The muon g-2: status from a theorist's point of view

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Theory of the g-2: the beginning

- Kusch and Foley 1948:

\[
\mu_{e}^{\text{exp}} = \frac{e\hbar}{2mc} \left(1.00119 \pm 0.00005\right)
\]

- Schwinger 1948 (triumph of QED!):

\[
\mu_{e}^{\text{th}} = \frac{e\hbar}{2mc} \left(1 + \frac{\alpha}{2\pi}\right) = \frac{e\hbar}{2mc} \times 1.00116
\]

- Keep studying the lepton–\(\gamma\) vertex:

\[
\bar{u}(p')\Gamma_{\mu}u(p) = \bar{u}(p')\left[\gamma_{\mu}F_{1}(q^{2}) + \frac{i\sigma_{\mu\nu}q^{\nu}}{2m}F_{2}(q^{2}) + \ldots\right]u(p)
\]

\[
F_{1}(0) = 1 \quad F_{2}(0) = \alpha_{l}
\]

A pure “quantum correction” effect!
The muon g-2: experimental status

Today: \[ a_\mu^{\text{EXP}} = (116592089 \pm 54_{\text{stat}} \pm 33_{\text{sys}}) \times 10^{-11} \] [0.5ppm].

Future: new muon g-2 experiments at:

- **Fermilab E989**: aiming at \( \pm 16 \times 10^{-11} \), ie 0.14ppm. Beam expected next year. First result expected in 2018 with a precision comparable to that of BNL E821.

- **J-PARC proposal**: aiming at 2019 Phase 1 start with 0.4ppm.

Are theorists ready for this (amazing) precision? Not yet.
The muon g-2: the QED contribution

\[ a_\mu^{\text{QED}} = (1/2)(\alpha/\pi) \quad \text{Schwinger 1948} \]

\[ + 0.765857426 \left(10^{16}\right) (\alpha/\pi)^2 \]

Sommerfield; Petermann; Suura&Wichmann '57; Elend '66; MP '04

\[ + 24.05050988 \left(10^{28}\right) (\alpha/\pi)^3 \]

Remiddi, Laporta, Barbieri … ; Czarnecki, Skrzypek; MP '04; Friot, Greynat & de Rafael '05, Mohr, Taylor & Newell 2012

\[ + 130.8773 \left(10^{61}\right) (\alpha/\pi)^4 \]


\[ + 752.85 \left(10^{93}\right) (\alpha/\pi)^5 \quad \text{COMPLETED!} \]

Kinoshita et al. ‘90, Yelkhovsky, Milstein, Starshenko, Laporta,… Aoyama, Hayakawa, Kinoshita, Nio 2012 & 2015

Adding up, we get:

\[ a_\mu^{\text{QED}} = 116584718.941 \left(10^{-11}\right) \]

from coeffs, mainly from 4-loop unc \(\downarrow\) \(\rightarrow\) from \(\delta\alpha(Rb)\)

with \(\alpha = 1/137.035999049(90) \quad [0.66 \text{ ppb}]\)
The muon g-2: the electroweak contribution

**One-loop term:**

$$a_{\mu}^{EW}(1\text{-loop}) = \frac{5G_{\mu}m_{\mu}^{2}}{24\sqrt{2}\pi^{2}} \left[ 1 + \frac{1}{5} \left( 1 - 4\sin^{2}\theta_{W} \right)^{2} + O \left( \frac{m_{\mu}^{2}}{M_{Z,W,H}^{2}} \right) \right] \approx 195 \times 10^{-11}$$

1972: Jackiv, Weinberg; Bars, Yoshimura; Altarelli, Cabibbo, Maiani; Bardeen, Gastmans, Lautrup; Fujikawa, Lee, Sanda; Studenikin et al. '80s

**One-loop plus higher-order terms:**

$$a_{\mu}^{EW} = 153.6 (1) \times 10^{-11}$$

with $$M_{Higgs} = 125.6 (1.5) \text{ GeV}$$

Hadronic loop uncertainties and 3-loop nonleading logs.

Kukhto et al. '92; Czarnecki, Krause, Marciano '95; Knecht, Peris, Perrottet, de Rafael '02; Czarnecki, Marciano and Vainshtein '02; Degrassi and Giudice '98; Heinemeyer, Stockinger, Weiglein '04; Gribouk and Czarnecki '05; Vainshtein '03; Gnendiger, Stockinger, Stockinger-Kim 2013.
The muon g-2: the hadronic LO contribution (HLO)

\[ K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)(s/m^2)} \]

\[ a_\mu^{\text{HLO}} = \frac{1}{4\pi^3} \int_{4m^2_\pi}^{\infty} ds K(s) \sigma^{(0)}(s) = \frac{\alpha^2}{3\pi^2} \int_{4m^2_\pi}^{\infty} \frac{ds}{s} K(s)R(s) \]

\[ a_\mu^{\text{HLO}} = 6870 \ (42)_{\text{tot}} \times 10^{-11} \]
\[ = 6928 \ (33)_{\text{tot}} \times 10^{-11} \]
\[ = 6949 \ (37)_{\text{exp}} \ (21)_{\text{rad}} \times 10^{-11} \]


Lots of progress in lattice calculations. T. Blum et al, PRL116 (2016) 232002

See D. Nomura’s talk

F. Jegerlehner and A. Nyffeler, Phys. Rept. 477 (2009) 1

F. Jegerlehner, arXiv:1511.04473 (includes BESIII 2π)

Davier et al, Tau2016, Beijing, Sep 2016, Preliminary

Hagiwara et al, JPG 38 (2011) 085003

F. Jegerlehner, arXiv:1511.04473 (includes BESIII 2π)
New space-like proposal for HLO

- Alternatively, exchanging the x and s integrations in $a_{\mu}^{\text{HLO}}$:

$$a_{\mu}^{\text{HLO}} = \frac{\alpha}{\pi} \int_0^1 dx \ (1 - x) \Delta \alpha_{\text{had}}[t(x)]$$

where $t(x) = \frac{x^2 m_{\mu}^2}{x - 1} < 0$

which involves $\Delta \alpha_{\text{had}}(t)$, the hadr. contrib. to the running of $\alpha$ in the space-like region. It can be extracted from Bhabha scattering data!
● $\Delta \alpha_{\text{had}}(t)$ can also be measured via the elastic scattering $\mu e \rightarrow \mu e$.

● Scattering a beam of muons of 150 GeV, available at CERN’s North Area, on a fixed electron target, $0<x<0.93$ (peak at 0.91).

With CERN’s 150 GeV muon beam (1.3 x $10^7 \mu$/s average) a statistical uncertainty of $\sim 0.3\%$ ($\sim 20 \times 10^{-11}$) can be reached on $a_\mu^{\text{HLO}}$ with 2 years of data taking. 10ppm systematic accuracy needed at peak.

G. Abbiendi et al, arXiv:1609.08987
The muon g-2: the hadronic NLO contributions (HNLO) - VP

HNLO: Vacuum Polarization

\[ a_{\mu}^{\text{HNLO}(vp)} = -98 \times 10^{-11} \]

\[ O(\alpha^3) \] contributions of diagrams containing hadronic vacuum polarization insertions:

Krause '96, Alemany et al. '98, Hagiwara et al. 2011
The muon g-2: the hadronic NLO contributions (HNLO) - LBL

**HNLO: Light-by-light contribution**

Unlike the HLO term, the hadronic l-b-l term relies at present on theoretical approaches.

This term had a troubled life! Latest values:

\[
\begin{align*}
    a_\mu^{\text{HNLO}}(\text{lbl}) &= +80 \ (40) \times 10^{-11} \quad \text{Knecht & Nyffeler '02} \\
    a_\mu^{\text{HNLO}}(\text{lbl}) &= +136 \ (25) \times 10^{-11} \quad \text{Melnikov & Vainshtein '03} \\
    a_\mu^{\text{HNLO}}(\text{lbl}) &= +105 \ (26) \times 10^{-11} \quad \text{Prades, de Rafael, Vainshtein '09} \\
    a_\mu^{\text{HNLO}}(\text{lbl}) &= +102 \ (39) \times 10^{-11} \quad \text{Jegerlehner, arXiv:1511.04473}
\end{align*}
\]

Results based also on Hayakawa, Kinoshita '98 & '02; Bijnens, Pallante, Prades '96 & '02

Improvements expected in the $\pi^0$ transition form factor A. Nyffeler 1602.03398


Progress on the lattice: $+53.5(13.5)\times10^{-11}$. Statistical error only, finite-volume and finite lattice-spacing errors being studied. Omitted subleading disconnected graphs still need to be computed.


See M. Hayakawa’s talk
The muon g-2: the hadronic NNLO contributions (HNNLO)

- **HNNLO: Vacuum Polarization**

\[ a_{\mu}^{\text{HNNLO (vp)}} = 12.4 (1) \times 10^{-11} \]

Kurz, Liu, Marquard, Steinhauser 2014

- **HNNLO: Light-by-light**

\[ a_{\mu}^{\text{HNNLO (lbl)}} = 3 (2) \times 10^{-11} \]

Colangelo, Hoferichter, Nyffeler, MP, Stoffer 2014
The muon g-2: SM vs. Experiment

Comparisons of the SM predictions with the measured g-2 value:

\[ a_\mu^{\text{EXP}} = 116592091 \, (63) \times 10^{-11} \]

<table>
<thead>
<tr>
<th>( a_\mu^{\text{SM}} \times 10^{11} )</th>
<th>( \Delta a_\mu = a_\mu^{\text{EXP}} - a_\mu^{\text{SM}} \times 10^{-11} )</th>
<th>( \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>116 591 761 (57)</td>
<td>330 (85) \times 10^{-11}</td>
<td>3.9 [1]</td>
</tr>
<tr>
<td>116 591 820 (51)</td>
<td>271 (81) \times 10^{-11}</td>
<td>3.3 [2]</td>
</tr>
<tr>
<td>116 591 841 (58)</td>
<td>250 (86) \times 10^{-11}</td>
<td>2.9 [3]</td>
</tr>
</tbody>
</table>

with the recent “conservative” hadronic light-by-light \( a_\mu^{\text{HNLO(lbl)}} = 102 (39) \times 10^{-11} \) of F. Jegerlehner arXiv:1511.04473, and the hadronic leading-order of:

Brief digression: the electron g-2
The electron g-2: SM vs Experiment

The 2008 measurement of the electron g-2 is:

\[ a_e^{\text{EXP}} = 11596521807.3 \pm 2.8 \times 10^{-13} \]

Hanneke, Fogwell, Gabrielse
PRL100 (2008) 120801

Using \( \alpha = 1/137.035 \, 999 \, 049 \) (90) from h/M measurement of \(^{87}\text{Rb}\) (2011), the SM prediction for the electron g-2 is

\[ a_e^{\text{SM}} = 115 \, 965 \, 218 \, 16.5 \pm 0.2 \times 10^{-13} \]

\( \delta C_4^{\text{qed}} = \delta C_5^{\text{qed}} = \delta a_e^{\text{had}} \) from \( \delta \alpha \)

The EXP-SM difference is (note the negative sign):

\[ \Delta a_e = a_e^{\text{EXP}} - a_e^{\text{SM}} = -9.2 \pm 8.1 \times 10^{-13} \]

The SM is in very good agreement with experiment (1\( \sigma \)).
The electron g-2 sensitivity and NP tests

- The present sensitivity is \( \delta \Delta a_e = 8.1 \times 10^{-13} \), i.e. (10\(^{-13}\) units):

\[
(0.2)_{\text{QED4}}, \quad (0.2)_{\text{QED5}}, \quad (0.2)_{\text{HAD}}, \quad (0.4)_{\text{TH}}, \quad (7.6)\delta \alpha, \quad (2.8)\delta a_e^{\text{EXP}}
\]

\( (0.4)_{\text{TH}} \) may drop to 0.2

- The (g-2)\(_e\) exp. error may soon drop below 10\(^{-13}\) and work is in progress for a significant reduction of that induced by \( \delta \alpha \).

\( \rightarrow \) sensitivity of 10\(^{-13}\) may be reached with ongoing exp. work

- In a broad class of BSM theories, contributions to \( a_l \) scale as

\[
\frac{\Delta a_{\ell_i}}{\Delta a_{\ell_j}} = \left( \frac{m_{\ell_i}}{m_{\ell_j}} \right)^2
\]

This Naive Scaling leads to:

\[
\Delta a_e = \left( \frac{\Delta a_\mu}{3 \times 10^{-9}} \right) 0.7 \times 10^{-13}; \quad \Delta a_\tau = \left( \frac{\Delta a_\mu}{3 \times 10^{-9}} \right) 0.8 \times 10^{-6}
\]
The experimental sensitivity in $\Delta a_e$ is not too far from what is needed to **test if the discrepancy in** $(g-2)_\mu$ **also manifests itself in** $(g-2)_e$ **under the naive scaling hypothesis.**

NP scenarios exist which **violate Naive Scaling.** They can lead to larger effects in $\Delta a_e$ and contributions to EDMs, LFV or lepton universality breaking observables.

**Example:** In the MSSM with non-degenerate but aligned sleptons (vanishing flavor mixing angles), $\Delta a_e$ can reach $10^{-12}$ (at the limit of the present exp sensitivity). For these values one typically has breaking effects of lepton universality at the few per mil level (within future exp reach).

Giudice, Paradisi, MP  JHEP 2012
Back to the muon g-2
\( \Delta a_\mu: \) could it be an error in the hadronic cross section?

- Can \( \Delta a_\mu \) be due to hypothetical mistakes in the hadronic \( \sigma(s) \)?
- An upward shift of \( \sigma(s) \) also induces an increase of \( \Delta \alpha_{\text{had}}^{(5)}(M_Z) \).
- Consider:

\[
\begin{align*}
\alpha_{\text{HLO}} & \to a = \int_{4m_\pi^2}^{s_u} ds \, f(s) \, \sigma(s), \quad f(s) = \frac{K(s)}{4\pi^3}, \quad s_u < M_Z^2; \\
\Delta \alpha_{\text{had}}^{(5)} & \to b = \int_{4m_\pi^2}^{s_u} ds \, g(s) \, \sigma(s), \quad g(s) = \frac{M_Z^2}{(M_Z^2 - s)(4\alpha \pi^2)};
\end{align*}
\]

and the increase

\[
\Delta \sigma(s) = \epsilon \sigma(s)
\]

(\( \epsilon > 0 \)), in the range:

\[
\sqrt{s} \in [\sqrt{s_0} - \delta/2, \sqrt{s_0} + \delta/2]
\]
How much does the $M_H$ upper bound from the EW fit change when we shift $\sigma(s)$ by $\Delta\sigma(s)$ [and thus $\Delta\alpha_{\text{had}}^{(5)}(M_Z)$] to accommodate $\Delta a_\mu$?
Given the quoted exp. uncertainty of $\sigma(s)$, the possibility to explain the muon g-2 with these very large shifts $\Delta\sigma(s)$ appears to be very unlikely.

Also, given a 125 GeV SM Higgs, these hypothetical shifts $\Delta\sigma(s)$ could only occur at very low energy (below ~ 1 GeV) where $\sigma(s)$ is precisely measured.

Vice versa, assuming we now have a SM Higgs with $M_H = 125$ GeV, if we bridge the $M_H$ discrepancy in the EW fit decreasing the low-energy hadronic cross section, the muon g-2 discrepancy increases.

W.J. Marciano, A. Sirlin, MP, 2008 & 2010
Limiting 2HDMs with Natural Flavor Conservation

- Study of the parameter regions explaining $\Delta a_\mu$ with constraints from EW precision observables, vacuum stability & perturbativity, B physics, lepton universality in $Z$ & $\tau$ decays, and direct searches at colliders:

- Type I and Y models: cannot account for the present value of $\Delta a_\mu$ due to their lack of $\tan^2\beta$ enhancements.

- Type II models: $\tan^2\beta$ enhancements, but the bound on $\text{BR}(B_s \to \mu^+\mu^-)$ forbids a light $A$ required to explain $\Delta a_\mu$.

- Type X models: still viable at $2\sigma$ for large $\tan\beta$ and $10 \text{ GeV} < M_A \ll 200 \text{ GeV} \lesssim M_{H^\pm} \sim M_H \lesssim 400 \text{GeV}$.

- L. Wang, X.-F. Han, JHEP 2015
Figure 1. Allowed at 1σ and 2σ CL are the regions inside the green (inner) and yellow (outer) shaded areas by the muon $g - 2$; below the red dashed (lower) and red solid (upper) lines by the lepton universality test in $Z$ decays; below the blue dashed (lower) and blue solid (upper) lines by the lepton universality test with $\tau$ decays, respectively.

See Stöckinger-Kim’s talk for full 2loop results
ALPs contributions to the muon g-2

Light spin 0 scalars & pseudoscalars (axion-like-particles or ALPs), contribute to $a_\mu$. We consider ALPs in the mass range $\sim[0.1–1]$ GeV, where experimental constraints are rather loose.

A possible resolution of $\Delta a_\mu$ by 1-loop contributions from scalar particles with relatively large Yukawa couplings to muons, of $O(10^{-3})$, was analyzed by Chen, Davoudiasl, Marciano & Zhang, PRD 93, 035006 (2016):

For a pseudoscalar, the 1-loop contribution has the wrong sign (negative) to resolve the discrepancy on its own.
Consider ALP-$\gamma\gamma$ couplings as well as Yukawa couplings:

\[ \mathcal{L}_a = \frac{1}{4} g_{a\gamma\gamma} a F_{\mu\nu} \tilde{F}^{\mu\nu} + i y_{a\psi} a \bar{\psi}\gamma_5 \psi , \]
\[ \mathcal{L}_s = \frac{1}{4} g_{s\gamma\gamma} s F_{\mu\nu} F^{\mu\nu} + y_{s\psi} s \bar{\psi}\psi \]

New, potentially important, ALP contributions to $a_\mu$:

ALPs contributions to the muon g-2 (3)

- **Y**:
  \[ a_{\ell,a}^Y < 0 \]

- **BZ**: 
  \[ a_{\ell,a}^{BZ} \approx \left( \frac{m_\ell}{4\pi^2} \right) g_{a\gamma\gamma} y_\alpha \log \frac{\Lambda}{m_a} \]

- **LbL**: 
  \[ a_{\ell,a}^{LbL} \approx 3 \frac{\alpha}{\pi} \left( \frac{m_\ell g_{a\gamma\gamma}}{4\pi} \right)^2 \log^2 \frac{\Lambda}{m_a} > 0 \]

- **VP**: 
  \[ a_{\ell,a}^{VP} \approx \frac{\alpha}{\pi} \left( \frac{m_\ell g_{a\gamma\gamma}}{12\pi} \right)^2 \log \frac{\Lambda}{m_a} > 0 \]

- **For a scalar ALP**, change the signs of Y & LbL.

- **The sign of BZ depends on the couplings. We assume it’s > 0.**

- **VP is positive both for scalar & pseudoscalar, but negligible.**

---

Both pseudoscalar and scalar ALPs can solve $\Delta a_\mu$ for masses and couplings allowed by current exp. constraints.

They can be tested at present low-energy $e^+e^-$ colliders through dedicated $e^+e^- \rightarrow e^+e^- +$ ALP searches.
Both pseudoscalar and scalar ALPs can solve $\Delta a_\mu$ for masses and couplings allowed by current exp. constraints.

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Conclusions

Muon g-2: $\Delta a_\mu \sim 3.5 \, \sigma$. New upcoming experiment: QED & EW ready. Lots of progress in the hadr. sector, but not yet ready!

New proposal to measure the leading hadronic contribution to the muon g-2 via $\mu$-e elastic scattering at CERN.

Electron g-2: Does the discrepancy in $(g-2)_\mu$ also manifests in $(g-2)_e$? NP sensitivity limited by exp. uncertainties, but a strong exp. program is under way to improve both $\alpha$ & $a_e$.

$\Delta a_\mu$ due to mistakes in the hadronic $\sigma(s)$? Very unlikely!

$\Delta a_\mu$ solved by 2HDMs? Not by type I, II, and Y. Type X viable at $2\sigma$ for large $\tan \beta$, $10 \, \text{GeV} < M_A \ll 200 \,\text{GeV} \lesssim M_{H^\pm} \sim M_H \lesssim 400\,\text{GeV}$.

Light spin 0 scalars & pseudoscalars can solve $\Delta a_\mu$ for masses and couplings allowed by current experimental bounds. Dedicated searches can test them at low-energy $e^+e^-$ colliders.
The End