The muon g-2: status from a theorist's point of view

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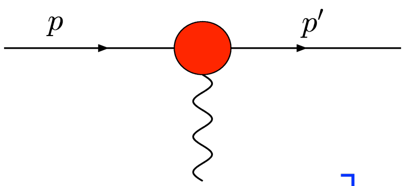
- **Kusch and Foley 1948:**

$$\mu_e^{\exp} = \frac{e\hbar}{2mc}\,(1.00119 \pm 0.00005)$$

- **Schwinger 1948 (triumph of QED!):**

$$\mu_e^{\rm th} = \frac{e\hbar}{2mc}\left(1+\frac{\alpha}{2\pi}\right) = \frac{e\hbar}{2mc} \times 1.00116$$

- **Keep studying the lepton–γ vertex:**

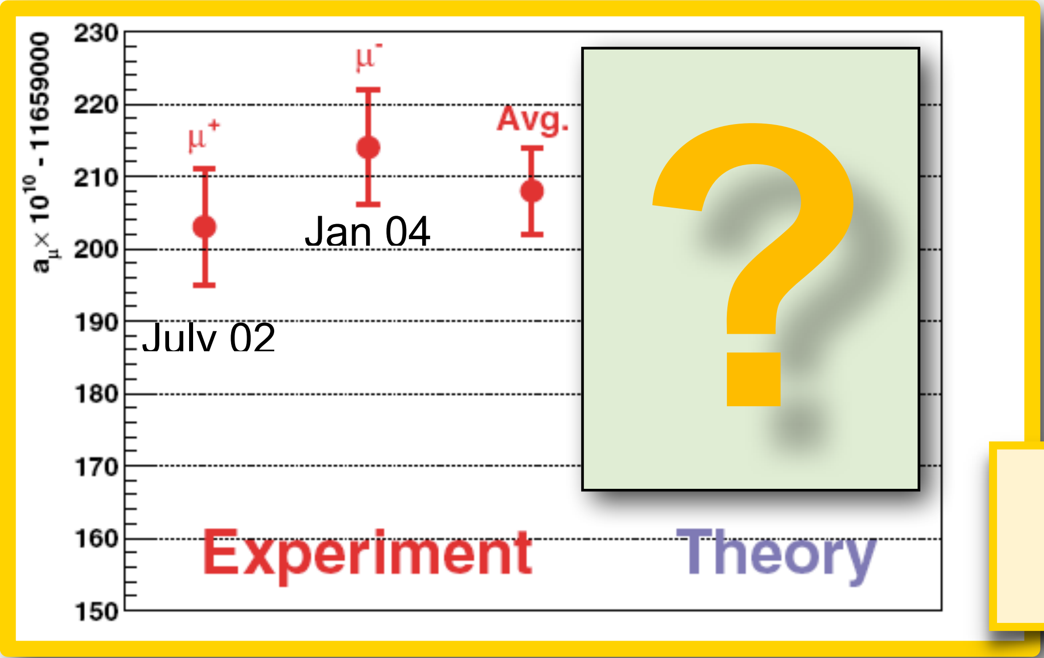


$$\bar{u}(p')\Gamma_\mu u(p) = \bar{u}(p')\left[\gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m}F_2(q^2) + \ldots\right]u(p)$$

$$\boxed{F_1(0) = 1 \qquad F_2(0) = a_l}$$

A pure “quantum correction” effect!

The muon g-2: experimental status



See Yl. Kim's and Y. Sato's talks

- **Today:** $a_\mu^{EXP} = (116592089 \pm 54_{stat} \pm 33_{sys})\mathrm{x}10^{-11}$ [0.5ppm].
- **Future:** new muon g-2 experiments at:
  - **Fermilab E989:** aiming at $\pm 16\mathrm{x}10^{-11}$, ie 0.14ppm.
    Beam expected next year. First result expected in 2018 with a precision comparable to that of BNL E821.
  - **J-PARC proposal:** aiming at 2019 Phase 1 start with 0.4ppm.
- **Are theorists ready for this (amazing) precision? Not yet**

The muon g-2: the QED contribution

a_μ^{QED} = $(1/2)(\alpha/\pi)$ Schwinger 1948

+ $0.765857426\ (16)\ (\alpha/\pi)^2$

Sommerfield; Petermann; Suura&Wichmann ’57; Elend ’66; MP ’04

+ $24.05050988\ (28)\ (\alpha/\pi)^3$

Remiddi, Laporta, Barbieri … ; Czarnecki, Skrzypek; MP ’04;
Friot, Greynat & de Rafael ’05, Mohr, Taylor & Newell 2012

+ $130.8773\ (61)\ (\alpha/\pi)^4$

Kinoshita & Lindquist ’81, … , Kinoshita & Nio ’04, ’05;
Aoyama, Hayakawa,Kinoshita & Nio, 2007, Kinoshita et al. 2012 & 2015;
Lee, Marquard, Smirnov^2, Steinhauser 2013 (electron loops, analytic),
Kurz, Liu, Marquard, Steinhauser 2013 (τ loops, analytic);
Steinhauser et al. 2015 & 2016 (all electron & τ loops, analytic).

+ $752.85\ (93)\ (\alpha/\pi)^5$ **COMPLETED!**

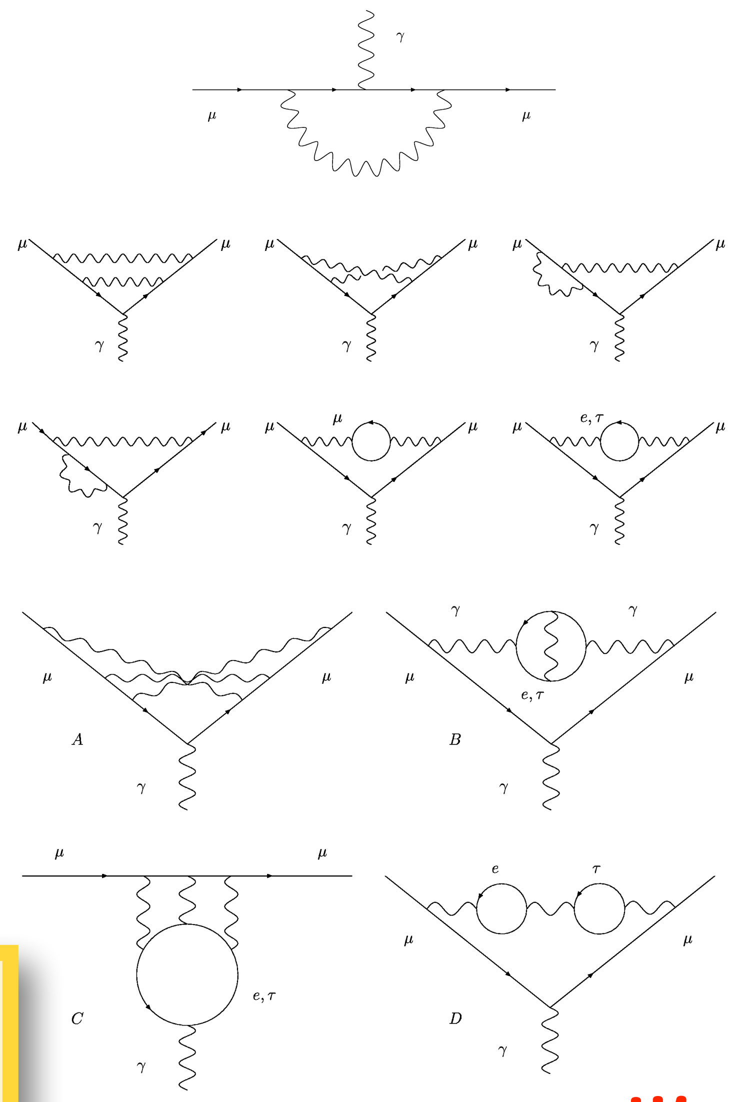
Kinoshita et al. ‘90, Yelkhovsky, Milstein, Starshenko, Laporta,…
Aoyama, Hayakawa, Kinoshita, Nio 2012 & 2015

Adding up, we get:

$a_\mu^{QED} = 116584718.941\ (21)(77) \times 10^{-11}$

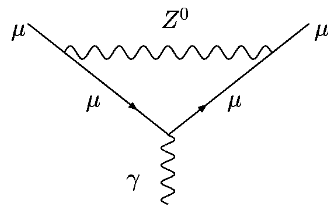
from coeffs, mainly from 4-loop unc   from $\delta\alpha$(Rb)

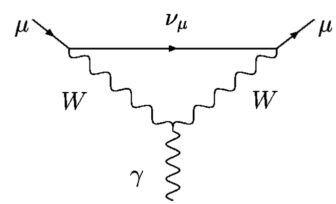
with $\alpha = 1/137.035999049(90)$ [0.66 ppb]

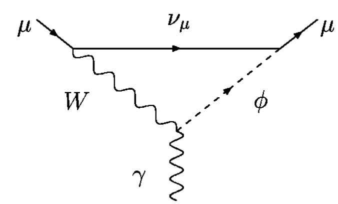


The muon g-2: the electroweak contribution

- **One-loop term:**







$$a_\mu^{\mathrm{EW}}(1\text{-loop}) = \frac{5 G_\mu m_\mu^2}{24\sqrt{2}\pi^2}\left[1 + \frac{1}{5}\left(1 - 4\sin^2\theta_W\right)^2 + O\left(\frac{m_\mu^2}{M_{Z,W,H}^2}\right)\right] \approx 195\times 10^{-11}$$

1972: Jackiv, Weinberg; Bars, Yoshimura; Altarelli, Cabibbo, Maiani; Bardeen, Gastmans, Lautrup; Fujikawa, Lee, Sanda;

Studenikin et al. '80s

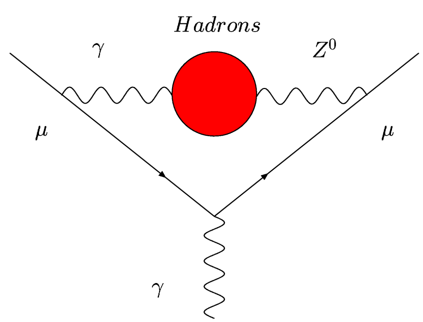
- **One-loop plus higher-order terms:**

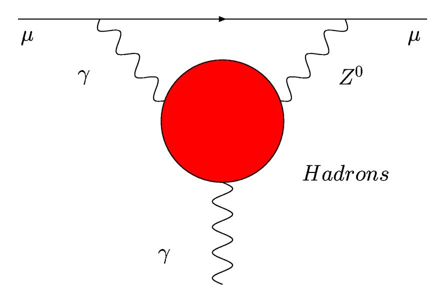
$$\boxed{a_\mu^{\mathrm{EW}} = 153.6\ (1) \times 10^{-11}}$$

with M_{Higgs} = 125.6 (1.5) GeV

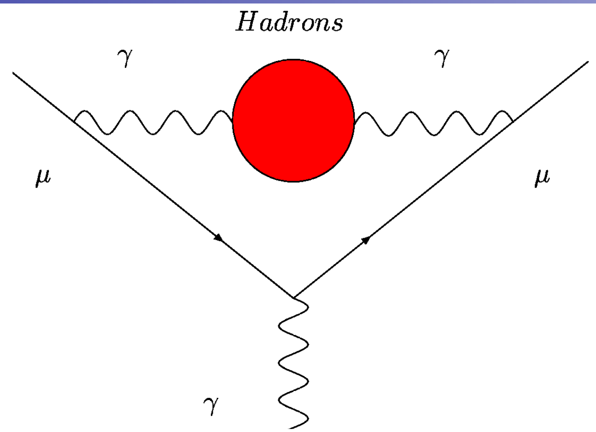
Hadronic loop uncertainties and 3-loop nonleading logs.

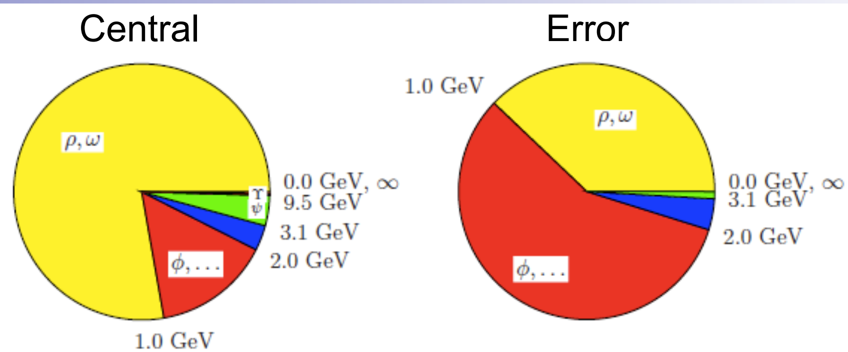
Kukhto et al. '92; Czarnecki, Krause, Marciano '95; Knecht, Peris, Perrottet, de Rafael '02; Czarnecki, Marciano and Vainshtein '02; Degrassi and Giudice '98; Heinemeyer, Stockinger, Weiglein '04; Gribouk and Czarnecki '05; Vainshtein '03; Gnendiger, Stockinger, Stockinger-Kim 2013.





The muon g-2: the hadronic LO contribution (HLO)





F. Jegerlehner and A. Nyffeler, Phys. Rept. 477 (2009) 1

$$K(s) = \int_0^1 dx \, \frac{x^2(1-x)}{x^2 + (1-x)(s/m^2)} \qquad a_\mu^{\mathrm{HLO}} = \frac{1}{4\pi^3} \int_{4m_\pi^2}^\infty ds\, K(s)\, \sigma^{(0)}(s) = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^\infty \frac{ds}{s} K(s) R(s)$$

$a_\mu^{\mathrm{HLO}} = 6870\ (42)_{\mathrm{tot}} \times 10^{-11}$ — F. Jegerlehner, arXiv:1511.04473 (includes BESIII 2π)

$= 6928\ (33)_{\mathrm{tot}} \times 10^{-11}$ — Davier et al, Tau2016, Beijing, Sep 2016, Preliminary

$= 6949\ (37)_{\mathrm{exp}}\ (21)_{\mathrm{rad}} \times 10^{-11}$ — Hagiwara et al, JPG 38 (2011) 085003

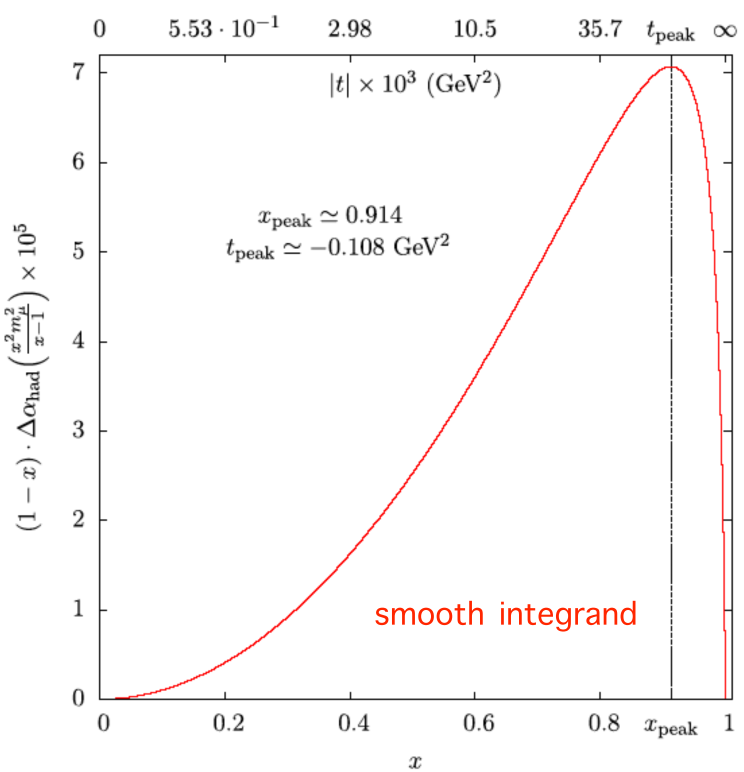
- **Radiative Corrections are crucial.** S. Actis et al, Eur. Phys. J. C66 (2010) 585
- **Lots of progress in lattice calculations.** T. Blum et al, PRL116 (2016) 232002

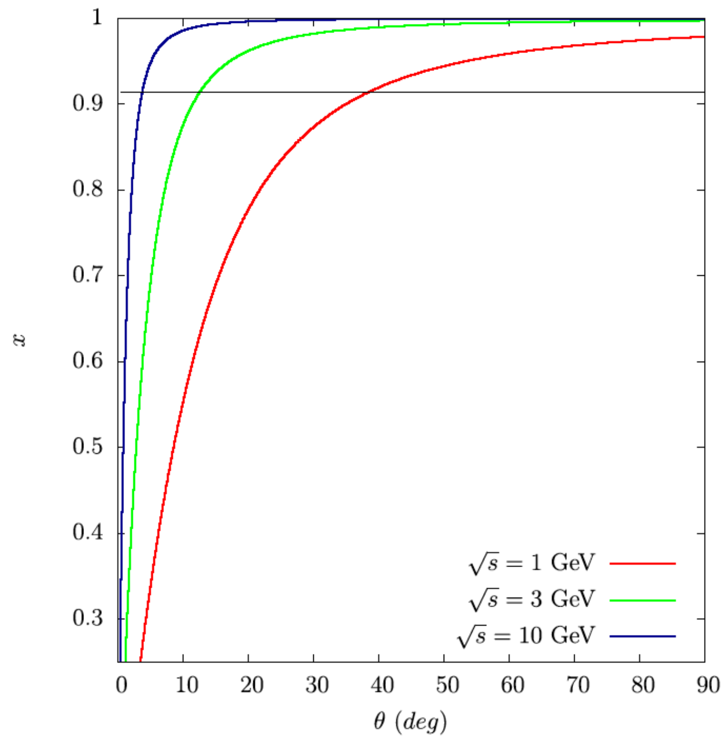
See D. Nomura's talk

- Alternatively, exchanging the x and s integrations in a_μ^{HLO}:

$$a_\mu^{\mathrm{HLO}} = \frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\,\Delta\alpha_{\mathrm{had}}[t(x)] \qquad t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$

which involves $\Delta\alpha_{had}(t)$, the hadr. contrib. to the running of α in the space-like region. It can be extracted from Bhabha scattering data!



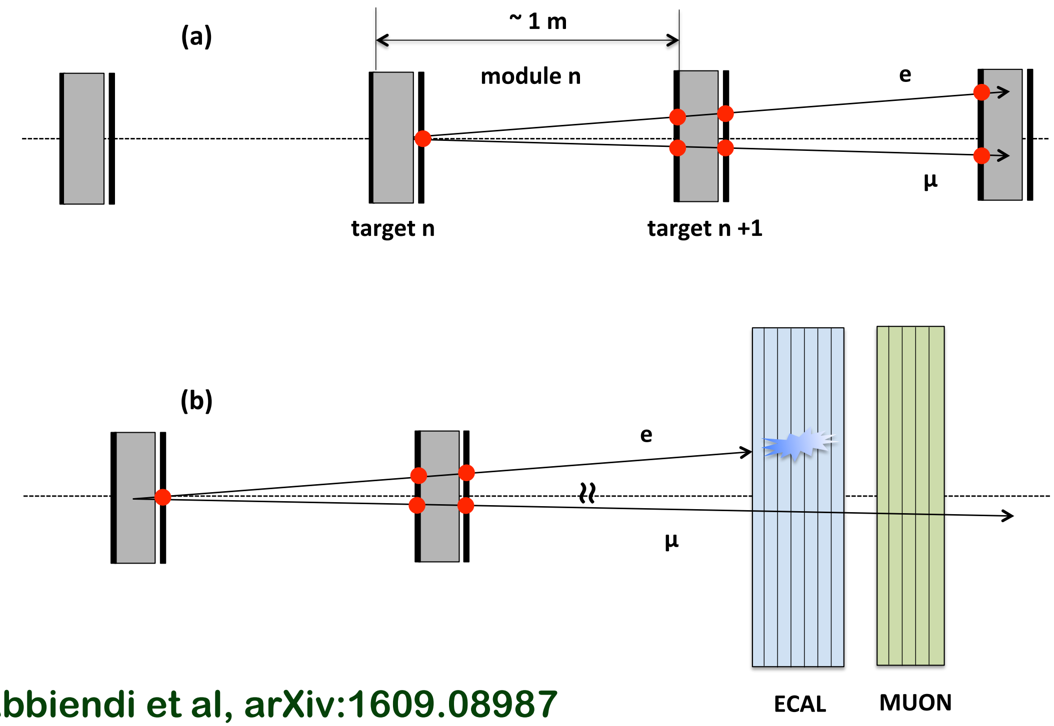


Carloni Calame, MP, Trentadue, Venanzoni, PLB 2015

New space-like proposal for HLO (2)

- $\Delta\alpha_{had}(t)$ can also be measured via the elastic scattering $\mu\, e \to \mu\, e$.

- Scattering a beam of muons of 150 GeV, available at CERN's North Area, on a fixed electron target, 0<x<0.93 (peak at 0.91).

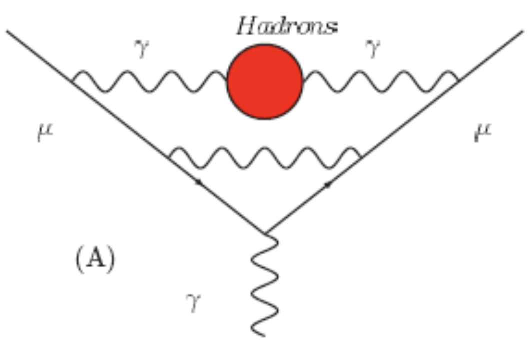


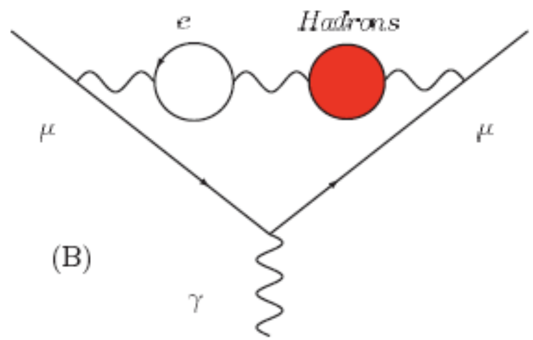
G. Abbiendi et al, arXiv:1609.08987

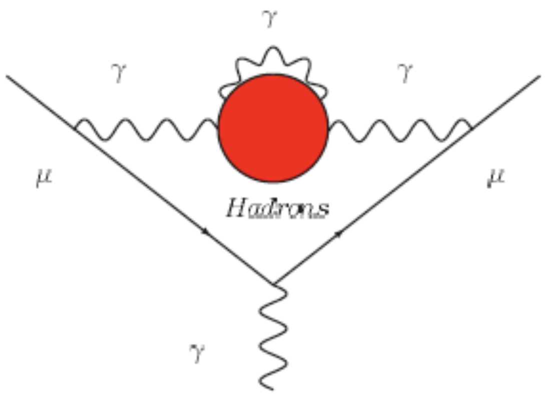
- With CERN's 150 GeV muon beam (1.3×10^7 μ/s average) a statistical uncertainty of ~0.3% (~20×10^{-11}) can be reached on a_μ^{HLO} with 2 years of data taking. 10ppm systematic accuracy needed at peak.

HNLO: Vacuum Polarization

Already included in a_μ^{HLO}





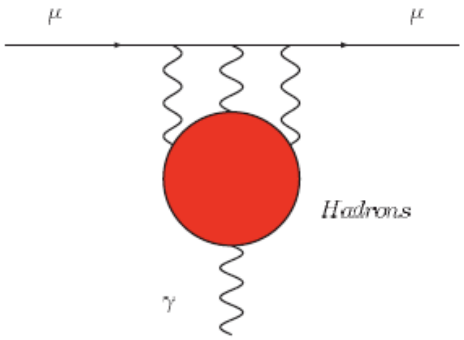


O(α^3) contributions of diagrams containing hadronic vacuum polarization insertions:

$$a_\mu^{HNLO}(vp) = -98\ (1) \times 10^{-11}$$

Krause '96, Alemany et al. '98, Hagiwara et al. 2011

HNLO: Light-by-light contribution



- Unlike the HLO term, the hadronic l-b-l term relies at present on theoretical approaches.

- This term had a troubled life! Latest values:

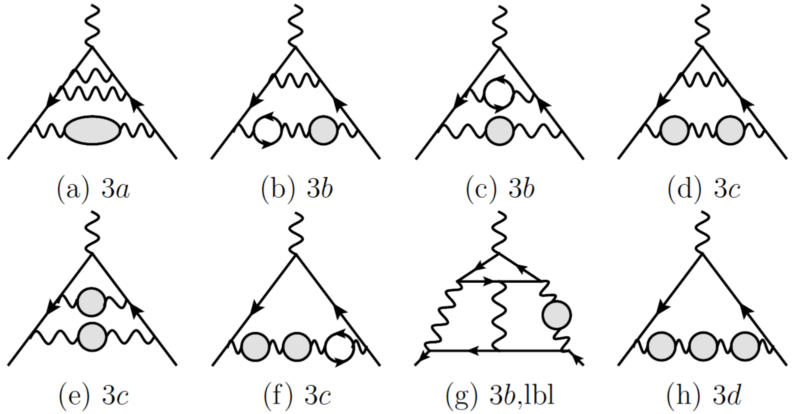
$a_\mu^{HNLO}(lbl) = +80\ (40) \times 10^{-11}$	Knecht & Nyffeler ’02
$a_\mu^{HNLO}(lbl) = +136\ (25) \times 10^{-11}$	Melnikov & Vainshtein ’03
$a_\mu^{HNLO}(lbl) = +105\ (26) \times 10^{-11}$	Prades, de Rafael, Vainshtein ’09
$a_\mu^{HNLO}(lbl) = +102\ (39) \times 10^{-11}$	Jegerlehner, arXiv:1511.04473

Results based also on Hayakawa, Kinoshita ’98 & ’02; Bijnens, Pallante, Prades ’96 & ’02

- Improvements expected in the π^0 transition form factor A. Nyffeler 1602.03398
- Dispersive approach proposed Colangelo et al, 2014 & 2015, Pauk & Vanderhaeghen 2014.
- Progress on the lattice: $+53.5(13.5)\times 10^{-11}$. Statistical error only, finite-volume and finite lattice-spacing errors being studied. Omitted subleading disconnected graphs still need to be computed.
Blum, Christ, Hayakawa, Izubuchi, Jin, Jung, Lehner, arXiv:1610.04603

See M. Hayakawa’s talk

- ## HNNLO: Vacuum Polarization



O(α^4) contributions of diagrams containing hadronic vacuum polarization insertions:

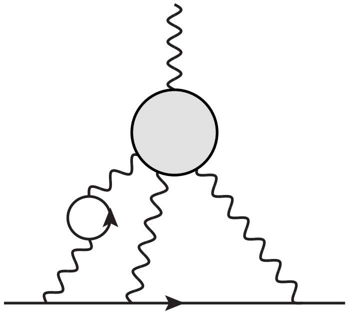
$$a_\mu^{\mathrm{HNNLO}}(\mathrm{vp}) = 12.4\ (1) \times 10^{-11}$$

Kurz, Liu, Marquard, Steinhauser 2014

- ## HNNLO: Light-by-light

$$a_\mu^{\mathrm{HNNLO}}(\mathrm{lbl}) = 3\ (2) \times 10^{-11}$$

Colangelo, Hoferichter, Nyffeler, MP, Stoffer 2014



Comparisons of the SM predictions with the measured g-2 value:

$a_\mu^{EXP} = 116592091\ (63) \times 10^{-11}$

E821 – Final Report: PRD73 (2006) 072 with latest value of $\lambda = \mu_\mu / \mu_p$ from CODATA'10

$a_\mu^{\mathrm{SM}} \times 10^{11}$	$\Delta a_\mu = a_\mu^{\mathrm{EXP}} - a_\mu^{\mathrm{SM}}$	σ
116 591 761 (57)	$330\ (85) \times 10^{-11}$	3.9 [1]
116 591 820 (51)	$271\ (81) \times 10^{-11}$	3.3 [2]
116 591 841 (58)	$250\ (86) \times 10^{-11}$	2.9 [3]

with the recent "conservative" hadronic light-by-light $a_\mu^{HNLO}(lbl) = 102\ (39) \times 10^{-11}$ of F. Jegerlehner arXiv:1511.04473, and the hadronic leading-order of:

[1] Jegerlehner, arXiv:1511.04473.
[2] Davier et al, Tau2016, Beijing, Sep 2016, Preliminary.
[3] Hagiwara et al, JPG38 (2011) 085003.

Brief digression: the electron g-2

- **The 2008 measurement of the electron g-2 is:**

$$a_e^{EXP} = 11596521807.3\ (2.8) \times 10^{-13}$$

Hanneke, Fogwell, Gabrielse
PRL100 (2008) 120801

- **Using α = 1/137.035 999 049 (90) from h/M measurement of ^{87}Rb (2011), the SM prediction for the electron g-2 is**

$$a_e^{SM} = 115\,965\,218\,16.5\ \underbrace{(0.2)}_{\delta C_4^{qed}}\ \underbrace{(0.2)}_{\delta C_5^{qed}}\ \underbrace{(0.2)}_{\delta a_e^{had}}\ \underbrace{(7.6)}_{\text{from } \delta\alpha} \times 10^{-13}$$

- **The EXP-SM difference is (note the negative sign):**

$$\Delta a_e = a_e^{EXP} - a_e^{SM} = -9.2\ (8.1) \times 10^{-13}$$

The SM is in very good agreement with experiment (1σ).

The electron g-2 sensitivity and NP tests

e

- The present sensitivity is $\delta\Delta a_e = 8.1 \times 10^{-13}$, ie ($10^{-13}$ units):

$$\underbrace{(0.2)_{\text{QED4}}, \quad (0.2)_{\text{QED5}}, \quad (0.2)_{\text{HAD}}}_{(0.4)_{\text{TH}}}, \quad (7.6)_{\delta\alpha}, \quad (2.8)_{\delta a_e^{\text{EXP}}}$$

← may drop to 0.2

- The $(g\text{-}2)_e$ exp. error may soon drop below 10^{-13} and work is in progress for a significant reduction of that induced by $\delta\alpha$.

→ sensitivity of 10^{-13} may be reached with ongoing exp. work

- In a broad class of BSM theories, contributions to a_l scale as

$$\frac{\Delta a_{\ell_i}}{\Delta a_{\ell_j}} = \left(\frac{m_{\ell_i}}{m_{\ell_j}}\right)^2$$

This Naive Scaling leads to:

$$\Delta a_e = \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right) 0.7 \times 10^{-13}; \qquad \Delta a_\tau = \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right) 0.8 \times 10^{-6}$$

The electron g-2 sensitivity and NP tests (2)

- The experimental sensitivity in Δa_e is not too far from what is needed to test if the discrepancy in $(g\text{-}2)_\mu$ also manifests itself in $(g\text{-}2)_e$ under the naive scaling hypothesis.

- NP scenarios exist which violate Naive Scaling. They can lead to larger effects in Δa_e and contributions to EDMs, LFV or lepton universality breaking observables.

- Example: In the MSSM with non-degenerate but aligned sleptons (vanishing flavor mixing angles), Δa_e can reach 10^{-12} (at the limit of the present exp sensitivity). For these values one typically has breaking effects of lepton universality at the few per mil level (within future exp reach).

Giudice, Paradisi, MP JHEP 2012

Back to the muon g-2

Δa_μ: could it be an error in the hadronic cross section?

- Can Δa_μ be due to hypothetical mistakes in the hadronic $\sigma(s)$?
- An upward shift of $\sigma(s)$ also induces an increase of $\Delta\alpha_{had}^{(5)}(M_Z)$.
- Consider:

$a_\mu^{HLO} \rightarrow$

$\Delta\alpha_{had}^{(5)} \rightarrow$

$$\boxed{\begin{aligned} a &= \int_{4m_\pi^2}^{s_u} ds\, f(s)\,\sigma(s), \qquad f(s) = \frac{K(s)}{4\pi^3},\ s_u < M_Z^2, \\ b &= \int_{4m_\pi^2}^{s_u} ds\, g(s)\,\sigma(s), \qquad g(s) = \frac{M_Z^2}{(M_Z^2 - s)(4\alpha\pi^2)}, \end{aligned}}$$

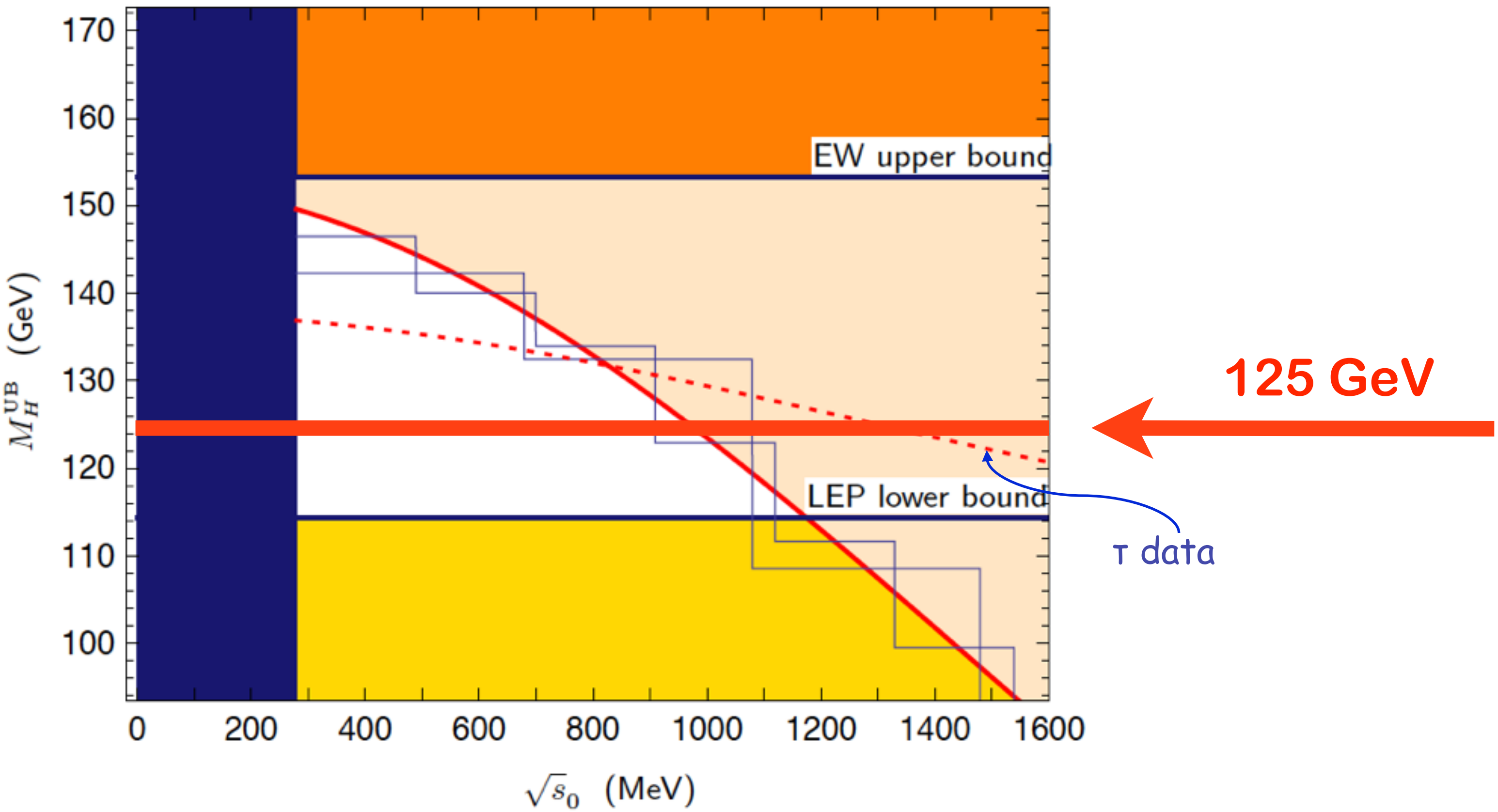
and the increase

$$\Delta\sigma(s) = \epsilon\sigma(s)$$

($\varepsilon>0$), in the range:

$$\sqrt{s} \in [\sqrt{s_0} - \delta/2, \sqrt{s_0} + \delta/2] \longrightarrow$$

- How much does the M_H upper bound from the EW fit change when we shift $\sigma(s)$ by $\Delta\sigma(s)$ [and thus $\Delta\alpha_{had}^{(5)}(M_Z)$] to accommodate Δa_μ ?



W.J. Marciano, A. Sirlin, MP, 2008 & 2010

The muon g-2: connection with the SM Higgs mass (2)

- Given the quoted exp. uncertainty of $\sigma(s)$, the possibility to explain the muon g-2 with these very large shifts $\Delta\sigma(s)$ appears to be very unlikely.

- Also, given a 125 GeV SM Higgs, these hypothetical shifts $\Delta\sigma(s)$ could only occur at very low energy (below ~ 1 GeV) where $\sigma(s)$ is precisely measured.

- Vice versa, assuming we now have a SM Higgs with M_H = 125 GeV, if we bridge the M_H discrepancy in the EW fit decreasing the low-energy hadronic cross section, the muon g-2 discrepancy increases.

W.J. Marciano, A. Sirlin, MP, 2008 & 2010

Limiting 2HDMs with Natural Flavor Conservation

- Study of the parameter regions explaining Δa_μ with constraints from EW precision observables, vacuum stability & perturbativity, B physics, lepton universality in Z & τ decays, and direct searches at colliders:

- Type I and Y models: cannot account for the present value of Δa_μ due to their lack of $\tan^2\beta$ enhancements.

- Type II models: $\tan^2\beta$ enhancements, but the bound on $BR(B_s \to \mu^+\mu^-)$ forbids a light A required to explain Δa_μ.

- Type X models: still viable at 2σ for large $\tan\beta$ and 10 GeV $< M_A \ll 200$ GeV $\lesssim M_{H\pm} \sim M_H \lesssim 400$GeV.

  - A. Broggio, E.J. Chun, MP, S. Vempati, JHEP 2014
  - L. Wang, X.-F. Han, JHEP 2015
  - T. Abe, R. Sato, K. Yagyu, JHEP 2015
  - E.J. Chun, Z. Kang, M. Takeuchi, Y.-L.S. Tsai, JHEP 2015
  - E.J. Chun, J. Kim, JHEP 2016

Limiting 2HDMs with Natural Flavor Conservation (2)

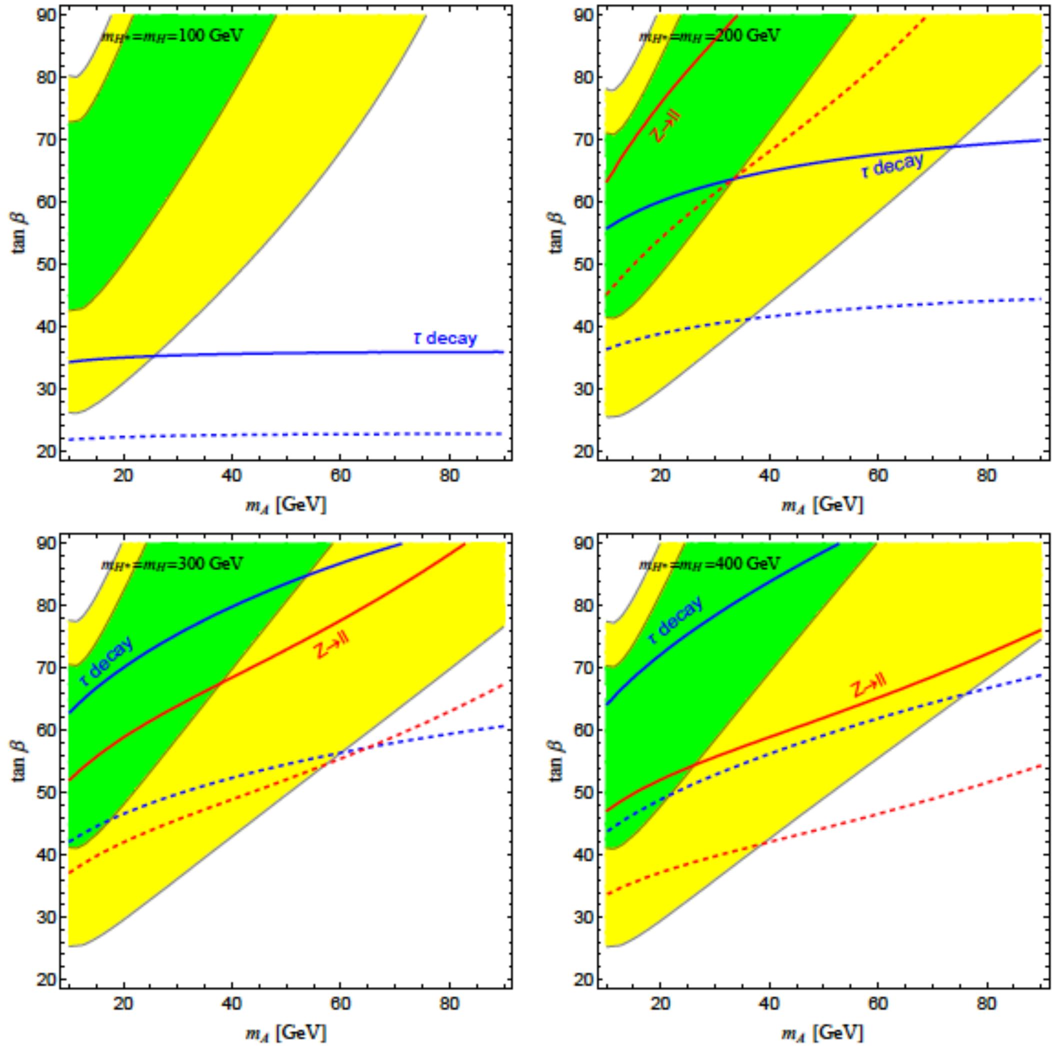


Figure 1. Allowed at 1σ and 2σ CL are the regions inside the green (inner) and yellow (outer) shaded areas by the muon $g-2$; below the red dashed (lower) and red solid (upper) lines by the lepton universality test in Z decays; below the blue dashed (lower) and blue solid (upper) lines by the lepton universality test with τ decays, respectively.

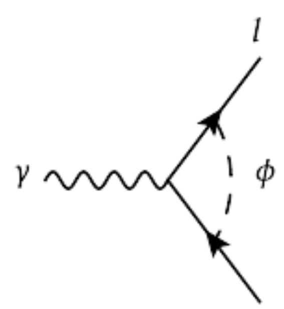
See Stöckinger-Kim's talk for full 2loop results

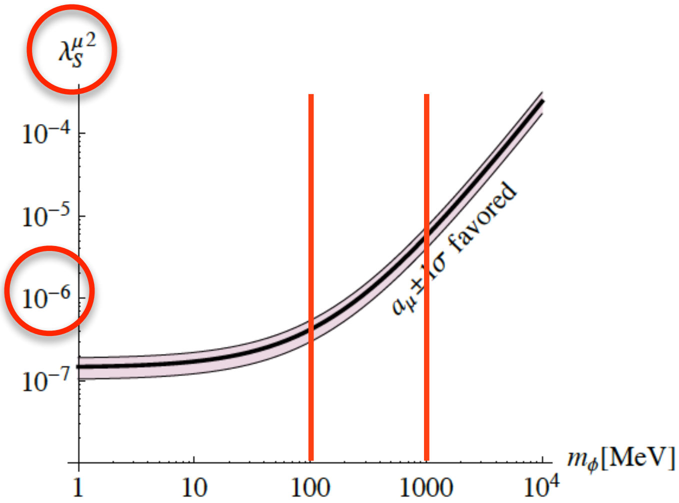
E.J. Chun, J. Kim, JHEP 2016

ALPs contributions to the muon g-2

- Light spin 0 scalars & pseudoscalars (axion-like-particles or ALPs), contribute to a_μ. We consider ALPs in the mass range ~[0.1–1] GeV, where experimental constraints are rather loose.

- A possible resolution of Δa_μ by 1-loop contributions from scalar particles with relatively large Yukawa couplings to muons, of $O(10^{-3})$, was analyzed by Chen, Davoudiasl, Marciano & Zhang, PRD 93, 035006 (2016):





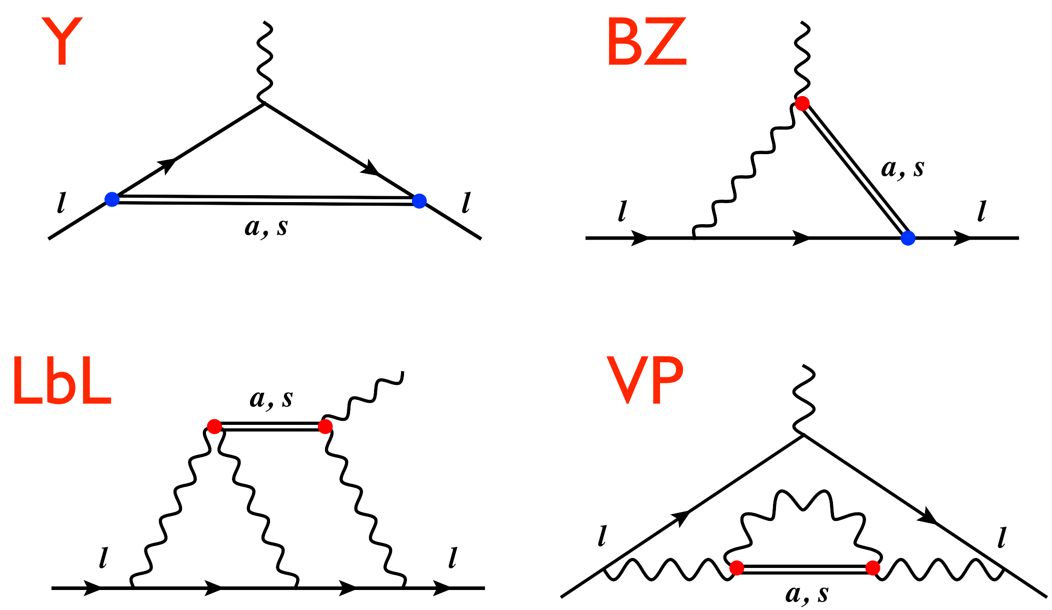
- For a pseudoscalar, the 1-loop contribution has the wrong sign (negative) to resolve the discrepancy on its own.

- **Consider ALP-γγ couplings as well as Yukawa couplings:**

$$\mathcal{L}_a = \frac{1}{4} g_{a\gamma\gamma}\, a\, F_{\mu\nu}\tilde{F}^{\mu\nu} + i\, y_{a\psi}\, a\, \bar{\psi}\gamma_5\psi\,,$$

$$\mathcal{L}_s = \frac{1}{4} g_{s\gamma\gamma}\, s\, F_{\mu\nu}F^{\mu\nu} + y_{s\psi}\, s\, \bar{\psi}\psi$$

- **New, potentially important, ALP contributions to a_μ:**



Marciano, Masiero, Paradisi, MP, arXiv:1607.01022

ALPs contributions to the muon g-2 (3)

μ

Y	$a_{\ell,a}^{\mathrm{Y}} < 0$	
BZ	$a_{\ell,a}^{\mathrm{BZ}} \simeq \left(\frac{m_\ell}{4\pi^2}\right) g_{a\gamma\gamma}\, y_{a\ell} \ln \frac{\Lambda}{m_a}$	leading log-enhanced terms
LbL	$a_{\ell,a}^{\mathrm{LbL}} \simeq 3\,\frac{\alpha}{\pi}\left(\frac{m_\ell\, g_{a\gamma\gamma}}{4\pi}\right)^2 \ln^2 \frac{\Lambda}{m_a} > 0$	Pseudoscalar
VP	$a_{\ell,a}^{\mathrm{VP}} \simeq \frac{\alpha}{\pi}\left(\frac{m_\ell\, g_{a\gamma\gamma}}{12\pi}\right)^2 \ln \frac{\Lambda}{m_a} > 0$	

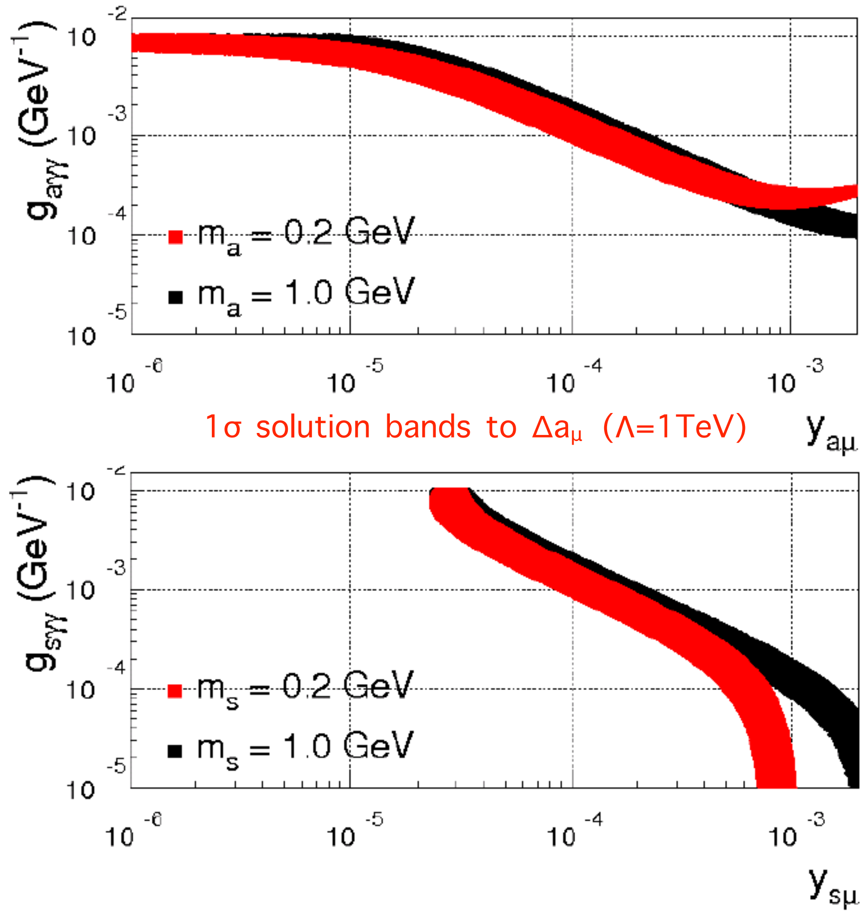
- **For a scalar ALP, change the signs of Y & LbL.**
- **The sign of BZ depends on the couplings. We assume it's > 0.**
- **VP is positive both for scalar & pseudoscalar, but negligible.**

Marciano, Masiero, Paradisi, MP, arXiv:1607.01022

ALPs contributions to the muon g-2 (4)

Pseudoscalar

Scalar

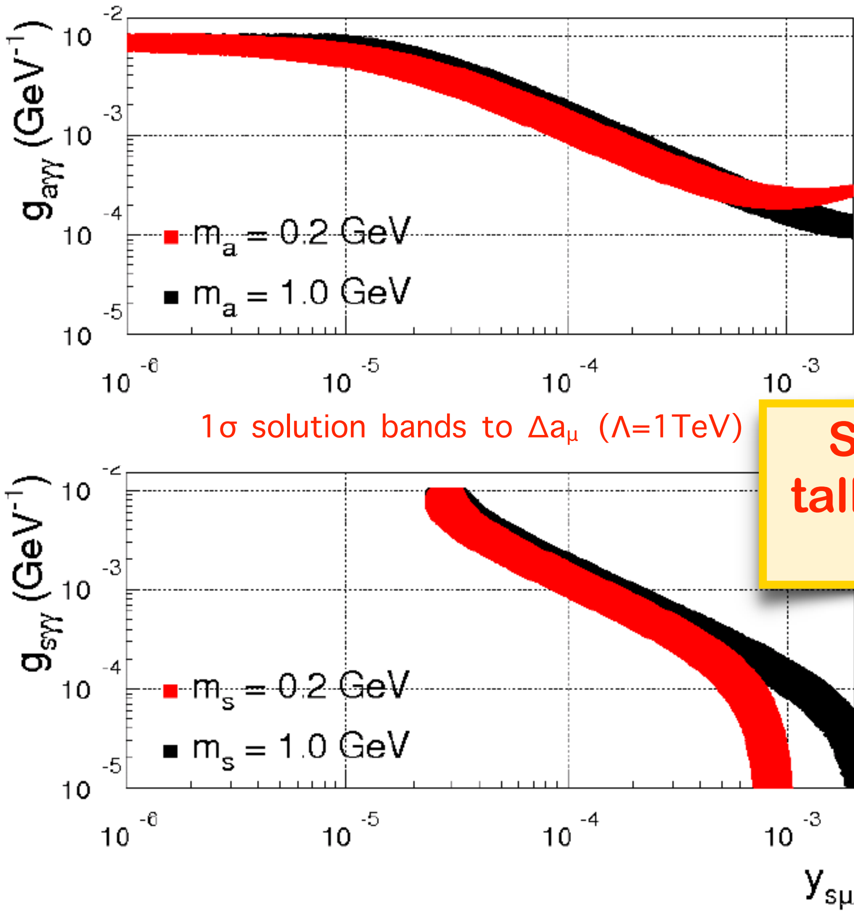


- **Both pseudoscalar and scalar ALPs can solve Δa_μ for masses and couplings allowed by current exp. constraints.**
- **They can be tested at present low-energy e^+e^- colliders through dedicated $e^+e^- \to e^+e^-$ + ALP searches.**

Marciano, Masiero, Paradisi, MP, arXiv:1607.01022

Pseudoscalar

Scalar



See Calibbi's talk for more NP scenarios

- Both pseudoscalar and scalar ALPs can solve Δa_μ for masses and couplings allowed by current exp. constraints.
- They can be tested at present low-energy e^+e^- colliders through dedicated $e^+e^- \rightarrow e^+e^-$ + ALP searches.

Marciano, Masiero, Paradisi, MP, arXiv:1607.01022

Conclusions

- Muon g-2: $\Delta a_\mu \sim 3.5\ \sigma$. New upcoming experiment: QED & EW ready. Lots of progress in the hadr. sector, but not yet ready!

- New proposal to measure the leading hadronic contribution to the muon g-2 via μ-e elastic scattering at CERN.

- Electron g-2: Does the discrepancy in $(g\text{-}2)_\mu$ also manifests in $(g\text{-}2)_e$? NP sensitivity limited by exp. uncertainties, but a strong exp. program is under way to improve both α & a_e.

- Δa_μ due to mistakes in the hadronic $\sigma(s)$? Very unlikely!

- Δa_μ solved by 2HDMs? Not by type I, II, and Y. Type X viable at 2σ for large $\tan\beta$, $10\ \text{GeV} < M_A \ll 200\ \text{GeV} \lesssim M_{H\pm} \sim M_H \lesssim 400\text{GeV}$.

- Light spin 0 scalars & pseudoscalars can solve Δa_μ for masses and couplings allowed by current experimental bounds. Dedicated searches can test them at low-energy e^+e^- colliders.

The End