Non-thermal Gravitino Production After Large-Field Inflation

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Supersymmetry, gravitino, and cosmology

- Supersymmetry (SUSY) is a well-motivated candidate of physics beyond the Standard Model.
- Supergravity (SUGRA) is its local version.
- Gravitino is the SUSY partner of graviton, and it is the gauge fermion field of SUSY with spin 3/2.
- Basically, its interaction is gravitational strength, and gravitino is long-lived. In general, long-lived particles affect cosmology.

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Cosmological gravitino problem

Stable gravitino

- It may exceed the observed dark matter abundance.
  (The annihilation cross-section is small.)

Unstable gravitino

- Its decay products alters light element abundance by affecting big-bang nucleosynthesis (BBN).
- They also affect cosmic spectrum including cosmic microwave background (CMB), and γ-ray background etc.
- The abundance of lightest SUSY particle (LSP) from gravitino decay must also satisfy the dark matter bound.

It's very important to estimate the gravitino abundance!

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BBN bound on gravitino abundance

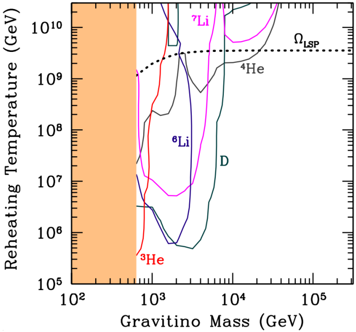

Figure: BBN constraints on the gravitino yield [Kawasaki et al., 2008].

Fitting formula for the gravitino yield $Y_{3/2}$ in [Kawasaki et al., 2008]:

$$Y_{3/2} \equiv \frac{n_{3/2}}{s} \simeq 2.3 \times 10^{-14} \times \left(\frac{T_{\rm R}}{10^8 \text{GeV}} \right) + \dots \quad (1)$$

Production mechanisms of gravitino

Thermal production

Production by scattering processes in a thermal bath.

Nonthermal production

Productions other than thermal production.
Typically, from decay of heavier particles like inflaton or moduli.

- Reheating: perturbative decay of the inflaton around the minimum of the inflaton potential
- Preheating: non-perturbative production of particles at the early stage of inflaton oscillation

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Earlier works and their problems

Gravitino production at preheating [$\sim$ 2000]

[Maroto and Mazumdar, 2000, Kallosh et al., 2000a, Giudice et al., 1999, Kallosh et al., 2000b, Nilles et al., 2001c, Nilles et al., 2001b]

- Inflation sector was a toy model.
- SUSY breaking by Polonyi model $\rightarrow$ Polonyi problem.
- Does not correspond to the late-time gravitino abundance.
- Lack of analytical understanding.

Gravitino from perturbative heavy scalar decay [$\sim$ 2006]

[Endo et al., 2006a, Nakamura and Yamaguchi, 2006, Kawasaki et al., 2006a, Asaka et al., 2006, Dine et al., 2006] and more.

Not so trivial (mixing between inflaton/moduli ϕ and SUSY breaking field z is important).

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Problems of steepness and negativity for inflaton potential

An exponential factor and a negative contribution.

$$V = e^K \left(g^{i\bar{j}} D_i W \bar{D}_{\bar{j}} \bar{W} - 3|W|^2 \right) + \frac{1}{2} f^{\mathrm{R}}_{AB} D^A D^B, \tag{2}$$

where $D_i W = \partial_i W + \partial_i K W$.

Shift symmetry to obtain a flat potential,

$$\phi \to \phi + c, \qquad K(\phi, \bar{\phi}) = K(i(\phi - \bar{\phi})), \tag{3}$$

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Two solutions to the steepness/negativity problem

More fields

A stabilizer field X is introduced which satisfies $\langle X \rangle \simeq 0$.

$$W = Xf(\phi), \qquad V \simeq |f(\phi)|^2. \tag{4}$$

Sometimes, a stabilization term for X is needed,

$$K = -\frac{1}{2}(\phi - \bar{\phi})^2 + |X|^2 - \frac{1}{\Lambda^2}|X|^4. \tag{5}$$

[Kawasaki et al., 2000, Kallosh and Linde, 2010, Kallosh et al., 2011]

More terms

Non-minimal terms are introduced.

$$K = ic(\phi - \bar{\phi}) - \frac{1}{2}(\phi - \bar{\phi})^2 - \frac{1}{\Lambda^2}(\phi - \bar{\phi})^4 + \dots, \tag{6}$$

$$W = e^{ic\phi} f(\phi), \tag{7}$$

with $|c| \gtrsim \sqrt{3}$. [Ketov and Terada, 2014b, Ketov and Terada, 2014a]

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Our work

- A shift-symmetric inflaton with or without a stabilizer field
  → Realistic large-field model.
- Strongly-stabilized Polonyi field → No Polonyi problem.
- Taking into account time-dependent mixing between gravitino and a fermion. → True gravitino abundance.
- Analytic estimation → Parametric dependence is clearer.
- Reproduces results of perturbative decay
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Analysis procedure

1. Start from the supergravity Lagrangian.
2. Rewrite it in terms of physical degrees of freedom. (Transverse and longitudinal modes of the gravitino.)
3. Canonically normalize them.
4. Remove mixing with other fermions. (diagonalization)
5. The form of the Lagrangian reduces to that of Majorana spinors. Apply the formula of fermion production by preheating.

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Production of Majorana fermion by preheating

The production rate is estimated by the background field method. Suppose that the fermion mass $m(t)$ oscillates with its amplitude $\widetilde{m}$ and frequency Ω. The number density grows as

$$n(t) \simeq \frac{\mathcal{C}}{16\pi}\Omega^2\widetilde{m}^2 t. \tag{8}$$

$\mathcal{C} = 1$ in the case of $m(t) \propto \cos(\Omega t)$.
Suppose the inflaton oscillates sinusoidally, $\phi(t) \propto \cos(m_\phi t)$.
$m \propto \phi^n$ leads to $\Omega = n m_\phi, (n-2)m_\phi, \ldots$.
$n = 1, 2$ can be interpreted as decay and annihilation, respectively:

$$\Gamma(n \times \phi \to \psi\psi) \sim \frac{n n_\psi}{2 n_\phi t} \sim \frac{n^3 \mathcal{C}}{32\pi}\frac{\widetilde{m}^2}{\phi_{\rm amp}^2} m_\phi. \tag{9}$$

[Greene and Kofman, 1999, Peloso and Sorbo, 2000, Asaka and Nagao, 2010] See also App. B of [Ema et al., 2016].

Summary of the section

It is very important to estimate the gravitino abundance!

Given a supergravity model, our tasks are

- To diagonalize the fermion mass matrix.
- To obtain the amplitude $\tilde{m}$ and frequency Ω of the oscillating mass eigenvalues.

$$\phi = \phi_{\rm amp}\cos(m_\phi t) \text{ and } m = \frac{m_\phi \phi^2}{2M_{\rm Pl}^2}$$

$$\to \tilde{m} = \frac{m_\phi \phi_{\rm amp}^2}{2M_{\rm Pl}^2} \text{ and } \Omega = 2m_\phi$$

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Assumptions

- Kinetic terms can be regarded as minimal, $K_{i\bar{j}} \simeq \delta_{i\bar{j}}$ after inflation.
- Scalar field configuration is real.
  (If all the parameters of the model and initial conditions are real, this assumption is satisfied.)
- D-term is not important.
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- Kinetic terms can be regarded as minimal, $K_{i\bar{j}} \simeq \delta_{i\bar{j}}$ after inflation.

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Supergravity Lagrangian

$$
\begin{aligned}
e^{-1}\mathcal{L} =& \frac{M_{\rm Pl}^2}{2}R - g_{i\bar{j}}\partial_\mu\phi^i\partial^\mu\phi^{*\bar{j}} - V \\
& - \frac{1}{2}g_{i\bar{j}}\left(\overline{\chi}_L^i\widehat{\not{D}}\chi_R^{\bar{j}} + \overline{\chi}_R^{\bar{j}}\widehat{\not{D}}\chi_L^i\right) - \frac{1}{2}\left(m_{ij}\overline{\chi}_L^i\chi_L^j + m_{\bar{i}\bar{j}}\overline{\chi}_R^{\bar{i}}\chi_R^{\bar{j}}\right) \\
& - \frac{1}{2}\overline{\psi}_\mu R^\mu + \frac{1}{2}\overline{\psi}_\mu\left(m_{3/2}P_R + m_{3/2}^*P_L\right)\widehat{\gamma}^{\mu\nu}\psi_\nu \\
& + \frac{\sqrt{2}}{M_{\rm Pl}}g_{i\bar{j}}\overline{\psi}_\mu\widehat{\gamma}^{\nu\mu}\left(\chi_L^i\partial_\nu\phi^{*\bar{j}} + \chi_R^{\bar{j}}\partial_\nu\phi^i\right) + \frac{1}{\sqrt{2}M_{\rm Pl}}\left(\overline{\psi}\cdot\widehat{\gamma}\right)v + e^{-1}\mathcal{L}_{4f},
\end{aligned}
$$

where

$$
R^\mu \equiv \widehat{\gamma}^{\mu\rho\sigma}D_\rho\psi_\sigma, \qquad D_\mu\psi_\nu = \left(\partial_\mu + \frac{1}{4}\omega_\mu{}^{ab}\gamma_{ab} - iA_\mu\gamma_*\right)\psi_\nu - \Gamma^\rho_{\mu\nu}\psi_\rho,
$$

$$
A_\mu \equiv \frac{i}{4M_{\rm Pl}^2}\left(\partial_i K\partial_\mu\phi^i - \partial_{\bar{i}}K\partial_\mu\phi^{*\bar{i}}\right), \quad v_L \equiv e^{K/2M_{\rm Pl}^2}D_iW\chi_L^i + g_{i\bar{j}}\not{\partial}\phi^i\chi_R^{\bar{j}}
$$

$$
m_{3/2} \equiv e^{K/2M_{\rm Pl}^2}\frac{W}{M_{\rm Pl}^2}, \quad m_{ij} \equiv e^{K/2M_{\rm Pl}^2}\left[\partial_i + \frac{\partial_i K}{M_{\rm Pl}^2}\right]D_jW - e^{K/2M_{\rm Pl}^2}\Gamma^k_{ij}D_kW.
$$

Equation of motion, constraints, and decomposition

We take unitary gauge $v = 0$, and assume the FLRW background.

Equation of motion

$$\Sigma^\mu \equiv R^\mu - \hat{\gamma}^{\mu\nu}\left(m_{3/2}\psi_\nu - \frac{\sqrt{2}}{M_{\rm Pl}} g_{i\bar{j}}\left(\chi_L^i \partial_\nu \phi^{*\bar{j}} + \chi_R^{\bar{j}} \partial_\nu \phi^i\right)\right) = 0. \qquad (10)$$

Constraints

$$0 = D_\mu \Sigma^\mu + \frac{m_{3/2}}{2}\hat{\gamma}_\mu \Sigma^\mu, \qquad 0 = \Sigma^0. \qquad (11)$$

Decomposition

$$\vec{\psi} = \vec{\psi}^t + \left(\frac{1}{2}\vec{\gamma} - \frac{1}{2k^2}\vec{k}\left(\vec{k}\cdot\vec{\gamma}\right)\right)\psi^\ell + \left(\frac{3}{2k^2}\vec{k} - \frac{1}{2k^2}\vec{\gamma}\left(\vec{k}\cdot\vec{\gamma}\right)\right)\vec{k}\cdot\vec{\psi}, \qquad (12)$$

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$$i\vec{k}\cdot\vec{\psi} = \left(i\vec{\gamma}\cdot\vec{k} - a\left(m_{3/2} + \gamma_0 H\right)\right)\psi^\ell, \qquad (13)$$

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Lagrangian in terms of physical degrees of freedom

$$e^{-1}\mathcal{L}_{3/2} = e^{-1}\mathcal{L}_t + e^{-1}\mathcal{L}_\ell + e^{-1}\mathcal{L}_{\rm mix}, \tag{14}$$

$$e^{-1}\mathcal{L}_t = -\frac{1}{2a^3}\overline{\vec{\psi}^t}\not{D}\vec{\psi}^t + \frac{H}{2a^2}\overline{\vec{\psi}^t}\gamma^0\vec{\psi}^t - \frac{1}{2a^2}\overline{\vec{\psi}^t}m_{3/2}\vec{\psi}^t, \tag{15}$$

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$$p_{\rm SB} \equiv \sum_i |\dot{\phi}_i|^2 - V_{\rm SB}, \quad p_W \equiv 2\dot{m}_{3/2}M_{\rm Pl}^2 = -\sum_i(\dot{\phi}_i^* F_i + \dot{\phi}_i F_i^*). \tag{20}$$

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Canonical normalization

canonical fields

$$\vec{\psi}^t_{\rm c} \equiv \sqrt{a}\vec{\psi}^t, \qquad \psi^\ell_{\rm c} \equiv -\frac{\sqrt{\rho_{\rm SB}}a^{3/2}}{\sqrt{2}k^2 M_{\rm Pl}} i\left(\vec{\gamma}\cdot\vec{k}\right)\psi^\ell, \qquad \chi'^i \equiv a^{3/2}\chi^i, \tag{21}$$

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$$\mathcal{L}_{\rm mix} = \frac{2}{\sqrt{\rho_{\rm SB}}}\overline{\psi^\ell_{\rm c}} i\left(\vec{\gamma}\cdot\vec{k}\right)\gamma^0 g_{i\bar{j}}\left[\dot{\phi}^i\chi'^{\bar{j}}_R + \dot{\phi}^{*\bar{j}}\chi'^i_L\right], \tag{24}$$

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Fermions χ_i $(i = 1, \ldots, N)$ can be decomposed as goldstino v and orthogonal fields $v_\perp^I$ $(I = 1, \ldots, N-1)$. They are packaged in $v_i \equiv (v, v_\perp^I)$.

goldstino

$$v = \sum_i \alpha_i(-F_i + \dot{\phi}_i \gamma^0)\chi_i \tag{26}$$

$$\equiv \sum_i \alpha_i(\cos\theta_i + \sin\theta_i \gamma^0)\chi_i = \sum_i \alpha_i e^{\gamma^0 \theta_i} \chi_i, \tag{27}$$

where $\alpha_i \equiv \sqrt{\rho_{\rm SB}^i/\rho_{\rm SB}}$ and $\rho_{\rm SB}^i \equiv |\dot{\phi}_i|^2 + |F_i|^2$.

relation of the two bases

$$v_i = \left(O^T\right)_{ij} e^{\gamma^0\theta_j}\chi_j \quad \leftrightarrow \quad \chi_i = e^{-\gamma^0\theta_i} O_{ij} v_j, \qquad O^T = \begin{pmatrix} \alpha_1, \ldots, \alpha_N \\ \widetilde{O}^T \end{pmatrix}. \tag{28}$$

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Preparation of diagonalization 2

In terms of these variables v_i, the matter fermion Lagrangian reads

$$\mathcal{L}_{f,\mathrm{kin}} = -\frac{1}{2}\overline{v_\perp^I}\left[\gamma^0\partial_0\delta_{IJ} + \widetilde{O}^T_{Ii}e^{\gamma^0\theta_i}i\left(\vec{\gamma}\cdot\vec{k}\right)e^{-\gamma^0\theta_i}\widetilde{O}_{iJ} + \widetilde{O}^T_{Ii}\left(\partial_0\theta_i\right)\widetilde{O}_{iJ} + \widetilde{O}^T_{Ii}\left(\gamma^0\partial_0\widetilde{O}_{iJ}\right)\right]v_\perp^J, \tag{29}$$

$$\mathcal{L}_{f,\mathrm{mass}} = -\frac{a}{2}\overline{\chi_i}m_{ij}\chi_j = -\frac{a}{2}\overline{v_\perp^I}\left(\widetilde{O}^T_{Ii}e^{\gamma^0\theta_i}m_{ij}e^{-\gamma^0\theta_j}\widetilde{O}_{jJ}\right)v_\perp^J. \tag{30}$$

The mixing term is now

$$\mathcal{L}_{\mathrm{mix}} = 2\overline{\psi^\ell_c}\,i\left(\vec{\gamma}\cdot\vec{k}\right)\gamma^0\left[\sum_i \alpha_i\sin\theta_i\,\chi_i\right] = 2\overline{\psi^\ell_c}\,i\left(\vec{\gamma}\cdot\vec{k}\right)\gamma^0\alpha_i\sin\theta_i\,e^{-\gamma^0\theta_i}\widetilde{O}_{iI}v_\perp^I. \tag{31}$$

Diagonalization of the gradient term

$$\mathcal{L}_{\rm grad} = -\frac{1}{2}\begin{pmatrix}\overline{\psi_{\rm c}^{\ell}} & \overline{v_{\perp}}\end{pmatrix}\left(i\vec{\gamma}\cdot\vec{k}\right)\widehat{\mathcal{A}}\begin{pmatrix}\psi_{\rm c}^{\ell} \\ v_{\perp}\end{pmatrix}, \tag{32}$$

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where we used $\widehat{A} = -\sum_i e^{2\gamma^0\theta_i}\alpha_i^2$.

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$$\begin{pmatrix}\psi_{\rm c}^{\ell\prime} \\ v_{\perp}'\end{pmatrix} \equiv e^{-\gamma^0\widehat{\theta}}\begin{pmatrix}\psi_{\rm c}^{\ell} \\ v_{\perp}\end{pmatrix}, \tag{34}$$

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$$\mathcal{L} = -\frac{1}{2}\begin{pmatrix}\overline{\psi_{\rm c}^{\ell\prime}} & \overline{v_{\perp}'}\end{pmatrix}\left(\gamma^0\partial_0 + i\vec{\gamma}\cdot\vec{k} + a\mathcal{M}\right)\begin{pmatrix}\psi_{\rm c}^{\ell\prime} \\ v_{\perp}'\end{pmatrix}. \tag{35}$$

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Master formula of the mass matrix

$$\mathcal{L} = -\frac{1}{2}\begin{pmatrix}\overline{\psi_{\rm c}^{\ell\prime}} & \overline{v'_\perp}\end{pmatrix}\left(\gamma^0\partial_0 + i\vec{\gamma}\cdot\vec{k} + a\mathcal{M}\right)\begin{pmatrix}\psi_{\rm c}^{\ell\prime} \\ v'_\perp\end{pmatrix}, \tag{36}$$

where

$$\mathcal{M} = e^{-\gamma^0\widehat{\theta}}\left[\gamma^0\frac{\partial}{\partial t} + \begin{pmatrix}\widehat{m}_{3/2} & 0 \\ 0 & \widetilde{O}^T\left(\widehat{m}_f + \dot{\theta}_i + \gamma^0\dot{\widetilde{O}}\widetilde{O}^T\right)\widetilde{O}\end{pmatrix}\right]e^{\gamma^0\widehat{\theta}}, \tag{37}$$

with

$$\widehat{m}_f \equiv e^{\gamma^0\theta_i}m_{ij}e^{-\gamma^0\theta_j}. \tag{38}$$

γ^0-dependent mass terms ($\gamma^0\mathcal{M}_2$ in $\mathcal{M} = \mathcal{M}_1 + \gamma^0\mathcal{M}_2$) can be removed, and do not contribute to the mass eigenvalues by time-dependent field redefinition $\psi_{\rm old} = L\psi_{\rm new}$ with L satisfying

$$(\partial_0 + \mathcal{M}_2)L = 0. \tag{39}$$

The final mass matrix is $L^{\rm T}\mathcal{M}_1 L$ [Nilles et al., 2001b].

The remaining task is to diagonalize the mass term, once specifying the model.

Master formula of the mass matrix

$$\mathcal{L} = -\frac{1}{2} \left(\overline{\psi_c^{\ell'}} \quad \overline{v'_\perp} \right) \left(\gamma^0 \partial_0 + i\vec{\gamma} \cdot \vec{k} + a\mathcal{M} \right) \begin{pmatrix} \psi_c^{\ell'} \\ v'_\perp \end{pmatrix}, \tag{36}$$

where

$$\mathcal{M} = e^{-\gamma^0 \widehat{\theta}} \left[\gamma^0 \frac{\partial}{\partial t} + \begin{pmatrix} \widehat{m}_{3/2} & 0 \\ 0 & \widetilde{O}^T \left(\widehat{m}_f + \dot{\theta}_i + \gamma^0 \dot{\widetilde{O}} \widetilde{O}^T \right) \widetilde{O} \end{pmatrix} \right] e^{\gamma^0 \widehat{\theta}}, \tag{37}$$

with

$$\widehat{m}_f \equiv e^{\gamma^0 \theta_i} m_{ij} e^{-\gamma^0 \theta_j}. \tag{38}$$

γ^0-dependent mass terms ($\gamma^0 \mathcal{M}_2$ in $\mathcal{M} = \mathcal{M}_1 + \gamma^0 \mathcal{M}_2$) can be removed, and do not contribute to the mass eigenvalues by time-dependent field redefinition $\psi_{\text{old}} = L\psi_{\text{new}}$ with L satisfying

$$(\partial_0 + \mathcal{M}_2)L = 0. \tag{39}$$

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Summary of this section

Assumptions

Minimal Kähler, real configuration, and irrelevant D-terms.

After canonical normalization,

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$$\mathcal{L} = -\frac{1}{2}\begin{pmatrix}\overline{\psi^{\ell\prime}_{\rm c}} & \overline{v'_\perp}\end{pmatrix}_{\rm new}\left(\gamma^0\partial_0 + i\vec{\gamma}\cdot\vec{k} + aL^{\rm T}\mathcal{M}_1 L\right)\begin{pmatrix}\psi^{\ell\prime}_{\rm c} \\ v'_\perp\end{pmatrix}_{\rm new}, \tag{41}$$

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$$K = -\frac{1}{2}(\phi - \phi^\dagger)^2 + |z|^2 - \frac{|z|^4}{\Lambda^2}, \quad (42)$$

$$W = \frac{1}{2}m_\phi \phi^2 + \mu^2 z + W_0. \quad (43)$$

Gravitino mass at the vacuum

$$m_{3/2}^0 \simeq \mu^2/\sqrt{3}M_{\rm Pl} \simeq W_0/M_{\rm Pl}^2. \quad (44)$$

Strongly stabilized Polonyi field

$$\langle z \rangle \simeq 2\sqrt{3}M_{\rm Pl}(m_{3/2}^0/m_z)^2, \quad \text{and} \quad m_z^2 = 12(m_{3/2}^0 M_{\rm Pl}/\Lambda)^2. \quad (45)$$

SUSY breaking and (transverse) gravitino mass

$$|\dot{\phi}| \sim |F_\phi| \sim m_\phi \phi_{\rm amp} \sim HM_{\rm Pl}, \quad |F_z| \sim \mu^2 \sim m_{3/2}^0 M_{\rm Pl} \gg |\dot{z}|, \quad (46)$$

$$m_{3/2} \simeq \frac{m_\phi \phi^2}{2M_{\rm Pl}^2} + m_{3/2}^0 \sim H\frac{\phi_{\rm amp}}{M_{\rm Pl}} + m_{3/2}^0. \quad (47)$$

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Who is the goldstino?

$$v \sim \begin{cases} \tilde{\phi} & \text{for} \quad H \gtrsim m^0_{3/2} \\ \tilde{z} & \text{for} \quad H \lesssim m^0_{3/2} \end{cases}, \qquad v_\perp \sim \begin{cases} \tilde{z} & \text{for} \quad H \gtrsim m^0_{3/2} \\ \tilde{\phi} & \text{for} \quad H \lesssim m^0_{3/2} \end{cases}. \tag{48}$$

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$$\begin{pmatrix} m_{\text{heavy}}, & m_{\text{light}} \end{pmatrix} \simeq \begin{cases} \begin{pmatrix} m_\phi, & -m_{3/2}(m^0_{3/2}/H)^2 \end{pmatrix} & \text{for} \quad H \gtrsim m^0_{3/2} \\ \begin{pmatrix} m_\phi, & -m_{3/2} \end{pmatrix} & \text{for} \quad H \lesssim m^0_{3/2} \end{cases}. \tag{50}$$

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Time dependence of various quantities

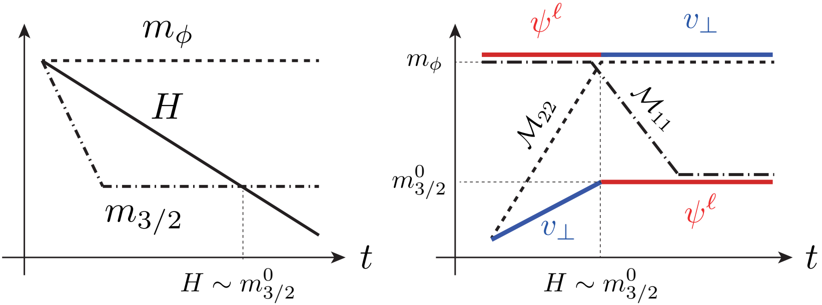

Figure: **Left**: Time dependence of m_ϕ, H and $m_{3/2}$ in single superfield inflation model. **Right**: Time evolution of mass eigenvalues of $(\psi^\ell, v_\perp)$ are shown by thick solid lines. The red (blue) segments show that the main composition of the mass eigenstate is ψ^ℓ $(v_\perp)$. Dashed and dot-dashed lines show $\mathcal{M}_{22}$ and $\mathcal{M}_{11}$, respectively.

Transverse gravitino production

The annihilation rate is

$$\Gamma(\phi\phi \to \psi^t\psi^t) \simeq \frac{\mathcal{C}}{4\pi}\frac{\widetilde{m}^2}{\phi_{\rm amp}^2}m_\phi \simeq \frac{3\mathcal{C}}{16\pi}\frac{H^2 m_\phi}{M_{\rm Pl}^2}, \tag{51}$$

where $\sqrt{3}\widetilde{m} \simeq H\phi_{\rm amp}/2M_{\rm Pl}$ is the oscillation amplitude of the mass of the produced fermion.

Transverse gravitino yield

$$\frac{n_{3/2}^{(t)}}{s} \simeq \left(\frac{\Gamma(\phi\phi \to \psi^t\psi^t)}{H}\right)_{H=H_{\rm inf}}\frac{3T_{\rm R}}{4m_\phi} \simeq \frac{9\mathcal{C}}{64\pi}\frac{H_{\rm inf}T_{\rm R}}{M_{\rm Pl}^2}$$
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Longitudinal gravitino production

Two sources of longitudinal gravitino

- $v_\perp$ produced when $H > m^0_{3/2}$ becomes ψ^ℓ later.

$$\Gamma(\phi\phi \to v_\perp v_\perp) \lesssim \frac{3\mathcal{C}}{16\pi}\left(\frac{m^0_{3/2}}{H}\right)^2 \frac{(m^0_{3/2})^2 m_\phi}{M^2_{\rm Pl}}. \quad (53)$$

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The mass of the heavier spinor $v_\perp$ has an oscillating part,

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This allows production at $H \sim H_{\text{inf}}$ if inflaton amplitude is Planck scale.

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Higher power inflaton potential

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Background expansion is different from the quadratic case.
For the quartic potential ($n = 3$), it is $\phi_{\rm amp} \propto a^{-1}$ and $H^2 \propto a^{-4}$, and the gravitino yields are

$$\frac{n_{3/2}^{(t)}}{s} \simeq \frac{\mathcal{C}}{64\pi}\left(\frac{90}{\pi^2 g_*}\right)^{1/4}\left(\frac{H_{\rm inf}}{M_{\rm Pl}}\right)^{3/2} \simeq 2\times 10^{-11}\mathcal{C}\left(\frac{H_{\rm inf}}{10^{13}\,{\rm GeV}}\right)^{3/2}, \tag{61}$$

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Summary of gravitino abundance

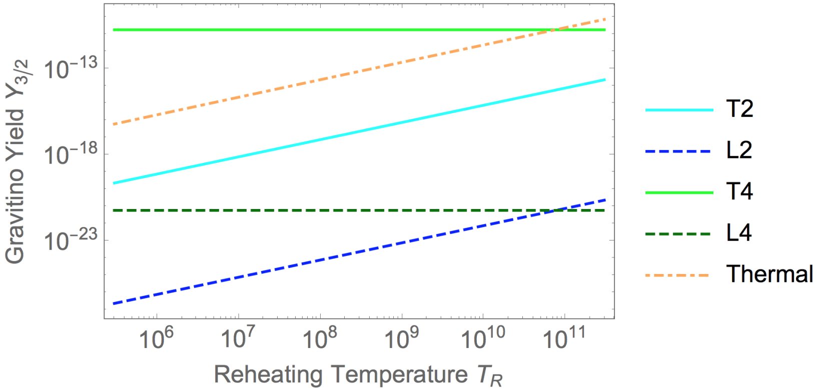

Figure: Dependence of gravitino abundance $Y_{3/2} = n_{3/2}/s$ on the reheating temperature $T_{\rm R}$. Parameters are set as $m_{3/2} = 10^6$ GeV and $H_{\rm inf} = 10^{13}$ GeV.

Summary of this section

Quadratic inflaton potential

- The transverse gravitino is produced dominantly at $H \sim H_{\text{inf}}$.
- The longitudinal gravitino is produced dominantly at $H \sim m^0_{3/2}$, and suppressed compared to the transverse mode by a factor $(m^0_{3/2}/H_{\text{inf}})^2$.
- The inflatino production is similar to the transverse gravitino.

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- Gravitino (mainly transverse) is copiously produced even for low reheating temperature.

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$$X_{\rm amp} \sim \frac{m^0_{3/2}}{H} \phi_{\rm amp}. \tag{65}$$

Time evolution of the gravitino mass $m_{3/2}$

$$m_{3/2} \simeq \begin{cases} \dfrac{m_\phi \phi^2}{M_{\rm Pl}^2} \dfrac{m^0_{3/2}}{H} + m^0_{3/2} & \text{for} \quad H > m^0_{3/2} \\ \dfrac{m_\phi \phi^2}{M_{\rm Pl}^2} + m^0_{3/2} & \text{for} \quad H < m^0_{3/2} \end{cases}. \tag{66}$$

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Rewrite the model

It is convenient to define $\Phi_\pm \equiv \frac{1}{\sqrt{2}}(\phi \pm X)$ so that

$$K = |\Phi_+|^2 + |\Phi_-|^2 - \frac{1}{4}\left[(\Phi_+ + \Phi_-)^2 + \text{h.c.}\right] + |z|^2 - \frac{|z|^4}{\Lambda^2}, \quad (67)$$

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longitudinal gravitino-fermion system

$$\mathcal{L}_f = -\frac{1}{2}\begin{pmatrix} \overline{\psi_c^{\ell\prime}} & \overline{v_\perp^{(1)\prime}} & \overline{v_\perp^{(2)\prime}} \end{pmatrix}\left[\gamma^0 \partial_0 + i\vec{\gamma}\cdot\vec{k} + a\mathcal{M}\right]\begin{pmatrix} \psi_c^{\ell\prime} \\ v_\perp^{(1)\prime} \\ v_\perp^{(2)\prime} \end{pmatrix}, \quad (69)$$

Mass eigenvalues

$$\begin{cases} \left(m_\phi, \quad -m_\phi, \quad -m_{3/2}(m_{3/2}^0/H)^2\right) & \text{for} \quad H \gtrsim m_{3/2}^0 \\ \left(m_\phi, \quad -m_\phi, \quad -m_{3/2}\right) & \text{for} \quad H \lesssim m_{3/2}^0 \end{cases}. \quad (70)$$

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Time dependence of various quantities

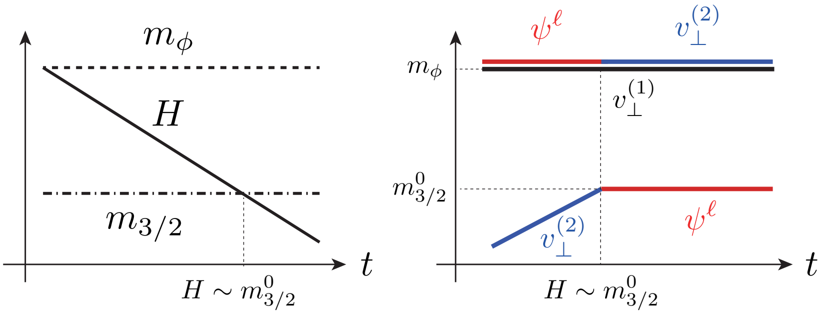

Figure: Same as Fig. 2 but for multi-superfield inflation models.

Transverse gravitino production

The production rate is suppressed by the small stabilizer amplitude.

$$\Gamma(\phi\phi \to \psi^t\psi^t) \simeq \begin{cases} \frac{\mathcal{C}}{4\pi}\frac{\phi_{\mathrm{amp}}^2}{M_{\mathrm{Pl}}^2}\frac{m_\phi^3}{M_{\mathrm{Pl}}^2}\left(\frac{m_{3/2}^0}{H}\right)^2 \simeq \frac{3\mathcal{C}}{4\pi}\frac{(m_{3/2}^0)^2 m_\phi}{M_{\mathrm{Pl}}^2} & \text{for} \quad H > m_{3/2}^0 \\ \frac{\mathcal{C}}{4\pi}\frac{\phi_{\mathrm{amp}}^2}{M_{\mathrm{Pl}}^2}\frac{m_\phi^3}{M_{\mathrm{Pl}}^2} \simeq \frac{3\mathcal{C}}{4\pi}\frac{H^2 m_\phi}{M_{\mathrm{Pl}}^2} & \text{for} \quad H < m_{3/2}^0 \end{cases}. \tag{71}$$

Transverse gravitino yield

$$\frac{n_{3/2}^{(t)}}{s} \simeq \left(\frac{\Gamma(\phi\phi \to \psi^t\psi^t)}{H}\right)_{H=m_{3/2}^0}\frac{3T_{\mathrm{R}}}{4m_\phi} = \frac{9\mathcal{C}}{16\pi}\frac{m_{3/2}^0 T_{\mathrm{R}}}{M_{\mathrm{Pl}}^2}$$

$$\simeq 3\times 10^{-22}\,\mathcal{C}\left(\frac{m_{3/2}^0}{10^6\,\mathrm{GeV}}\right)\left(\frac{T_{\mathrm{R}}}{10^{10}\,\mathrm{GeV}}\right). \tag{72}$$

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Longitudinal gravitino production

Two sources of logitudinal gravitino

- $v_\perp^{(2)}$ produced when $H > m_{3/2}^0$ becomes ψ^ℓ later.

$$\Gamma(\phi\phi \to v_\perp^{(2)} v_\perp^{(2)}) \lesssim \frac{\mathcal{C}}{4\pi}\left(\frac{m_{3/2}^0}{H}\right)^6 \frac{\phi_{\rm amp}^2}{M_{\rm Pl}^2}\frac{m_\phi^3}{M_{\rm Pl}^2} \simeq \frac{3\mathcal{C}}{4\pi}\left(\frac{m_{3/2}^0}{H}\right)^4 \frac{(m_{3/2}^0)^2 m_\phi}{M_{\rm Pl}^2}, \quad (73)$$

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Inflatino and stabilizino production

Heavy fermion states have oscillating parts in their masses.

$$m^{\pm}_{\text{heavy}} \simeq \pm m_\phi + 2\alpha^2_{\pm}\widehat{m}^{\pm}_{3/2}, \tag{76}$$

where $\widehat{m}_{3/2}$ has an oscillating term of $\mathcal{O}(H)$.

Inflatino/stabilizino yields

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Scalar mass matrix

$$V = (\phi\ X) \begin{pmatrix} (\lambda\phi^{n-1})^2 & -2m_{3/2}^0(\lambda\phi^{n-1}) \\ -2m_{3/2}^0(\lambda\phi^{n-1}) & n^2(\lambda\phi^{n-1})^2 \end{pmatrix} \begin{pmatrix} \phi \\ X \end{pmatrix}. \tag{80}$$

For $n \neq 1$, the masses are not degenerate.

Induced oscillation amplitude of stabilizer X ($n \neq 1$)

$$X_{\rm amp} \sim \frac{m_{3/2}^0}{m_\phi}\phi_{\rm amp}, \qquad \text{with} \qquad m_\phi \equiv \lambda\phi_{\rm amp}^{n-1}. \tag{81}$$

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$$m_{3/2} \simeq \begin{cases} \frac{m_{3/2}^0\phi^2}{M_{\rm Pl}^2} + m_{3/2}^0 & \text{for} \quad m_\phi > m_{3/2}^0 \\ \frac{m_\phi\phi^2}{M_{\rm Pl}^2} + m_{3/2}^0 & \text{for} \quad m_\phi < m_{3/2}^0 \end{cases}. \tag{82}$$

Higher power inflaton potential

$$K = -\frac{1}{2}(\phi - \phi^\dagger)^2 + |X|^2 + |z|^2 - \frac{|z|^4}{\Lambda^2}, \tag{78}$$

$$W = \lambda X \phi^n + \mu^2 z + W_0. \tag{79}$$

Scalar mass matrix

$$V = (\phi\ X) \begin{pmatrix} (\lambda\phi^{n-1})^2 & -2m_{3/2}^0(\lambda\phi^{n-1}) \\ -2m_{3/2}^0(\lambda\phi^{n-1}) & n^2(\lambda\phi^{n-1})^2 \end{pmatrix} \begin{pmatrix} \phi \\ X \end{pmatrix}. \tag{80}$$

For $n \neq 1$, the masses are not degenerate.

Induced oscillation amplitude of stabilizer X $(n \neq 1)$

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The transverse gravitino production is suppressed by $(m^0_{3/2}/m_\phi)^2$ compared to the case without a stabilizer field.

For a quartic potential ($n = 2$),

Gravitino yields

$$\frac{n^{(t)}_{3/2}}{s} \simeq \frac{9\mathcal{C}}{16\pi}\left(\frac{90}{\pi^2 g_*}\right)^{1/4}\left(\frac{(m^0_{3/2})^2}{H_{\rm inf}^{1/2} M_{\rm Pl}^{3/2}}\right) \simeq 7\times 10^{-24}\mathcal{C}\left(\frac{H_{\rm inf}}{10^{13}\,{\rm GeV}}\right)^{-1/2}\left(\frac{m^0_{3/2}}{10^6\,{\rm GeV}}\right) \quad (83)$$

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Summary of gravitino abundance

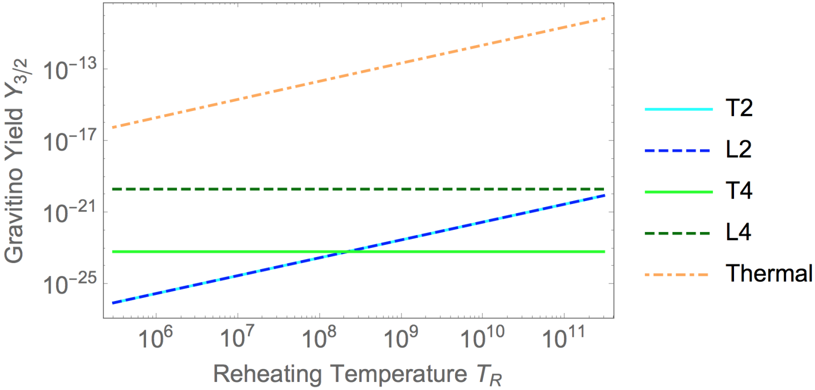

Figure: Dependence of gravitino abundance $Y_{3/2} = n_{3/2}/s$ on the reheating temperature $T_{\rm R}$. Parameters are set as $m_{3/2} = 10^6$ GeV and $H_{\rm inf} = 10^{13}$ GeV.

Summary of this section

quadratic inflaton potential

- The transverse gravitino is produced dominantly at $H \sim m^0_{3/2}$, and suppressed compared to the case without a stabilizer field.
- The longitudinal gravitino is produced dominantly at $H \sim m^0_{3/2}$.
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- We studied gravitino production by preheating with focuses on (Z_2-symmetric) large-field models.
- Our setup is way more realistic than the previous works: shift symmetry of the inflaton Kähler potential, with or without a stabilizer field, no Polonyi problem, and gravitino as a time-dependent mass eigenstate.
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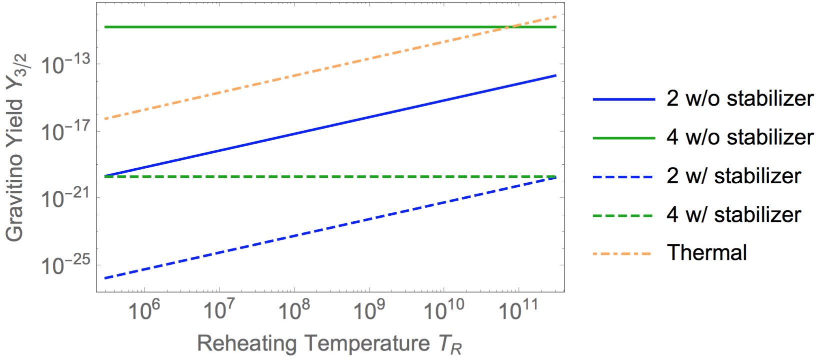

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Conclusion

Without a stabilizer field X

- Longitudinal gravitino abundance is well suppressed.
- For a quadratic potential, transverse gravitino abundance is less than thermal gravitino abundance.
- For a quartic potential, transverse gravitino abundance is large irrespective of the reheating temperature, and cosmologically problematic.

With a stabilizer field X

- Due to the small induced oscillation of X, the oscillation of the gravitino masses is suppressed.
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Prospects

We can generalize this work to the following cases:

- Non-minimal Kähler potential
  → General treatment becomes technically involved.
- Complex scalar configurations
  → We cannot neglect the auxiliary vector field in supergravity.
- D-term inflation
  → Gaugino plays the role of goldstino.
- Constrained superfields such as (orthogonal) nilpotent superfield(s)
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Notations and conventions

Metric

The sign is the $(-,+,+,+)$ convention.

Dirac γ matrices

$\{\widehat{\gamma}^\mu, \widehat{\gamma}^\nu\} = 2g^{\mu\nu}$.

Related to the flat space quantities by $\widehat{\gamma}_\mu = e_\mu^a \gamma_a$.

$(\gamma^0)^\dagger = -\gamma^0$ and $(\vec{\gamma})^\dagger = \vec{\gamma}$.

$\gamma_* \equiv i\gamma_0\gamma_1\gamma_2\gamma_3$, and $P_L \equiv \frac{1+\gamma_*}{2}$, $\quad P_R \equiv \frac{1-\gamma_*}{2}$.

Dirac/Majorana conjugate

$\overline{\psi} \equiv i\psi^\dagger\gamma^0$.

Charge conjugation matrix C: $\gamma^{\mu T} = -C\gamma^\mu C^{-1}$.

Majorana fermion λ satisfies $\lambda = -C^{-1}\overline{\lambda}^T$.

Curvature

$$\omega_\mu{}^{ab} = 2e^{\nu[a}\partial_{[\mu}e_{\nu]}{}^{b]} - e^{\nu[a}e^{b]\sigma}e_{\mu c}\partial_\nu e_\sigma{}^c,$$

$$R_{\mu\nu}{}^{ab} \equiv 2\partial_{[\mu}\omega_{\nu]}{}^{ab} + 2\omega_{[\mu}{}^{ac}\omega_{\nu]c}{}^b, \qquad R_{\mu\nu} \equiv R_{\mu\rho}{}^{ab}e_{a\nu}e_b{}^\rho, \qquad R \equiv g_{\mu\nu}R^{\mu\nu}.$$

More explanations on the derivation of yield quantities

Suppose that gravitinos are produced dominantly at $H \simeq H_*$.

Quadratic potential case

$$\begin{aligned} Y_{3/2} \equiv \frac{n_{3/2}}{s} &= \frac{\left(n_\phi \frac{\Gamma(\phi\phi\to\psi\psi)}{H}\right)\Big|_{H=H_*} \left(\frac{a_*}{a_\mathrm{R}}\right)^3}{\frac{4\rho_{\mathrm{rad,\ R}}}{3T_\mathrm{R}}} \\ &= \frac{3T_\mathrm{R}}{4\rho_{\mathrm{rad,\ R}}} \frac{\rho_{\phi,*}\left(\frac{a_*}{a_\mathrm{R}}\right)^3}{m_\phi} \left(\frac{\Gamma(\phi\phi\to\psi\psi)}{H}\right)\Big|_{H=H_*} \\ &= \frac{3T_\mathrm{R}}{4m_\phi}\left(\frac{\Gamma(\phi\phi\to\psi\psi)}{H}\right)\Big|_{H=H_*} \end{aligned} \tag{85}$$

because $\rho_{\phi,*}\left(\frac{a_*}{a_\mathrm{R}}\right)^3 = \rho_{\phi,\mathrm{R}}^{(\text{just before decay})} = \rho_{\mathrm{rad,\ R}}^{(\text{just after decay})}$.

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Here, we defined $T_* \equiv T_{\rm R}\left(\frac{a_{\rm R}}{a_*}\right) = \left(\frac{90}{\pi^2 g_*}\right)^{1/4}\sqrt{H_* M_{\rm Pl}}$.

Linear term in the inflaton Kahler potential

If the inflaton does not have a Z_2 symmetry, and the Kahler potential has the linear sinflaton term,

$$K = ic(\phi - \phi^\dagger) + \ldots, \tag{87}$$

the gravitino production is significantly enhanced.
For definiteness, consider the quadratic potential case ($n = 2$) without a stabilizer field. There is a mixing between ϕ and z induced by the following term,

$$V \supset (D_\phi W)\overline{(D_\phi W)} \supset \frac{K_\phi W}{M_{\rm Pl}^2} m_\phi \phi^* + \text{h.c.} \simeq \frac{\sqrt{3} i c m_{3/2} m_\phi}{M_{\rm Pl}} z\phi^* + \text{h.c.}. \tag{88}$$

Mixing angle

$$\theta_{\phi z} \sim \begin{cases} \dfrac{\sqrt{3} c m_{3/2} m_\phi}{m_z^2 M_{\rm Pl}} & \text{for} \quad m_\phi < m_z \\ \dfrac{\sqrt{3} c m_{3/2}}{m_\phi M_{\rm Pl}} & \text{for} \quad m_\phi > m_z \end{cases}. \tag{89}$$

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Gravitino production rate

$$\dot{n}_{3/2}^{(\ell)} \simeq \frac{2\rho_z}{m_z}\Gamma(z \to \psi^\ell \psi^\ell) \simeq \frac{2\rho_\phi}{m_\phi}\frac{m_\phi}{m_z}\theta_{\phi z}^2 \Gamma(z \to \psi^\ell \psi^\ell), \tag{92}$$

where

$$\Gamma(z \to \psi^\ell \psi^\ell) \simeq \frac{1}{96\pi}\frac{m_z^5}{(m_{3/2}^0)^2 M_{\rm Pl}^2}. \tag{93}$$

Inflaton partial decay rate

$$\Gamma(\phi \to \psi^\ell \psi^\ell) \simeq \frac{m_\phi}{m_z}\theta_{\phi z}^2 \Gamma(z \to \psi^\ell \psi^\ell) \simeq \begin{cases} \dfrac{c^2 m_\phi^3}{32\pi M_{\rm Pl}^4} & \text{for} \quad m_\phi < m_z \\ \dfrac{c^2 m_\phi^3}{32\pi M_{\rm Pl}^4}\left(\dfrac{m_z}{m_\phi}\right)^4 & \text{for} \quad m_\phi > m_z \end{cases}. \tag{94}$$

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If the inflaton does not have a Z_2 symmetry, and the Kähler potential has the linear sinflaton term,

$$K = ic(\phi - \phi^\dagger) + \dots, \tag{95}$$

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The result is consistent with [Endo et al., 2007, Nakayama et al., 2012].

(Longitudinal) gravitino yield

$$\frac{n_{3/2}^{(\ell)}}{s} \simeq \left(\frac{2\Gamma(\phi \to \psi^\ell \psi^\ell)}{H}\right)_{H=\Gamma_{\rm inf}} \frac{3T_{\rm R}}{4m_\phi} \simeq \frac{3c^2 m_\phi^2}{64\pi M_{\rm Pl}^3 T_{\rm R}} \left(\frac{90}{\pi^2 g_*}\right)^{1/2}$$
$$\simeq 1 \times 10^{-5} \left(\frac{c}{M_{\rm Pl}}\right)^2 \left(\frac{m_\phi}{10^{13}\,{\rm GeV}}\right)^2 \left(\frac{10^{10}\,{\rm GeV}}{T_{\rm R}}\right). \tag{96}$$

Very violent production occurs unless c is suppressed!

Quantization I

Consider a fermion with an oscillating mass $m(t)$ with its frequency Ω,

$$\mathcal{L} = -\frac{1}{2}\overline{\psi}\left(\partial\!\!\!/ - m(t)\right)\psi. \tag{97}$$

Creation and annihilation operators

The Fourier mode $\psi_{\vec{k}}(t) = \int \frac{\mathrm{d}^3 x}{(2\pi)^{3/2}} e^{-i\vec{k}\cdot\vec{x}}\psi(t,x)$ is expanded as

$$\psi_{\vec{k}}(t) = \sum_s \left[u_{\vec{k},s}(t)\hat{b}_{\vec{k},s} + v_{\vec{k},s}(t)\hat{b}^{\dagger}_{-\vec{k},s}\right], \tag{98}$$

where $v_{\vec{k},s}(t) = -C^{-1}\overline{u}^T_{-\vec{k},s}(t)$, and mode functions are orthonormal, and the creation/annihilation operators satisfy the standard canonical anti-commutation relations.

Helicity basis

$$u_{\vec{k},h}(t) = \begin{pmatrix} u^+_{\vec{k},h}(t) \\ u^-_{\vec{k},h}(t) \end{pmatrix} \otimes \xi_{\vec{k},h}, \quad v_{\vec{k},h}(t) = \begin{pmatrix} -u^-_{-\vec{k},h}(t) \\ u^+_{-\vec{k},h}(t) \end{pmatrix}^* \otimes \xi'_{-\vec{k},h}. \tag{99}$$

Quantization II

Here $\xi_{\vec{k},h}$ is the normalized eigenvector of helicity $h = \pm 1$, satisfying $(\vec{\sigma} \cdot \hat{\vec{k}})\xi_{\vec{k},h} = h\xi_{\vec{k},h}$. $\hat{\vec{k}} \equiv \vec{k}/k$ is a unit vector. We have also defined $\xi'_{\vec{k},h} \equiv -i\sigma^2 \xi^*_{\vec{k},h}$, which satisfies $(\vec{\sigma} \cdot \hat{\vec{k}})\xi'_{\vec{k},h} = -h\xi'_{\vec{k},h}$.

Now, the normalization condition becomes $\left|u^+_{\vec{k},h}\right|^2 + \left|u^-_{\vec{k},h}\right|^2 = 1$.

Equation of motion

$$i\partial_0 u^+_{\vec{k},h} + hku^-_{\vec{k},h} = m(t)u^+_{\vec{k},h}, \tag{100}$$

$$i\partial_0 u^-_{\vec{k},h} + hku^+_{\vec{k},h} = -m(t)u^-_{\vec{k},h}. \tag{101}$$

Combining them, we obtain

$$0 = \ddot{u}^+_{\vec{k},h}(t) + \widetilde{\omega}^2_{\vec{k}}(t)u^+_{\vec{k},h}(t). \tag{102}$$

$$0 = \ddot{u}^-_{\vec{k},h}(t) + \widetilde{\omega}^2_{\vec{k}}(t)^* u^-_{\vec{k},h}(t). \tag{103}$$

where $\widetilde{\omega}^2_{\vec{k}}(t) \equiv \omega^2_{\vec{k}}(t) + i\dot{m}(t)$ and $\omega^2_{\vec{k}}(t) \equiv m^2(t) + k^2$.

Quantization III

Vacuum initial conditions

$$u^{+}_{\vec{k},h}(t\to 0) =\sqrt{\frac{\omega_{\vec{k}}(0)+m(0)}{2\omega_{\vec{k}}(0)}}, \quad \dot{u}^{+}_{\vec{k},h}(t\to 0) = -i\omega_{k}(0)u^{+}_{\vec{k},h}(t\to 0), \quad (104)$$

$$u^{-}_{\vec{k},h}(t\to 0) = -h\sqrt{\frac{\omega_{\vec{k}}(0)-m(0)}{2\omega_{\vec{k}}(0)}}, \quad \dot{u}^{-}_{\vec{k},h}(t\to 0) = -i\omega_{\vec{k}}(0)u^{-}_{\vec{k},h}(t\to 0). \quad (105)$$

Particle production I

Hamiltonian density

$$\langle \mathcal{H}(t) \rangle = \left\langle \psi^\dagger i\partial_0 \psi \right\rangle$$
$$= \int \frac{\mathrm{d}^3k}{(2\pi)^3} \frac{1}{2} \sum_h \left[m(t) \left(\left| u^-_{\vec{k},h}(t) \right|^2 - \left| u^+_{\vec{k},h}(t) \right|^2 \right) + 2hk\Re \left(u^+_{\vec{k},h}(t) u^{-*}_{\vec{k},h}(t) \right) \right]$$
$$= 2 \times \int \frac{\mathrm{d}^3k}{(2\pi)^3} \omega_{\vec{k}}(t) \left(f_\psi(\vec{k};t) - \frac{1}{2} \right) \tag{106}$$

where the phase space density is given by

$$f_\psi(\vec{k};t) \equiv \frac{1}{2\omega(t)} \left[m(t) \left(\left| u^-_{\vec{k}}(t) \right|^2 - \left| u^+_{\vec{k}}(t) \right|^2 \right) + 2hk\Re \left(u^+_{\vec{k}}(t) u^{-*}_{\vec{k}}(t) \right) \right] + \frac{1}{2}$$
$$= \frac{1}{2\omega(t)} \left[m(t) + 2\Im \left(u^{+*}_{\vec{k}}(t) \partial_0 u^+_{\vec{k}}(t) \right) \right] + \frac{1}{2}. \tag{107}$$

Particle production II

Number density

$$n_\psi(t) = 2 \times \int \frac{\mathrm{d}^3 k}{(2\pi)^3} f_\psi(\vec{k}; t), \tag{108}$$

Ansatz of the mode function

$$u^{+}_{\vec{k},h}(t) = \frac{A_{k,h}(t)}{\sqrt{2\widetilde{\omega}_{\vec{k}}(t)}} e^{-i\int^t \mathrm{d}\tau \widetilde{\omega}_{\vec{k}}(\tau)} + \frac{B_{k,h}(t)}{\sqrt{2\widetilde{\omega}_{\vec{k}}(t)}} e^{i\int^t \mathrm{d}\tau \widetilde{\omega}_{\vec{k}}(\tau)}, \tag{109}$$

where

$$\dot{A}_k(t) = \frac{\dot{\widetilde{\omega}}_{\vec{k}}(t)}{2\widetilde{\omega}_{\vec{k}}(t)} e^{2i\int^t \mathrm{d}\tau \widetilde{\omega}_{\vec{k}}(\tau)} B_k(t), \quad \dot{B}_k(t) = \frac{\dot{\widetilde{\omega}}_{\vec{k}}(t)}{2\widetilde{\omega}_{\vec{k}}(t)} e^{-2i\int^t \mathrm{d}\tau \widetilde{\omega}_{\vec{k}}(\tau)} A_k(t). \tag{110}$$

This satisfies the equation of motion. We solve the time evolution of $A(t)$ and $B(t)$ perturbatively in time.

Particle production III

Initial conditions of A and B

$$A_k(t \to 0) = \sqrt{\omega_{\vec{k}}(0) + m(0)}, \quad B(t \to 0) = 0. \tag{111}$$

Initially, we expect $A_{\vec{k}} \simeq \sqrt{\omega_{\vec{k}} + m}$ and $B_{\vec{k}} \simeq 0$ at the leading order.

For modes with $k^2 \gg m^2$, we obtain

$$B_{\vec{k}}(t) \simeq \int_0^t \mathrm{d}t' \frac{m\dot{m} + i\ddot{m}/2}{2\widetilde{\omega}_{\vec{k}}^2} A_{\vec{k}}(0) e^{-2i\int^{t'} \mathrm{d}\tau \widetilde{\omega}_{\vec{k}}(\tau)} \simeq -iA_{\vec{k}}(0) \int_0^t \mathrm{d}t'\, m(t')\, e^{-2i\omega_{\vec{k}} t'}. \tag{112}$$

For given time t, the integration cancels out due to oscillations of the phase except for $\Omega - \Delta\Omega \lesssim 2\omega_{\vec{k}} \lesssim \Omega + \Delta\Omega$ with $\Delta\Omega \sim 1/t$.

$$B_{\vec{k}}(t) \simeq -\frac{i}{2} A_{\vec{k}}(0) \widetilde{m} t \quad \text{for} \quad \Omega - \frac{1}{t} \lesssim 2\omega_{\vec{k}} \lesssim \Omega + \frac{1}{t}, \tag{113}$$

Particle production IV

where $\widetilde{m}$ stands for the amplitude of oscillating $m(t)$. Similarly,

$$A_{\vec{k}}(t) \simeq A_{\vec{k}}(0) - iB'_{\vec{k}}(0)\widetilde{m}\frac{t^2}{4} \simeq A_{\vec{k}}(0)\left(1 - \frac{\widetilde{m}^2 t^2}{8}\right) \quad \text{for} \quad \Omega - \frac{1}{t} \lesssim 2\omega_{\vec{k}} \lesssim \Omega + \frac{1}{t}. \tag{114}$$

Growth of number density

$$f_\psi(\vec{k};t) \simeq \frac{\widetilde{m}^2 t^2}{4} \quad \text{for} \quad \Omega - \frac{1}{t} \lesssim 2\omega_{\vec{k}} \lesssim \Omega + \frac{1}{t}. \tag{115}$$

This expression is valid as long as $f_\psi \ll 1$, namely $q\Omega t \lesssim 1$ with the resonance parameter being $q \equiv \widetilde{m}^2/\Omega^2 \ll 1$. Integrating this,

$$n_\psi(t) \simeq \frac{\mathcal{C}}{16\pi}\Omega^2 \widetilde{m}^2 t. \tag{116}$$

$\mathcal{C}$ is an $\mathcal{O}(1)$ parameter depending on the details of the oscillation. For example, $\mathcal{C} = 1$ for $m(t) \propto \cos(\Omega t)$.

Particle production V

Oscillation induced by a scalar field

Suppose

$$\phi(t) \simeq \phi_{\rm amp}\cos(m_\phi t), \qquad m(t) \propto \phi^n(t). \tag{117}$$

This involves $\Omega = j m_\phi$ with $j = n, n-2, n-4, \ldots$.

Example

For $n = 1$, $\Omega = m_\phi$, and we can interpret the process as decay of ϕ,

$$\Gamma(\phi \to \psi\psi) \sim \frac{n_\psi}{2n_\phi t} \sim \frac{\mathcal{C}}{32\pi}\frac{\widetilde{m}^2}{\phi_{\rm amp}^2} m_\phi \tag{118}$$

Example

For $n = 2$, $\Omega = 2m_\phi$, and we can interpret the process as annihilation of ϕ,

$$\Gamma(\phi\phi \to \psi\psi) \sim \frac{n_\psi}{n_\phi t} \sim \frac{\mathcal{C}}{4\pi}\frac{\widetilde{m}^2}{\phi_{\rm amp}^2} m_\phi \tag{119}$$

Small-field model without a stabilizer field

Single-field new inflation model [Izawa and Yanagida, 1997]

$$K = |\phi|^2 + |z|^2 - \frac{|z|^4}{\Lambda^2}, \tag{120}$$

$$W = \phi\left(M^2 - \frac{\lambda\phi^n}{n+1}\right) + \mu^2 z. \tag{121}$$

Expansion around the vacuum: $\phi = \langle\phi\rangle + \delta\phi$ with $\langle\phi\rangle = (M^2/\lambda)^{1/n}$.

$$K = \langle\phi\rangle\left(\delta\phi + \delta\phi^\dagger\right) + |\delta\phi|^2 + |z|^2 - \frac{|z|^4}{\Lambda^2}, \tag{122}$$

$$W \simeq \frac{1}{2}m_\phi(\delta\phi)^2 + \mu^2 z + W_0 - m_{3/2}^0\langle\phi\rangle\delta\phi, \tag{123}$$

where $m_\phi = nM^2/\langle\phi\rangle$ and $W_0 = \frac{n}{n+1}\langle\phi\rangle M^2 = m_{3/2}^0 M_{\rm Pl}^2$.

This is similar to the chaotic inflation with a linear term in the Kähler potential with $c \sim \langle\phi\rangle$. The inflaton decays through the mixing with z, and the rate is consistent with [Endo et al., 2006b, Endo et al., 2007].

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where $m_\phi = nM^2/\langle\phi\rangle$ and $W_0 = \frac{n}{n+1}\langle\phi\rangle M^2 = m^0_{3/2}M^2_{\rm Pl}$.

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Small-field model with a stabilizer field

Multi-field new inflation model [Asaka et al., 2000, Senoguz and Shafi, 2004]:

$$K = |\phi|^2 + |X|^2 + |z|^2 - \frac{|z|^4}{\Lambda^2}, \quad (124)$$

$$W = X(M^2 - \lambda\phi^n) + \mu^2 z + W_0. \quad (125)$$

Expansion around the vacuum: $\phi = \langle\phi\rangle + \delta\phi$ with $\langle\phi\rangle \simeq (M^2/\lambda)^{1/n}$ and $\langle X\rangle \simeq 0$.

$$K = \langle\phi\rangle(\delta\phi + \delta\phi^\dagger) + |\delta\phi|^2 + |X|^2 + |z|^2 - \frac{|z|^4}{\Lambda^2}, \quad (126)$$

$$W \simeq m_\phi X\delta\phi + \mu^2 z + W_0, \quad (127)$$

where $m_\phi = nM^2/\langle\phi\rangle$.

This is similar to the chaotic inflation with a linear term in the Kähler potential with $c \sim \langle\phi\rangle$. Since ϕ and X mixes maximally, and there is a mixing term $\sim \langle\phi\rangle X z^*$, the decay rate is similar to the previous case at least for $H \lesssim m_{3/2}$. The inflaton decays through the mixing with z, and the rate is consistent with [Endo et al., 2006b, Endo et al., 2007].

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where $m_\phi = nM^2/\langle\phi\rangle$.

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Induction of oscillation through mixing I

Potential of two real scalars ϕ_1 and ϕ_2

$$V = \frac{1}{2}(\phi_1\ \phi_2)\mathcal{M}^2 \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \quad \mathcal{M}^2 = \begin{pmatrix} m_1^2 & m_{12}^2 \\ m_{12}^2 & m_2^2 \end{pmatrix}. \tag{128}$$

where we assume $|m_1 m_2| > m_{12}^2$ (no tachyon).

Initial condition: $(\phi_1, \phi_2) = (\phi_i, 0)$.

Mass eigenvalues

$$V = \frac{1}{2}(\phi_1'\ \phi_2')\mathcal{M}'^2 \begin{pmatrix} \phi_1' \\ \phi_2' \end{pmatrix}, \quad \mathcal{M}'^2 = \begin{pmatrix} m_1'^2 & 0 \\ 0 & m_2'^2 \end{pmatrix}, \tag{129}$$

where

$$m_1'^2 = \frac{1}{2}\left(m_1^2 + m_2^2 + \frac{|m_1^2 - m_2^2|}{m_1^2 - m_2^2}\sqrt{(m_1^2 - m_2^2)^2 + 4m_{12}^4}\right), \tag{130}$$

$$m_2'^2 = \frac{1}{2}\left(m_1^2 + m_2^2 - \frac{|m_1^2 - m_2^2|}{m_1^2 - m_2^2}\sqrt{(m_1^2 - m_2^2)^2 + 4m_{12}^4}\right), \tag{131}$$

Induction of oscillation through mixing II

Mass eigenstates

$$\begin{pmatrix} \phi_1' \\ \phi_2' \end{pmatrix} = \begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \tag{132}$$

where $c_\theta \equiv \cos\theta$ and $s_\theta \equiv \sin\theta$ with $-\pi/4 < \theta \leq \pi/4$.

$$c_\theta^2 = \frac{1}{2}\left(1 + \sqrt{1 - \frac{4m_{12}^4}{4m_{12}^4 + (m_1^2 - m_2^2)^2}}\right), \tag{133}$$

$$s_\theta^2 = \frac{1}{2}\left(1 - \sqrt{1 - \frac{4m_{12}^4}{4m_{12}^4 + (m_1^2 - m_2^2)^2}}\right). \tag{134}$$

with $\theta \geq 0$ for $(m_1^2 - m_2^2)/m_{12}^2 > 0$ and $\theta < 0$ for $(m_1^2 - m_2^2)/m_{12}^2 < 0$.

Induction of oscillation through mixing III

Solution of the equation of motion

$$\phi_1'(t) = c_\theta \phi_i \left(\frac{a_i}{a(t)}\right)^{3/2} \cos(m_1' t), \tag{135}$$

$$\phi_2'(t) = -s_\theta \phi_i \left(\frac{a_i}{a(t)}\right)^{3/2} \cos(m_2' t), \tag{136}$$

In the original basis, this becomes

$$\phi_1(t) = \phi_i \left(\frac{a_i}{a(t)}\right)^{3/2} \left[c_\theta^2 \cos(m_1' t) + s_\theta^2 \cos(m_2' t)\right], \tag{137}$$

$$\phi_2(t) = -\phi_i \left(\frac{a_i}{a(t)}\right)^{3/2} \sin(2\theta) \sin\left(\frac{(m_1' + m_2')t}{2}\right) \sin\left(\frac{(m_1' - m_2')t}{2}\right). \tag{138}$$

Induction of oscillation through mixing IV

$$\phi_1(t) = \phi_i \left(\frac{a_i}{a(t)}\right)^{3/2} \left[c_\theta^2 \cos(m_1' t) + s_\theta^2 \cos(m_2' t)\right],$$

$$\phi_2(t) = -\phi_i \left(\frac{a_i}{a(t)}\right)^{3/2} \sin(2\theta) \sin\left(\frac{(m_1' + m_2')t}{2}\right) \sin\left(\frac{(m_1' - m_2')t}{2}\right).$$

Induced oscillation (non-degenerate case)

$$\phi_2(t) \sim \sin(2\theta)\phi_i, \tag{139}$$

after a few oscillation.

Induced oscillation (degenerate case)

In the degenerate limit $m_1 = m_2$, we have $m_1' - m_2' \simeq m_{12}^2/m_1$, and

$$\phi_2(t) \simeq -\phi_i \left(\frac{a_i}{a(t)}\right)^{3/2} \sin(m_1 t) \frac{m_{12}^2}{2m_1} t \quad \text{for} \quad t \lesssim \frac{2m_1}{m_{12}^2}. \tag{140}$$

Some definition for later convenience

Remember the definition of θ_i, $\tan\theta_i = -\dot{\phi}_i/F_i$. We can estimate its time derivative as

$$\dot{\theta}_i = \frac{1}{F_i}\left(-\ddot{\phi}_i + \frac{\dot{\phi}_i\dot{\rho}^i_{\rm SB}}{2\rho^i_{\rm SB}}\right) = \frac{\partial_{\phi_i}V}{F_i} - \frac{3m_{3/2}\dot{\phi}_i^2}{\rho^i_{\rm SB}} + \frac{3H\dot{\phi}_iF_i}{\rho^i_{\rm SB}}, \tag{141}$$

which is of the order of $\sim \mathcal{O}(m_{\phi_i}) + \mathcal{O}(m_{3/2}) + \mathcal{O}(H)$. It is also conveniently expressed as

$$\dot{\theta}_i = \frac{\partial_{\phi_i}V}{F_i} - m_{3/2} - \widehat{m}^i_{3/2}, \tag{142}$$

where we have decomposed

$$\widehat{m}_{3/2} = \sum_i \alpha_i^2\widehat{m}^i_{3/2}, \qquad \widehat{m}^i_{3/2} \equiv \frac{3Hp^i_W + m_{3/2}(\rho^i_{\rm SB} + 3p^i_{\rm SB})}{2\rho^i_{\rm SB}}, \tag{143}$$

where

$$p^i_{\rm SB} \equiv |\dot{\phi}_i|^2 - |F_i|^2, \quad p^i_W \equiv -(\dot{\phi}_i^*F_i + \dot{\phi}_iF_i^*). \tag{144}$$

Note that $(\rho^i_{\rm SB})^2 = (p^i_{\rm SB})^2 + |p^i_W|^2$.

Single-superfield model I

Trace and determinant

$$\begin{aligned}\mathrm{Tr}\,\mathcal{M} =& \widehat{m}_{3/2} + m_f - \alpha_1^2\dot{\theta}_1 - \alpha_2^2\dot{\theta}_2 \\ \simeq& m_\phi + 2\alpha_1^2\widehat{m}^1_{3/2} + (\alpha_1^2 - \alpha_2^2)m_{3/2}, \end{aligned} \tag{145}$$

$$\begin{aligned}\det\,\mathcal{M} =& \alpha_1^2\alpha_2^2\sin^2(\theta_1-\theta_2)(\widehat{m}_{3/2} - m_f^c)^2 - \alpha_2\dot{\alpha}_2(\widehat{m}_{3/2} - m_f^c)\sin(2(\theta_1-\theta_2)) \\ & - (\dot{\alpha}_1^2 + \dot{\alpha}_2^2)\sin^2(\theta_1-\theta_2) + \alpha_2^2(\widehat{m}_{3/2} - m_f^c)(\dot{\theta}_2 - \dot{\theta}_1) \\ & + (\widehat{m}_{3/2} - \dot{\theta}_1)(m_f^c - \dot{\theta}_2) \\ \simeq& -\alpha_2^2 m_\phi m_{3/2} + \alpha_1^2\alpha_2^2\sin^2(\theta_1-\theta_2)(\widehat{m}^1_{3/2})^2 - \alpha_1^2\alpha_2^2\left(\widehat{m}^1_{3/2} + m_{3/2}\right)^2 \end{aligned} \tag{146}$$

where $\dot{\theta}_\phi \simeq -m_\phi - m_{3/2} - \widehat{m}^1_{3/2}$, $\dot{\theta}_z \simeq 0$,
$\widehat{m}^1_{3/2} = (m_{3/2} + 3H\sin 2\theta_1 - 3m_{3/2}\cos 2\theta_1)/2$, $\widehat{m}^2_{3/2} \simeq -m_{3/2}$,
$m_f^c \equiv m_f + \alpha_2^2\dot{\theta}_1 + \alpha_1^2\dot{\theta}_2 \simeq -\alpha_2^2(m_{3/2} + \widehat{m}^1_{3/2})$ and it satisfies
$\widehat{m}_{3/2} - m_f^c \simeq \widehat{m}^1_{3/2}$. $\dot{\alpha}_i$ have been neglected because
$\alpha_1\dot{\alpha}_1 = -\alpha_2\dot{\alpha}_2 = \mathcal{O}(\min\left[(m^0_{3/2})^2/H, H^2/(m^0_{3/2}\right]) \lesssim \mathcal{O}(m^0_{3/2})$.

Single-superfield model II

Mass eigenvalues

Since $(\mathrm{Tr}\,\mathcal{M})^2 \gg \det\,\mathcal{M}$, the mass eigenvalues of $\mathcal{M}$ are given by

$$\frac{1}{2}\left(\mathrm{Tr}\,\mathcal{M} \pm \sqrt{(\mathrm{Tr}\,\mathcal{M})^2 - 4\det\,\mathcal{M}}\right) = \begin{cases} \mathrm{Tr}\,\mathcal{M} & \text{for the heavy state,} \\ \det\,\mathcal{M}/\mathrm{Tr}\,\mathcal{M} & \text{for the light state.} \end{cases} \tag{147}$$

Thus, we obtain the eigenvalues
$(m_{\mathrm{heavy}}, m_{\mathrm{light}}) \simeq (m_\phi + 2\alpha_1^2 \widehat{m}_{3/2}^1, -\alpha_2^2 m_{3/2})$.

Multi-superfield model I

Matter fermion mass matrix

$$\widehat{m}_f \simeq m_f \simeq \mathrm{diag}(m_\phi, -m_\phi, 0), \tag{148}$$

in the "light-cone" basis (Φ_+, Φ_-, z) with $\Phi_\pm = (\Phi \pm X)/\sqrt{2}$.

$\dot{\theta}_i$ and $\widehat{m}^i_{3/2}$

$\dot{\theta}_\pm \simeq \mp m_\phi - m_{3/2} - \widehat{m}^\pm_{3/2}$ and $\widehat{m}^\pm_{3/2} \simeq (m_{3/2} - 3m_{3/2}\cos 2\theta_1 \pm 3H \sin 2\theta_1)/2$.

Using these, we can straightforwardly calculate the following quantities.

Trace, determinant, and one more combination

$$\mathrm{Tr}\mathcal{M} = (1-2\alpha_z^2)m_{3/2} + 2\left(\alpha_+^2 \widehat{m}^+_{3/2} + \alpha_-^2 \widehat{m}^-_{3/2}\right), \tag{149}$$

$$\det \mathcal{M} = s_2^2 m_{3/2} m_\phi^2 + \cdots = \alpha_z^2 m_{3/2} m_\phi^2 + \ldots, \tag{150}$$

$$\begin{aligned} m_1 m_2 + m_2 m_3 + m_3 m_1 =& \mathcal{M}_{11}\mathcal{M}_{22} + \mathcal{M}_{22}\mathcal{M}_{33} + \mathcal{M}_{33}\mathcal{M}_{11} \\ &- \mathcal{M}_{12}^2 - \mathcal{M}_{23}^2 - \mathcal{M}_{31}^2 \\ =& - m_\phi^2 - 2(\alpha_+^2 \widehat{m}^+_{3/2} - \alpha_-^2 \widehat{m}^-_{3/2} + \mathcal{O}(m_{3/2}))m_\phi + \ldots, \end{aligned} \tag{151}$$

where m_1, m_2, and m_3 are mass eigenvalues.

Multi-superfield II

Mass eigenvalues

The three mass eigenvalues are
$(m_\phi + 2\alpha_+^2 \widehat{m}_{3/2}^+, -m_\phi + 2\alpha_-^2 \widehat{m}_{3/2}^-, -\alpha_z^2 m_{3/2})$.

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