Fragmentation contributions to $J/\psi$ production and the polarization at the Tevatron and the LHC

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Outline

• Nonrelativistic QCD factorization
• $J/\psi$ polarization puzzle
• Our approach to the puzzle
• Calculation details
• Result and Conclusion
Nonrelativistic QCD factorization
According to NRQCD factorization conjecture, a heavy quarkonium $H$ production cross section can be factorized into

short-distance coefficient (SDC, perturbative, $\alpha_s$)

$$d\sigma_{ij \rightarrow H(q\bar{q})+X} = d\hat{\sigma}_{ij \rightarrow q\bar{q}(2S+1 L_J^{[\alpha]})+X}|0\rangle\langle 0| O^H (2S+1 L_J^{[\alpha]})|0\rangle$$

long-distance matrix element (LDME, nonperturbative, $\nu$)

- $^3S_1^{[8]}$, $^3P_J^{[8]}$, $^1S_0^{[8]}$, and $^3S_1^{[1]}$ channels contribute to $J/\psi$ production through order $\nu^4$
$J/\psi$
polarization
puzzle
• Color-singlet (CS) model failed to explain the $\psi'(J/\psi)$ surplus
• The introduction of the color-octet (CO) mechanism from NRQCD has resolved the surplus problem. [Braaten, Fleming, PRL74, 3327 (1994)]
• Here, LDMEs are determined by fitting with the experimental data
• One can predict other independent physical quantities by making use of these determined LDMEs because LDME is global → Polarization
Prompt $J/\psi (\psi')$ polarization

- In 2000, the theoretical prediction to the polarization of the prompt $J/\psi$ and $\psi'$ has been published:

  - The dominance of the CO spin-triplet S-wave channel predicts the transverse polarization at large $p_T$.
  - But, the experiment disproved this theoretical prediction.

Braaten, Lee, Kneihl, PRD62,094005

- The dominance of the CO spin-triplet S-wave channel predicts the transverse polarization at large $p_T$.
QCD NLO correction

• Full NLO QCD corrections to the $J/\psi$ production

[Kneihl, Buthenchoen]
PRL106, 022003 (2011)
PRL108, 172002 (2012)

[Chao, Ma, Shao, Wang]
PRL106, 042002 (2011)
PRL108, 242002 (2012)

[Gong, Wan, Wang, Zhang]
PRL110, 042002 (2013)

• All groups have failed to predict the polarization

$\rightarrow$ we need to consider the NNLO corrections
Our approach to the puzzle
Leading-power (LP) factorization and NRQCD

- According to LP factorization, leading power terms in $1/p_T^2$ for quarkonium $H$ production can be factorized into

$$d\sigma_{A+B\to H+X} = \sum_i \int_0^1 dz \, d\hat{\sigma}_{A+B\to i+X} \left( k^+, \mu_f \right) \times D_{i\to H} \left( z = \frac{p^+}{k^+}, \mu_f \right) \equiv \sum_i d\hat{\sigma}_{A+B\to i+X} \otimes D_{i\to H}$$

where,

$d\hat{\sigma}_{A+B\to i+X}$: single parton $i$ production cross section

$D_{i\to H}$: single parton fragmentation function, nonperturbative

$k^+$: light-cone momentum of parent parton $i$

$p^+$: light-cone momentum of parent parton $i$

$\mu_f$: factorization scale

- If we apply LP factorization to NRQCD factorization, we get

$$d\sigma_{A+B\to H+X} = \sum_{n,i} d\sigma_{A+B\to i+X} \otimes [D_{i\to Q\bar{Q}(n)} \langle \mathcal{O}^H(n) \rangle]$$

perturbative

- Dominant at large $p_T$, relatively easy to evaluate
Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP) equation

- Leading-log (LL) terms can be summed over all orders in $\alpha_s$ by solving DGLAP equation.

- LO DGLAP evolution equation is given by

$$\frac{d}{d \log \mu_f^2} \left( \frac{D_S}{D_g} \right) (z, \mu_f) = \int \frac{dx}{x} \frac{\alpha_s(\mu_f)}{2\pi} \left( \frac{P_{qq}}{P_{gq}} - \frac{2n_f P_{gq}}{P_{gg}} \right) \left( \frac{z}{x} \right) \times \left( \frac{D_S}{D_g} \right) (x, \mu_f)$$

where,

- $P_{ij}$: splitting function
- $D_S = \sum_f (D_{q_f \rightarrow Q\bar{Q}}(n) + D_{\bar{q}_f \rightarrow Q\bar{Q}}(n))$
- $D_g = D_{g \rightarrow Q\bar{Q}}(n)$
- $n_f$: active quark flavor
Our approach to the puzzle

• We ignored the CS contributions to the $J/\psi$ production

• At large $p_T$, the gluon fragmentation process that is leading power in $1/p_T^2$ expansion is dominant. ($\propto 1/p_T^4$)

• With LP factorization, we evaluate the NNLO or higher-order corrections including the all-orders leading-log (LL) resummation that are not considered in NLO result.
Calculation details
Parton subprocess

- The parton subprocesses that are LO in $\alpha_s$ is proportional to $\alpha_s^2$.

- The parton subprocesses that are NLO in $\alpha_s$ is proportional to $\alpha_s^3$.

\[ 3 S_1^{[8]} \] Gluon fragmentation function

\[ D[g \to Q\bar{Q}(3 S_1^{[8]})]^{\text{NLO}}(z, \mu_f) \]

\[
= \frac{\pi \alpha_s(\mu_f)}{24m^3} \left\{ \delta(1 - z) + \frac{\alpha_s(\mu_f)}{\pi} \left[ A(\mu_f)\delta(1 - z) + \left( \log \frac{\mu_f}{2m} - \frac{1}{2} \right) P_{gg}(z) \right.ight.
\]
\[
+ 6(2 - z + z^2) \log(1 - z) - \left. \frac{6}{z} \left[ \frac{\log(1 - z)}{1 - z} \right]_+ + \frac{3(1 - z)}{z} \right\} \]

where,

\[ A(\mu_f) = \beta_0 \left( \log \frac{\mu_f}{2m} + \frac{13}{6} \right) + \frac{2}{3} - \frac{\pi^2}{2} + 8 \log 2 \]

\[ \beta_0 = \frac{1}{6} (11C_A - 4N_fT_R). \]
\[ 3 P_J^{[8]} 1 \ S_0^{[8]} \] Gluon fragmentation function

\[
D[g \to Q\bar{Q}(3P^{[8]})](z, \mu_f) = \frac{8\alpha_s^2(\mu_f)}{3(d-1)(N_c^2 - 1)m^5} \frac{N_c^2 - 4}{4N_c} \left\{ \frac{(5z - 8)(2z - 1)}{8} \right. \\
+ \left( \frac{1}{6} - \log \frac{\mu_A}{2m} \right) \delta(1 - z) - \frac{7z - 13}{4} \log(1 - z) + \left[ \frac{1}{1 - z} \right]_+ \right. \\
D[g \to Q\bar{Q}(1S_0^{[8]})](z, \mu_f) = \frac{\alpha_s^2(\mu_f)}{8m^3} \frac{N_c^2 - 4}{4N_c} \left[ 3z - 2z^2 + 2(1 - z) \log(1 - z) \right]
\]

Bodwin, Kim, Lee, JHEP1211(2012)020
Other quark fragmentation functions are proportional to $\alpha_s^3$ which we exclude.
LP $J/\psi$ production process

- Order $\alpha_s^3$ diagrams:

- Order $\alpha_s^4$ diagrams:

- Because the previous NLO result also contains LP contributions through order $\alpha_s^4$, we should eliminate these corrections to avoid the double counting.
LP $J/\psi$ production process

• Order $\alpha_s^5$ diagrams:

We cannot evaluate the following process now

NNLO parton subprocess
LP $J/\psi$ production process

- Order $\alpha_s^5$ diagrams:

<table>
<thead>
<tr>
<th>Results</th>
<th>Order</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NLO</td>
<td>$\sim \alpha_s^4$</td>
<td>$\sigma_{\text{NLO}}$</td>
</tr>
<tr>
<td>Our result</td>
<td>$\sim \alpha_s^4$</td>
<td>$\sigma_{\text{LP}}$ $\sigma_{\text{overlapped}}$</td>
</tr>
<tr>
<td></td>
<td>$\alpha_s^5 + \text{higher (LL resum)}$</td>
<td></td>
</tr>
</tbody>
</table>

$$\sigma_{\text{result}} = \sigma_{\text{NLO}} + \sigma_{\text{LP}} - \sigma_{\text{overlapped}}$$

[NEW] NNLO +higher(LL) LP corrections
Result
Our result fits well both the CMS data and the CDF data.

We determine CO LDMEs by fitting with Exp.

- Our result fits well both the CMS data and the CDF data.
- We determine CO LDMEs by fitting with Exp.
Large cancellation between $^3S_1^{[8]}$ and $^3P_J^{[8]}$
\( \mathcal{J}/\psi \) Polarization

- we predict the polarization parameter \( \lambda_\theta \) at the Tevatron and the LHC:

- The CDF data is quite well explained
- The CMS data is perfectly explained
Prompt \( J/\psi \) production

- Prompt \( J/\psi \) production includes the production from decays of \( \psi(2S') \) and \( \chi_{cJ} \)

- We determined \( \psi(2S') \) LDME by fitting with CMS and CDF cross section data
  CDF, PRD80, 031103 (2009)
  CMS, JHEP02, 011 (2012)
  CMS-PAS-BPH-14-001

- We determined \( \chi_{cJ} \) LDME by fitting with ATLAS cross section data
  ATLAS, JHEP1407, 154 (2014)
psi(2S) and chicj production and polarization

- Our result fits $\psi(2S)$ and $\chi_{cJ}$ cross section data
- Our result well predicts $\psi(2S)$ polarization at the CMS but fails at the CDF
- $\chi_{c1}, \chi_{c2}$ and $\psi(2S)$ are slightly transverse at large $p_T$
Prompt $J/\psi$ production cross section

- Our result fits well both the CMS data and the CDF data
- At large $p_T$, the feed down contribution is not negligible
- The cancellation between $3S_1^{[8]}$ and $3P_J^{[8]}$ still occurs

Hee Sok Chung’s talk at 10th International Workshop on Heavy Quarkonium
Prompt $J/\psi$ polarization

- Feed down contributions are slightly transverse
  $\rightarrow$
  Prompt $J/\psi$ becomes slightly transverse

- The prediction agrees with the CMS data, but fails to explain the CDF data

Hee Sok Chung's talk at 10th International Workshop on Heavy Quarkonium
Conclusion

• We evaluated the higher-order corrections by making use of LP factorization formula and DGLAP.

• Our results well predicted the prompt $J/\psi$ polarization at the CMS (world-first successful prediction)

• Dominant channel is not $^{3}S^{[8]}_{1}$ but $^{1}S^{[8]}_{0}$
Backups
LDMEs
LDMEs

Kniehl  

Chao  

Wang

Our result for direct production

<table>
<thead>
<tr>
<th>$\langle O^{J/\psi (1S_0^{[8]})} \rangle$</th>
<th>$\langle O^{J/\psi (3P_0^{[8]})} \rangle$</th>
<th>$\langle O^{J/\psi (3S_1^{[8]})} \rangle$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(4.50 \pm 0.72) \times 10^{-2}$ GeV$^3$</td>
<td>$(3.12 \pm 0.93) \times 10^{-3}$ GeV$^3$</td>
<td>$(-1.21 \pm 0.35) \times 10^{-2}$ GeV$^3$</td>
</tr>
<tr>
<td>$10^{-2}$ GeV$^3$</td>
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</tr>
<tr>
<td>$1.16$</td>
<td>$8.9 \pm 0.98$</td>
<td>$0.30 \pm 0.12$</td>
</tr>
<tr>
<td>$0.56 \pm 0.21$</td>
<td></td>
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</tbody>
</table>

$3 \text{ GeV} < p_T$  

$7 \text{ GeV} < p_T$  

$10 \text{ GeV} < p_T$
Polarization parameter
Polarization parameters

• In experiment, by observing the angular distribution of the lepton pair that are decayed from $J/\psi$ meson, we determine the polarization of $J/\psi$:

$$\frac{d\Gamma(J/\psi \rightarrow l^+l^-)}{d \cos \theta \ d\phi} \propto 1 + \lambda_\theta \cos^2 \theta + \lambda_\phi \sin^2 \theta \cos 2\phi + \lambda_\theta \phi \sin 2\theta \cos \phi$$

• Here, $\theta$ is the polar angle and $\phi$ is the azimuthal angle of the lepton $l^+$ in the $J/\psi$ rest frame.

$\lambda_\theta = 1$: Transversely polarized
$\lambda_\theta = 0$: Unpolarized
$\lambda_\theta = -1$: Longitudinally polarized.
Polarization parameters

• In theory, the polarization parameter $\lambda_\theta$ is determined by

$$\lambda_\theta = \frac{\sigma_T - 2\sigma_L}{\sigma_T + 2\sigma_L}$$

• Here, $\sigma_T$ is the transverse components of the cross section and $\sigma_L$ is the longitudinal components of the cross section respectively.
Polarization parameter \( \lambda_\theta \)

- \( ^3S^8_1 \) and \( ^3P^8_J \) contributions become 100% transversely polarized at high \( p_T \).
  \( \rightarrow \) assumed that \( ^3S^8_1 \) and \( ^3P^8_J \) are 100% transverse.

- \( ^1S^8_0 \) contribution is unpolarized.
  \( \rightarrow \) 1/3 Longitudinal, 2/3 transverse.

\[
\sigma_T = \frac{d\sigma \left[ ^3S^8_1 \right]}{dp_T} + \frac{d\sigma \left[ ^3P^8_J \right]}{dp_T} + \frac{2}{3} \frac{d\sigma \left[ ^1S^8_0 \right]}{dp_T}
\]

\[
\sigma_L = \frac{1}{3} \frac{d\sigma \left[ ^1S^8_0 \right]}{dp_T}
\]

\[
\lambda_\theta = \frac{\sigma_T - 2\sigma_L}{\sigma_T + 2\sigma_L}
\]
How to solve DGLAP equation
Mellin transformation

- The Mellin transformation is defined by

\[ \mathcal{M}[f](n) \equiv \tilde{f}(n) = \int_0^1 dz \ z^{n-1} f(z) \]

- And its inverse can be made by

\[ f(z) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} dn \ z^{-n} \tilde{f}(n) \]

or we can rewrite the inverse formula by analytic continuation as

\[ f(z) = \frac{1}{\pi} \int_0^\infty dn \ \text{Im} \left[ z^{-c-ne^{i\phi}} e^{i\phi} \hat{f}(c+ne^{i\phi}) \right] \]
Mellin transformation-convolution

- The Mellin transformation makes the convolution
  \[ f \otimes g(z) \equiv \int_{z}^{1} \frac{dy}{y} f \left( \frac{z}{y} \right) g(y) \]
  be linear.

\[ \mathcal{M}[f \otimes g(z)](s) \]
\[ = \int_{0}^{1} dz \ z^{s-1} [f \otimes g](z) \]
\[ = \int_{0}^{1} dz \ z^{s-1} \int_{z}^{1} \frac{dy}{y} f(\frac{z}{y}) g(y) \quad \leftrightarrow \quad z = yt, \quad 0 \leq z = yt \leq y \leq 1 \]
\[ = \int_{0}^{1} dt \ t^{s-1} f(t) \int_{0}^{1} dy \ y^{s-1} g(y) \]
\[ = \hat{f}(s) \hat{g}(s) \]

\[ \therefore \quad \mathcal{M}[f \otimes g](n) = \tilde{f}(n) \times \tilde{g}(n) \]
Mellin transformation of DGLAP

- DGLAP equation is also the convolution form.
- DGLAP equation in the Mellin space:

$$\frac{d}{d \log \mu_f^2} \begin{pmatrix} \tilde{D}_S(n) \\ \tilde{D}_g(n) \end{pmatrix} = \frac{\alpha_s(\mu_f)}{2\pi} \begin{pmatrix} \tilde{P}_{qq}(n) & 2n_f \tilde{P}_{gq}(n) \\ \tilde{P}_{qg}(n) & \tilde{P}_{gg}(n) \end{pmatrix} \begin{pmatrix} \tilde{D}_S(n) \\ \tilde{D}_g(n) \end{pmatrix}$$

- Here, we use 1-loop $\alpha_s$:

$$\alpha_s(\mu_f) = \frac{4\pi}{b_0 \log(\mu_f^2/\Lambda^2)}, \quad b_0 = \frac{11}{3} N_c - \frac{2}{3} n_f$$

- DGLAP equation becomes solvable.
Solution

• Without $q - g$ mixing, the equation become more simpler:

$$\frac{d}{d \log \mu_f^2} \tilde{D}_g(n, \mu_f) = \frac{\alpha_s(\mu_f)}{2\pi} \tilde{P}_{gg}(n) \tilde{D}_g(n, \mu_f)$$

• Because we used one-loop $\alpha_s$, it is obvious that

$$\frac{d}{d \log \mu_f^2} = -\frac{b_0}{4\pi} \frac{\alpha_s^2}{\alpha_s} \frac{d}{d \alpha_s}$$

• Therefore, the solution is given by

$$\tilde{D}_g(n, \mu_f) = \left( \frac{\alpha_s(\mu_{f_0})}{\alpha_s(\mu_f)} \right)^{2\tilde{P}_{gg}(n)/b_0} \tilde{D}_g(n, \mu_{f_0})$$
Fitting NRQCD LDMEs
Fitting CO LDMEs

- We decided CO LDMEs by least $\chi^2$ fitting where

$$
\chi^2 \equiv \sum_i \frac{(O_i - E_i)^2}{\sigma_i^2}
$$

here, at $p_T = i$,

$O_i$: The results of CMS and CDF

Here, we took $p_T \geq 10$ GeV data only.

$E_i$: Theoretical prediction.

$\langle \mathcal{O}^{J/\psi (3S_1^{[8]})} \rangle$, $\langle \mathcal{O}^{J/\psi (1S_0^{[8]})} \rangle$, and $\langle \mathcal{O}^{J/\psi (3P_0^{[8]})} \rangle$ are unknown.

$\sigma_i^2$: Total variance including systematic, statistical and theoretical errors
NRQCD LDMEs
NRQCD LDMEs for $J/\psi$

- $\nu^0$: $\langle 0 | O_H^0 (3 S_1^{[1]}) | 0 \rangle = \langle 0 | \chi^\dagger \sigma^i \psi \mathcal{P}_H \psi^\dagger \sigma^i \chi | 0 \rangle$

- $\nu^2$: $\langle 0 | O_2^H (3 S_1^{[1]}) | 0 \rangle = \frac{1}{2} \langle 0 | \chi^\dagger \sigma^i (-\frac{i}{2} \vec{D})^2 \psi \mathcal{P}_H \psi^\dagger \sigma^i \chi + \text{H. c.} | 0 \rangle$

- $\nu^3$: $\langle 0 | O_0^H (1 S_0^{[8]}) | 0 \rangle = \langle 0 | \chi^\dagger T^a \psi \mathcal{P}_H \psi^\dagger T^a \chi | 0 \rangle$

- $\nu^4$ (color octet):
  - $\langle 0 | O_0^H (3 S_1^{[8]}) | 0 \rangle = \langle 0 | \chi^\dagger \sigma^i T^a \psi \mathcal{P}_H \psi^\dagger \sigma^i T^a \chi | 0 \rangle$
  - $\langle 0 | O_0^H (3 P_0^{[1]}) | 0 \rangle = \frac{1}{d-1} \langle 0 | \chi^\dagger (-\frac{i}{2} \vec{D} \cdot \sigma) \psi \mathcal{P}_H \psi^\dagger (-\frac{i}{2} \vec{D} \cdot \sigma) \chi | 0 \rangle$
  - $\langle 0 | O_0^H (3 P_1^{[1]}) | 0 \rangle = \langle 0 | \chi^\dagger (-\frac{i}{2} \vec{D}^{[i, \sigma^j]} \psi \mathcal{P}_H \psi^\dagger (-\frac{i}{2} \vec{D}^{[i, \sigma^j]} \chi | 0 \rangle$
  - $\langle 0 | O_0^H (3 P_2^{[1]}) | 0 \rangle = \langle 0 | \chi^\dagger (-\frac{i}{2} \vec{D}^{(i \sigma^j)} \psi \mathcal{P}_H \psi^\dagger (-\frac{i}{2} \vec{D}^{(i \sigma^j)} \chi | 0 \rangle$
  - $\langle 0 | O_0^H (3 P^{[8]}) | 0 \rangle = \sum_{J=0,1,2} \langle 0 | O_0^H (3 P_j^{[8]}) | 0 \rangle$
NRQCD LDMEs for $J/\psi$

- Order-$\nu^4$(color singlet)

\[
\begin{align*}
\langle 0 | O_{4,1}^H (^3S_{1}^{[1]}) | 0 \rangle &= \langle 0 | \chi^\dagger \sigma^i (-\frac{i}{2} \overset{\leftarrow}{D})^2 \psi \mathcal{P}_H \psi^\dagger \sigma^i (-\frac{i}{2} \overset{\rightarrow}{D})^2 \chi | 0 \rangle \\
\langle 0 | O_{4,2}^H (^3S_{1}^{[1]}) | 0 \rangle &= \frac{1}{2} \langle 0 | \chi^\dagger \sigma^i (-\frac{i}{2} \overset{\rightarrow}{D})^4 \psi \mathcal{P}_H \psi^\dagger \sigma^i \chi + \text{H. c.} | 0 \rangle \\
\langle 0 | O_{4,3}^H (^3S_{1}^{[1]}) | 0 \rangle &= \frac{1}{2} \langle 0 | \chi^\dagger \sigma^i \psi \mathcal{P}_H \psi^\dagger \sigma^i (\overset{\rightarrow}{D} \cdot gE + gE \cdot \overset{\rightarrow}{D}) \chi \\
&\quad - \chi^\dagger \sigma^i (\overset{\rightarrow}{D} \cdot gE + gE \cdot \overset{\rightarrow}{D}) \psi \mathcal{P}_H \psi^\dagger \sigma^i \chi | 0 \rangle
\end{align*}
\]

- Equation of motion eliminates $\langle 0 | O_{4,3}^H (^3S_{1}^{[1]}) | 0 \rangle$

- $\langle 0 | O_{4,1}^H (^3S_{1}^{[1]}) | 0 \rangle = \langle 0 | O_{4,2}^H (^3S_{1}^{[1]}) | 0 \rangle + \mathcal{O}(\nu^2)$
Charmonium spectroscopy

\[ J^{PC} = \begin{array}{ccccccc}
0^{-+} & 1^{--} & 0^{++} & 1^{++} & 1^{--} & 2^{++} \\
\end{array} \]
Another attempt

- Relativistic corrections to CS fragmentation contribution

At large $p_T$, one can roughly estimate the relative size of the contribution by making use of the following approximation:

$$\frac{d\sigma_{\text{frag}}}{dp_T} = \int_0^1 dz \frac{d\sigma_{\text{parton}}}{dp_T}(p_T/z)D(z) \propto \int_0^1 dz z^\kappa D(z) \equiv I_\kappa(D)$$

By making use of the Kneihl's result, we got $\kappa = 5.2$.

<table>
<thead>
<tr>
<th>$I_\kappa(d_n) \mid d_n[g \to Q\bar{Q}(^3S_1^{[1]})]$</th>
<th>$d_0$</th>
<th>$d_2$</th>
<th>$d_{4,1} + d_{4,2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{5.2}(d_n) \mid \mu_A=m \times R_n$</td>
<td>0.743</td>
<td>2.18</td>
<td>1.35</td>
</tr>
<tr>
<td>$I_{5.2}(d_n) \mid \mu_A=2m \times R_n$</td>
<td>0.743</td>
<td>2.18</td>
<td>3.19</td>
</tr>
</tbody>
</table>