Very high multiplicity Higgs production at \(~100\) TeV

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• The model: the Weak sector of the SM: SU(2) + Higgs

• Investigate scattering processes at ~100 TeV CoM energies $E$

• Concentrate on $n \sim 100$s of Higgses and W & Z's produced in the final state. $n$ times $\lambda$ >> 1 or $n$ times $\alpha_{\text{weak}}$ >> 1.

• Two distinct classes of high-$E$ processes with such final states are of interest:
  
  • 1) Non-perturbative (B+L)-violating processes (sphalerons and instantons $\Rightarrow$ tunneling)

  • 2) Ordinary perturbative high-$n$ processes (expansion around standard perturbative vacuum)

• in (1): Sphaleron mass is a new scale in the SM at ~10 TeV so that at > 30 TeV a possibility of new non-perturbative dynamics in the SM

• in (2) perturbative high-$E$ behaviour presents an easier problem to tackle

• Our trusted weakly coupled perturbation theory breaks down: Amplitudes $\sim n!$
1. Baryon + Lepton number violation

- Electroweak vacuum has a nontrivial structure (!) [SU(2)-sector]
- The saddle-point at the top of the barrier is the sphaleron. New EW scale $\sim 10$ TeV
- Transitions between the vacua change $B+L$ (result of the ABJ anomaly):
  $\Delta (B+L) = 3 \times (1+1)$; $\Delta (B-L) = 0$
- Instantons are tunnelling solutions between the vacua. They mediate $B+L$ violation
- $3 \times (1 \text{ lepton} + 3 \text{ quarks}) = 12$ fermions
  12 left-handed fermion doublets are involved
- There are EW processes which are not described by perturbation theory!

$$ q + q \rightarrow 7\bar{q} + 3\bar{l} + n_WW + n_ZZ + n_HH $$
Instanton approach

- All instanton contributions come with an exponential suppression due to the instanton action:

\[ \mathcal{A}^{\text{inst}} \propto e^{-S^{\text{inst}}} = e^{-2\pi/\alpha_w - \pi^2 \rho^2 v^2}, \quad \sigma^{\text{inst}} \propto e^{-4\pi/\alpha_w} \simeq 5 \times 10^{-162} \]

- This is precisely the expected semiclassical price to pay for a quantum mechanical tunnelling process.

- For the B+L violating process

\[ q + q \rightarrow 7\bar{q} + 3\bar{l} + n_W W + n_Z Z + n_h H \]

- at leading order, the instanton acts as a point-like vertex with a large number \( n \) of external legs => \( n! \) factors in the amplitude.

- As the number of W's, Z's and H's produced in the final state at sphaleron-like energies is allowed to be large, \( \sim 1/\alpha \), the instanton cross-section receives exponential enhancement with energy

Ringwald 1990 => McLerran, Vainshtein, Voloshin 1990 => ….
• Cross-section is obtained by squaring the instanton $|Amplitude|$.  

• Final states are instrumental in combatting the exponential suppression.  

• Now also the interactions between the final states (and the improvement on the point-like I-vertex) are taken into account.  

• Use the Optical Theorem to compute $\text{Im}$ part of the FES amplitude in around the Instanton-Antiinstanton configuration.  

• Higher and higher energies correspond to shorter and shorter I-Ibar separations $R$. At $R=0$ they annihilate to perturbative vacuum.  

• The suppression of the cross-section is gradually reduced with energy.
Instanton-Antiinstanton valley [soft-soft interactions]

- Instanton — anti-instanton valley configuration has Q=0; it interpolates between infinitely separated instanton—anti-instanton and the perturbative vacuum at z=0

\[ G_{4\text{Eucl}} \sim \int d^4 R \, d\rho_I \, d\rho_{\bar{I}} \ldots \exp \left[ i(p_1 + p_2) \cdot R - S_{I\bar{I}}(z) - \pi^2 v^2 (\rho_I^2 + \rho_{\bar{I}}^2) \right] \]

- Exponential suppression is gradually reduced with energy <= valley configuration
- no radiative corrections from hard initial states included in this approximation
Instanton-Antiinstanton optimistic estimate

\[ \hat{\sigma}_{\text{inst}}^{qq} \approx \frac{1}{m_W^2} \left( \frac{2\pi}{\alpha_W} \right)^{7/2} \times \exp \left[ -\frac{4\pi}{\alpha_W} F_{\text{hg}} \left( \frac{\sqrt{s}}{4\pi m_W/\alpha_W} \right) \right] \]

\[ \simeq (5.28 \times 10^{15} \text{ fb}) \times \exp \left[ -\frac{4\pi}{\alpha_W} F_{\text{hg}} \left( \frac{\sqrt{s}}{4\pi m_W/\alpha_W} \right) \right] \]

First few terms in the expansion:

\[ F_W(\epsilon) = 1 - \frac{3^{4/3}}{2} \epsilon^{4/3} + \frac{3}{2} \epsilon^2 + O(\epsilon^{8/3}) + \ldots \]

\[ \epsilon = \sqrt{s}/(4\pi m_W/\alpha_W) \simeq \sqrt{s}/(30 \text{ TeV}) \]
Pessimistic view:

The sphaleron is a semiclassical configuration with

\[
\text{Size}_{\text{sph}} \sim m_W^{-1}, \quad E_{\text{sph}} = \text{few} \times m_W/\alpha_W \approx 10 \text{ TeV}.
\]

It is ‘made out’ of \( \sim 1/\alpha_W \) particles (i.e. it decays into \( \sim 1/\alpha_W \) W’s, Z’s, H’s).

\[
2_{\text{initial hard partons}} \rightarrow \text{Sphaleron} \rightarrow (\sim 1/\alpha_W)_{\text{soft final quanta}}
\]

The sphaleron production out of 2 hard partons is unlikely.

Assumptions:
(1) the intermediate state had to be the sphaleron;
(2) the initial state was a 2-particle state;
(3) that one cannot create \( (\sim 1/\alpha_W)_{\text{soft final quanta}} \) from \( 2_{\text{initial hard partons}} \).
The BLRRT approach (from 1/alpha to 2 initial quanta)

Construct an auxiliary solution with the initial data chosen that:

1. the initial state has $N = \tilde{N}/\alpha_W$ particles with $\tilde{N}$ fixed and $\alpha_W \to 0$
2. the energy also scales as $E = \tilde{E}/\alpha_W$
3. for simplicity also assume spherical symmetry.

The probability of tunnelling from such multiparticle state is computed semi-classically:

$$\sigma \sim \exp\left(-\frac{4\pi}{\alpha_W} F_{\tilde{N}}(\tilde{E})\right)$$

For fixed $\tilde{N}$ and $E \sim E_{sph}$ the rate will be unsuppressed. But this is not the 2-particle in-state.

**Conjecture** that the holy grail function relevant for the 2-particle initial state is obtained by taking the $\tilde{N} \to 0$ limit of the overall rate,

$$\lim_{\tilde{N} \to 0} F_{\tilde{N}}(\tilde{E}) = F_0(\tilde{E}) \sim F_{hg}(\tilde{E})$$

The suppression will arise from this limit (not from the lack of Energy!)

Bezrukov, Levkov, Rebbi, Rubakov & Tinyakov 2003
Electroweak sector of the SM is always seen as perturbative. If these instanton processes can be detected —> a truly remarkable breakthrough in realising & understanding non-perturbative EW dynamics

Numbers of W’s, Z’s and H’s produced in the final state at 30-100 TeV energies is allowed to be large, \(~1/\alpha\) => a technical consequence of this fact is that the instanton crossection receives an **exponential enhancement with energy**

The B+L processes are accompanied by \(~50\) EW vector & H bosons; charged Lepton number can also be measured —> unique experimental signature of the final state

The **rate** of the B+L processes is still not known theoretically. There are optimistic phenomenological models with \(~pb\) or \(~fb\) crossections, and there are pessimistic models with unobservable rates even at infinite energy.

Very hard theoretical problem, new computational methods are needed.

Since the final state is essentially backgroundless, the observability of the rate can be always settled experimentally.

B+L processes provide **physics opportunities** which are completely unique to the **very high energy pp machine** (100 TeV FCC pp).
Perturbative large-n amplitudes

• $1^* \rightarrow n$ on mass threshold at tree level: Recursion relations & classical solutions general technique - Brown 1992

Results in factorial growth of amplitudes in:

• (a) unbroken phi$^4$ theory

• (b) scalar theory with the VEV

• (c) Gauge-Higgs theory (spontaneously broken gauge theory)

• Perturbative growth generalises to realistic $2 \rightarrow n$ processes
Tree-level n-point Amplitudes on mass threshold

The amplitude $A_{1 \rightarrow n}$ for the field $\phi$ to create $n$ particles in the $\phi^4$ theory,

$$L_p(\phi) = \frac{1}{2} (\partial \phi)^2 - \frac{1}{2} M^2 \phi^2 - \frac{1}{4} \lambda \phi^4 + \rho \phi,$$

is derived by applying the LSZ reduction technique:

$$\langle n|\phi(x)|0 \rangle = \lim_{\rho \to 0} \left[ \prod_{j=1}^{n} \lim_{p_j^2 \to M^2} \int d^4 x_j e^{i p_j \cdot x_j} \frac{\delta}{\delta \rho(x_j)} \right] \langle 0_{\text{out}}|\phi(x)|0_{\text{in}} \rangle_{\rho}.$$

Tree-level approximation is obtained via $\langle 0_{\text{out}}|\phi(x)|0_{\text{in}} \rangle_{\rho} \longrightarrow \phi_{\text{cl}}(x)$ where $\phi_{\text{cl}}(x)$ is a solution to the classical field equation.

On mass threshold limit all outgoing particles are produced at rest, $\vec{p}_j = 0$ and we set all $p_j^{\mu} = (\omega, 0)$ and $\rho(x) = \rho(t) = \rho_0(\omega) e^{i \omega t}$. Hence,

$$(M^2 - p_j^2) \frac{\delta}{\delta \rho(x_j)} \longrightarrow (M^2 - \omega^2) \frac{\delta}{\delta \rho(t_j)} = \frac{\delta}{\delta z(t_j)},$$

$$z(t) := \frac{\rho_0(\omega) e^{i \omega t}}{M^2 - \omega^2 - i \epsilon} := z_0 \ e^{i \omega t}, \quad z_0 = \text{finite const}$$
Tree-level amplitudes in $\phi^4$ on mass threshold

The generating function of tree amplitudes on multiparticle thresholds is a classical solution. It solves an ordinary differential equation with no source term,

$$d_t^2 \phi + M^2 \phi + \lambda \phi^3 = 0.$$

The solution contains only positive frequency harmonics, i.e. the Taylor expansion in $z(t)$,

$$\phi_{cl}(t) = z(t) + \sum_{n=2}^{\infty} d_n z(t)^n, \quad z := z_0 e^{iM t}$$

Coefficients $d_n$ determine the actual amplitudes by differentiation w.r.t. $z$,

$$A_{1 \to n} = \left( \frac{\partial}{\partial z} \right)^n \phi_{cl} \bigg|_{z=0} = n! d_n \quad \text{Factorial growth!!}$$

$$\phi_{cl}(t) = \frac{z(t)}{1 - \frac{\lambda}{8M^2} z(t)^2} \quad A_{1 \to n} = n! \left( \frac{\lambda}{8M^2} \right)^{n-\frac{1}{2}}$$
Example 2: apply to \( \phi^4 \) with SSB (Higgs-like)

\[
\mathcal{L}(h) = \frac{1}{2} (\partial h)^2 - \frac{\lambda}{4} (h^2 - v^2)^2 ,
\]

The classical equation for the spatially uniform field \( h(t) \),

\[
d_t^2 h = -\lambda h^3 + \lambda v^2 h ,
\]

again has a closed-form solution with correct initial conditions \( h_{cl} = v + z + \ldots \)

\[
h_{cl}(t) = v \frac{1 + \frac{z(t)}{2v}}{1 - \frac{z(t)}{2v}} , \quad \text{where} \quad z(t) = z_0 e^{iM_h t} = z_0 e^{i\sqrt{2\lambda} v t}
\]

\[
h_{cl}(t) = 2v \sum_{n=0}^{\infty} \left( \frac{z(t)}{2v} \right)^n d_n = v + 2v \sum_{n=1}^{\infty} \left( \frac{z(t)}{2v} \right)^n ,
\]

i.e. with \( d_0 = 1/2 \) and all \( d_{n \geq 1} = 1 \).

\[
\mathcal{A}_{1 \rightarrow n} = \left( \frac{\partial}{\partial z} \right)^n h_{cl} \bigg|_{z=0} = n! \left(2v\right)^{1-n} \quad \text{Factorial growth!!}
\]
These equations are solved by iterations (numerically) with Mathematica. The double Taylor expansion of the generating functions takes the form:

\begin{align*}
    h_{c1}(z, w^a) &= 2v \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} d(n, 2k) \left( \frac{z}{2v} \right)^n \left( \frac{w^a w^a}{(2v)^2} \right)^k, \\
    A^a_{Lc1}(z, w^a) &= w^a \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} a(n, 2k) \left( \frac{z}{2v} \right)^n \left( \frac{w^a w^a}{(2v)^2} \right)^k,
\end{align*}

where \( d(n, 2k) \) and \( a(n, 2k) \) are determined from the iterative solution of EOM.

By repeatedly differentiating these with respect to \( z \) and \( w^a \) for the Higgs to \( n \) Higgses and \( m \) longitudinal Z bosons threshold amplitude we get,

\[ A(h \to n \times h + m \times Z_L) = (2v)^{1-n-m} n! \cdot m! \cdot d(n, m), \]

and for the longitudinal Z decaying into \( n \) Higgses and \( m + 1 \) vector bosons,

\[ A(Z_L \to n \times h + (m + 1) \times Z_L) = \frac{1}{(2v)^{n+m}} n! (m + 1)! \cdot a(n, m). \]

Factorial growth reemains (in \( n \) and in \( m \))!
Off the multi-particle threshold

- Tree level recursion relations & classical equations
- Non-relativistic kinematics in the multi-particle final state
- Integration over the n-particle phase space
- Total cross-section & the holy grail function

- Starting point: the approach of Libanov, Rubakov, Son & Troitsky 9407381 and Son 9505338 in unbroken phi^4
Off the mass-threshold:

\[-(\partial^\mu \partial_\mu + M_h^2) \varphi = 3\lambda v \varphi^2 + \lambda \varphi^3\]

This classical equation for \( \varphi(x) = h(x) - v \) determines directly the structure of the recursion relation for tree-level scattering amplitudes:

\[
(P_{\text{in}}^2 - M_h^2) A_n(p_1 \ldots p_n) = 3\lambda v \sum_{n_1, n_2}^n \delta_{n_1+n_2} \sum_{P} A_{n_1}(p^{(1)}_1, \ldots, p^{(1)}_{n_1}) A_{n_2}(p^{(2)}_1 \ldots p^{(2)}_{n_2}) + \lambda \sum_{n_1, n_2, n_3}^n \delta_{n_1+n_2+n_3} \sum_{P} A_{n_1}(p^{(1)}_1 \ldots p^{(1)}_{n_1}) A_{n_2}(p^{(2)}_1 \ldots p^{(2)}_{n_2}) A_{n_3}(p^{(3)}_1 \ldots p^{(3)}_{n_2})
\]

Away from the multi-particle threshold, the external particles 3-momenta \( \vec{p}_i \) are non-vanishing. In the non-relativistic limit, the leading momentum-dependent contribution to the amplitudes is proportional to \( E_n^{\text{kin}} \) (Galilean Symmetry),

\[
A_n(p_1 \ldots p_n) = A_n + M_n E_n^{\text{kin}} := A_n + M_n n \varepsilon,
\]

\[
\varepsilon = \frac{1}{n M_h} E_n^{\text{kin}} = \frac{1}{n} \frac{1}{2 M_h^2} \sum_{i=1}^n \vec{p}_i^2.
\]

In the non-relativistic limit we have \( \varepsilon \ll 1 \).

This is all we need to find. Use the same recursion relations
Solution of the recursion relations:
\[ A_n(p_1 \ldots p_n) = n! (2v)^{1-n} \left( 1 - \frac{7}{6} n \varepsilon - \frac{1}{6} \frac{n}{n-1} \varepsilon + O(\varepsilon^2) \right). \]

An important observation is that by exponentiating the order-\(n\varepsilon\) contribution, one obtains the expression for the amplitude which solves the original recursion relation to all orders in \((n\varepsilon)^m\) in the large-\(n\) non-relativistic limit,
\[ A_n(p_1 \ldots p_n) = n! (2v)^{1-n} \exp \left[ -\frac{7}{6} n \varepsilon \right], \quad n \to \infty, \quad \varepsilon \to 0, \quad n\varepsilon = \text{fixed}. \]

Simple corrections of order \(\varepsilon\), with coefficients that are not-enhanced by \(n\) are expected, but the expression is correct to all orders \(n\varepsilon\) in the double scaling large-\(n\) limit. The exponential factor can be absorbed into the \(z\) variable so that
\[ \varphi(z) = \sum_{n=1}^{\infty} d_n \left( z e^{-\frac{7}{6} \varepsilon} \right)^{n}, \]
remains a solution to the classical equation and the original recursion relations.

Can now integrate over the phase-space
Cross-section & the Holy Grail function

In the non-rel. limit for perturbative Higgs bosons only production we obtained:

\[
\sigma_n \propto \exp \left[ n \left( \log \frac{\lambda n}{4} - 1 \right) + \frac{3n}{2} \left( \log \frac{\varepsilon}{3\pi} + 1 \right) - \frac{25}{12} n \varepsilon \right]
\]

More generally, in the large-\(n\) limit with \(\lambda n = \text{fixed}\) and \(\varepsilon = \text{fixed}\), one expects

\[
\sigma_n \propto \exp \left[ \frac{1}{\lambda} F_{\text{h.g.}}(\lambda n, \varepsilon) \right]
\]

[e.g. Libanov, Rubakov, Troitsky review 1997]

where the holy grail function \(F_{\text{h.g.}}\) is of the form,

\[
\frac{1}{\lambda} F_{\text{h.g.}}(\lambda n, \varepsilon) = \frac{\lambda n}{\lambda} (f_0(\lambda n) + f(\varepsilon))
\]

In our higgs model, i.e. the scalar theory with SSB,

\[
f_0(\lambda n) = \log \frac{\lambda n}{4} - 1 \quad \text{at tree level}
\]

\[
f(\varepsilon) \to \frac{3}{2} \left( \log \frac{\varepsilon}{3\pi} + 1 \right) - \frac{25}{12} \varepsilon \quad \text{for } \varepsilon \ll 1
\]
Large-$n$ limit with $\lambda n = \text{fixed}$ and $\varepsilon = \text{fixed},$

\[
\frac{1}{n} \log \sigma_n = \frac{1}{\lambda n} F_{\text{h.g.}}(\lambda n, \varepsilon) = f_0(\lambda n) + f(\varepsilon)
\]

\[
f_0(\lambda n) = \log \frac{\lambda n}{4} - 1 \quad \text{at tree level}
\]
\[
f_{\text{asympt}}(\varepsilon) = \frac{3}{2} \left( \log \frac{\varepsilon}{3\pi} + 1 \right) - \frac{25}{12} \varepsilon \quad \text{for } \varepsilon \ll 1
\]

f(eps) asymptotes to a const at large eps
(highly relativistic final state)

Compute w MadGraph for n=7

and scale to large n using known n-dependence of the holy grail

computed w MadGraph 2-> 7 for any epsilon and scaled to large n
Loop corrections to tree-level amplitudes @ threshold

The 1-loop corrected threshold amplitude for the pure $n$ Higgs production:

$$\phi^4 \text{ with SSB: } A_{1\rightarrow n}^{\text{tree+1loop}} = n! \left(2v\right)^{1-n} \left(1 + n(n-1)\frac{\sqrt{3}\lambda}{8\pi}\right)$$

There are strong indications, based on the analysis of leading singularities of the multi-loop expansion around singular generating functions in scalar field theory, that the 1-loop correction exponentiates,

Libanov, Rubakov, Son, Troitsky 1994

$$A_{1\rightarrow n} = A_{1\rightarrow n}^{\text{tree}} \times \exp \left[B \lambda n^2 + O(\lambda n)\right]$$

in the limit $\lambda \rightarrow 0$, $n \rightarrow \infty$ with $\lambda n^2$ fixed. Here $B$ is determined from the 1-loop calculation (as above) – Smith; Voloshin 1992:

$$\phi^4 \text{ with SSB: } B = + \frac{\sqrt{3}}{8\pi},$$

$$\phi^4 \text{ w. no SSB: } B = - \frac{1}{64\pi^2} \left(\log(7+4\sqrt{3}) - i\pi\right),$$

In the Higgs model, 1st equation leads to the exponential enhancement of the tree-level threshold amplitude at least in the leading order in $n^2 \lambda$. 

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In the non-rel. limit for perturbative Higgs bosons only production we obtained:

$$\sigma_n \propto \exp \left[ n \left( \log \frac{\lambda n}{4} - 1 \right) + \frac{3n}{2} \left( \log \frac{\varepsilon}{3\pi} + 1 \right) - \frac{25}{12} n \varepsilon \right] + 2\lambda n^2 B$$

More generally, in the large-\(n\) limit with \(\lambda n = \) fixed and \(\varepsilon = \) fixed, one expects

$$\sigma_n \propto \exp \left[ \frac{1}{\lambda} F_{\text{h.g.}}(\lambda n, \varepsilon) \right]$$

where the *holy grail* function \(F_{\text{h.g.}}\) is of the form,

$$\frac{1}{\lambda} F_{\text{h.g.}}(\lambda n, \varepsilon) = \frac{\lambda n}{\lambda} \left( f_0(\lambda n) + f(\varepsilon) \right)$$

In our higgs model, i.e. the scalar theory with SSB,

$$f_0(\lambda n) = \log \frac{\lambda n}{4} - 1 + \lambda n \frac{\sqrt{3}}{4\pi} \quad \text{a significant enhancement though higher orders unknown!}$$

$$f(\varepsilon) \rightarrow \frac{3}{2} \left( \log \frac{\varepsilon}{3\pi} + 1 \right) - \frac{25}{12} \varepsilon \quad \text{for } \varepsilon \ll 1$$
Gluon fusion process

\[ A_{gg \to n \times h} = \sum_{\text{polygons}} A_{\text{polygons}}^{gg \to k \times h^*} \sum_{n_1 + \ldots + n_k = n} \prod_{i=1}^{k} A_{h_i^* \to n_i \times h} \]

1-loop polygons: triangles, boxes, pentagons, hexagons, etc.
Compute numerically in the high-energy limit
MadGraph5_aMC@NLO

Tree-level 1->n multi-Higgs processes. Compute at fixed multiplicities n=5,6,7 at all energies (i.e. arbitrary epsilon) and scale to large n using known n-dependence of the holy grail function.

And include the leading-order loop correction in the holy-grail function f in the exponent

\[ + \lambda n \frac{\sqrt{3}}{4\pi} \]

- Degrande-VVK-Mattelaer 1605.06372
**Polygon contributions:**

\[ \sigma_{gg\to hh} \]

\[ \sigma_{gg\to hhh} \]

\[ \sigma_{gg\to hhhh} \]

\[ s \gg m_t, M_h \text{ limit} \]

<table>
<thead>
<tr>
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<th>( \sigma_{gg\to hh} )</th>
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<tbody>
<tr>
<td>Triangles</td>
<td>( y_t \frac{m_t^2 M_h^2}{s^3} \log^4 \left( \frac{m_t}{\sqrt{s}} \right) \frac{M_h^2}{v^2} )</td>
<td>( y_t \frac{m_t^2}{s^2} \log^4 \left( \frac{m_t}{\sqrt{s}} \right) \frac{M_h^4}{v^4} )</td>
<td>( y_t \frac{m_t^2}{s^2} \log^4 \left( \frac{m_t}{\sqrt{s}} \right) \frac{M_h^6}{v^6} )</td>
</tr>
<tr>
<td>Boxes</td>
<td>( y_t^{\frac{4}{3}} \frac{1}{s} )</td>
<td>( y_t^{\frac{4}{3}} \frac{1}{s} \frac{M_h^2}{v^2} )</td>
<td>( y_t^{\frac{4}{3}} \frac{1}{s} \frac{M_h^4}{v^4} )</td>
</tr>
<tr>
<td>Pentagons</td>
<td>(-)</td>
<td>( y_t^{\frac{6}{3}} \frac{m_t^2}{s^2} \log^4 \left( \frac{m_t}{\sqrt{s}} \right) )</td>
<td>( y_t^{\frac{6}{3}} \frac{m_t^2}{s^2} \log^4 \left( \frac{m_t}{\sqrt{s}} \right) \frac{M_h^2}{v^2} )</td>
</tr>
<tr>
<td>Hexagons</td>
<td>(-)</td>
<td>(-)</td>
<td>( y_t^{\frac{8}{3}} \frac{1}{s} )</td>
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Polygon contributions:

We thus determine Effective Vertices (with Form-factors) describing contributions of polygons with k-sides:

\[ \mathcal{V}_k = C_k \frac{\alpha_s(\sqrt{s})}{\pi} \text{tr}(G_{\mu\nu}G^{\mu\nu}) \left( \frac{y_t h}{\sqrt{s}} \right)^k \times \begin{cases} 1 \\ \frac{m_t}{\sqrt{s}} \log^2 \left( \frac{m_t}{\sqrt{s}} \right) \end{cases} \quad : \quad k = \text{even } \geq 2 \]

\[ : \quad k = \text{odd } \geq 3 \]

Constants (computed)

And for h we now substitute the classical generating function describing subsequent tree-level branchings.
Gluon fusion process

\[ \sigma_n = K_k \frac{n^2}{s} \left( \frac{1}{1+\varepsilon} \right)^{2(k-2)} e^{n(f_0(\lambda n) + f(\varepsilon))} = K_k \frac{1}{M_h^2} \left( \frac{1}{1+\varepsilon} \right)^{2k-2} e^{n(f_0(\lambda n) + f(\varepsilon))} \]

\[ K_k \sim \left( \frac{C_k \alpha_s}{\kappa_k \pi} \right)^2 \left( \frac{2\sqrt{2}m_t}{M_h} \right)^{2k} \approx \left\{ \begin{array}{ll}
0.1 & : k = 2 \\
20 & : k = 4
\end{array} \right. \]

\[ f_0(\lambda n)^{\text{NLO-resummed}} = \log \left( \frac{\lambda n}{4} \right) - 1 + \sqrt{3} \frac{\lambda n}{4\pi} \]

Figure 3: The logarithm of the cross-section (3.8) (in picobarns) is plotted as the function of energy for a range of final-state multiplicities between 7 and 130. The plot in Fig. 3 plots these cross-sections as the function of energy for a range of final-state multiplicities between 7 and 130.

- Degrande-VVK-Mattelaer 1605.06372
Finally:
Convolving with Parton Distribution Functions

Left panel: Cross-sections for multi-Higgs production at proton colliders including the PDFs for different energies of the proton-proton collisions plotted as the function of the Higgs multiplicity. Only the contributions from the boxes are included. The right panel illustrates the dependence on average energy variable $\epsilon$ by applying a sequence of cuts on $\epsilon$ at 100 TeV.

- Degrange-VVK-Mattelaer 1605.06372
Conclusions for the perturbative part

- At (not too high) high energies perturbative Standard Model exhibits a formal breakdown. Perturbative unitarity is broken. OPTIONS:

- At high energies (multiplicities) the Standard Model is fundamentally non-perturbative (?)

- The theory classicalizes: the ultra-high multiplicity processes will completely dominate everything else (?)

- New physics beyond the Standard Model has to set in before the cross-sections become large, i.e. as early as at ~50 TeV (?)

- New theoretical approaches & computational techniques have to be developed to determine the relevant energy scale - almost as exciting as probing this at the Future proton Collider -