What does condensed matter tell us about general relativity?

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Based on works done by NI with A. Ishibashi (Kinki U.) and K. Maeda (Shibaura U.)

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Organization of the talk

- What does CM (condensed matter) tells about gravity through holography?
- Paradox !?

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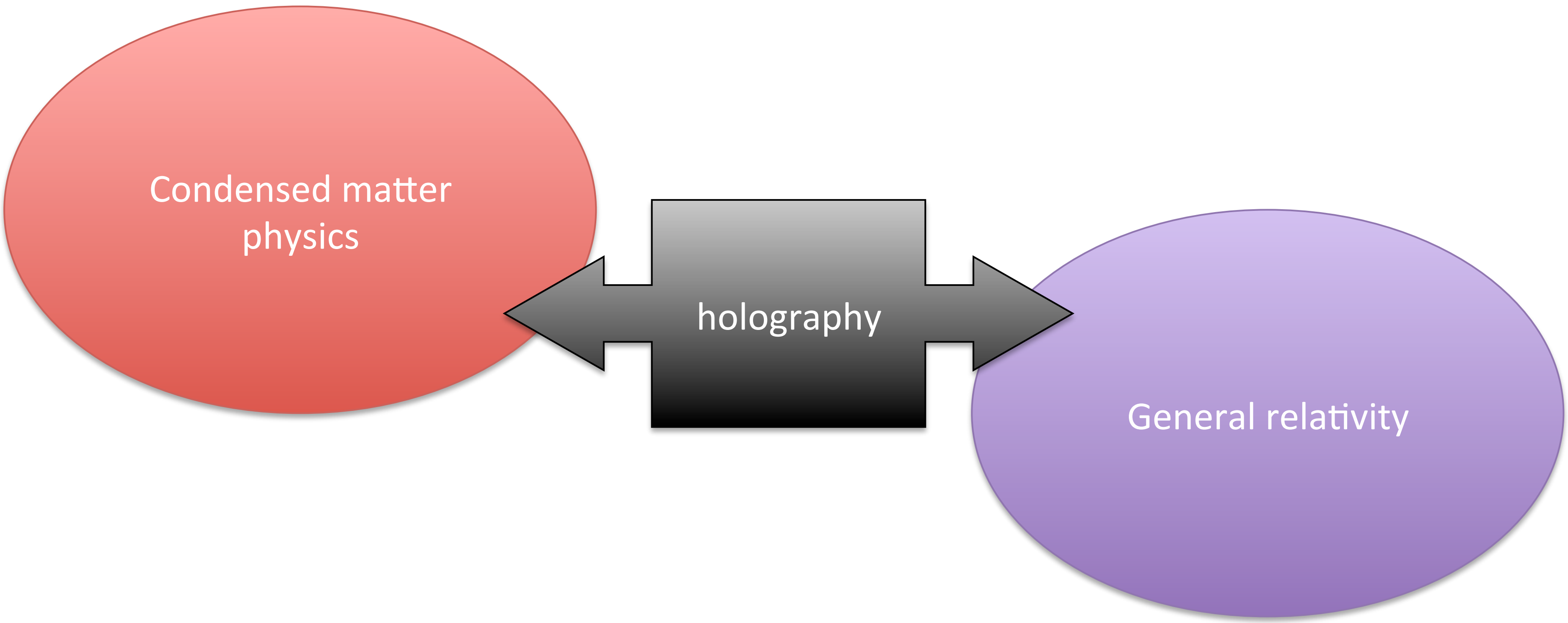
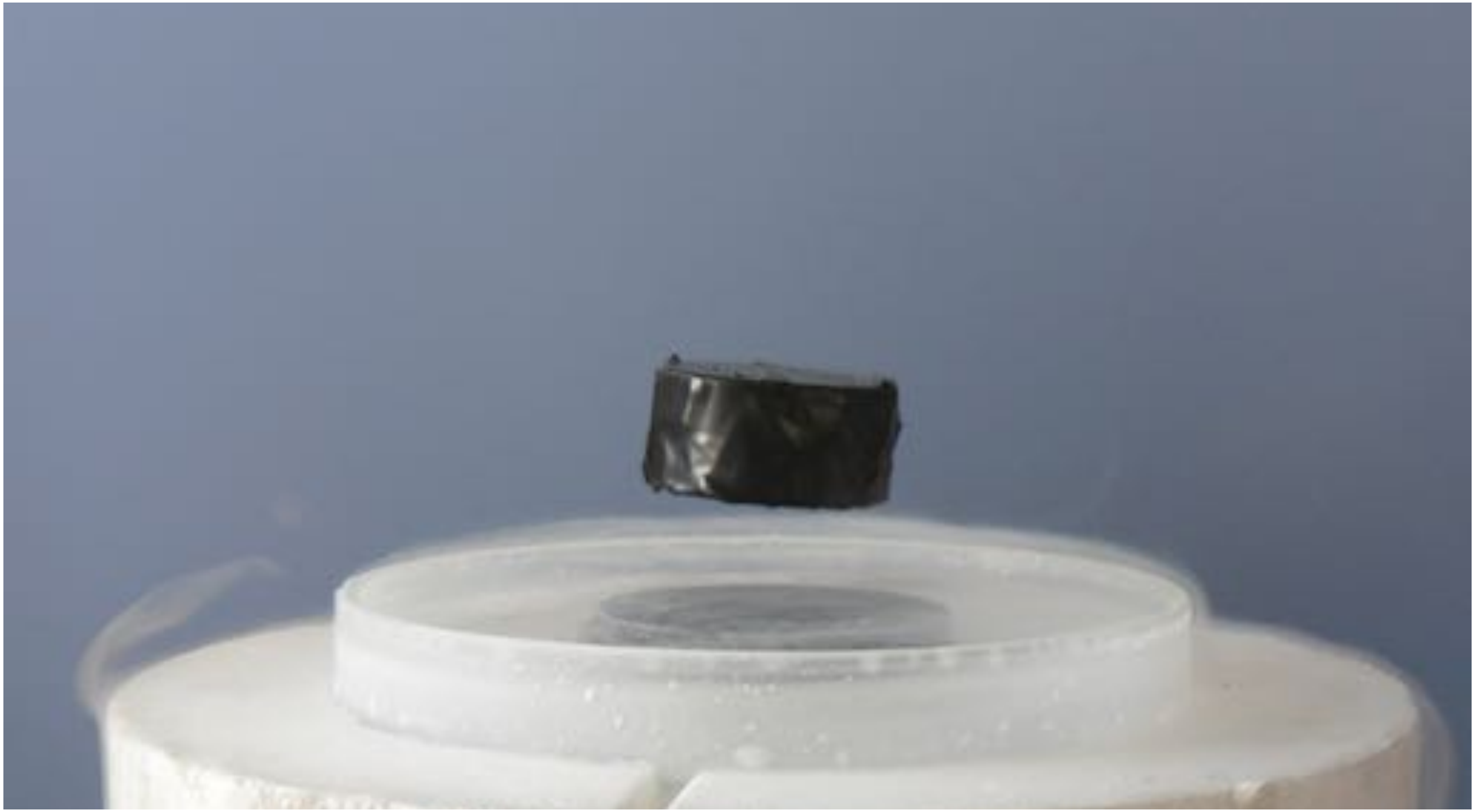
- Paradox !?
- Solution
- Conclusion

Main topic


Condensed matter physics
holography
General relativity

Condensed matter physics

Superconductor/superfluid



Superconductor/superfluid

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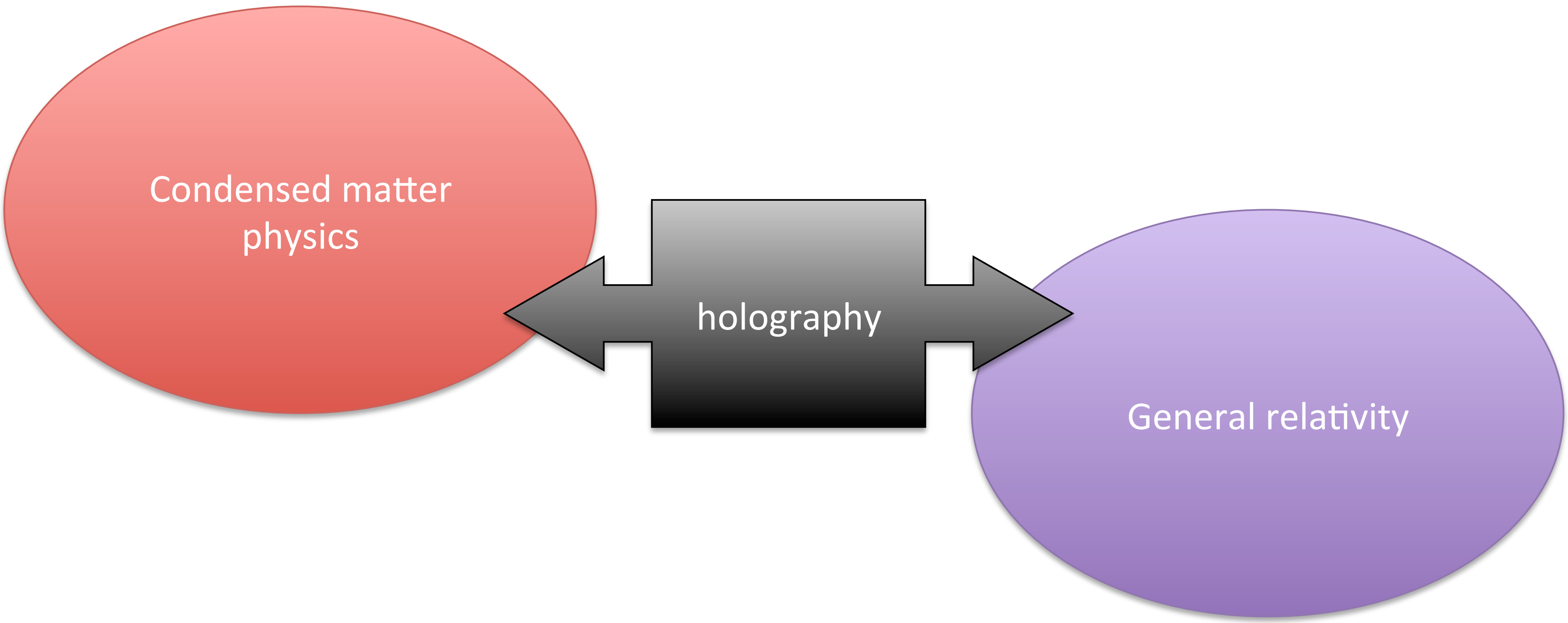
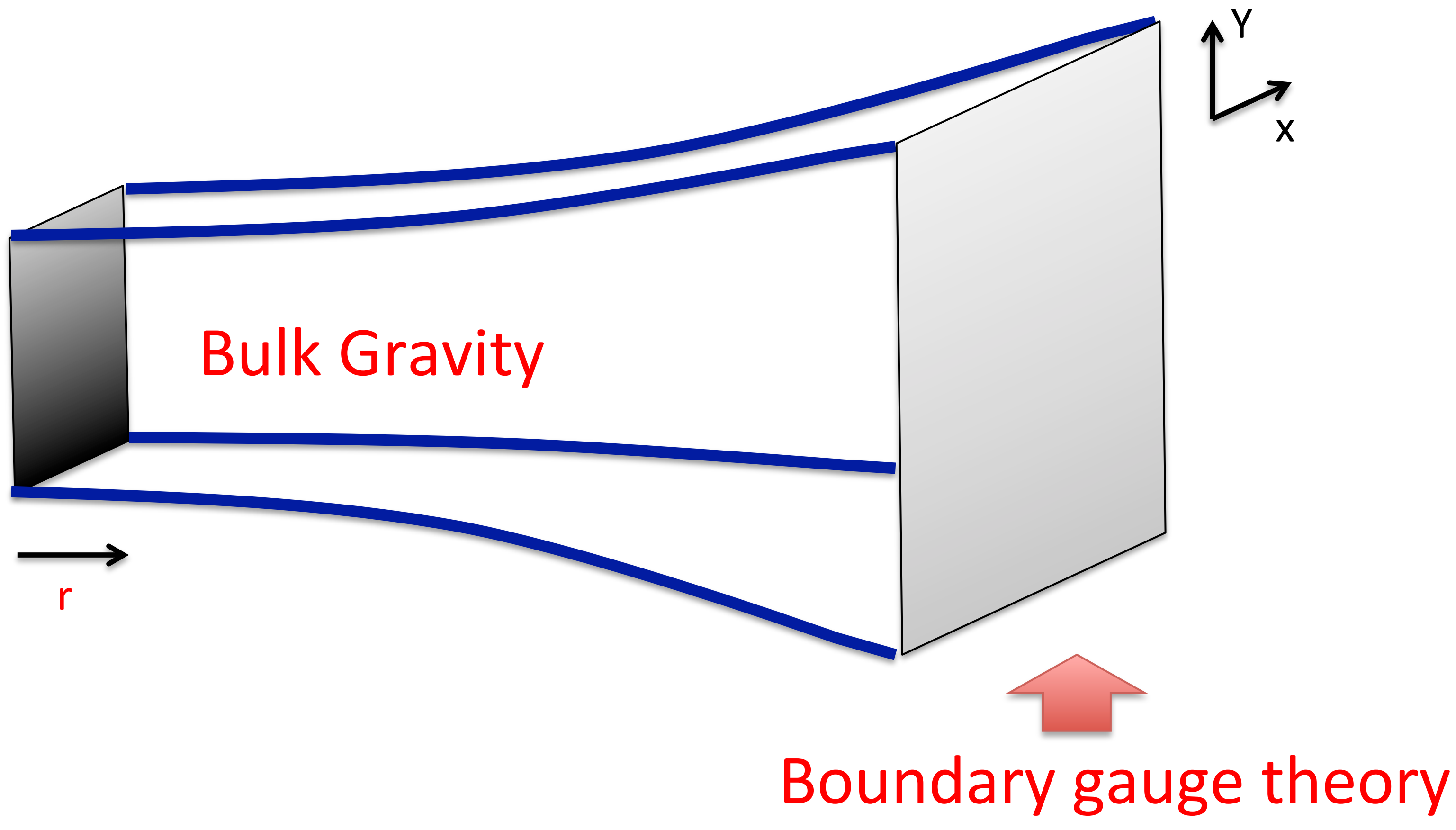
- No resistivity even at nonzero temperature
- What is its bulk dual ?

This talk


Condensed matter physics
holography
General relativity

Holographic view of this



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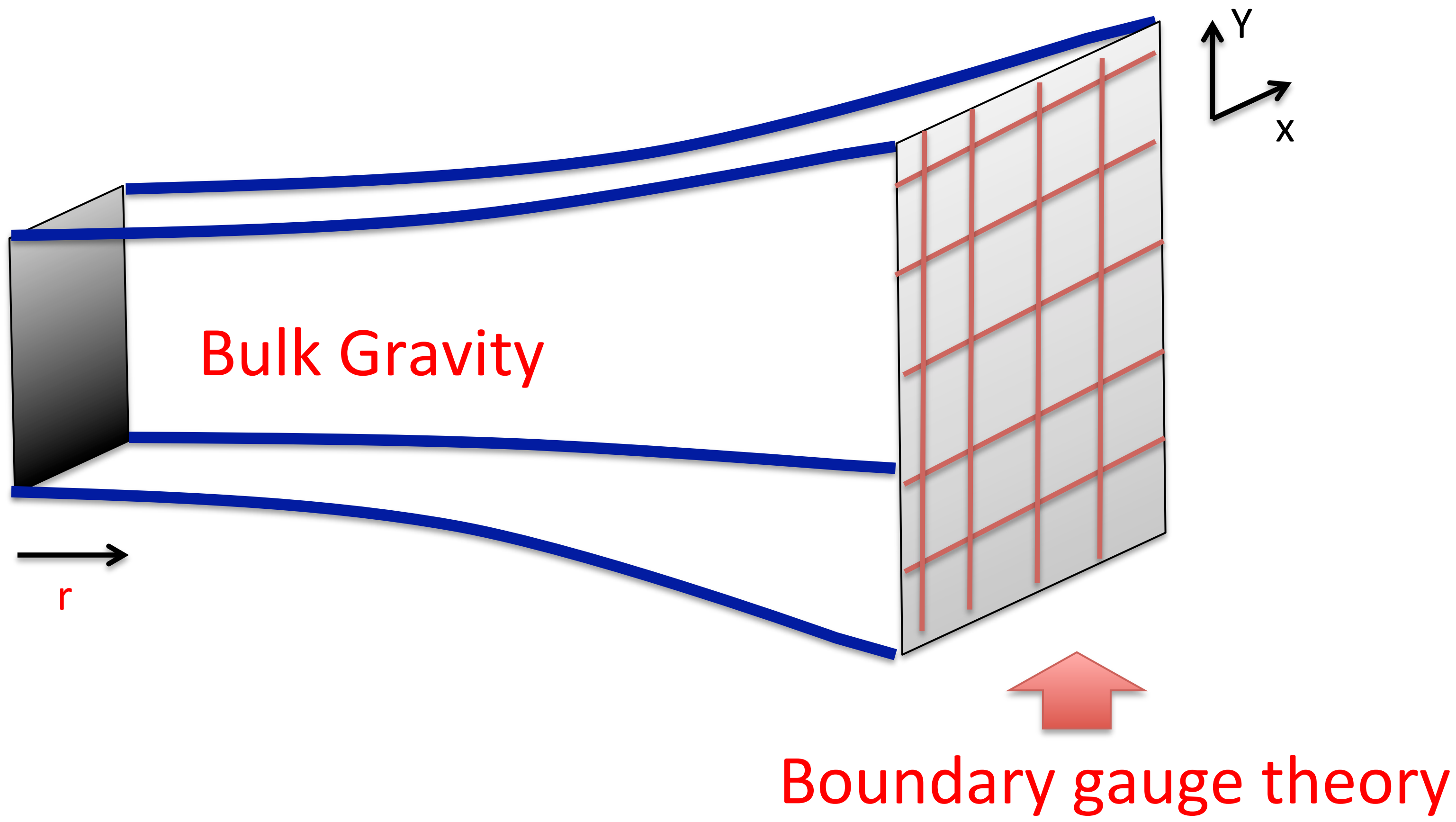

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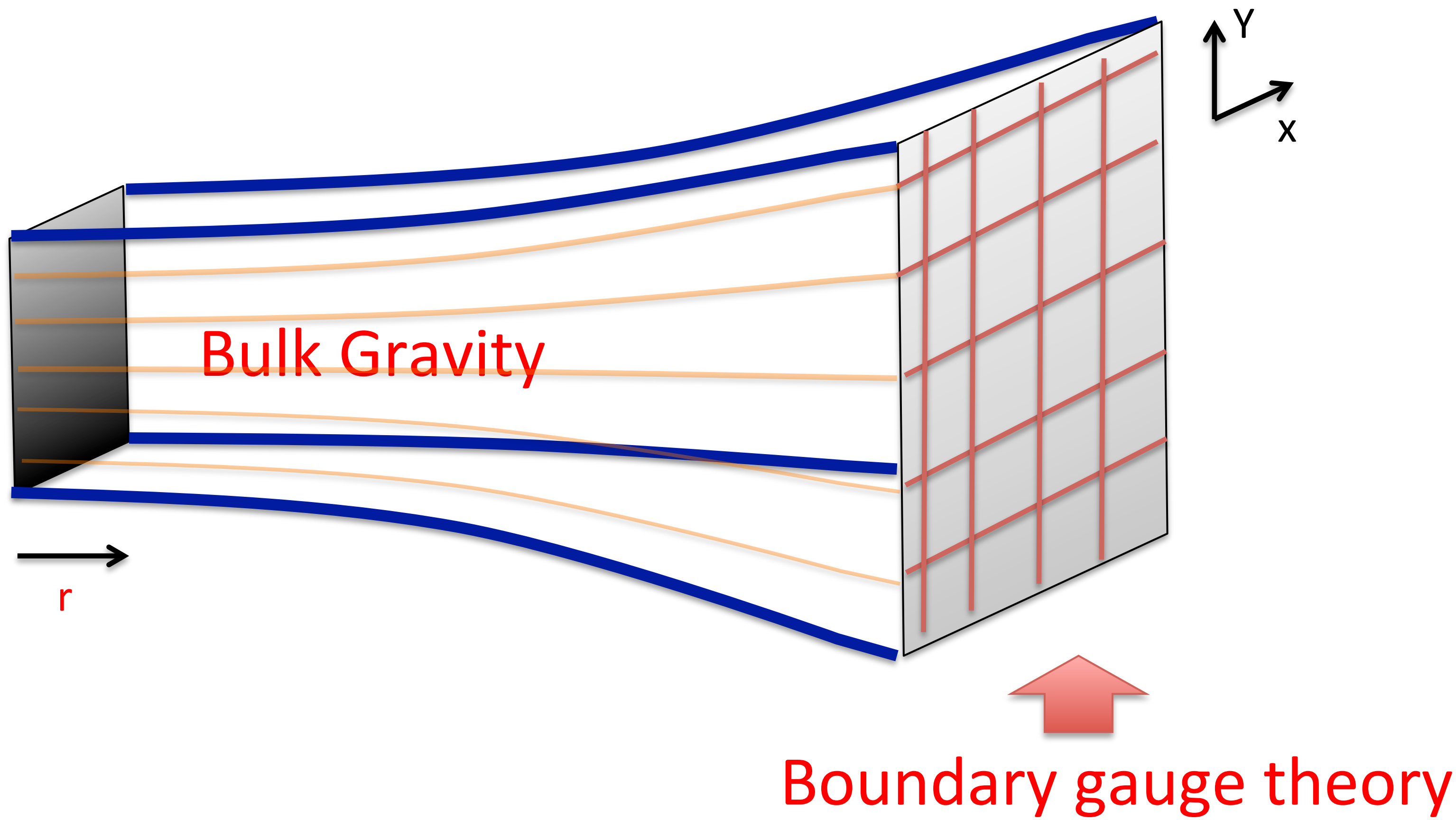

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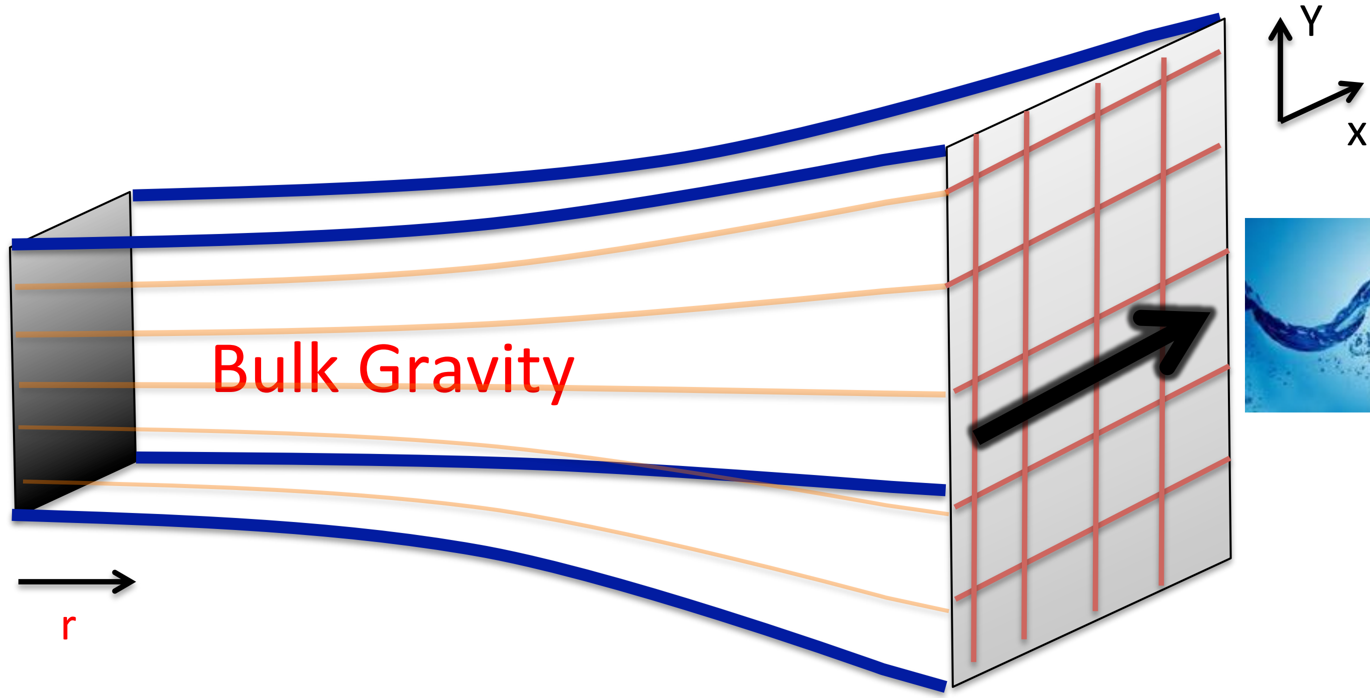
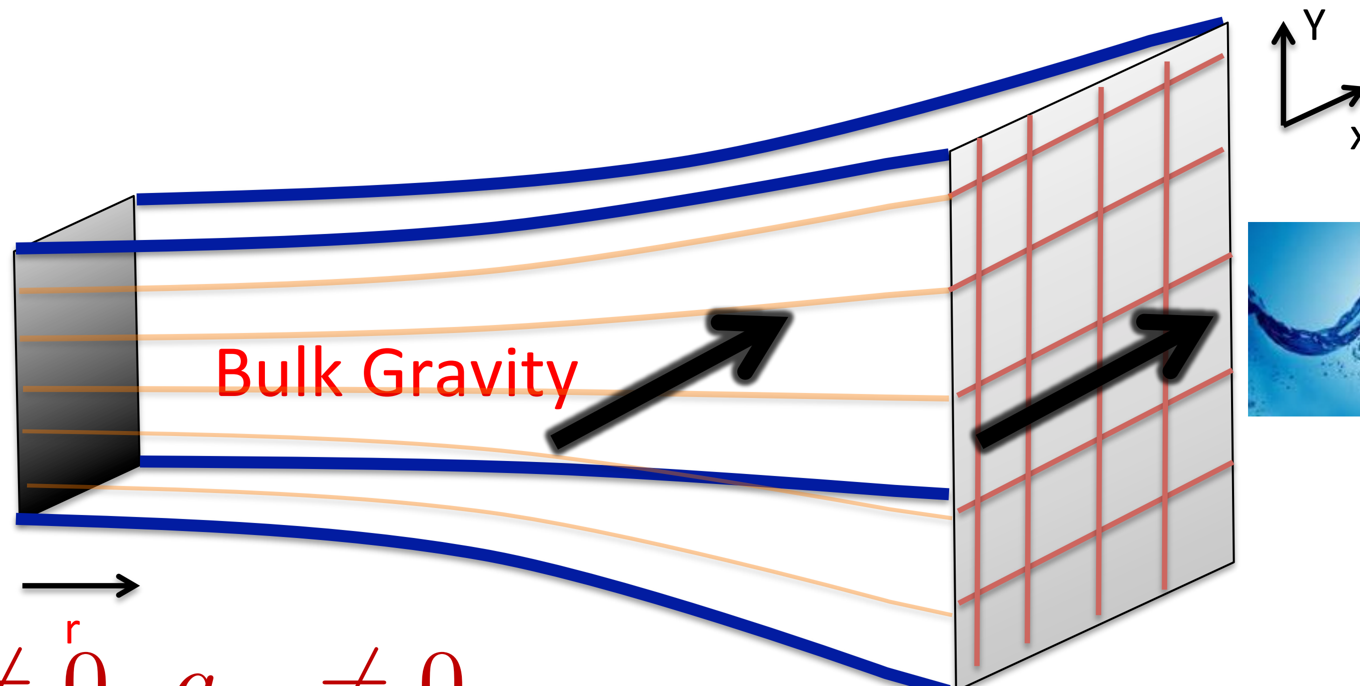


$$J_x \neq 0\,, T_{tx} \neq 0$$

Boundary gauge theory

Holographic view of this



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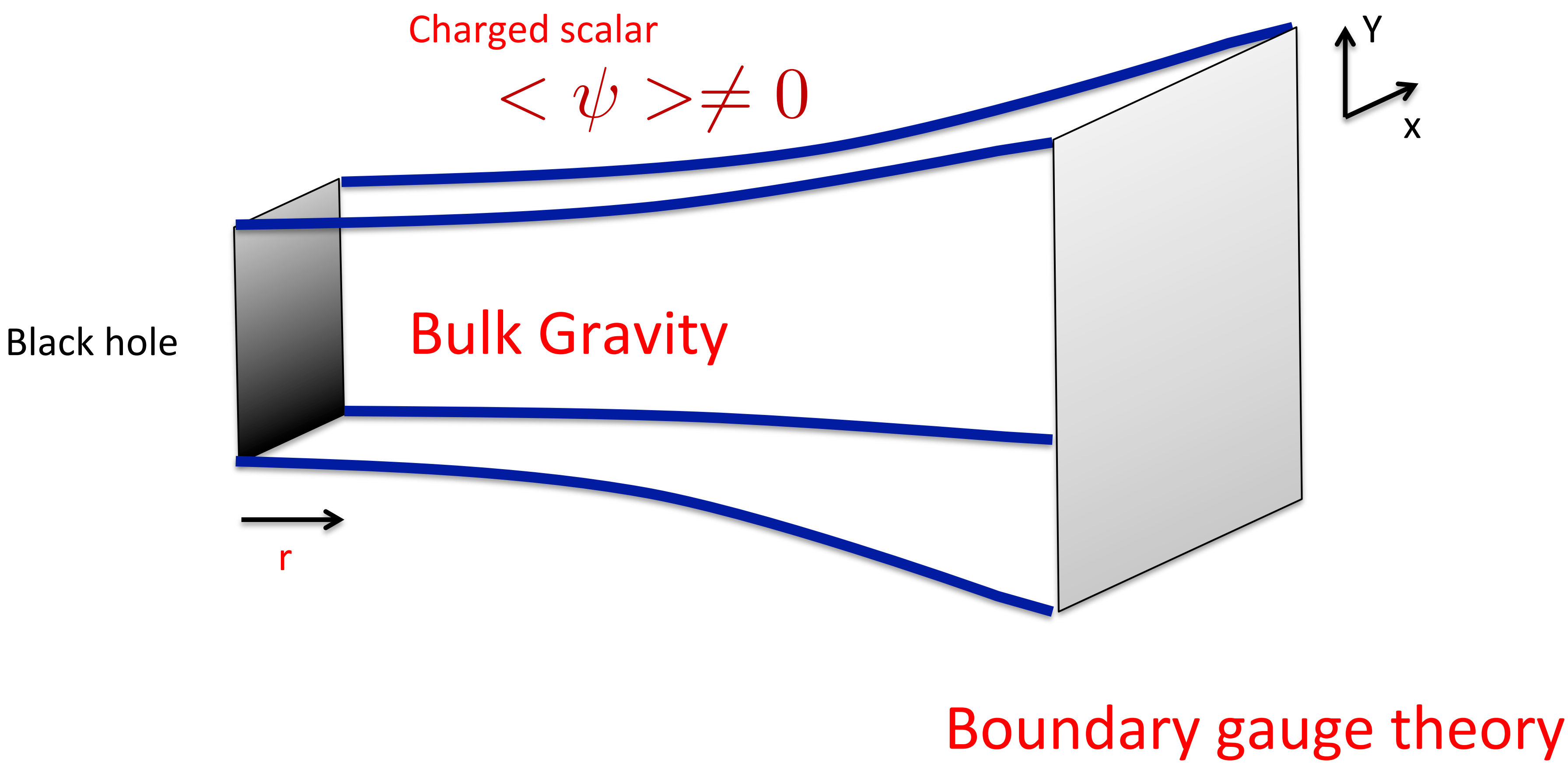
Boundary gauge theory

Holographic super-conductor/fluid

Superconducting phase;
Spontaneous sym. breaking

Infinite conductivity; no
Electric field but nonzero J_x



$$m_{eff}^2(r) = m^2 + e^2 g^{tt}(r)(A_t(r))^2$$

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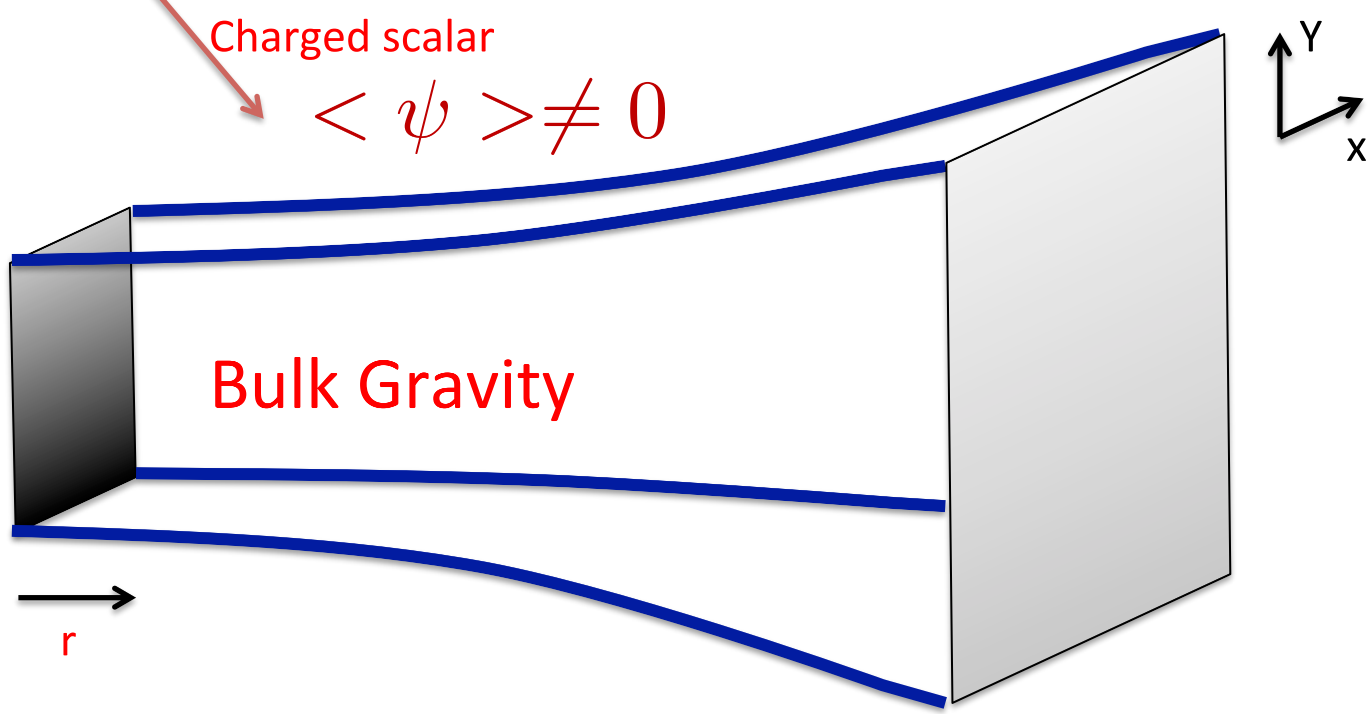


Black hole

Boundary gauge theory

Holographic view of this

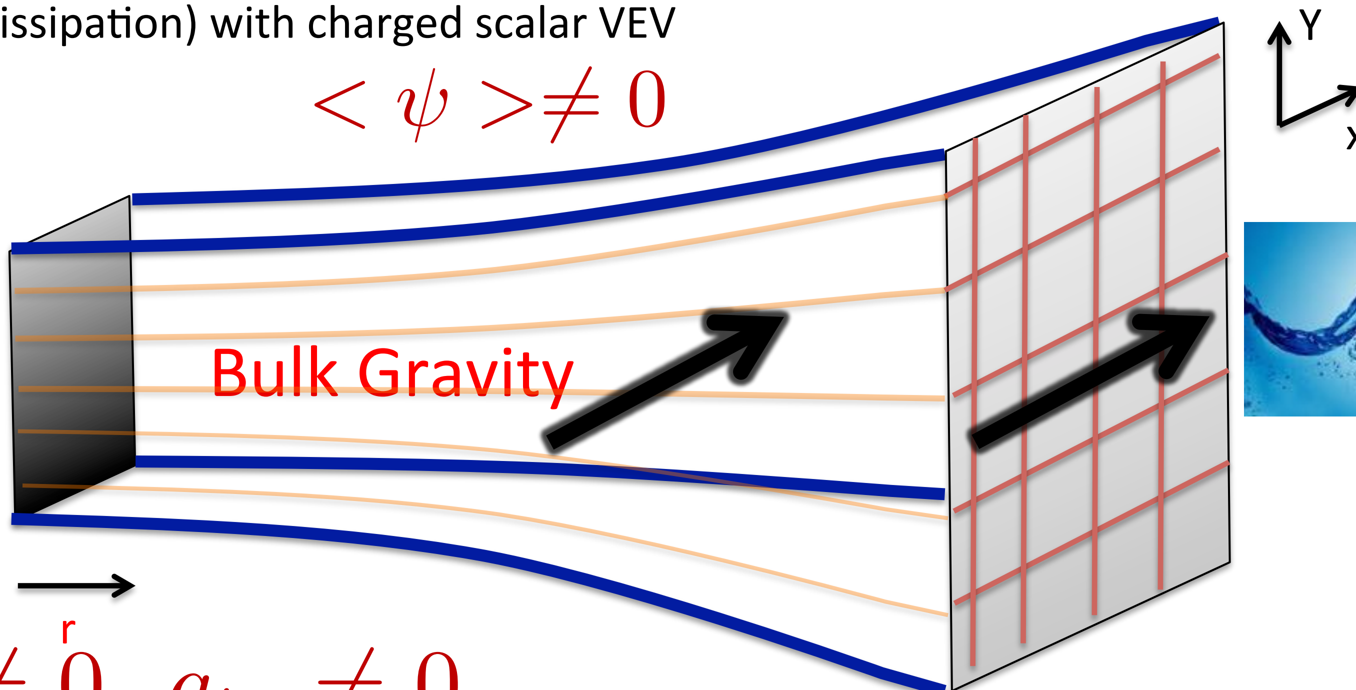
Superconducting phase;
Spontaneous sym. breaking

Infinite conductivity; no
Electric field but nonzero J_x

Black hole is rotating; nonzero g_{tx};
No Source (dissipation) with charged scalar VEV

hairy

Black hole



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Boundary gauge theory

Holographic view of this

- Holographic dual of persistent superconductor current predict the existence of stationary rotating hairy black hole along the direction of no translational symmetry; without source field in bulk, i.e., no outer energy input and with no dissipation

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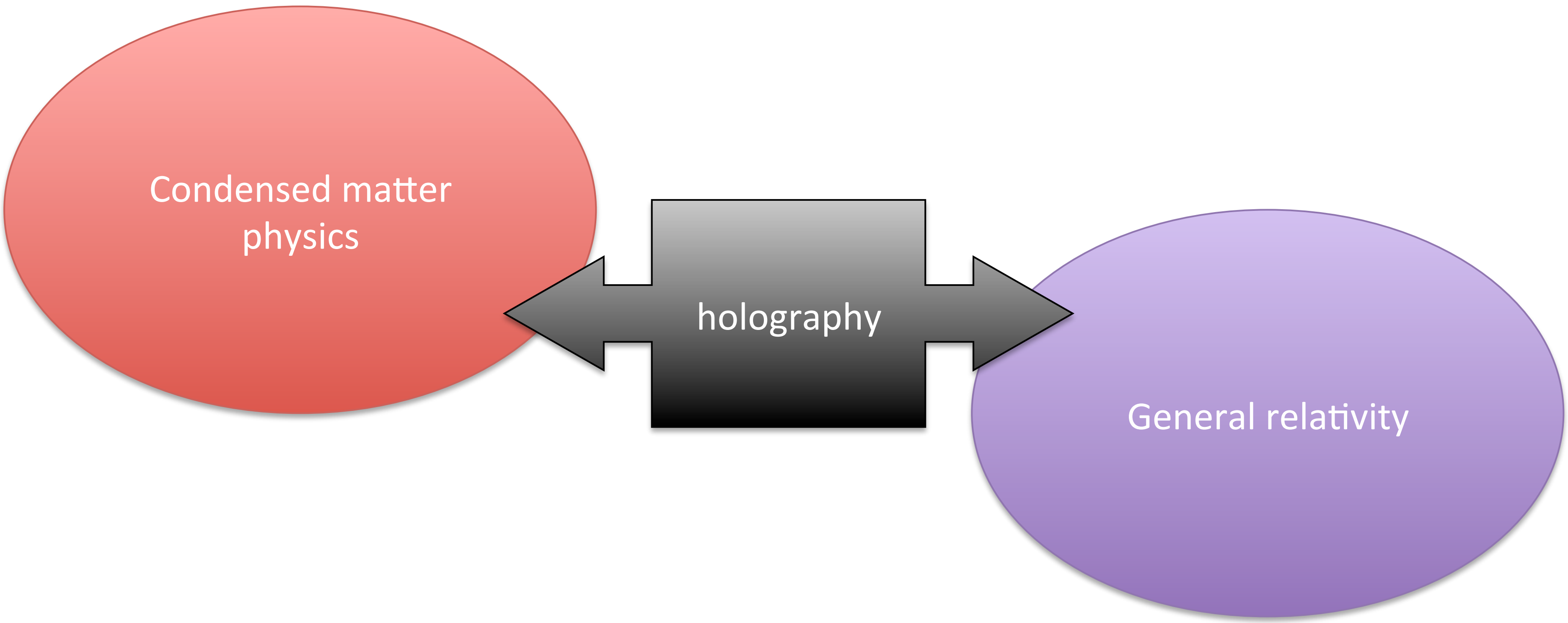
- However no such solution is known so far …

Contradiction!?



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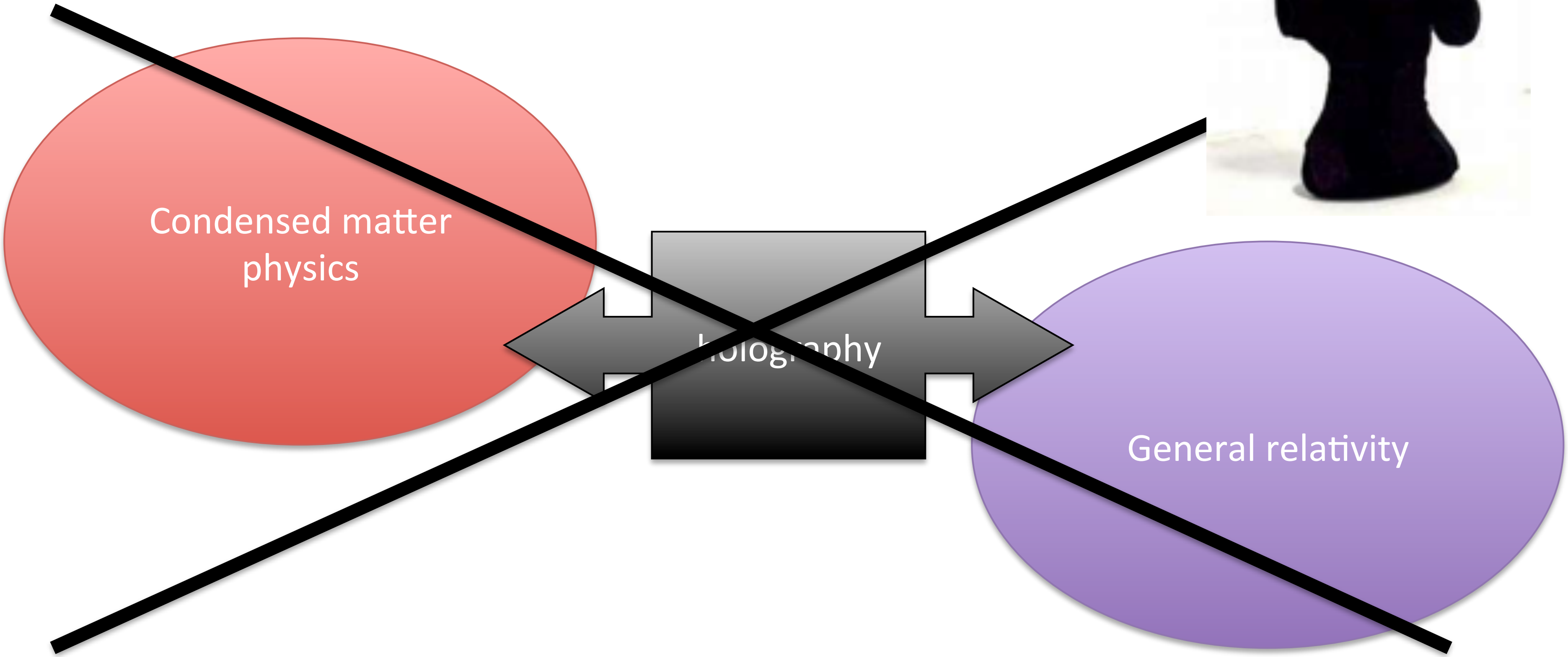
- There is a mathematical proof that no such solution is allowed in GR;

  this is called *black hole rigidity theorem*
- *If black hole is rotating along the direction of no symmetry, then it loses its angular momentum by the emission of gravitational waves*
- More rigorously, one can show that such a solution violates Raychaudhuri eq. of GR

Given such theorem;

- We have seen that holography (and string theory) is <u>wrong!</u>?



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- ~~We have seen that holography (or string theory) is wrong?~~
- General relativity theorem is classical physics, so in quantum gravity, it doesn't hold. And one can do holography without large N, where we don't care GR theorem?

Unlikely

Given such statement;

- Persistent superconductor does not exist in the large N limit? Unlikely
- Something is wrong with our understanding?

Possible !

So what is my mistake?

- So let's go back to the rigidity theorem…

- Black hole rigidity theorem –

``*If black hole is rotating* along the direction of no symmetry, then it loses its angular momentum by the emission of gravitational waves''

Holographic view of this

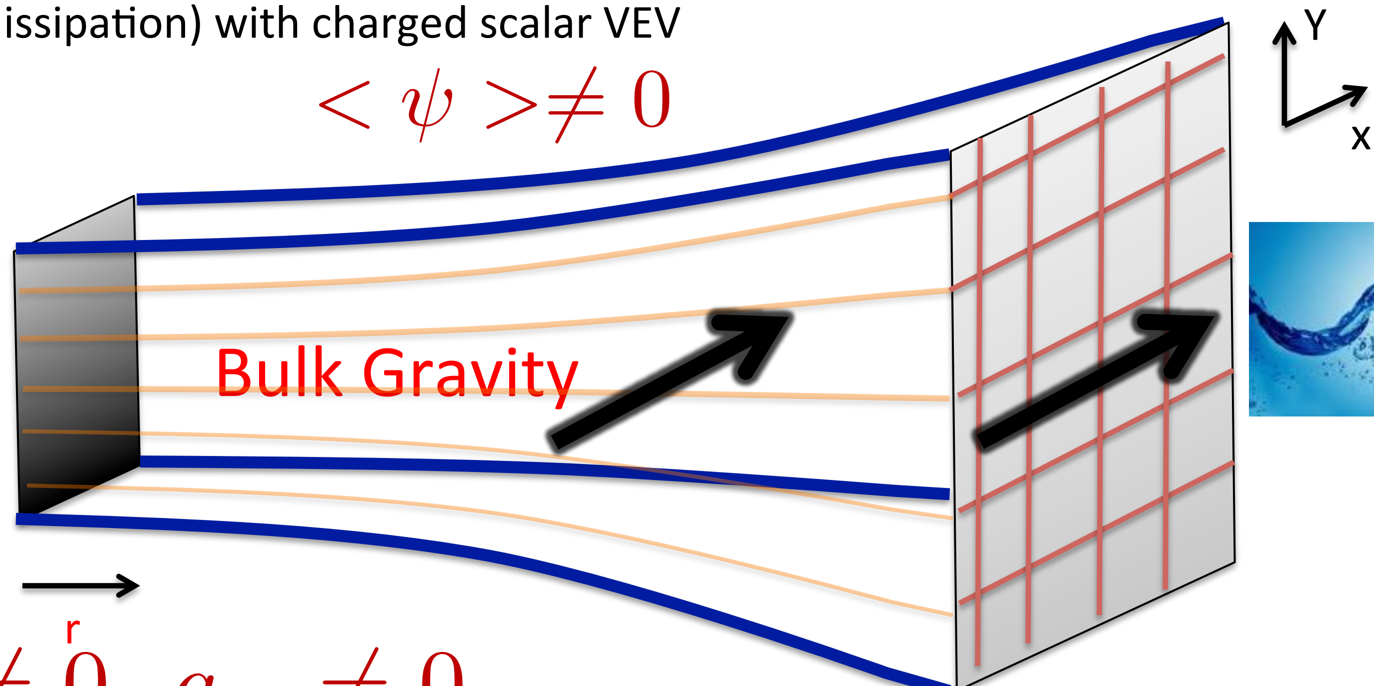
Superconducting phase;
Spontaneous sym. breaking

Infinite conductivity; no
Electric field but nonzero J_x

Black hole is rotating; nonzero g_{tx};
No Source (dissipation) with charged scalar VEV

$$< \psi > \neq 0$$

Black hole



$$A_x \neq 0 \, , g_{tx} \neq 0$$

$$J_x \neq 0 \, , T_{tx} \neq 0$$

Boundary gauge theory

We propose that

- The dual of persistent superconductor is not rotating black hole. But rather it is a *stationary non-rotating but not static black hole*.
- In other words, $g_{tx} = 0$ at the horizon but nonzero outside
- Total momentum is only carried by the matter field outside
- This teaches which dof can carry supercurrent

- We construct such novel solutions!
- Our solution has no dissipation and no source (no energy input, so horizon size doesn't change).
- This corresponds to persistent current without electric field!
- This is (as far as we know) the first solution of such example

For the rest of my talk…

- The action and our set-up
- Solutions
- Comparison with Superfluid hydrodynamics (by Landau Tisza)
- No go without charged scalar
- Dual interpretation
- Luttinger Theorem and Final Comments
- Conclusion & summary

Our set-up: a holographic model

$$\mathcal{L} = R + \frac{12}{L^2} - \frac{1}{4}F^2 - \frac{1}{4}W^2 - |D\Phi|^2 - m^2|\Phi|^2$$

- 5 dim Einstein-Maxwell-charged scalar model
- Two gauge bosons: U(1) x U(1) sym.

$$F = dA\,, \quad W = dB$$

- But charged scalar Φ is charged under only one U(1)

$$D_\mu = \nabla_\mu - iqA_\mu$$

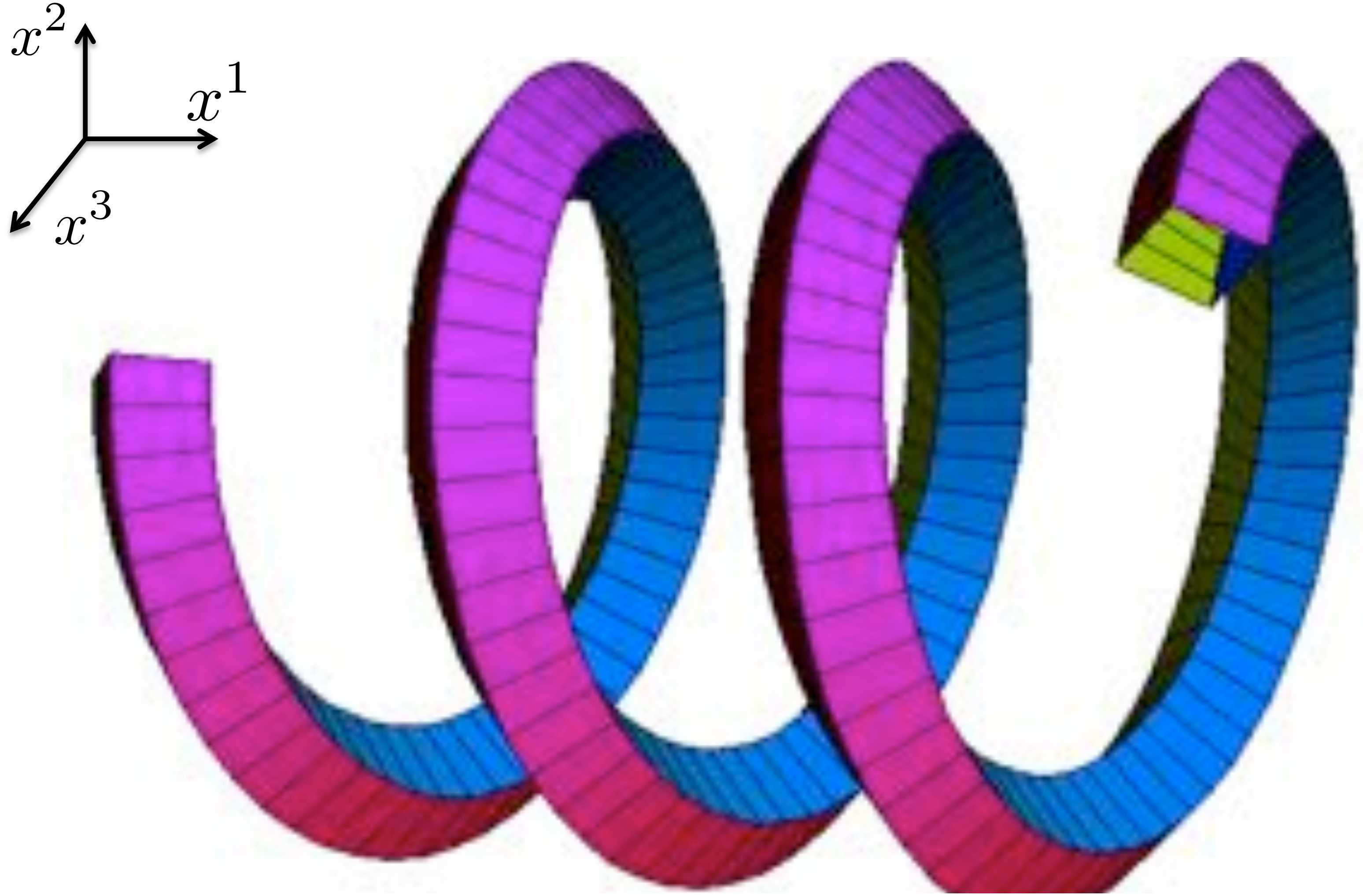
Our set-up: a holographic model

- We solve the system with the metric ansatz

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + e^{2v_3(r)}(\omega^3 - \Omega(r)dt)^2 + e^{2v_1(r)}(\omega^1)^2 + e^{2v_2(r)}(\omega^2)^2\,.$$

$$\omega^1 = \cos(x^1)dx^2 + \sin(x^1)dx^3\,,$$

$$\omega^2 = -\sin(x^1)dx^2 + \cos(x^1)dx^3\,, \omega^3 = dx^1$$



x^2
x^1
x^3

Our set-up: a holographic model

- We solve the system with the metric ansatz

$$A_\mu dx^\mu = A_{x^1}(r)\,\omega^3 + A_t(r)dt\,,$$

$$B_\mu dx^\mu = b(r)\,\omega^1\,, \quad \Phi = \phi(r)$$

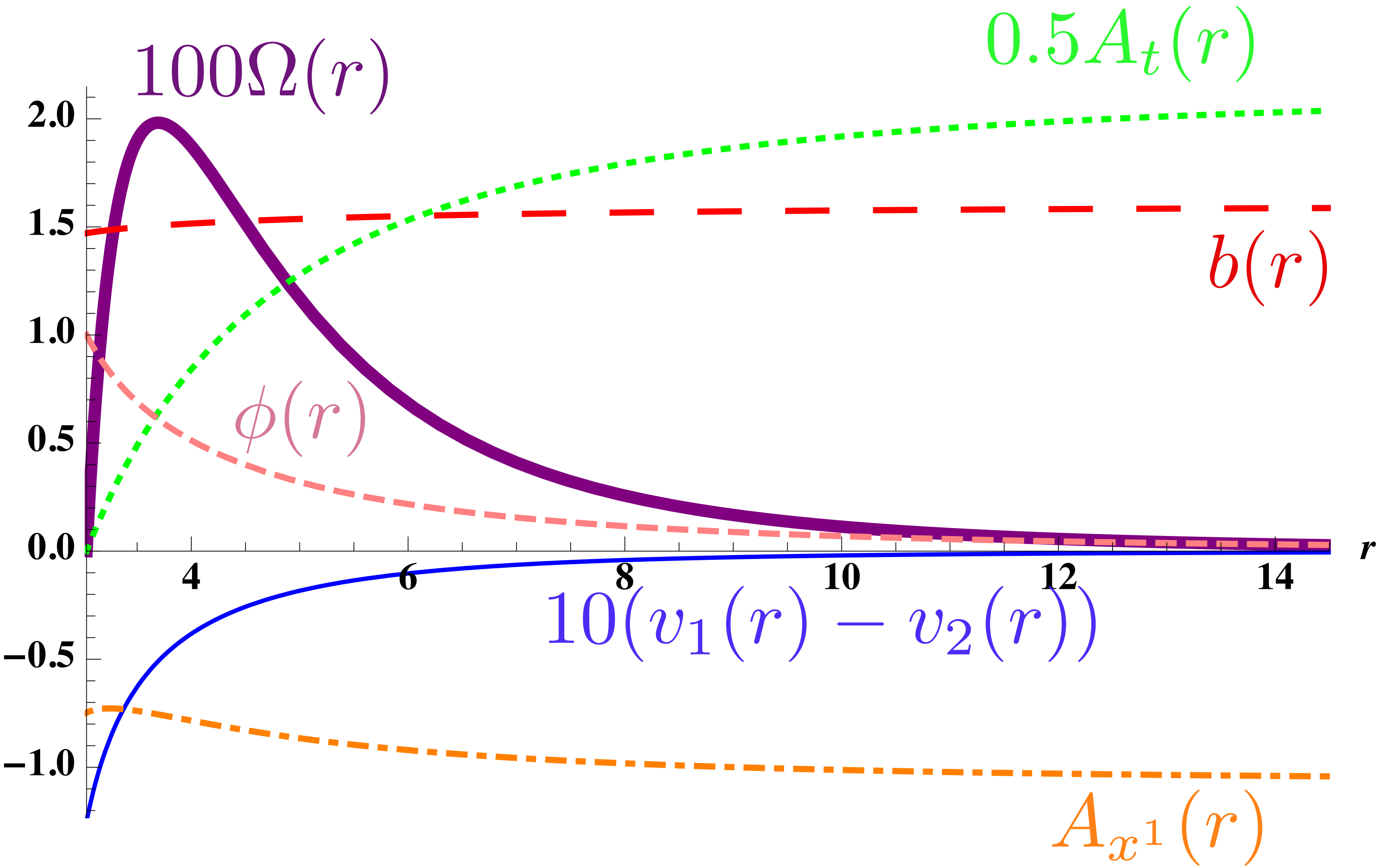
$$\omega^1 = \cos(x^1)dx^2 + \sin(x^1)dx^3\,,$$

$$\omega^2 = -\sin(x^1)dx^2 + \cos(x^1)dx^3\,, \omega^3 = dx^1$$

Our set-up: a holographic model

- A_μ is to introduce a chemical potential
- We take an ansatz for the other one form B_μ to be proportional to type VII_0 Bianchi form
- This induces holographic ``helical lattice'' effects
- If we set $B_\mu = 0$, then this reduces to the normal holographic superconductor model

Our Solutions



$$g_{tx^1} \sim \frac{< T_{tx^1} >}{r^2}|_{r\to\infty}$$

Our Solutions

$$A_{x^1} \sim \mu\,\zeta + \frac{< j_{x^1} >}{r^2}|_{r\to\infty}$$

$\phi(r_h)$	T	μ	$b(\infty)$	$-\zeta$	$< T_{tx^1} >$	$< j_{x^1} >$
1	0.08138	4.325	5.927	0.5489	−43.554	10.07
1	0.1450	4.295	8.012	0.2491	−25.225	5.873
2/3	0.03570	4.071	4.955	0.7103	−16.993	4.174
2/3	0.1059	3.919	7.057	0.5018	−16.308	4.161
4/5	0.1513	4.003	7.048	0.2524	−14.474	3.616

$$\frac{< T_{tx^1} >}{\mu < j_{x^1} >} = -1.000 \pm O(10^{-4}),$$

Hydrodynamics by Landau & Tisza

- Stress tensor and current including normal and superfluid component (set normal = zero)

$$T_{\mu\nu} = (\epsilon + P)u_\mu u_\nu + P\eta_{\mu\nu} + \mu\rho_s v_\mu v_\nu \,,$$

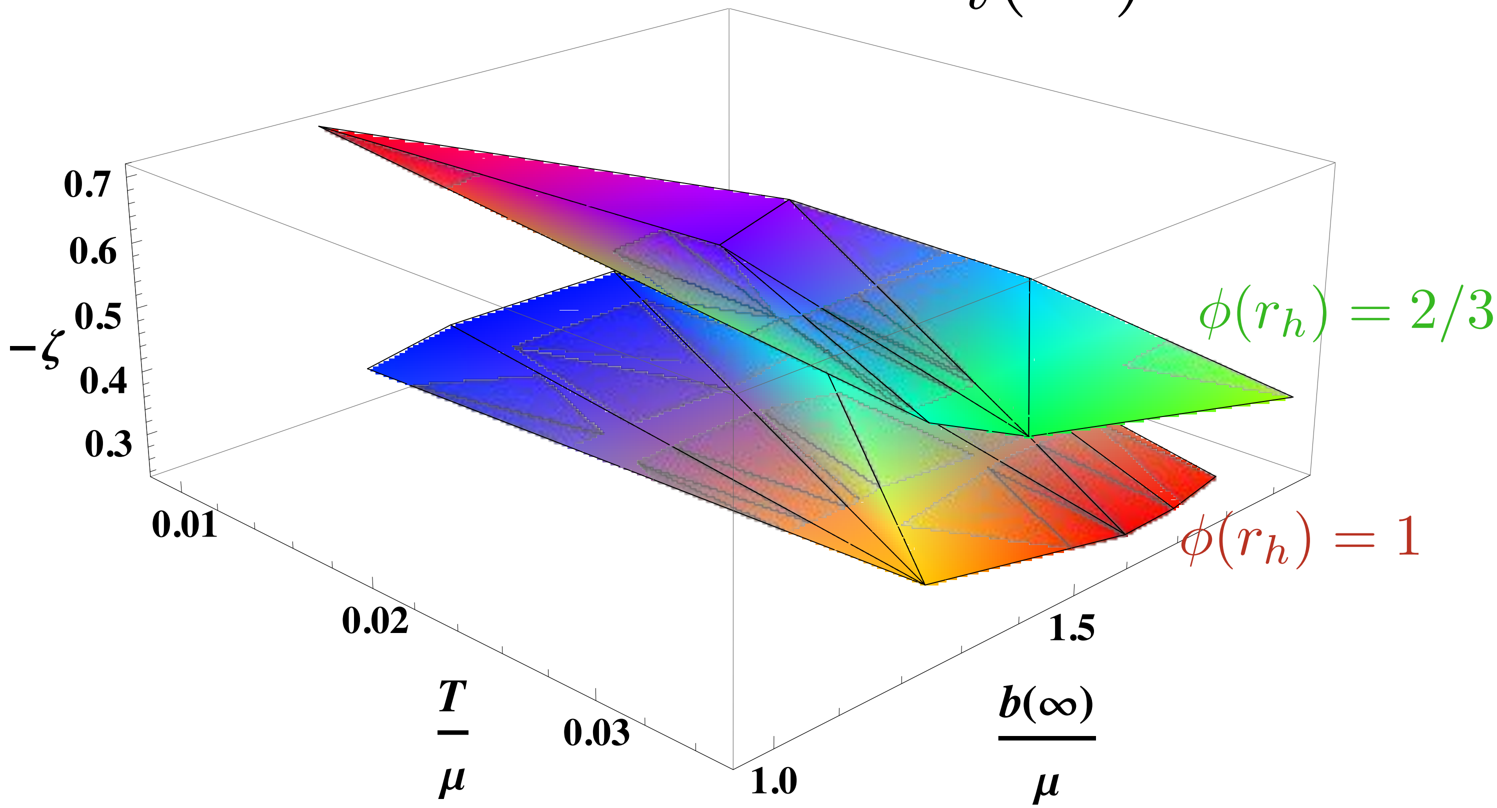
$$j_\mu = \rho_n u_\mu + \rho_s v_\mu \;, \qquad v_\mu u^\mu = -1$$

$$\Rightarrow \quad \frac{T_{tx^1}}{\mu\, j_{x^1}} = v_t = -(u^t)^{-1} = -1$$

General Relativity Knows Superfluidity

3D plot of dimensionless parameters

$$(\, T/\mu \, , b(\infty)/\mu \, , \zeta\left(= \frac{A_{x^1}(\infty)}{A_t(\infty)}\right) \,)$$



- As we increase $T/\mu \, , b(\infty)/\mu \, , |\zeta|$, condensate VEV $\phi(r_h)$ decreases (s-conductor breaking)

Summary

- Our solutions has no non-normalizable mode except for constant term for gauge boson
- No source corresponding to electric field
- Stationary, no time-dependence
- Black hole is non-rotating but geometry outside horizon is rotating along the direction of no symmetry
- Our solution shows no dissipation

Summary

- Charged scalar condensate is crucial
- Without that, one can show that there is no-go theorem which shows that such solutions do not exist   See our paper : arXiv: 1403.0752
- Symmetry breaking is crucial

Summary

- Black hole

  = non-fermi liquids dof

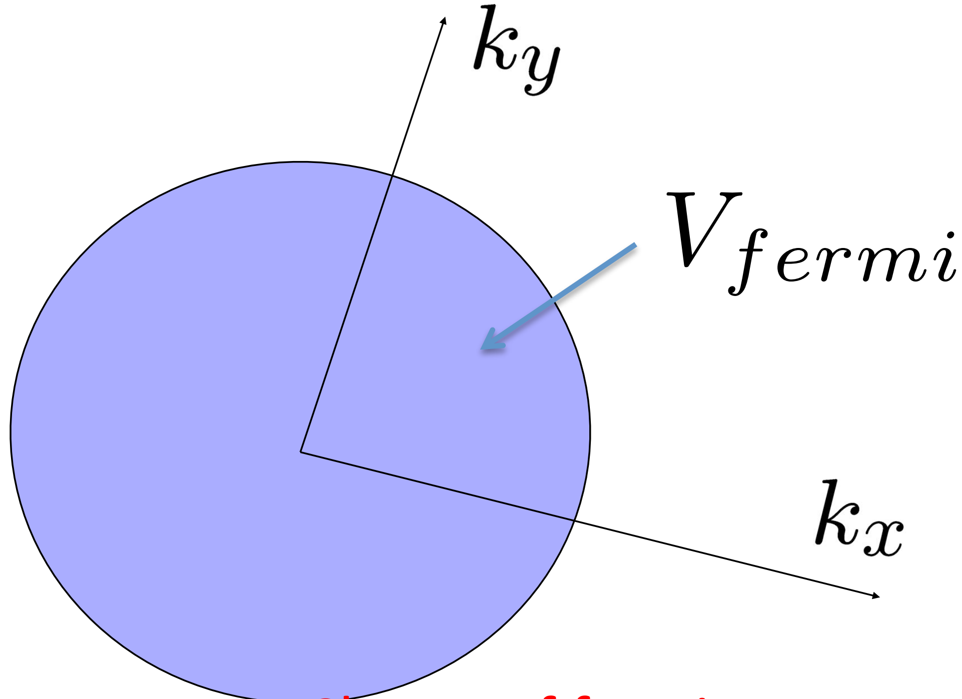
  = `fractionalized' dof

  which violates Luttinger theorem

- Graviton = normal dof

  satisfying Luttinger theorem

Luttinger theorem



Charge density

Charge of fermion

$$Q = qV_{fermi}$$

For free fermions (electrons), this is trivial

Luttinger theorem

- Normal materials (Landau Fermi liquids)
  （Luttinger, Ward ’60）

$$Q = qV_{fermi}$$

- This relation holds beyond the perturbation!!
- Luttinger’s theorem: V_{fermi} is not renormalized by the interaction (Oshikawa ’00)

Luttinger theorem

- Normal materials (Landau Fermi liquids)

（Luttinger '60, Ward '60, Oshikawa '00, …）

$$Q = qV_{fermi}$$

- Exotic materials（fractionalized Fermi liquids）

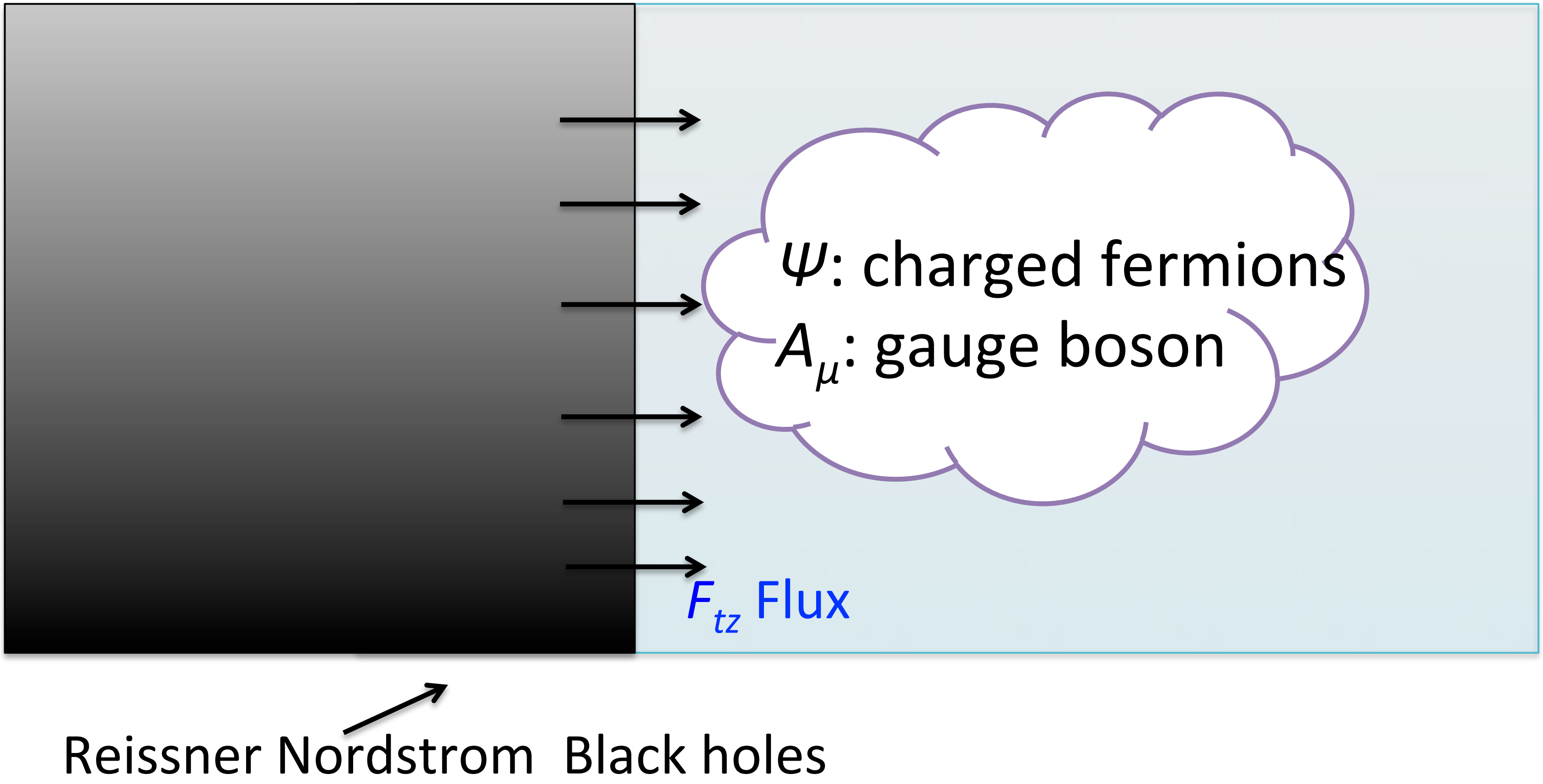
（Senthil, Sachdev, Vojta '03）

$$Q \neq qV_{fermi}$$

Holographic Luttinger theorem

(Sachdev '11, Hartnoll '11, Iqbal-Liu ',11 Hashimoto, N.I.'12)

$$Q \neq qV_{fermi}$$



Conclusion

Black hole is responsible for the ``exotic phase” of the condensed matter physics

“Holographic” construction is one peculiar way of strongly coupled system