Instanton Operators in 5D Gauge Theories

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Motivation

Dramatic result coming out of string/M-theory:

$\exists$ interacting SCFTs in 5D and 6D

$\Rightarrow$ Provide UV completions for a variety of 5D gauge theories [Seiberg]

UV theories enjoy global or Lorentz symmetry enhancement:

- $\diamond$ $\mathcal{N}=1$ SYM theories with $N_f \leq 7$ and $\mathrm{SO}(2N_f)$ symmetry
  $\Rightarrow$ $\mathcal{N}=1$ 5D SCFT with E_{N_f+1} symmetry
- $\diamond$ $\mathcal{N}=2$ SYM theory $\Rightarrow$ $(2,0)$ 6D SCFT

Indirect ways of seeing this enhancement, mainly using index calculations [Kim^2-Lee, Bashkirov, Hwang-Kim^2-Park,...]

Can we find a simpler way? $\Rightarrow$ Draw upon our knowledge of 3D theories where local monopole operators play important role

$\Rightarrow$ Global symmetry and susy enhancement in the IR [Borokhov-Kapustin-Wu, Gaiotto-Witten, ABJM, ...]

Q: Is there an analogue in 5D?

A: We can construct local instanton operators

Outline

- Definition of instanton operators
- Supersymmetry
- Chern-Simons terms
- Applications
- Summary

Definition

Well-known that 5D SYM has conserved current

$$J = \frac{1}{8\pi^2} \mathrm{Tr} \star (F \wedge F)$$

Charged BPS-particle solutions: instanton solitons

Both global and Lorentz symmetry enhancement associated with instanton charge.

Preliminary: An instanton operator is a local operator which creates instanton solitons out of vacuum

The OPE of this current with $\mathcal{I}_n(0)$ is given by

$$J^\mu(x)\mathcal{I}_n(0) \sim \frac{3n}{8\pi^2}\frac{x^\mu}{|x|^5}\mathcal{I}_n(0) + \cdots$$

More formally: Instanton operators, $\mathcal{I}_n(x)$, modify boundary conditions for gauge field at infinity in Eucliden path integral:

$$\langle \mathcal{I}_n(x)\mathcal{O}_{01}(x_1)\ldots\mathcal{O}_{0k}(x_k)\rangle =$$
$$= \int_{\frac{1}{8\pi^2}\mathrm{Tr}\oint_{S^4_x} F\wedge F=n} [DXDAD\psi]\,\mathcal{O}_{01}(x_1)\ldots\mathcal{O}_{0k}(x_k)e^{-S_E}$$

Fields need to satisfy classical eom near insertion point in $\mathbb{R}^5$

$$D^\mu F_{\mu\nu} = 0\,, \qquad D_{[\mu}F_{\nu\lambda]} = 0$$

but with non-vanishing

$$I = \frac{1}{8\pi^2}\mathrm{Tr}\oint_{S^4} F\wedge F$$

In spherical coordinates a simple solution is given by taking $A_r = F_{ri} = 0$ and the angular components satisfying

$$F = \pm \star_{S^4} F$$

This solution for $\mathrm{SU}(2)$ theory was found long ago by Yang, as static $\mathrm{SO}(5)$-symmetric particle in 6D $\Rightarrow$ Yang monopole

A DBI generalisation for $\mathrm{SU}(N)$ later given by Constable-Myers-Tafjord in context of D1$\perp$D5 intersections

Alternatively: Instanton operators defined by the condition that the gauge field has a Yang monopole singularity at the insertion point

Instantons on S^4 can be straightforwardly constructed by stereographic projection from $\mathbb{R}^4$

The solutions exhibit some amusing properties:

$$F \wedge F = \frac{8\rho^4 \ \sum_{i=1}^{3} T_i^2}{\Big(1 + \rho^2 + (1-\rho^2)\cos\theta^1\Big)^4} \sqrt{\gamma}\; d^4\theta$$

with $[T_i, T_j] = 2i\epsilon_{ijk}T_k$ an $N \times N$ representation of $\mathfrak{su}(2)$

When $\rho = 1$ this reduces to the $\mathrm{SO}(5)$-symmetric

$$F \wedge F = \frac{1}{2} \sum_{i=1}^{3} T_i^2 \sqrt{\gamma}\; d^4\theta$$

When the T_i are irreps of $\mathfrak{su}(2)$ then

$$\sum_{i=1}^{3} T_i^2 = (N^2 - 1)\ \mathbb{1}_{N\times N}$$

and $F \wedge F$ is gauge invariant without the trace

By further considering (for generic ρ)

$$I = \frac{1}{8\pi^2}\mathrm{Tr}\int F \wedge F = \frac{N(N^2-1)}{6}$$

Supersymmetry

Are these solutions supersymmetric?

The supervariation of a fermion in the background of the Yang monopole is

$$\delta\psi = \frac{1}{2}\Gamma^{\mu\nu}F_{\mu\nu}\Gamma_5\varepsilon$$

The ε are 32-component spinors which also satisfy

$$\Gamma_{012345}\varepsilon = \varepsilon$$

Using that $F = \pm \star_{S^4} F$ one can satisfy $\delta\psi = 0$ if

$$\left(\frac{x^\mu}{|x|^2}\Gamma_\mu\Gamma_5 \pm i\right)\varepsilon = 0$$

This cannot hold for all x^μ and supersymmetry is broken.

Q: How can a non-susy instanton operator create a $\frac{1}{2}$-BPS instanton soliton state?

A: The operator creates a tower of states; the BPS instanton soliton can be found at the bottom of that tower

The 5D susy algebra contains a central charge

$$Z_5 = -\frac{1}{2g^2}\mathrm{Tr}\int F \wedge F$$

The instanton-solitons with charge n can be found after projecting out all other states

$$|n\rangle = \lim_{\tau\to\infty} e^{-(H-Z_5)\tau}\mathcal{I}_n(0)|0\rangle$$

Analogous to 3D where monopole operators and BPS vortices annihilated by different combinations of supercharges [Intriligator-Seiberg]

CS terms

In $\mathcal{N} = 1$ theories we can also add Chern-Simons terms

$$S_{CS} = \frac{k}{24\pi^2}\mathrm{Tr}\int (F \wedge F \wedge A + \frac{i}{2} F \wedge A \wedge A \wedge A - \frac{1}{10} A \wedge A \wedge A \wedge A \wedge A)$$

In the presence of such a term the instanton operators are not always gauge invariant:

$$\delta S_{CS} = \frac{k}{8\pi^2}\mathrm{Tr}\int F \wedge F \wedge \delta A$$

and by considering $\delta A = D\omega$ with $\omega = 0$ at ∞ one finds

$$\delta S_{CS} = -\frac{k}{8\pi^2}\mathrm{Tr}\left[\omega(x)\oint_{S^4_x} F \wedge F\right]$$

Inserting this into a correlator

$$\delta\langle \mathcal{I}_n(x)\mathcal{O}_{01}(x_1)\dots\mathcal{O}_{0k}(x_k)\rangle =$$

$$= -\int_{\frac{1}{8\pi^2}\mathrm{Tr}\oint_{S_x^4} F\wedge F=n} [DXDAD\psi]\,\mathcal{O}_{01}(x_1)\dots\mathcal{O}_{0k}(x_k)\delta S_{CS}e^{-S_E}$$

$$= \frac{k}{8\pi^2}\mathrm{Tr}\left[\omega(x)\oint_{S_x^4} F\wedge F\right]\langle \mathcal{I}_n(x)\mathcal{O}_{01}(x_1)\dots\mathcal{O}_{0k}(x_k)\rangle$$

and one needs to understand the properties of

$$Q_I = \frac{1}{8\pi^2}\oint_{S_x^4} F\wedge F$$

Could this play a similar role to $Q_M = \frac{1}{2\pi}\oint_{S^2} F$ in GNO?

For the single instanton irreducible case

$$Q_I = \frac{1}{6}\sum_{i=1}^{3} T_i^2 = \frac{1}{6}(N^2-1)\ \mathbb{1}_{N\times N}$$

and Q_I is gauge invariant.

Introducing a basis t_a of full gauge group Lie algebra with metric $\kappa_{ab} = \mathrm{Tr}(t_a t_b)$ and symmetric tensor

$$d_{abc} = \mathrm{Tr}(t_{(a}t_{b)}t_c)$$

so more generally we have

$$\delta\mathcal{I}_n = k d_{abc} Q_I^{ab}\omega^c\mathcal{I}_n$$

Application: Lorentz symmetry enhancement

The $(2,0)$ SCFT is completely described in terms of operator spectrum and OPE coefficients.

Consider

$$\langle \hat{\mathcal{O}}_1(\hat{x}_1)\hat{\mathcal{O}}_2(\hat{x}_2)\rangle = \frac{c_{12}}{|\hat{x}_1 - \hat{x}_2|^{\Delta_1+\Delta_2}}$$

$\Rightarrow$ Use instanton operators to relate 6D to 5D $\mathcal{N}=2$ correlators by compactifying on S^1

Implement this by viewing S^1 as orbifold $\mathbb{R}/\Gamma$ and

$$\Gamma : (x, y) \mapsto (x, y + 2\pi R n) \text{ with } n \in \mathbb{Z}$$

For an operator on $\mathbb{R}^5 \times S^1$ write

$$\mathcal{O}(x, y) := \sum_{n \in \mathbb{Z}} \hat{\mathcal{O}}(x, y + 2\pi R n) = \sum_{m \in \mathbb{Z}} e^{imy/R} \mathcal{O}_m(x)$$

where $\mathcal{O}_m$ are Fourier modes

$\Rightarrow$ Not all operators can be related in this way as they have to satisfy linear equations

Starting from the two-point function $\langle \mathcal{O}_1(x_1, y_1)\mathcal{O}_2(x_2, y_2)\rangle$, using these facts and the identification $R = g^2/4\pi^2$ one arrives at the following:

- For the zero modes (pertubative)

$$\langle \mathcal{O}_0(x_1)\mathcal{O}_0(x_2)\rangle = -\frac{c_{12}\pi^{\frac{3}{2}}}{g^2}\frac{\Gamma(\frac{\Delta_1+\Delta_2-1}{2})}{\Gamma(\frac{\Delta_1+\Delta_2}{2})}\frac{1}{|x_{12}|^{\Delta_1+\Delta_2-1}}$$

- For the non-zero modes (non-pertubative)

$$\langle \mathcal{O}_n(x_1)\mathcal{O}_{-n}(x_2)\rangle$$
$$= -\frac{c_{12}\pi^{\frac{\Delta_1+\Delta_2}{2}}}{2|n|\Gamma(\frac{\Delta_1+\Delta_2}{2})}\left(\frac{2\pi|n|}{g^2|x_{12}|}\right)^{\frac{\Delta_1+\Delta_2}{2}} e^{-\frac{4\pi^2}{g^2}|n||x_{12}|}\left(1+\mathcal{O}\Big(\frac{g^2}{|n||x_{12}|}\Big)\right)$$

We propose that the non-zero modes are related to instanton operators

$$\mathcal{O}_n(x) := \mathcal{I}_n(x)\mathcal{O}_0(x)$$

Compatible with:

- Momentum conservation on S^1
- Lack of susy
- The characteristic e^{-S_n} dependence of the non-zero mode correlators with

$$S_n = \frac{4\pi^2}{g^2}|n||x_1 - x_2|$$

Momentum Conservation

We have that

$$S_I = \frac{4\pi^2|n|}{g^2}R$$

where $R \to \infty$. Therefore the action blows up unless $n = 0$.

This implies that for correlators

$$\langle \mathcal{O}_{n_1 1}(x_1)\mathcal{O}_{n_2 2}(x_2)...\mathcal{O}_{n_k k}(x_2)\rangle = 0$$

unless

$$\sum_{i=1}^{k} n_i = 0$$

$\Rightarrow$ Momentum conservation on S^1

Supersymmetry

Consider a BPS operator in 6D satisfying $[Q, \hat{\mathcal{O}}] = 0$.

Introduce a superspace with

$$Q = \frac{\partial}{\partial\theta} + i\bar{\theta}\Gamma^m\partial_m$$

Then using

$$\mathcal{O}_m(x) = \frac{1}{2\pi R}\int_0^{2\pi R} dy\; e^{-imy/R}\sum_{n\in\mathbb{Z}}\hat{\mathcal{O}}(x, y + 2\pi Rn)$$

we find that

$$[Q, \mathcal{O}_n] = \frac{n}{2\pi R^2}\bar{\theta}\Gamma^y\mathcal{O}_n \neq 0$$

Non-perturbative dependence

Finally, consider a correlator involving insertions of two instanton operators at x_1 and x_2 with charges n and $-n$.

By dimensional analysis the minimum action is:

$$S_{min} = \frac{1}{4g^2}\int d^5x F_{\mu\nu}F^{\mu\nu} = \frac{K}{g^2}|x_1 - x_2|$$

The coefficient K can be pinned down by taking the second point to ∞ and comparing with $S_I = \frac{4\pi^2|n|}{g^2}R$ as $R \to \infty$

We get: $S_{min} = \frac{4\pi^2}{g^2}|n||x_1 - x_2| = S_n$ as required

Summary

- Introduced instanton operators in 5D gauge theories
- Looked at some properties
- These are non-BPS in $\mathbb{R}^5$
- Application: Relating correlators of $(2,0)$ theory on $\mathbb{R}^5 \times S^1$ and $\mathcal{N} = 2$ SYM in 5D

Q: Can the instanton operators be made supersymmetric?

$\Rightarrow$ Yes, by employing an R-symmetry twist [Rodríguez-Gómez, Schmude]

Q: Application: Flavour symmetry enhancement in UV of $\mathcal{N} = 1$ 5D SYM?

$\Rightarrow$ The fermionic zero-modes of the instanton operators would seem to account for this [Tachikawa]