.) and New Physics at LHC.

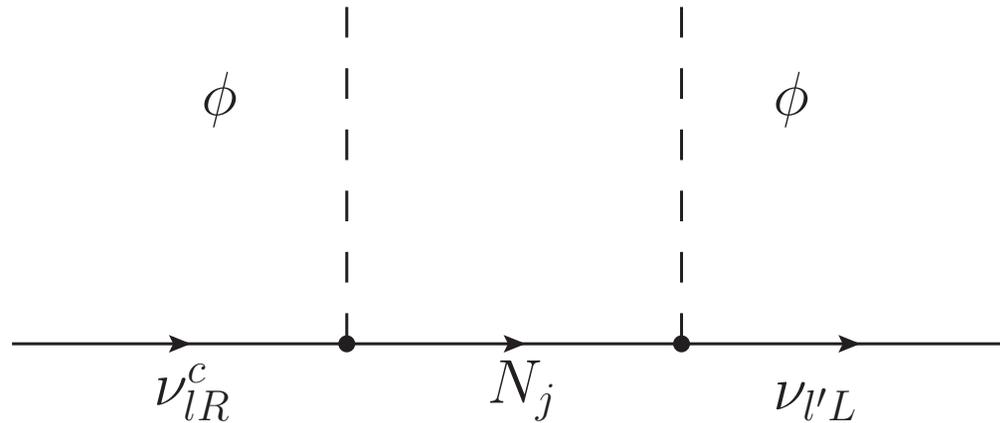
Type I Seesaw Mechanism

- Requires both $\nu_{lL}(x)$ and $\nu_{l'R}(x)$.
- Dirac+Majorana Mass Term: $M^{LL} = 0$, $|M_D = vY^\nu/\sqrt{2}| \ll |M^{RR}|$.
- Diagonalising M^{RR} : N_j - heavy Majorana neutrinos, $M_j \sim \text{TeV}$; or $(10^9 - 10^{13})$ GeV in GUTs.

For sufficiently large M_j , Majorana mass term for $\nu_{lL}(x)$:

$$M_\nu \cong v_u^2 (Y^\nu)^T M_j^{-1} Y^\nu = U_{\text{PMNS}}^* m_\nu^{\text{diag}} U_{\text{PMNS}}^\dagger .$$

$v_u Y^\nu = M_D$, $M_D \sim 1$ GeV, $M_j = 10^{10}$ GeV: $M_\nu \sim 0.1$ eV.



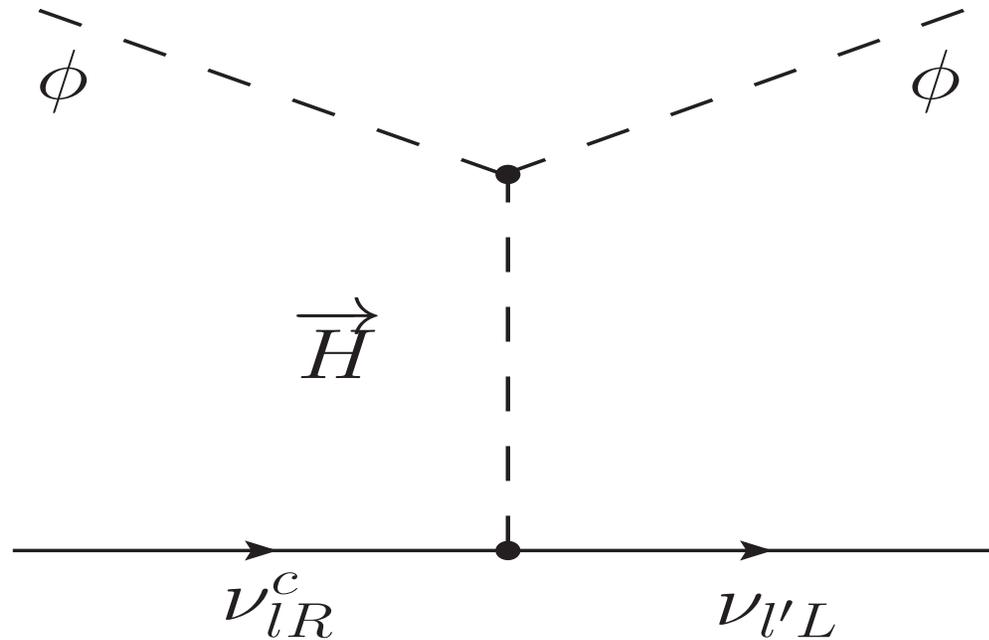
- $\nu_{lR}(x)$: Majorana mass term at “high scale” ($\sim \text{TeV}$; or $(10^9 - 10^{13}) \text{ GeV}$ in $SO(10)$ GUT)

$$\mathcal{L}_M^\nu(x) = + \frac{1}{2} \nu_{lR}^\top(x) C^{-1} (M^{RR})_{ll}^\dagger \nu_{lR}(x) + h.c. = - \frac{1}{2} \sum_j \bar{N}_j M_j N_j ,$$

- Yukawa type coupling of $\nu_{lL}(x)$ and $\nu_{lR}(x)$ involving $\Phi(x)$:

$$\begin{aligned} \mathcal{L}_Y(x) &= \bar{Y}_{ll}^\nu \overline{\nu_{lR}}(x) \Phi^T(x) (i\tau_2) \psi_{lL}(x) + h.c. , \\ &= Y_{jl}^\nu \overline{N_{jR}}(x) \Phi^T(x) (i\tau_2) \psi_{lL}(x) + h.c. , \\ M_D &= \frac{v}{\sqrt{2}} Y^\nu , \quad v = 246 \text{ GeV} . \end{aligned}$$

Type II Seesaw Mechanism

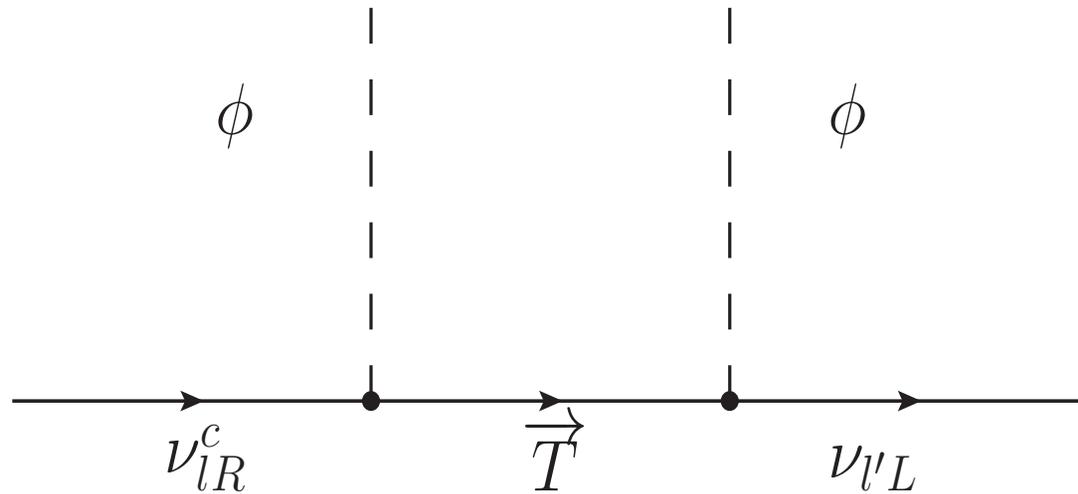


Due to I. Girardi

$$M_\nu \cong h v^2 M_H^{-1} = U_{\text{PMNS}}^* m_\nu^{\text{diag}} U_{\text{PMNS}}^\dagger .$$

$$h \sim 10^{-2}, v = 246 \text{ GeV}, M_H \sim 10^{12} \text{ GeV}: M_\nu \sim 0.6 \text{ eV}.$$

Type III Seesaw Mechanism



$$M_\nu \cong v^2 (Y_T)^T M_T^{-1} Y_T = U_{\text{PMNS}}^* m_\nu^{\text{diag}} U_{\text{PMNS}}^\dagger .$$

$$v Y_T \sim 1 \text{ GeV}, M_T \sim 10^{10} \text{ GeV}: M_\nu \sim 0.1 \text{ eV}.$$

TeV Scale Type I See-Saw Mechanism

Type I see-saw mechanism, heavy Majorana neutrinos N_j at the TeV scale:

$$m_\nu \simeq -M_D \hat{M}_N^{-1} M_D^T, \quad \hat{M} = \text{diag}(M_1, M_2, M_3), \quad M_j \sim (100 - 1000) \text{ GeV}.$$

$$\mathcal{L}_{CC}^N = -\frac{g}{2\sqrt{2}} \bar{\ell} \gamma_\alpha (RV)_{\ell k} (1 - \gamma_5) N_k W^\alpha + \text{h.c.}, \quad (RV)_{\ell k} \equiv U_{\ell 3+k},$$

$$\mathcal{L}_{NC}^N = -\frac{g}{2c_w} \bar{\nu}_{\ell L} \gamma_\alpha (RV)_{\ell k} N_{kL} Z^\alpha + \text{h.c.}$$

- All low-energy constraints can be satisfied in a scheme with two heavy Majorana neutrinos $N_{1,2}$, which form a pseudo-Dirac pair:

$$M_2 = M_1(1 + z), \quad 0 < z \ll 1.$$

- Only NH and IH ν mass spectra possible: $\min(m_j) = 0$.

- Requirements: $|(RV)_{\ell k}|$ “sizable”

+ reproducing correctly the neutrino oscillation data:

$$|(RV)_{e1}|^2 = \frac{1}{2} \frac{y^2 v^2}{M_1^2} \frac{m_3}{m_2 + m_3} \left| U_{e3} + i\sqrt{m_2/m_3} U_{e2} \right|^2, \quad \text{NH},$$

$$|(RV)_{e1}|^2 = \frac{1}{2} \frac{y^2 v^2}{M_1^2} \frac{m_2}{m_1 + m_2} \left| U_{e2} + i\sqrt{m_1/m_2} U_{e1} \right|^2 \cong \frac{1}{4} \frac{y^2 v^2}{M_1^2} |U_{e2} + iU_{e1}|^2, \quad \text{IH},$$

$$(RV)_{\ell 2} = \pm i (RV)_{e1} \sqrt{\frac{M_1}{M_2}}, \quad \ell = e, \mu, \tau,$$

y - the maximum eigenvalue of Y^ν , $v_u \simeq 174$ GeV.

4 parameters: M , z , y and a phase ω . A. Ibarra, E. Molinaro, S.T.P., 2010 and 2011

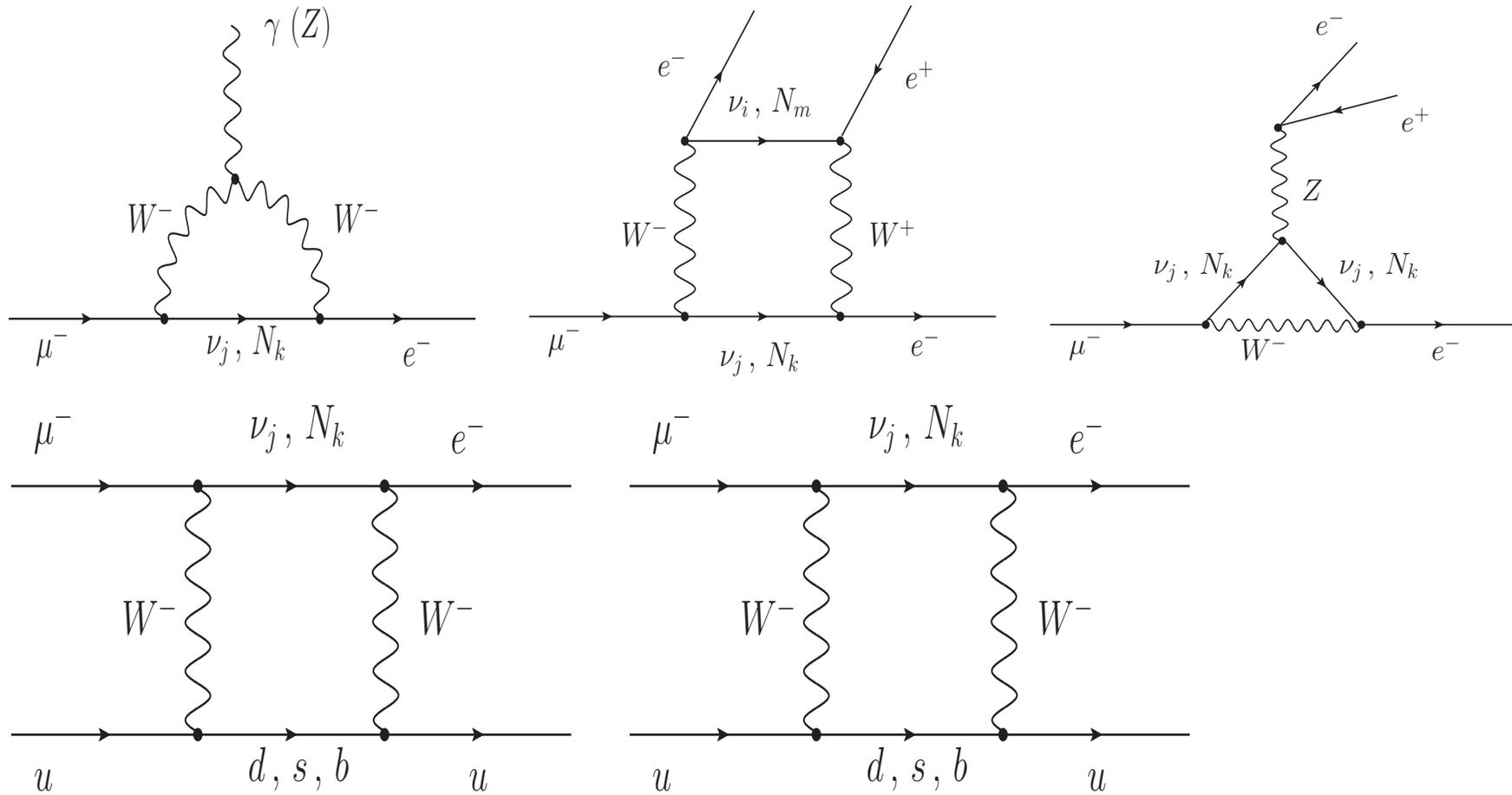
Low energy data:

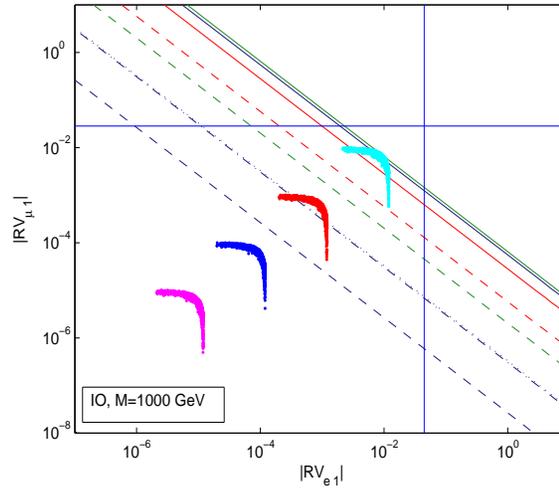
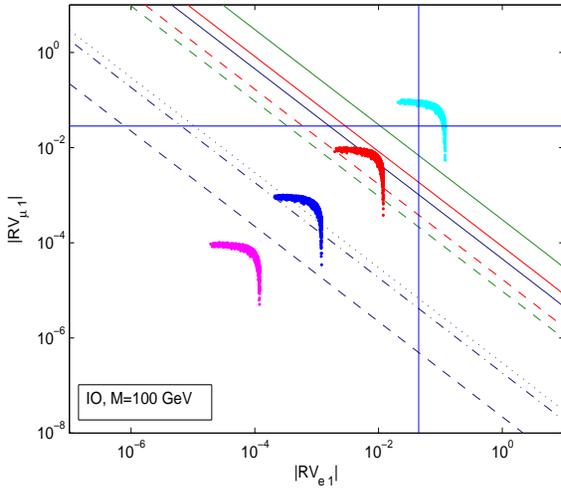
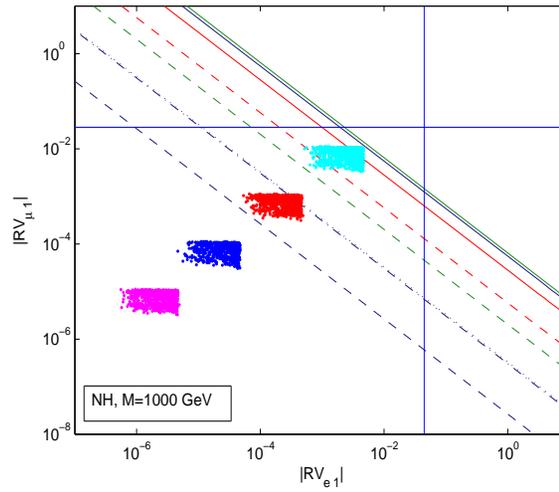
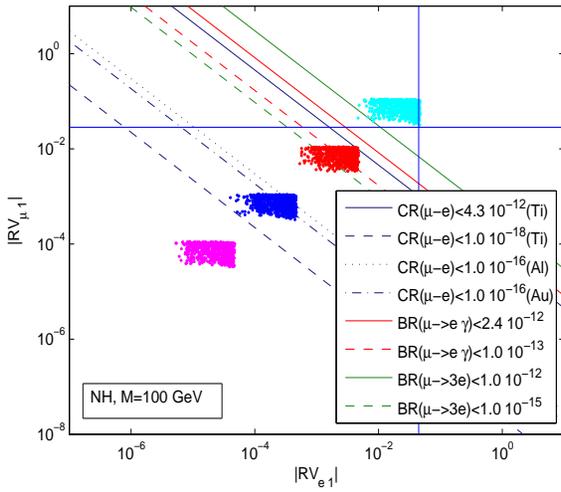
$$\begin{aligned} |(RV)_{e1}|^2 &\lesssim 2 \times 10^{-3}, \\ |(RV)_{\mu 1}|^2 &\lesssim 0.8 \times 10^{-3}, \\ |(RV)_{\tau 1}|^2 &\lesssim 2.6 \times 10^{-3}. \end{aligned}$$

S. Antusch et al., 2008

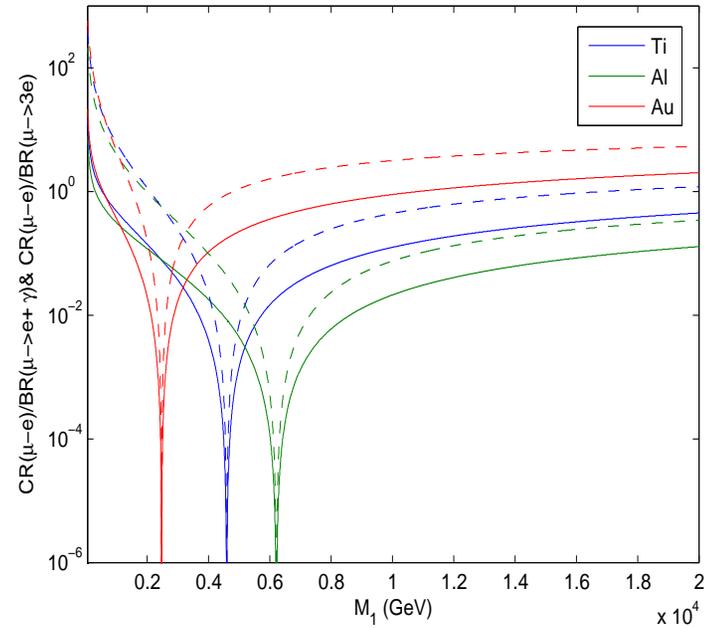
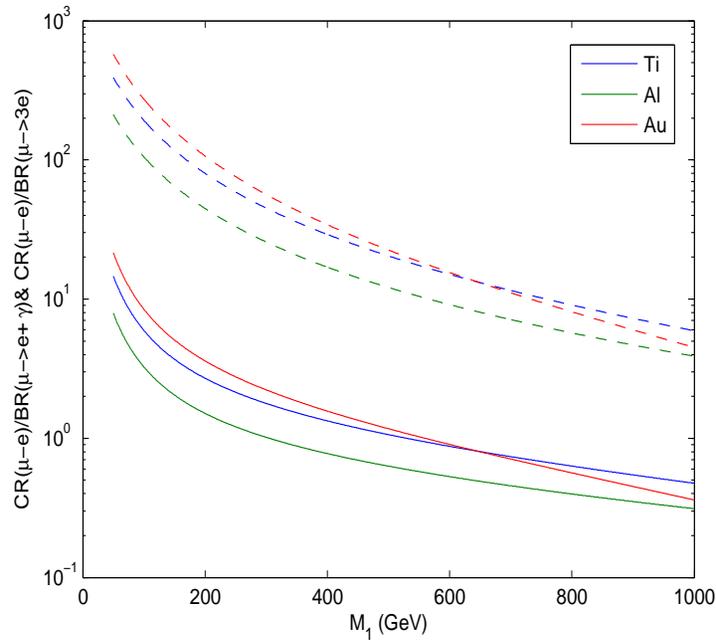
Observation of $N_{1,2}$ at LHC - problematic.

LFV processes: $\mu \rightarrow e + \gamma$, $\mu \rightarrow 3e$, $\mu^- + \mathcal{N} \rightarrow e^- + \mathcal{N}$: can proceed with exchange of virtual N_j :





Current limits and potential sensitivity to $|(RV)_{e1}|$ and $|(RV)_{\mu 1}|$ from data on LFV processes for NH (upper panels) and IH (lower panels) spectra, for $M_1 = 100$ (1000) GeV and, *i*) $y = 0.0001$ (magenta pts), *ii*) $y = 0.001$ (blue pts), *iii*) $y = 0.01$ (red pts) and *iv*) $y = 0.1$ (cyan pts).



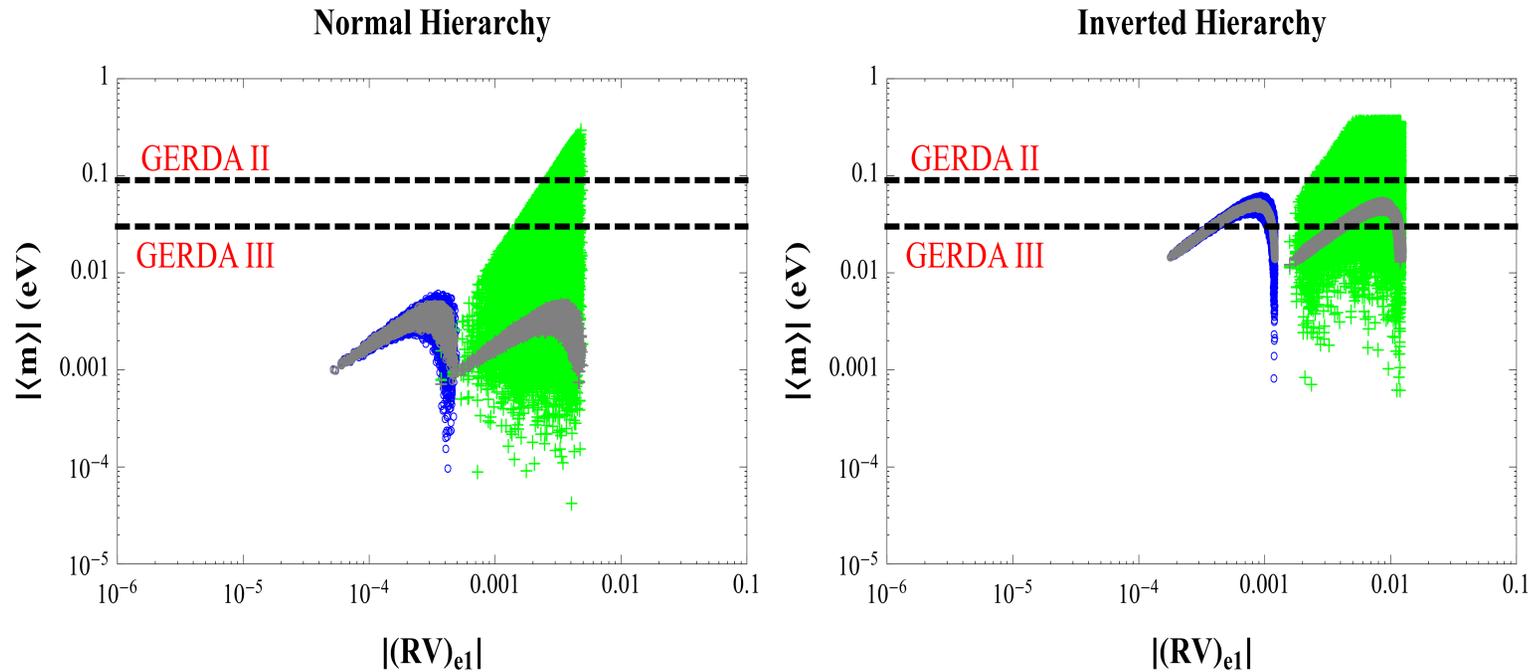
The ratio of the $\mu - e$ relative conversion rate and the branching ratio of the
i) $\mu \rightarrow e\gamma$ decay (solid lines), ii) $\mu \rightarrow 3e$ decay (dashed lines), versus the type I
see-saw mass scale M_1 , for three different nuclei: $^{48}_{22}\text{Ti}$ (blue lines), $^{27}_{13}\text{Al}$ (green
lines) and $^{197}_{79}\text{Au}$ (red lines).

The exchange of virtual N_j gives a contribution to $|\langle m \rangle|$:

$$|\langle m \rangle| \cong \left| \sum_i (U_{PMNS})_{ei}^2 m_i - \sum_k f(A, M_k) (RV)_{ek}^2 \frac{(0.9 \text{ GeV})^2}{M_k} \right| ,$$
$$f(A, M_k) \cong f(A) .$$

For, e.g., ^{48}Ca , ^{76}Ge , ^{82}Se , ^{130}Te and ^{136}Xe , the function $f(A)$ takes the values $f(A) \cong 0.033, 0.079, 0.073, 0.085$ and 0.068 , respectively.

- **The Predictions for $|\langle m \rangle|$ can be modified considerably.**



$|\langle m \rangle|$ vs $|(RV)_{e1}|$ for ^{76}Ge in the cases of NH (left panel) and IH (right panel) light neutrino mass spectrum, for $M_1 = 100$ GeV and *i*) $y = 0.001$ (blue), *ii*) $y = 0.01$ (green). The gray markers correspond to $|\langle m \rangle^{\text{std}}| = |\sum_i (U_{PMNS})_{ei}^2 m_i|$.

A. Ibarra, E. Molinaro, S.T.P., 2010 and 2011

Understanding the Pattern of Neutrino Mixing

Predictions for the CPV Phase δ .

The Quest for Precision Measurement of Neutrino
Mixing Angles and Dirac CP Violation Phase
(or The Quest for Nature's Message)

With the observed pattern of neutrino mixing Nature is sending us a message. The message is encoded in the values of the neutrino mixing angles, leptonic CP violation phases and neutrino masses. The message can have two completely different contents: it can read

ANARCHY or SYMMETRY.

PMNS Matrix: Standard Parametrization

$$U = V P, \quad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix},$$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- $s_{ij} \equiv \sin \theta_{ij}$, $c_{ij} \equiv \cos \theta_{ij}$, $\theta_{ij} = [0, \frac{\pi}{2}]$,
- δ - Dirac CPV phase, $\delta = [0, 2\pi]$; CP inv.: $\delta = 0, \pi, 2\pi$;
- α_{21}, α_{31} - Majorana CPV phases; CP inv.: $\alpha_{21(31)} = k(k')\pi$, $k(k') = 0, 1, 2, \dots$
S.M. Bilenky, J. Hosek, S.T.P., 1980
- $\Delta m_{\odot}^2 \equiv \Delta m_{21}^2 \cong 7.54 \times 10^{-5} \text{ eV}^2 > 0$, $\sin^2 \theta_{12} \cong 0.308$, $\cos 2\theta_{12} \gtrsim 0.28$ (3σ),
- $|\Delta m_{31(32)}^2| \cong 2.47$ (2.42) $\times 10^{-3} \text{ eV}^2$, $\sin^2 \theta_{23} \cong 0.437$ (0.455), NO (IO),
- θ_{13} - the CHOOZ angle: $\sin^2 \theta_{13} = 0.0234$ (0.0240), Capozzi et al. NO (IO).
F. Capozzi et al. (Bari Group), arXiv:1312.2878v2 (May 5, 2014)

- $\Delta m_{\odot}^2 \equiv \Delta m_{21}^2 \cong 7.54 \times 10^{-5} \text{ eV}^2 > 0$, $\sin^2 \theta_{12} \cong 0.308$, $\cos 2\theta_{12} \gtrsim 0.28$ (3σ),
- $|\Delta m_{31(32)}^2| \cong 2.47$ (2.42) $\times 10^{-3} \text{ eV}^2$, $\sin^2 \theta_{23} \cong 0.437$ (0.455), NO (IO),
- θ_{13} - the CHOOZ angle: $\sin^2 \theta_{13} = 0.0234$ (0.0240), NH (IH).
- $1\sigma(\Delta m_{21}^2) = 2.6\%$, $1\sigma(\sin^2 \theta_{12}) = 5.4\%$;
- $1\sigma(|\Delta m_{31(23)}^2|) = 2.6\%$, $1\sigma(\sin^2 \theta_{23}) = 9.6\%$;
- $1\sigma(\sin^2 \theta_{13}) = 8.5\%$;
- $3\sigma(\Delta m_{21}^2) : (6.99 - 8.18) \times 10^{-5} \text{ eV}^2$; $3\sigma(\sin^2 \theta_{12}) : (0.259 - 0.359)$;
- $3\sigma(|\Delta m_{31(23)}^2|) : 2.27(2.23) - 2.65(2.60) \times 10^{-3} \text{ eV}^2$;
 $3\sigma(\sin^2 \theta_{23}) : 0.374(0.380) - 0.628(0.641)$;
- $3\sigma(\sin^2 \theta_{13}) : 0.0176(0.0178) - 0.0295(0.0298)$.

F. Capozzi et al. (Bari Group), arXiv:1312.2878v2 (May 5, 2014)

Neutrino Mixing: New Symmetry?

- $\theta_{12} = \theta_{\odot} \cong \frac{\pi}{5.4}$, $\theta_{23} = \theta_{\text{atm}} \cong \frac{\pi}{4}(\text{?})$, $\theta_{13} \cong \frac{\pi}{20}$

$$U_{\text{PMNS}} \cong \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & \epsilon \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}}(\text{?}) \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}}(\text{?}) \end{pmatrix};$$

Very different from the CKM-matrix!

- $\theta_{12} \cong \pi/4 - 0.20$, $\theta_{13} \cong 0 + \pi/20$, $\theta_{23} \cong \pi/4 \mp 0.10$.
- U_{PMNS} due to new approximate symmetry?

A Natural Possibility (vast literature):

$$U = U_{\text{lep}}^\dagger(\theta_{ij}^\ell, \delta^\ell) Q(\psi, \omega) U_{\text{TBM, BM, LC, ...}} \bar{P}(\xi_1, \xi_2),$$

with

$$U_{\text{TBM}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}; \quad U_{\text{BM}} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \pm \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \pm \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \mp \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

- $U_{\text{lep}}^\dagger(\theta_{ij}^\ell, \delta^\ell)$ - from diagonalization of the l^- mass matrix;
- $U_{\text{TBM, BM, LC, ...}} \bar{P}(\xi_1, \xi_2)$ - from diagonalization of the ν mass matrix;
- $Q(\psi, \omega)$, - from diagonalization of the l^- and/or ν mass matrices.

P. Frampton, STP, W. Rodejohann, 2003

$U_{LC}, U_{GRAM}, U_{GRBM}, U_{HGM}$:

$$U_{LC} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{c_{23}^\nu}{\sqrt{2}} & \frac{c_{23}^\nu}{\sqrt{2}} & s_{23}^\nu \\ \frac{s_{23}^\nu}{\sqrt{2}} & -\frac{s_{23}^\nu}{\sqrt{2}} & c_{23}^\nu \end{pmatrix}; \quad \mu - \tau \text{ symmetry: } \theta_{23}^\nu = \mp \pi/4;$$

$$U_{GR} = \begin{pmatrix} c_{12}^\nu & s_{12}^\nu & 0 \\ -\frac{s_{12}^\nu}{\sqrt{2}} & \frac{c_{12}^\nu}{\sqrt{2}} & -\sqrt{\frac{1}{2}} \\ -\frac{s_{12}^\nu}{\sqrt{2}} & \frac{c_{12}^\nu}{\sqrt{2}} & \sqrt{\frac{1}{2}} \end{pmatrix}; \quad U_{HGM} = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad \theta_{12}^\nu = \pi/6.$$

U_{GRAM} : $\sin^2 \theta_{12}^\nu = (2 + r)^{-1} \cong 0.276$, $r = (1 + \sqrt{5})/2$
(GR: $r/1$; $a/b = a + b/a$, $a > b$)

U_{GRBM} : $\sin^2 \theta_{12}^\nu = (3 - r)/4 \cong 0.345$.

- U_{TBM} : $s_{12}^2 = 1/3$, $s_{23}^2 = 1/2$, $s_{13}^2 = 0$; $s_{13}^2 = 0$ must be corrected; if $\theta_{23} \neq \pi/4$, $s_{23}^2 = 0.5$ must be corrected .
- U_{BM} : $s_{12}^2 = 1/2$, $s_{23}^2 = 1/2$, $s_{13}^2 = 0$; $s_{13}^2 = 0$, $s_{12}^2 = 1/2$ and possibly $s_{23}^2 = 1/2$ must be corrected.

$U_{\text{TBM(BM)}}$: Groups A_4 , $T' (S_4)$, ... (vast literature)

(Reviews: G. Altarelli, F. Feruglio, arXiv:1002.0211; M. Tanimoto et al., arXiv:1003.3552; S. King and Ch. Luhn, arXiv:1301.1340)

- U_{GRA} : Group A_5, \dots ; $s_{13}^2 = 0$ and possibly $s_{12}^2 = 0.276$ and $s_{23}^2 = 1/2$ must be corrected.

L. Everett, A. Stuart, arXiv:0812.1057;...

- U_{LC} : alternatively $U(1)$, $L' = L_e - L_\mu - L_\tau$

S.T.P., 1982

- U_{LC} : $s_{12}^2 = 1/2$, $s_{13}^2 = 0$, s_{23}^{ν} - free parameter; $s_{13}^2 = 0$ and $s_{12}^2 = 1/2$ must be corrected.

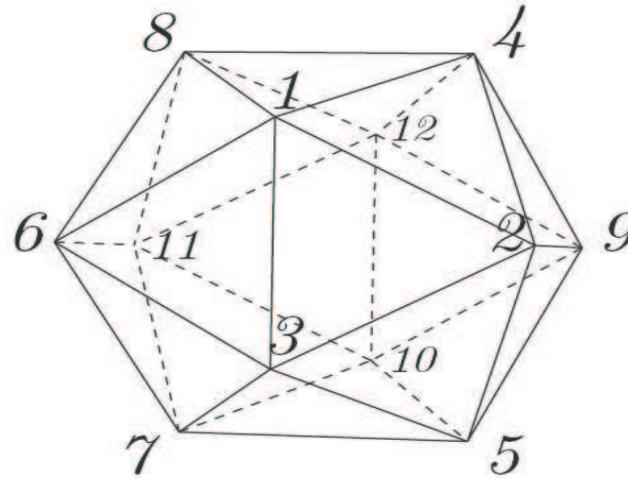
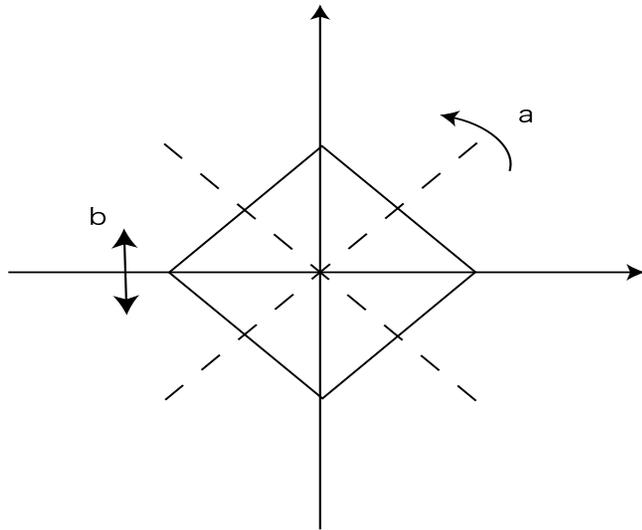
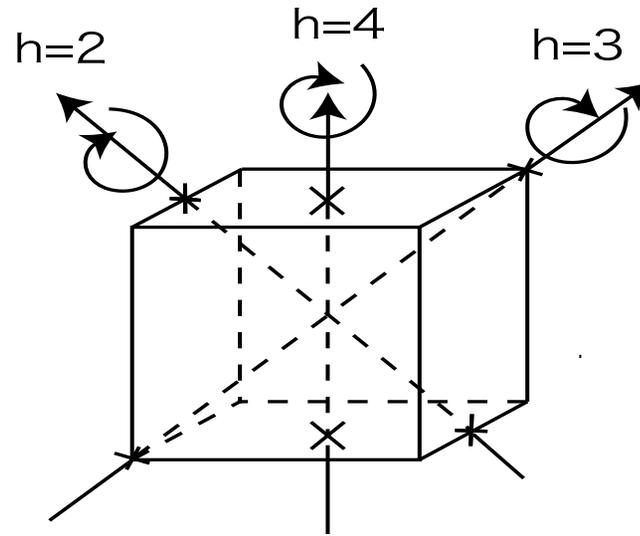
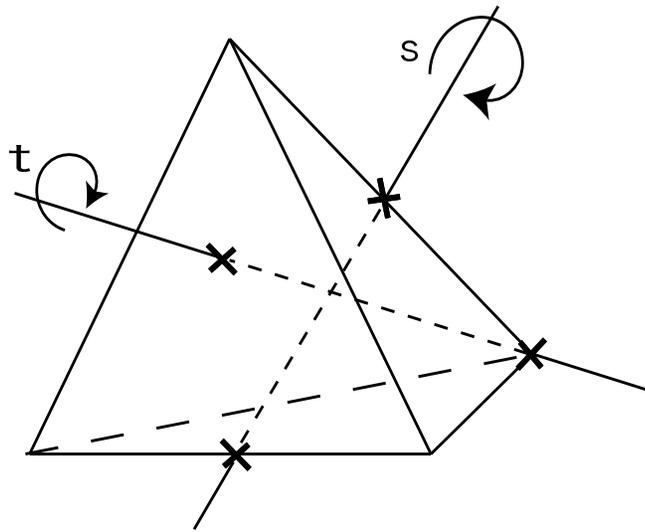
• U_{GRB} : Group $D_{10, \dots}$; $s_{13}^2 = 0$ and possibly $s_{12}^2 = 0.345$ and $s_{23}^2 = 1/2$ must be corrected.

• U_{HG} : Group $D_{12, \dots}$; $s_{13}^2 = 0$, $s_{12}^2 = 0.25$ and possibly $s_{23}^2 = 1/2$ must be corrected.

For all symmetry forms considered we have: $\theta_{13}^\nu = 0$, $\theta_{23}^\nu = \mp \pi/4$.

They differ by the value of θ_{12}^ν :

TBM, BM, GRA, GRB and HG forms correspond to $\sin^2 \theta_{12}^\nu = 1/3; 0.5; 0.276; 0.345; 0.25$.



Examples of symmetries: A_4 , S_4 , D_4 , A_5

From M. Tanimoto et al., arXiv:1003.3552

Group	Number of elements	Generators	Irreducible representations
S_4	24	S, T, U	$1, 1', 2, 3, 3'$
A_4	12	S, T	$1, 1', 1'', 3$
T'	24	$S, T (R)$	$1, 1', 1'', 2, 2', 2'', 3$
A_5	60	S, T	$1, 3, 3', 4, 5$
D_{10}	20	A, B	$1_1, 1_2, 1_3, 1_4, 2_1, 2_2, 2_3, 2_4$
D_{12}	24	A, B	$1_1, 1_2, 1_3, 1_4, 2_1, 2_2, 2_3, 2_4, 2_5$

Number of elements, generators and irreducible representations of some discrete groups.

None of the symmetries leading to U_{TBM} , U_{BM} or other approximate forms of U_{PMNS} can be exact.

Which is the correct approximate symmetry, i.e., approximate form of U_{PMNS} (if any)?

In the cases of U_ν given by U_{TBM} , U_{BM} , etc. the requisite corrections of some of the mixing angles are small and can be considered as perturbations to the corresponding symmetry values.

Depending on the symmetry leading to $U_{\text{TBM, BM}}$, etc. and on the form of U_{lep} , one obtains different experimentally testable predictions for the sum of the neutrino masses, the neutrino mass spectrum, the nature (Dirac or Majorana) of ν_j and the CP violating phases in the neutrino mixing matrix. Future data will help us understand whether there is some new fundamental symmetry behind the observed patterns of neutrino mixing and Δm_{ij}^2 .

What is the minimal U_{lep} providing the requisite corrections to $U_{\text{TBM,BM,LC,GRM,HGM}}$?

Predictions for δ

Assume:

- $U_{PMNS} = U_{\text{lep}}^\dagger(\theta_{ij}^\ell, \delta^\ell) Q(\psi, \omega) U_{\text{TBM, BM, GR, HG}} \bar{P}(\xi_1, \xi_2),$

- U_{lep}^\dagger - minimal, such that

- i) $\sin \theta_{13} \cong 0.16$; BM: $\sin^2 \theta_{12} \cong 0.31$;

- ii) $\sin^2 \theta_{23}$ can deviate significantly (by more than $\sin^2 \theta_{13}$) from 0.5 (b.f.v. = 0.40-0.45) .

The “minimal” = simplest case ($SU(5) \times T', \dots$)

$U_{\text{lep}} \cong O_{12}^{\ell}(\theta_{12}^{\ell})$; now $Q = \text{diag}(e^{i\varphi}, 1, 1)$;

$\sin^{\ell} \theta_{13}, \sin^{\ell} \theta_{23}$ - negligibly small ($SU(5) \times T', \dots$).

Thus, $\theta_{12}, \theta_{23}, \theta_{13}, \delta$ - functions of $\theta_{12}^{\ell}, \varphi$ and θ_{12}^{ν} .

$\theta_{23} = \theta_{23}(\theta_{13}), \quad \delta = \delta(\theta_{12}, \theta_{13}, \theta_{12}^{\nu})(!)$

$U_{\text{BM}, \dots}: \sin^2 \theta_{12} \cong \sin^2 \theta_{12}^{\nu} + \sin 2\theta_{12}^{\nu} \sin \theta_{13} \cos \delta + \dots$,

The exact sum rule will be given later. The preceding approximate sum rule leads to a rather precise prediction for $\cos \delta$ **only** in the BM (LC) case.

$U_{\text{BM}}: \sin 2\theta_{12}^{\nu} = 1$ and $\sin^2 \theta_{12} = (0.31 - 0.33)$ require $\cos \delta \cong -1$.

$$\sin^2 \theta_{23} = \frac{1 - 2 \sin^2 \theta_{13}}{2(1 - \sin^2 \theta_{13})} \cong 0.5(1 - \sin^2 \theta_{13}).$$

Problem for $U_{\text{TBM}, \text{BM}, \text{GRA(B)}, \text{HG}}$ if $\sin^2 \theta_{23} \cong 0.44 - 0.45$:

Larger correction to $\sin^2 \theta_{23}^\nu = 0.5$ might be needed.

Next-to-Minimal case: $U_{\text{lep}} \cong U_{\text{lep}}(\theta_{12}^\ell, \theta_{23}^\ell)$,
 $Q = \text{diag}(1, e^{-i\psi}, e^{-i\omega})$.

“Standard” Ordering:

$U_{\text{lep}}(\theta_{12}^\ell, \theta_{23}^\ell) = O_{23}^T(\theta_{23}^\ell) O_{12}^T(\theta_{12}^\ell)$ (GUTs typically);
in many theories - a consequence of $m_e^2 \ll m_\mu^2 \ll m_\tau^2$.

Standard Ordering

$$U = O_{12}(\theta_{12}^{\ell})O_{23}(\theta_{23}^{\ell})\text{diag}(1, e^{-i\psi}, e^{-i\omega})O_{23}(\theta_{23}^{\nu})O_{12}(\theta_{12}^{\nu})\bar{P},$$

$$\bar{P} = \text{diag}(1, e^{i\xi_1}, e^{i\xi_2}).$$

Can be shown to be equivalent to:

$$U = O_{12}(\theta_{12}^{\ell})\text{diag}(1, e^{i\phi}, 1)O_{23}(\hat{\theta}_{23})O_{12}(\theta_{12}^{\nu})\bar{P}(\xi_1, \xi_2 + \beta)$$

$$\hat{\theta}_{23} = \hat{\theta}_{23}(\theta_{23}^{\ell}, \psi - \omega, \theta_{23}^{\nu}), \quad \phi = \phi(\theta_{23}^{\ell}, \psi, \omega, \theta_{23}^{\nu}).$$

$$\theta_{12}^{\nu} = \pi/4 \text{ (BIM, LC), or } \sin^{-1}(1/\sqrt{3}) \text{ (TBM), etc.}$$

Thus, $\theta_{12}, \theta_{23}, \theta_{13}, \delta$ - functions of $\theta_{12}^{\ell}, \phi, \hat{\theta}_{23}$.

ϕ serves as a “source” for δ .

Expect $\delta = \delta(\theta_{12}, \theta_{23}, \theta_{13})(!)$

For arbitrary fixed θ_{12}^ν and any θ_{23}
(“minimal” and “next-to-minimal” cases):

$$\cos \delta = \frac{\tan \theta_{23}}{\sin 2\theta_{12} \sin \theta_{13}} \left[\cos 2\theta_{12}^\nu + (\sin^2 \theta_{12} - \cos^2 \theta_{12}^\nu) (1 - \cot^2 \theta_{23} \sin^2 \theta_{13}) \right].$$

S.T.P., arXiv:1405.6006

This result is exact. For TBM and BM cases derived first in:

D. Marzocca, S.T.P., A. Romanino, M.C. Sevilla, arXiv:1302.0423

“Minimal” case: $\sin^2 \theta_{23} = \frac{1}{2} \frac{1 - 2 \sin^2 \theta_{13}}{1 - \sin^2 \theta_{13}}.$

In a complete self-consistent theory corrections are possible: $\theta_{13}^e \neq 0$ (see further), RG running effects (negligible for $m_0 \lesssim 0.01$ eV), etc.

Comparing the imaginary and real parts of $U_{e1}^* U_{\mu 3}^* U_{e3} U_{\mu 1}$ in the two parametrisations:

$$\sin \delta = - \frac{\sin 2\theta_{12}^\nu}{\sin 2\theta_{12}} \sin \phi ,$$

$$\cos \delta = \frac{\sin 2\theta_{12}^\nu}{\sin 2\theta_{12}} \cos \phi \left(\frac{2 \sin^2 \theta_{23}}{\sin^2 \theta_{23} \cos^2 \theta_{13} + \sin^2 \theta_{13}} - 1 \right) + \frac{\cos 2\theta_{12}^\nu}{\sin 2\theta_{12}} \frac{\sin 2\theta_{23} \sin \theta_{13}}{\sin^2 \theta_{23} \cos^2 \theta_{13} + \sin^2 \theta_{13}} .$$

S.T.P., arXiv:1405.6006

The relations are exact.

In all cases BM, LC, TBM, GRA, GRB, HGM:

- $J_{CP} = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}, \delta) = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}; \theta_{12}^{\nu})$.
- TBM case: $\delta \cong 3\pi/2$ or $\pi/2$; b.f.v. of θ_{ij} :
 $\delta \cong 263.5^\circ$ or 96.5° , $\cos \delta = -0.114$, $J_{CP} \cong \mp 0.034$.
- GRAM case, b.f.v. of θ_{ij} : $\delta \cong 286.8^\circ$ or 73.2° ;
 $\cos \delta = 0.289$, $J_{CP} \cong \mp 0.0327$.
- GRBM case, b.f.v. of θ_{ij} : $\delta \cong 258.5^\circ$ or 101.5° ;
 $\cos \delta = -0.200$, $J_{CP} \mp 0.0333$.
- HGM case, b.f.v. of θ_{ij} : $\delta \cong 298.4^\circ$ or 61.6° ;
 $\cos \delta = 0.476$, $J_{CP} \cong \mp 0.0299$.
- BM, LC cases: $\delta \cong \pi$, $\cos \delta \cong -0.978$, $J_{CP} \cong \mp 0.008$

The results shown - for NO neutrino mass spectrum; the results are practically the same for IO spectrum. (Best fit values of θ_{ij} : F. Capozzi et al., arXiv:1312.2878v1.)

S.T.P., arXiv:1405.6006

By measuring $\cos \delta$ or δ one can distinguish between different symmetry forms of \tilde{U}_ν !

Relatively high precision measurement of δ will be performed at the future planned neutrino oscillation experiments, see, e.g., A. de Gouvea *et al.*, arXiv:1310.4340, P. Coloma *et al.*, arXiv:1203.5651.

Alternative “minimal” case

$U_{\text{lep}} \cong O_{13}^{\ell}(\theta_{13}^{\ell})$; also in this case $Q = \text{diag}(e^{i\varphi_{13}}, 1, 1)$;

Now, $\theta_{12}, \theta_{23}, \theta_{13}, \delta$ - functions of $\theta_{13}^{\ell}, \varphi_{13}$ and θ_{12}^{ν} .

Similarly we have: $\sin^2 \theta_{23} = \frac{1}{2(1 - \sin^2 \theta_{13})}$.

Alternative Next-to-Minimal case: $U_{\text{lep}} \cong U_{\text{lep}}(\theta_{13}^{\ell}, \theta_{23}^{\ell})$,
 $Q = \text{diag}(1, e^{-i\tilde{\psi}}, e^{-i\tilde{\omega}})$.

“Standard” Ordering:

$$U_{\text{lep}}(\theta_{12}^{\ell}, \theta_{23}^{\ell}) = O_{23}^T(\theta_{23}^{\ell}) O_{13}^T(\theta_{13}^{\ell})$$

For arbitrary fixed θ_{12}^ν and any θ_{23}
(alternative “minimal” and “next-to-minimal” cases):

$$\cos \delta = -\frac{\cot \theta_{23}}{\sin 2\theta_{12} \sin \theta_{13}} \left[\cos 2\theta_{12}^\nu + (\sin^2 \theta_{12} - \cos^2 \theta_{12}^\nu) (1 - \tan^2 \theta_{23} \sin^2 \theta_{13}) \right].$$

I. Girardi, S.T.P., A. Titov, arXiv:1504.00658

This result is exact. Differs only by the overall sign and by $\tan \theta_{23} \leftrightarrow \cot \theta_{23}$ from the sum rule for $\cos \delta$ discussed earlier.

Alternative “Minimal” case: $\sin^2 \theta_{23} = \frac{1}{2(1 - \sin^2 \theta_{13})}$.

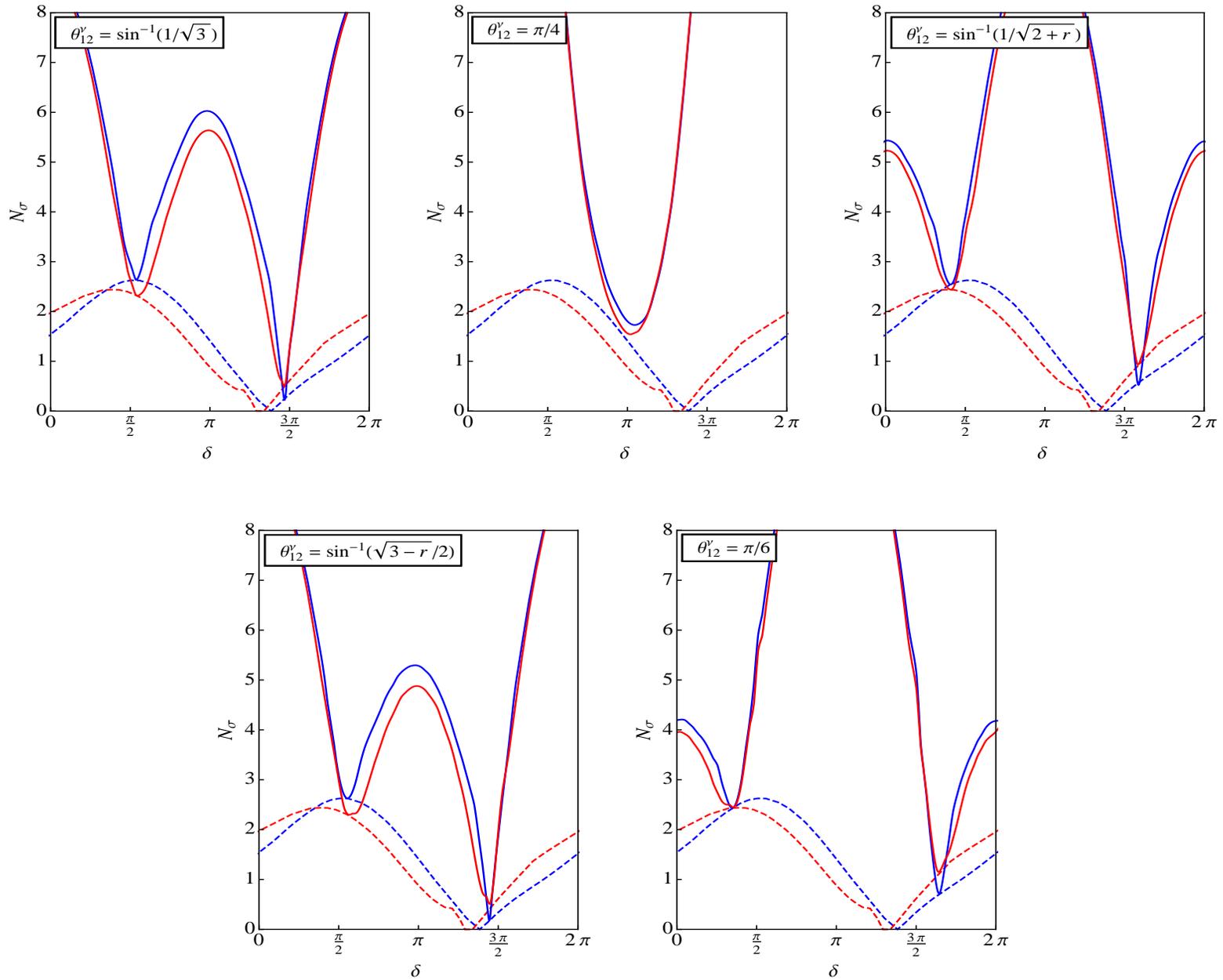
Statistical analysis, likelihood method;
input “data”: $\sin^2 \theta_{13}, \sin^2 \theta_{12}, \sin^2 \theta_{12}, \delta$
from F. Capozzi et al., arXiv:1312.2878v2 (May 5, 2014).

$$L(\cos \delta) \propto \exp\left(-\frac{\chi^2(\cos \delta)}{2}\right)$$

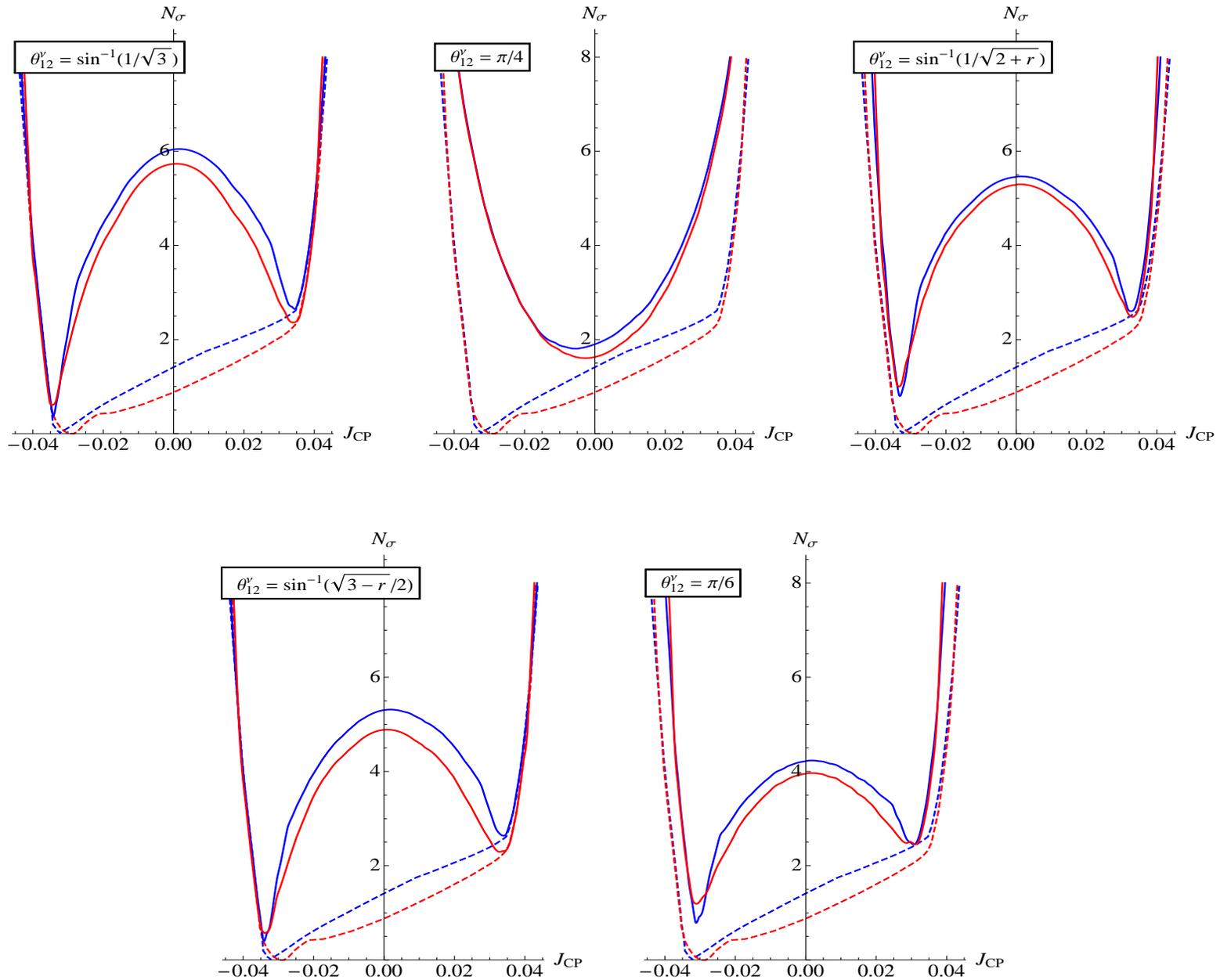
$n\sigma$ confidence level interval of values of $\cos \delta$:

$$L(\cos \delta) \geq L(\chi_{\min}^2) \cdot L(\chi^2 = n^2)$$

I. Girardi, S.T.P., A. Titov, arXiv:1410.8056



I. Girardi, S.T.P., A. Titov, arXiv:1410.8056



I. Girardi, S.T.P., A. Titov, arXiv:1410.8056

TBM, GRA, GRB, HG: $J = 0$ excluded at 5σ , 4σ , 4σ , 3σ confidence level.

At 3σ : $0.020 \leq |J_{CP}| \leq 0.039$.

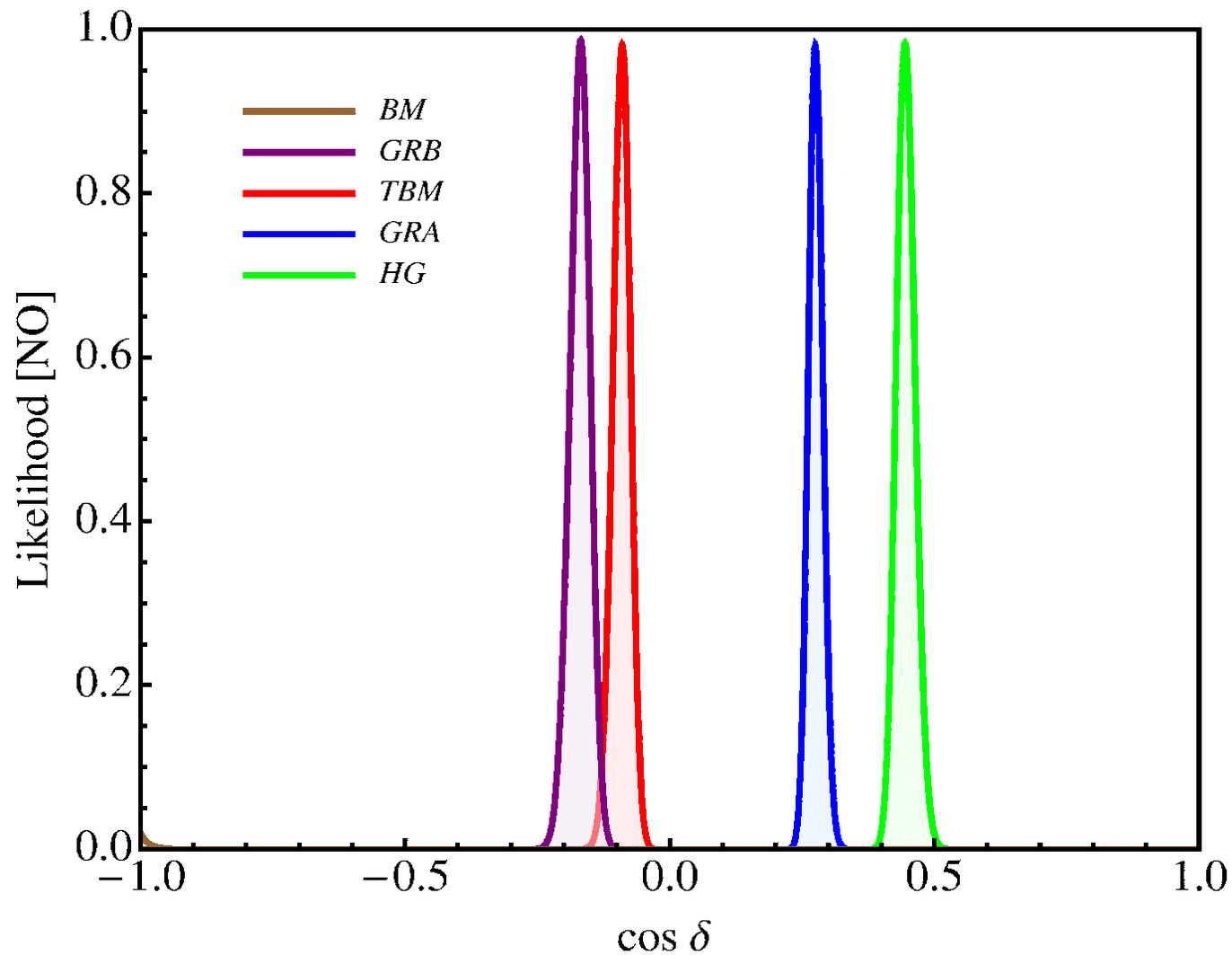
**BM (LC), b.f.v.: $J_{CP} = 0$;
at 3σ : -0.026 (-0.025) $\leq J_{CP} \leq 0.021$ (0.023) for NO
(IO) neutrino mass spectrum.**

Prospective precision:

$$\delta(\sin^2 \theta_{12}) = 0.7\% \text{ (JUNO)},$$

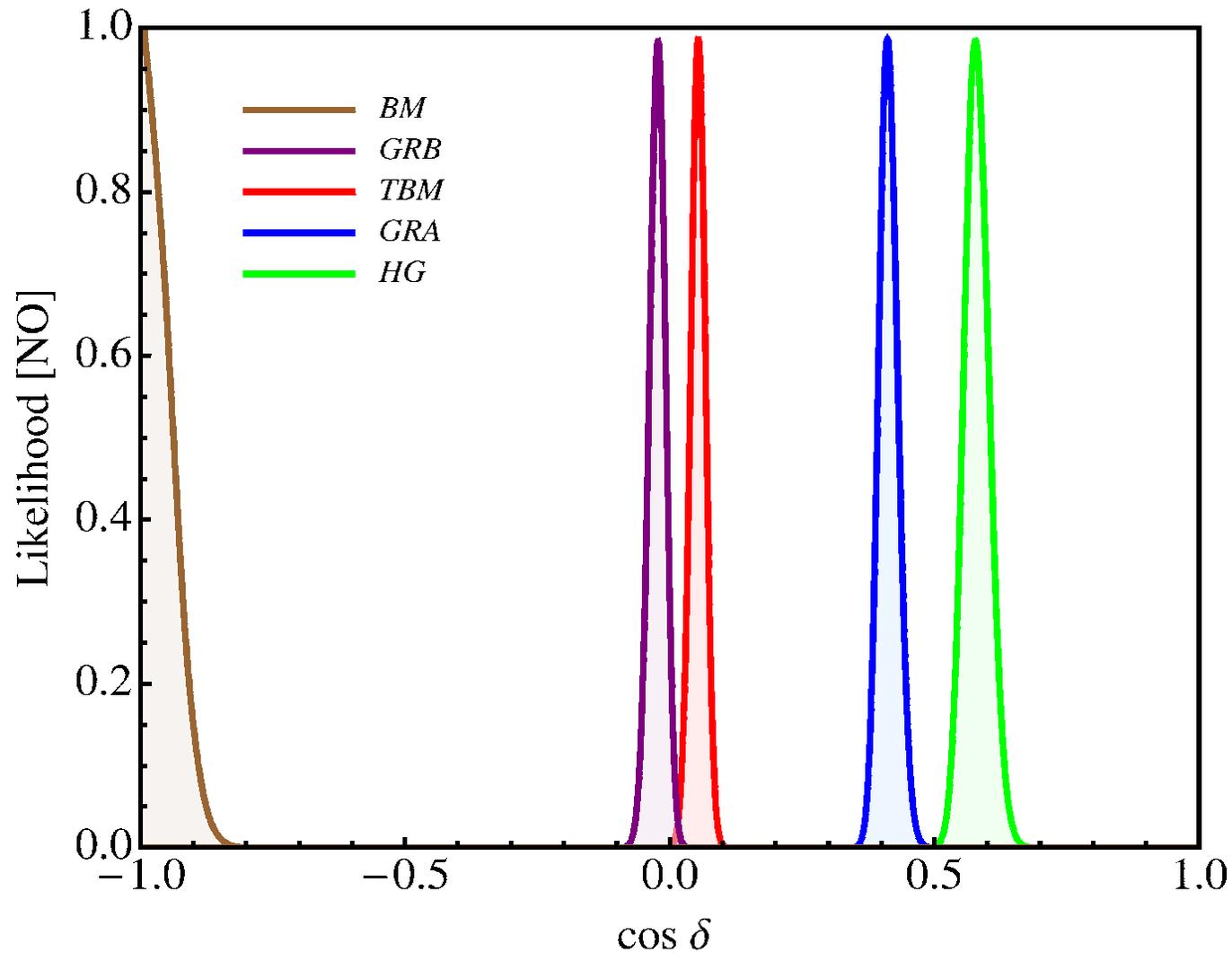
$$\delta(\sin^2 \theta_{13}) = 3\% \text{ (Daya Bay)},$$

$$\delta(\sin^2 \theta_{23}) = 5\% \text{ (T2K, NO}\nu\text{A combined)}.$$



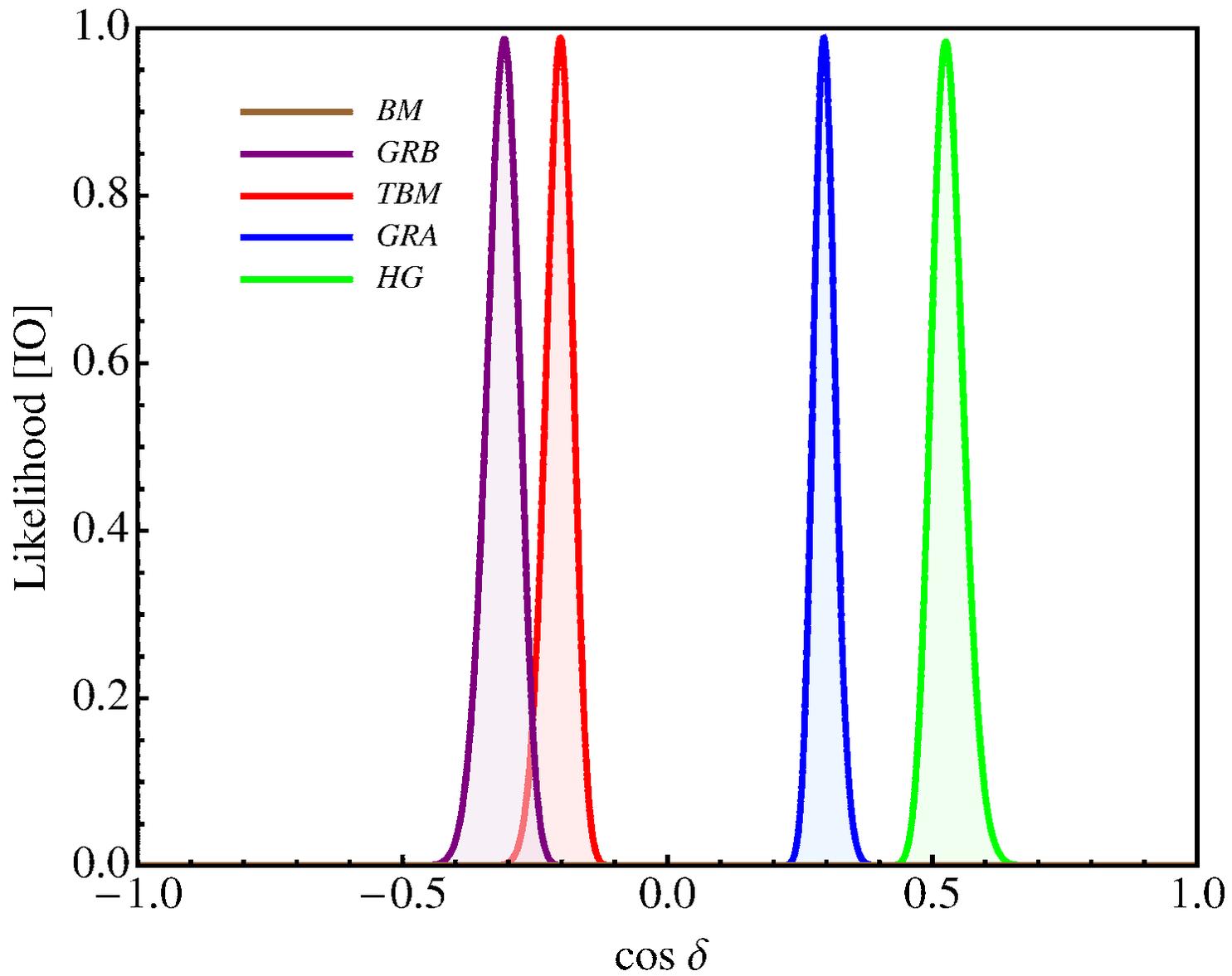
I. Girardi, S.T.P., A. Titov, arXiv:1410.8056

b.f.v. of $\sin^2 \theta_{ij}$ (Capozzi et al., 2014) + the prospective precision used.



I. Girardi, S.T.P., A. Titov, arXiv:1410.8056

The same, but for $\sin^2 \theta_{12} = 0.33$ (the BM prediction dependence on $\sin^2 \theta_{12}$).



I. Girardi, S.T.P., A. Titov, arXiv:1410.8056

$\sin^2 \theta_{23} = 0.557$ (b.f.v.: C. Gonzales-Garcia et al., 2014, IO case).

The predictions obtained for $\cos \delta$ are valid in a large class of theoretical models of (lepton) flavour based on discrete symmetries.

J. Gehrlein *et al.*, “An $SU(5) \times A_5$ Golden Ratio Flavour Model”, arXiv:1410.2095;

I. Girardi *et al.*, “Generalised Geometrical CP Violation in a T' Lepton Flavour Model”, arXiv:1312.1966, JHEP 1402 (2014) 050.

T' model of lepton flavour: U_{TBM} , $\delta \cong 3\pi/2$ or $\pi/2$.

I. Girardi, A. Meroni, STP, M. Spinrath, arXiv:1312.1966

- Light neutrino masses: type I seesaw mechanism.
- ν_j - Majorana particles.
- Diagonalisation of M_ν : $U_{\text{TBM}}\Phi$, $\Phi = \text{diag}(1, 1, 1(i))$
- U_{TBM} “corrected” by
 $U_{\text{lep}}^\dagger Q = R_{12}(\theta_{12}^\ell) R_{23}(\theta_{23}^\ell) Q$, $Q = \text{diag}(1, e^{i\phi}, 1)$

T' model of lepton flavour: U_{TBM} , $\delta \cong 3\pi/2$ or $\pi/2$.

- T' : double covering of A_4 (tetrahedral symmetry group).
- T' : $\mathbf{1}, \mathbf{1}', \mathbf{1}''; \mathbf{2}, \mathbf{2}', \mathbf{2}''; \mathbf{3}$.
- T' model: $\psi_{eL}(x), \psi_{\mu L}(x), \psi_{\tau L}(x)$ - triplet of T' ;
 $e_R(x), \mu_R(x)$ - a doublet, $\tau_R(x)$ - a singlet, of T' ;
 $\nu_{eR}(x), \nu_{\mu R}(x), \nu_{\tau R}(x)$ - a triplet of T' ;
the Higgs doublets $H_u(x), H_d(x)$ - singlets of T' .
- The discrete symmetries of the model are $T' \times H_{\text{CP}} \times Z_8 \times Z_4^2 \times Z_3^2 \times Z_2$, the Z_n factors being the shaping symmetries of the superpotential required to forbid unwanted operators.

Predictions of the T' Model

- $m_{1,2,3}$ determined by 2 real parameters + Φ^2 :

NO spectrum A : $(m_1, m_2, m_3) = (4.43, 9.75, 48.73) \cdot 10^{-3} \text{ eV}$

NO spectrum B : $(m_1, m_2, m_3) = (5.87, 10.48, 48.88) \cdot 10^{-3} \text{ eV}$

IO spectrum : $(m_1, m_2, m_3) = (51.53, 52.26, 17.34) \cdot 10^{-3} \text{ eV}$

$$\text{NO A : } \sum_{j=1}^3 m_j = 6.29 \times 10^{-2} \text{ eV ,}$$

$$\text{NO B : } \sum_{j=1}^3 m_j = 6.52 \times 10^{-2} \text{ eV ,}$$

$$\text{IO : } \sum_{j=1}^3 m_j = 12.11 \times 10^{-2} \text{ eV .}$$

- $\theta_{12}, \theta_{23}, \theta_{13}, \delta, \alpha_{21}, \alpha_{31}$ determined by 3 real parameters.

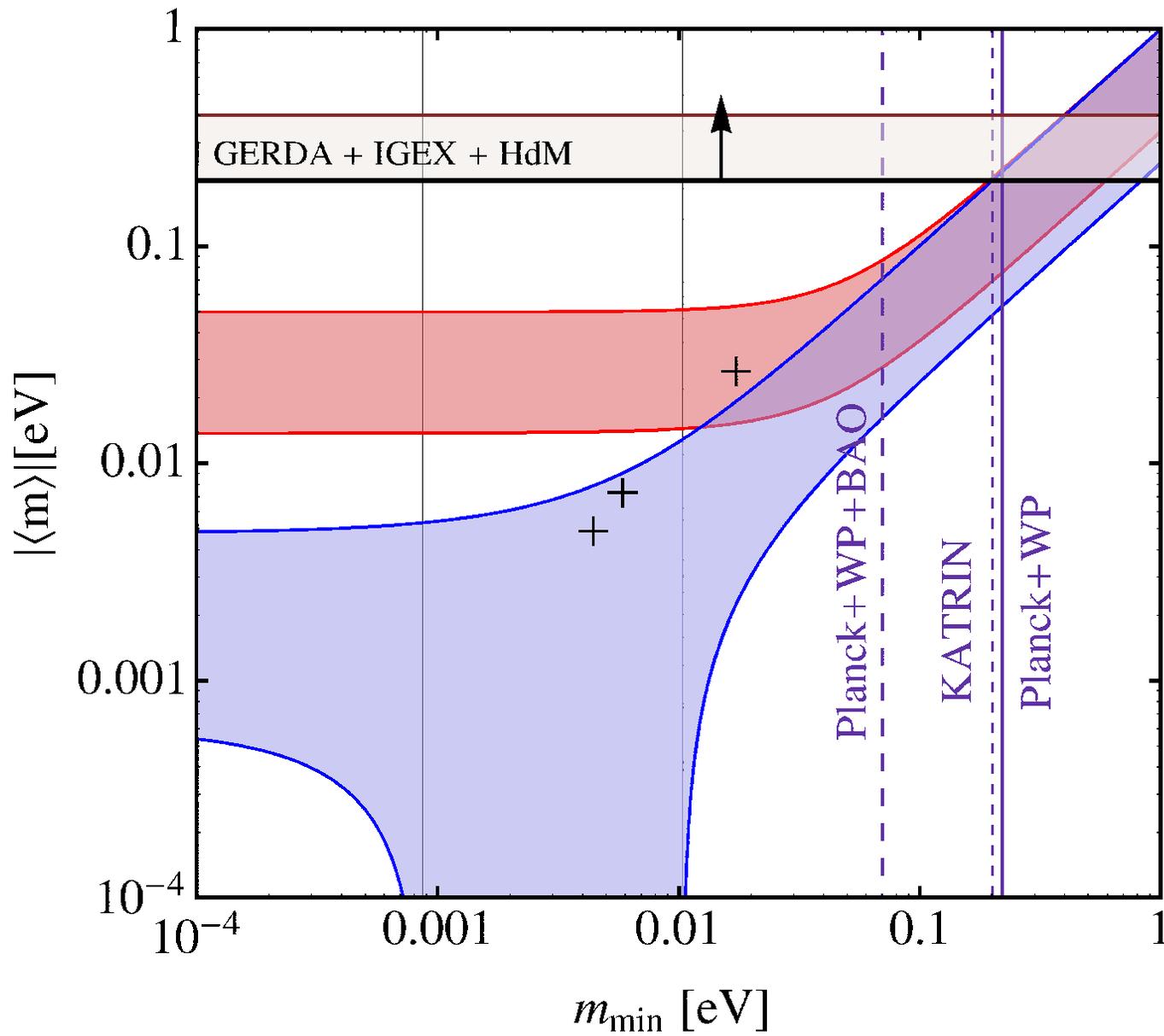
Given the values of $\theta_{12}, \theta_{23}, \theta_{13}, \delta, \alpha_{21}, \alpha_{31}$ are predicted:

$$\delta \cong 3\pi/2 (266^\circ) \text{ (or } \pi/2 (94^\circ)\text{);}$$

$$\text{NO A: } \alpha_{21} \cong +47.0^\circ \text{ (or } -47.0^\circ) (+2\pi),$$

$$\alpha_{31} \cong -23.8^\circ \text{ (or } +23.8^\circ) (+2\pi).$$

The model is falsifiable.



Instead of Conclusions

We are at the beginning of the Road...

The next 5, 10, 15,... years will be very exciting in Neutrino (Lepton) Physics!

The future of neutrino physics is bright.