Natural Low-Scale Inflation and the Relaxion

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JLE, Gherghetta, Nagata, Thomas arXiv:1602.04812

JLE, Gherghetta, Nagata, Peloso arXiv:1608.xxxxx

Outline

Naturalness Problem

- QFT: Radiative corrections generate mass
  - Quantum Gravity $\rightarrow$ all masses of order M_P
- Fermion masses composed left and right handed fields
  - $Q(\psi_L) \neq Q(\psi_R)$ forbids mass term
  - m_f order parameter of chiral symmetry breaking
  - All mass corrections proportional to m_f
  - $m_f / M_P \ll 1$ technically natural
- Scalar masses built of single field
  - Only shift symmetry can forbid mass term

    Shift symmetry $\rightarrow$ Nambu-Goldstone
  - Symmetry relating bosons and fermions

    Chiral symmetry protects boson as well
  - Dynamically selected mass

Dynamical Relaxation

- Two distinct contributions to Higgs mass
  - M^2: All radiative corrections plus tree-level piece
  - $g\phi$: scalar which couples to the Higgs
- $g\phi = M^2$ special dynamically
  - $g\phi > M^2 \rightarrow \langle H \rangle = 0$
  - $g\phi < M^2 \rightarrow \langle H \rangle \neq 0$
- Dynamical selection of Higgs mass
  - $\phi > M^2/g$ for IC to be natural
  - Shift symmetry breaking $\rightarrow$ slowly relax back to minimum
  - $\phi < M^2/g$, $\langle H \rangle \rightarrow$ growing $m_\phi^2(\langle H \rangle)$ (Axion mass)

$$\mathcal{L} \supset \left(-M^2 + g\phi\right)|H|^2 + \frac{1}{2}(g\phi)^2 + 2y\langle H \rangle\langle \bar{q}_L q_R \rangle \cos\left(\frac{\phi}{f}\right)$$

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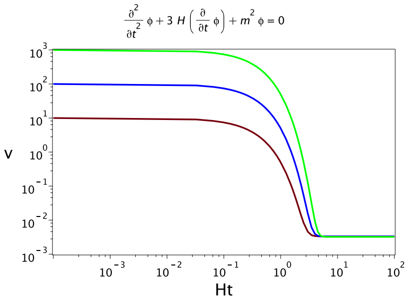
Insensitivity to Initial Conditions

- $\langle H \rangle$ sensitive to ϕ's approach to $m_H^2 = 0$
  - ϕ must stop with $(-M^2 + g\phi)/M^2 \ll 1$
- Hubble friction makes approach to $m_H^2 = 0$ similar
  - $\dot{\phi} \simeq v_{term}$ (Independent of IC)
  - Slow $v_{term} \to (-M^2 + g\phi)/M^2 \ll 1 \qquad (\phi \sim M^2/g)$



CP Violation

- Axion relaxion has $\theta_{QCD} \sim 1$

$$\frac{\partial V}{\partial \phi} = gM^2 + \frac{m_\pi^2(\langle H \rangle) f_\pi^2}{f} \sin\left(\frac{\phi}{f}\right) + .. \sim 0$$

  - Non-QCD axion can have $\theta \sim 1$
  - Radiative corrections require $f \sim v$

$$\mathcal{L} \supset \left(m_N + \lambda \frac{|H|^2}{M_L}\right) NN^c \to \left(m_N + \lambda \frac{|H|^2}{M_L}\right) \Lambda^3 \cos\left(\frac{\phi}{f}\right)$$

- θ still can have consequences

$$m_\phi \sim v \left(\frac{\Lambda^3}{M_L f^2}\right)^{1/2} \lesssim 4\pi v \left(\frac{v}{f}\right) \qquad \theta_{\phi h} \sim \frac{m_\phi^2}{m_h^2 - m_\phi^2} \frac{f}{v} \sin\theta$$

  - $\theta_{h\phi} \to$ CP violation in MSSM (EDM)
  - Two field models have $\theta \simeq 0$

Supersymmetric Axion Relaxion

- Shift/SUSY symmetry breaking in superpotential

$$\frac{1}{2}(g\phi)^2 \qquad \to \qquad W \supset \frac{m}{2}S^2 \qquad \& \qquad V = |F_S|^2 = \frac{1}{2}|m_S a|^2 \neq 0$$

Batell, Giudice, and McCullough

- SUSY breaking generates relaxion dependent Higgs soft masses

$$g\phi|H|^2 \qquad \to \qquad \frac{|F_S|^2}{M^{*2}}|H_{u,d}|^2 = \frac{m^2 a^2}{M^{*2}}|H_{u,d}|^2$$

- Supersymmetric Higgs mass sets natural realxion scale

$$-M^2|H|^2 \qquad \to \qquad W = \mu_0 H_u H_d$$

- Instanton potential from gauge kinetic function

$$\frac{\phi}{32\pi f_\phi} G^{a\mu\nu} G^a_{\mu\nu} \qquad \to \qquad \int d\theta^2 c_a \frac{S}{16\pi^2 f_\phi} \mathrm{Tr}(\mathcal{W}_a \mathcal{W}_a) + \mathrm{h.c.} \supset$$

Two-Field Supersymmetric Relaxion Model

- Explicit breaking of shift symmetry

$$W_{S,T} = \frac{m_S}{2} S^2 + \frac{m_T}{2} T^2$$

- $N\bar{N}$ confine giving Higgs dependent back reaction

$$W_N = \boxed{\left(m_N + i g_S S + i g_T T + \tfrac{\lambda}{M_L} H_u H_d \right)} N\bar{N}$$

- Two-field relaxion evolution potential

$$V_{\phi,\sigma}(\phi, \sigma, H_u H_d) = \frac{1}{2}|m_S|^2 \phi^2 + \frac{1}{2}|m_T|^2 \sigma^2 + \boxed{\mathcal{A}(\phi, \sigma, H_u H_d)} \Lambda_N^3 \cos\left(\frac{\phi}{f_\phi}\right)$$

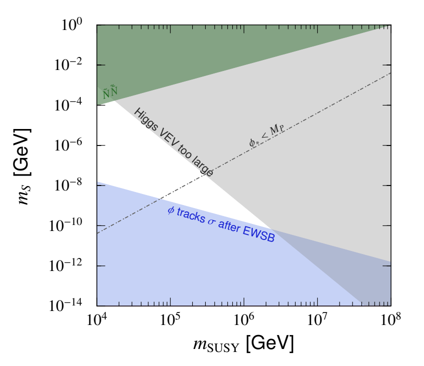
- Relaxion dependent Higgs mass

$$m^2_{H_{u,d}} = c_{u,d} \frac{m^2 a^2}{f^2} \qquad \mu = \mu_0 - c_\mu \frac{ma}{f} \qquad B_\mu = c_0 \mu \frac{ma}{f} + c_B \frac{m^2 a^2}{f^2}$$

$$\mathrm{Det}(M_H^2) = \left(m^2_{H_u} + |\mu|^2\right)\left(m^2_{H_d} + |\mu|^2\right) - |B_\mu|^2$$

Constraint Summary

- $\zeta = 10^{-8} \quad r_{TS} = 0.1 \quad r_\Lambda = 1 \quad r_{\rm SUSY} = 1.$



- Parameters

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$$r_\Lambda \equiv \frac{\Lambda_N}{f} \quad r_{\rm SUSY} \equiv \frac{m_{\rm SUSY}}{f}$$

$$M_L = m_{\rm SUSY} \ ,$$

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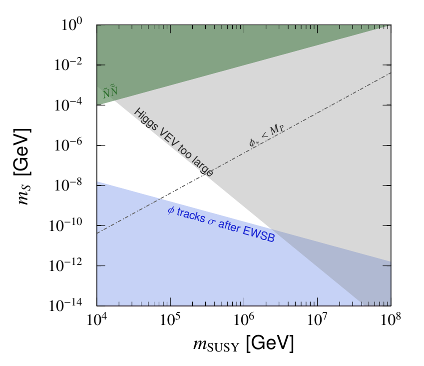


- Barrier too small to stop ϕ

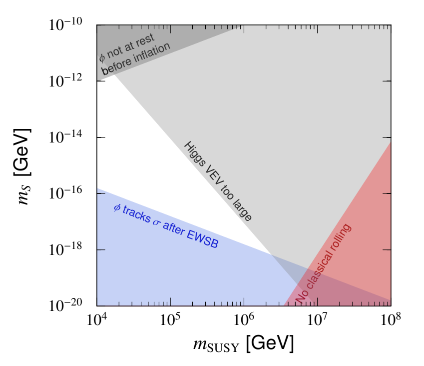
$$V' = \frac{m^2}{2}\phi + \frac{\lambda v^2 \sin(2\beta)}{4M_L}\frac{\Lambda_N^3}{f} \neq 0$$

- A changes too slowly with v

$$\frac{\partial A}{\partial t} \simeq \frac{\partial A}{\partial v}\frac{\partial v}{\partial t}$$

Constraint Summary

- $\zeta = 10^{-8} \quad r_{TS} = 0.1 \quad r_\Lambda = 10^{-4} \quad r_{\mathrm{SUSY}} = 10^{-4}.$



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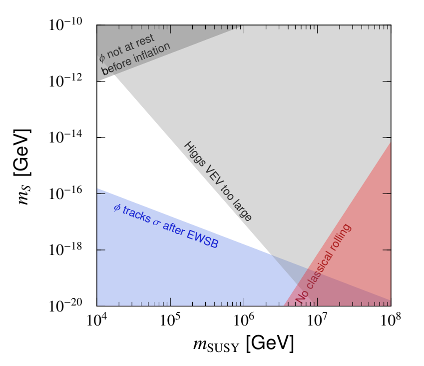
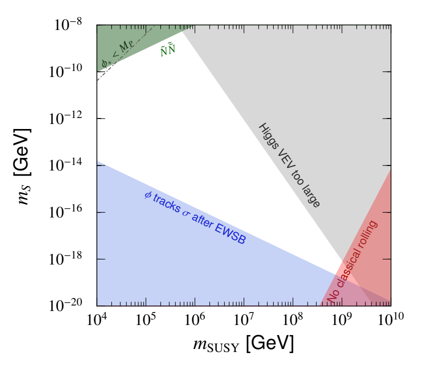


- Quantum spread of σ

$$\frac{\dot{\sigma}}{H_I} > H_I$$

Constraint Summary

- $\zeta = 10^{-14} \quad r_{TS} = 0.1 \quad r_\Lambda = 1 \quad r_{\rm SUSY} = 1.$



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$$M_L = m_{\rm SUSY} \; ,$$

Inflation for Relaxions

- Axion+SM
  - Classical rolling beats quantum spreading

$$H_I < (gM^2)^{1/3} \simeq \left(\frac{m_\pi^2 f_\pi^2}{f}\right)^{1/3} = 6 \times 10^{-5}\ \mathrm{GeV}\ \left(\frac{10^9\ \mathrm{GeV}}{f}\right)^{1/3}$$

- Axion +SUSY
  - Classical rolling beats quantum spreading

$$H_I < (m\mu_0 f)^{1/3} \simeq \Lambda_{QCD}\left(\frac{\Lambda_{QCD}}{f}\right)^{1/3} = 2 \times 10^{-4}\ \mathrm{GeV}\ \left(\frac{10^9\ \mathrm{GeV}}{f}\right)^{1/3}$$

- Two Field +SUSY
  - Classical rolling beats quantum spreading

$$H_I < (m m_{SUSY} f_\phi)^{1/3} \simeq v\left(\frac{v}{f_\phi}\right)^{1/3} = 33\ \mathrm{GeV}\ \left(\frac{10^9\ \mathrm{GeV}}{f}\right)^{1/3}$$

Low-Scale Natural Inflation

- Power Spectrum

$$(P_S)^{1/2} = \frac{1}{2\pi}\frac{H}{M_P}\frac{1}{\sqrt{2\epsilon}} \simeq 5 \times 10^{-5}$$

- Difficulties of low scale inflation

$$\epsilon = 8.6 \times 10^{-27}\left(\frac{H}{100\ \mathrm{GeV}}\right)^2$$

- Potential must be very flat

$$\epsilon = \frac{M_P^2}{2}\left(\frac{V_\phi}{V}\right)^2 \qquad \eta = M_P^2\frac{V_{\phi\phi}}{V}.$$

D-term Inflation

- Inflaton couples directly to $U(1)$ charged particles

$$\Delta W = \kappa T \phi_+ \phi_-$$

- D-term with FI term

$$V_D = \frac{g^2}{2}\left[|\phi_+| - |\phi_-| - \xi\right]^2$$

- Tree-level potential perfectly flat

$$|\kappa T| > \sqrt{g^2 \xi}$$

- $U(1)$ breaking ends inflation

$$\phi_+ = \xi \qquad\qquad F_i = 0$$

Coleman-Weinberg Potential

- One-loop potential for $|\kappa T| > \sqrt{g\xi}$

$$V = \frac{g^2\xi^2}{2}\left(1 + \frac{g^2}{8\pi^2}\ln\left[\frac{|\kappa T|^2}{Q^2}\right]\right)$$

- Slow-roll parameters

$$\epsilon = \frac{g^4}{32\pi^4}\left(\frac{M_P}{\phi}\right)^2 \qquad \eta = -\frac{g^2}{4\pi^2}\left(\frac{M_P}{\phi}\right)^2$$

The Generation of the CMB

- CMB determined in the final 30ish e-folds

$$N_{CMB} = \int H dt = \int \frac{d\phi}{\sqrt{2\epsilon}} \quad \rightarrow \quad \phi_*^2 = \frac{g^2}{2\pi^2} M_P^2 N_{CMB}$$

- The slow-roll parameters of the CMB

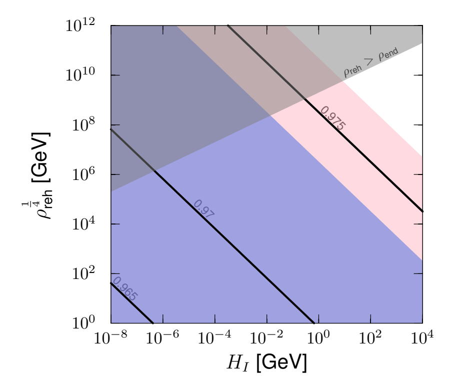
$$\boxed{\epsilon = \frac{g^2}{16\pi^2} \frac{1}{N_{CMB}}} \qquad \eta = -\frac{1}{2N_{CMB}}$$

- For small g, ϵ small enough

$$P_s^{1/2} = 5 \times 10^{-5} \left(\frac{N_{CMB}}{36}\right) \left(\frac{H_I}{100\ \mathrm{GeV}}\right) \boxed{\left(\frac{6.8 \times 10^{-12}}{g}\right)}$$

The Spectral Tilt

- Reduced N_{CMB} puts n_s in $1-\sigma$ lines



– Spectral tilt

$$n_s - 1 = 2\eta = -0.02\left(\frac{50}{N_{CMB}}\right)$$

– Number of e-folds

$$N_{CMB} = 35.8 + \frac{1}{3}\ln\left(\frac{H_I}{1\ \mathrm{GeV}}\right) + \frac{1}{3}\ln\left(\frac{\rho_{reh}^{1/4}}{1\ \mathrm{TeV}}\right)$$

Cosmic Strings

- $U(1)$ breaking after inflation leads to topological strings
  - Vacuum manifold S^1
  - Loops are non-contractible
  - String tension large $\rightarrow$ effect in CMB ($\mu = 2\pi\beta\langle\phi_+\rangle^2$)

$$\langle\phi_+\rangle^2 = \xi \sim \left(10^{16}\ \mathrm{GeV}\right)^2$$

- $U(1)$ must be broken before inflation
  - Inflation inflates away cosmic strings
- Dynamical D-terms generate FI-term
  - Additional U(1) breaking generates FI term
  - $\sqrt{\xi} \ll M_P$ more natural
  - Superpotential interactions tie two sectors together

Inflaton as the Second Field

- The inflaton can play the roll of the amplitudon

$$W_{S,T} = \frac{m_S}{2} S^2 + \frac{m_T}{2} \boxed{T}^2 \qquad W_{inf} = \kappa \boxed{T} \phi_+ \phi_-$$

- Inflation and relaxation happen at very different field values

$$\sigma_{CMB} = 2.3 \times 10^5 \text{ GeV} \left(\frac{H}{1 \text{ GeV}}\right) \qquad \sigma_* = 10^{16} \text{ GeV} \left(\frac{m_{SUSY}}{10^5 \text{ GeV}}\right)^2 \left(\frac{1}{r_{SUSY}}\right) \left(\frac{10^{-6} \text{ GeV}}{m_S}\right)$$

- D-term and relaxion have very different energies

$$\sqrt{g\xi} = 2.3 \times 10^9 \text{ GeV} \left(\frac{H}{1 \text{ GeV}}\right)^{1/2} \qquad \sqrt{m_T \sigma^*} = 10^5 \text{ GeV} \left(\frac{m_{SUSY}}{10^5 \text{ Gev}}\right) \left(\frac{1}{r_{SUSY}}\right)$$

Energy and Rolling Dominance

- ξ Dominates energy for small σ values
- $m_T^2\sigma$ dominates slope only for large σ

$$H = 1\ \mathrm{GeV} \qquad m_S = 10^{-6}\ \mathrm{GeV}$$

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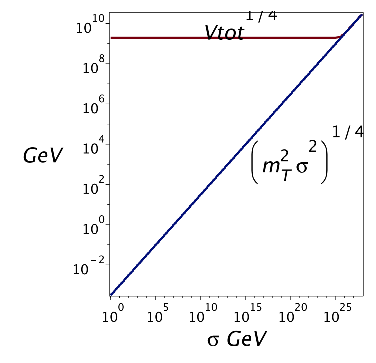
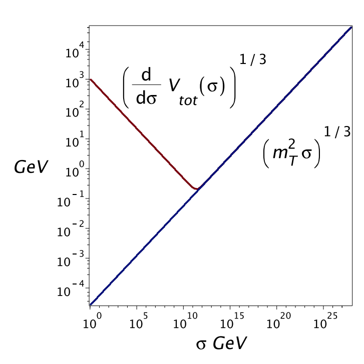


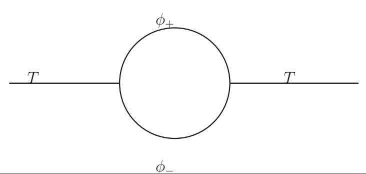


Shift Symmetry Breaking of Inflaton Sector

- Inflaton has relatively large shift symmetry breaking

$$W = \kappa T \phi_+ \phi_- \qquad \kappa \gtrsim 10^{-2}$$

- Loop correction transmit shift symmetry breaking to Kähler



$$K \supset \frac{|\kappa|^2}{16\pi^2} |T|^2$$

- SUGRA corrections to scalar potential generate mass for T

$$V \supset e^{\frac{K}{M_P}} |F_S|^2 + \ldots \qquad \rightarrow \qquad V \supset \frac{|\kappa|^2}{16\pi^2} \frac{|F_S|^2}{M_P^2} |T|^2$$

- Kähler corrections give lower bound on m_T

$$m_T \gtrsim \frac{\kappa}{4\pi} \frac{|F_S|}{M_P} = \frac{\kappa}{4\pi} \frac{m_{SUSY} f}{M_P}$$

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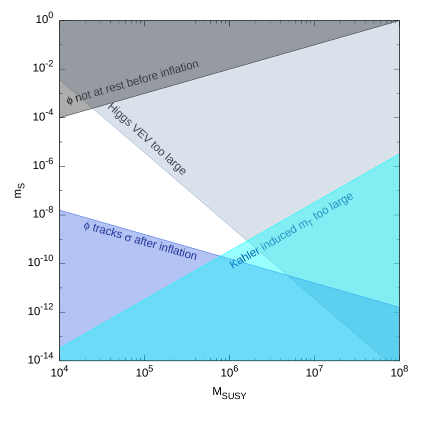

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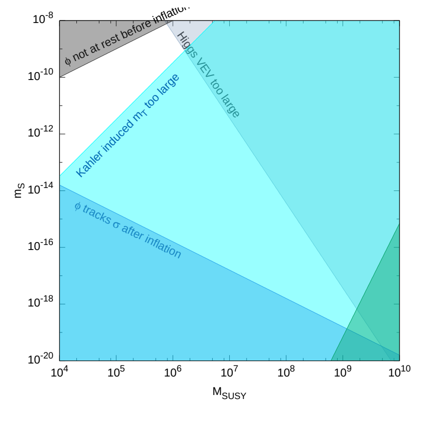

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Conclusions

- Dynamical relaxion gives new handle on naturalness
- Relaxion naturalness scale $\ll M_P$
  - Supersymmetry can UV complete naturalness
- Inflation scale very low $H \lesssim v$
- D-term inflation accommodates very low-scale inflation
  - $g \ll 1$ technically natural
- SUSY two field method
  - Inflaton can be identified as amplitudon

Reheating for $SU(2) \times U(1)$ Model

- Inflaton decays to $\tilde{\phi}_+\tilde{\phi}_-$ kinematically forbidden
- Inflaton heavy after inflation

$$|m_T|^2 = \xi \sim 10^{16}\ \mathrm{GeV}$$

- Couple right-handed neutrinos to inflaton to reheat to SM

$$\Delta W = \frac{\kappa_{ij}}{T} N_i N_j + \frac{M_{ij}}{2} N_i N_j$$

- $\Gamma_H \gg H \to$ instantaneous reheating
  - $\kappa_{ij} \ll 1$ reheat could be lower

$$\rho_{reh}^{1/4} = \frac{1}{2^{1/4}} \sqrt{\xi}$$

Inflationary Constraints: Continued

- Classical rolling dominates ϕ, σ evolution ($g_S\phi^* \sim -g_T\sigma*$)

$$\frac{|\dot{\sigma}|}{H_I} \ll H_I \rightarrow \quad H_I^3 \ll \frac{g_S|m_T|^2 f_\phi m_{SUSY}}{|g_T||m_S|}$$

- Stopping condition on relaxion

$$\frac{\lambda v^2 \Lambda^3}{f M_L} \sim m_S^2 \phi^* = m_S m_{SUSY} f \quad \rightarrow \quad m_S = \frac{\lambda v^2 \Lambda^2}{f^2 m_{SUSY}}$$

- ϕ must roll sufficiently long to be natural

$$\phi_* \sim 10^{17}\ \mathrm{GeV} \times \left(\frac{m_{\mathrm{SUSY}}}{10^5\ \mathrm{GeV}}\right)\left(\frac{f_\phi}{10^5\ \mathrm{GeV}}\right)\left(\frac{10^{-7}\ \mathrm{GeV}}{m_S}\right)$$

- Number e-folds constrained

$$N_e \simeq \frac{H_I \Delta\phi}{\left|\frac{d\phi}{dt}\right|} \simeq \frac{3H_I^2 \Delta\phi}{\left|\frac{\partial V}{\partial \phi}\right|} \gtrsim \frac{H_I^2}{|m_S|^2} = 10^{14} \times \left(\frac{H_I}{1\ \mathrm{GeV}}\right)^2 \left(\frac{10^{-7}\ \mathrm{GeV}}{|m_S|}\right)^2$$

SUGRA and the Relaxion

- Planck suppressed corrections to potential

$$V = \epsilon^{K/M_P^2}\left(D^i W D_i W - 3\frac{|W|^2}{M_P^2}\right)$$

- For $\sigma, \phi > M_P$ the $|W|^2$ term dominates
- Relaxion process allong $(m_S\phi^2)^2/M_P^2$ which does work
- Larger sequestered no-scale SUSY breaking

$$V = e^{K/M_P^2}\left(W^{*i}W_i + \frac{1}{M_P^2}(W^{*i}K_iW + \mathrm{h.c.}) + (K^iK_i - 3M_P^2)\frac{|W|^2}{M_P^4}\right).$$

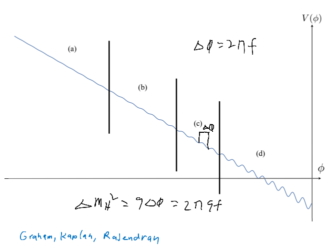
- Exact no-scale means $K^iK_i = 3M_P^2$.
  - Break no scale a little bit
  - Corrections to Flat SUSY small

Stopping of the relaxion

- Barrier beyond classical stopping point very small (See Picture)

$$\Lambda^4 \sim \langle \bar{q}_L q_R \rangle v \qquad v \sim |m_H| \sim gf \sim \frac{\Lambda^4}{M^2} \ll m_W^2$$

- Higgs masses roughly the same for adjacent minimum
- Small barrier $\rightarrow \phi$ spread over many periods all with $v \sim m_W$



- ϕ make jumps order H
  - Walks ϕ to stable minimum
  - Barrier large, jumps cease
- Inflation $\rightarrow$ patches $\mathcal{O}(1/H_0)$

UV Completion: Clockwork Axion

- $N+1$ U(1) with explicit breaking to a single U(1)

Choi,Hui Im; Kaplan,Ratazzi

$$W_{\rm UV} = \sum_{i=0}^{N} \lambda_i S_i \left(\phi_i \bar{\phi}_i - f_i^2\right) + \epsilon \sum_{i=0}^{N-1} \left(\bar{\phi}_i \phi_{i+1}^2 + \phi_i \bar{\phi}_{i+1}^2\right)$$

- Parameterize light superfield

$$\phi_i = f_i e^{\frac{\Pi_i}{f_i}} \, , \qquad \bar{\phi}_i = f_i e^{-\frac{\Pi_i}{f_i}}$$

- Effective superpotetial

$$W_{\rm eff} = 2\epsilon \sum_{i=0}^{N-1} f_i f_{i+1}^2 \cosh\left[\frac{\Pi_i}{f_i} - \frac{2\Pi_{i+1}}{f_{i+1}}\right]$$

- Massless mode, S, corresponding to remaining U(1)

$$S = c_N \sum_{i=0}^{N} \frac{f_i}{2^i f_0} \Pi_i \, ,$$

UV Completion: Continued

- Couple ϕ_0 to SU(N) charged field

$$W \supset \phi_0 \bar{Q} Q \quad \rightarrow \quad c_a \frac{S}{16\pi^2 f_\phi} \mathrm{Tr}(\mathcal{W}_a \mathcal{W}_a)$$

- When $N\bar{N}$ condense generates relaxion potential
- Soft masses from coupling ϕ_N to additional SU(N)

$$V_N \sim \widetilde{\Lambda}_N^4 \cos\left(\frac{\phi}{2^N f_0}\right) \quad \supset \quad \tilde{\Lambda}^4 \frac{\phi^2}{2^{N+1} f_0}$$

- g's generated from coupling in Kähler

$$i\frac{\kappa}{\widetilde{M}_N^2} \int d^4\theta N\bar{N}\Xi^*\bar{\Xi}^* + \mathrm{h.c.} \simeq i\frac{\kappa}{\widetilde{M}_N^2} \int d^2\theta\, \widetilde{\Lambda}_N^3 e^{\frac{\Pi_N}{f_N}} N\bar{N} + \mathrm{h.c.} \simeq \int d^2\theta \frac{i\kappa\widetilde{\Lambda}_N^3}{f_\phi 2^N \widetilde{M}_N^2} SN\bar{N} + \mathrm{h.c.}$$