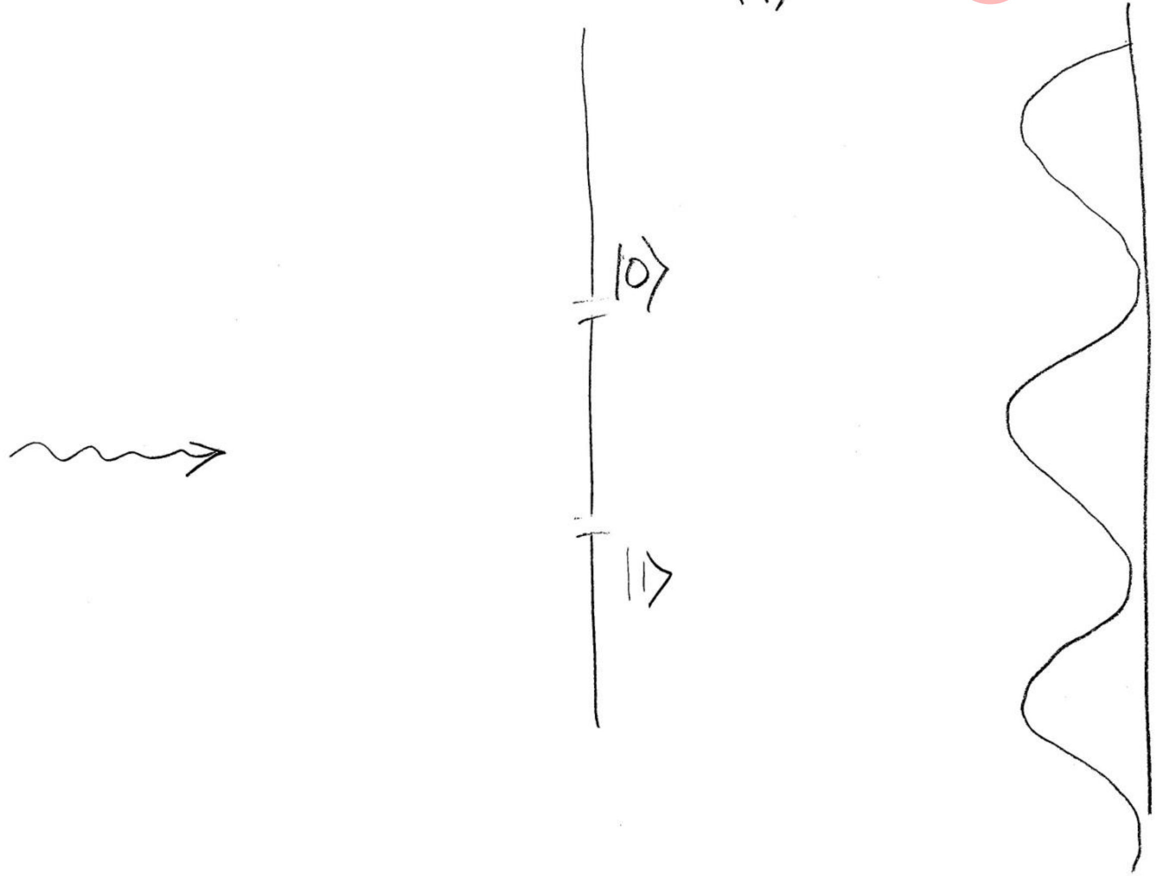
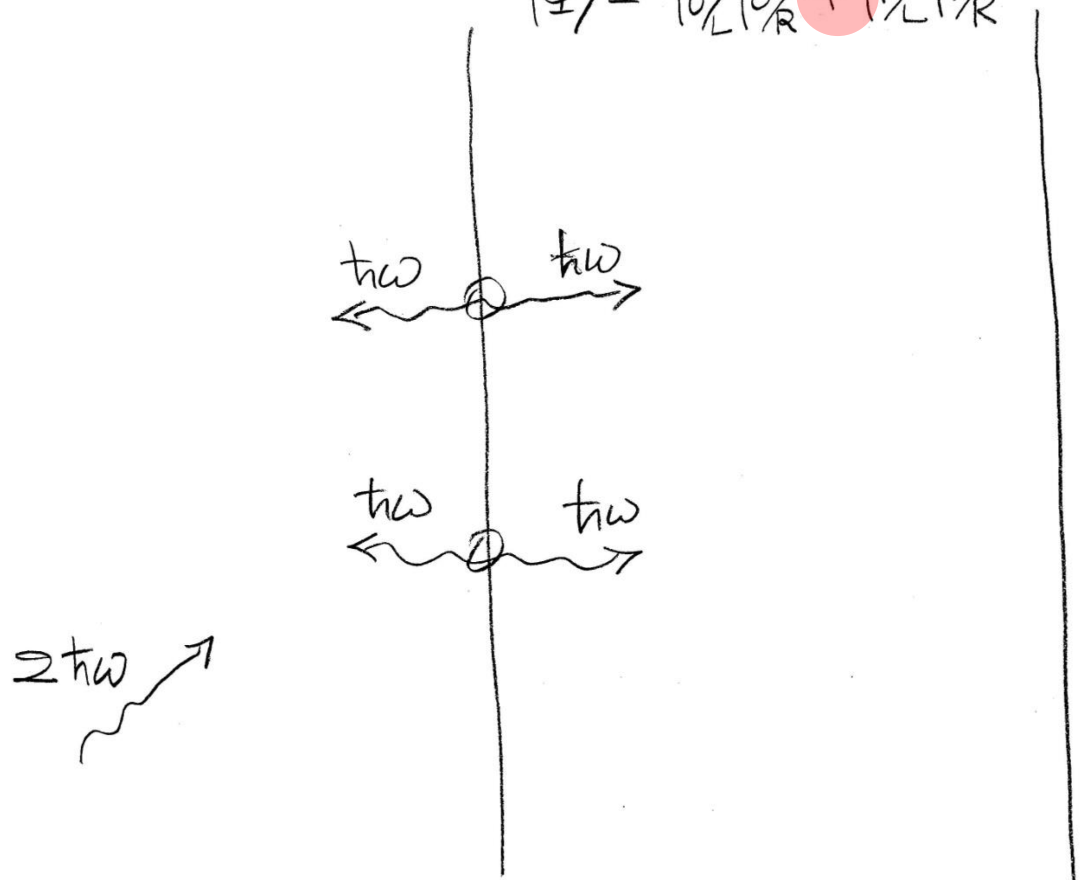
Numerical demonstration of Quantum Thermodynamics of Spin System

Jaewan Kim
Korea Institute for Advanced Study

$$|\phi\rangle = |0\rangle + |1\rangle$$

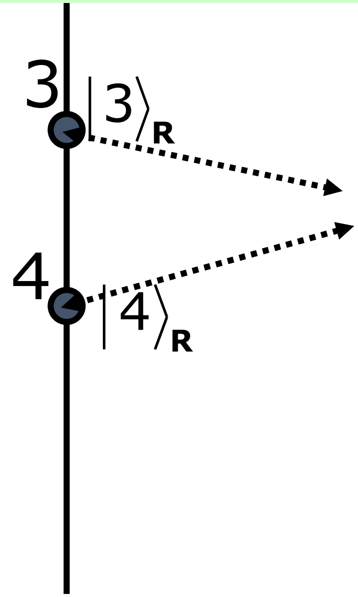


$$|\Phi\rangle = |0\rangle_L |0\rangle_R + |1\rangle_L |1\rangle_R$$



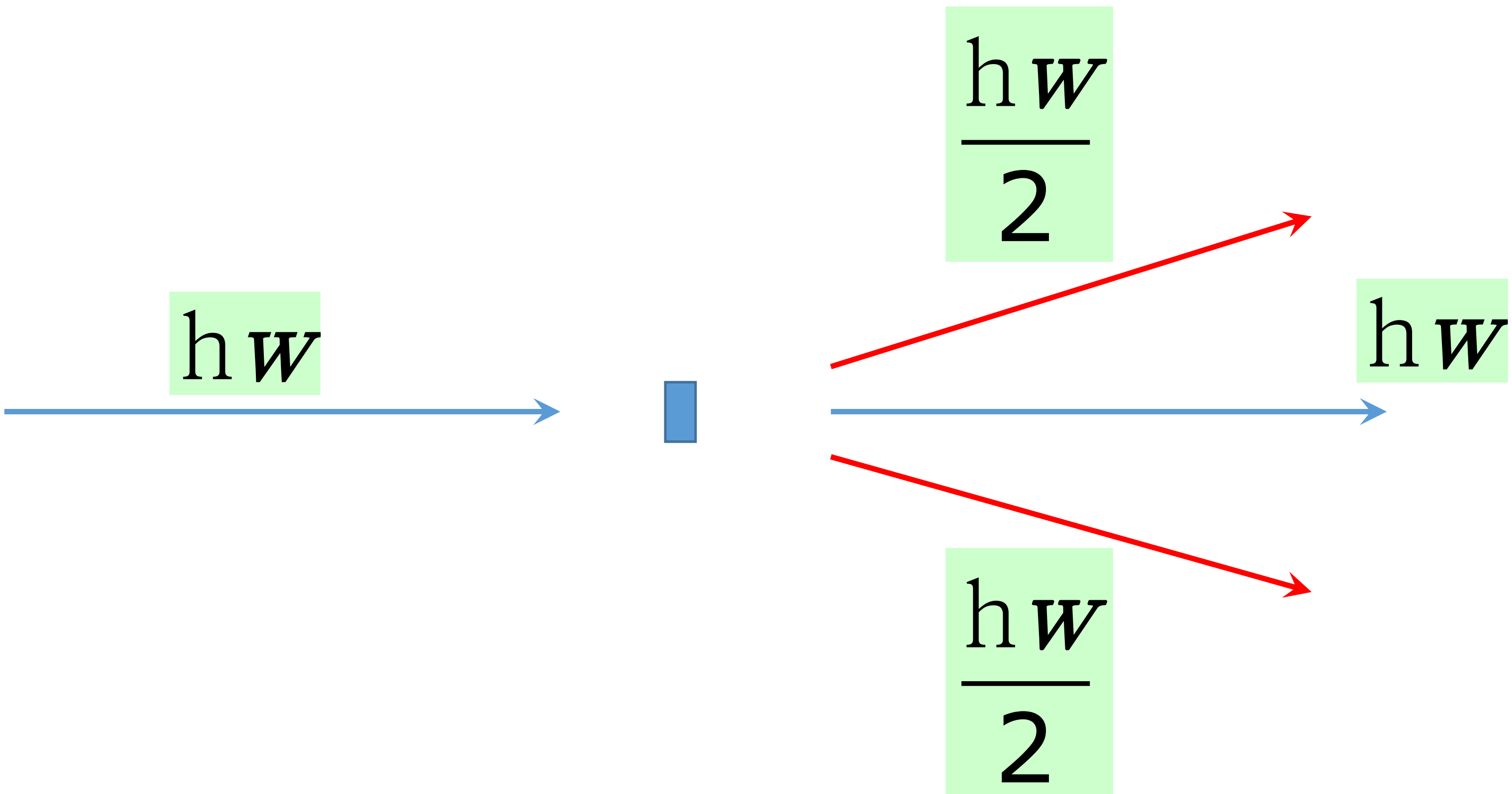
$$|j\rangle = \frac{1}{\sqrt{2}}\left(|3\rangle_{R} + |4\rangle_{R}\right)$$

$$r_{LR} = |j\rangle\langle j| = \begin{array}{c} \\ |3\rangle \\ |4\rangle \end{array} \begin{array}{cc} \langle 3| & \langle 4| \\ \left(\begin{array}{cc} 1/2 & 1/2 \\ 1/2 & 1/2 \end{array}\right) \end{array}$$



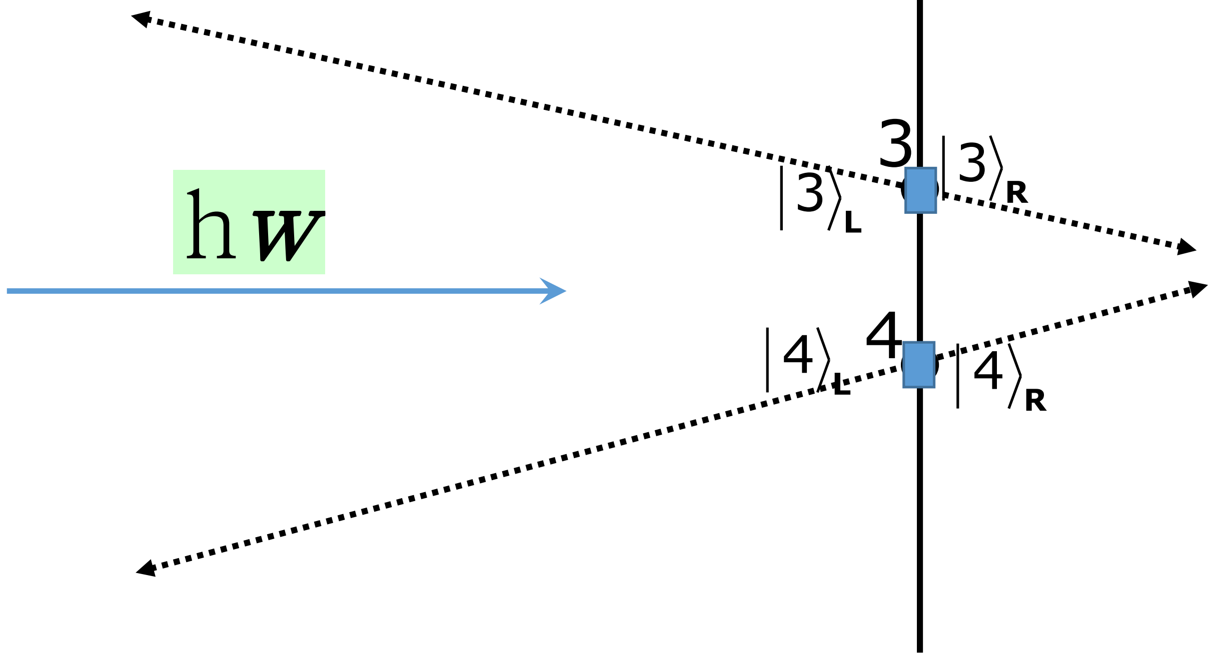
Parametric Down Conversion

: Splitting a photon into two



$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|3\rangle_{L}|3\rangle_{R} + |4\rangle_{L}|4\rangle_{R}\right)$$

$$\rho_{LR} = |\psi\rangle\langle\psi|$$

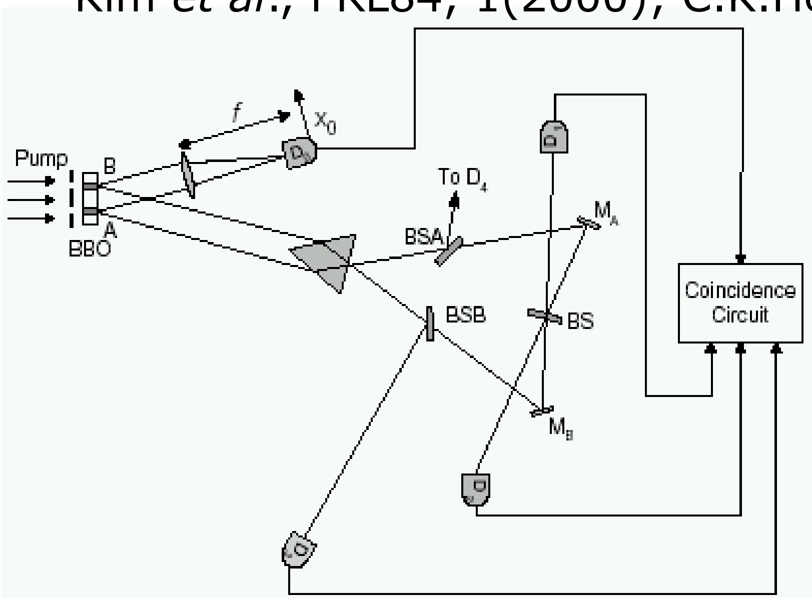


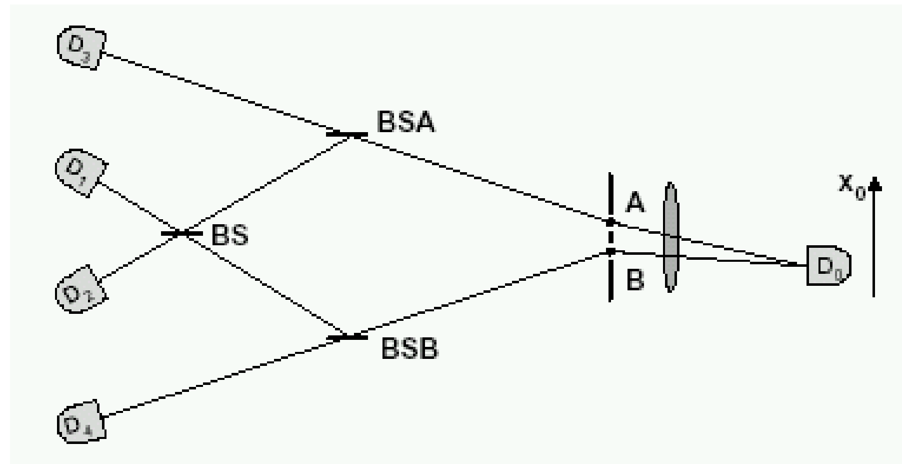
C K Hong and T Noh (1998)

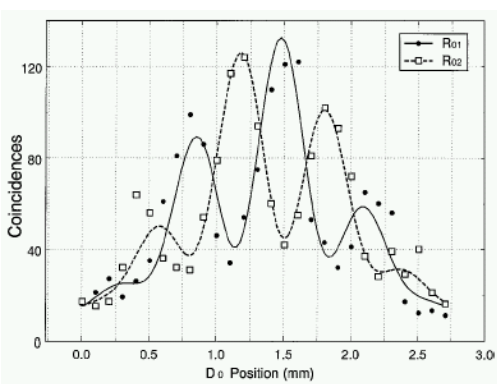
Y-H Kim and Y Shi (2000)

Delayed Choice Quantum Erasure **REVISITED**

Kim *et al*., PRL84, 1(2000); C.K.Hong and T.G.Noh, JOSA B15, 1192(1998)







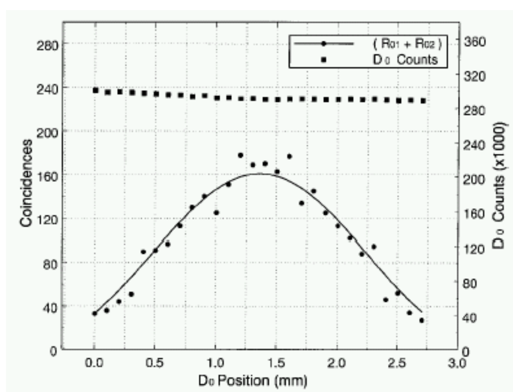


FIG. 4. $R_{01} + R_{02}$ is shown. The solid line is a fit to the sinc function given in Eq. (6). The single counting rate of D_0 is constant over the scanning range.

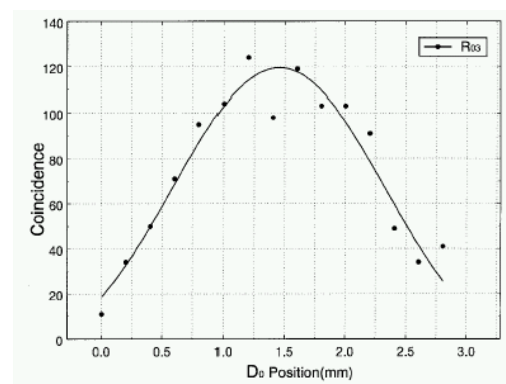
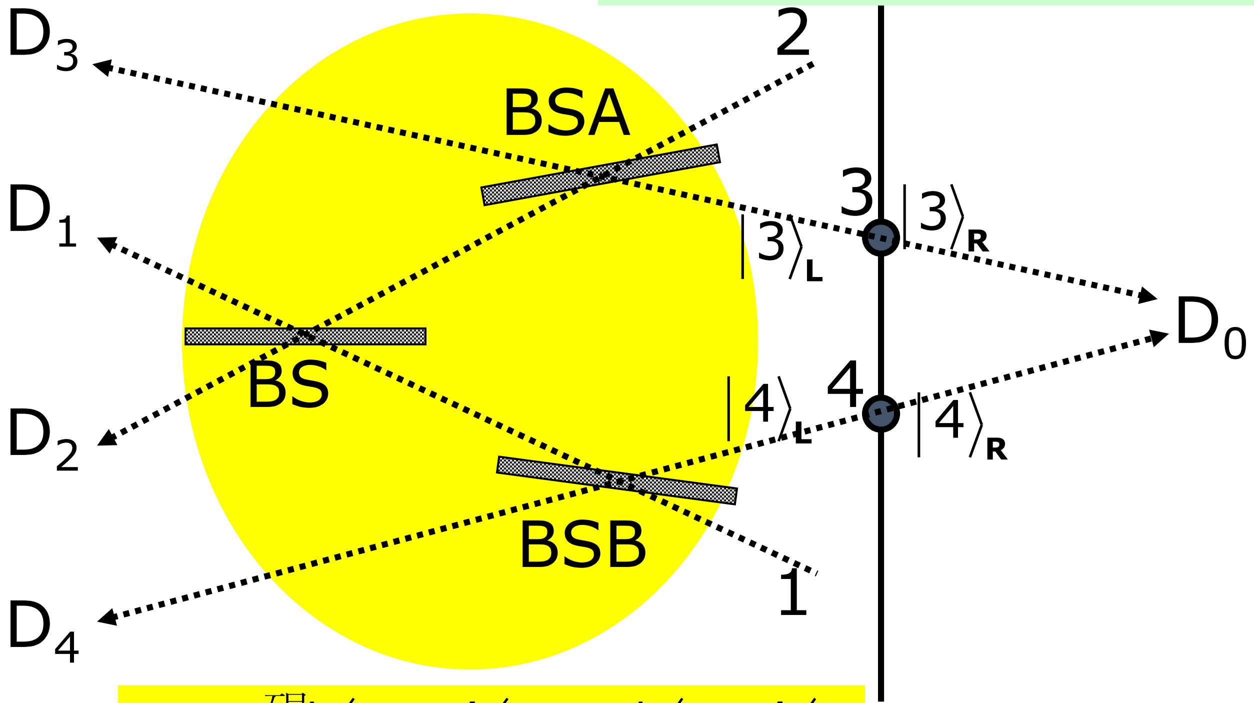


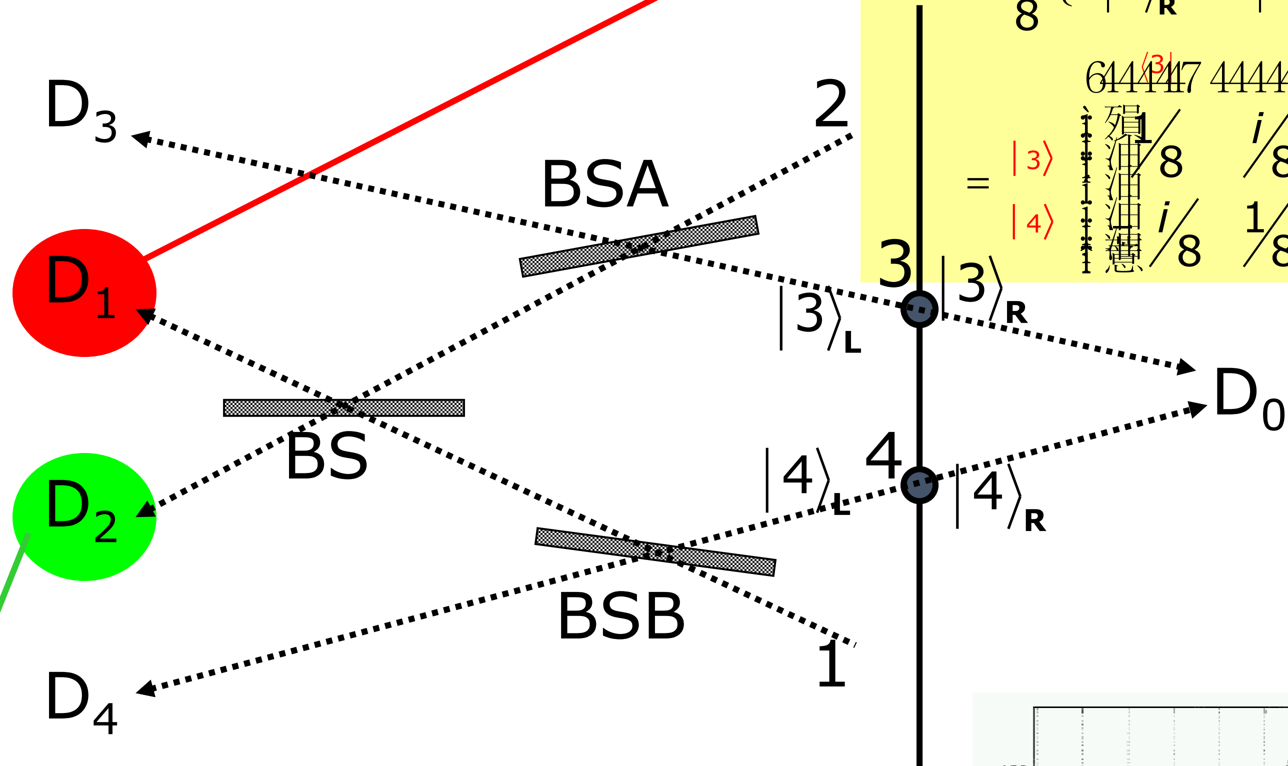
FIG. 5. R_{03} is shown. Absence of interference is clearly demonstrated. The solid line is a fit to the sinc function given in Eq. (6).

$$|j\rangle = \frac{1}{\ldots}\left(|3\rangle|3\rangle + |4\rangle|4\rangle\right)$$

$$r'_{LR} = U_L|j\rangle\langle j|U_L^{+} = \frac{1}{2}\left[\left(\frac{|1\rangle_L}{2} + i\frac{|2\rangle_L}{2} + \frac{|3\rangle_L}{\sqrt{2}}\right)|3\rangle_R + i\left(\frac{|1\rangle_L}{2} - \frac{|2\rangle_L}{2} + \frac{|4\rangle_L}{\sqrt{2}}\right)|4\rangle_R\right] + h.c.$$



$$U_L = \begin{pmatrix} 1/2 & i/2 & -1/2 & i/2 \\ i/2 & 1/2 & i/2 & -1/2 \\ 0 & i/\sqrt{2} & 1/\sqrt{2} & 0 \\ i/\sqrt{2} & 0 & 0 & 1/\sqrt{2} \end{pmatrix}$$

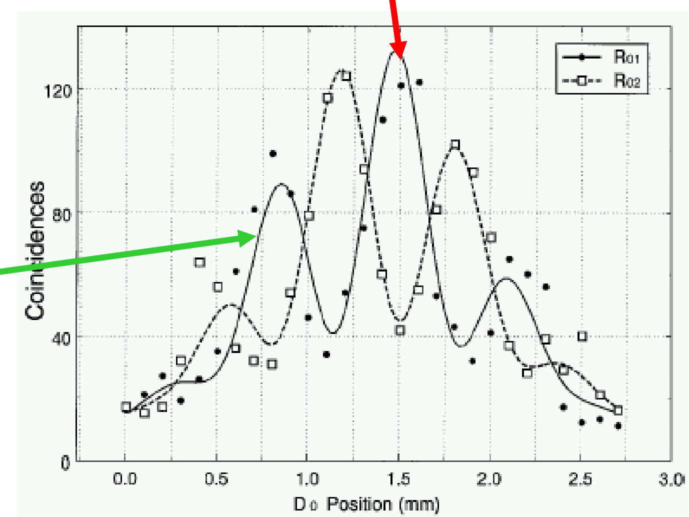


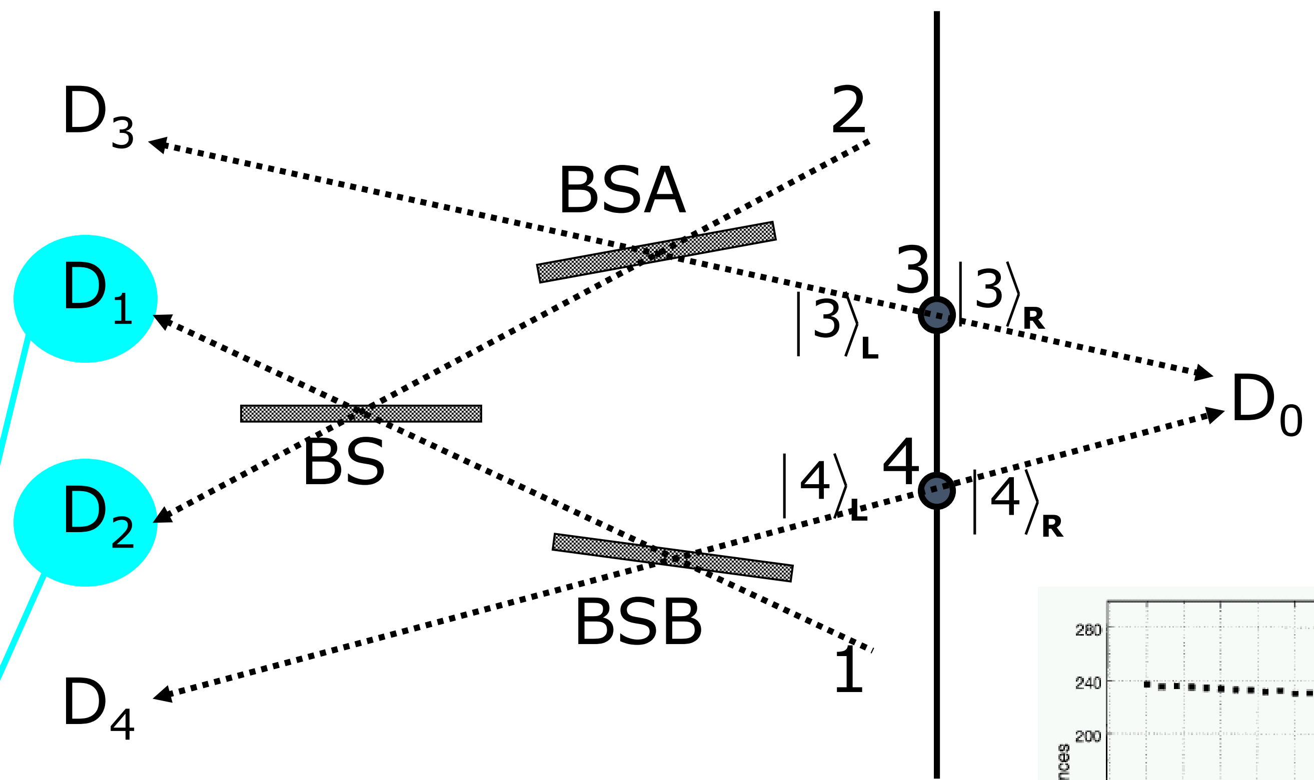
$$\rho'_{R|D_1} = {}_L\langle 1|U_L|\varphi\rangle\langle\varphi|U_L^{+}|1\rangle_L = \frac{1}{8}\{-|3\rangle_R + i|4\rangle_R\}\{-{}_R\langle 3| - i\,{}_R\langle 4|\}$$

$$= \begin{array}{c|cc} & \langle 3| & \langle 4| \\ \hline |3\rangle & 1/8 & i/8 \\ |4\rangle & i/8 & 1/8 \end{array}$$

$$\rho'_{R|D_2} = {}_L\langle 2|U_L|\varphi\rangle\langle\varphi|U_L^{+}|2\rangle_L = \frac{1}{8}\{i|3\rangle_R - |4\rangle_R\}\{-i\,{}_R\langle 3| - {}_R\langle 4|\}$$

$$= \begin{array}{c|cc} & \langle 3| & \langle 4| \\ \hline |3\rangle & 1/8 & -i/8 \\ |4\rangle & i/8 & 1/8 \end{array}$$





$$\rho'_{R|(D_1+D_2)} = \begin{pmatrix} \tfrac{1}{8} & \tfrac{i}{8} \\ -\tfrac{i}{8} & \tfrac{1}{8} \end{pmatrix} + \begin{pmatrix} \tfrac{1}{8} & -\tfrac{i}{8} \\ \tfrac{i}{8} & \tfrac{1}{8} \end{pmatrix} = \begin{pmatrix} \tfrac{1}{4} & 0 \\ 0 & \tfrac{1}{4} \end{pmatrix}$$

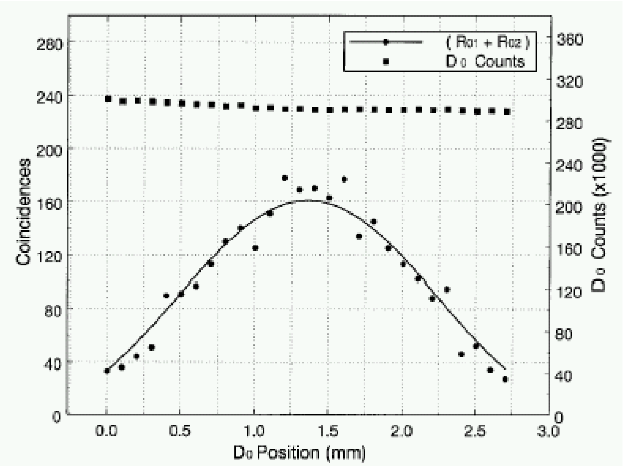
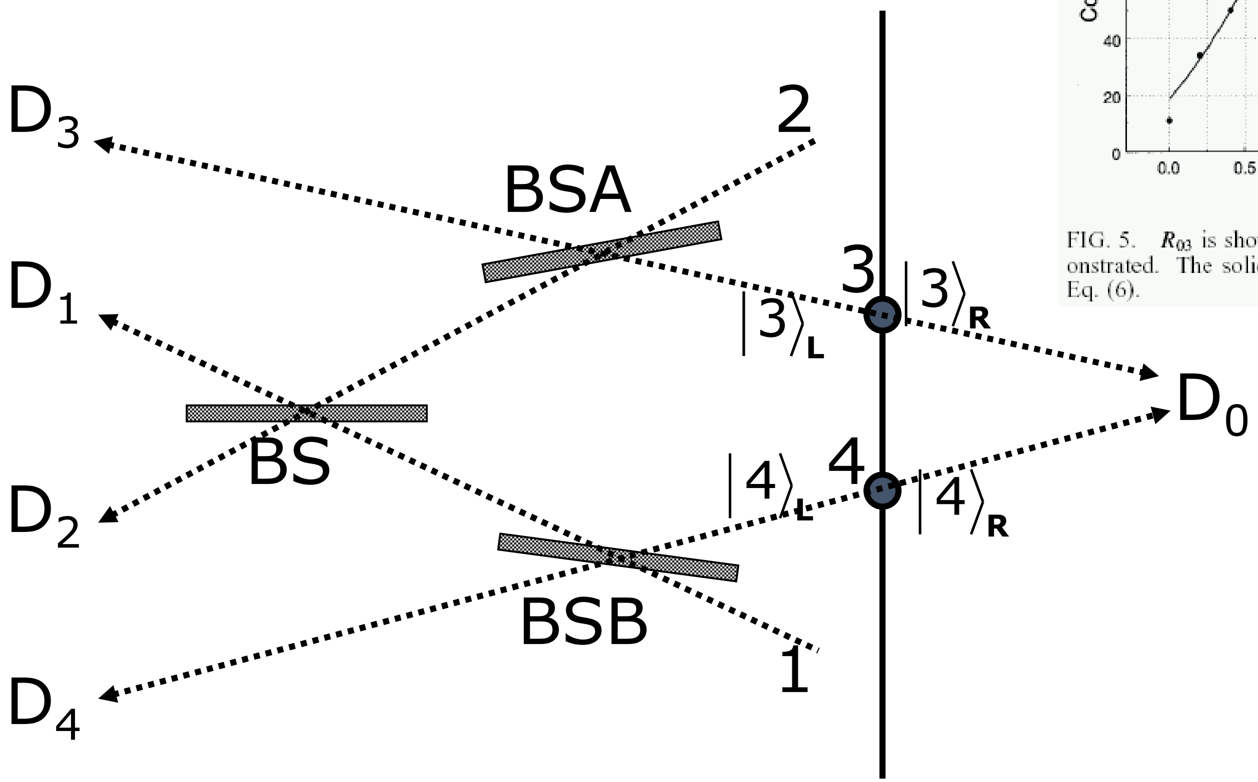


FIG. 4. $R_{01} + R_{02}$ is shown. The solid line is a fit to the sinc function given in Eq. (6). The single counting rate of D_0 is constant over the scanning range.

$$r'_{R|D_3} = {}_L\langle 3|U_L|j\rangle\langle j|U_L^{+}|3\rangle_L = \frac{1}{4}|3\rangle_R\,{}_R\langle 3| = \begin{pmatrix} 1/4 & 0 \\ 0 & 0 \end{pmatrix}$$



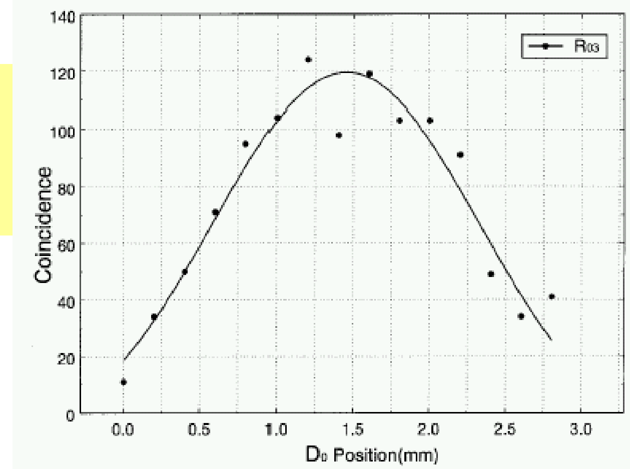


FIG. 5. R_{03} is shown. Absence of interference is clearly demonstrated. The solid line is a fit to the sinc function given in Eq. (6).

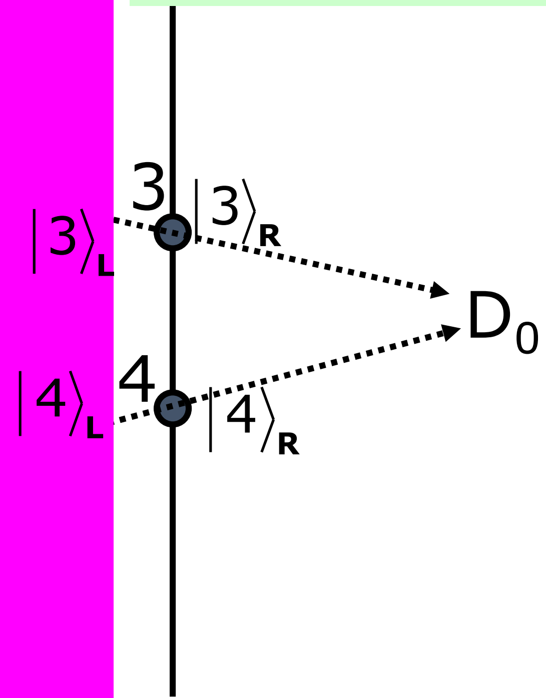
$$r'_{R|D_4} = {}_L\langle 4|U_L|j\rangle\langle j|U_L^{+}|4\rangle_L = \frac{1}{4}|4\rangle_R\,{}_R\langle 4| = \begin{pmatrix} 0 & 0 \\ 0 & 1/4 \end{pmatrix}$$

Environment

Decoherence

$$|j\rangle = \frac{1}{\sqrt{2}}\left(|3\rangle_{\mathbf{L}}|3\rangle_{\mathbf{R}} + |4\rangle_{\mathbf{L}}|4\rangle_{\mathbf{R}}\right)$$

$$r_{\mathbf{LR}} = |j\rangle\langle j|$$



$$r_{\mathbf{R}} = \mathbf{tr}_{\mathbf{L}} r_{\mathbf{LR}} = \frac{1}{2}\left\{|3\rangle_{\mathbf{R}\,\mathbf{R}}\langle 3| + |4\rangle_{\mathbf{R}\,\mathbf{R}}\langle 4|\right\} = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$r'_{\mathbf{R}|(\mathbf{D1+D2+D3+D4})} = \begin{pmatrix} 1/4 & 0 \\ 0 & 1/4 \end{pmatrix} + \begin{pmatrix} 1/4 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 1/4 \end{pmatrix} = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$$

Entanglement

- Systems A and B are interacting.

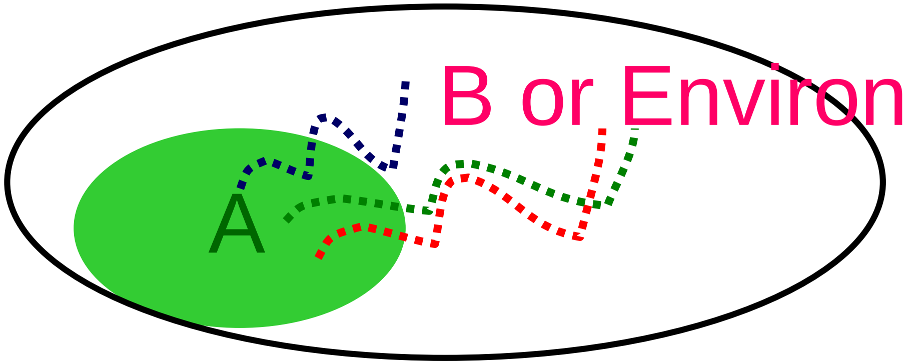
$$|\Psi\rangle_{\mathrm{AB}} \neq |\phi\rangle_{\mathrm{A}} \otimes |\chi\rangle_{\mathrm{B}}$$

$$\rho_{\mathrm{AB}} \neq \sum_n p_n \sigma_{n,\mathrm{A}} \otimes \tau_{n,\mathrm{B}}$$

- There is no state vector description of system A. Only the density matrix description of system A is available by tracing over the rest of the total system.

$$\rho_{\mathrm{A}} = \mathrm{tr}_{\mathrm{B}} \rho_{\mathrm{AB}}$$

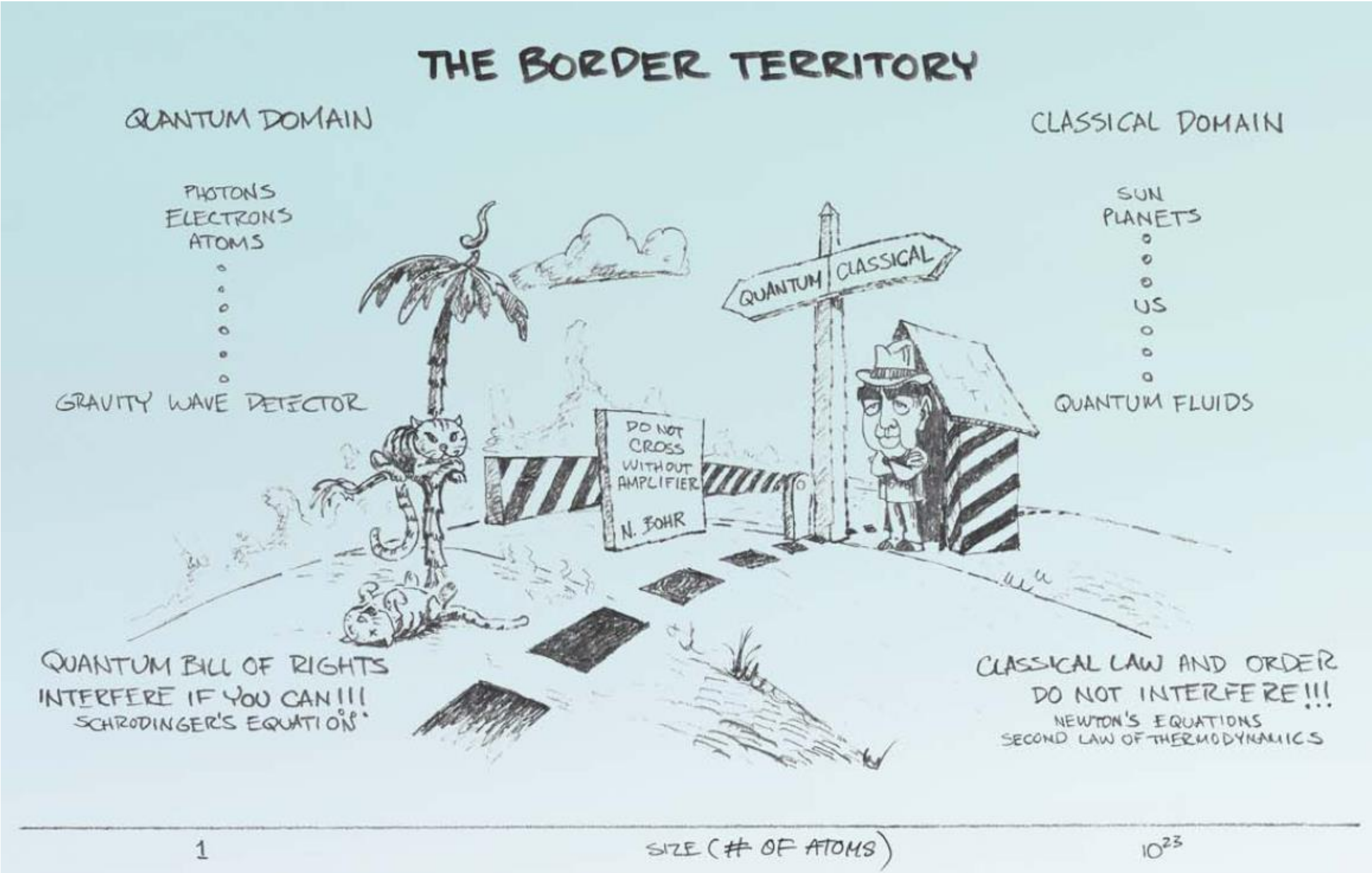
Entanglement and Decoherence



When system A is entangled with environment, state of A cannot be described by a state vector, but by a density matrix.

$$|\Psi\rangle_{\mathrm{AB}} = a|0\rangle_{\mathrm{A}}|0'\rangle_{\mathrm{B}} + b|1\rangle_{\mathrm{A}}|1'\rangle_{\mathrm{B}} \neq |\phi\rangle_{\mathrm{A}} \otimes |\chi\rangle_{\mathrm{B}}$$

$$\rho_{\mathrm{A}} = \mathrm{tr}\rho_{\mathrm{AB}} = \begin{pmatrix} |a|^2 & 0 \\ 0 & |b|^2 \end{pmatrix}$$



Zurek, QP/0306072

Quantum to Classical

- Isolated, closed system scenario:

$$\hbar \to 0$$

- Open system scenario:

Decoherence

Quantum to Statistical

- Isolated, closed system scenario:

  ETH (Eigenstate Thermalization Hypothesis)

- Open system scenario:

  Decoherence

- Classical Equilibrium Statistical Mechanics maximizing probability

$$e^{-\beta H}$$

- Quantum Statistical Mechanics inherits this.

?

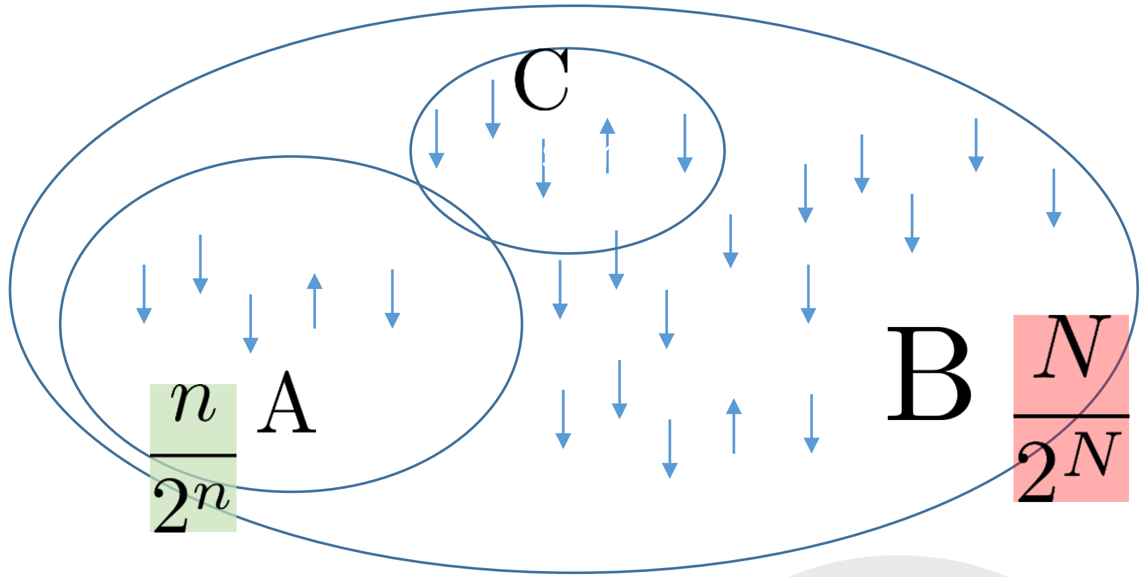
Decoherence Picture

cf. G. Mahler *et al*., Physica E, 29, 53-65 (2005)

- An equilibrium state [steady] corresponds to an energy eigenstate [steady] of the total system.
- By tracing out the environment (bath), the system A can be described by a density matrix

$$\rho_A \sim e^{-H/kT}$$

Model



$$H_{\mathrm{AB}} = \sum_{i \in \mathrm{A}} h_i \sigma_{iz} + \sum_{i,j \in \mathrm{A}} \epsilon_{ij} \sigma_{iz} \sigma_{jz} + \sum_{i,j \in \mathrm{A}} \eta_{ij} \sigma_{ix} \sigma_{jx}$$

$$+ \sum_{l \in \mathrm{B}} h_l \sigma_{lz} + \sum_{l,m \in \mathrm{B}} \epsilon_{lm} \sigma_{lz} \sigma_{mz} + \sum_{l,m \in \mathrm{B}} \eta_{lm} \sigma_{lx} \sigma_{mx}$$

$$+ \sum_{i \in \mathrm{A}, l \in \mathrm{B}} \epsilon_{il} \sigma_{iz} \sigma_{lz} + \sum_{i \in \mathrm{A}, l \in \mathrm{B}} \eta_{il} \sigma_{ix} \sigma_{lx}$$

Energy conserving

Random walk in conjugate basis space

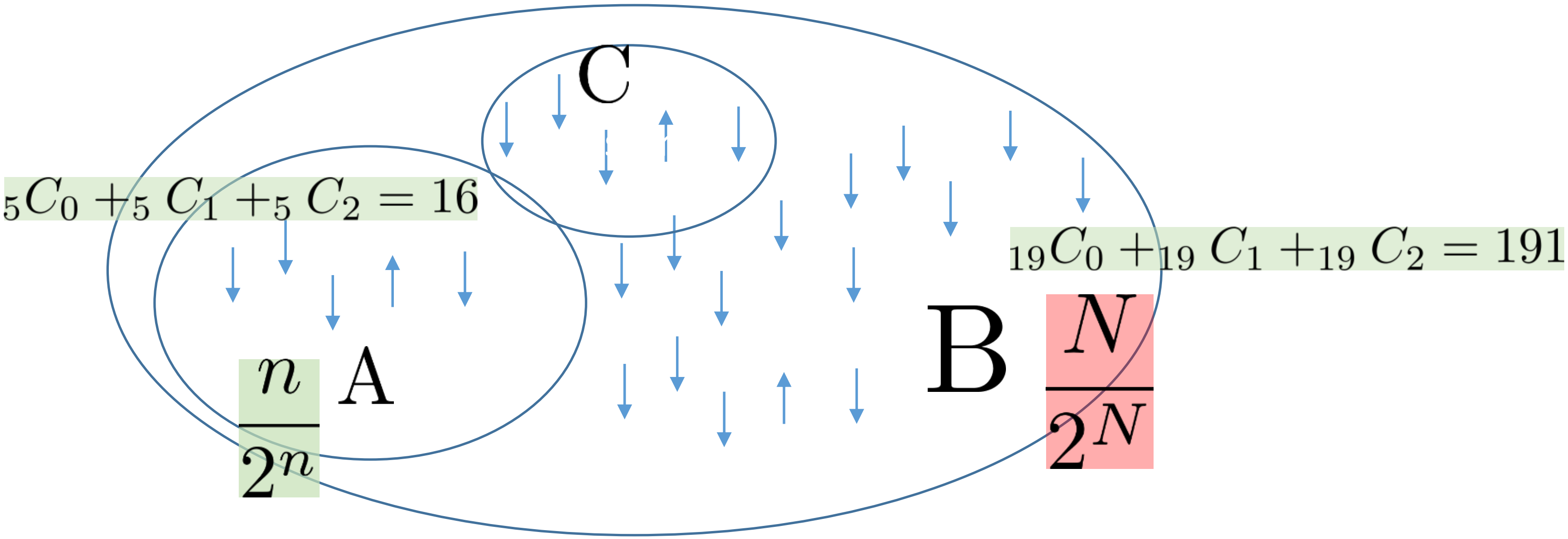
- Temperature ~ eigenenergy of the total system

- Entropies of subsystems ~ extensive/additive

  Entropy of the total system = 0

Using lowest eigenstates

… highest states ~ negative temp



Approximate Hamiltonian matrix 3056×3056

instead of $2^{(5+19)} \times 2^{(5+19)}$

16,777,216 x 16,777,216

Using lowest eigenstates

$$|\Psi\rangle_{\mathrm{AB}} = \sum_{p\in\mathrm{A},u\in\mathrm{B}} a_{pu}|p\rangle|u\rangle$$

$$\begin{aligned}\rho_{\mathrm{A}} &= \mathrm{tr}_{\mathrm{B}} \sum_{p,q\in\mathrm{A},u,v\in\mathrm{B}} a^*_{qv}a_{pu}|p\rangle|u\rangle\langle v|\langle q| \\ &= \sum_{p,q\in\mathrm{A}}\sum_{u\in\mathrm{B}} a^*_{qu}a_{pu}|p\rangle\langle q| \\ &= \sum_{p\in\mathrm{A}}\left(\sum_{u\in\mathrm{B}}|a_{pu}|^2\right)|p\rangle\langle p|\end{aligned}$$

What I expect:

- Demonstrate equilibrium as a decoherence.
- Clarify Temperature and Entropy.
- Understand ‘Isolated vs Open’
  in the emergence of
  classical mechanics and statistical physics.