Neutrino masses from SUSY breaking in radiative seesaw models

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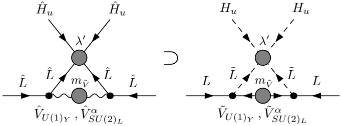
Introduction: m_ν from ~~SUSY~~ $\quad \langle \hat{X} \rangle \sim \theta^2 F_X$

- Hard-~~SUSY~~ terms and only MSSM visible fields: [Frere *et al* '99]
  - $\tilde{L}$-number breaking provided by hard-~~SUSY~~ terms that respect R-parity

$$\lambda' \sim \left(\frac{m_{\rm soft}}{M_X}\right)^{1\ \text{or}\ 2} \lambda : \quad (\tilde{L}H_u)^2 \subset \frac{1}{M_X^2}\int d^2\theta \hat{X}(\hat{L}\hat{H}_u)^2 \ \text{ or } \ \tilde{L}H_d^\dagger \tilde{L}H \subset \frac{1}{M_X^2}\int d^4\theta \hat{X}^\dagger \hat{X}\hat{L}\hat{H}_d^\dagger \hat{L}\hat{H}$$

  - Turned into L-number breaking via gaugino fermion-number violation



$$\Rightarrow m_\nu \sim \frac{\alpha}{4\pi}\frac{v^2}{M_X}$$

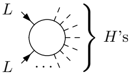
  - But, if ~~SUSY~~ sector breaks L-number, why wouldn't generate $\frac{1}{M_X}\hat{L}\hat{L}\hat{H}_u\hat{H}_u \subset \mathcal{W}_{\rm eff}$?
- Suppose ~~SUSY~~ sector carries visible sector symmetries (e.g. forbid H_uH_d so that $\mu \sim m_{\rm soft}$). May forbid RH neutrino masses/couplings to visible sector: [Arkani-Hamed *et al* '00]
  - Majorana:
    $\frac{1}{M_X}\int d^4\theta \hat{X}^\dagger \hat{N}\hat{N} \supset m_{\rm soft}NN$ and $\frac{1}{M_X}\int d^2\theta \hat{X}\hat{L}\hat{N}\hat{H}_u \supset \sqrt{\frac{m_{\rm soft}}{M_X}}LNH_u \Rightarrow m_\nu \sim \frac{v^2}{M_X}$
  - Dirac: $\frac{1}{M_X^2}\int d^4\theta \hat{X}^\dagger \hat{L}\hat{N}\hat{H}_u \supset \frac{m_{\rm soft}}{M_X}LNH_u \Rightarrow m_\nu \sim \frac{v\, m_{\rm soft}}{M_X}$
- New visible sector symmetry may forbid unsuppressed terms & allow suppressed ones:
  - E.g. $U(1)'$ gauge symmetry may forbid LH_uN and $m_{\rm soft}\tilde{L}H_u\tilde{N}$ [Demir *et al* '07]
  - and allow $\frac{m_{\rm soft}}{M_X}LH_d^\dagger N \subset \frac{1}{M_X^2}\int d^4\theta \hat{X}^\dagger \hat{L}\hat{H}_d^\dagger \hat{N}$ and $\frac{m_{\rm soft}^2}{M_X}\tilde{L}H_d^\dagger \tilde{N} \subset \frac{1}{M_X^3}\int d^4\theta \hat{X}^\dagger \hat{X}\hat{L}\hat{H}_d^\dagger \hat{N}$
  - Tree- and loop-level $LH_d^\dagger N \Rightarrow m_\nu \sim \frac{v\, m_{\rm soft}}{M_X}$

Radiative m_ν in SUSY

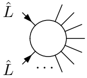
Radiative m_ν $\xrightarrow[\text{(non-renormalisation theorem)}]{\text{SUSY}}$ m_ν from ~~SUSY~~

- Dimensionless suppression
- Maybe Dark Matter

- Dimensional suppression $\sim \left(\frac{m_{\text{soft}}}{M}\right)^n$
- ~~SUSY~~ sector needs not carry visible symmetries



$$\text{OP}_\nu \subset \int d^4\theta\, \widehat{\text{OP}}_\nu$$

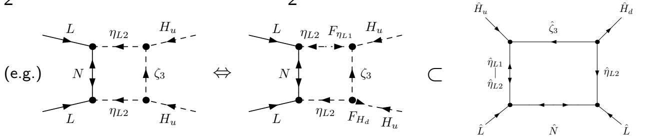


- *L*-number breaking can be independent of ~~SUSY~~ and still $m_\nu \propto$ ~~SUSY~~

- Not entirely obvious in component field calculations
- Examples in the literature:
  - Genuine radiative seesaw

[Ma *et al* '14]

$$\frac{M_N}{2}\hat{N}\hat{N} + \mu \hat{H}_u \hat{H}_d + \mu_{L2}\hat{\eta}_{L1}\hat{\eta}_{L2} + \frac{\mu_{s3}}{2}\hat{\zeta}_3\hat{\zeta}_3 + f_9 \hat{H}_d \hat{\eta}_{L2}\hat{\zeta}_3 + f_{10}\hat{H}_u\hat{\eta}_{L1}\hat{\zeta}_3 + f_{16}\hat{L}\hat{N}\hat{\eta}_{L2} \subset \mathcal{W}$$



  - *R*-parity violation

$$\frac{1}{M^2}LLH_uF^\dagger_{H_d} \quad \subset \quad \frac{1}{M^2}\int d^4\theta\, \hat{L}\hat{L}\hat{H}_u\hat{H}^\dagger_d$$

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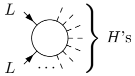
Radiative m_ν —SUSY (non-renormalisation theorem)→ **m_ν from $\cancel{\text{SUSY}}$**

- Dimensionless suppression
- Maybe Dark Matter



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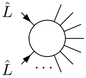

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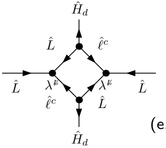
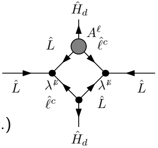


(e.g.)



$$\Rightarrow \left.\begin{array}{l} \hat{L}\hat{L}\hat{H}_d^\dagger\hat{H}_d^\dagger \\ \hat{X}^\dagger\hat{L}\hat{L}\hat{H}_d^\dagger\hat{H}_d^\dagger \end{array}\right\} \subset \mathcal{K}_{\text{eff}} \to m^\nu \sim \frac{A^{\ell,d} - \mu\, c_\xi\, t_\beta}{16\pi^2}\left(\frac{m^{\ell,d}}{m^{\tilde{\ell},\tilde{d}}}\right)^2$$

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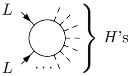

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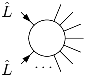
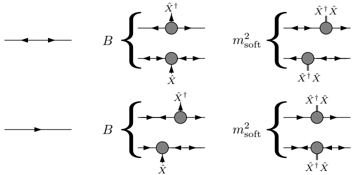


- L-number breaking can be independent of $\cancel{\text{SUSY}}$ and still $m_\nu \propto \cancel{\text{SUSY}}$

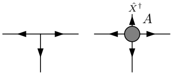
- Can classify the $\cancel{\text{SUSY}}$ contributions as:
  - Related to EWSB

$$\langle F^\dagger \rangle = \sum_H \mu_H \langle H \rangle + \sum_H \lambda_H \langle HH' \rangle \neq 0 \text{ or } \langle D \rangle = g \sum_H \langle H^\dagger \otimes_H H \rangle \neq 0$$

  - Otherwise: insertions of $\cancel{\text{SUSY}}$ spurions into internal lines or vertices



(e.g. soft-$\cancel{\text{SUSY}}$)



Radiative seesaws in SUSY

- Assume
  - L-number broken by 2 units at SUSY level with a radiative seesaw at scale M
  - Low energy Higgs sector of the MSSM

$$\langle F^{\dagger}_{H_{u,d}} \rangle = \mu \langle H_{d,u} \rangle \, ,$$

$$\langle D_{U(1)_Y} \rangle = \frac{g'}{2} \left(|\langle H_u \rangle|^2 - |\langle H_d \rangle|^2 \right) \, , \quad \langle D^3_{SU(2)_L} \rangle = \frac{g}{2} \left(-|\langle H_u \rangle|^2 + |\langle H_d \rangle|^2 \right)$$

- Expectations

$$\frac{m_\nu}{v^2} \propto \frac{\mu}{M^2} \oplus \frac{g^2 v^2}{M^3} \oplus \frac{m_{\text{soft}}}{M^2}$$

- If $\frac{m_{\text{soft}}}{M}$ is small, supergraphs very convenient calculation tool
  - Automatic component field cancellations
  - Non-renormalisation theorem becomes manifest
  - Simpler Lorentz structure
  - Fewer diagrams

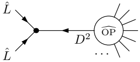
Radiative seesaws in SUSY: Understanding ~~SUSY~~ contributions

- In general $\mathrm{OP}_\nu = LL \otimes \text{Higgses} \subset \int d^4\theta\, \widehat{\mathrm{OP}}_\nu$:

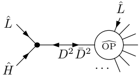
$$\widehat{\mathrm{OP}}_\nu \in \left[\hat{A}\, D^2(\hat{L}\hat{L}\hat{H}^k) \text{ or } \hat{B}^\dagger\, \hat{L}\hat{L}\right] \otimes \left\{\hat{H}, \hat{H}^\dagger, D^2 Z, \bar{D}^2\hat{Z}^\dagger, D\bar{D}^2 D\hat{V}\right\}^n$$

$$D\bar{D}^2 D\hat{V}| = D \supset g\hat{H}^\dagger \otimes H\,, \quad \bar{D}^2\hat{Z}^\dagger| = F_Z^\dagger \supset \mu H \text{ or } \lambda H \otimes H'$$

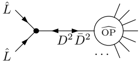
$$\int d^4\theta \hat{A} \;\text{ and }\; \int d^2\bar{\theta}\hat{B}^\dagger \sim \text{Constants and/or Higgses}$$



$\int d^4\theta D^2(\hat L\hat L)\widehat{\mathrm{OP}}$



$\int d^4\theta\hat L\hat H\,\widehat{\mathrm{OP}}[\hat L]$



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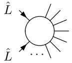
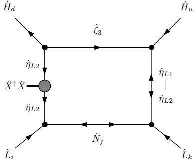


- Pure-~~SUSY~~$_{\text{EWSB}}$ contributions $\hat{A} \sim \hat{V}$ or $\hat{B}^\dagger\hat{B}$ and $\hat{B}^\dagger \sim \hat{Z}^\dagger$ or $D^2\hat{V}$
- ~~SUSY~~$_{\text{EWS}}$ contributions
  - Insertions of spurionic constants $D^2\hat{X}$ and $\bar{D}^2\hat{X}^\dagger$

(e.g.)



$$\frac{1}{M^4}\int d^4\theta\, \bar{D}^2 D^2 \left(\frac{\hat{X}^\dagger\hat{X}}{M_X^2}\right)_{m^2_{\text{soft}}} \hat{L}\hat{L}\hat{H}_u\hat{H}_d^\dagger \supset \frac{\mu m^2_{\text{soft}}}{M^4} LLH_uH_u$$

  - Insertions of ~~SUSY~~ spurions $\hat{X}$ and $\hat{X}^\dagger$

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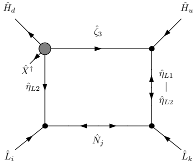
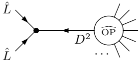


$$\frac{1}{M^2}\int d^4\theta \left(\frac{\hat{X}^\dagger}{M_X}\right)_A \hat{L}\hat{L}\hat{H}_u\hat{H}_d^\dagger \supset \frac{A^*}{M^2} LLH_uH_d^\dagger$$



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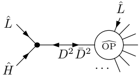
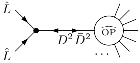


$\int d^4\theta \hat{L}\hat{H}\, \widehat{\mathrm{OP}}[\hat{L}]$



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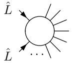
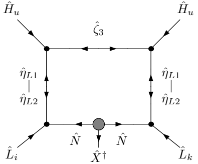


Radiative seesaws in SUSY: $\not{\text{SUSY}}_{\text{EWS}}$ superoperators

- Identically zero topologies in SUSY limit can contribute through internal $\not{\text{SUSY}}$ effects

(e.g.)



$$\frac{1}{M^3}\int d^4\theta\,(\hat{X}^\dagger)_B \hat{L}\hat{L}\hat{H}_u\hat{H}_u \supset \frac{B^*}{M^3} L L H_u H_u$$

- Ask in general: which superoperators only contribute to OP_ν by insertions of $\not{\text{SUSY}}$ spurions?

1. $\widehat{\text{OP}} = D^2(\hat{L}\hat{L}\hat{H}^n) \otimes \Big($a superoperator whose D-term is zero at $p_{\text{ext}} = 0\Big)$
2. $\widehat{\text{OP}} = \hat{L}\hat{L} \otimes \Big($a superoperator whose $F^\dagger$-term is zero at $p_{\text{ext}} = 0\Big)$

- If in a radiative seesaw model all leading superoperators are of this type:

$$m_\nu^{\text{leading}} \propto \text{soft-}\not{\text{SUSY}} \text{ effects in seesaw mediators}$$

- $m_\nu^{\text{leading}} \propto \frac{m_{\text{soft}}}{M}$ without resorting to non-standard $\not{\text{SUSY}}$ terms

Radiative seesaws in SUSY: ~~SUSY~~$_{\mathrm{EWS}}$ one-loop topologies for $LLHH$

- 1 1PI + 4 based on radiative vertices:
  - $D^2(\hat{L}\hat{L})\hat{H}_u\hat{H}_u$, $\hat{L}\hat{L}D^2(\hat{H}_u\hat{H}_u)$, $D^2(\hat{L}\hat{L})\hat{H}_d^\dagger\hat{H}_d^\dagger$ and $\hat{L}\hat{L}\bar{D}^2(\hat{H}_d^\dagger\hat{H}_d^\dagger)$
    – type-II without a chirality flip
  - $\hat{L}\hat{L}\hat{H}_u\hat{H}_u$ (1PR)
    – type-II with a chirality flip, type-I and -III
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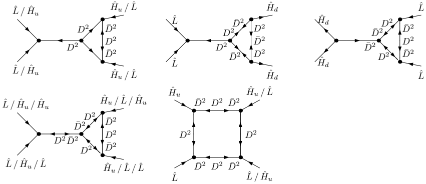
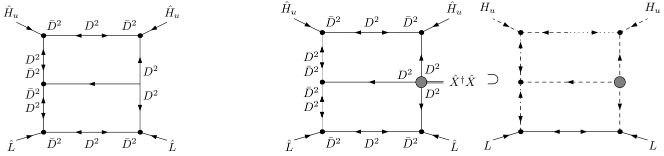


- 14 based on self-energies
- Dimensional suppressions:
  - $\mu m_{\mathrm{soft}}/M^3$ or m_{soft}^2/M^3 to $\mu^2 m_{\mathrm{soft}}/M^4$, $\mu m_{\mathrm{soft}}^2/M^4$ or m_{soft}^3/M^4
  - Non-minimal least suppressions consequence of holomorphy: higher loops contain m_{soft}/M^2 contributions:

(e.g.)



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  - $\hat{L}\hat{L}\hat{H}_u\hat{H}_u$ (1PR) – type-II with a chirality flip, type-I and -III
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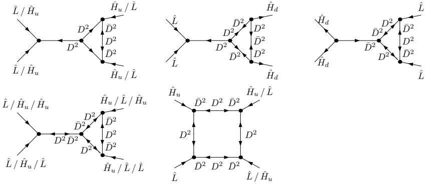

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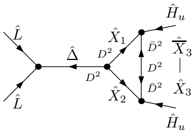
Construction of models

- Pick a symmetry that forbids undesired leading topologies and allow desired
- General criterion: radiative seesaw either couples to $\hat{H}_u$ or to $\hat{H}_d^\dagger$, but not both
- However, this structure is not respected by
  - Higher-loop contributions to operators of leading dimension
  - Operators of higher dimension

A Model Example

- Base topology



$$\hat{\rho}^\dagger \hat{\Delta}^\dagger \hat{H}_u \hat{H}_u \to \langle \rho^\dagger \rangle \hat{\Delta}^\dagger \hat{H}_u \hat{H}_u + \hat{\rho}^\dagger \hat{\Delta}^\dagger \hat{H}_u \hat{H}_u$$

- Charges

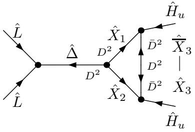
	$SU(2)_L \otimes U(1)_Y$	$U(1)_X$	$U(1)_L$
$\hat{\Delta}$	$(\mathbf{3}, 1)$	0	-2
$\hat{\rho}$	$(\mathbf{1}, 0)$	0	2
$\hat{X}_1$	$(\mathbf{2}, -1/2)$	1	1
$\hat{X}_2$	$(\mathbf{2}, -1/2)$	-1	1
$\hat{X}_3$	$(\mathbf{1}, 0)$	1	-1
$\hat{\overline{X}}_3$	$(\mathbf{1}, 0)$	-1	-1

- Superpotential

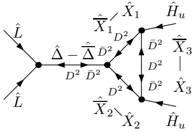
$$\begin{aligned} \mathcal{W} &= \mathcal{W}_{\text{MSSM}} + M_\Delta \hat{\Delta} \hat{\overline{\Delta}} + \sum_{i=1}^{2} M_{X_i} \hat{X}_i \hat{\overline{X}}_i + \lambda \hat{\rho} \hat{X}_3 \hat{\overline{X}}_3 \\ &\quad + \hat{H}_u \left(\lambda_1 \hat{X}_1 \hat{\overline{X}}_3 + \lambda_2 \hat{X}_2 \hat{X}_3 \right) + \hat{\Delta} \left(\lambda_L \hat{L} \hat{L} + \lambda_X \hat{X}_1 \hat{X}_2 \right) + \bar{\lambda}_X \hat{\overline{\Delta}} \hat{\overline{X}}_1 \hat{\overline{X}}_2 \end{aligned}$$

A Model Example

- L-number recovered if any in $\{\lambda_1, \lambda_2, \lambda_L\}$, or both λ_X and any in $\{\bar{\lambda}_X, M_\Delta, M_{X_1}, M_{X_2}\}$ goes to zero
  - Contribution to neutrino masses proportional to either $\lambda_1\lambda_2\boldsymbol{\lambda_L}\lambda_X^*$ or $\lambda_1\lambda_2\boldsymbol{\lambda_L}\bar{\lambda}_X M_\Delta M_{X_1} M_{X_2}$
  - Leading perturbation theory contributions:



$\frac{1}{M^2} D^2(\hat{L}\hat{L})\hat{H}_u\hat{H}_u$



$\frac{1}{M}\hat{L}\hat{L}\hat{H}_u\hat{H}_u$

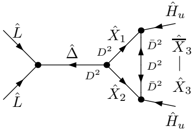
- Leading $LLHH$ in the absence of $\not{\text{SUSY}}$ spurions:

$$\int d^4\theta\, D^2(\hat{L}\hat{L})\hat{H}_u\hat{H}_u \;=\; -\Box(\tilde{L}\tilde{L})\left[\tilde{H}_u\tilde{H}_u + 2F_{H_u}H_u\right] - \Box(H_uH_u)\left[LL + 2F_L\tilde{L}\right]$$
$$+\,4\,(p_L + p_{\tilde{L}})^2\, L\tilde{H}_u\tilde{L}H_u$$
$$\int d^4\theta\,\hat{L}\hat{L}\hat{H}_u\hat{H}_u \;=\; 0 \quad \left(\text{or} \quad \int d^4\theta\,\hat{\bar{\Delta}}\hat{H}_u\hat{H}_u = 0\right)$$

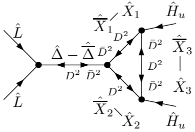
$m_\nu^{\text{leading}} = 0$ in the absence of $\not{\text{SUSY}}$ spurions

A Model Example

- L-number recovered if any in $\{\lambda_1, \lambda_2, \lambda_L\}$, or both λ_X and any in $\{\bar{\lambda}_X, M_\Delta, M_{X_1}, M_{X_2}\}$ goes to zero
  - Contribution to neutrino masses proportional to either $\lambda_1\lambda_2\boldsymbol{\lambda_L}\lambda_X^*$ or $\lambda_1\lambda_2\boldsymbol{\lambda_L}\bar{\lambda}_X M_\Delta M_{X_1} M_{X_2}$
  - Leading perturbation theory contributions:

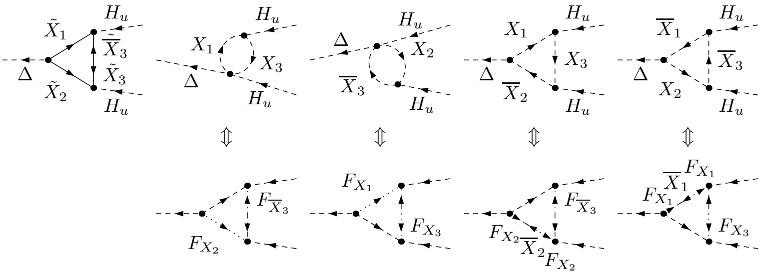


$\frac{1}{M^2} D^2(\hat{L}\hat{L})\hat{H}_u\hat{H}_u$



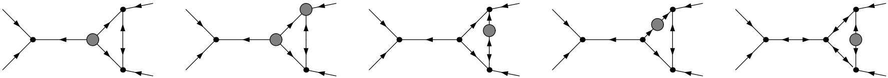
$\frac{1}{M}\hat{L}\hat{L}\hat{H}_u\hat{H}_u$

- Leading $LLHH$ in the absence of ~~SUSY~~ spurions:



A Model Example

- Leading soft-~~SUSY~~ effects

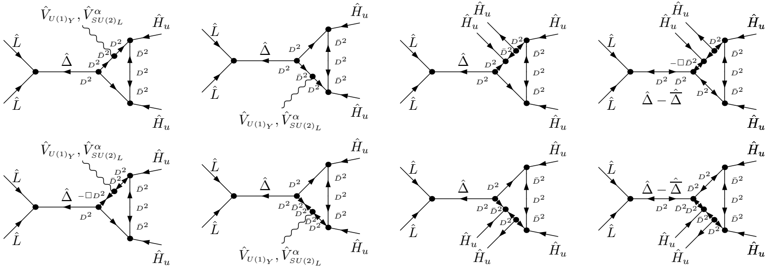


$$\frac{\mu^* A^*}{M^3} LLH_u H_d^\dagger \oplus \frac{A^* A \oplus m_{\rm soft}^2 \oplus B^*}{M^3} LLH_u H_u \subset \mathcal{L}_{\rm eff}$$

- Or up to order 3

$$\frac{1}{64\pi^2 M_\Delta^2}\left(\mathbf{a}\left[\frac{2m_{\rm soft}^2}{M_X} + \frac{2A}{M_X}\left(A_X^* - \frac{B_\Delta}{M_\Delta}\right) - \frac{A_X^* B_X}{M_X^2}\right] + \mathbf{b} M_\Delta \frac{B_X}{M_X^2}\right) LLH_u H_u$$
$$-\frac{\mathbf{a}}{32\pi^2 M_\Delta^2}\left(\frac{\mu^*}{M_X}\right)\left[A_X^*\left(1 - \frac{m_{\rm soft}^2}{M_X^2} - \frac{(m_{\rm soft}^2)_\Delta}{M_\Delta^2}\right) - \frac{B_\Delta}{M_\Delta}\right] LLH_u H_d^\dagger$$
$$-\frac{\mathbf{a}}{192\pi^2 M_\Delta^2}\left(\frac{\mu^*}{M_X}\right)^2 \frac{A_X^* B_X}{M_X^2} LLH_d^\dagger H_d^\dagger$$

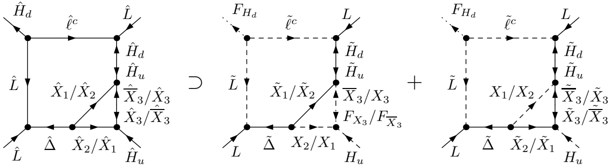
A Model Example



$$\frac{1}{M^3} D^2(\hat{L}\hat{L})\hat{H}_u\hat{H}_u\hat{V}_{U(1)_Y,SU(2)_L}\ , \quad \frac{1}{M^5} D^2(\hat{L}\hat{L})\hat{H}_u\hat{H}_u\hat{H}_u^\dagger\hat{H}_u\ , \quad \frac{1}{M^4}\hat{L}\hat{L}\hat{H}_u\hat{H}_u\hat{H}_u^\dagger\hat{H}_u$$

- At one-loop order $\not{SUSY}_{EWSB}$ contributions have dimension-7: $\frac{g'^2 \oplus g^2}{M^3}$, $\frac{|\mu|^2}{M^5}$, $\frac{\mu}{M^4}$
- At higher-loops there are contributions to dimension-5 operators that are not proportional to $\not{SUSY}$ effects involving the seesaw mediators

(e.g.)



Conclusions

- L-number breaking can take place at SUSY level and still $m_\nu \propto \not{\text{SUSY}}$
- There are models in which m_ν^{leading} is proportional to soft-$\not{\text{SUSY}}$ involving the seesaw mediators, say m. Schematically

$$\frac{m_\nu^{\text{leading}}}{v^2} \propto \frac{\mu\, m \oplus m^2}{M^3}$$

- In these models the smallness m can be partially responsible for the smallness of m_ν
- Ballpark figure

$$M \sim 10 \text{ TeV}\,,\ \lambda \sim 0.1\,,\ \mu \sim 2 \text{ TeV}\ : \quad m \lesssim 100 \text{ GeV}\,,\ m_\nu \lesssim 1 \text{ eV}$$

- Explicative power of small m is limited by the size of next-to-leading order contributions that are independent of m
- Caveat: how small can m be does depend on the amount of hierarchy in soft-$\not{\text{SUSY}}$ terms, i.e. on how small is $m/m_{\text{SM-sparticles}}$. Complete understanding would require an actual model of $\not{\text{SUSY}}$
- Phenomenological virtues:
  - Small m_ν generated by new physics near the TeV scale
  - No need for small superpotential couplings
  - No need for large hierarchies in superpotential mass scales

Looking for low scale SUSY seesaws signals in cLFV

- In addition to LFV mediated by radiative seesaw mediators, there will in general be the traditional contributions from slepton-EWino/Higgisno loops

[Borzumati and Masiero '86]

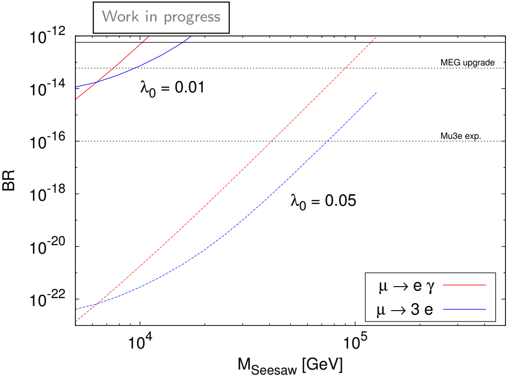
$$\beta^{(1)}_{m_L^2} \supset 6\left(m^2_{\text{soft}}\right)_\Delta \lambda_L^*\lambda_L + 6A_L^*A_L + 3m_L^2\lambda_L^*\lambda_L + 3\lambda_L^*\lambda_L m_L^2 + 6\lambda_L^* m_L^{2*}\lambda_L$$

- Sleptonic LFV ratios:

[Kim *et al* '98] [Hisano and Tobe '01]

$$\frac{\text{BR}(\ell\to 3\ell')}{\text{BR}(\ell\to\ell'\gamma)} \simeq \frac{\alpha}{3\pi}\left(\log\frac{m_\ell^2}{m_{\ell'}^2} - \frac{11}{4}\right), \qquad \frac{\text{CR}(\mu - e, \text{Ti})}{\text{BR}(\mu\to e\gamma)} \approx 5\times 10^{-3}$$

Deviations would suggest the existence of other low-lying states with lepton-flavour



$m^2_{\tilde\ell} \approx 6$ TeV, $m_{\widetilde{\text{EW}}} \approx 1.5$ TeV

At ~~M_{SUSY}~~ $= 10^{10}$ GeV:

$\lambda_1 = \lambda_2 = \lambda_X = \bar\lambda_X := \lambda_0$

$M_\Delta = M_{X_{1,2,3}} := M_{\text{Seesaw}}$

$(m_{\text{soft}})_\Delta = (m_{\text{soft}})_X := (m_{\text{soft}})_{\text{Seesaw}} = 0$

$m_0 = 6$ TeV, $M_{1/2} = 2$ TeV

$\tan\beta = 10$, $A_0 = 0$, $\mu > 0$

(sparticle constraints & m_h✓)

$$\text{BR}_{\text{LFV}(\tilde\ell)} \propto \lambda_F^4 \propto \begin{cases} \lambda^{-12} \\ \left(\frac{M^3_{\text{seesaw}}}{B_X}\right)^4 \sim M^8_{\text{Seesaw}} \end{cases}$$

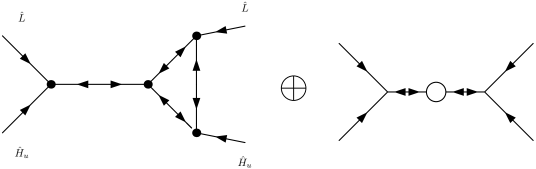
$$\text{BR}_{\text{LFV}(\Delta)} \propto \frac{\lambda_F^4}{M^4_{\text{Seesaw}}} \propto M^4_{\text{Seesaw}}$$

Thank you

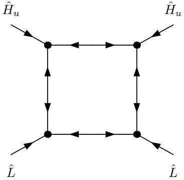
Work in progress

Characteristic models

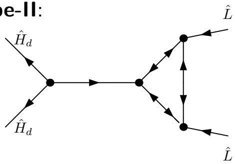
- **3 models** to span the **different topologies**:
  - type-II :: $D^2(\hat{L}\hat{L})\hat{H}_u\hat{H}_u$
  - type-I/III :: $\hat{L}\hat{L}\hat{H}_u\hat{H}_u$



  - 1PI :: $\hat{L}\hat{L}\hat{H}_u\hat{H}_u$



- **Plus** the especially **suppressed** version of **type-II**:
  - $\hat{L}\hat{L}\bar{D}^2(\hat{H}_d^\dagger\hat{H}_d^\dagger)$



Characteristic models

$$\mathcal{W}_{\mathrm{I}} := M_N \hat{N}\hat{\bar{N}} + \sum_{i=1}^{2} \lambda_i \hat{\rho}_i \hat{X}_i \hat{\bar{X}}_i + M_{X_3} \hat{X}_3 \hat{\bar{X}}_3$$
$$+\hat{L}\left(Y^N \hat{N}\hat{H}_u + Y^X \hat{X}_1 \hat{X}_3\right) + \hat{\bar{N}}\left(\bar{\lambda}_X \hat{\bar{X}}_1 \hat{\bar{X}}_2 + \lambda_X \hat{X}_1 \hat{X}_2\right) + \lambda_H \hat{H}_u \hat{X}_2 \hat{\bar{X}}_3 \,.$$

$$\mathcal{W}_{\mathrm{1PI}} := \frac{1}{2} M_{X_1} \hat{X}_1^2 + \lambda \hat{\rho} \hat{X}_2 \hat{\bar{X}}_2 + \frac{1}{2} M_{X_3} \hat{X}_3^2 + \lambda_L \hat{L} \hat{X}_1 \hat{X}_2 + \lambda_H \hat{H}_u \hat{\bar{X}}_2 \hat{X}_3 \,.$$

$$\mathcal{W}_{\mathrm{II}_d} := M_\Delta \hat{\Delta}\hat{\bar{\Delta}} + \sum_{i=1}^{2} M_{X_i} \hat{X}_i \hat{\bar{X}}_i + \lambda \hat{\rho} \hat{X}_3 \hat{\bar{X}}_3$$
$$+\hat{L}\left(\lambda_3 \hat{\bar{X}}_1 \hat{X}_3 + \bar{\lambda}_3 \hat{\bar{X}}_2 \hat{\bar{X}}_3\right) + \hat{\Delta}\left(\lambda_H \hat{H}_d \hat{H}_d + \lambda_X \hat{X}_1 \hat{X}_2\right)$$
$$+\hat{\bar{\Delta}}\left(\bar{\lambda}_X \hat{\bar{X}}_1 \hat{\bar{X}}_2 + \bar{\lambda}_H \hat{H}_u \hat{H}_u\right) \,.$$

Characteristic models

- **3 models** to span the **different topologies**:
  - type-II :: $D^2(\hat{L}\hat{L})\hat{H}_u\hat{H}_u$
  - type-I/III :: $\hat{L}\hat{L}\hat{H}_u\hat{H}_u$
  - 1PI :: $\hat{L}\hat{L}\hat{H}_u\hat{H}_u$
- **Plus** the especially **suppressed** version of **type-II**:
  - $\hat{L}\hat{L}\bar{D}^2(\hat{H}_d^\dagger\hat{H}_d^\dagger)$

- Model scheme:

Seesaw/flavour Mediator $\oplus$ Internal/loop Sector ($\hat{X}$'s)

- Involved in the effective tree-level topology
- Reponsible for flavour structure

- Involved in the fundamental topology
- The source of soft-~~SUSY~~ effects for m_ν

Model	Seesaw Mediator	Flavour Mediator	$\hat{X}$'s	m_ν/v^2
II_u	$\hat{\Delta}\oplus\hat{\bar{\Delta}}$		$\hat{X}_{1,2,3}\oplus\hat{\bar{X}}_{1,2,3}$	$\frac{B_X\oplus(m^2_{\mathrm{soft}})_X}{M^3}$
II_d	$\hat{\Delta}\oplus\hat{\bar{\Delta}}$	$\hat{X}_3\oplus\hat{\bar{X}}_3$		$\frac{\mu B_X\oplus A_H B_X}{M^4}$
I	$\hat{N}\oplus\hat{\bar{N}}$	$\hat{N}\oplus\hat{\bar{N}},\,\hat{X}_1\oplus\hat{\bar{X}}_1$		$\frac{B_X}{M^3}$
1PI	——	$\hat{X}_1$	$\hat{X}_1,\,\hat{X}_2\oplus\hat{\bar{X}}_2,\,\hat{X}_3$	

Characteristic models

$$\text{II}_u: \qquad \frac{m_\nu}{v^2} \sim \frac{\boldsymbol{\lambda_L} \lambda^2 s_\beta^2}{16\pi^2 M^3} \left(\bar{\lambda}_X B_X^* \oplus \lambda_X (m_{\text{soft}}^2)_X \right)$$

$$\text{II}_d: \qquad \frac{m_\nu}{v^2} \sim \frac{(\boldsymbol{\lambda}\boldsymbol{\lambda}^{\boldsymbol{T}}) \lambda_{H_d}^* \lambda_X B_X^* c_\beta}{16\pi^2 M^4} \left(\mu s_\beta \oplus A_{H_d}^* c_\beta \right)$$

$$\text{I \& 1PI}: \qquad \frac{m_\nu}{v^2} \sim \frac{(\boldsymbol{\lambda}\boldsymbol{\lambda}^{\boldsymbol{T}}) \lambda^2 B_X^* s_\beta^2}{16\pi^2 M^3}$$

Minimal choices:

II_d: $1 \times (\hat{X}_3 \oplus \hat{\bar{X}}_3)$

I: $1 \times (\hat{N} \oplus \hat{\bar{N}}) + 1 \times (\hat{X}_1 \oplus \hat{\bar{X}}_1)$

1PI: $2 \times \hat{X}_1$

$$\text{II}_d: \qquad (\boldsymbol{\lambda}\boldsymbol{\lambda}^{\boldsymbol{T}}) = \left(\Lambda_3 \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \Lambda_3^T \right), \qquad \Lambda_3 := \begin{pmatrix} | & | \\ \lambda_3 & \bar{\lambda}_3 \\ | & | \end{pmatrix}$$

$$\text{I}: \qquad (\boldsymbol{\lambda}\boldsymbol{\lambda}^{\boldsymbol{T}}) = \left(Y^\nu \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} Y^{\nu\, T} \right), \qquad Y^\nu := \frac{1}{\sqrt{\kappa_1}} \begin{pmatrix} | & | \\ \kappa_1 Y^N & \kappa_2 Y^X \\ | & | \end{pmatrix}$$

$$\text{1PI}: \qquad (\boldsymbol{\lambda}\boldsymbol{\lambda}^{\boldsymbol{T}}) = \Lambda_L \Lambda_L^T, \qquad \Lambda_L := \begin{pmatrix} | & | \\ \kappa_1 (\lambda_L)_{i1} & \kappa_2 (\lambda_L)_{i2} \\ | & | \end{pmatrix}$$

Some preliminary comments

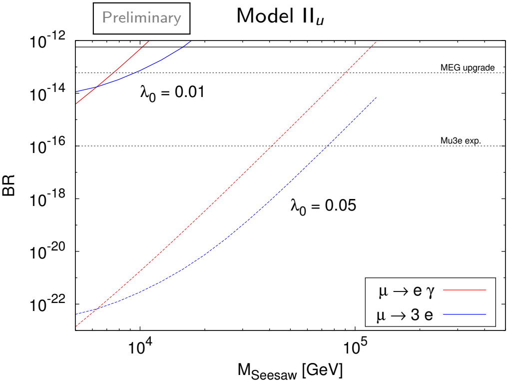
- Take $\not{S}U\not{S}Y$ at some input scale $M_{\not{SUSY}} > M := M_{\text{seesaw}} \sim M_X$
- Running will generate $(m_{\text{soft}})_{\text{seesaw}}(M_Z) \neq (m_{\text{soft}})_{\text{seesaw}}(\text{input})$
  - This means that
  $$\frac{m_\nu M^3}{c_{\text{loop}} v^2} \sim (m^2_{\text{soft}})_{\text{seesaw}}$$
  is sensitive at leading order running to EWinos masses $M_{1,2}$.
  Example (for Model 1PI):
  $$\begin{aligned} \beta^{(1)}_{B_{X_1}} &= 2\left(2A_L^T\lambda_L^* M_{X_1} + 2M_{X_1}\lambda_L^\dagger A_L + B_{X_1}\lambda_L^\dagger\lambda_L + \lambda_L^T\lambda_L^* B_{X_1}\right) \\ \beta^{(1)}_{B_{X_2}} &= \frac{2}{5}M_{X_2}\left(15g_2^2 M_2 + 3g_1^2 M_1 + 5\lambda_H A_H + 5\text{Tr}\left(\lambda_L^\dagger A_L\right)\right) + B_{X_2}\,(\ldots) \\ \beta^{(1)}_{B_{X_3}} &= 4\lambda_H\left(2M_{X_3}A_H + \lambda_H B_{X_3}\right) \end{aligned}$$
- It will also imprint LFV in the slepton soft-$\not{S}U\not{S}Y$ sector. [Borzumati and Masiero '86]
  Example (for Model II$_u$):
  $$\beta^{(1)}_{m_L^2} \supset 6\left(m^2_{\text{soft}}\right)_\Delta \lambda_L^*\lambda_L + 6A_L^*A_L + 3m_L^2\lambda_L^*\lambda_L + 3\lambda_L^*\lambda_L m_L^2 + 6\lambda_L^* m_L^{2*}\lambda_L$$
  - In addition to LFV mediated by radiative seesaw mediators, there will in general be the traditional contributions from slepton-EWino/Higgisno loops
  - Sleptonic LFV ratios: [Kim *et al* '98] [Hisano and Tobe '01]
  $$\frac{\text{BR}(\ell \to 3\ell')}{\text{BR}(\ell \to \ell'\gamma)} \simeq \frac{\alpha}{3\pi}\left(\log\frac{m_\ell^2}{m_{\ell'}^2} - \frac{11}{4}\right), \qquad \frac{\text{CR}(\mu - e, \text{Ti})}{\text{BR}(\mu \to e\gamma)} \approx 5\times 10^{-3}$$
  Deviations would suggest the existence of other low-lying states with lepton-flavour number

E.g. of a feature of II_u model

- Model II_u vs ordinary SUSY type-II model:

$$\frac{m_\nu}{v^2} \sim \frac{\lambda_F \lambda^3}{16\pi^2} \frac{m^2_{\mathrm{soft}}}{M^3_{\mathrm{Seesaw}}} \quad \text{vs} \quad \frac{m_\nu}{v^2} \sim \frac{\lambda_F \lambda}{M_{\mathrm{Seesaw}}}$$

$\hat{L}\hat{L}\hat{\Delta}$ coupling (as in Mode II_u) may enhance $\ell \to 3\ell'$ compared to $\ell \to \ell'\gamma$



$m^2_{\tilde{\ell}} \approx 6$ TeV, $m_{\widetilde{\mathrm{EW}}} \approx 1.5$ TeV
At $M_{\mathrm{SUSY}} = 10^{10}$ GeV:
$\lambda_1 = \lambda_2 = \lambda_X = \bar{\lambda}_X := \lambda_0$
$M_\Delta = M_{X_{1,2,3}} := M_{\mathrm{Seesaw}}$
$(m_{\mathrm{soft}})_\Delta = (m_{\mathrm{soft}})_X := (m_{\mathrm{soft}})_{\mathrm{Seesaw}} = 0$

$m_0 = 6$ TeV, $M_{1/2} = 2$ TeV
$\tan\beta = 10$, $A_0 = 0$, $\mu > 0$
(sparticle constraints & m_h✓)

$$\mathrm{BR}_{\mathrm{LFV}(\tilde{\ell})} \propto \lambda_F^4 \propto \begin{cases} \lambda^{-12} \\ \left(\frac{M^3_{\mathrm{seesaw}}}{B_X}\right)^4 \end{cases} \sim M^8_{\mathrm{Seesaw}}$$

$$\mathrm{BR}_{\mathrm{LFV}(\Delta)} \propto \frac{\lambda_F^4}{M^4_{\mathrm{Seesaw}}} \propto M^4_{\mathrm{Seesaw}}$$

Backup

II_u

$$\mathcal{W}_{\mathrm{II}_u} \quad := \quad M_\Delta \hat{\Delta}\hat{\bar{\Delta}} + \sum_{i=1}^{2} M_{X_i}\hat{X}_i\hat{\bar{X}}_i + \lambda\hat{\rho}\hat{X}_3\hat{\bar{X}}_3$$
$$+\hat{H}_u\left(\lambda_1\hat{X}_1\hat{\bar{X}}_3 + \lambda_2\hat{X}_2\hat{X}_3\right) + \hat{\Delta}\left(\lambda_L\hat{L}\hat{L} + \lambda_X\hat{X}_1\hat{X}_2\right) + \bar{\lambda}_X\hat{\bar{\Delta}}\hat{\bar{X}}_1\hat{\bar{X}}_2\,.$$

In the absence of the last term the model acquires the R-symmetry shown in the last column of the table. The broken L-number phase corresponds to

$$\lambda\hat{\rho}\hat{X}_3\hat{\bar{X}}_3 \to M_{X_3}\hat{X}_3\hat{\bar{X}}_3 + \lambda\hat{\rho}\hat{X}_3\hat{\bar{X}}_3\,,\quad M_{X_3} := \lambda\langle\rho\rangle\,.$$

	$SU(2)_L \otimes U(1)_Y$	$U(1)_X$	$U(1)_L$	$U(1)_R$
$\hat{\Delta}$	$(\mathbf{3}, 1)$	0	-2	4
$\hat{\rho}$	$(\mathbf{1}, 0)$	0	2	0
$\hat{X}_1$	$(\mathbf{2}, -1/2)$	1	1	-2
$\hat{X}_2$	$(\mathbf{2}, -1/2)$	-1	1	0
$\hat{X}_3$	$(\mathbf{1}, 0)$	1	-1	0
$\hat{\bar{X}}_3$	$(\mathbf{1}, 0)$	-1	-1	2

Table: Extension of the MSSM in model II_u. We omitted the conjugates of $\hat{\Delta}$ and $\hat{X}_{1,2}$. $U(1)_R$ stands for an R-symmetry acquired as $\bar{\lambda}_X \to 0$.

II_d

$$
\begin{aligned}
\mathcal{W}_{\mathrm{II}_d} \quad := \quad & M_\Delta \hat{\Delta}\hat{\bar{\Delta}} + \sum_{i=1}^{2} M_{X_i} \hat{X}_i \hat{\bar{X}}_i + \lambda \hat{\rho} \hat{X}_3 \hat{\bar{X}}_3 \\
& + \hat{L} \left(\lambda_3 \hat{\bar{X}}_1 \hat{X}_3 + \bar{\lambda}_3 \hat{\bar{X}}_2 \hat{\bar{X}}_3 \right) + \hat{\Delta} \left(\lambda_H \hat{H}_d \hat{H}_d + \lambda_X \hat{X}_1 \hat{X}_2 \right) \\
& \cancel{+ \hat{\bar{\Delta}} \left(\bar{\lambda}_X \hat{\bar{X}}_1 \hat{\bar{X}}_2 + \bar{\lambda}_H \hat{H}_u \hat{H}_u \right)} \,.
\end{aligned}
$$

	$SU(2)_L \otimes U(1)_Y$	$U(1)_X$	$U(1)_L$	$U(1)_R$	R_p
$\hat{\Delta}$	$(\mathbf{3}, 1)$	0	0	4	$+1$
$\hat{\rho}$	$(\mathbf{1}, 0)$	0	2	0	$+1$
$\hat{X}_1$	$(\mathbf{2}, -1/2)$	1	0	-2	$+1$
$\hat{X}_2$	$(\mathbf{2}, -1/2)$	-1	0	0	$+1$
$n \times \hat{X}_3$	$(\mathbf{1}, 0)$	1	-1	0	-1
$n \times \hat{\bar{X}}_3$	$(\mathbf{1}, 0)$	-1	-1	2	-1

Table: Extension of the MSSM in model II_d. We omitted the conjugates of $\hat{\Delta}$ and $\hat{X}_{1,2}$. We assume an $U(1)_R$ symmetry (with charges as in the last column) that is broken in such a way that the last two terms are negligible.

II_d (cont'd)

We will take $n = 1$. Then,

$$(\lambda_3)_i(\bar{\lambda}_3)_j + (\lambda_3)_j(\bar{\lambda}_3)_i = \left(\Lambda_3 \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \Lambda_3^T\right)_{ij}, \qquad \Lambda_3 := \begin{pmatrix} | & | \\ \lambda_3 & \bar{\lambda}_3 \\ | & | \end{pmatrix},$$

$$\hat{m}_\nu = U^T m_\nu U = v^2 \kappa\, U^T \left(\Lambda_3 \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \Lambda_3^T\right) U$$

$$\Rightarrow \Lambda_3 = \frac{1}{\sqrt{2} v \sqrt{\kappa}} U^* \sqrt{\hat{m}_\nu} R^T \begin{pmatrix} e^{-i\pi/4} & e^{i\pi/4} \\ e^{i\pi/4} & e^{-i\pi/4} \end{pmatrix},$$

where R is a general 2×2 complex orthogonal matrix. We can check that this parameterisation does in fact span the 10 independent real parameters of the matrix $\lambda_3 \bar{\lambda}_3^T + \bar{\lambda}_3 \lambda_3^T$.

$$
\begin{aligned}
\mathcal{W}_{\mathrm{I}} \quad := \quad & M_N \hat{N}\hat{\bar{N}} + \sum_{i=1}^{2} \lambda_i \hat{\rho}_i \hat{X}_i \hat{\bar{X}}_i + M_{X_3} \hat{X}_3 \hat{\bar{X}}_3 \\
& + \hat{L}\left(Y^N \hat{N} \hat{H}_u + Y^X \hat{X}_1 \hat{X}_3 \right) + \hat{\bar{N}} \left(\bar{\lambda}_X \hat{\bar{X}}_1 \hat{\bar{X}}_2 + \lambda_X \hat{X}_1 \hat{X}_2 \right) + \lambda_H \hat{H}_u \hat{X}_2 \hat{\bar{X}}_3 \,.
\end{aligned}
$$

(One can show that there exists no $U(1)_R$ symmetry that would forbid either $\bar{\lambda}_X$ or λ_X, while retaining the rest of the superpotential.)
We take $n = n' = 1$. Then:

$$
\kappa_1 Y_i^N Y_j^N + \kappa_2 \left(Y_i^X Y_j^N + Y_j^X Y_i^N \right) = \left(Y^\nu \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} Y^{\nu\, T} \right)_{ij} \,,
$$

$$
Y^\nu := \frac{1}{\sqrt{\kappa_1}} \begin{pmatrix} | & | \\ \kappa_1 Y^N & \kappa_2 Y^X \\ | & | \end{pmatrix} \,,
$$

$$
\hat{m}_\nu = U^T m_\nu U = v^2 \kappa\, U^T \left(Y^\nu \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} Y^{\nu\, T} \right) U
$$

$$
\Rightarrow Y^\nu = \frac{1}{v\sqrt{\kappa}} U^* \sqrt{\hat{m}_\nu} R^T \begin{pmatrix} 1 & 0 \\ -i & i \end{pmatrix} \,,
$$

$$
\Rightarrow Y_i^N = \frac{1}{\sqrt{\kappa_1}} Y_{i1}^\nu \,, \quad Y_i^X = \frac{\sqrt{\kappa_1}}{\kappa_2} Y_{i2}^\nu \,.
$$

I (cont'd)

	$SU(2)_L \otimes U(1)_Y$	$U(1)_X$	$U(1)_L$	R_p
$n \times \hat{N}$	$(\mathbf{1}, 0)$	0	-1	-1
$\hat{\rho}_1$	$(\mathbf{1}, 0)$	0	$2/3$	$+1$
$\hat{\rho}_2$	$(\mathbf{1}, 0)$	0	$4/3$	$+1$
$n' \times \hat{X}_1$	$(\mathbf{1}, 0)$	1	-1	-1
$n' \times \hat{\bar{X}}_1$	$(\mathbf{1}, 0)$	-1	$1/3$	-1
$\hat{X}_2$	$(\mathbf{1}, 0)$	-1	0	$+1$
$\hat{\bar{X}}_2$	$(\mathbf{1}, 0)$	1	$-4/3$	$+1$
$\hat{X}_3$	$(\mathbf{2}, 1/2)$	-1	0	$+1$
$\hat{\bar{X}}_3$	$(\mathbf{2}, -1/2)$	1	0	$+1$

Table: Extension of the MSSM in model I. We omitted the conjugate of $\hat{N}$.

1PI

$$\mathcal{W}_{\text{1PI}} \quad := \quad \frac{1}{2}M_{X_1}\hat{X}_1^2 + \lambda\hat{\rho}\hat{X}_2\hat{\bar{X}}_2 + \frac{1}{2}M_{X_3}\hat{X}_3^2 + \lambda_L\hat{L}\hat{X}_1\hat{X}_2 + \lambda_H\hat{H}_u\hat{\bar{X}}_2\hat{X}_3\,.$$

	$SU(2)_L \otimes U(1)_Y$	Z_2^X	$U(1)_L$	R_p
$n \times \hat{X}_1$	$(\mathbf{1}, 0)$	-1	0	-1
$\hat{X}_2$	$(\mathbf{2}, 1/2)$	-1	-1	$+1$
$\hat{\bar{X}}_2$	$(\mathbf{2}, -1/2)$	-1	0	$+1$
$\hat{X}_3$	$(\mathbf{1}, 0)$	-1	0	$+1$
$\hat{\rho}$	$(\mathbf{1}, 0)$	1	1	$+1$

Table: Extension of the MSSM in model 1PI. All MSSM superfields are even under the Z_2.

1PI (cont'd)

We take $n = 2$. Then:

$$\hat{m}_\nu = U^T m_\nu U = v^2 \kappa\, U^T \left(\Lambda_L \Lambda_L^T\right) U\,, \qquad \Lambda_L = \begin{pmatrix} | & | \\ \kappa_1 (\lambda_L)_{i1} & \kappa_2 (\lambda_L)_{i2} \\ | & | \end{pmatrix}\,,$$

$$\Rightarrow \Lambda_L = \frac{1}{v\sqrt{\kappa}} U^* \sqrt{\hat{m}_\nu} R^T\,,$$

$$\Rightarrow (\lambda_L)_{ij} = \frac{1}{\kappa_j} (\Lambda_L)_{ij}\,.$$

Observable	Old Bound	Current bound	Future sensitivity
$\mathrm{BR}(\mu \to e\gamma)$	1.2×10^{-11}	5.7×10^{-13}	6×10^{-14}
$\mathrm{BR}(\tau \to e\gamma)$	3.3×10^{-8}	—	$\mathcal{O}(10^{-9})$
$\mathrm{BR}(\tau \to \mu\gamma)$	4.4×10^{-8}	—	$\mathcal{O}(10^{-9})$
$\mathrm{BR}(\mu \to 3\,e)$	1.0×10^{-12}	—	$\mathcal{O}(10^{-16})$
$\mathrm{BR}(\tau \to 3\,e)$	2.7×10^{-8}	—	$\mathcal{O}(10^{-9})$
$\mathrm{BR}(\tau \to 3\,\mu)$	2.1×10^{-8}	—	$\mathcal{O}(10^{-9})$
$\mathrm{CR}(\mu - e,\ \mathrm{Ti})$	4.3×10^{-12}	—	$\mathcal{O}(10^{-18})$
$\mathrm{CR}(\mu - e,\ \mathrm{Au})$	7×10^{-13}	—	—
$\mathrm{CR}(\mu - e,\ \mathrm{Al})$	—	—	$\mathcal{O}(10^{-16})$

Table: Bounds and future sensitivities for several cLFV observables.