Pair Production in Near Extremal Kerr-Newman Black Holes

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Outline

- Motivation
- Scalar Pair Production
- Thermal Interpretation
- Dual CFT Description
- Summary

RN black holes:

CMC, Kim, Lin, Sun, Wu, PRD 85 (2012) 124041 [arXiv:1202.3224 [hep-th]]
CMC, Sun, Tang, Tsai, CQG 32 (2015) 195003 [arXiv:1412.6876 [hep-th]]
Kim, Lee, Yoon, arXiv:1503.00218 [hep-th]

Motivation

- The spontaneous pair production from charged black holes mixtures two independent processes:
  - Schwinger mechanism by electromagnetic force

$$N_S \sim \mathrm{e}^{-\frac{m}{T_S}}, \qquad T_S = 2T_U, \quad T_U = \frac{1}{2\pi}\frac{qE}{m}$$

  - Hawking radiation by tunneling through horizon

$$N_H \sim \mathrm{e}^{-\frac{\omega}{T_H}}, \qquad T_H = \frac{\kappa}{2\pi}$$

- Near-extremal Kerr-Newman black holes: dual CFT description & exactly solvable
- Near-horizon region: GR solution & pair production occurs

Near Horizon Geometry of Kerr-Newman

- Kerr-Newman (KN) black holes: $M, \ Q, \ a = J/M$
- Near horizon limits: $\varepsilon \to 0, \quad r_0^2 \equiv Q^2 + a^2$

$$\hat{\varphi} \to \varphi + \frac{a}{r_0^2 + a^2}\hat{t}, \quad \hat{r} \to r_0 + \varepsilon r, \quad \hat{t} \to \frac{r_0^2 + a^2}{\varepsilon} t, \quad M \to r_0 + \varepsilon^2 \frac{B^2}{2r_0}$$

- Near horizon of near extremal KN black holes

$$\begin{aligned} ds^2 &= (r_0^2 + a^2\cos^2\theta)\left[-(r^2 - B^2)dt^2 + \frac{dr^2}{r^2 - B^2} + d\theta^2\right] \\ &\quad + \frac{(r_0^2 + a^2)^2 \sin^2\theta}{r_0^2 + a^2\cos^2\theta}\left(d\varphi + \frac{2ar_0}{r_0^2 + a^2} r dt\right)^2 \\ A &= -Q\left(\frac{r_0^2 - a^2\cos^2\theta}{r_0^2 + a^2\cos^2\theta} r dt + \frac{r_0 a \sin^2\theta}{r_0^2 + a^2\cos^2\theta} d\varphi\right) \end{aligned}$$

Scalar Pair Production

- Klein-Gordon equation

$$(\nabla_\alpha - iqA_\alpha)(\nabla^\alpha - iqA^\alpha)\Phi - m^2\Phi = 0$$

- ansätz: $\Phi(t, r, \theta, \varphi) = \mathrm{e}^{-i\omega t + in\varphi} R(r)\Theta(\theta)$
- decoupled equations

$$\partial_r \left[(r^2 - B^2)\partial_r R\right] + \left(\frac{\left[\omega(r_0^2 + a^2) - qQ^3 r + 2nar_0 r\right]^2}{(r_0^2 + a^2)^2 (r^2 - B^2)} - m^2(r_0^2 + a^2) - \lambda\right) R = 0$$

$$\frac{1}{\sin\theta}\partial_\theta(\sin\theta\partial_\theta\Theta) - \left(\frac{\left[n(r_0^2 + a^2\cos^2\theta) + qQar_0\sin^2\theta\right]^2}{(r_0^2 + a^2)^2\sin^2\theta} - m^2a^2\sin^2\theta - \lambda\right)\Theta = 0$$

Scalar Pair Production

- Exact Solution:

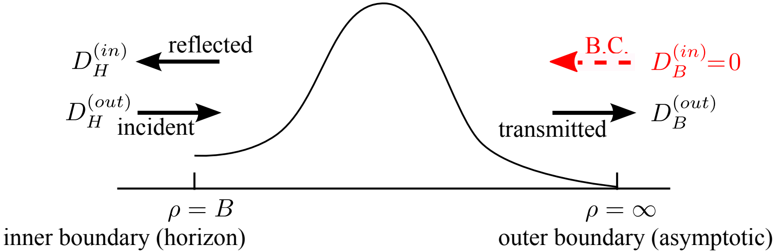
$$
\begin{aligned}
R(r) &= c_1 (r-B)^{\frac{i}{2}(\tilde{\kappa}-\kappa)} (r+B)^{\frac{i}{2}(\tilde{\kappa}+\kappa)} \\
& \quad F\left(\frac{1}{2}+i\tilde{\kappa}+i\mu, \frac{1}{2}+i\tilde{\kappa}-i\mu; 1+i\tilde{\kappa}-i\kappa; \frac{1}{2}-\frac{r}{2B}\right) \\
&+ c_2 (r-B)^{-\frac{i}{2}(\tilde{\kappa}-\kappa)} (r+B)^{\frac{i}{2}(\tilde{\kappa}+\kappa)} \\
& \quad F\left(\frac{1}{2}+i\kappa+i\mu, \frac{1}{2}+i\kappa-i\mu; 1-i\tilde{\kappa}+i\kappa; \frac{1}{2}-\frac{r}{2B}\right)
\end{aligned}
$$

- three essential parameters

$$
\tilde{\kappa} = \frac{\omega}{B}, \quad \kappa = \frac{qQ^3 - 2nar_0}{r_0^2 + a^2}, \quad \mu = \sqrt{\kappa^2 - m^2(r_0^2 + a^2) - \lambda - \frac{1}{4}}
$$

Scalar Pair Production

- Outer boundary condition: no incoming flux at asymptotic



CMC, Kim, Lin, Sun, Wu, PRD 85 (2012) 124041 [arXiv:1202.3224 [hep-th]]

Scalar Pair Production

- Flux conservation for bosonic particles

$$|D_{\text{incident}}| = |D_{\text{reflected}}| + |D_{\text{transmitted}}|$$

- Bogoliubov relation

$$|\alpha|^2 - |\beta|^2 = 1$$

- Vacuum persistence amplitude $|\alpha|^2$ and Mean number of produced pairs $|\beta|^2$

$$|\alpha|^2 \equiv \frac{D_{\text{incident}}}{D_{\text{reflected}}}, \quad |\beta|^2 \equiv \frac{D_{\text{transmitted}}}{D_{\text{reflected}}}$$

- Absorption cross section

$$\sigma_{\text{abs}} \equiv \frac{D_{\text{transmitted}}}{D_{\text{incident}}} = \frac{|\beta|^2}{|\alpha|^2}$$

Scalar Pair Production

- Expansion around $r = B$

$$R_H(r) \approx c_H^{(\mathrm{in})}(2B)^{\frac{i}{2}(\tilde{\kappa}+\kappa)}(r-B)^{-\frac{i}{2}(\tilde{\kappa}-\kappa)} + c_H^{(\mathrm{out})}(2B)^{\frac{i}{2}(\tilde{\kappa}+\kappa)}(r-B)^{\frac{i}{2}(\tilde{\kappa}-\kappa)}$$

  - parameters: $c_H^{(\mathrm{in})} = c_2$ and $c_H^{(\mathrm{out})} = c_1$

- Expansion around $r \to \infty$

$$\begin{aligned} R_B(r) &\approx c_B^{(\mathrm{in})}(r-B)^{-\frac{1}{2}-\frac{i}{2}(\tilde{\kappa}+\kappa)-i\mu}(r+B)^{\frac{i}{2}(\tilde{\kappa}+\kappa)} \\ &\quad + c_B^{(\mathrm{out})}(r-B)^{-\frac{1}{2}-\frac{i}{2}(\tilde{\kappa}+\kappa)+i\mu}(r+B)^{\frac{i}{2}(\tilde{\kappa}+\kappa)} \\ &\approx c_B^{(\mathrm{in})} r^{-\frac{1}{2}-i\mu} + c_B^{(\mathrm{out})} r^{-\frac{1}{2}+i\mu} \end{aligned}$$

- Condition for existence of propagating modes

$$\mu^2 = \kappa^2 - m^2(r_0^2 + a^2) - \lambda - \frac{1}{4} > 0$$

Scalar Pair Production

- relations of parameters

$$
\begin{aligned}
c_B^{(\mathrm{in})} &= c_1(2B)^{\frac{1}{2}+i\tilde{\kappa}+i\mu}\frac{\Gamma(1+i\tilde{\kappa}-i\kappa)\Gamma(-2i\mu)}{\Gamma\left(\frac{1}{2}-i\kappa-i\mu\right)\Gamma\left(\frac{1}{2}+i\tilde{\kappa}-i\mu\right)} \\
&\quad +c_2(2B)^{\frac{1}{2}+i\kappa+i\mu}\frac{\Gamma(1-i\tilde{\kappa}+i\kappa)\Gamma(-2i\mu)}{\Gamma\left(\frac{1}{2}+i\kappa-i\mu\right)\Gamma\left(\frac{1}{2}-i\tilde{\kappa}-i\mu\right)} \\
c_B^{(\mathrm{out})} &= c_1(2B)^{\frac{1}{2}+i\tilde{\kappa}-i\mu}\frac{\Gamma(1+i\tilde{\kappa}-i\kappa)\Gamma(2i\mu)}{\Gamma\left(\frac{1}{2}-i\kappa+i\mu\right)\Gamma\left(\frac{1}{2}+i\tilde{\kappa}+i\mu\right)} \\
&\quad +c_2(2B)^{\frac{1}{2}+i\kappa-i\mu}\frac{\Gamma(1-i\tilde{\kappa}+i\kappa)\Gamma(2i\mu)}{\Gamma\left(\frac{1}{2}+i\kappa+i\mu\right)\Gamma\left(\frac{1}{2}-i\tilde{\kappa}+i\mu\right)}
\end{aligned}
$$

Scalar Pair Production

- Flux formula (radial direction)

$$D = \int d\theta d\varphi \, i\sqrt{-g} g^{rr} (\Phi D_r \Phi^* - \Phi^* D_r \Phi)$$

- Relevant fluxes:

$$D_B^{(\text{in})} = -2(r_0^2 + a^2)\mu \left| c_B^{(\text{in})} \right|^2 \mathfrak{S}$$

$$D_H^{(\text{in})} = -2B(r_0^2 + a^2)(\tilde{\kappa} - \kappa) \left| c_H^{(\text{in})} \right|^2 \mathfrak{S}$$

$$D_B^{(\text{out})} = 2(r_0^2 + a^2)\mu \left| c_B^{(\text{out})} \right|^2 \mathfrak{S}$$

$$D_H^{(\text{out})} = 2B(r_0^2 + a^2)(\tilde{\kappa} - \kappa) \left| c_H^{(\text{out})} \right|^2 \mathfrak{S}$$

- angular part (irrelevant): $\mathfrak{S} = 2\pi \int d\theta \, \sin\theta \, \Theta\Theta^*$

Scalar Pair Production

- Outer boundary condition: $c_B^{(\mathrm{in})} = 0$

$$c_1 = -c_2 (2B)^{i(\kappa-\tilde{\kappa})} \frac{\Gamma(1 - i\tilde{\kappa} + i\kappa)\Gamma(\frac{1}{2} + i\tilde{\kappa} - i\mu)\Gamma(\frac{1}{2} - i\kappa - i\mu)}{\Gamma(\frac{1}{2} + i\kappa - i\mu)\Gamma(\frac{1}{2} - i\tilde{\kappa} - i\mu)\Gamma(1 - i\kappa + i\tilde{\kappa})}$$

- parameter $c_B^{(\mathrm{out})}$:

$$\begin{aligned} c_B^{(\mathrm{out})} &= -c_2 (2B)^{\frac{1}{2}+i\kappa-i\mu} \frac{\sinh(2\pi\mu)\sinh(\pi\tilde{\kappa} - \pi\kappa)}{\cosh(\pi\tilde{\kappa} - \pi\mu)\cosh(\pi\kappa + \pi\mu)} \\ &\times \frac{\Gamma(1 - i\tilde{\kappa} + i\kappa)\Gamma(2i\mu)}{\Gamma(\frac{1}{2} - i\tilde{\kappa} + i\mu)\Gamma(\frac{1}{2} + i\kappa + i\mu)} \end{aligned}$$

Scalar Pair Production

- Bogoliubov coefficients:

$$|\alpha|^2 = \frac{D_{\text{incident}}}{D_{\text{reflected}}} = \frac{\left|D_H^{(\text{out})}\right|}{\left|D_H^{(\text{in})}\right|} = \frac{\cosh(\pi\kappa - \pi\mu)\cosh(\pi\tilde{\kappa} + \pi\mu)}{\cosh(\pi\kappa + \pi\mu)\cosh(\pi\tilde{\kappa} - \pi\mu)}$$

$$|\beta|^2 = \frac{D_{\text{transmitted}}}{D_{\text{reflected}}} = \frac{\left|D_B^{(\text{out})}\right|}{\left|D_H^{(\text{in})}\right|} = \frac{\sinh(2\pi\mu)\sinh(\pi\tilde{\kappa} - \pi\kappa)}{\cosh(\pi\kappa + \pi\mu)\cosh(\pi\tilde{\kappa} - \pi\mu)}$$

- Absorption cross section:

$$\sigma_{\text{abs}} = \frac{D_{\text{transmitted}}}{D_{\text{incident}}} = \frac{\left|D_B^{(\text{out})}\right|}{\left|D_H^{(\text{out})}\right|} = \frac{\sinh(2\pi\mu)\sinh(\pi\tilde{\kappa} - \pi\kappa)}{\cosh(\pi\kappa - \pi\mu)\cosh(\pi\tilde{\kappa} + \pi\mu)}$$

Thermal Interpretation

- Hamilton-Jacobi action: $R(r) = \mathrm{e}^{iS(r)}$ (WKB)

$$S(r) = \int \frac{dr}{r^2 - B^2} \sqrt{\frac{[\omega(r_0^2 + a^2) - qQ^3 r + 2nar_0 r]^2}{(r_0^2 + a^2)^2} - \bar{m}^2 (r_0^2 + a^2)(r^2 - B^2)}$$

$$\bar{m} = m\sqrt{1 + \frac{\lambda + 1/4}{m^2 (r_0^2 + a^2)}}$$

- Residue contributions of the contour integrate at three simple poles: $r = \pm B$ and $r = \infty$

$$\begin{aligned} S_a &= S_- + S_+ = 2\pi \frac{qQ^3 - 2nar_0}{r_0^2 + a^2} = 2\pi\kappa \\ \tilde{S}_a &= S_- - S_+ = 2\pi \frac{\omega}{B} = 2\pi\tilde{\kappa} \\ S_b &= S_\infty = 2\pi \sqrt{\frac{(qQ^3 - 2nar_0)^2}{(r_0^2 + a^2)^2} - \bar{m}^2 (r_0^2 + a^2)} = 2\pi\mu \end{aligned}$$

Thermal Interpretation

- Phase-integral formula:

$$N_S = \mathrm{e}^{-S_a + S_b} = \mathrm{e}^{-\frac{\bar{m}}{T_{\text{KN}}}}$$

- Temperature for Schwinger effect:

$$T_{\text{KN}} = T_U + \sqrt{T_U^2 + \frac{\mathcal{R}}{8\pi^2}}$$

- Unruh temperature and effective curvature:

$$T_U = \frac{qQ^3 - 2nar_0}{2\pi\bar{m}(r_0^2 + a^2)^2}, \qquad \mathcal{R} = -\frac{2}{r_0^2 + a^2}$$

Thermal Interpretation

- Mean number of produced pairs: extremal limit $\tilde{S}_a \to \infty$

$$|\beta|^2 = \left(\frac{e^{-S_a+S_b} - e^{-S_a-S_b}}{1+e^{-S_a-S_b}}\right)\left(\frac{1-e^{-\tilde{S}_a+S_a}}{1+e^{-\tilde{S}_a+S_b}}\right)$$

- New parameters: $(\mathcal{R} \to 0 \ \Rightarrow \ T_{\text{KN}} = 2T_U, \mathcal{T}_{\text{KN}} = 0)$

$$\mathcal{T}_{\text{KN}} = T_U - \sqrt{T_U^2 + \frac{\mathcal{R}}{8\pi^2}}, \quad \frac{2}{T_M} = \frac{1}{\mathcal{T}_{\text{KN}}} + \frac{1}{T_{\text{KN}}}, \quad \frac{2}{\mathcal{T}_M} = \frac{1}{\mathcal{T}_{\text{KN}}} - \frac{1}{T_{\text{KN}}}$$

- Mean number of produced pairs:

$$\mathcal{N} = \left(\frac{e^{-\frac{\bar{m}}{T_{\text{KN}}}} - e^{-\frac{\bar{m}}{\mathcal{T}_{\text{KN}}}}}{1+e^{-\frac{\bar{m}}{\mathcal{T}_{\text{KN}}}}}\right)\left(\frac{1-e^{-\frac{\omega_t}{T_H}+\frac{\bar{m}}{T_M}}}{1+e^{-\frac{\omega_t}{T_H}+\frac{\bar{m}}{\mathcal{T}_M}}}\right)$$

  - ω_t: near horizon frequencies $\frac{\varepsilon\omega}{r_0^2+a^2}$
  - T_H: Hawking temperature

Dual CFT Description

- Absorption cross section (pair production)

$$\sigma_{\text{abs}} = \frac{\sinh(2\pi\mu)}{\pi^2} \sinh(\pi\tilde{\kappa} - \pi\kappa) \left|\Gamma\left(\frac{1}{2} + i(\mu - \kappa)\right)\right|^2 \left|\Gamma\left(\frac{1}{2} + i(\mu + \tilde{\kappa})\right)\right|^2$$

- from 2-point correlator in 2D CFT

$$\sigma_{\text{abs}} \sim T_L^{2h_L - 1} T_R^{2h_R - 1} \sinh\left(\frac{\tilde{\omega}_L}{2T_L} + \frac{\tilde{\omega}_R}{2T_R}\right) \left|\Gamma\left(h_L + i\frac{\tilde{\omega}_L}{2\pi T_L}\right)\right|^2 \times \left|\Gamma\left(h_R + i\frac{\tilde{\omega}_R}{2\pi T_R}\right)\right|^2$$

- Conformal weight: $h_L = h_R = \frac{1}{2} + i\mu$

Dual CFT Description

- Twofold CFT descriptions for Kerr-Newman black holes

CMC, Huang, Sun, Wu, Zou, PRD 82 (2010) 066004 [arXiv:1006.4097 [hep-th]]

- J-picture: $c_L^J = c_R^J = 12J$ (central charges) temperatures

$$T_L^J = \frac{r_+^2 + r_-^2 + 2a^2}{4\pi a(r_+ + r_-)}, \; T_R^J = \frac{r_+ - r_-}{4\pi a} \quad \Rightarrow \quad T_L^J \sim \frac{r_0^2 + a^2}{4\pi a r_0}, \; T_R^J \sim \frac{B}{2\pi a}$$

- Q-picture: $c_L^Q = c_R^Q = \frac{6Q^3}{\ell}$ (central charges) temperatures

$$T_L^Q = \frac{(r_+^2 + r_-^2 + 2a^2)\ell}{4\pi Q(r_+ r_- - a^2)}, \quad T_R^Q = \frac{(r_+^2 - r_-^2)\ell}{4\pi Q(r_+ r_- - a^2)}$$

$$\Rightarrow \qquad T_L^Q \sim \frac{(r_0^2 + a^2)\ell}{2\pi Q^3}, \quad T_R^Q \sim \frac{r_0 B\ell}{\pi Q^3}$$

- geometrical meaning of ℓ: radius of embedded extra circle

Dual CFT Description

- CFT entropy: (for both pictures)

$$S_{\text{CFT}} = \frac{\pi^2}{3}(c_L T_L + c_R T_R) \sim \pi(r_0^2 + a^2 + 2r_0 B)$$

- Black hole entropy and temperature

$$S_{\text{BH}} = \pi(r_+^2 + a^2) \quad \Rightarrow \quad S_{\text{BH}} \sim \pi(r_0^2 + a^2 + 2r_0 B)$$

$$\hat{T}_H = \frac{r_+ - r_-}{4\pi(r_+^2 + a^2)} \quad \Rightarrow \quad T_H \sim \frac{B}{2\pi}$$

- Identification via first law of thermodynamics ($\delta S_{\text{BH}} = \delta S_{\text{CFT}}$)

$$\frac{\delta M - \Omega_H \delta J - \Phi_H \delta Q}{T_H} = \frac{\tilde{\omega}_L}{T_L} + \frac{\tilde{\omega}_R}{T_R}$$

Dual CFT Description

- angular velocity and chemical potential (at $r = B$)

$$\Omega_H = \frac{2ar_0}{r_0^2 + a^2} B, \qquad \Phi_H = -\frac{Q^3 B}{r_0^2 + a^2}$$

- variation of parameters: $\delta M = \omega, \ \delta J = -n, \ \delta Q = -q$
- CFT "frequencies":

$$J\text{-picture}: \quad \tilde{\omega}_R^J = \frac{\omega}{a}, \qquad \tilde{\omega}_L^J = -\frac{qQ^3 - 2nar_0}{2ar_0}$$

$$Q\text{-picture}: \quad \tilde{\omega}_R^Q = \frac{2r_0 \ell \omega}{Q^3}, \qquad \tilde{\omega}_L^Q = -\frac{(qQ^3 - 2nar_0)\ell}{Q^3}$$

- for both pictures

$$\frac{\tilde{\omega}_L}{2T_L} = -\pi\kappa, \qquad \frac{\tilde{\omega}_R}{2T_R} = \pi\tilde{\kappa}$$

Summary

- Spontaneous pair production of scalar field in near extremal KN black holes: exact Bogoliubov coefficients
- There is a remarkable thermal interpretation.
- The pair production (unstable mode) is holographically dual to an operator with complex conformal weight.