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# EFFECT OF NEUTRINO ON CMB

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*KIAS-NCTS JOINT WORKSHOP ON PARTICLE  
PHYSICS, STRING THEORY AND COSMOLOGY*



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# OUTLINE

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- ❖ PROBES FOR NEUTRINO IN COSMOLOGY
- ❖ MASSIVE NEUTRINO IN COSMOLOGY
- ❖ MASSLESS NEUTRINO IN COSMOLOGY
- ❖ EFFECT ON CMB
- ❖ CONCLUSION



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# PROBES FOR NEUTRINOS IN COSMOLOGY

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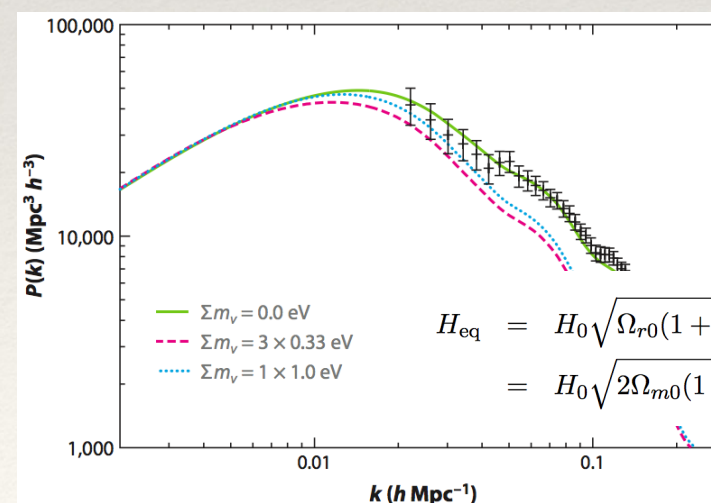
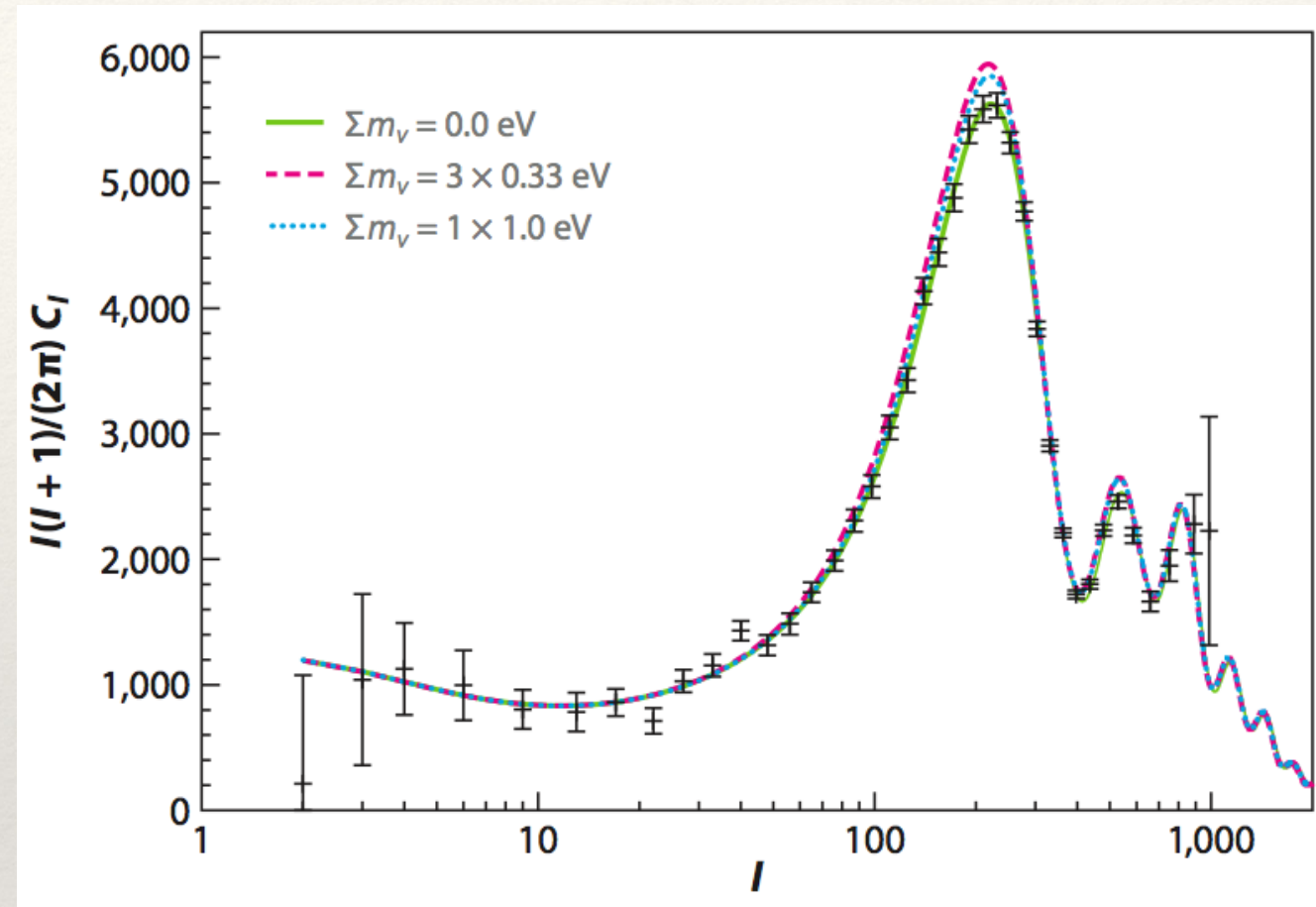
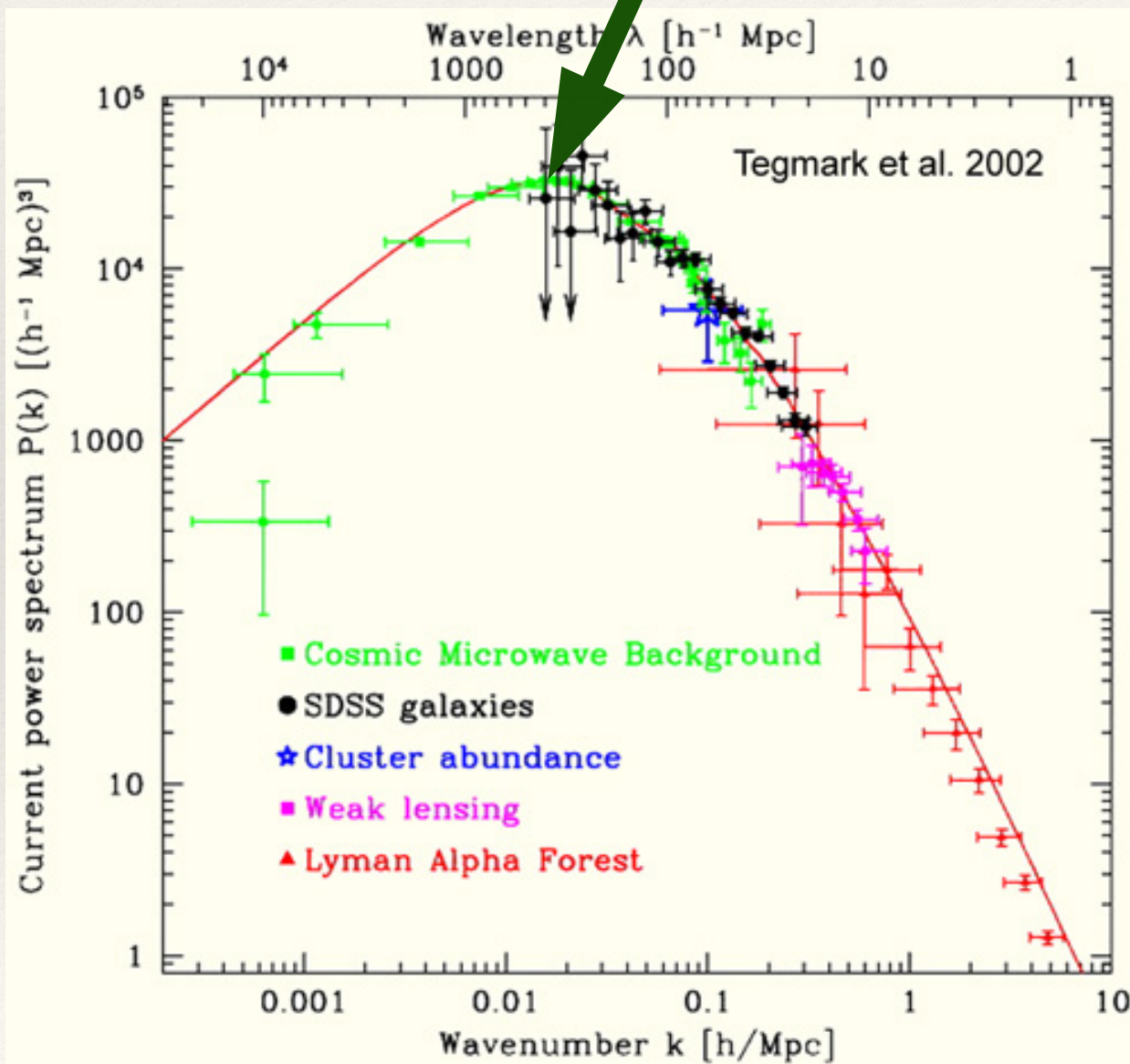
- ❖ Radiation and Matter equality epoch ( $z_{\text{eq}} > z$  : no matter perturbation can grow) : LSS (both massive and massless  $\nu_s$ )
- ❖ Amplitude of matter power spectrum : LSS (massive neutrino)
- ❖ Acoustic angular size at the last scattering surface ( $z_{\text{ls}}$ ) : CMB T and P anisotropies
- ❖ Amplitude of primordial gravitational waves : CMB P anisotropies + GWs



# COSMOLOGICAL PROBES

## ❖ LSS vs CMB

$$ck_{\text{eq}} = a_{\text{eq}} H_{\text{eq}}$$



Y. Wong et.al 11

$$H_{\text{eq}} = H_0 \sqrt{\Omega_{r0}(1+z_{\text{eq}})^4 + \Omega_{m0}(1+z_{\text{eq}})^3 + \Omega_{w0}(1+z_{\text{eq}})^{3(1+w)}}$$

$$= H_0 \sqrt{2\Omega_{m0}(1+z_{\text{eq}})^3 + \Omega_{w0}(1+z_{\text{eq}})^{3(1+w)}} \simeq H_0 \sqrt{2\Omega_{m0}(1+z_{\text{eq}})^3}$$



# MASSIVE NEUTRINO IN COSMOLOGY I

- ❖ For cold dark matter (CDM), perturbations of sub-horizon scales ( $aH \ll k < k_{nl}$ ),  $\delta_{cdm}$  grow uniformly ( $D(a)$  no  $k$  dependence)

- ❖ Eq for  $\delta_{cdm}$   $\ddot{\delta}_{cdm} + \frac{\dot{a}}{a}\dot{\delta}_{cdm} - 4\pi G a^2 \delta\rho = 0$  where  $\dot{\delta} \equiv \frac{d\delta}{d\eta}$   $\delta\rho \equiv \delta\rho_{cdm} + \delta\rho_b$

- ❖ Growing mode solution for  $\delta_{cdm}$   $\delta_{cdm} \propto a$  (in MD) , or  $D(a)$  (with DE)

- ❖ Matter power spectrum of CDM

$$P_{cdm}(k, a) \equiv \left\langle \left| \delta_{cdm}(k, a) \right|^2 \right\rangle = AT^2(k) \left( \frac{k}{k_0} \right)^{n_s-1} \left( \frac{D_{cdm}(a)}{D_{cdm0}} \right)^2$$

However this is **not** an observed quantity (galaxy PS)



# MASSIVE NEUTRINO IN COSMOLOGY II

- ❖ For cold dark matter (CDM) with  $\nu$ , perturbations of sub-horizon scales smaller than free-streaming scale ( $k > k_{\text{fs}}$ ),  $\delta_{\text{cdm}}$  is **suppressed** ( $P > g$ )

- ❖ Eq for  $\delta_{\text{cdm}}$  with massive  $\nu$ , 
$$\ddot{\delta}_{\text{cdm}} + \frac{\dot{a}}{a} \dot{\delta}_{\text{cdm}} - 4\pi G a^2 (\rho_{\text{m}} - \rho_{\nu}) \delta_{\text{cdm}} = 0$$

- ❖ Growing mode solution for  $\delta_{\text{cdm}}$

$$\delta_{\text{cdm}} \propto a^{1-\frac{3}{5}f_{\nu}} \text{ (in MD) , or } (aD_{\text{cdm}})^{1-\frac{3}{5}f_{\nu}} \text{ (with DE)} \quad f_{\nu} = \frac{\Omega_{\nu}}{\Omega_{\text{m}}}$$

$$\Omega_{\nu 0} = \frac{8\pi G \rho_{\nu 0}^{\text{nr}}}{3H_0^2} = \frac{8\pi G n_{\nu 0}}{3H_0^2} \sum_{i=1}^{N_{\nu}^{\text{nr}}} m_{\nu,i} \simeq \frac{\sum_i m_{\nu,i}}{94.1 h^2 \text{eV}}$$

can constrain sum of  $\nu$  masses



# MASSIVE NEUTRINO IN COSMOLOGY III

- ❖ Suppression factor,  $(1-f_\nu)^2$  in the matter power spectrum (for  $k > k_{\text{nr}}$ ), give the limit on the sum of masses of  $\nu_s$
- ❖ Overall suppression factor in CDM PS is

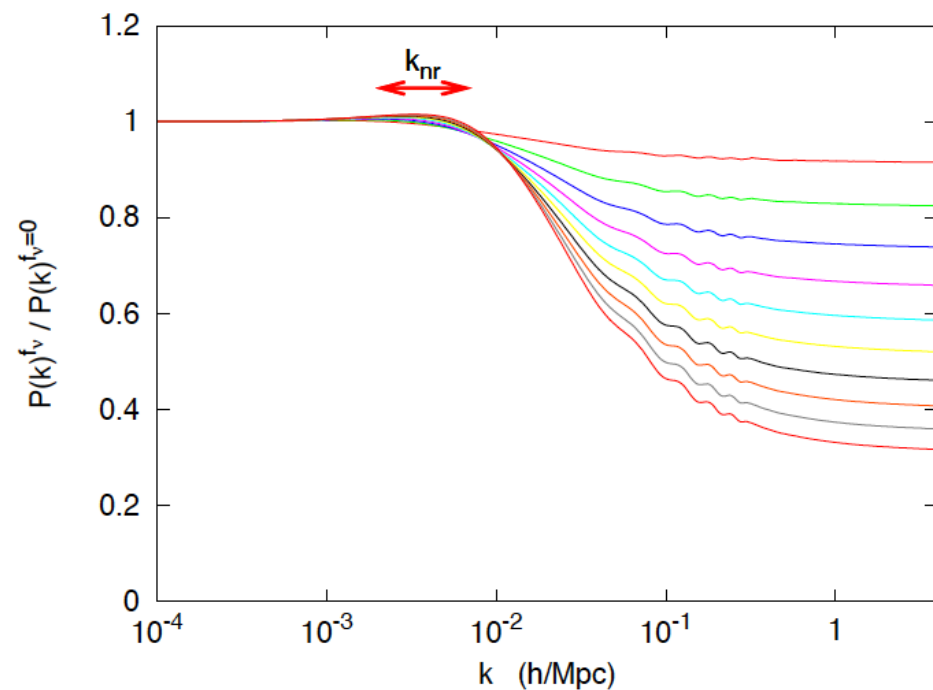


Fig. 13. Ratio of the matter power spectrum including three degenerate massive neutrinos with density fraction  $f_\nu$  to that with three massless neutrinos. The parameters  $(\omega_m, \Omega_\Lambda) = (0.147, 0.70)$  are kept fixed, and from top to bottom the curves correspond to  $f_\nu = 0.01, 0.02, 0.03, \dots, 0.10$ . The individual masses  $m_\nu$  range from 0.046 eV to 0.46 eV, and the scale  $k_{\text{nr}}$  from  $2.1 \times 10^{-3} h \text{ Mpc}^{-1}$  to  $6.7 \times 10^{-3} h \text{ Mpc}^{-1}$  as shown on the top of the figure.  $k_{\text{eq}}$  is approximately equal to  $1.5 \times 10^{-2} h \text{ Mpc}^{-1}$ .

Lesgourgues & Pastor 06

$$\left(\frac{\Delta P}{P}\right) \approx -8 \frac{\Omega_\nu}{\Omega_m} \approx -0.8 \left(\frac{m_\nu}{1 \text{ eV}}\right) \left(\frac{0.1 N}{\Omega_m h^2}\right)$$

at  $z = 0$

$$P(k, z) = \begin{cases} \left(\frac{g(z)}{(1+z)g(0)}\right)^2 P(k, 0) & \text{for } aH < k < k_{\text{nr}} \\ \left(\frac{g(z)}{(1+z)g(0)}\right)^{2-6/5 f_\nu} P(k, 0) & \text{for } k \gg k_{\text{nr}} \end{cases}$$



# MASSIVE NEUTRINO IN COSMOLOGY IV

Jimenez et.al 10

- ❖ Massive neutrino with mass hierarchy
- ❖ Perturbation of massive neutrinos can grow only for scale bigger than free-streaming scale ( $k < k_{\text{fs}}$ )
- ❖ If we consider two degenerated neutrinos with normal and inverted hierarchy, then PS

$$\frac{k^3 P(k; z)}{2\pi^2} = \Delta_R^2 \frac{2k^2}{5H_0^2 \Omega_m^2} D_\nu^2(k, z) T^2(k) \left( \frac{k}{k_0} \right)^{(n_s-1)}$$

$$f_{\nu,i} = \frac{\Omega_{\nu,i}}{\Omega_m} = 0.05 \left( \frac{m_{\nu_i}}{0.658 \text{eV}} \right) \left( \frac{0.14}{\Omega_m h^2} \right)$$

$$k_{\text{fs},i} = 0.113 \left( \frac{m_{\nu_i}}{1 \text{eV}} \right)^{1/2} \left( \frac{\Omega_m h^2}{0.14} \frac{5}{1+z} \right)^{1/2} \text{Mpc}^{-1}$$

$$D_{\nu_i}(k, z) \propto (1 - f_{\nu_i}) D(z)^{1-p_i}$$

$$D_\nu(k, z) = D(k, z) \quad k < k_{\text{fs},m}$$

$$D_\nu(k, z) = (1 - f_{\nu,m}) D(z)^{(1-p_m)} \quad k_{\text{fs},m} < k < k_{\text{fs},\Sigma}$$

$$D_\nu(k, z) = (1 - f_{\nu,\Sigma}) D(z)^{(1-p_\Sigma)} \quad k > k_{\text{fs},\Sigma},$$

where  $k \gg k_{\text{fs},i}(z)$  and  $p_i = (5 - \sqrt{25 - 24f_{\nu_i}})/4$ .

$$\text{NH : } \quad \Sigma = 2m + M \quad \Delta = (M - m)/\Sigma$$

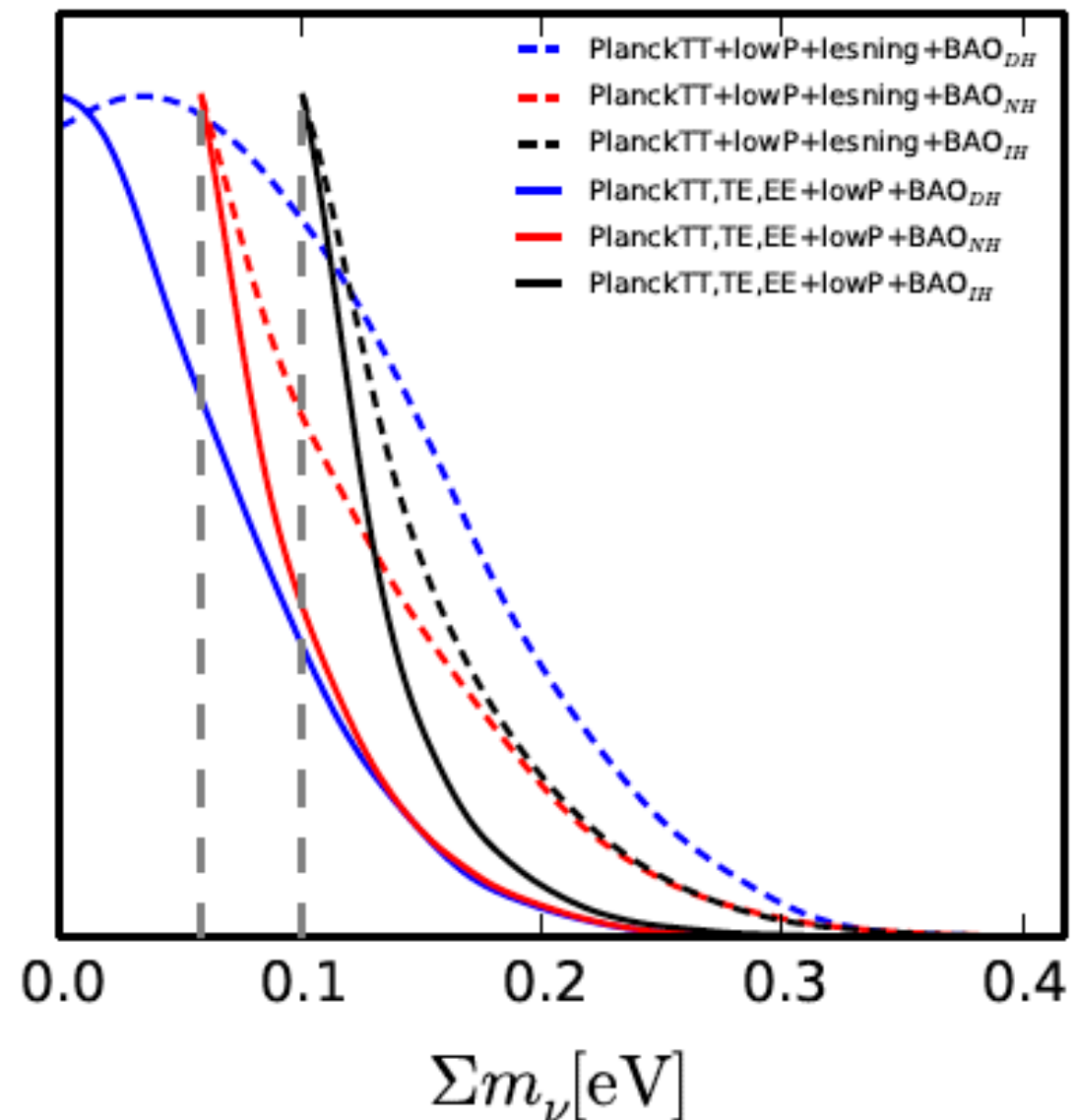
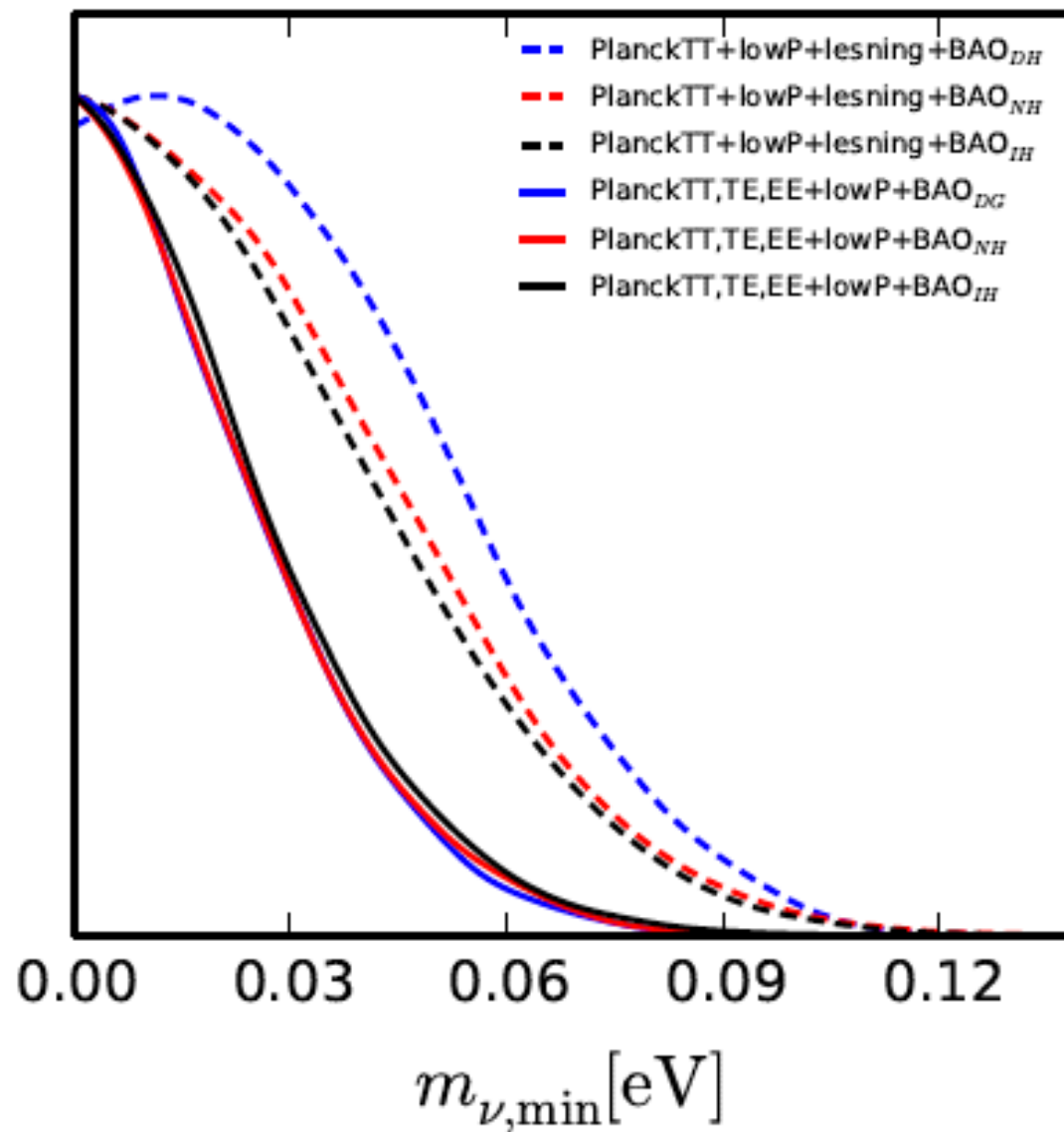
$$\text{IH : } \quad \Sigma = m + 2M \quad \Delta = (m - M)/\Sigma$$



# MASSIVE NEUTRINO IN COSMOLOGY V

## ❖ Current constraints

Huang 15





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# MASSLESS NEUTRINO IN COSMOLOGY I

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SL 14

- ❖ Acoustic angular scale (for location of first few peaks)
- ❖ Diffusion angular scale (for location of higher peaks)

$$\frac{\theta_s[z, N_{\text{eff}}, w, h]}{\theta_d(z_*)[z, N_{\text{eff}}, w, h, Y_P]} = \frac{r_s(z_*)[z, N_{\text{eff}}, w, h]}{r_d(z_*)[z, N_{\text{eff}}, w, h, Y_P]}$$

- ❖ Degeneracy between # of massless neutrino and other parameters ( $w, h, Y_p$ )



# MASSLESS NEUTRINO IN COSMOLOGY II

## ❖ Observationally indistinguishable CMB

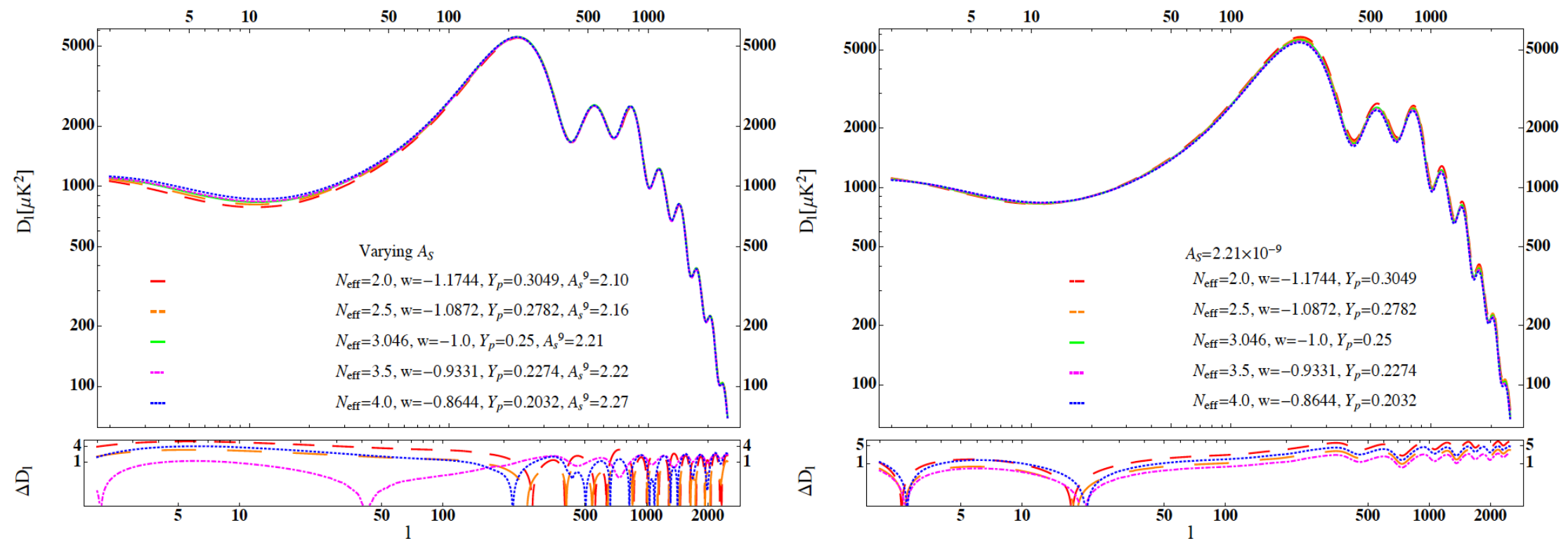


FIG. 1: CMB angular power spectra for different models and their differences from the fiducial model with different normalization. *Top left*) CMB angular power spectra for  $N_{\text{eff}} = 2$  (dashed), 2.5 (long-dashed), 3.046 (solid), 3.5 (dot-dashed), and 4 (dotted), respectively. *Bottom left*) The differences of CMB power spectra between  $N_{\text{eff}} = 2$  (2.5, 3.5, 4.0) model and the fiducial one depicted by dashed (long-dashed, dot-dashed, dotted) line. *Top right*) CMB angular power spectra using the same  $A_s(10^9)$ . *Bottom right*) The differences of CMB power spectra between models with the same notation as the left panel.



# MASSLESS NEUTRINO IN COSMOLOGY III

## ❖ Degeneracies between $N_{\text{eff}}$ , $w$ , and $Y_p$

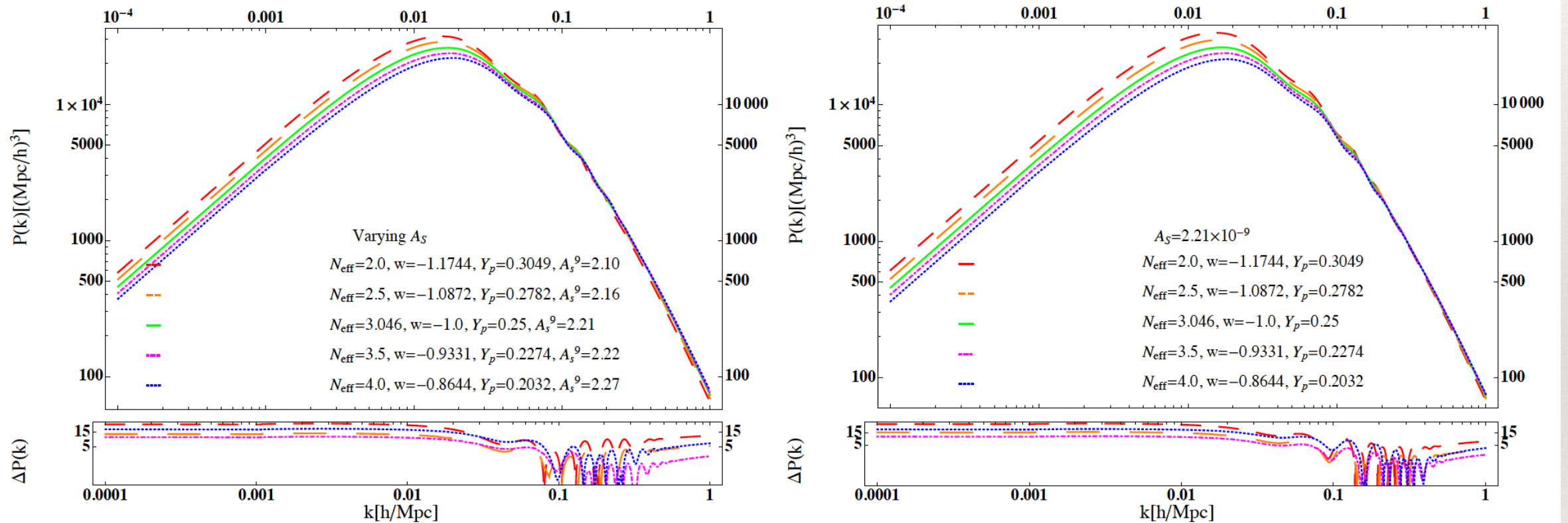


FIG. 2: Linear matter power spectra for different models and their differences from the fiducial model with different normalization. *Top left*) CMB angular power spectra for  $N_{\text{eff}} = 2$  (dashed), 2.5 (long-dashed), 3.046 (solid), 3.5 (dot-dashed), and 4 (dotted), respectively. *Bottom left*) The differences of CMB power spectra between  $N_{\text{eff}} = 2$  (2.5, 3.5, 4.0) model and the fiducial one depicted by dashed (long-dashed, dot-dashed, dotted) line. *Top right*) CMB angular power spectra using the same  $A_s(10^9)$ . *Bottom right*) The differences of CMB power spectra between models with the same notation as the left panel.



# MASSLESS NEUTRINO IN COSMOLOGY IV

## ❖ Degeneracies between $N_{\text{eff}}$ , $w$ , and $Y_p$

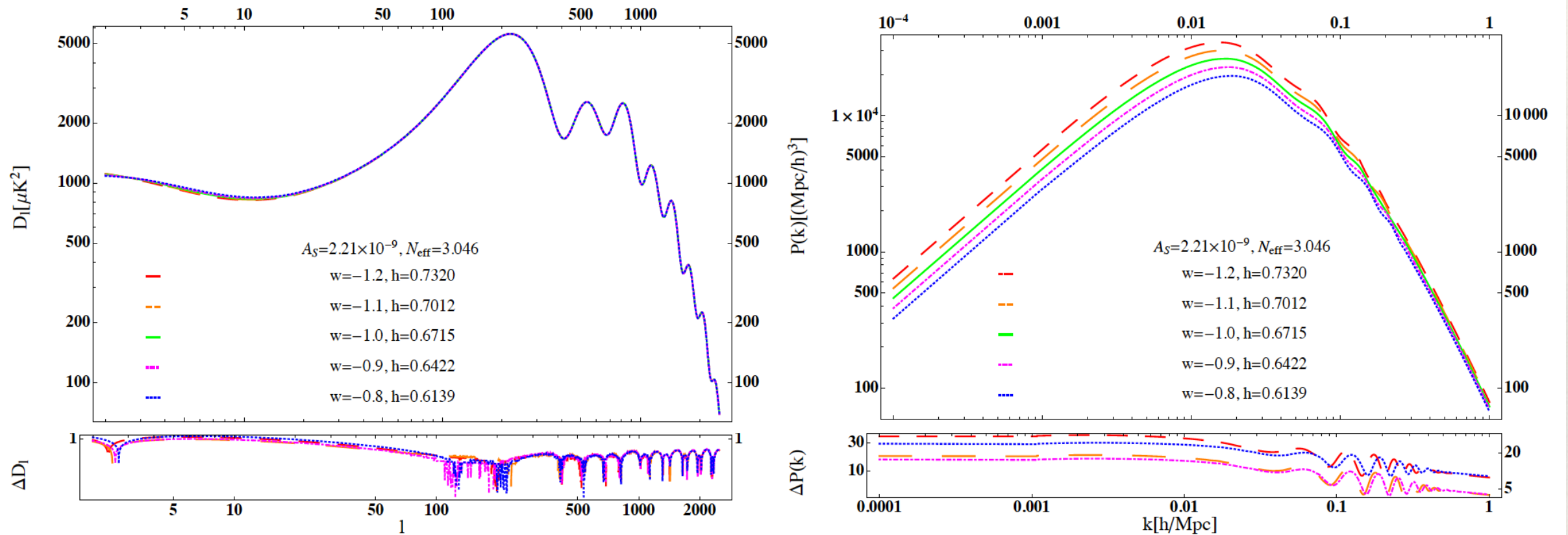


FIG. 3: CMB angular power spectra for different models and their differences from the fiducial model with different normalization. *Top left*) CMB angular power spectra for  $N_{\text{eff}} = 2$  (dashed), 2.5 (long-dashed), 3.046 (solid), 3.5 (dot-dashed), and 4 (dotted), respectively. *Bottom left*) The differences of CMB power spectra between  $N_{\text{eff}} = 2$  (2.5, 3.5, 4.0) model and the fiducial one depicted by dashed (long-dashed, dot-dashed, dotted) line. *Top right*) CMB angular power spectra using the same  $A_S(10^9)$ . *Bottom right*) The differences of CMB power spectra between models with the same notation as the left panel.



# MASSLESS NEUTRINO IN COSMOLOGY V

## ❖ Degeneracies between $N_{\text{eff}}$ , $h$ , and $A_s$

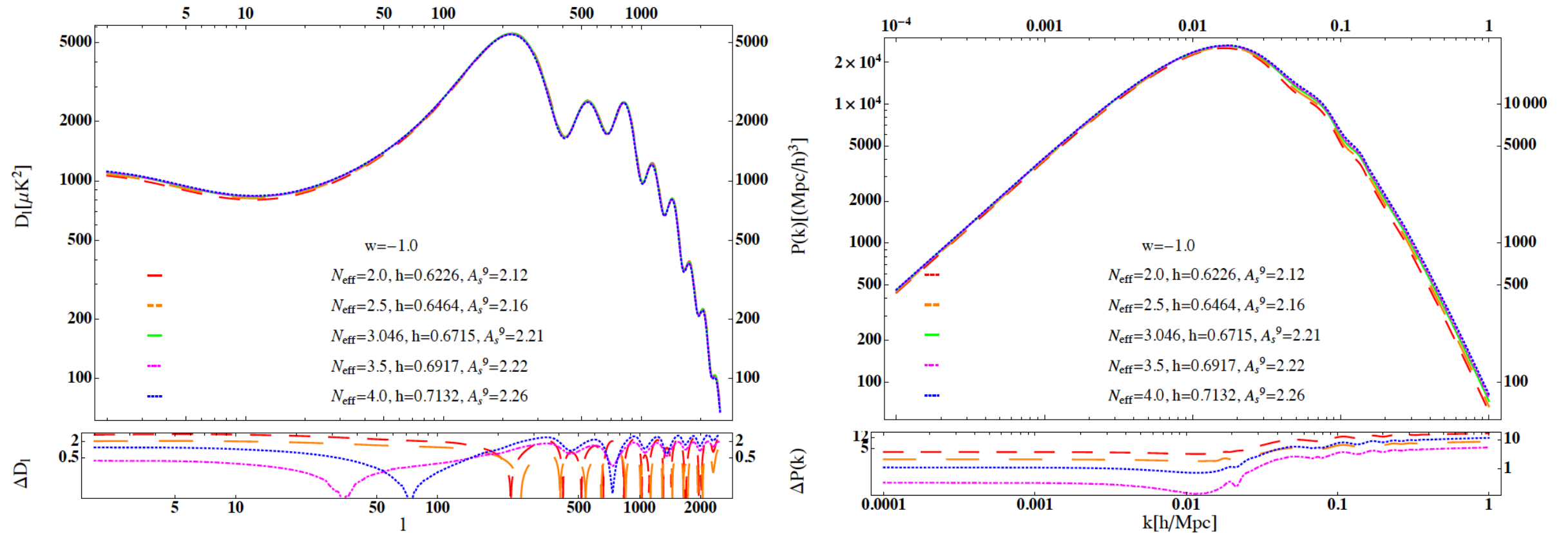
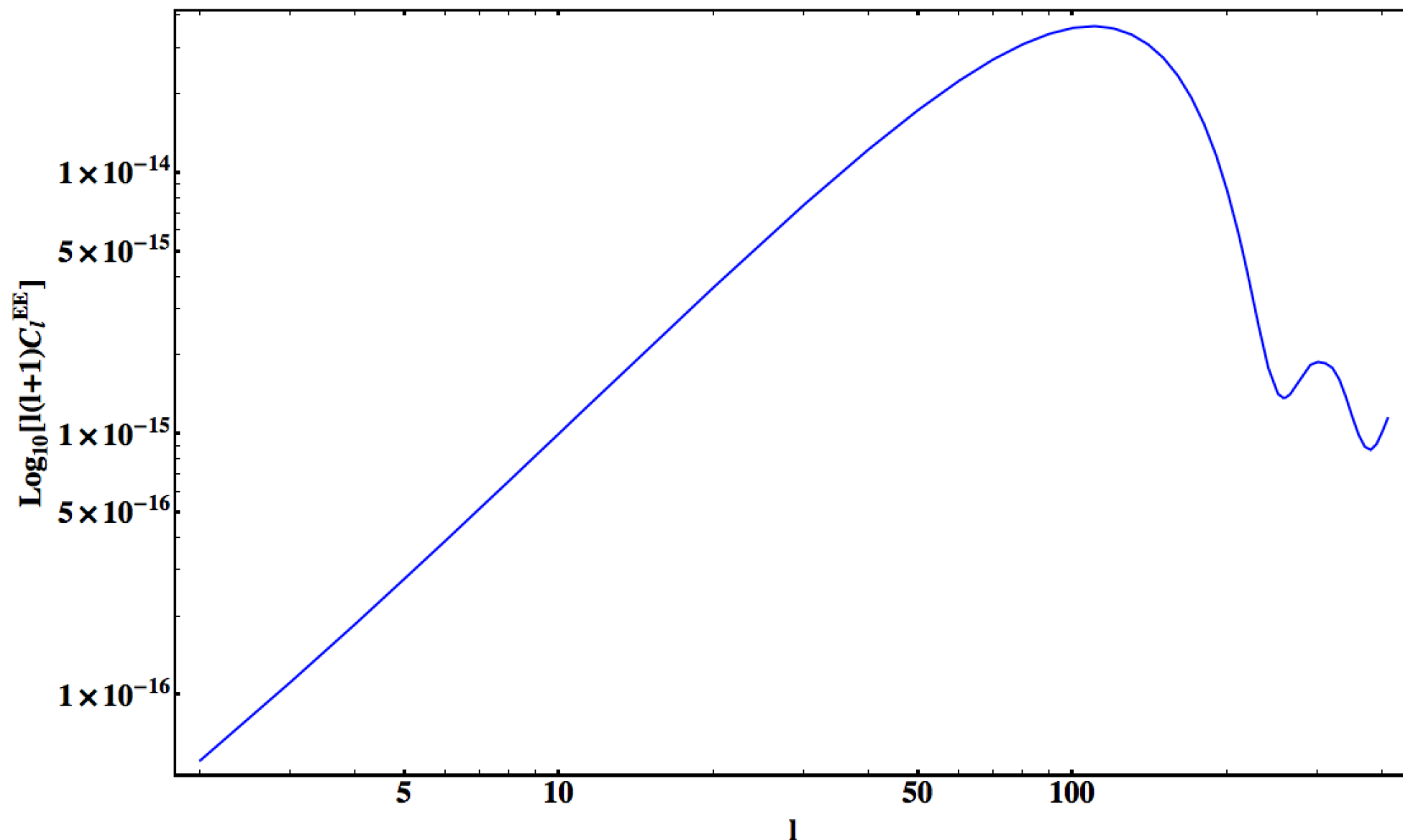


FIG. 4: CMB angular power spectra for different models and their differences from the fiducial model with different normalization. *Top left*) CMB angular power spectra for  $N_{\text{eff}} = 2$  (dashed), 2.5 (long-dashed), 3.046 (solid), 3.5 (dot-dashed), and 4 (dotted), respectively. *Bottom left*) The differences of CMB power spectra between  $N_{\text{eff}} = 2$  (2.5, 3.5, 4.0) model and the fiducial one depicted by dashed (long-dashed, dot-dashed, dotted) line. *Top right*) CMB angular power spectra using the same  $A_s(10^9)$ . *Bottom right*) The differences of CMB power spectra between models with the same notation as the left panel.



# MASSLESS $\nu$

❖ In CMB Polarization





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# REALITY I

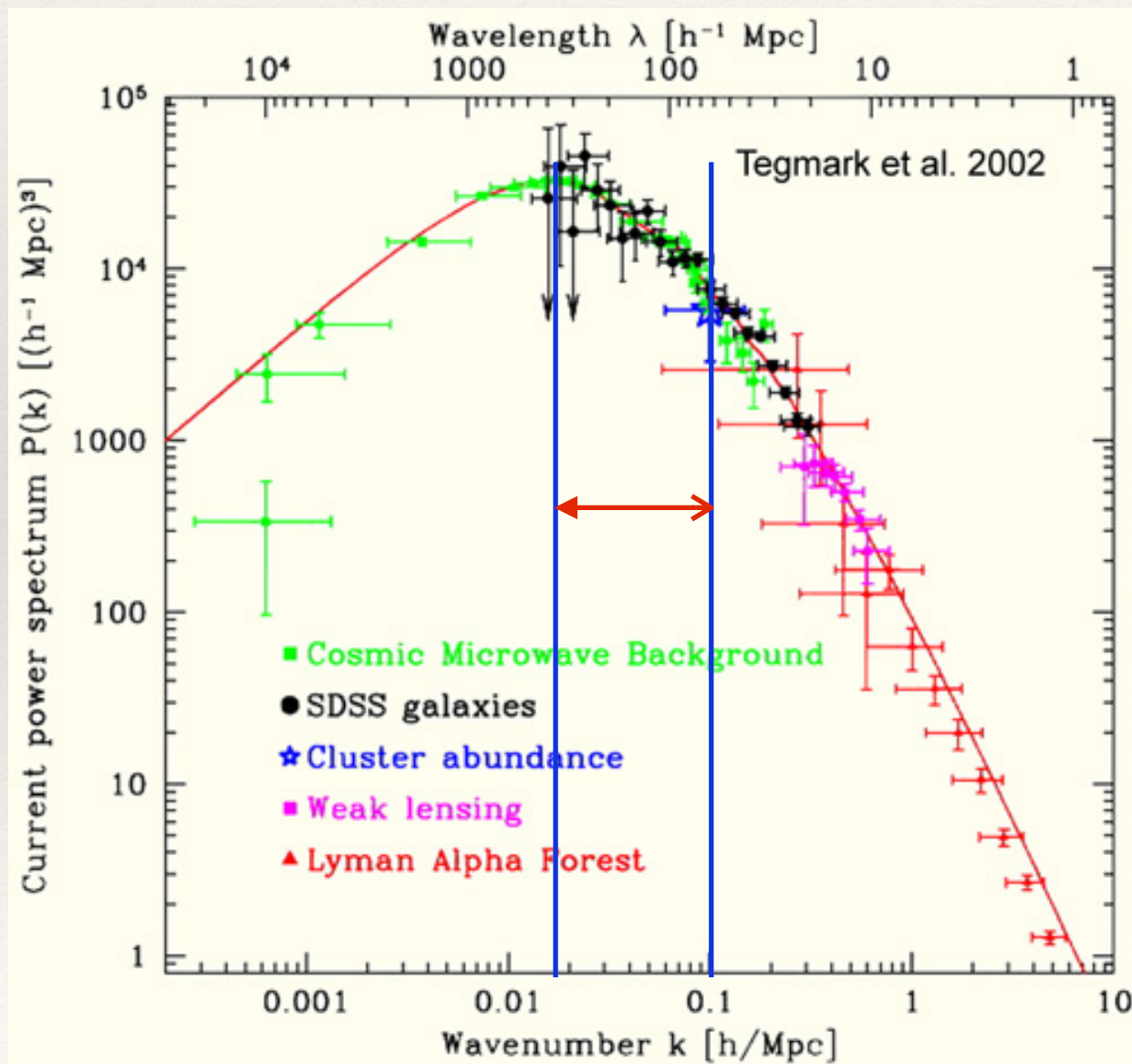
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- ❖ Both massless and massive neutrinos is degenerated by CMB
- ❖ Theoretically, both massless and massive neutrino can be distinguished by LSS
- ❖ However, what we measure is the galaxy (baryon) matter power spectrum not the DM PS (bias factor)
- ❖  $k > 0.1 \text{ h/Mpc}$  is non-linear regime (difficulty in handle on perturbation)



# REALITY II

## ❖ baryon vs DM perturbation



$$\delta_b(k, a) = b(k, a) \delta_{\text{DM}}(k, a)$$

What we calculate

$$P_b(k, a) = b(k, a)^2 P_{\text{DM}}(k, a)$$

What we measure

$$b(k, a) \simeq b(a) \text{ , in linear regime}$$

Mode coupling in non-linear regime

$$\langle \delta(k_1, a) \delta(k_2, a) \rangle \neq P(k, a) \delta_D(k_1 - k_2)$$



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# CONCLUSION

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- ❖ Massive neutrino mainly affects on LSS
- ❖ Massless neutrino mainly affects on CMB
- ❖ However, effects of neutrino mass and number are degenerated with other cosmological parameters
- ❖ LSS with PS are unknown both in theory and in observation
- ❖ Better to use the standard ruler or candle methods (**distance** instead of amplitude) or **bias free** observation