Thermal transport of the solar captured dark matter and its implication

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Outline

- Motivation
- Heat exchange process:
  - Injection
  - Dissipation
  - Thermal contact
- How it evolves inside the Sun
- Numerical results
- Summary
Motivation

- Recent studies suggest DM is not collisionless
- DM can *talk* to others. Energy transport is possible
- The DM captured in the Sun is not necessarily to have $T_\chi = T_c$
- What condition makes $T_\chi \neq T_c$ after the equilibrium state achieves
- The consequence of $T_\chi \neq T_c$ to the indirect DM search
- Outlook on the stellar evolution
What’s the story?
Halo of the Milky Way

Solar system says hello!

Soaking in a DM reservoir

~30 kpc

~40 - ? kpc
Boundary yet unknown
Infalling DM scattered with nucleus or captured DM at $r$

DM has velocity $w(r)$ at layer $r$ due to angular momentum conservation:

$$w(r) = \sqrt{u^2 + v_{esc}(r)}$$
A probability that DM will lose enough kinetic and being bounded gravitationally inside the Sun

\[ \frac{u^2}{w^2} \leq \frac{-\Delta E}{E} \leq \frac{4m_A}{(m_\chi + m_A)^2} \]
Each DM carries initial kinetic energy $E_k$

If captured via nucleus, each DM carries *average* kinetics

$$E_k = \frac{m_\chi}{4} \left( \frac{m_\chi - m_A}{m_\chi + m_A} \right)^2 u^2 + \frac{m_\chi}{2} \frac{m_\chi^2 + m_A^2}{(m_\chi + m_A)^2} v_{esc}^2$$

For DM self-captured

$$E_k = \frac{1}{4} m_\chi v_{esc}^2$$
Microscopically, we see scatterings among particles.

Thermodynamical viewpoint:
- Heat exchange between particles
- Heat flows: $T_{\text{high}}$ to $T_{\text{low}}$
Which collides more frequently?

- Smaller mean collision time $\Rightarrow$ Collides frequently $\Rightarrow$ Exchange heat more efficient $\Rightarrow$ Thermal equilibrium faster

$$
\begin{align*}
\tau_{xx}(t) &= \frac{V_\odot}{N(t)\sigma_{xx} \bar{v}} & \text{DM-DM} \\
\tau_{x\odot} &= \frac{V_\odot}{\sum_A N_A \sigma_{xA} \bar{v}} & \text{DM-nucleus}
\end{align*}
$$

- The time scale for DM to reach thermal equilibrium is $t = \tau_{eq}$, thus we have

$$\tau_{xx}(\tau_{eq}) = \tau_{eq} = \sqrt{\frac{V_\odot}{(C_c \sigma_{xx} \bar{v})}}$$

- Numerically, we found that under current constraints:

$$\frac{\tau_{eq}}{\tau_{x\odot}} \approx \mathcal{O}(0.1) \quad \text{DM gets its own thermal equilibrium faster}$$

$$\frac{\tau_{eq}}{t_\odot} \approx \mathcal{O}(10^{-3}) \quad \text{Thermal equilibrium happens far earlier than the age of the Sun}$$
All DMs in the Sun form a system with each has an average temperature $T_\chi$.

The system now exchanges heat with nuclei in the Sun.
Each annihilation takes $2E_\chi$ away from the DM system.
The physical quantities

- $J_c$ & $J_s$ determine the energy injection of the halo DM
- $J_a$ determines the energy taken away by the annihilation process
- $J_x$ describes the energy transport efficiency between DM and nuclei
- The energy flows between DM and the Sun is easier than with smaller $\sigma_{\chi p}$
- The sign is given by:

$$\begin{align*}
T_\chi > T_c, & \quad J_\chi < 0 \quad \text{energy flows out} \\
T_\chi < T_c, & \quad J_\chi > 0 \quad \text{energy flows into}
\end{align*}$$
Physical analysis
The evolution equations

- In principle, we describe the DM number in the Sun $N_\chi$ by
  \[ \frac{dN_\chi}{dt} = C_c + C_s N_\chi - C_a N_\chi^2 \]

- In order to incorporate the energy flows in the system, additional transport equation should be added
  \[ \frac{d(N_\chi E_\chi)}{dt} = J_c + (J_s + J_x) N_\chi - J_a N_\chi^2 \]

- Thus the DM temperature $T_\chi$ is linked with energy by the Boltzmann constant $k_B$ and $s$ is the degree of freedom
  \[ E_\chi = \frac{s}{2} k_B T_\chi \]
Energy injection

- Rate of average energy injection per volume
  \[
  \int du f(u) \Omega \times E_k \]
  kinetic energy carried by DM during the capture
  rate being captured per volume

- Energy injection via the gravitational capture of the entire Sun
  \[
  J_c \propto \int dV \left[ \int du f(u) \Omega E_k \right] dV
  \]
  integrate over the solar volume

- So does the DM self-injection \( J_s \): \( m_A = m_\chi \)
Dissipation and thermal contact

- The dissipation effect
  \[ J_a \propto n_{\chi}^2 \langle \sigma v \rangle \times 2E_{\chi} \]  
  \text{energy cost per annihilation}
  \text{rate of annihilation}

- Heat flow due to thermal contact
  \[ J_x \propto n_A n_{\chi} \sigma_{\chi A} \bar{v} \times \Delta T \]  
  \text{temp. diff.}
  \text{rate of DM collision}
  \text{with the nuclei}

- Sign of \( J_x \) determines that the energy flows in or out the DM system
Connection between the two

- The annihilation coefficient $C_a$
  \[ C_a \propto T_\chi^{-3/2} \]
- How temperature evolving will affect the DM number in the Sun
- The $E_\chi$ here is taken to be the average DM energy
- The transport equation is valid when $t > \tau_{eq}$
- Otherwise, we need to deal with non-equilibrium thermodynamics
The implications
$N_\chi$ and $T_\chi$: larger $\sigma_{\chi p}$

- $(\sigma_{\chi p},\sigma_{\chi\chi})=(10^{-45},10^{-23})$ cm$^2$
- In this case, $J_x$ is efficient thus it can overcome the energy injection due to $J_c$ and $J_s$
- Eventually $T_\chi$ will be balanced by the solar temperature $T_c$
$N_\chi$ and $T_\chi$: smaller $\sigma_{\chi p}$

- $(\sigma_{\chi p}, \sigma_{\chi\chi}) = (10^{-47}, 10^{-23})$ cm$^2$
- $\sigma_{\chi p}$ is smaller by 2 orders. $J_x$ is less efficient than the previous one
- Energy injection due to $J_c$ and $J_s$ can overcome the thermal contact and dissipation
- Eventually $T_\chi$ will differ from the solar temperature $T_c$
Consequence of temp. correct.

- It is the total annihilation rate $\Gamma_A$ plays the significant role in the indirect DM search
  
  \[ N \propto \Gamma_A \times t \]

- $N$ is the event number and $t$ the detector operating time

- For $\sigma_{\chi p} = 10^{-45} \text{ cm}^2$, the temperature correction is insignificant

- It’s on the contrary for $\sigma_{\chi p} = 10^{-47} \text{ cm}^2$

- However, in the choice of $\sigma_{\chi \chi} = 10^{-24} \text{ cm}^2$, it makes the DM not in a maximum $N_{\chi}$ stage. Thus $\Gamma_A$ is smaller than w/o temperature correction
To summarize so far...
Summary

❖ Solar captured DM:
  ❖ The assumption $T_\chi = T_c$ is unnecessary. They could be different eventually
  ❖ To accurately calculate $\Gamma_A$, $T_\chi$ correction should be involved

❖ Outlook on the stellar evolution:
  ❖ The Sun is not essentially a heat bath during its early stage of formation
  ❖ Energy injection and heat exchange of DM may alter the evolution process and its final temperature