Cosmic Inflation without Inflaton

Hyun Seok Yang
CQUeST, Sogang University

KIAS-NCTS Joint workshop on Particle Physics, String theory and Cosmology

January 31 ~ February 04 (Thu), 2016
High 1 Resort

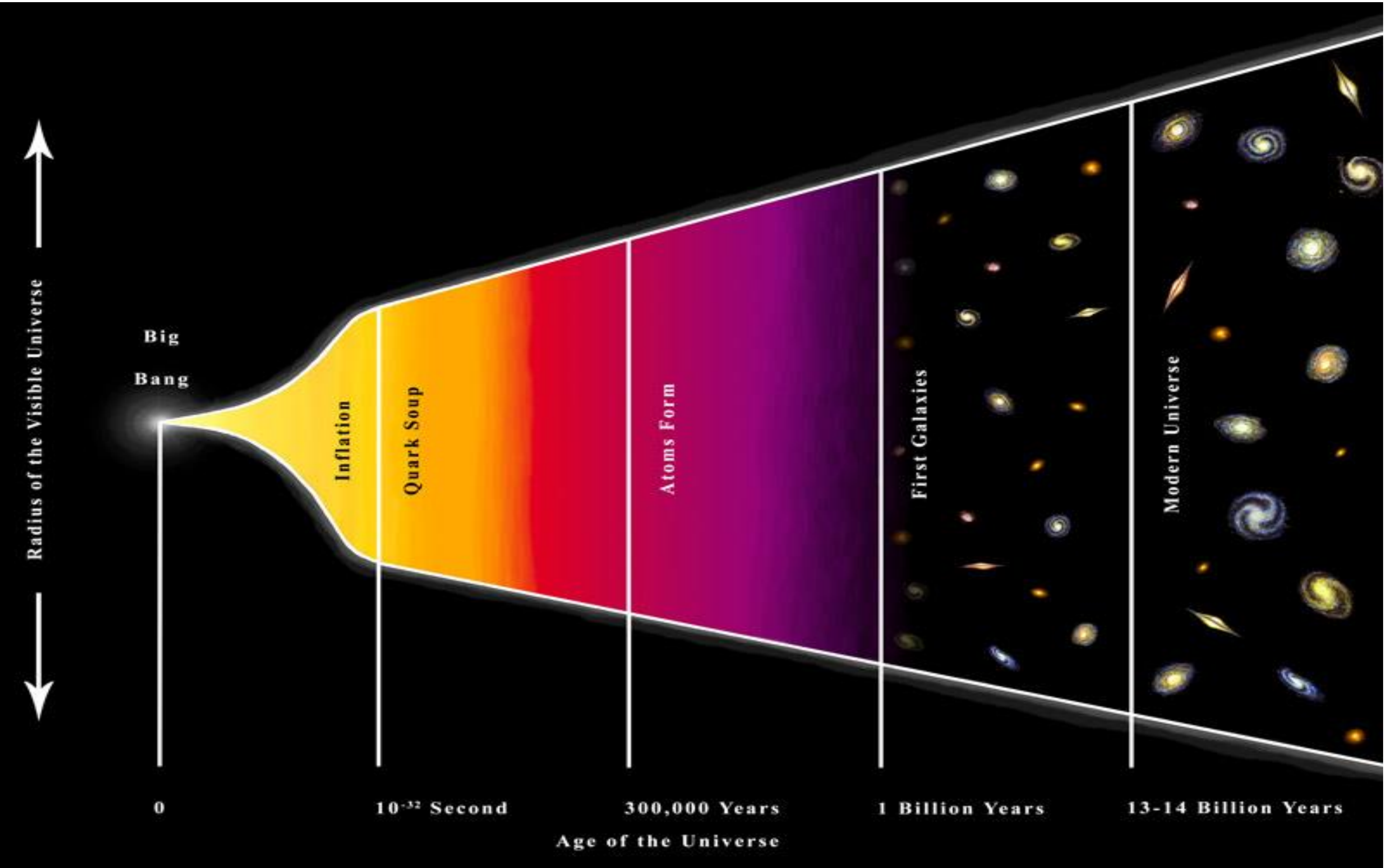
In this talk

- I want to emphasize that cosmic inflation must be a dynamical process generating space and time. Thus we need a background-independent formulation of cosmic inflation.
- I will give you an overall picture why NC spacetime necessarily implies emergent spacetime and provides the desired machinery for the background-independent formulation of cosmic inflation.

Every mathematical details can be found in my paper. ("Emergent Spacetime and Cosmic Inflation," arXiv:1503.00712).

- The emergent spacetime opens a new prospect that may cripple all the rationales to introduce the multiverse hypothesis. ("Emergent Spacetime: Reality or Illusion?", arXiv:1504.00464).

History of our Universe



Why emergent spacetime is necessary for cosmic inflation?

1. Inflating spacetime is past incomplete (Borde, Guth & Vilenkin).
   Cosmic inflation suffers initial singularity in which space and time cease to exist.

2. This means that cosmic inflation must be a dynamical process generating space and time as well as matters. Thus we need a background-independent formulation of cosmic inflation.

3. Remove spacetime and replace it by something more fundamental. How and what?

4. NC spacetime: $[x^\mu, x^\nu] = i\theta^{\mu\nu}$ takes the superb mission for how and what.
   Recall that $f(x+a) = U(a)^\dagger f(x) U(a)$ where $U(a) = e^{-ipa/\hbar}$,
   and $p_\mu = (\theta^{-1})_{\mu\nu}\, x^\nu$. There is no space but an algebra $\mathcal{A}_\theta$ only.
   Thus the NC *space* is a misnomer.

5. NC spacetime implies a paradigm shift: Geometry $\longrightarrow$ Algebra
   Hilbert space $\mathcal{H}$: dynamical variables $\longrightarrow$ $N \times N$ matrices
   where $N = \dim(\mathcal{H}) \to \infty$.
   NC spacetime arises as a vacuum solution of matrix model in the Coulomb branch.

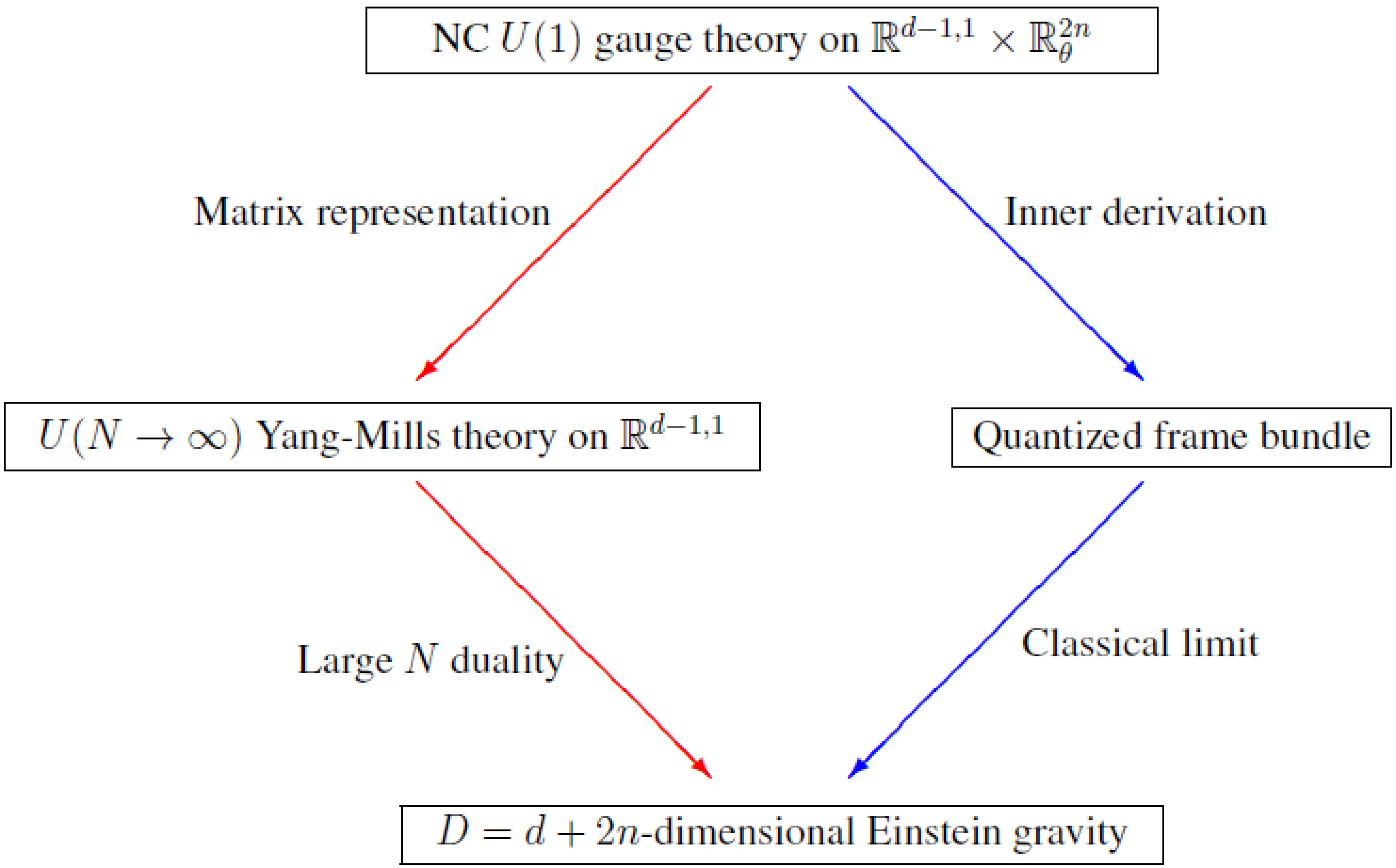


Figure 1: Flowchart for emergent gravity

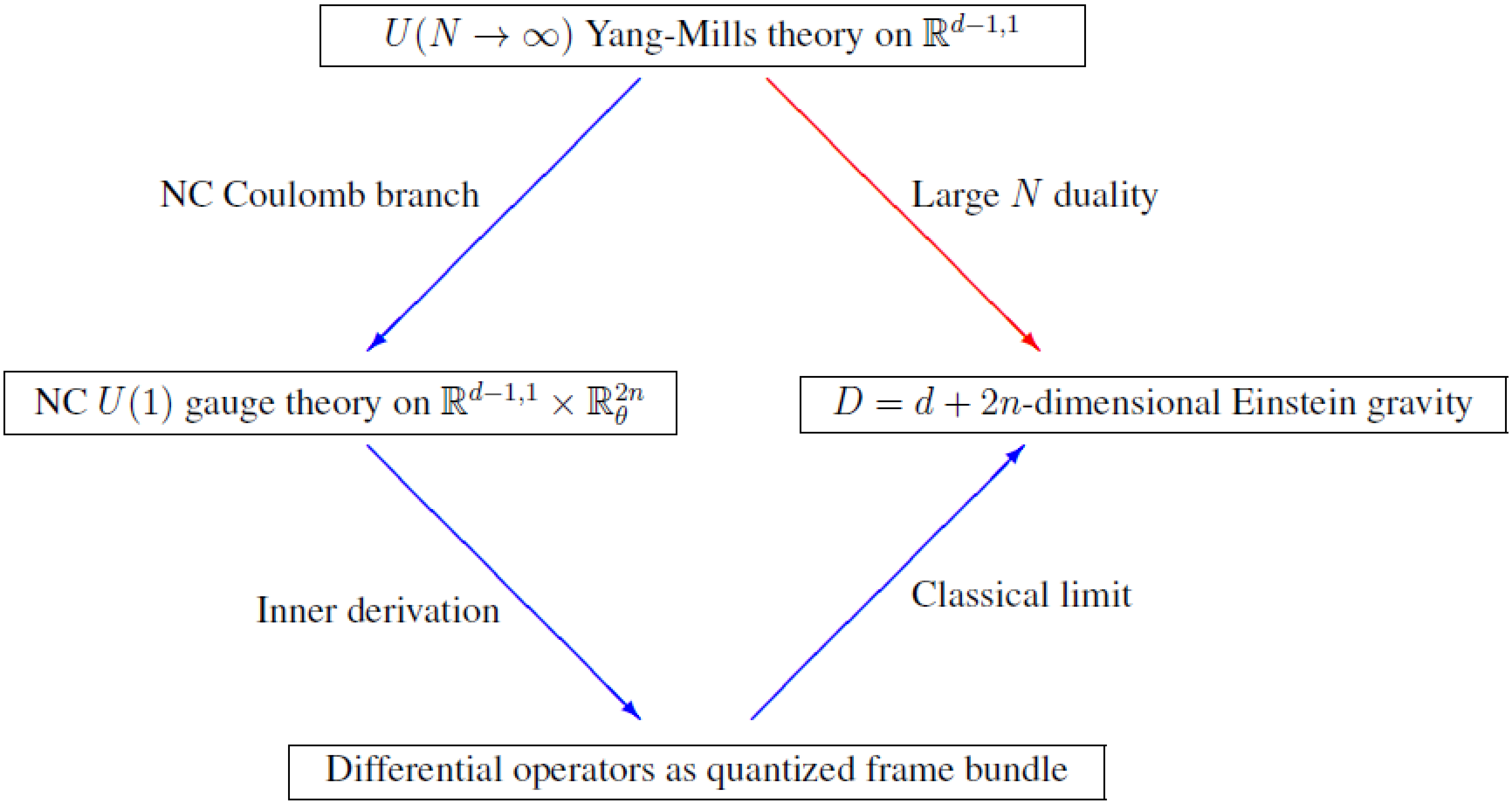


Figure 2: Flowchart for large N duality

Quantum gravity = Electromagnetism on NC spacetime!

NC spacetime : $[x^\mu, x^\nu] = i\theta^{\mu\nu}$

Note that the mathematical structure of NC spacetime is essentially the same as the NC phase space in quantum mechanics. But NC spacetime is conceptually much more radical and mysterious than quantum mechanics. I think next revolution after quantum mechanics should come from the NC spacetime. Let me explain why.

1. NC spacetime admits a (dynamical) diffeomorphism symmetry which precisely plays a role of the novel form of the equivalence principle for electromagnetic force. So gravity emerges from the NC spacetime: *Emergent gravity*.

2. NC spacetime necessarily implies *emergent spacetime* because there is no space but an algebra $\mathcal{A}_\theta$ only. A classical spacetime must be derived from the NC algebra.

3. Large N duality or gauge/gravity duality such as the AdS/CFT correspondence is an inevitable consequence of the NC spacetime.

4. A novel unification between spacetime geometry and matters: *Emergent quantum mechanics.*

Matrix quantum mechanics (MQM)

Let us consider the BFSS matrix model- matrix quantum mechanics, whose action is given by

$$S_{BFSS} = \frac{1}{g^2}\int dt\, Tr\left(-\frac{1}{2}(D_0\phi_a)^2 + \frac{1}{4}[\phi_a, \phi_b]^2\right)$$

$$= \frac{1}{4g^2}\int dt\, \eta^{AC}\eta^{BD}\, Tr([\phi_A, \phi_B][\phi_C, \phi_D]) \qquad (1)$$

where $D_0 = \frac{\partial}{\partial t} - iA_0 \equiv -i\phi_0, \quad \phi_A = (\phi_0, \phi_a), \; a = 1, \cdots, 2n,$ and $\eta^{AB} = \mathrm{diag}(-1, 1, \cdots 1), \; A, \; B = 0, 1, \cdots, 2n.$

Equation of motion:

$$D_0^2\phi_a + [\phi_b, [\phi_a, \phi_b]] = 0,$$

Gauss constraint:

$$[\phi_a, D_0\phi_a] = 0.$$

The matrix action (1) respects the Poincaré automorphism: $\phi_A \mapsto \phi'_A = \Lambda_A{}^B\, \phi_B + c_A$, $\Lambda_A{}^B \in SO(2n, 1)$, that will be realized as the Poincaré symmetry of an emergent spacetime manifold.

A vacuum solution of MQM (1) in the Coulomb branch:

$$\langle\phi_a\rangle_{vac} = p_a = B_{ab}x^b, \qquad \langle A_0\rangle_{vac} = 0, \qquad (2)$$

where $B_{ab} = (\theta^{-1})_{ab}$ and $x^a \in \mathbb{R}^{2n}_\theta$.

Let us consider fluctuations around the vacuum (2):

$$\phi_0 = iD_0 = i\frac{\partial}{\partial t} + \hat{A}_0(x,t), \quad \phi_a = B_{ab}x^b + \hat{A}_a(x,t). \tag{3}$$

where the fluctuations give rise to a deformation of the vacuum algebra given by

$$[\phi_A, \phi_B] = -i\big(B_{AB} - \hat{F}_{AB}\big),$$

and $\widehat{F}_{AB} = \partial_A \widehat{A}_B - \partial_B \widehat{A}_A - i[\widehat{A}_A, \widehat{A}_B]_\star \in \mathcal{A}_\theta^1$ is the field strength of NC U(1) gauge fields $\hat{A}_A = (\hat{A}_0, \hat{A}_a)(x,t)$. Plugging the fluctuations (3) into the action (1) leads to the (2n+1)-dimensional NC U(1) gauge theory whose action is given by

$$\begin{aligned} S &= \frac{1}{g^2}\int dt \mathrm{Tr}\left(\frac{1}{2}(D_0\phi_a)^2 + \frac{1}{4}[\phi_a, \phi_b]^2\right) \\ &= -\frac{1}{4g_{YM}^2}\int d^{2n+1}y\big(\widehat{F}_{AB} - B_{AB}\big)^2, \end{aligned}$$

where $g_{YM}^2 = (2\pi)^n |\mathrm{Pf}\theta| g^2$ is the $(2n+1)$-dimensional gauge coupling constant.

Dynamical origin of flat spacetime

Recall that we have considered a translation invariant vacuum defined by

$$\langle\phi_a\rangle_{vac} = p_a = B_{ab}\, x^b \in \mathcal{A}_N, \qquad \langle A_0\rangle_{vac} = 0,$$

where $[x^a, x^b] = i\theta^{ab}\mathbb{1}_{N\times N}$ and $B_{ab} = (\theta^{-1})_{ab}$. The Heisenberg-Moyal algebra is a consistent vacuum of the BFSS matrix model. Fluctuations are introduced around the vacuum (2)

Then we see that the emergent geometry for the fluctuations is described by the Lorentzian metric (4) and the flat spacetime is emergent from a uniform condensate of gauge fields in vacuum:

$$\langle\phi_A\rangle_{vac} = p_A = (p_0, p_a) \Rightarrow E_A^{(0)} = \delta_A^M \partial_M \Rightarrow \langle g_{MN}\rangle_{vac} = \eta_{MN}.$$

We can calculate the energy density for the vacuum condensate which is responsible for the generation of flat spacetime:

$\rho_{vac} \sim \frac{1}{g_{YM}^2} |B_{ab}|^2 \sim g_{YM}^2 M_P^4 \sim 10^{-2} M_P^4$ where $G\hbar^2/c^2 \sim g_{YM}^2|\theta|$ and

$M_P = (8\pi G)^{-\frac{1}{2}} \sim 10^{18}$ GeV.

Emergent spacetime from BFSS matrix model

By applying the duality chain in Fig. 2 and the nontrivial inner automorphism of $\mathcal{A}_\theta$, it is straightforward to show that the emergent geometry dual to the BFSS matrix model is precisely given by the $(2n+1)$-dimensional Lorentzian metric

$$ds^2 = \lambda^2(-dt^2 + V_\mu^a V_\nu^a\,(dy^\mu - \mathbf{A}^\mu)\,(dy^\nu - \mathbf{A}^\nu)) \tag{4}$$

where $\mathbf{A}^\mu = A_0^\mu(t,y)dt$ and $\lambda^2 = \tilde{\nu}\,(V_0, \cdots, V_{2n})$ with volume form $\tilde{\nu} = dt \wedge \nu$.

If all fluctuations are turned off, $V_\mu^a = \delta_\mu^a$, $\mathbf{A}^\mu = 0$ and $\lambda^2 = 1$,
the vacuum geometry reduces to flat Minkowski space $\mathbb{R}^{2n,1}$.

And the global Lorentz symmetry should be emergent too as the isometry of the vacuum geometry $\mathbb{R}^{2n,1}$, which is a direct consequence of the Poincaré automorphism.
The Poincaré automorphism also explains the local Lorentz symmetry in a Darboux frame (i.e., a locally inertial frame).

No cosmological constant problem

A striking fact is that the vacuum responsible for the generation of flat spacetime is not empty. Rather the flat spacetime is originated from the uniform vacuum energy known as the cosmological constant in general relativity.

The huge vacuum energy was simply used to create the flat spacetime and so the cosmological constant in general relativity does not gravitate in emergent gravity.

But the spacetime in our theory was not existent at the beginning. Thus the vacuum condensate must be regarded as a dynamical process. Since the dynamical scale for the condensate is about of the Planck energy, the typical time scale for the condensate will be roughly of the Planck time $\sim 10^{-44}$ sec.

Thus it is natural to consider the instantaneous condensate of vacuum energy enormously spreading out spacetime as the cosmic inflation of our universe in which the metric (4) corresponds to a final state of the inflation.

So the problem is how to describe the dynamical process for the vacuum condensate. Now I will show that the cosmic inflation of our universe arises as a solution of BFSS matrix model.

Cosmic Inflation from matrix model

We can show that the inflating vacuum defined by

$$\langle \phi_a \rangle_{vac} = e^{Ht}\, p_a, \quad \langle \mathrm{A}_0 \rangle_{vac} = a_0(y) = H \int_0^1 d\sigma\, \frac{dy^\mu(\sigma)}{d\sigma} p_\mu(\sigma) \qquad (5)$$

satisfies the equations of motion as well as the Gauss constraint. $a_0(y)$ is an open Wilson line along a path parametrized by the curve $y^a(\sigma) = y_0^a + \zeta^a(\sigma)$ where

$\zeta^a(\sigma) = \theta^{ab} k_b \sigma$ with $0 \leq \sigma \leq 1$ and $y^a(\sigma = 0) \equiv y_0^a$ and $y^a(\sigma = 1) \equiv y^a$.

The inflating vacuum (5) is a consistent vacuum of the BFSS matrix model. Fluctuations are introduced around the vacuum (5)

$$D_0 = \frac{\partial}{\partial t} - i(a_0(y) + \hat{A}_0(y,t)), \qquad \Phi_a = e^{Ht}\,(p_a + \hat{A}_a(y,t)).$$

The spacetime metric for the inflating vacuum (5) is determined by (4):

$$ds^2 = -dt^2 + e^{2Ht}(dx^\mu - a^\mu)(dx^\mu - a^\mu), \qquad (6)$$

where $a^\mu = H\, e^{-2Ht}\, x^\mu\, dt$. This geometry quickly transits to the FRW universe for the cosmic inflation at late times. Including all fluctuations around the inflation background, the general Lorentzian metric for inflating spacetime is given by

$$ds^2 = \lambda^2(-dt^2 + e^{2Ht} v_\mu^a v_\nu^a (dx^\mu - A^\mu)(dx^\nu - A^\nu)). \qquad (7)$$

Picture of emergent inflation

- Old inflation is simply an (exponential) expansion of a preexisting spacetime triggered by the potential energy carried by inflaton(s):
$$H^2 = \frac{8\pi G}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right)$$
- Inflation is a time-dependent dynamical system and corresponds to a non-Hamiltonian system

$\ddot{\phi} + 3H\dot{\phi} + \frac{\delta V}{\delta \phi} = 0 \;\; \rightarrow$ frictional force ≈ external force during inflation.

NC spacetime means that spacetime becomes a phase space with a symplectic structure $B = \frac{1}{2} B_{\mu\nu} dy^\mu \wedge dy^\nu$. In a conservative Hamiltonian system, we have the Liouville theorem stating the invariance of phase space volume under Hamiltonian time evolution.

But, the cosmic inflation means that the volume of spacetime phase space has to exponentially expand. It can be achieved by the vacuum (5) which defines a locally conformal symplectic manifold and its vector fields obey $\mathcal{L}_X B = H B$.

Predictions of emergent inflation

- Inflation metric described by the conformal Hamiltonian vector field X_I is given by

  $$ds^2 = -dt^2 + e^{2Ht} dx^\mu dx^\mu,$$

  after ignoring all fluctuations around the inflation background.

  This geometry precisely describes the FRW universe for the cosmic inflation.

- Emergent gravity predicts the existence of dark energy $\rho_{DE} \sim M_H^4$ which puts the size $L_H = M_H^{-1} \sim (10^{-3} eV)^{-1}$ of the cosmic horizon (observable Universe).

  This implies that the size of entire universe is extremely large, $L_U \sim e^{60} L_H$, compared to our visible universe L_H.

- When including fluctuations, the inflationary metric is generalized to

  $$ds^2 = -(1+\phi)dt^2 + e^{2Ht}\left(\delta_{\mu\nu} + h_{\mu\nu}\right) dx^\mu dx^\nu .$$

  ∃ reasonable size of density and tensor perturbations.

- Inflation is a single event due to the exclusion principle of NC spacetime.

  Our universe is (almost) stable. Chaotic, eternal inflations and cyclic universe seem to be inconsistent with our picture. Our visible universe is an extremely small part $\sim 10^{-27}$, but the emergent spacetime picture strongly suggests the orthodox universe (not multiverse) picture.

Cosmic inflation: orthodox vs. emergent spacetime

- Existence of spacetime is a priori assumed.
- Effective field theory: (modified) gravity $\oplus$ (quantum) field theory
- Inflation is triggered by a potential energy carried by inflaton.
- Inflation is an expansion of a preexisting spacetime.
- $E \leq 10^{16}$ GeV (BICEP 2)
- Inflation is (almost) eternal. E.g., chaotic and eternal inflations, cyclic universe, etc.
- Slow roll inflation with enough e-folding $\geq 50 \sim 60$. Multiverse is almost inevitable.

Inflation is an expansion of a preexisting spacetime

- Existence of spacetime is not assumed but defined by the theory.
- Background independent theory: matrix models
- Inflation is triggered by a condensate of Planck energy into vacuum.
- Inflation is a dynamical process generating spacetime.
- 10^{15} GeV $\sim E \sim 10^{17}$ GeV
- Inflation is a single event due to the exclusion principle of NC spacetime. Our universe is (almost) stable.
- Neither inflaton nor inflation potential. Universe with our visible part $\sim 10^{-27}$ after 60 e–foldings.

Inflation is a dynamical process generating spacetime