A Detailed Analysis of FCNC Decays of the Top Quark

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Introduction to Top Decays

• The top quark decays before hadronisation due to its very large mass

• The decay thus doesn’t involve non-perturbative processes such as parton showering

• Dominant decay mode: \( t \rightarrow b \, W \) (almost 100% BR)

• Flavour Changing Neutral Current (FCNC) decays – rare decays like \( t \rightarrow c \, h \)

• Consider the indirect contributions of BSM particles through loop diagrams

• Won’t be considering any flavour changing couplings for Higgs
LHC Reach

• CMS Collaboration places a 95% CL using Run-I data

\[ BR(t \rightarrow ch) < 5.6 \times 10^{-3} \]
\[ BR(t \rightarrow cZ) < 5.0 \times 10^{-4} \]

• Can reach the exclusion limit (2-sigma) of (at 3000 fb\(^{-1}\) of 13 TeV data):

\[ BR_{3ab^{-1}}(t \rightarrow ch) = 1.5 \times 10^{-4} \]
\[ BR_{3ab^{-1}}(t \rightarrow cZ) = 7.0 \times 10^{-5} \]

• Optimistic limit :

\[ BR_{3ab^{-1}}(t \rightarrow ch) \sim 10^{-5} \]
Outline

• Standard Model Process

• Analysis using a toy model

• Process in cMSSM

• Process in RPV SUSY
Calculating Top FCNC Decay

\[ t(p) \rightarrow c(p + k) + h(-k) \]

**Scalar**: \[ iM = \bar{c}(p + k) i\Gamma t(p) \]

- Calculated the process \( t \rightarrow c h \) with generic couplings in the Feynman gauge. Form factors have been used:

\[
i \Gamma_n = \frac{ig^3}{16\pi^2} \sum_{i=1}^{3} \lambda_i \left( F_{1i}^{(n)} P_L + F_{2i}^{(n)} P_R \right)
\]

\[ \Gamma = \sum_n \Gamma_n \]

- Contribution to effective vertex of the \( n \)th diagram is:

- GIM suppression comes from the factor:

\[ \lambda_i = V_{ci} V_{ti}^* \]
Feynman Diagrams in the SM
Result (in the SM)

In the SM

\[ BR(t \rightarrow ch) \sim 10^{-15} \]

No way of seeing a SM signal
Result (in the SM)

In the SM

\[ BR(t \rightarrow ch) \sim 10^{-15} \]

No way of seeing a SM signal

Sources of Suppression

1. GIM Cancellation
2. MFV structure of the quark sector
3. Low value of the coupling constant
GIM Suppression

- First proposed by Glashow, Iliopoulos and Miani to explain the suppression of the FCNC decay amplitudes seen in $\Delta S = 2$ decays.

- Led to the prediction of the *charm* quark.

- Directly related to the unitarity of the CKM matrix.

- Unitarity Condition: $\sum_i \lambda_i = \sum_i V_{ti}^* V_{ci} = 0$  
  \[ \lambda_i = V_{ti}^* V_{ci} \]

- The FCNC amplitude is of the form:

\[
\mathcal{A} = C \left[ \sum_i \lambda_i A(x_i) \right]
\]

\[
x_i = \left( \frac{m_i}{M_W} \right)^2
\]
GIM Suppression

• Taylor expand the amplitude in $x_i's$ since they are small numbers

\[ A(x_i) = A(0) + x_i A'(0) + \ldots \]

• Put this back:

\[ \left[ \sum_i \lambda_i x_i \right] = A(0) \sum_i \lambda_i + A'(0) \sum_i \lambda_i x_i \]

Unitarity of CKM

• Thus the leading amplitude is proportional to $x_i's$

Avoiding GIM Suppression

• If FC coupling depends on the mass, we can violate GIM

\[ \left[ \sum_i \lambda_i m_i A(x_i) \right] = A(0) \sum_i \lambda_i m_i + A'(0) \sum_i \lambda_i m_i x_i \]
MFV structure of the Quark sector

- **MFV hypothesis**: Yukawas are the only source of flavour violation in the SM and in any BSM models
  

- Yukawas might have a high energy dynamical origin

**Implications:**
- SM flavour structure is all that there is
- Produces additional suppression for NP flavour transitions
- Inherits the hierarchical nature of the CKM matrix

\[
CKM \approx \begin{pmatrix}
1 & \lambda & A\lambda^3 \\
-\lambda & 1 & A\lambda^2 \\
A\lambda^3 & -A\lambda^2 & 1
\end{pmatrix} \approx \begin{pmatrix}
1 & 0.2 & 0.003 \\
0.008 & 0.04 & 1
\end{pmatrix}
\]
Toy Model

- Introduce a charged scalar – a scalar version of the $W$-boson

\[ \mathcal{L}_{int} = \xi \omega^+ \omega^- h + \sum_{i,j=1}^{3} \eta V_{ij} \bar{u}_i P_L d_j \omega^+ + h.c. \]
Toy Model

- Used helicity amplitude techniques to calculate the Branching Ratios
- Amplitudes for each helicity combination of top, charm: $A_i(h_c, h_t)$
- Only two combinations non-zero: $A_i(+, +); A_i(−, −)$

“SM Like” amplitudes

$$\mathcal{A} = \sum_i \lambda_i A(m_i, X) \quad \lambda_i = V_{ti}^* V_{ci}$$
Suppression Mechanisms

Break GIM

\[ \mathcal{M} = \sum_i \lambda_i A_i (m_i, X) \times \left( \frac{m_i}{m_b} \right)^2 \quad m_i = (m_d, m_s, m_b) \]

GIM Relaxation:

Gives us an enhancement of \( \sim \left[ \left( \frac{m_b}{M_\omega} \right)^2 \right]^{-1} \approx 10^2 - 10^4 \quad M_\omega \sim M_W = 80.4 \text{ GeV} \)

Go beyond MFV

\[
V'_{CKM} = \begin{pmatrix}
1 & 0 & 0 \\
0 & \cos \theta & \sin \theta \\
0 & -\sin \theta & \cos \theta
\end{pmatrix}
\]

For \( \theta = \frac{\pi}{4} \); \( \lambda_3 = \left( 0, -\frac{1}{2}, \frac{1}{2} \right) \)

Departure from MFV:

Gives us an enhancement of 1 order of magnitude

Maximize coupling

Increase Coupling Constant \quad \text{Factor of 3 - 5}
Toy Model Amplitudes for $t \rightarrow ch$

(a)

(b)
Top Rare Decays in the cMSSM

Introduction to cMSSM

• The simplest version of the MSSM

• cMSSM contains 5 free parameters – $m_0$, $m_{1/2}$, $A_0$, $\tan \beta$, $\text{sgn}(\mu)$

• GIM would be broken by the charged Higgs

• MFV structure retained

• Couplings similar to those of the SM particles, scaled by factors like $\tan \beta$
Top Rare Decays in the cMSSM

Parameter Space - Constraints

• Theory Constraints
  – Issues like vacuum stability, proper LSP etc.

• Higgs Mass constraint
  – light Higgs mass taken between 124 to 127 GeV (2σ interval)
  – Constraints $m_0$ values ⇒ charged Higgs mass is large

• Direct mass constraints
  – Latest results by ATLAS
  – $m_{\tilde{g}} \geq 1.7$ TeV; $m_{\tilde{q}} \geq 1000$ GeV; $m_{\tilde{t}}(b\chi^+) > 380$ GeV

• Flavour Physics constraints
  – FCNC processes involving $b$-quark are also GIM-violating
  – $B \to K^*\gamma$ and $B_s \to \mu^+\mu^-$ are measured very close to SM
  – Although, we don’t take the anomalies like $R_D$, $R_{D^*}$ and $B \to D^*\tau\nu$

LHCb: 1211.2674; Belle: 1208.4678
Top Rare Decays in the cMSSM

Parameter Space - Constraints

Black – ruled out by ‘theory constraints’:
- Shape of the EW potential
- LSP is DM candidate – not coloured or charged

Blue – ruled out by mass constraints
- Higgs mass: $124 \leq m_H \leq 127$
- Direct searches bounds for sparticle masses

Red – ruled out by Flavour Physics constraints
- b-quark physics; FCNC processes
- $B \rightarrow K^*\gamma$ and $B \rightarrow \mu^+\mu^-$ are almost SM-like

LHCb: 1211.2674; Belle: 1208.4678
SUSY Contributions

- Additional diagrams with (a) charged Higgs bosons  
  (b) charginos  
  (c) d-squarks
Top Rare Decays in the cMSSM

Blue – Mass forbidden:
  • Higgs and sparticle mass disallowed

Red – Flavour forbidden:
  • Ruled out by flavour data

Black – Allowed points:
  • Gives us a tiny branching ratio
R-parity Violating SUSY

- R-parity is a $\mathbb{Z}_2$ symmetry which differentiates between SM and SUSY particles
  \[ R = (-1)^{2s+3B+L} \]

- SUSY particles are odd and SM particles are even under R-parity: R-parity conservation gives a viable dark matter candidate, the LSP

- R-parity violating SUSY superpotential -
  \[ W_{RPV} = \mu_i \hat{H}_u \hat{L}_i + \frac{1}{2} \lambda_{ijk} \hat{L}_i \hat{L}_j \hat{E}^c_k + \lambda'_{ijk} \hat{L}_i \hat{Q}_j \hat{D}^c_k + \frac{1}{2} \lambda''_{ijk} \hat{U}^c_i \hat{D}^c_j \hat{D}^c_k \]

- LQD - Lagrangian
  \[ \mathcal{L}_{LQD} = -\lambda'_{ijk} (\tilde{\nu}_i L \bar{d}_k R d_j L + \tilde{d}_j L \bar{d}_k R \nu_i L + \tilde{d}^*_k R \tilde{\nu}^c_i R d_j L - (\nu_L \rightarrow l_L, d_L \rightarrow u_L)) + h. c. \]
RPV-SUSY Couplings

• Having both $\lambda'$ and $\lambda''$ terms leads to proton decay – severely constrained ($10^{33}$ yrs)

• Have either $\lambda'$ or $\lambda''$, but not both – either L- violating or B- violating

• Our process: require couplings of the form: $\lambda'_{i2k}\lambda'_{i3k}$ (LQD) or $\lambda''_{2jk}\lambda''_{3jk}$ (UDD)
RPV SUSY Contributions
RPV SUSY Results

Current Bounds (with 20 fb\(^{-1}\))

- \(B(t \to cH) < 5.6 \times 10^{-3}\)  
  CMS PAS-Hig-13-034
- \(B(t \to cZ) < 5.0 \times 10^{-4}\)  
  PRL 112 (2014) 171802

Projections (with 3000 fb\(^{-1}\))

- \(B(t \to cH) < 1.5 \times 10^{-4}\)  
  ATL-PHYS-PUB-2013-012
- \(B(t \to cZ) < 7.0 \times 10^{-5}\)  
  hep-ex/1311.2028
Conclusion and Summary

- Studied the different suppression mechanisms and possible enhancement factors for $t \rightarrow ch$ using a toy model

- Collider reach $BR \sim 10^{-5}$

- Impossible to probe in SM or cMSSM framework

- Might be observable in RPV-SUSY scenarios the near future
Conclusion and Summary

- Studied the different suppression mechanisms and possible enhancement factors for $t \rightarrow ch$ using a toy model

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Thank You!
BACKUP SLIDES
MFV structure of the Quark sector

• Total SM fermion flavour structure

\[ G_f = SU(3)_q \otimes SU(3)_u \otimes SU(3)_d \otimes SU(3)_l \otimes SU(3)_E \]

• Introduce spurions like Yukawa fields to break \( G_f^Q \)

\[ Y_u \sim (3, \overline{3}, 1); \quad Y_d \sim (3, 1, \overline{3}) \]

\[ \mathcal{L} = \bar{Q}Y_d D\phi + \bar{Q}Y_u U\phi + h.c. \quad \text{Invariant under } G_f \]

• Source of Yukawa fields – some high energy dynamics

• Dim-5 terms in EFT
Passarino Veltman Integrals

- Algorithm provided by Passarino and Veltman


\[
B_0 (m_1, m_2; M) = \int \frac{d^4 k}{\pi^2} \frac{1}{(k^2 + m_1^2)((k + p)^2 + m_2^2)}
\]

\[
p_\mu B_1 (m_1, m_2; M) = \int \frac{d^4 k}{\pi^2} \frac{k_\mu}{(k^2 + m_1^2)((k + p)^2 + m_2^2)}
\]

\[
P^2 = -M^2
\]

\[
C_0 (m_1, m_2, m_3; M_1, M_2, M_3) = \int \frac{d^4 k}{\pi^2} \frac{1}{(k^2 + m_1^2)((k + p_2)^2 + m_2^2)((k + p_2 + p_3)^2 + m_3^2)}
\]

\[
C_{11} p_2_\mu + C_{12} p_3_\mu = \int \frac{d^4 k}{\pi^2} \frac{k_\mu}{(k^2 + m_1^2)((k + p_2)^2 + m_2^2)((k + p_2 + p_3)^2 + m_3^2)}
\]

\[
P_i^2 = -M_i^2
\]
## RPV Couplings

<table>
<thead>
<tr>
<th>Strongest Constraint arises from</th>
<th>Scales as mass of</th>
<th>Scaling Exponent</th>
<th>Upper bound (100 GeV)</th>
<th>Sfermion mass (GeV)</th>
<th>Current upper bound</th>
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<tbody>
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### Results

<table>
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<tr>
<th>Parameter</th>
<th>Value</th>
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<td>$\lambda'^{c}_{121}$</td>
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