The IR obstruction to UV completion for Dante’s Inferno Model with Higher-Dimensional Gauge Theory Origin

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based on arXiv:1511.06818

January 31, 2016, 3rd KIAS-NCTS Joint Workshop at High 1
Plan of talk

1. Introduction and Motivation

2. Dante’s Inferno model with 5D gauge theory origin

3. Prediction of the model

4. Summary
1. Introduction and Motivation

- Recent observation for the tensor to scalar ratio:
  \[ r \approx 0.16 \rightarrow r < 0.12 \]
  \[ \text{[BICEP2, Planck collaboration, 15]} \]
  \[ \text{More recently, } r < 0.09 \text{. [BICEP/Keck Array,15]} \]

There is still a possibility of large field inflation (Lyth bound)
\[ \Delta \phi > M_P (r \gtrsim 10^{-3}) \]

- We consider large field inflation model based on higher-dimensional (5D) gauge theories which is a possible solution to the fine-tuning problem in inflation

- In extra natural inflation, the weak gravity conjecture (WGC) restricts axion field range to be sub-Planckian.
  \[ \text{[Arkani-Hamed et al, 06]} \]
Taking into account of WGC, previously, we studied the models that realize the super-Plankian field excursion effectively from the sub-Planckian field excursion of the original fields. [Furuuchi & YK, 14]

Dante’s Inferno (DI) model is the most preferred model in point of view of the naturalness of the 5D massive gauge theory parameters.

However DI model for a simple chaotic inflation $V(\phi) = m^2 \phi^2$ is (modestly) disfavored by the current observational bound $r < 0.12$ since $V(\phi) = m^2 \phi^2 \rightarrow r \simeq 0.16$. 
What we have done in this work

- We consider $V = m^2 \phi^2 - \lambda \phi^4$ in order to accommodate DI model with 5D massive gauge theory origin to the updated upper bound on $r$.

- In our 5D model, we needed to examine a criterion for effective field theories to be embedded in a consistent UV theory:

  The IR obstruction to UV completion  

  [Adams et al, 06]

- Connection to DBI action
Dante’s Inferno (DI) [Berg et al, 08]

- DI model is described by the potential

\[
V_{DI}(A, B) = V_A(A) + \Lambda^4 \left\{ 1 - \cos \left( \frac{A}{f_A} - \frac{B}{f_B} \right) \right\}
\]

- To see the inflation, it is convenient to rotate the fields as

\[
\begin{pmatrix} \tilde{B} \\ \tilde{A} \end{pmatrix} = \begin{pmatrix} \cos \xi & \sin \xi \\ -\sin \xi & \cos \xi \end{pmatrix} \begin{pmatrix} B \\ A \end{pmatrix}, \quad \sin \xi = \frac{f_A}{\sqrt{f_A^2 + f_B^2}}, \quad \cos \xi = \frac{f_B}{\sqrt{f_A^2 + f_B^2}}.
\]

Then the potential takes the form,

\[
V(\tilde{A}, \tilde{B}) = V_A(\tilde{A} \cos \xi + \tilde{B} \sin \xi) + \Lambda^4 (1 - \cos \frac{\tilde{A}}{f}) \quad f \equiv \frac{f_A f_B}{\sqrt{f_A^2 + f_B^2}}
\]

In this model, the regime of interest is consistent with WGC

\[
2\pi f_A \ll 2\pi f_B \lesssim M_P \quad \cos \xi \approx 1, \quad \sin \xi \approx \frac{f_A}{f_B}, \quad f \approx f_A
\]
The potential of DI model: \( V_{DI}(A, B) = V_A(A) + \Lambda^4 \left\{ 1 - \cos \left( \frac{A}{f_A} - \frac{B}{f_B} \right) \right\} \)

\( \tilde{B} \) will be identified with the inflaton.

\[ V_{eff}(\phi) = V_A \left( \frac{f_A}{f_B} \tilde{B} \right) \]

Under the conditions, DI model shows the layered structure (inferno) in the potential.

\[ \frac{\Lambda^4}{f} \gg \partial_{\tilde{A}} V_A(\tilde{A}, \tilde{B}) , \quad \frac{\partial^2}{\partial \tilde{A}^2} V(\tilde{A}, \tilde{B}) > H^2 \]
2. Dante’s Inferno model with 5D gauge theory origin

- Dante’s Inferno model:  \[ V_{DI}(A, B) = V_A(A) + \Lambda^4 \left\{ 1 - \cos \left( \frac{A}{f_A} - \frac{B}{f_B} \right) \right\} \]

- This potential is derived from a 5D \( U(1) \) gauge theory on \( M^4 \times S^1 \)

\[
S = \int d^5x \left[ -\frac{1}{4} F^{(A)}_{MN} F^{(A)MN} - V_A(A_M) - \frac{1}{4} F^{(B)}_{MN} F^{(B)MN} \right. \\
\left. - i \bar{\psi} \gamma^M (\partial_M + ig_{A5} A_M - ig_{B5} B_M) \psi \right] \quad (M, N = 0, 1, 2, 3, 5)
\]

where

\[
A_M = A_M - g_{A5} \partial_M \theta \quad \theta : \text{Stueckelberg field}
\]

and a matter has two kinds of charge belonging to \( U_A(1) \) and \( U_B(1) \).

- Two scalar fields \( A, B \) are identified with \( A^{(0)}_5, B^{(0)}_5 \), respectively.

\[
A, B \equiv \sqrt{2\pi L_5} A^{(0)}_5, \ \sqrt{2\pi L_5} B^{(0)}_5
\]
Note that the form of $V_A(A)$ in $V_{DI}(A, B)$ will have the same form as $V_A(A_M)$ after the dimensional reduction.

We consider the potential for the massive gauge field:

$$V_A(A_M) = v_2 A_M A^M + v_4 (A_M A^M)^2$$

* Inclusion of higher order terms $\rightarrow$ we have more parameters to tune

It is here that the IR obstruction to UV completion is relevant.

The following sign constraints are derived from the condition that massive gauge theory to be embedded to a UV theory:

$$v_2, v_4 < 0$$

[A. Hashimoto, 08]

* Our metric convention is $\eta_{MN} = \text{diag}(+ - - - -)$
The IR obstruction to UV completion for massive gauge field
[A. Hashimoto, 08]

i) To UV completion

Let us focus on the forward scattering amplitude
\[ M(s, t = 0) \equiv \mathcal{A}(s) \] of \( 2 \rightarrow 2 \) scattering.

The analytic property of the S-matrix: \( \text{Im} \mathcal{A}(s) \) appears as a discontinuity across the branch cut singularity on the real axis.

\[ \text{Disc}[\mathcal{A}(s)] = 2i \text{Im} \mathcal{A}(s) \]

Using the analytic property and the unitarity (optical theorem) of the S-matrix, in relativistic field theories, it holds that

\[ \frac{1}{2} \frac{d^2}{s^2} \mathcal{A}(s \rightarrow 0) = \oint_C \frac{ds}{2\pi i} \frac{\mathcal{A}(s)}{s^3} \rightarrow \frac{d^2}{s^2} \mathcal{A}(s \rightarrow 0) = \frac{4}{\pi} \int_0^\infty ds \frac{\sigma(s)}{s^2} \]

[Adams et al, 06]
[Baumann et al, 15]
\[ \partial_s^2 A(s \to 0) = \frac{4}{\pi} \int_0^\infty ds \frac{\sigma(s)}{s^2} \]

LHS is the IR limit of the forward scattering. RHS, which is manifestly positive since \( \sigma(s) > 0 \), is given by the pole structure of UV theory.

The sign of coupling constants of IR effective theory is constrained by the property of the S-matrix in UV theory

[Adams et al, 06]

ii) **The IR obstruction**

- Superluminal propagation in a certain background.

  i) and ii) give the same constraint on the sign of coupling constant of effective theory

  [Adams et al, 06]
• In the current massive gauge theory case

i) **To UV completion**

\[ \mathcal{A}(s) \text{ for } 2 \rightarrow 2 \text{ scattering of the longitudinal mode of } A_M \text{ is} \]

\[ \mathcal{A}(s) \propto -\frac{v_4}{v_2^2} (s^2 + \mathcal{O}(s)) \]

\[ v_4 < 0 \quad \text{for UV completion} \]

[A. Hashimoto, 08]

ii) **The IR obstruction**

If \( v_4 > 0 \), the superluminal fluctuation appears around Lorentz symmetry breaking backgrounds.

[Velo et al, 79]

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For a model which has a sound IR behavior as well as an origin in sane UV theory, we assume that \( v_2, v_4 < 0 \) is satisfied.
Dante’s Inferno model with 5D gauge theory origin

The potential for Dante’s Inferno is obtained from our 5D gauge theory as the one-loop effective potential for $A, B$:

$$V_{DI}(A, B) = \frac{m^2}{2} A^2 - \frac{\lambda}{4!} A^4 + \Lambda^4 \left\{ 1 - \cos \left( \frac{A}{f_A} - \frac{B}{f_B} \right) \right\}$$

The parameters in $V_{DI}(A, B)$ relate to 5D gauge theory parameters as

$$-v_2 = \frac{m^2}{2} > 0, \quad -\frac{v_4}{2\pi L_5} = \frac{\lambda}{4!} > 0$$

$$f_A = \frac{1}{g_A(2\pi L_5)}, \quad f_B = \frac{1}{g_B(2\pi L_5)}, \quad g_A = \frac{g_{A5}}{\sqrt{2\pi L_5}}, \quad g_B = \frac{g_{B5}}{\sqrt{2\pi L_5}}$$

Now, the two condition should be satisfied in DI model:

condition 1 \quad f_A \ll f_B \ll 1. \quad \rightarrow \quad g_A \gg g_B, \quad \cos \gamma \simeq 1, \quad \sin \gamma \simeq \frac{f_A}{f_B}, \quad f \simeq f_A

condition 2 \quad |\partial_A V_A(A)|_{A=A_{in}}| \ll \frac{\Lambda^4}{f}$
The potential of DI model: \( V_{DI}(A, B) = V_A(A) + \Lambda^4 \left\{ 1 - \cos \left( \frac{A}{f_A} - \frac{B}{f_B} \right) \right\} \)

\( \tilde{B} \) is identified with the inflaton.

Super-Planckian field excursion \( \phi > 1 \) is effectively realized by the sub-Planckian fields \( A, B < 1 \).
The inflaton potential

\[ V_{\text{eff}}(\phi) = \frac{m_{\text{eff}}}{2} \left( \frac{f_A}{f_B} \tilde{B} \right)^2 - \frac{\lambda}{4!} \left( \frac{f_A}{f_B} \tilde{B} \right)^4 = \frac{m_{\text{eff}}^2}{2} m^2 - \frac{\lambda_{\text{eff}}}{4!} \phi^4 \]

\[ = \frac{m_{\text{eff}}^2}{2} \phi^2 \left( 1 - c\phi^2 \right) \]

where \[ m_{\text{eff}}^2 \approx \left( \frac{f_A}{f_B} \right)^2 m^2, \quad \lambda_{\text{eff}} \approx \left( \frac{f_A}{f_B} \right)^4 \lambda, \quad c := \frac{\lambda_{\text{eff}}}{12m_{\text{eff}}^2} \]

The potential is not bounded below.

The slow-roll inflation is described inside of the region \(|\phi| < \phi_{\text{max}}\).
Remark

- We will not worry about the potential beyond $|\phi| > \phi_{max}$. Massive vector field theories which can be embedded to a UV theory whose S-matrix satisfies canonical analyticity constraints do not have a Lorentz-symmetry-breaking vacuum. Hashimoto (08)

$$V''_A(\langle A_M \rangle) > 0$$

- In such theories, before the potential starts to go down, the contribution from higher order terms in the potential should come in to prevent Lorentz-symmetry-breaking local minimum.
3. Prediction of the model

- For the slow-roll inflation we have two parameters in the inflaton potential $V_{eff}(\phi)$: $m_{eff}$ and $c$.

We use $P_s \simeq 2.2 \times 10^{-9}$ to determine $m_{eff}$ as a function of $c$. 

$$V_{eff}(\phi) = \frac{m_{eff}^2}{2} \phi^2 (1 - c\phi^2)$$
Parameters of the 5D gauge theory

Finally, the following constraints on the parameters of the 5D gauge theory should be satisfied for of DI model:

\[ g_A \gtrsim 15g_B, \quad (f_B \gtrsim 15f_A), \]

\[ 7 \times 10^{-3} g_B^{-1/3} < \frac{1}{L_5} \lesssim 2\pi g_B, \quad (N_* = 60, \ c = 0.001). \]
Connection to DBI action

- We start with DBI action of D5-brane. After double dimensional reduction twice, in four dimension, we obtain the potential of DI model

\[
V_{\text{eff}}(\phi) \sim m_{\text{eff}}^2 \int d^4x \sqrt{1 + \left( \frac{\phi}{\phi_c} \right)^2} \quad m_{\text{eff}}^2 \propto \frac{1}{\phi_c^2} \quad \lambda_{\text{eff}} \propto \frac{3}{\phi_c^4}
\]

\( \phi_c \) is the convergence radius of the Taylor expansion of the DBI action.

\[
\phi_c = \frac{1}{2\sqrt{c}} \quad c = \frac{\lambda_{\text{eff}}}{12m_{\text{eff}}^2}
\]

- our phenomenological parametrization of the potential is a good approximation
- linear approximation → axion monodromy model
4. Summary

- We considered $V = m^2 \phi^2 - \lambda \phi^4$ in order to accommodate Dante’s Inferno model with 5D massive gauge theory origin to the updated upper bound on $r < 0.12$.

- We examined a criterion for effective field theories to be embedded in a consistent UV theory

\[
V_A(A_M) = v_2 A_M A^M + v_4 (A_M A^M)^2 \quad \text{with} \quad v_2, v_4 < 0
\]

- We gave a possible connection of our DL model to DBI action of D5-brane.
Backup slides
\textbullet\ v_2, v_4 the bare couplings in the 5D theory. In the theory at lower energy, the loop corrections to v_2, v_4 are very small for $2\pi L_5 > 100$

\textbullet\ In general, in $v_2, v_4 \rightarrow 0$ limit, the 5D gauge symmetry without the Stueckelberg field will restore. Then the tiny v_2 and v_4 may be natural at low energy in the sense of 't Hooft.
The effective potential for $A, B$ at the one-loop level is

$$V_{1\text{-loop}}(A, B) = V_{\text{cl}}(A) + V_g(A) + V_f(A, B)$$

$$A, B \equiv \sqrt{2\pi L_5} A_5^{(0)}, \sqrt{2\pi L_5} B_5^{(0)}$$

where $V_{\text{cl}}(A)$ is the contribution from the 5D classical potential

$$V_{\text{cl}}(A) = \frac{1}{2} m^2 A^2 - \frac{\lambda}{4!} A^4, \quad -v_2 = \frac{m^2}{2} > 0, \quad -\frac{v_4}{2\pi L_5} = \frac{\lambda}{4!} > 0$$

$V_g$ is the contribution from the gauge field $A_M$, and is sub-leading compared with $V_{\text{cl}}(A)$ when

$$2\pi L_5 \gtrsim 1 \times 10^2 \quad (M_P = 1)$$

this is a natural assumption to make since we do not expect the compactification radius $L_5$ to be very close to the Planck scale.
The one-loop diagram:

\[ \sum_{A,B} \sum_{m} \psi(m) \]  

+ gauge loop due to the self coupling from \( V_A(A_M) \)

At one-loop level, the fermion contribution \( V_f(A, B) \) is

\[ V_f(A, B) = \Lambda^4 \sum_{n=1}^{\infty} \frac{1}{n^5} \cos \left\{ n \left( \frac{A}{f_A} - \frac{B}{f_B} \right) \right\}, \quad \Lambda^4 \propto \frac{1}{L_5^4} \]

The potential has a discrete shift symmetry \( B \rightarrow B + 2\pi f_B \)

Taking \( n = 1 \), an appropriate constant shift of \( B \) and adding a constant yields Dante’s Inferno.

The parameters in \( V_{DI}(A, B) \) relate to 5D gauge theory parameters as

\[ f_A = \frac{1}{g_A(2\pi L_5)}, \quad f_B = \frac{1}{g_B(2\pi L_5)}, \quad g_A = \frac{g_{A5}}{\sqrt{2\pi L_5}}, \quad g_B = \frac{g_{B5}}{\sqrt{2\pi L_5}} \]
Connection to DBI action

- DBI action of D5-brane is
  \[ S_{D5} = -T_{D5} \int d^6 \sigma \sqrt{- \det (G_{ab} + F_{ab})}, \quad (a, b = 0, 1, 2, 3, 5, 6) \]
  \[ F_{ab} = B_{ab} - \partial_a C_b + \partial_b C_a \]

- We consider the following background:
  \[ G_{MN} = \text{diag}(+ - - - - -), \quad B_{MN} = 0 \quad \sigma^a = x^a \text{ static gauge} \]
  \[ F_{a6} \propto (a_M - \partial_M \theta) \] will be the 5D gauge and the Stueckelberg fields

- After double dimensional reduction twice, in four dimension, we obtain the potential of \( A \sim a_5 \),
  \[ V_A(A) \sim \int d^4 x \sqrt{1 + \text{const.} \times A^2} \quad \sqrt{1 + A^2} = 1 + \frac{1}{2} A^2 - \frac{1}{8} (A^2)^2 + \frac{1}{16} (A^2)^3 - \cdots. \]
Through the DI model, the inflaton potential is

\[ V_{eff}(\phi) \sim m_{eff}^2 \int d^4x \sqrt{1 + \left( \frac{\phi}{\phi_c} \right)^2} \]

\[ m_{eff} \propto \frac{1}{\phi_c^2} \quad \lambda_{eff} \propto \frac{3}{\phi_c^4} \]

\( \phi_c \) is the radius of convergence of the Taylor expansion of the DBI action.

\[ \phi_c = \frac{1}{2\sqrt{c}} \quad c = \frac{\lambda_{eff}}{12m_{eff}^2} \]

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1-1. Our previous study

- Previously we have studied large field inflation models from higher-dim. gauge theories.

- Important theoretical ingredients in our EFT approach were
  
  - naturalness of gauge theory parameters
  - weak gravity conjecture (WGC) \[ 2\pi f \lesssim M_P \]

\[ f = 1/(2\pi gL) \]
\[ (M_P L)^{-1} \lesssim g \]

- The models we studied were those in which the defining theories are sub-Planckian but inflation effectively travels trans-Planckian field range. Good inflaton potentials had already been proposed.

  - Single axion monodromy
  - Dante’s Inferno
  - Axion alignment
  - Axion hierarchy

  (Typical models)
**Expected parameter range**

<table>
<thead>
<tr>
<th>Gauge couplings</th>
<th>Compactification radius</th>
<th>Charges</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-\log_{10}[(LM_P)^2] \lesssim \log_{10}[g^2] \lesssim 0$</td>
<td>$\log_{10}[1/(L \text{GeV})] \sim 3 - 17$</td>
<td>$n \sim \mathcal{O}(1)$</td>
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</tbody>
</table>

- Expected parameter ranges from higher dimensional gauge theory.

- The lower bound in $g$ is imposed by WGC, while the upper bound comes from applicability of perturbation theory.

- The expected value of charges is in unit of the minimal charge in the model. (Extraordinary large charge is unlikely or rare in nature)
## Resultant parameter range in the previous study

<table>
<thead>
<tr>
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<th>Compactification radius</th>
</tr>
</thead>
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<tr>
<td>AM</td>
<td>$-8 \lesssim \log_{10}[g^2] \lesssim 0$</td>
<td>$\log_{10}[1/(L \text{ GeV})] \sim 14 - 16$</td>
</tr>
<tr>
<td>DI</td>
<td>$-1 \lesssim \log_{10}[g_A^2] \lesssim 0$, $-3 \lesssim \log_{10}[g_B^2] \lesssim -2$</td>
<td>$\log_{10}[1/(L \text{ GeV})] \sim 17$</td>
</tr>
<tr>
<td>AA</td>
<td>$-10 \lesssim \log_{10}[g_A^2]$, $\log_{10}[g_B^2] \lesssim -4$</td>
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</tr>
<tr>
<td>AH</td>
<td>$-10 \lesssim \log_{10}[g_A^2] \lesssim -4$, $-10 \lesssim \log_{10}[g_B^2] \lesssim 0$</td>
<td>$\log_{10}[1/(L \text{ GeV})] \sim 14 - 17$</td>
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<td>m_1, m_2</td>
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- Axion Monodromy (AM)
- Dante’s Inferno (DI)
- Axion Alignment (AA)
- Axion Hierarchy (AH)
Single Axion monodromy

\[ V(A) = \frac{1}{2} m^2 A^2 + \Lambda^4 \left( 1 - \cos \left( \frac{A}{f} \right) \right) \]

- Due to the quadratic term, the potential energy does not return the same under the shift \( A \rightarrow A + 2\pi f \)

- Even if the fundamental field has sub-Planckian period \( 2\pi f < M_P \), the trans-Planckian excursion can be effectively achieved by traverses many cycle. The potential energy increases over each cycle but much of the remaining physics essentially repeats itself.

- This model effectively reduces to chaotic model \( V \sim \frac{1}{2} m^2 A^2 \) when the slope of the sinusoidal potential is much smaller than that of the mass term during inflation.

\[ \Lambda^4 / f \ll m^2 A_* \quad *: \text{at horizon exit} \]
The potential can be derived from a 5D $U(1)$ gauge theory on $M^4 \times S^1$

$$S = \int d^5x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} m^2 (A_\mu - g_5 \partial_\mu \theta)^2 + \text{(matters)} \right] \quad (\mu = 0, \cdots, 3, 5)$$

- The inflaton field $A$ comes from the zero mode of $A_5$ after $S^1$ compactification.
  $$A = \sqrt{2\pi L} A_5^{(0)} \quad L : S^1 \text{ radius}$$

- We introduced the Stueckelberg filed $\theta$ and the Stueckelberg mass term which gives rise to the quadratic term in the effective potential.
  $$A_\mu \to A_\mu + \partial_\mu \Lambda, \quad \theta \to \theta + 1/g_5 \Lambda \quad \text{gauge trf.}$$

- The one-loop effective potential of $A_5^{(0)}$ is obtained as
  $$V(A_5^{(0)})_{1}\text{-loop} = \frac{m^2}{2} A_5^{(0)2} + \frac{3}{\pi^2(2\pi L)^4} \sum_{n=1}^{\infty} \frac{1}{n^5} \cos \left( nA_5^{(0)} (2\pi gL) \right) + \text{const.}$$

$$g = \frac{g_5}{\sqrt{2\pi L}} : 4d \text{ gauge coupling}$$

$$\sum_{A} \sum_{m} A_{\psi} (m) \quad \text{All KK modes } m \text{ are taken into account.}$$
• At one-loop, the Stueckelberg mass $m$ is not renormalized.

• Stueckelberg field does not contribute to the potential at one loop

• The parameters in the axion monodromy model are related to the parameters in the 5D gauge theory as

$$f = \frac{1}{g(2\pi L)}, \quad \Lambda^4 = \frac{c}{\pi^2(2\pi L)^4}, \quad c \sim \mathcal{O}(1), \quad m = m \quad \text{Stueckelberg mass}$$

Then, $\Lambda^4/f \ll m^2 A_\ast$ (effectively chaotic) and CMB data with $r = 0.16$ require

$$1.0 \times 10^{14} \text{ GeV} < \frac{1}{L} < 3.2 \times 10^{16} \text{ GeV}, \quad m^2 \sim 10^{26} \text{ GeV} \ll H_\ast^2 \quad (H_\ast \simeq 10^{14} \text{ GeV})$$

and $g \sim \mathcal{O}(1)$

$\Delta A > M_P$ : trans-Planckian field excursion of fundamental field
The small $m^2$ is natural in the sense of 't Hooft if the shift symmetry $A \rightarrow A + C$ is a good symmetry at the Planck scale. But it is beyond the scope of higher-dim. gauge theory so we can not ensure the naturalness of small $m < H$. 
Axion Alignment & Axion Hierarchy:
 improvement of natural inflation

[Kim et al, 08, Ben-Dayan et al. 14]

Both models can be described by the potential of the form

\[ V(A, B) = \Lambda_1^4 \left( 1 - \cos \left( \frac{m_1}{f_A} A + \frac{n_1}{f_B} B \right) \right) + \Lambda_2^4 \left( 1 - \cos \left( \frac{m_2}{f_A} A + \frac{n_2}{f_B} B \right) \right) \]

The main feature of these two models is to acquire a large effective decay const. \( f_{\text{eff}} > M_P \) from the (small) scales \( f_A \) and \( f_B \) by defining the eigenvectors of the mass matrix. \( \Delta A, \Delta B < M_P \) is satisfied.

\[
\begin{pmatrix}
\phi_s \\
\phi_l
\end{pmatrix} =
\begin{pmatrix}
\cos \zeta & \sin \zeta \\
-\sin \zeta & \cos \zeta
\end{pmatrix}
\begin{pmatrix}
A \\
B
\end{pmatrix}
\]

where

\[
\cos \zeta = \frac{f_s}{f_A} m_1, \quad \sin \zeta = \frac{f_s}{f_B} n_1, \quad f_s = \frac{1}{\sqrt{\frac{m_1^2}{f_A^2} + \frac{n_1^2}{f_B^2}}}
\]
Dante’s Inferno with quadratic potential from 5D gauge theory

- The constraints for Dante’s Inferno is written in terms of the parameters of the 5D gauge theory as

\[ g_A > 14 g_B, \quad g_B^{-1/3} \times 3.2 \times 10^{16} \text{ GeV} < \frac{1}{L} \lesssim g_B \times 2.4 \times 10^{18} \text{ GeV} \]

The allowed values of the gauge couplings and the compactification radius are rather restricted.

The gauge couplings are in the range \(0.04 - \mathcal{O}(1)\).
In terms of two physical fields

\[ V(\phi_s, \phi_l) = \Lambda_1^4 \left(1 - \cos \left(\frac{\phi_s}{f_s}\right)\right) + \Lambda_2^4 \left(1 - \cos \left(\frac{\phi_s}{f'_s} + \frac{\phi_l}{f_l}\right)\right), \]

effective decay constant: \[ f_l = \frac{\sqrt{m_1^2 f_B^2 + n_1^2 f_A^2}}{m_1 n_2 - m_2 n_1} \]

**Axion alignment model**

\[ |m_1 n_2 - m_2 n_1| \ll |m_1|, |n_1| \Rightarrow |f_l| \gg f_A, f_B \ (f_s, f'_s) \]

\(\phi_s\) : heavy, \(m_{\phi_s} > H\),
irrelevant to inflation

\(\phi_l\) : light, \(m_{\phi_l} < H\),
identified with the inflaton
**Axion hierarchy model** \((n_2 = 0)\)

\[
\frac{f_A}{m_1} \ll \frac{f_A}{m_2}, \frac{f_B}{n_1} \Rightarrow |f_i| = \frac{\sqrt{m_1^2 f_B^2 + n_1^2 f_A^2}}{|m_2 n_1|} \simeq \frac{m_1}{n_1 m_2} f_B
\]

This model requires simple hierarchy \(m_1 \gg m_2\) to obtain \(f_i \gg f_B \gg f_A\)

- The two models reduce to natural inflation with the effective decay constant \(f_i\).
The potential is derived from the 5D action with two kinds of matters.

\[ S = \int d^5x \left[ -\frac{1}{4} F^A_{\mu\nu} F^{A\mu\nu} - \frac{1}{4} F^B_{\mu\nu} F^{B\mu\nu} ight. 
\left. - i\bar{\psi} \gamma^\mu (\partial_\mu + ig_A m_1 A_\mu + ig_B n_1 B_\mu) \psi - i\bar{\chi} \gamma^\mu (\partial_\mu + ig_A m_2 A_\mu - ig_B n_2 B_\mu) \chi \right] \]

\[ V(A, B) = \Lambda_1^4 \left( 1 - \cos \left( \frac{m_1}{f_A} A + \frac{n_1}{f_B} B \right) \right) + \Lambda_2^4 \left( 1 - \cos \left( \frac{m_2}{f_A} A + \frac{n_2}{f_B} B \right) \right) \]

In the 5D gauge theory, \( m_1, m_2, n_1, n_2 \) correspond to the charges of \( U_{A,B}(1) \).

Assumption: \( m_1, m_2, n_1, n_2 \) are all integers.
We assume that charges are quantized.

The model parameters are given by

\[ \Lambda_{1,2} \sim \frac{3}{\pi^2} \frac{1}{2\pi L^4} , \quad f_{A,B} = \frac{1}{g_{A,B}(2\pi L)} , \quad g_{A,B} = \frac{g_{A5,B5}}{\sqrt{2\pi L}} \]
• WGC and natural inflation + \( r = 0.16 \) require \( 2\pi f_{A,B} \lesssim M_P \) and \( |f_l| \gtrsim 20M_P \)

**Axion alignment**

\[
|m_1n_2 - m_2n_1| \ll |m_1|, |n_1| \quad f_l = \frac{\sqrt{m_1^2f_B^2 + n_1^2f_A^2}}{m_1n_2 - m_2n_1}
\]

\( \max(|m_1|, |n_1|) \gtrsim 20 \times 2\pi \), matter with large charge at least \( \sim \mathcal{O}(100) \)

A matter with such a large charge seems to us quite unnatural, considering that the energy scale under consideration is rather high (\( H \simeq 10^{14}\text{GeV} \)).

**Axion hierarchy**

\[
|f_l| \simeq \left|\frac{m_1}{n_1m_2}\right| f_B \quad \Rightarrow \quad |m_1| \gtrsim 20|m_1m_2| \times 2\pi
\]

A large hierarchy between the charges in the same gauge group \( U_A(1) \)

Such a large hierarchy (\( \mathcal{O}(100) \)) between the charges in the same gauge group seems quite unnatural.
- Various axion inflation models can be derived from the higher-dimensional (5D) gauge theories.

- The allowed range of the gauge theory parameters are quite constrained. - CMB data and WGC

- Among the models studied, Dante’s Inferno model appears as the most natural model in this framework, the gauge couplings are in the range \( 0.04 - \mathcal{O}(1) \).

- Single field axion monodromy leaves the problem that whether the shift symmetry is a good symmetry or not to its UV completion theory.
Comment on the anionic coupling $\frac{\alpha}{4f} \phi \tilde{F}_{\mu\nu} F^{\mu\nu}$

In 5D U(1) gauge theory we could get the same type of interaction from the CS action which breaks the $\mathbb{Z}_2$ symmetry in 5th direction. After $S^1$ compactification, [Furuuchi and Jackson (13)]

$$\frac{g_4^2 \kappa}{8\pi} \frac{A_5^{(0)}}{2\pi f} \tilde{F}_{\mu\nu}^{(0)} F^{\mu\nu(0)}$$

It results in the corrections to the power spectrum and the non-Gaussian parameter [Barnaby et al (2010), Ferreira et al (2014)]

$$\mathcal{P}_{\zeta \text{ one-loop}} = 7.5 \times 10^{-5} \cdot \mathcal{P}_{\zeta} \frac{e^{4\pi\xi}}{\xi^6} \quad f_{\text{eq}}^{\text{NL}} = 4.4 \times 10^{10} \cdot \mathcal{P}_{\zeta}^3 \frac{e^{6\pi\xi}}{\xi^9}$$

$$\xi_i = \frac{\alpha_i \dot{\phi}_i}{2f_i H} \quad \text{with} \quad \alpha_i = \frac{k_i g_i^2}{\pi 4\pi}, \quad f = \frac{1}{2\pi g_i L}$$
If we require
\[ P_\zeta \simeq P_\zeta^{\text{total}} \simeq 2.2 \times 10^{-9} \quad (P_\zeta \gg P_\zeta^{\text{one-loop}}) \]
\[ |f_{NL}^{\text{equil}}| < 117 \quad \text{by PLANCK} \]

Dantes Inferno case, \( L \sim 10^{-17}\text{GeV}^{-1} \)
\[ \xi_i = \frac{k_i g_i^3}{4\pi} \phi_i \cdot 10^{-31}\text{GeV}^{-2} \quad (i = A, B) \]

It is evaluated as follows \[ |\xi_A| \simeq \frac{f_A}{f_B} \frac{k_A g_A^3}{4\pi}, \quad |\xi_B| \simeq \frac{k_B g_B^3}{4\pi} \]

We find that \( \xi_i < 3 \) can easily be satisfied within our parameters space for
\[ k_A \sim \mathcal{O}(1) - \mathcal{O}(10), \quad k_B \sim \mathcal{O}(1) - \mathcal{O}(100) \]
\[ \therefore \quad f_A/f_B = g_B/g_A < 0.07, \quad g_B \lesssim 0.2 \]