Interacting Dark Matter and Dark Radiation

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based on P. Ko & Y.T., 1608.01083 (PLB), 1609.02307; Y.T., 1603.00165 (PLB)
Outline

• Introduction & Motivation
  • Dark Matter evidence
  • Hubble constant and structure growth
• Interacting Dark Matter & Dark Radiation
  • U(1) dark photon
  • Residual Yang-Mills Dark Matter
• Summary
Dark Matter Evidence

- Rotation Curves of Galaxies
- Gravitational Lensing
- Large Scale Structure
- CMB anisotropies, …

All confirmed evidence comes from gravitational interaction

CDM: negligible velocity, WIMP
WDM: keV sterile neutrino
HDM: active neutrino
Merger History of Dark Halo

- Standard picture
- DM halo grow hierarchically
- Small scale structures form first
- then merge into larger halo

Fig. 1.3. A schematic merger tree, illustrating the merger history of a dark matter halo. It shows, at three different epochs, the progenitor halos that at time $t_4$ have merged to form a single halo. The size of each circle represents the mass of the halo. Merger histories of dark matter halos play an important role in hierarchical theories of galaxy formation.
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The formation history of a dark matter halo can be described by a 'merger tree' that traces all its progenitors, as illustrated in Fig. 1.3. Such merger trees play an important role in modern galaxy formation theory. Note, however, that illustrations such as Fig. 1.3 can be misleading. In CDM models part of the growth of a massive halo is due to merging with a large number of much smaller halos, and to a good approximation, such mergers can be thought of as smooth accretion.

When two similar mass dark matter halos merge, violent relaxation rapidly transforms the orbital energy of the progenitors into the internal binding energy of the quasi-equilibrium remnant. Any hot gas associated with the progenitors is shock-heated during the merger and settles back into hydrostatic equilibrium in the new halo. If the progenitor halos contained central galaxies, the galaxies also merge as part of the violent relaxation process, producing a new central galaxy in the final system. Such a merger may be accompanied by strong star formation or AGN activity if the merging galaxies contained significant amounts of cold gas.

If two merging halos have very different mass, the dynamical processes are less violent. The smaller system orbits within the main halo for an extended period of time during which two processes compete to determine its eventual fate. Dynamical friction transfers energy from its orbit to the main halo, causing it to spiral inwards, while tidal effects remove mass from its outer regions and may eventually dissolve it completely (see Chapter 12). Dynamical friction is more effective for more massive satellites, but if the mass ratio of the initial halos is large enough, the smaller object (and any galaxy associated with it) can maintain its identity for a long time. This is the process for the build-up of clusters of galaxies: a cluster may be considered as a massive dark matter halo hosting a relatively massive galaxy near its center and many satellites that have not yet dissolved or merged with the central galaxy.
Weakly Interacting Massive Particle

- Mass around $\sim 100\text{GeV}$
- Coupling $\sim 0.5$
- Correct relic abundance $\Omega \sim 0.3$
- Thermal History
  - Equilibrium $XX<>ff$
  - Equilibrium $XX >ff$
  - Freeze-out
- Cold Dark Matter (CDM)
$\Lambda$CDM: successful on large scales
Why Interacting DM

• Theoretically interesting
  • Atomic DM, Mirror DM, Composite DM
  • Eventually, all DM is *interacting* in some way, the question is how strongly?
• Self-Interacting DM \( \frac{\sigma}{M_X} \sim \text{cm}^2/\text{g} \sim \text{barn/GeV} \)
• Possible new testable signatures
  • *CMB, LSS, BBN*
  • Other astrophysical effects,…
• Solution of CDM controversies
  • *Cusp-vs-Core, Too-big-to-fail, missing satellite,…*
  • \( H_0, \sigma_8 \) 2-3\( \sigma \), systematic uncertainty
• Hubble Constant $H_0$ defined as the present value of
  \[ H = \frac{1}{a} \frac{da}{dt} = \sqrt{\frac{\rho_r + \rho_m + \rho_\Lambda}{M_p}} \]

- Planck(2015) gives $67.8 \pm 0.9$ km s$^{-1}$Mpc$^{-1}$
- HST(2016) gives $73.24 \pm 1.74$ km s$^{-1}$Mpc$^{-1}$

![Diagram showing the tension in the Hubble constant with different scenarios and data points.

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Tension in $\sigma_8$?

- Variance of perturbation field for collapsed objects

$$\sigma^2(R) = \frac{1}{2\pi^2} \int W^2_R(k) P(k) k^2 dk,$$

- where the filter function

$$W_R(k) = \frac{3}{(kR)^3} [\sin(kR) - kR \cos(kR)],$$

$P(k)$ is matter power spectrum.

- $\sigma_8 \equiv \sigma(8h^{-1}\text{Mpc})$
Tension in $\sigma_8$?

**Planck2015, Sunyaev–Zeldovich cluster counts**

<table>
<thead>
<tr>
<th>Data</th>
<th>$\sigma_8 \left( \frac{\Omega_m}{0.31} \right)^{0.3}$</th>
<th>$\Omega_m$</th>
<th>$\sigma_8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>WtG + BAO + BBN</td>
<td>0.806 ± 0.032</td>
<td>0.34 ± 0.03</td>
<td>0.78 ± 0.03</td>
</tr>
<tr>
<td>CCCP + BAO + BBN [Baseline]</td>
<td>0.774 ± 0.034</td>
<td>0.33 ± 0.03</td>
<td>0.76 ± 0.03</td>
</tr>
<tr>
<td>CMBlens + BAO + BBN</td>
<td>0.723 ± 0.038</td>
<td>0.32 ± 0.03</td>
<td>0.71 ± 0.03</td>
</tr>
<tr>
<td>CCCP + $H_0$ + BBN</td>
<td>0.772 ± 0.034</td>
<td>0.31 ± 0.04</td>
<td>0.78 ± 0.04</td>
</tr>
</tbody>
</table>

**Planck2015, Primary CMB**

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>$\Omega_b h^2$</td>
<td>0.02222 ± 0.00023</td>
<td>0.02228 ± 0.00025</td>
<td>0.0240 ± 0.0013</td>
<td>0.02225 ± 0.00016</td>
</tr>
<tr>
<td>$\Omega_c h^2$</td>
<td>0.1197 ± 0.0022</td>
<td>0.1187 ± 0.0021</td>
<td>0.1150$^{+0.0048}_{-0.0055}$</td>
<td>0.1198 ± 0.0015</td>
</tr>
<tr>
<td>$100\theta_{MC}$</td>
<td>1.04085 ± 0.00047</td>
<td>1.04094 ± 0.00051</td>
<td>1.03988 ± 0.00094</td>
<td>1.04077 ± 0.00032</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.078 ± 0.019</td>
<td>0.053 ± 0.019</td>
<td>0.059$^{+0.022}_{-0.019}$</td>
<td>0.079 ± 0.017</td>
</tr>
<tr>
<td>$\ln(10^{10}A_s)$</td>
<td>3.089 ± 0.036</td>
<td>3.031 ± 0.041</td>
<td>3.066$^{+0.046}_{-0.041}$</td>
<td>3.094 ± 0.034</td>
</tr>
<tr>
<td>$n_s$</td>
<td>0.9655 ± 0.0062</td>
<td>0.965 ± 0.012</td>
<td>0.973 ± 0.016</td>
<td>0.9645 ± 0.0049</td>
</tr>
<tr>
<td>$H_0$</td>
<td>67.31 ± 0.96</td>
<td>67.73 ± 0.92</td>
<td>70.2 ± 3.0</td>
<td>67.27 ± 0.66</td>
</tr>
<tr>
<td>$\Omega_m$</td>
<td>0.315 ± 0.013</td>
<td>0.300 ± 0.012</td>
<td>0.286$^{+0.027}_{-0.038}$</td>
<td>0.3156 ± 0.0091</td>
</tr>
<tr>
<td>$\sigma_8$</td>
<td>0.829 ± 0.014</td>
<td>0.802 ± 0.018</td>
<td>0.796 ± 0.024</td>
<td>0.831 ± 0.013</td>
</tr>
<tr>
<td>$10^9A_s e^{-2\tau}$</td>
<td>1.880 ± 0.014</td>
<td>1.865 ± 0.019</td>
<td>1.907 ± 0.027</td>
<td>1.882 ± 0.012</td>
</tr>
</tbody>
</table>
A Light Dark Photon

- **Lagrangian**
  \[
  \mathcal{L} = -\frac{1}{4} V_{\mu\nu} V^{\mu\nu} + D_\mu \Phi^\dagger D^\mu \Phi + \bar{\chi} (i\partial \Phi - m_\chi) \chi + \bar{\psi} i\partial \psi \\
  - (y_\chi \Phi^\dagger \bar{\chi}^c \chi + y_\psi \Phi \bar{\psi} N + h.c.) - V(\Phi, H),
  \]

- **DM** $\chi$ (+1), dark radiation $\psi$ (+2), scalar (+2)

- **$U(1)$** symmetry (**unbroken**), massless dark photon $V_\mu$

- **$\Phi$** is responsible for the DM relic density
  \[
  \Omega h^2 \approx 0.1 \times \left( \frac{y_\chi}{0.7} \right)^{-4} \left( \frac{m_\chi}{\text{TeV}} \right)^2.
  \]

- **$\Phi$** can decay into $\psi$ and $N$. 

P.Ko, [YT,1608.01083(PLB)]
Dark Radiation $\delta N_{\text{eff}}$

- **Effective Number of Neutrinos, $N_{\text{eff}}$**
  \[
  \rho_R = \left[ 1 + N_{\text{eff}} \times \frac{7}{8} \left( \frac{4}{11} \right)^{4/3} \right] \rho_\gamma,
  \]
  \[\rho_\gamma \propto T_\gamma^4\]

- In SM cosmology, $N_{\text{eff}}=3.046$, Neutrinos decouple around MeV, and then stream freely.

- **Cosmological bounds**

  Joint CMB+BBN, 95% CL preferred ranges [Planck 2015, arXiv:1502.01589]

  \[
  N_{\text{eff}} = \begin{cases}
  3.11^{+0.59}_{-0.57} & \text{He+Planck TT+lowP}, \\
  3.14^{+0.44}_{-0.43} & \text{He+Planck TT+lowP+BAO}, \\
  2.99^{+0.39}_{-0.39} & \text{He+Planck TT,TE,EE+lowP},
  \end{cases}
  \]

  **Constraint on New Physics**

  \[
  \begin{align*}
  N_{\text{eff}} &< 3.7 \\
  m_{\nu, \text{sterile}}^{\text{eff}} &< 0.52 \text{ eV}
  \end{align*}
  \]

  95%, Planck TT+lowP+lensing+BAO.
Dark Radiation $\delta N_{\text{eff}}$

- Massless dark photon and fermion will contribute

$$
\delta N_{\text{eff}} = \left( \frac{8}{7} + 2 \right) \left[ \frac{g_{*s} (T_{\nu})}{g_{*s} (T_{\text{dec}})} \frac{g_{*s}^D (T_{\text{dec}})}{g_{*s}^D (T_D)} \right]^{\frac{4}{3}},
$$

where $T_{\nu}$ is neutrino’s temperature,

$g_{*s}$ counts the effective number of dof for entropy density in SM,

$g_{*s}^D$ denotes the effective number of dof being in kinetic equilibrium with $V_\mu$.

For instance, when $T_{\text{dec}} \gg m_t \simeq 173\text{GeV}$ for $|\lambda_{\Phi H}| \sim 10^{-6}$, we can estimate $\delta N_{\text{eff}}$ at the BBN epoch as

$$
\delta N_{\text{eff}} = \frac{22}{7} \left[ \frac{43/4}{427/4} \frac{11}{9/2} \right]^{\frac{4}{3}} \simeq 0.53,
$$

(1)

$\delta N_{\text{eff}}$=0.4~1 for relaxing tension in Hubble constant
Interacting Radiation

- All components are connected by Einstein's equation
  \[ R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \]

- First-order perturbation of Boltzmann equation
  - Anisotropy in CMB
  - Matter power spectrum for LSS

- (Self-)Interaction sometimes also matters
Interacting Radiation

- **free-streaming**

\[
\delta_v = -\frac{4}{3} \theta_v + 4\phi ,
\]

\[
\dot{\theta}_v = k^2 \left( \frac{1}{4} \delta_v - \sigma_v \right) + k^2 \psi ,
\]

\[
\dot{F}_{vl} = \frac{k}{2l + 1} \left[ lF_{v(l-1)} - (l + 1)F_{v(l+1)} \right] ,
\]

- **perfect fluid** \( \Gamma \gg \mathcal{H} \)

\[
\delta_v = -\frac{4}{3} \theta_v + 4\phi ,
\]

\[
\dot{\theta}_v = k^2 \left( \frac{1}{4} \delta_v - \sigma_v \right) + k^2 \psi ,
\]

\( \sigma_v = 0 \)

---

Fig. 4. Effects of \( N^e = 0.1 \) on CMB temperature anisotropy. Dashed (Long-dashed) line corresponds to the case with perfect fluid (free-streaming) radiation, as shown in the upper plot for the overall effect. In the lower plot, we show the relative difference from the standard \( \Lambda \) CDM, at order of \( O(1\%) \). See text for details.

3. \( \mu_1 = 0 \) and \( \sigma_1 = 0 \); \( \mu_1 \) is streaming freely after its kinetic decoupling just like neutrinos in standard cosmology.

The above discussion can be best illustrated with a schematic plot in Fig. 3, where \( \mathcal{H} \) and \( \Gamma \) are shown as functions of photon temperature \( T \) in log-scale. As \( T \) decreases towards the right-hand side, \( \mathcal{H} \) experiences first radiation dominant (RD) era as black solid line, then through the matter dominant (MD) epoch shown in dotted line, and finally dark energy (DE) dominant time with dashed line. Evolutions of \( \mu \)-term and \( \sigma \)-term in \( \mathcal{H} \), Eq. 3.2, are shown in blue median-dashed line and long-dashed line with arrow, respectively. Increasing or decreasing \( \mu \) and \( \sigma \) will shift the corresponding arrowed line upwards or downwards globally. All the above mentioned cases can be understood by shifting the arrowed lines.

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Neutrinos as perfect fluid excluded, Audren et al 1412.5948
Diffusion Damping

- Dark Matter scatters with radiation, which induces new contributions in the cosmological perturbation equations,

$$
\dot{\delta}_\chi = -\theta_\chi + 3\dot{\Phi},
$$

$$
\dot{\theta}_\chi = k^2 \Psi - \mathcal{H} \theta_\chi + S^{-1} \dot{\mu} (\theta_\psi - \theta_\chi),
$$

$$
\dot{\theta}_\psi = k^2 \Psi + k^2 \left( \frac{1}{4} \delta_\psi - \sigma_\psi \right) - \dot{\mu} (\theta_\psi - \theta_\chi),
$$

where dot means derivative over conformal time $d\tau \equiv dt/a$ ($a$ is the scale factor), $\theta_\psi$ and $\theta_\chi$ are velocity divergences of radiation $\psi$ and DM $\chi$’s, $k$ is the comoving wave number, $\Psi$ is the gravitational potential, $\delta_\psi$ and $\sigma_\psi$ are the density perturbation and the anisotropic stress potential of $\psi$, and $\mathcal{H} \equiv \dot{a}/a$ is the conformal Hubble parameter. Finally, the scattering rate and the density ratio are defined by $\dot{\mu} = an_\chi \langle \sigma_{\chi\psi} c \rangle$ and $S = 3\rho_\chi/4\rho_\psi$, respectively.
Scattering Cross Section

The averaged cross section $\langle \sigma_{\chi\psi} \rangle$ can be estimated from the squared matrix element for $\chi\psi \rightarrow \chi\psi$:

$$|\mathcal{M}|^2 \equiv \frac{1}{4} \sum_{\text{pol}} |\mathcal{M}|^2 = \frac{2g_X^4}{t^2} \left[ t^2 + 2st + 8m^2_\chi E^2_\psi \right], \quad (9)$$

where the Mandelstam variables are $t = 2E^2_\psi (\cos \theta - 1)$ and $s = m^2_\chi + 2m_\chi E_\psi$, where $\theta$ is the scattering angle, and $E_\psi$ is the energy of incoming $\psi$ in the rest frame of $\chi$. Integrated with a temperature-dependent Fermi-Dirac distribution for $E_\psi$, we find that $\langle \sigma_{\chi\psi} \rangle$ goes roughly as $g^4_X/(4\pi T_D^2)$.

- **In general, the cross section could have different temperature dependence, depending on the underlying particle models.**
Parametrize the cross section ratio

\[ u_0 \equiv \left[ \frac{\sigma_{\chi\psi}}{\sigma_{\text{Th}}} \right] \left[ \frac{100\text{GeV}}{m_{\chi}} \right] , \quad u_\beta(T) = u_0 \left( \frac{T}{T_0} \right)^\beta , \]

where \( \sigma_{\text{Th}} \) is the Thomson cross section, \( 0.67 \times 10^{-24}\text{cm}^{-2} \).

\[ \delta N_{\text{eff}} = 0.1 \]

\[ \beta = +0 \]
\[ \beta = +2 \]
\[ \beta = -2 \]
\[ \beta = -4 \]
Numerical Results

We take the central values of six parameters of $\Lambda$CDM from Planck,

\[ \Omega_b h^2 = 0.02227, \]
\[ \Omega_c h^2 = 0.1184, \]
\[ 100\theta_{MC} = 1.04106, \]
\[ \tau = 0.067, \]
\[ \ln (10^{10} A_s) = 3.064, \]
\[ n_s = 0.9681, \]

Baryon density today

CDM density today

100 $\times$ approximation to $r_*/D_A$

Thomson scattering optical depth

Log power of primordial curvature perturbations

Scalar Spectrum power-law index

which gives $\sigma_8 = 0.817$ in vanilla $\Lambda$CDM cosmology.

With the same input as above, now take

\[ \delta N_{\text{eff}} \simeq 0.53, \]
\[ m_\chi \simeq 100 \text{GeV} \text{ and } g^2_X \simeq 10^{-8} \]

in the interacting DM case, we have $\sigma_8 \simeq 0.744$. 

Modified Boltzmann code CLASS (Blas&Lesgourgues&Tram)
DM-DR scattering causes diffuse damping at relevant scales, resolving $\sigma_8$ problem.
Residual Non-Abelian DM&DR

• Consider $SU(N)$ Yang-Mills gauge fields and a Dark Higgs field $\Phi$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F^{a,\mu\nu} + (D_\mu \Phi)\dagger (D^\mu \Phi) - \lambda_\phi (|\Phi|^2 - v_\phi^2/2)^2,$$

• Take $SU(3)$ as an example,

$$A^a_{\mu} t^a = \frac{1}{2} \begin{pmatrix}
A^3_{\mu} + \frac{1}{\sqrt{3}} A^8_{\mu} & A^1_{\mu} - i A^2_{\mu} & A^4_{\mu} - i A^5_{\mu} \\
A^1_{\mu} + i A^2_{\mu} & -A^3_{\mu} + \frac{1}{\sqrt{3}} A^8_{\mu} & A^6_{\mu} - i A^7_{\mu} \\
A^4_{\mu} + i A^5_{\mu} & A^6_{\mu} + i A^7_{\mu} & -\frac{2}{\sqrt{3}} A^8_{\mu}
\end{pmatrix}.$$

• $SU(3) \rightarrow SU(2)$

$$\langle \Phi \rangle = \begin{pmatrix} 0 & 0 \\ v_\phi & \sqrt{2} \end{pmatrix}^T, \quad \Phi = \begin{pmatrix} 0 & 0 \\ v_\phi + \phi(x) & \sqrt{2} \end{pmatrix}^T,$$

The massive gauge bosons $A^4, \ldots, 8$ as dark matter obtain masses,

$$m_{A^4,5,6,7} = \frac{1}{2} g v_\phi, \quad m_{A^8} = \frac{1}{\sqrt{3}} g v_\phi,$$

and massless gauge bosons $A^{1,2,3}_\mu$. The physical scalar $\phi$ can couple to $A^{4,\ldots,8}_\mu$ at tree level and to $A^{1,2,3}_\mu$ at loop level.
\[ SU(N) \rightarrow SU(N - 1) \]

- **2N-1** massive gauge bosons: Dark Matter
- \((N-1)^2-1\) massless gauge bosons: Dark Radiation
- mass spectrum

\[
m_{A(N-1)^2}, \ldots, N^2-2 = \frac{1}{2} g v_\phi, \quad m_{A N^2-1} = \frac{\sqrt{N-1}}{\sqrt{2N}} g v_\phi,
\]

This can be proved by looking at the structure of \(f^{abc}\). Divide the generators \(t^a\) into two subset,

\[ a \subset [1, 2, \ldots, (N - 1)^2 - 1], \quad a \subset [(N - 1)^2, \ldots, N^2 - 1]. \]

Since \([t^a, t^b] = i f^{abc} t^c\) for the first subset forms closed \(SU(N - 1)\) algebra, we have \(f^{abc} = 0\) when only one of \(a, b\) and \(c\) is from the second subset. If one index is \(N^2 - 1\), then other two must be among the second subset to give no vanishing \(f^{abc}\), because \(t^{N^2-1}\) commutes with \(t^a\) from \(SU(N - 1)\).
Phenomenology

- Self-scattering processes
  \[ \delta N_{\text{eff}} = \frac{8}{7} \left[ (N - 1)^2 - 1 \right] \times 0.055, \]
  \[ g^2 \lesssim \frac{T_\gamma}{T_A} \left( \frac{m_A}{M_P} \right)^{1/2} \sim 10^{-7}, \]
  \[ \frac{m_A}{T_{\text{reh}}} \sim \ln \left[ \frac{\Omega_b M_P g_4^4}{\Omega_X m_p \eta} \right] \sim \mathcal{O}(30). \]

- Constraints
  - \(N<6\) if thermal
  - small coupling,
  - non-thermal production,
  - low reheating temperature

Schmaltz et al.(2015) EW charged DM

P.Ko&YT, 1609.02307

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Summary

- We discussed some cosmological effects with *interacting* Dark Matter and Dark Radiation.
- This scenario is motivated theoretically and also from observational tensions, $H_0$ and $\sigma_8$.
- We present two particle physics models:
  - A massless *dark photon* with *unbroken* $U(1)$ gauge symmetry.
  - *Residual* *non-Abelian* Dark Matter and Dark Radiation.
- It is possible to resolve tensions simultaneously.
Thanks for your attention.