Heavy Quark Observables in Gauge/Gravity Duality

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Today's talk: Study of Heavy Quark Observables
using generic properties of the gravity dual theories.

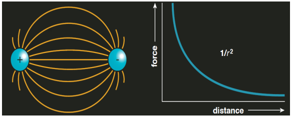
- The width of the chromo-electric flux tube in the confining phase.
- The momentum change of a moving quark in the deconfined phase of the theory.
- "Model Independent"=Universal results!

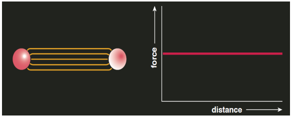
Gauge/Gravity Duality

- Gauge/Gravity correspondence: Quantum questions are mapped to Gravity questions.
- The initial AdS/CFT correspondence ($\mathcal{N} = 4$ sYM $\leftrightarrow AdS_5 \times S^5$) is the harmonic oscillator of the gauge/gravity dualities.
- Example of the Mapping:At 't Hooft limit, the Wilson Loop:

$$W = Tr\left(\mathcal{P}exp \oint_{\mathcal{C}} A_\mu dx^\mu\right) \rightarrow \langle W(\mathcal{C})\rangle = e^{-S(\mathcal{C})}$$

- The static potential: $\langle W(\mathcal{C})\rangle \propto e^{-V_{Q\bar{Q}}T}$.
- Reminder:





Set-up

- Assume the existence of the gravity dual of a theory.
- For most of the observables we can work in full generality by grouping all the backgrounds/theories in

$$ds_{\mathcal{X}}^2 = g_{00}(u)dx_0^2 + \sum g_{ii}(u)dx_i^2 + g_{uu}(u)du^2 \ ,$$

u is the holographic direction and x_μ are the space-time coordinates.

- The background corresponds to several dual field theories, by specifying the metric elements.
  Example: If $\mathcal{X} = AdS_5$ then the dual field theory is $\mathcal{N} = 4$ sYM.

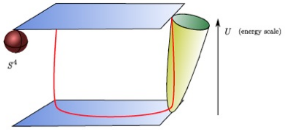
Some Theories

- ✓ Finite Temperature Field Theory in the deconfined phase: Presence of Black hole

$$ds^2 = \frac{1}{u^2}\left(-f(u)dt^2 + d\vec{x}^2 + \frac{du^2}{f(u)}\right) , \quad f(u) = 1 - \frac{u^4}{u_h^4} .$$

Where $u_h = (\pi T)^{-1}$ and T is the temperature of the dual field theory.

- ✓ Confinement. Common background property $g_{00}g_{uu} \to \infty$.
Example: $D4$ Witten model.



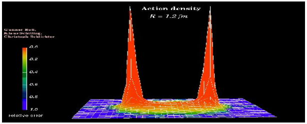
(Fig: 0708.1502)

$$ds^2 = \left(\frac{u}{R}\right)^{3/2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + f(u)dx_4{}^2\right) + \left(\frac{R}{u}\right)^{3/2}\left(\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right) ,$$

$$f(u) = 1 - \left(\frac{u_k}{u}\right)^3$$

Width of the Chromoelectric flux tube

The chromoelectric field energy density between the $Q\bar{Q}$ is confined.



(*Fig:Bali, Schilling, Schlichter, 1995*)

• To measure it we use a small probe Wilson loop $P(c)$, at distance Δx_3 above a large Wilson loop $W(C)$ that corresponds to the heavy quark pair

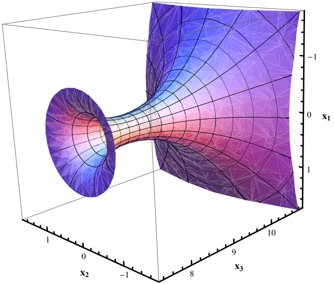
$$\mathcal{S}(x) = \frac{\langle W(C)P(c)\rangle - \langle W(C)\rangle\langle P(c)\rangle}{\langle W(C)\rangle} \ .$$

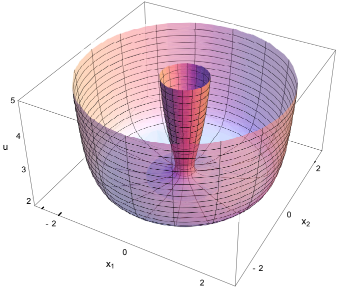
(*Lüsher, Munster, Weisz 1980*)

•The mean square width of the flux tube is then defined as

$$w^2 = \frac{\int d(\Delta x_3)\Delta {x_3}^2 \ \mathcal{S}}{\int d(\Delta x_3) \ \mathcal{S}} \ .$$

Holographically we compute the connected minimal surface between two loops with radii $R \gg r_0$. R is the distance between the $Q\bar{Q}$ pair.





The eoms in static gauge: $(x_1, x_2) \to (r(\sigma), \theta);\ \theta = \tau,\ x_3 = \sigma;\ u(\sigma)$.
With $r(\sigma)$ the radii of the circles, $u(\sigma)$ the holographic coordinate

$$r'' - hr = 0\ ,$$
$$2u'' + u'^2 \partial_u (\ln f) - r^2 \frac{\partial_u h}{f} = 0\ ,$$
$$r'^2 + f u'^2 = hr^2 - 1\,,$$
$$\text{and} \qquad h(u) := \frac{g_{11}^2}{c^2}, \qquad f(u) := \frac{g_{uu}}{g_{11}}\ . \tag{1}$$

For any confining background using its properties ($g_{00} g_{uu} \sim$ *diverges*...)

$$S \simeq \sigma \left(\frac{\Delta x_3^2}{\log \frac{R}{r_0}} + R^2 - r_0^2 \right) .$$

Resulting the logarithmic broadening

$$w^2 \simeq \frac{1}{2\pi\sigma} \log \frac{R}{r_0} .$$

- Reminder: R is the distance between Q and $\bar{\mathrm{Q}}$.

✓Universal feature for any confining holographic theory! (*D.G.,Irges 2015*)

✓ Logarithmic Broadening in lattice Computations. e.g. (*Gliozzi, Pepe, Wiese, 2010; Caselle, Panero, Vadacchino 2016, ...*)

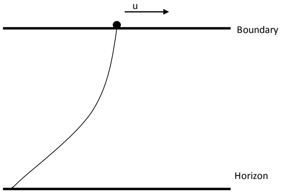
Deconfined phase

- Deconfined Phase: Supergravity Background has a black hole.
- Fundamental Question: How a moving quark interacts with the medium?
  Answer:
- The dynamics and the interactions of the heavy quark can be described by a diffusion treatment.
- The thermal momentum of the heavy quark($\sim m_Q T$) $\gg$ momentum transfer of the medium($\sim T^2$).
- Brownian motion of the heavy quark in a light particle fluid

$$\frac{dp}{dt} = F_{drag} + F(t) \ ,$$

where $F_{drag}(t)$ is the "friction" force and $F(t)$ is the stochastic factor that causes the momentum broadening.

AdS/CFT picture: A trailing string extending from the boundary of $\mathcal{X}^5$ where the probe quark moves with the constant speed v, to the horizon of the black hole. (*Gubser, 2006*)



The string is static, touches the horizon of the background black hole and since it is "bended" it has its own induced black hole.
The world-sheet is parametrized as

$$t = \tau, \quad u = \sigma, \quad x_3 = v\ t + \xi(\sigma).$$

The drag force of a quark moving along the x_3 direction, is given by the momentum flowing from the boundary to the bulk

$$F_{drag} = -\sqrt{\lambda}\frac{\sqrt{-g_{00}g_{33}}}{(2\pi)}\Big|_{u=u_0}$$

where u_0 is the horizon of the induced worldsheet metric given by

$$\left(g_{uu}(g_{00} + g_{33}v^2)\right) |_{u=u_0} = 0 \ .$$

Quick Test: At $v = 0$, the w-s b.h. horizon= background b.h. horizon.

The "effective w-s temperature" is

$$T_{ws}^2 = \left|\frac{1}{16\pi^2}\frac{1}{g_{00}g_{uu}}(g_{00}\, g_{33})'\left(\frac{g_{00}}{g_{33}}\right)'\right|\Big|_{u=u_0} \ .$$

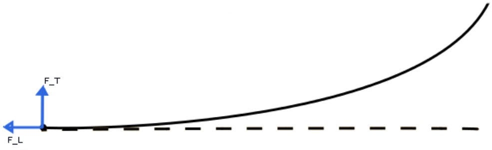
Momentum Broadening

The $F(t)$ is the factor that causes the momentum broadening, which leads to

$$\frac{\left\langle p_{L,T}^2 \right\rangle}{\mathcal{T}} = (2)\kappa_{L,T} \ , \quad \langle F_{L,T}(t) F_{L,T}(t') \rangle = \kappa_{L,T}\,\delta(t-t')$$

$\kappa =$ Mean Squared Momentum Transfer per Time.

- The index L refers to the direction along the motion of quark, the index T is the direction transverse to the velocity of quark.



- In strong coupling limit for a quark moving along x_3 direction, we compute the two point function by studying the effect of fluctuations to the Wilson line

$$t = \tau, \quad u = \sigma, \quad x_3 = v\ t + \xi(\sigma) + \delta x_3(\tau,\sigma), \quad x_{1,2} = \delta x_{1,2}(\tau,\sigma)\ .$$

Their ratio of the Langevin coefficients can be simplified to

$$\frac{\kappa_L}{\kappa_T} = \frac{1}{g_{33} g_{11}} \frac{(g_{00} g_{33})'}{(g_{00}/g_{33})'}\Bigg|_{u=u_0}$$

Reminder: Quark moves along the direction x_3, and the transverse direction to motion is x_1.

- For any isotropic theory $g_{33} = g_{11}$ and $g_{00} = g_{00,bh}\ g_{11} f$, we prove $\kappa_L > \kappa_T$.
- This is a Universal Inequality independent of the background used!

(D.G, Soltanpanahi, 2013a; Gursoy, Kiritsis, Mazzanti, Nitti 2010)

- The only possibility to have violation of the inequality is in the anisotropic theories! In the axion anisotropic theories

$$\frac{\kappa_L}{\kappa_T} = \gamma^2 + \frac{\alpha^2}{24\pi^2 T^2}\left((\gamma-1)(4+\gamma+2\gamma^2) - 2\gamma^2(2+\gamma^2)\log\left(1+\gamma^{-1}\right)\right) < 1$$

(D.G, Soltanpanahi, 2013b)

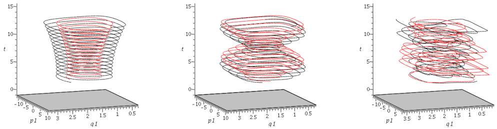
Conclusions

Working with a large class of theories we find universal behaviors among them.

- In confining theories, all the backgrounds give the logarithmic flux tube broadening.
- The Universal Langevin coefficients inequality $\kappa_L > \kappa_T$ proved to hold for isotropic backgrounds is violated for the anisotropic theories!

Similar treatment:

- The energy of the *k*-strings is proportional to the energy of a meson for a large class of theories. *(D.G. 2015)*
- Non-integrability and chaotic observables for classes of theories.



(D.G., Sfetsos 2014; D.G, Pando-Zayas, Zoubos 2013)

Thank you