The 3rd KIAS Workshop on Quantum Information and Thermodynamics

MACROSCOPIC SUPERPOSITIONS AND QUANTUM INFORMATION PROCESSING

Center for Macroscopic Quantum Control

Hyunseok Jeong
Department of Physics and Astronomy
Seoul National University

NRF 한국연구재단 National Research Foundation of Korea

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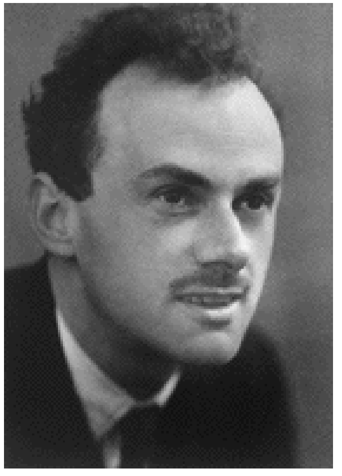
Quantum superposition principle

The principle of quantum superposition: Any two states may be superposed to give a new state.

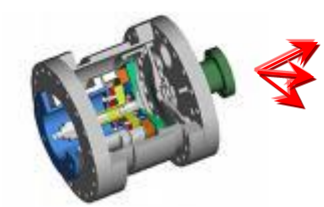


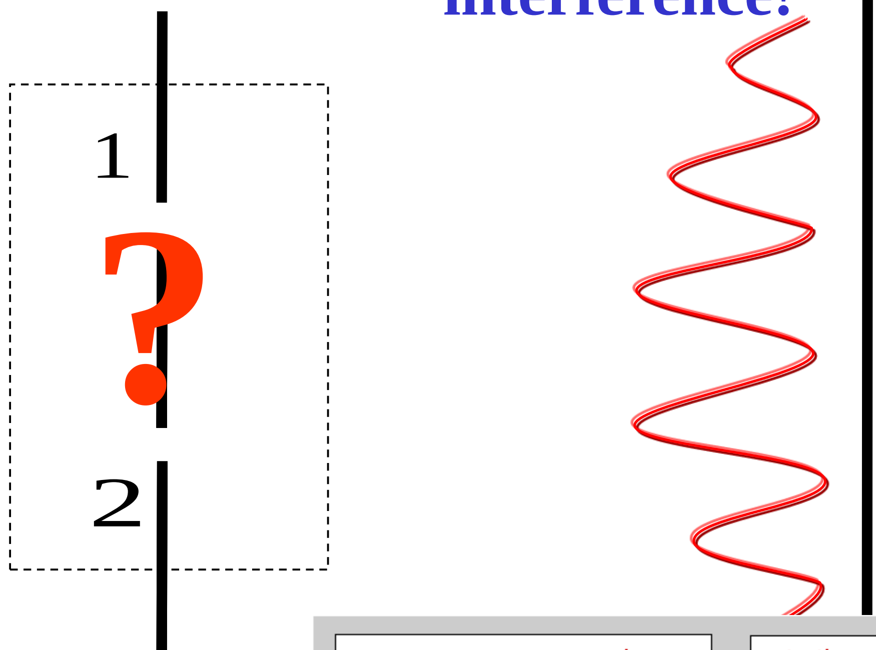
P. A. M. Dirac

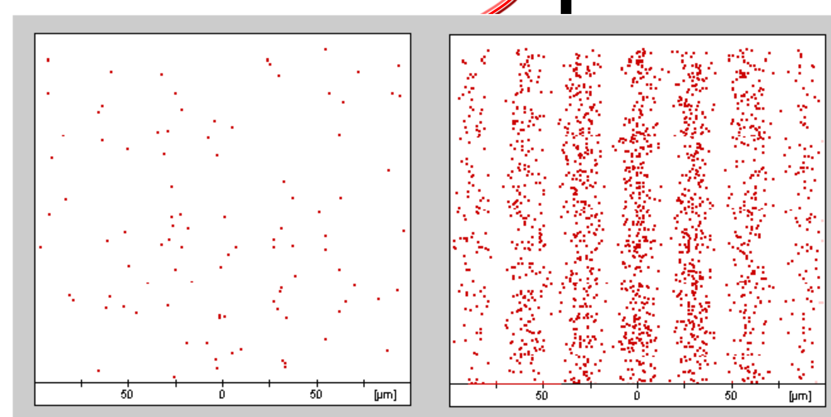
"The superposition principle lies at the very heart of quantum mechanics."

Double split experiment with electrons

Quantum interference!



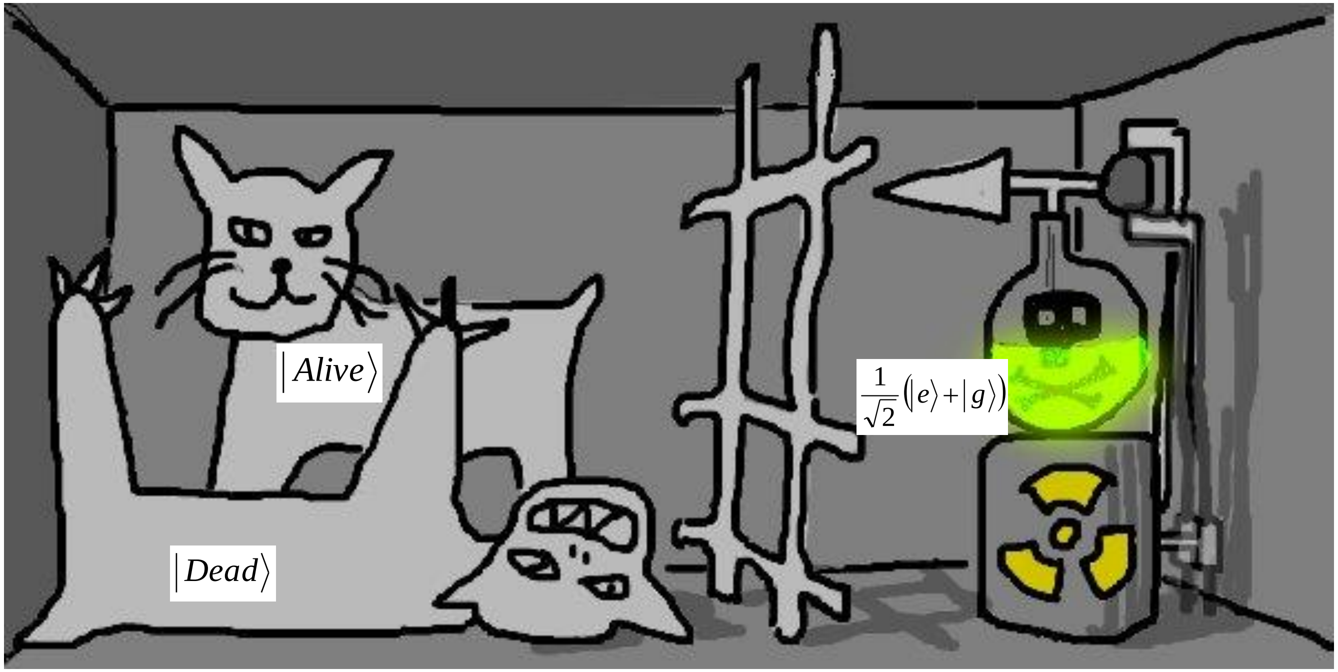




- Researchers led by Pier Giorgio Merli, the University of Milan (1974).
- Tonomura *et al*., Hitachi in Japan (1989).

Schrödinger's cat paradox

E. Schrödinger, *Naturwissenschaftern*. 23 (1935)

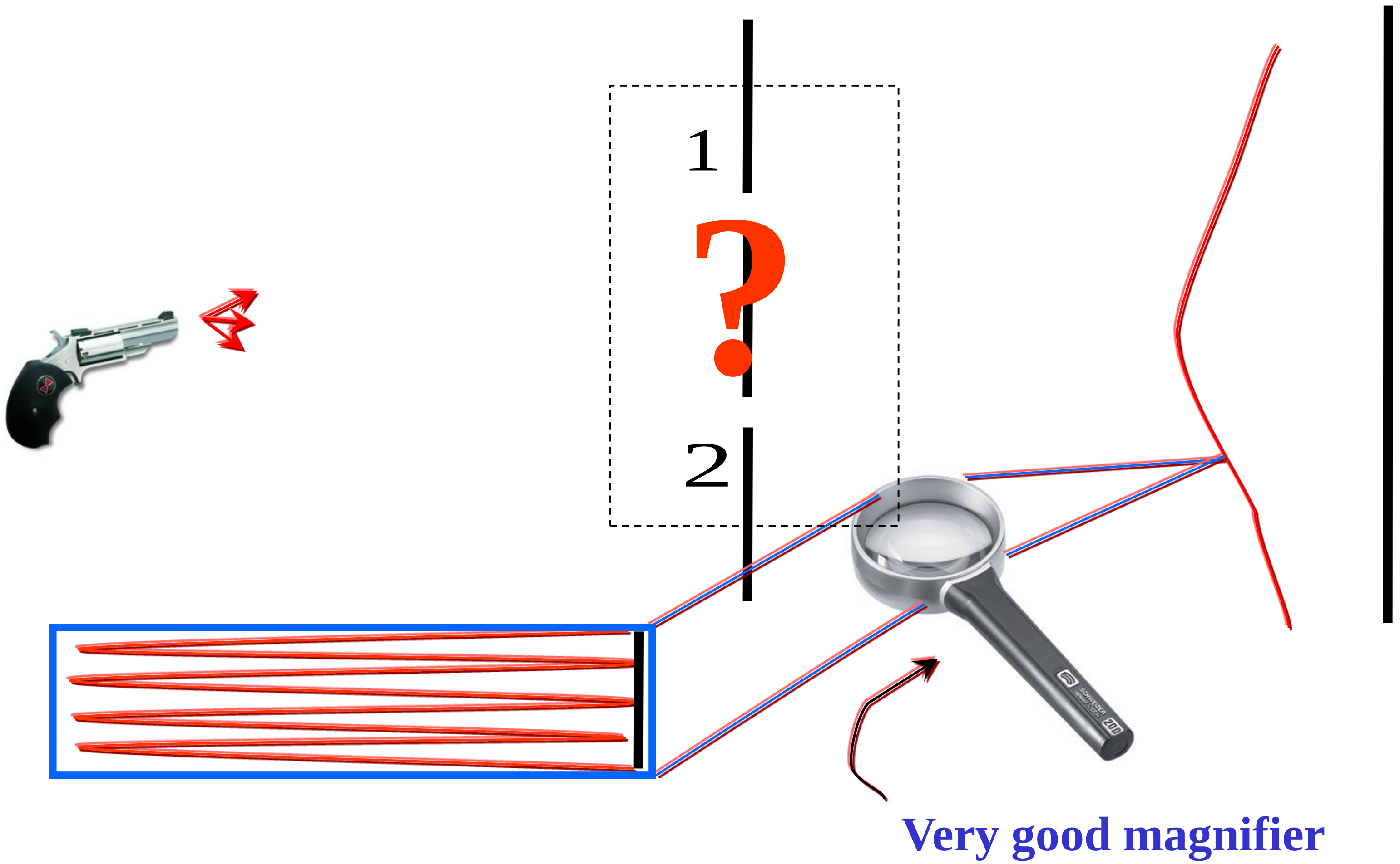


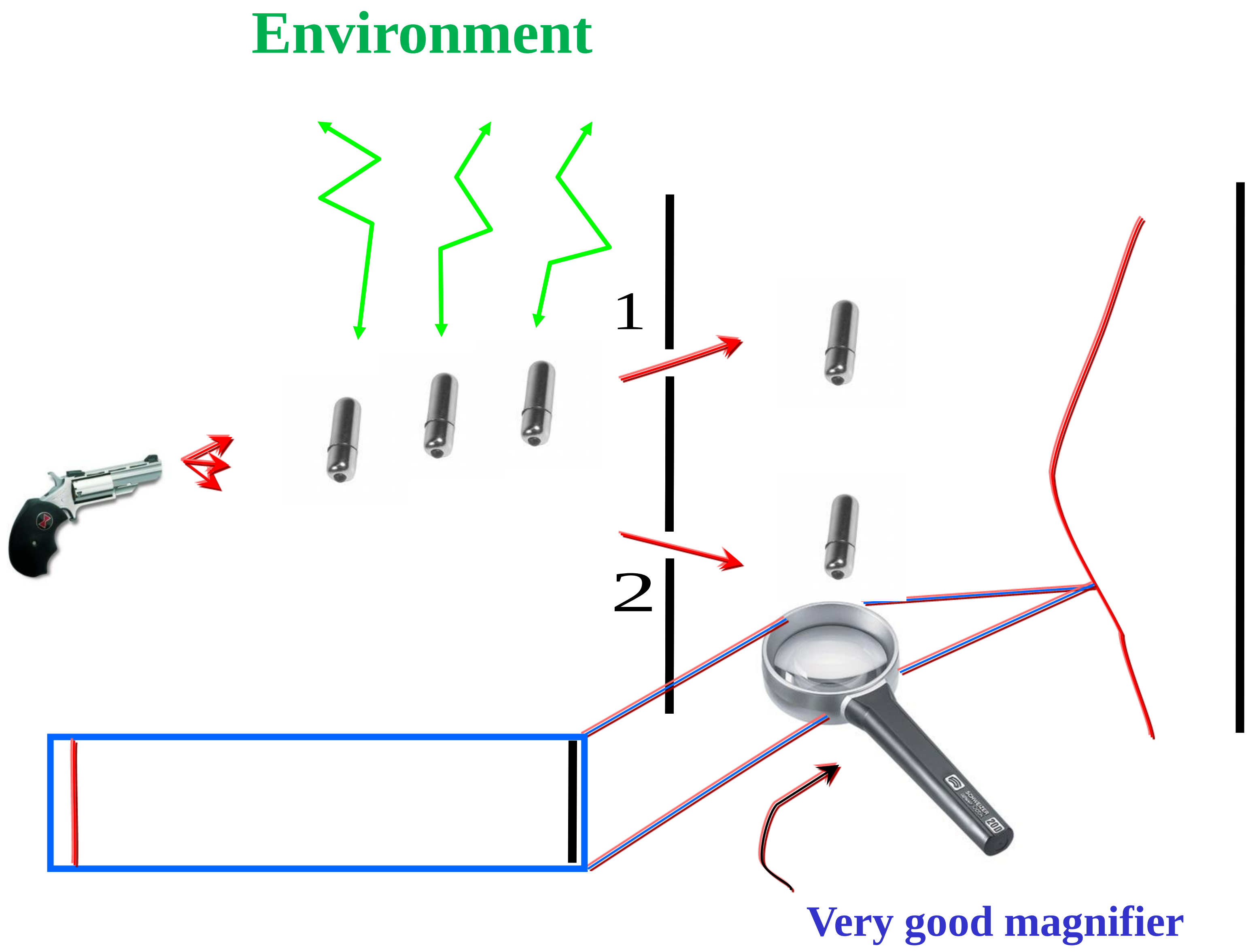
The Brussels Journal (29 October 2007)

$$\frac{1}{\sqrt{2}}(|e\rangle+|g\rangle)|Alive\rangle \longrightarrow \boxed{\frac{1}{\sqrt{2}}(|e\rangle|Alive\rangle+|g\rangle|Dead\rangle)}$$

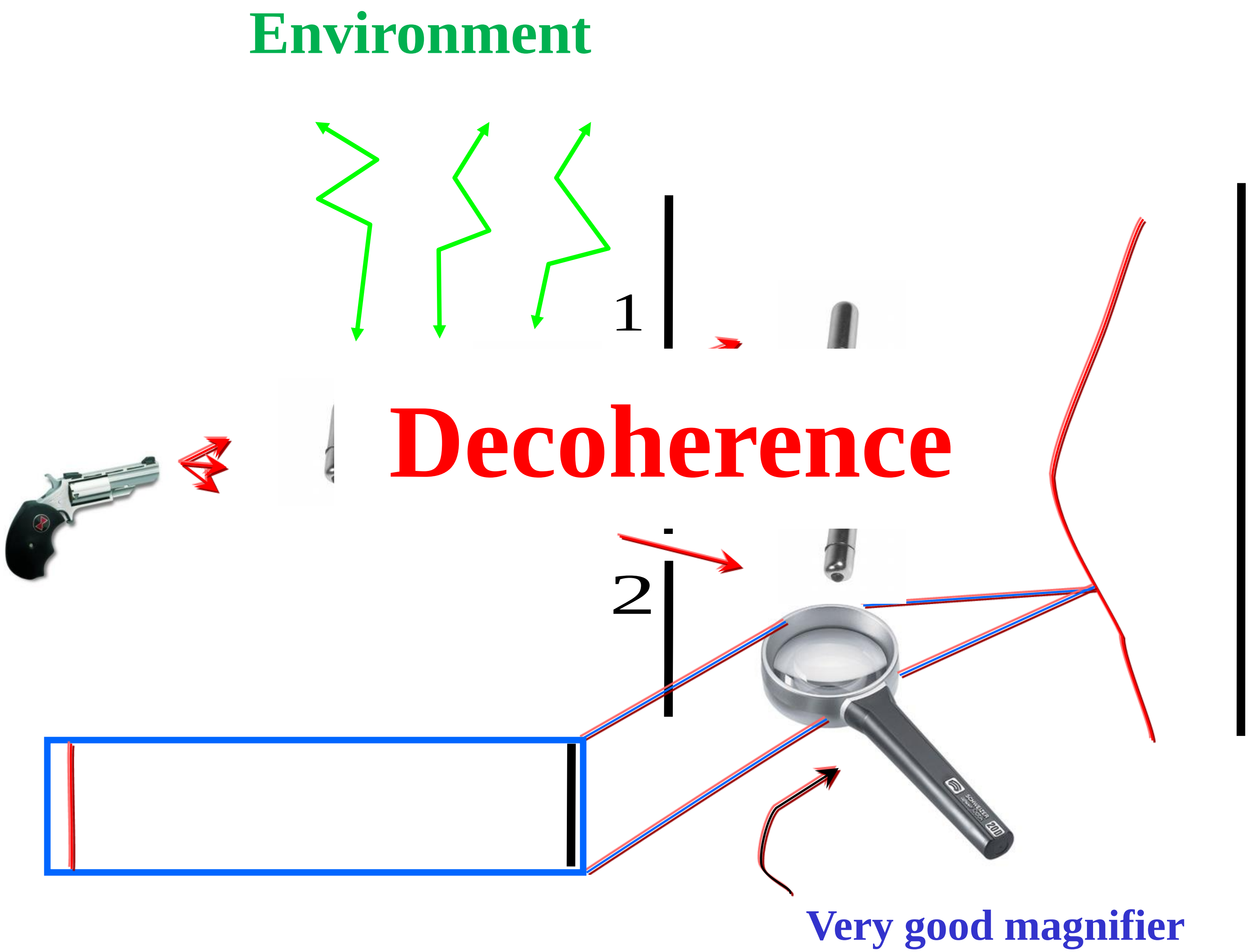
Quantum superposition or entanglement of macroscopic objects?

Classical gun – if we are really living in a *quantum* universe…



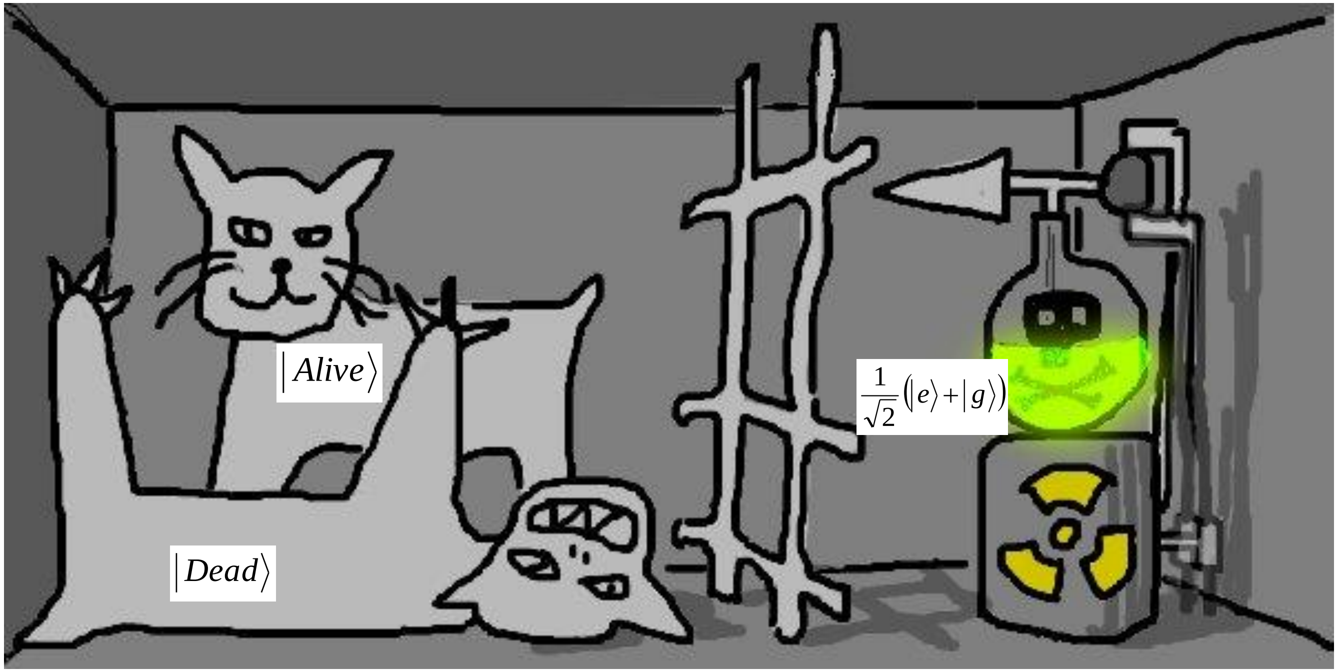


Environment
1
2
Very good magnifier


Environment
1
Decoherence
2
Very good magnifier

Schrödinger's cat paradox

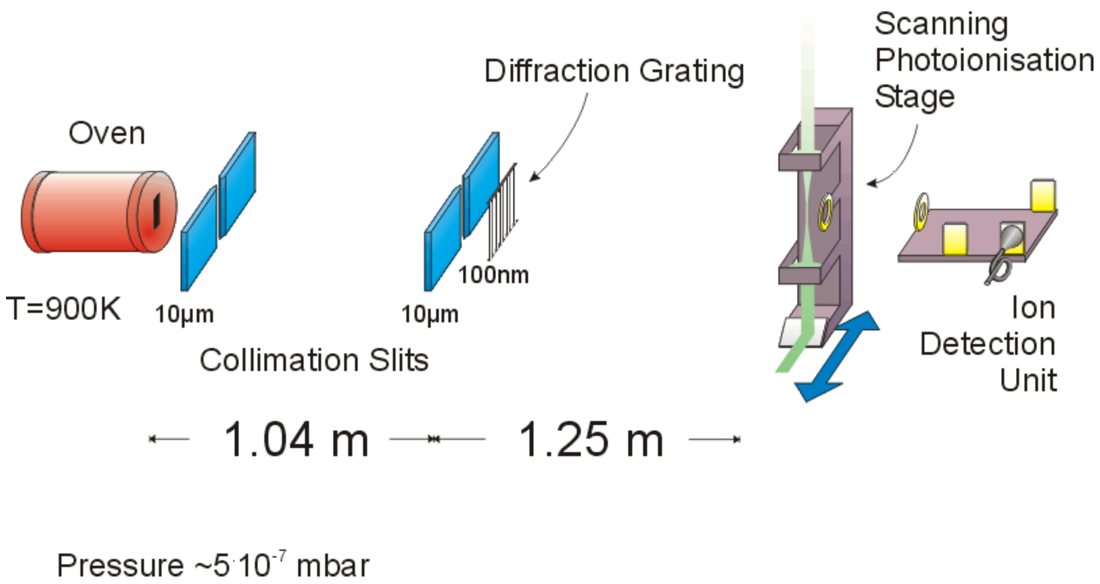
E. Schrödinger, *Naturwissenschaftern*. 23 (1935)



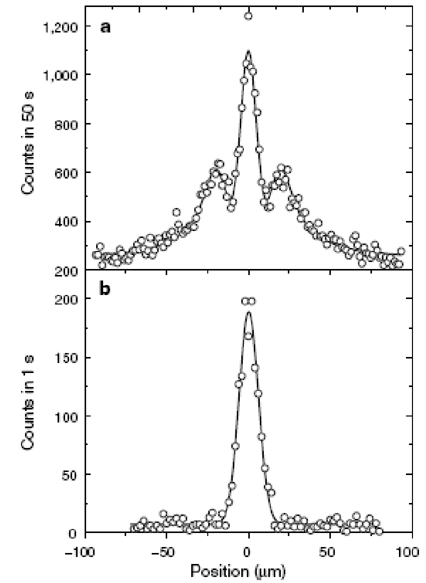
The Brussels Journal (29 October 2007)

Macroscopic ***and*** quantum?

Interference of large molecules



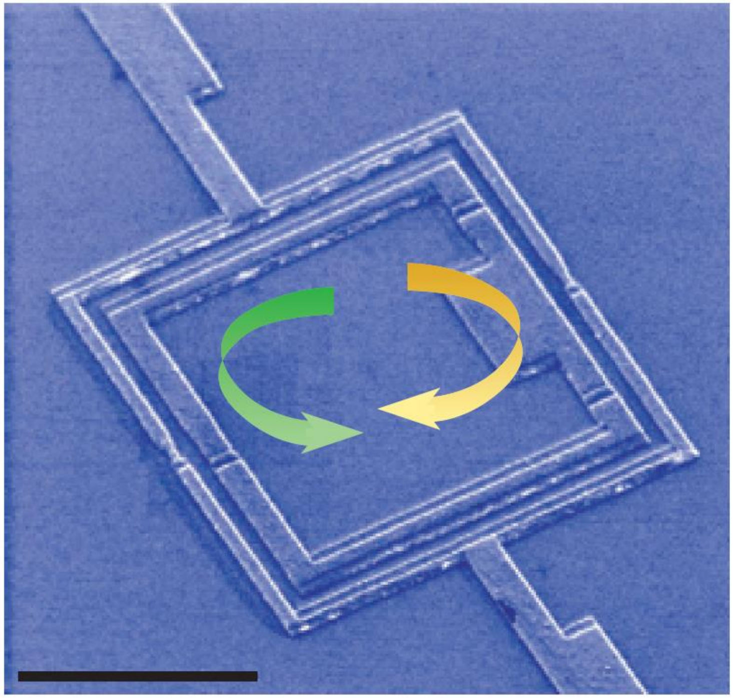
https://www.univie.ac.at/qfp/research/matterwave/c60/



(Interference with C60 molecules, *Nature* 401, 680, 1999)

- Interference with C60 molecules [M. Arndt et al., Nature 401, 680 (1999)]

Superposition of supercurrents



Copyright: C. Kohstall and R. Grimm, University of Innsbruck

- Quantum superposition of left- and right- circulating supercurrents [R. Friedman *et al*., *Nature* 406, 43 (2000)]

Schrödinger cat states of light

A. Ourjoumtsev, H. Jeong, R. Tualle-Brouri and Ph. Grangier, *Nature* 448, 784 (2007)

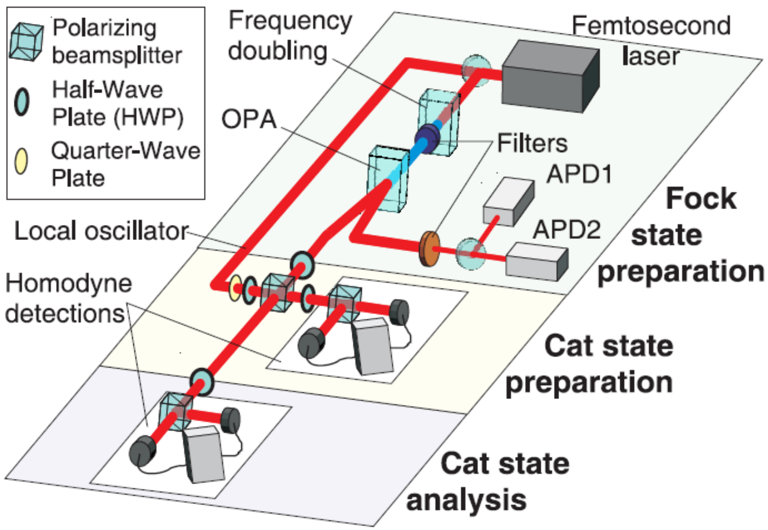
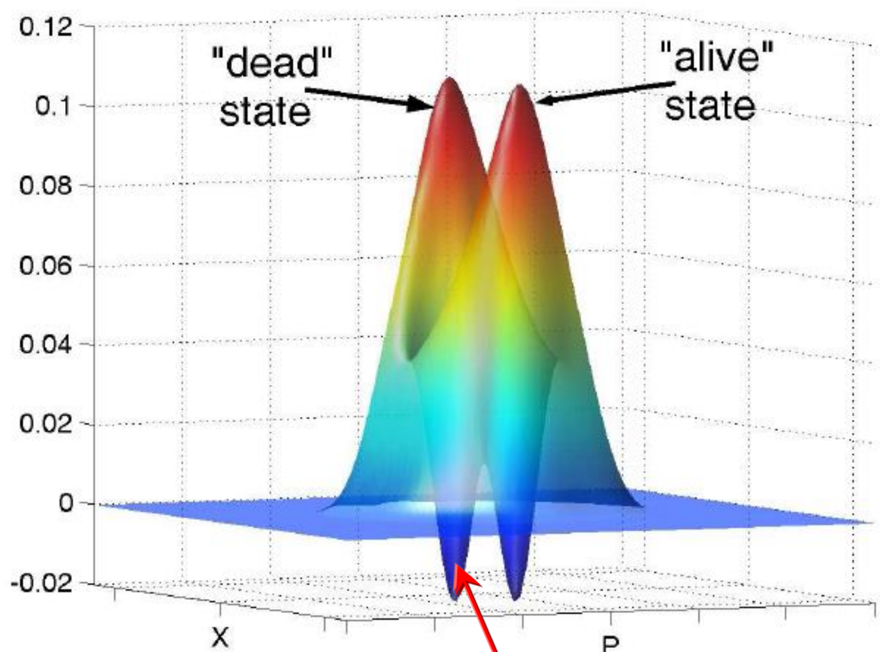


FIG. 3: Experimental setup.

Wigner function



- Cavity field: S. Deléglise *et al*., Nature 455, 510-514 (2008)
- Cavity field (circuit QED): Vlastakis *et al*. Science 342, 607 (2013)
- **And many more... (atomic systems, mechanical systems *etc*...)**

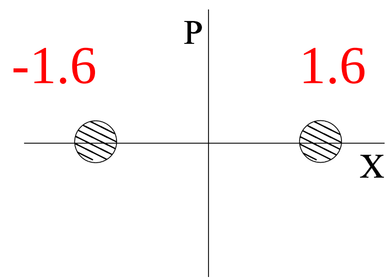
Evidence of quantum interference

Schrödinger cat states of light

- **Schrödinger cat states:**

$$\boxed{|cat\rangle = N\big(|\alpha\rangle + e^{i\varphi}|-\alpha\rangle\big)} \qquad \boxed{\alpha >> 1}$$

$$\boxed{\begin{aligned}\hat{X} &= \hat{a} + \hat{a}^{+} \\ \hat{P} &= -i(\hat{a} - \hat{a}^{+})\end{aligned}}$$



- **Coherent state:**

$$|\alpha\rangle = e^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle$$

✓ Coherent states are most classical among all pure states: an analogy of ***classical*** point-particles in the quantum phase space [Schrödinger, *Naturwissenschaften* 14 (1926)] and most robust against decoherence.

✓ The two coherent states | α > and | -α > are "classically" (or macroscopically) distinguishable for α >>1, *i.e.*, they can be well discriminated by a homodyne measurement (HD) with ***limited*** efficiency.

(For 70% of HD efficiency: *D*≈99.7% for α=1.6 and *D*>99.9% for α=2.0.)

Wigner function

$$W(\beta) = \frac{1}{\pi^2} \int \exp[\xi^* \beta - \xi \beta^*] C(\xi) d^2 \xi$$

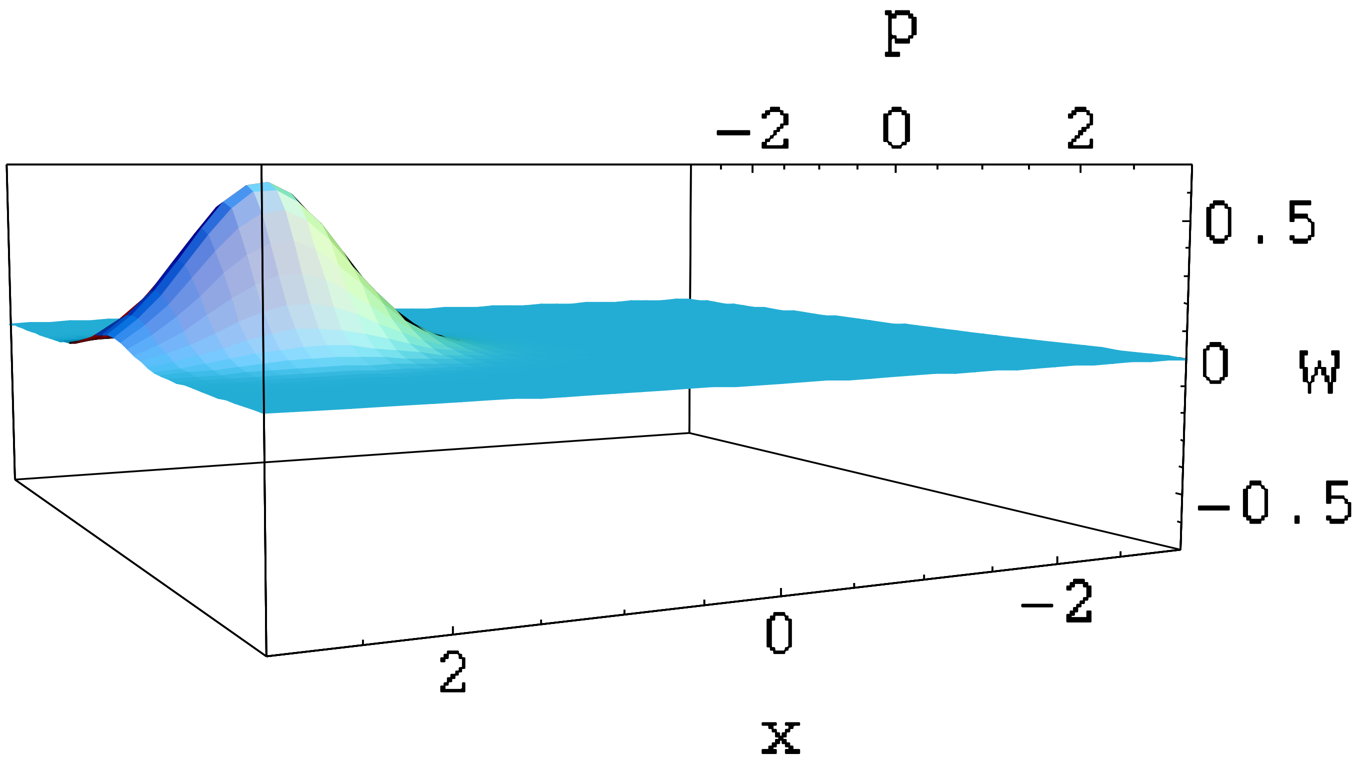
$$\boxed{\hat{D}(\xi) = e^{\xi \hat{a}^+ - \xi^* \hat{a}}}$$

$$\boxed{C(\xi) = \mathrm{Tr}[\hat{D}(\xi) \hat{\rho}]}$$

$$\boxed{\beta = x + ip}$$

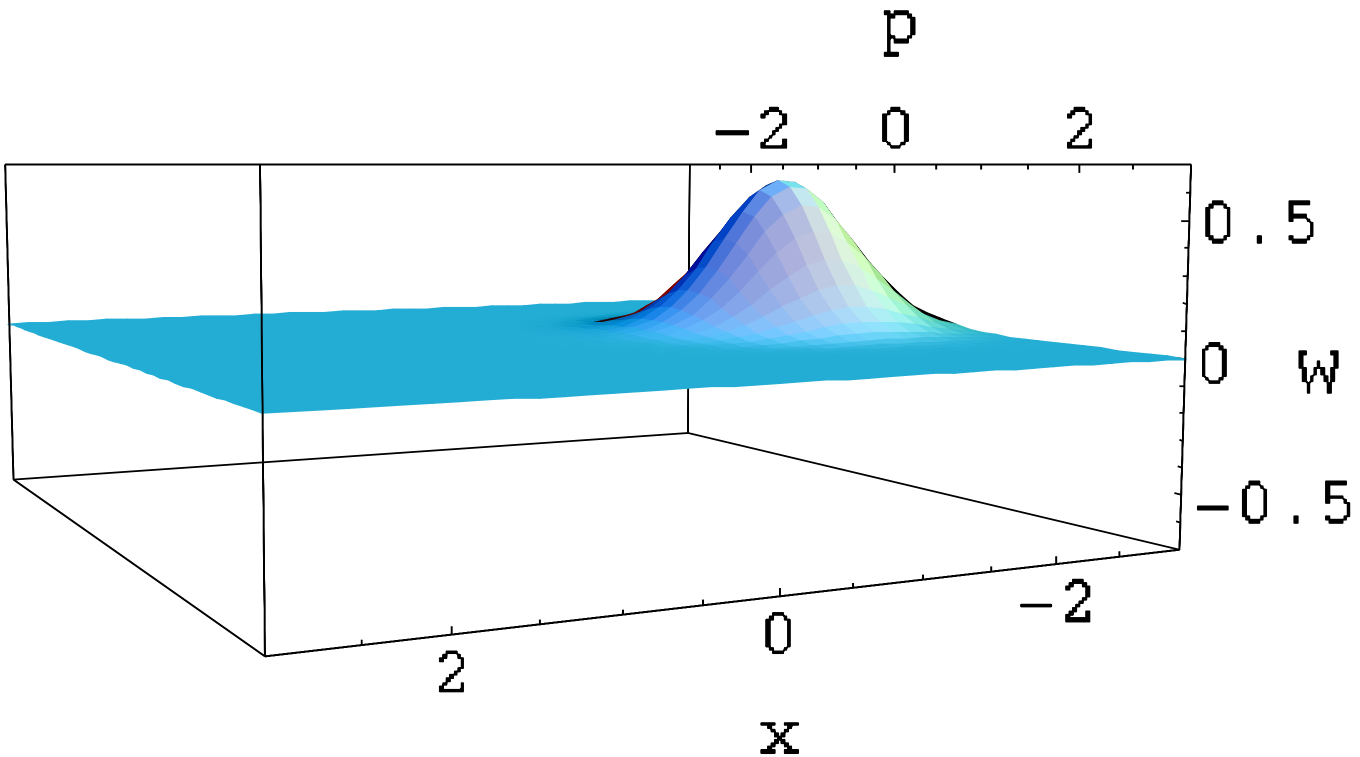
- The Wigner function is a quasi-probability distribution: an analogy of the classical probability distribution in the quantum phase space.
- Negative values in the Wigner function are a definite sign of non-classicality.

Coherent state α=2



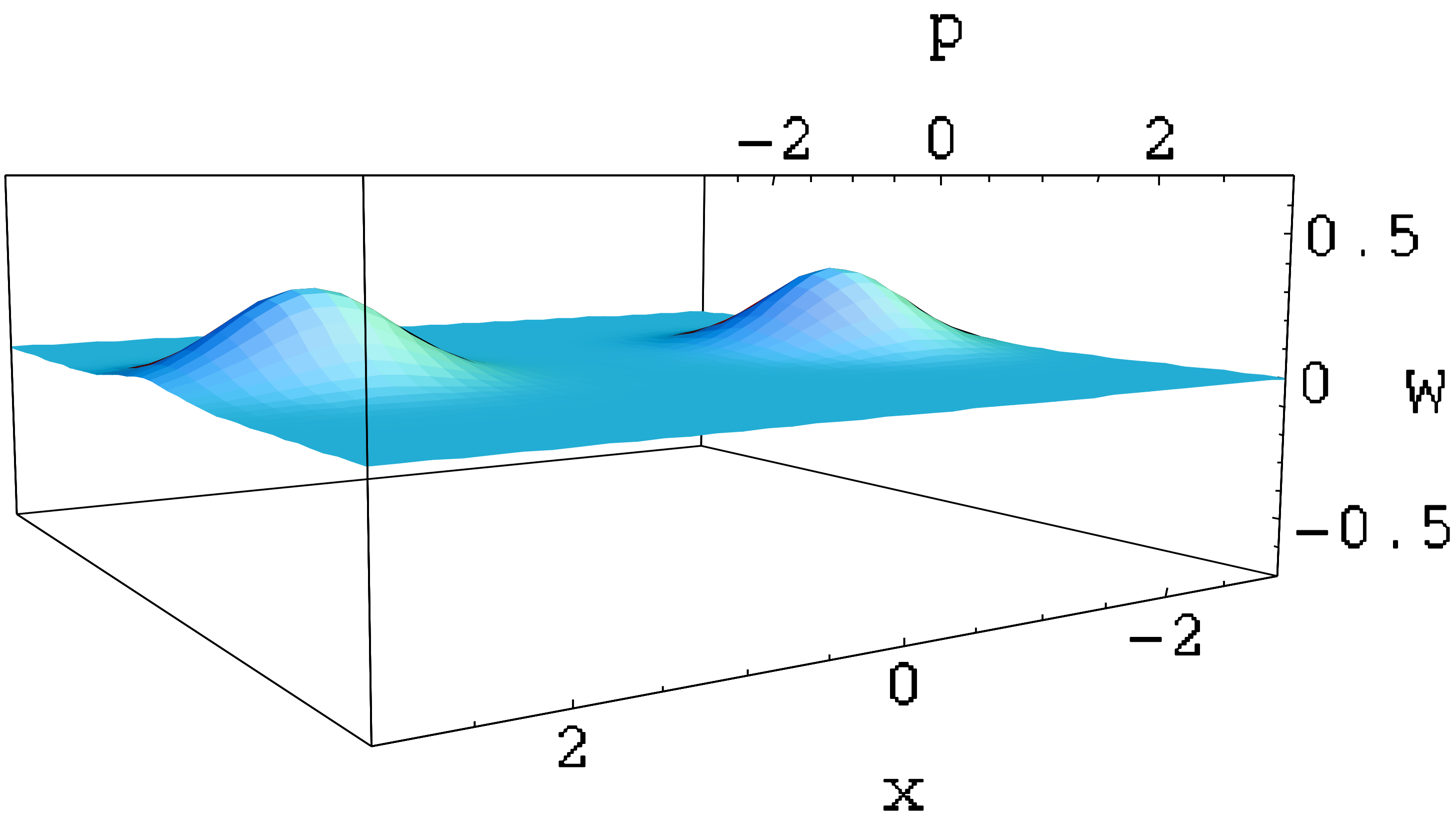
$$|\alpha\rangle \text{ with } \alpha = 2$$

Coherent state α=-2



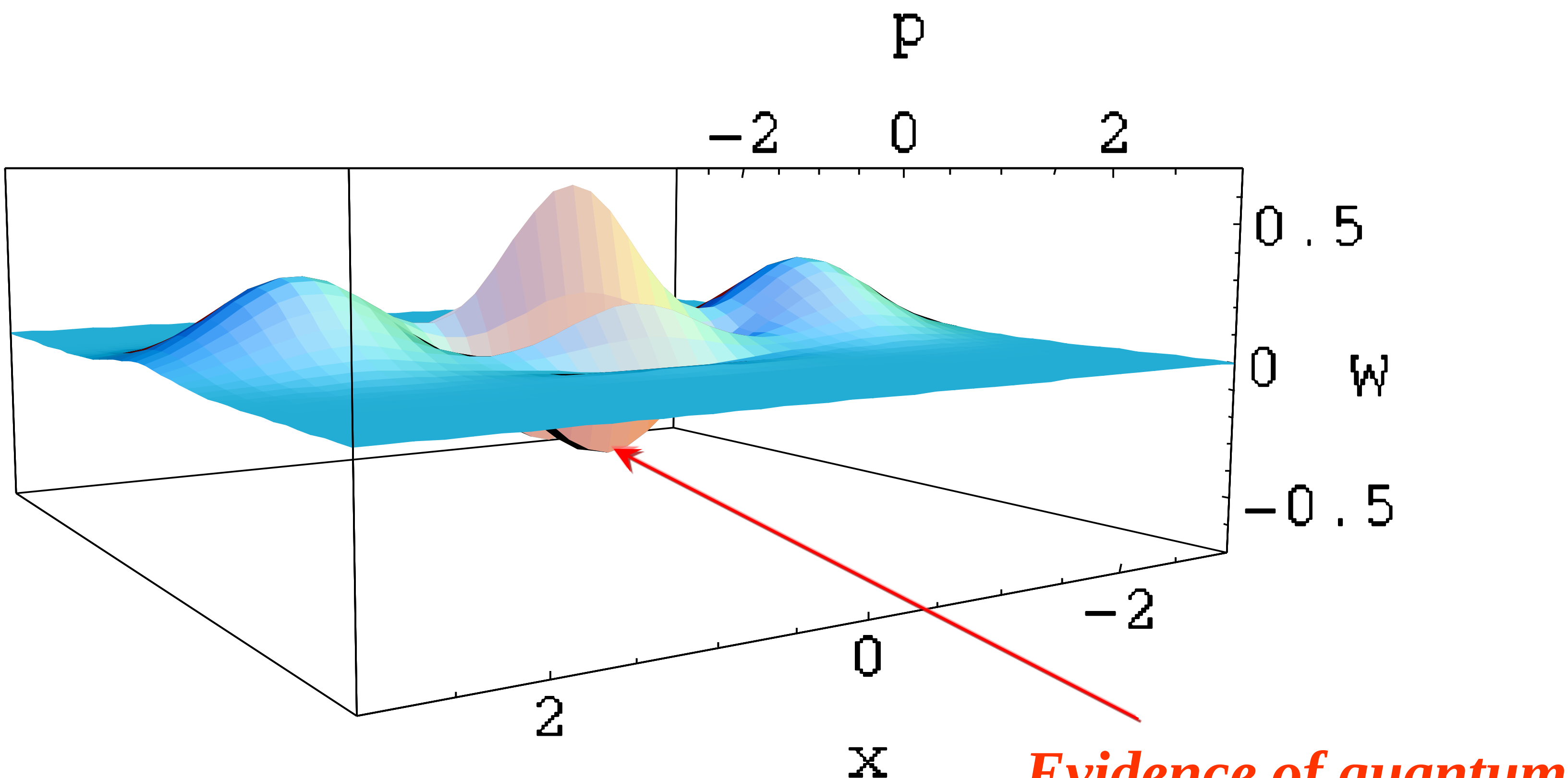
$$|\alpha\rangle \text{ with } \alpha = -2$$

Statistical mixture



$$N_m\left(|\alpha\rangle\langle\alpha| + |-\alpha\rangle\langle-\alpha|\right)$$

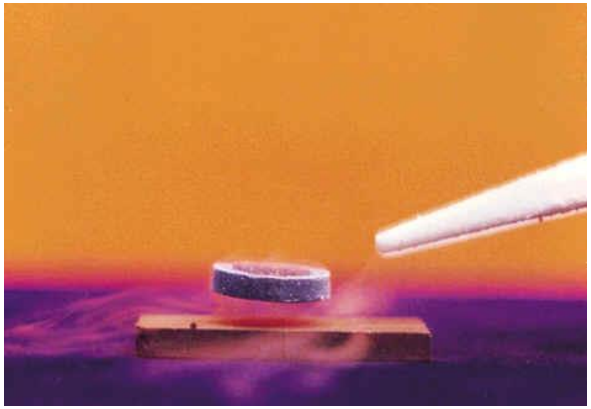
Quantum superposition
(Schrödinger cat state)

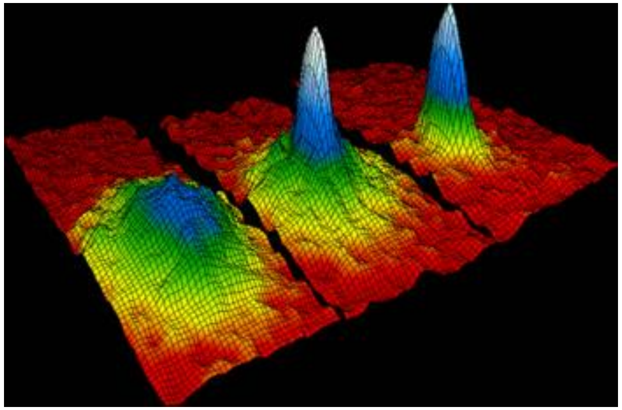


$$N_s\left(\left|\alpha\right\rangle + \left|-\alpha\right\rangle\right)$$

Macroscopic quantum phenomena

- Superconductivity
- Superfluidity
- Bose-Einstein Condensates





Macroscopic quantum phenomena

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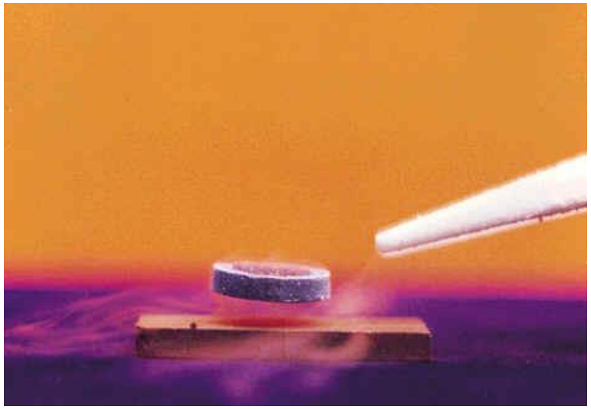

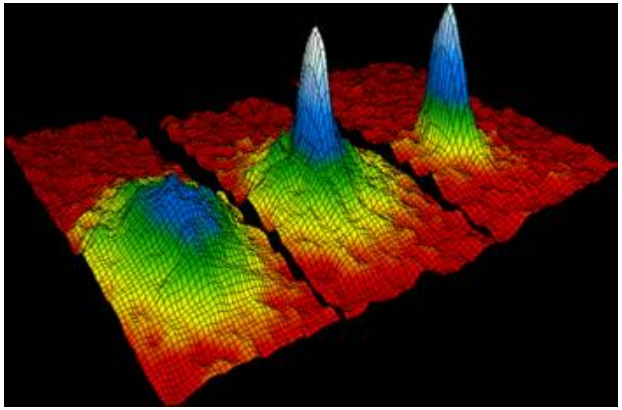

- However, these are ***not*** macroscopic superpositions nor macroscopic entanglement.

Can we quantify 'Schrödinger's-cattiness' or 'macroscopic quantumness'?

"*What is the correct measure of 'Schrödinger's-cattiness'? Ideally, one would like* ***a quantitative measure which corresponds to our intuitive sense****; I shall attempt one below, but would emphasize that the choice between this and a number of similar and perhaps equally plausible definitions is, with one important exception (see below),* ***very much a matter of personal taste****, and that I very much doubt that 50 years from now anything of importance will be seen to have hung on it.*"

(A. J. Leggett, J. Phys.: Condens. Matter 14 (2002) R415–R451)

Disconnectivity

A. J. Leggett, Prog. Theor. Phys. Suppl. 69, 80 (1980); J. Phys. 14, R415 (2002)

- D quantifies genuine multipartite quantum correlation such as:

$$|\phi\rangle^{\otimes N} + |\phi^{\perp}\rangle^{\otimes N}$$

- D for ρ_N is defined as the largest number n that makes δ_n the smallest where

$$\delta_n = \frac{S_n}{\min_m(S_m + S_{n-m})} \qquad S_n = -\mathrm{Tr}[\rho_n \ln \rho_n]$$

  and ρ_n $(n < N)$ is a reduced density operator from ρ_N.

- Leggett pointed out that so-called "macroscopic quantum phenomena" such as superconductivity or superfluidity do not require the existence of a high-D state.
- Superfluidity can be explained by a product of identical bosonic states of which disconnectivity is obviously 1.
- A superconducting system described by Cooper pairs also shows a small value of D.

Disconnectivity

A. J. Leggett, Prog. Theor. Phys. Suppl. 69, 80 (1980); J. Phys. 14, R415 (2002)

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  and ρ_n $(n < N)$ is a reduced density operator from ρ_N.
- Applicable only to certain types of pure states.

Previous studies

- "Distance" or "distinguishability" between the component states (e.g. Bjork and Mana, J. Opt. B 2004; Korsbakken *et al*., PRA 2007)
- Number of effective particles that involve the superposition (e.g. Leggett (1980); Dur, Simon and Cirac, PRL 2002)

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References taken from

H. Jeong, M. Kang and H. Kwon, Special Issue on Macroscopic Quantumness, Optics Communications 337, 12 (2015) (Review Article).

Distinguishability-based measure

J.I. Korsbakken, K.B. Whaley, J. Dubois, J.I. Cirac, Phys.Rev.A75, 042106 (2007).

- For an N-partite superposition state |A>+|B>
- n_{min}: number of measurements (with limited efficiency δ) required to distinguish between |A> and |B>.

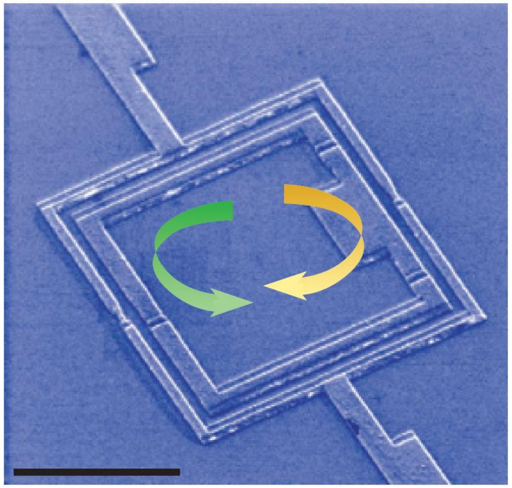
$$C_\delta(|\psi_N\rangle) = \frac{N}{n_{\min}}$$

Effective size of *N*-particle superposition state

J.I. Korsbakken, K.B. Whaley, J. Dubois, J.I. Cirac, Phys.Rev.A75, 042106 (2007).

$$C_\delta(|\psi_N\rangle) = \frac{N}{n_{\min}}$$

- Example: the effective size of the flux qubits, as a genuine macroscopic superposition, is surprisingly (but not trivially) ***small*** despite the apparent large difference in macroscopic observables with billions of electrons.



Just another “kitten”…

Distinguishability-based measure

J.I. Korsbakken, K.B. Whaley, J. Dubois, J.I. Cirac, Phys.Rev.A75, 042106 (2007).

- For an *N*-partite superposition state |A>+|B>
- n_{min}: number of measurements (with limited efficiency δ) required to distinguish between |A> and |B>.

$$C_\delta(|\psi_N\rangle) = \frac{N}{n_{\min}}$$

- *C* does not distinguish between a pure superposition and a classical mixture.
- *C* is decomposition-dependent.

General measure?

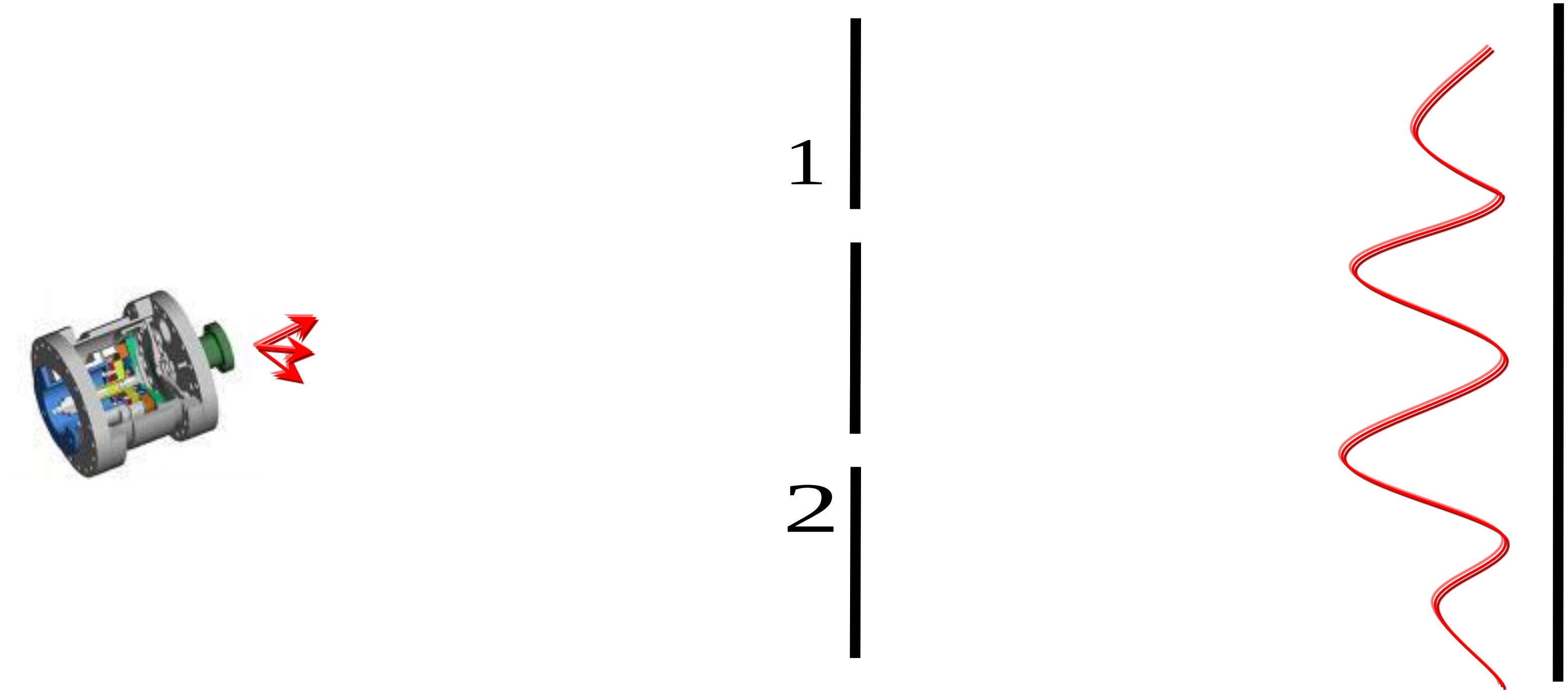
- *It should be applicable to a wide range of states, not limited to a specific type of states.*
- *It should be able to quantify the degree of a genuine superposition against a classical mixture together with its effective size factor.*

General (and useful) measure?

- *It should be applicable to a wide range of states, not limited to a specific type of states.*
- *It should be able to quantify the degree of a genuine superposition against a classical mixture together with its effective size factor.*
- *Independent of decomposition of the superposition*
- *Easy to calculate*
- *Experimentally measurable (without full tomography)*

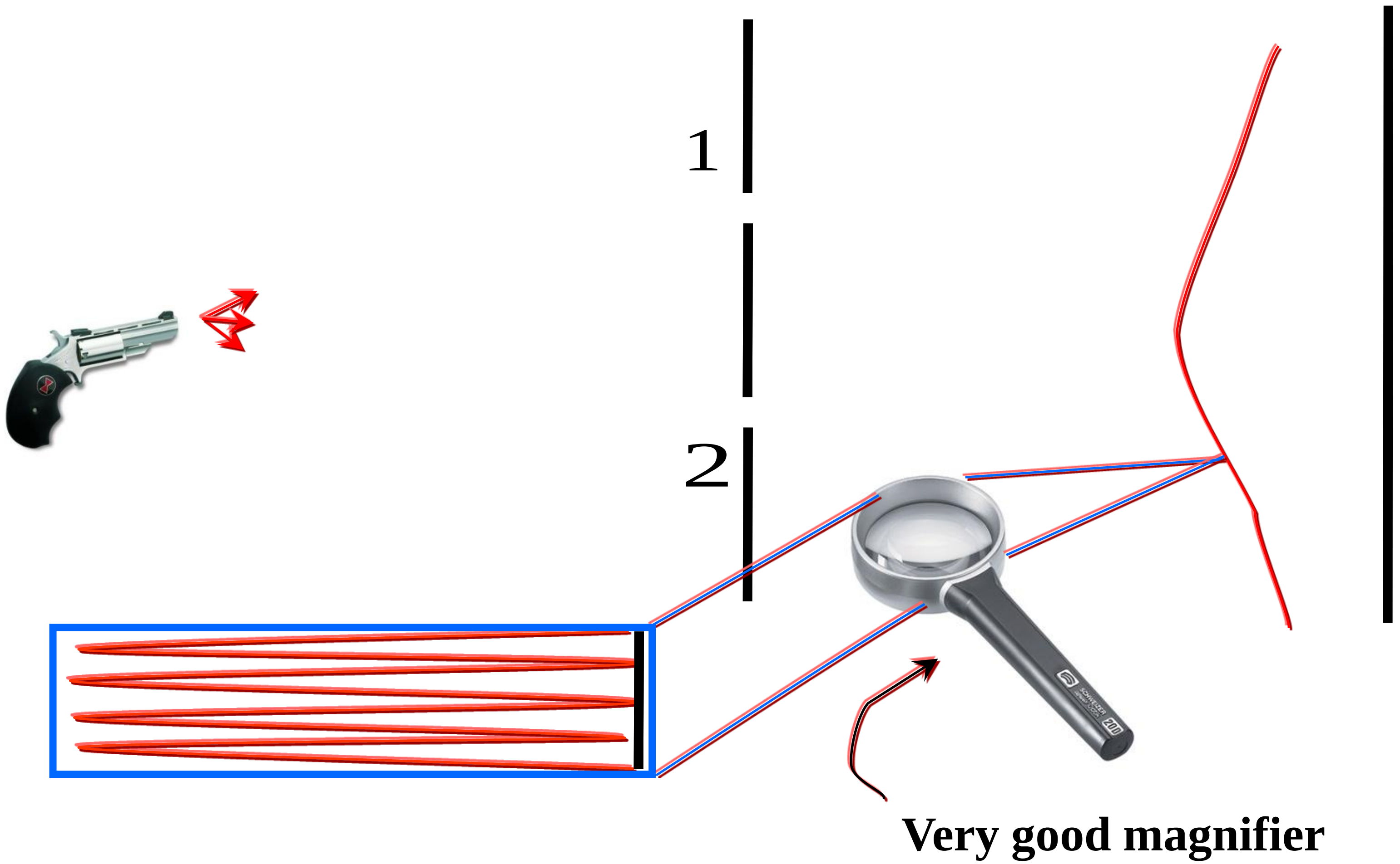
Double slit experiment

Feynman, *Lectures on Physics*, *Volume 3*

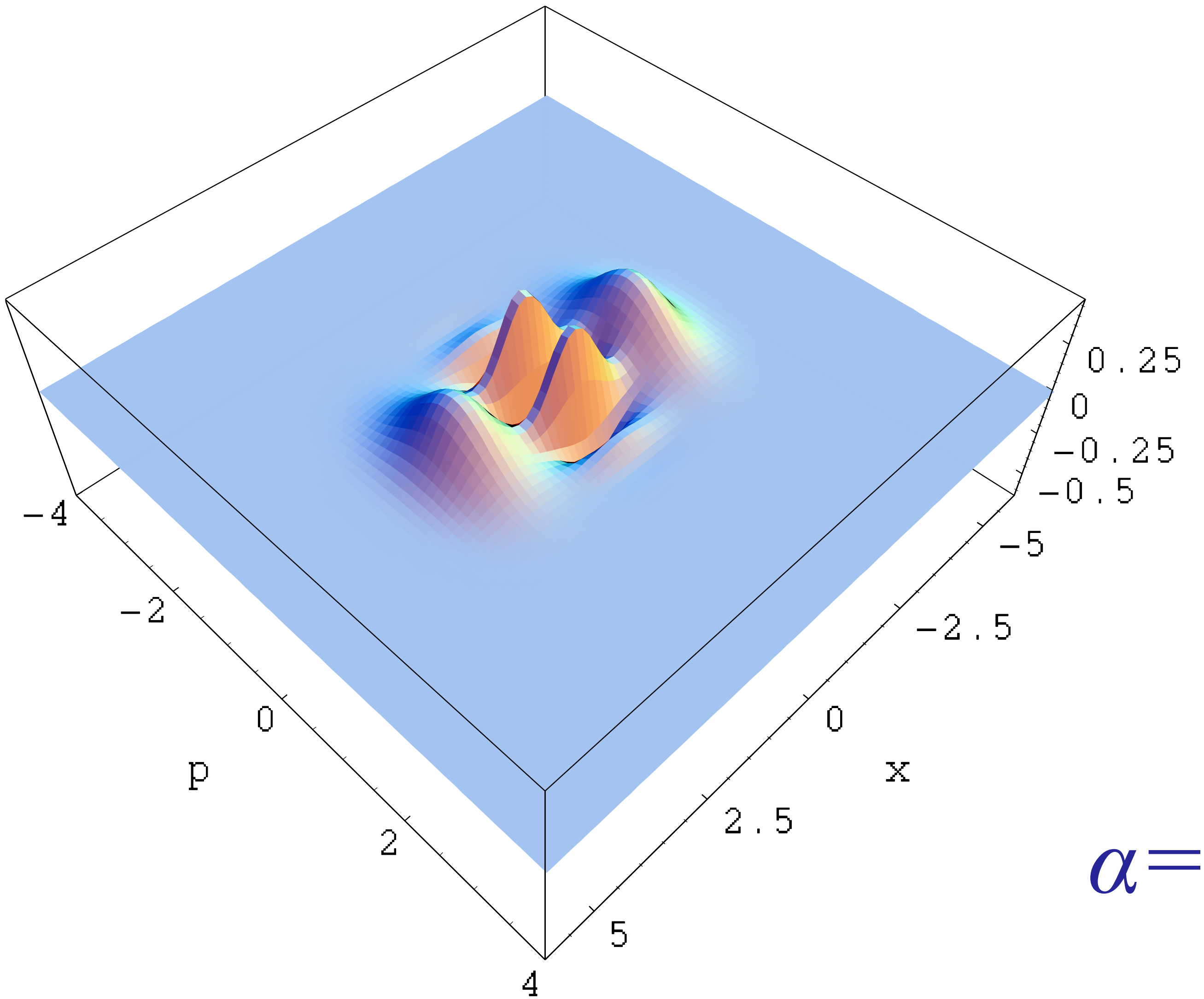


Double slit experiment

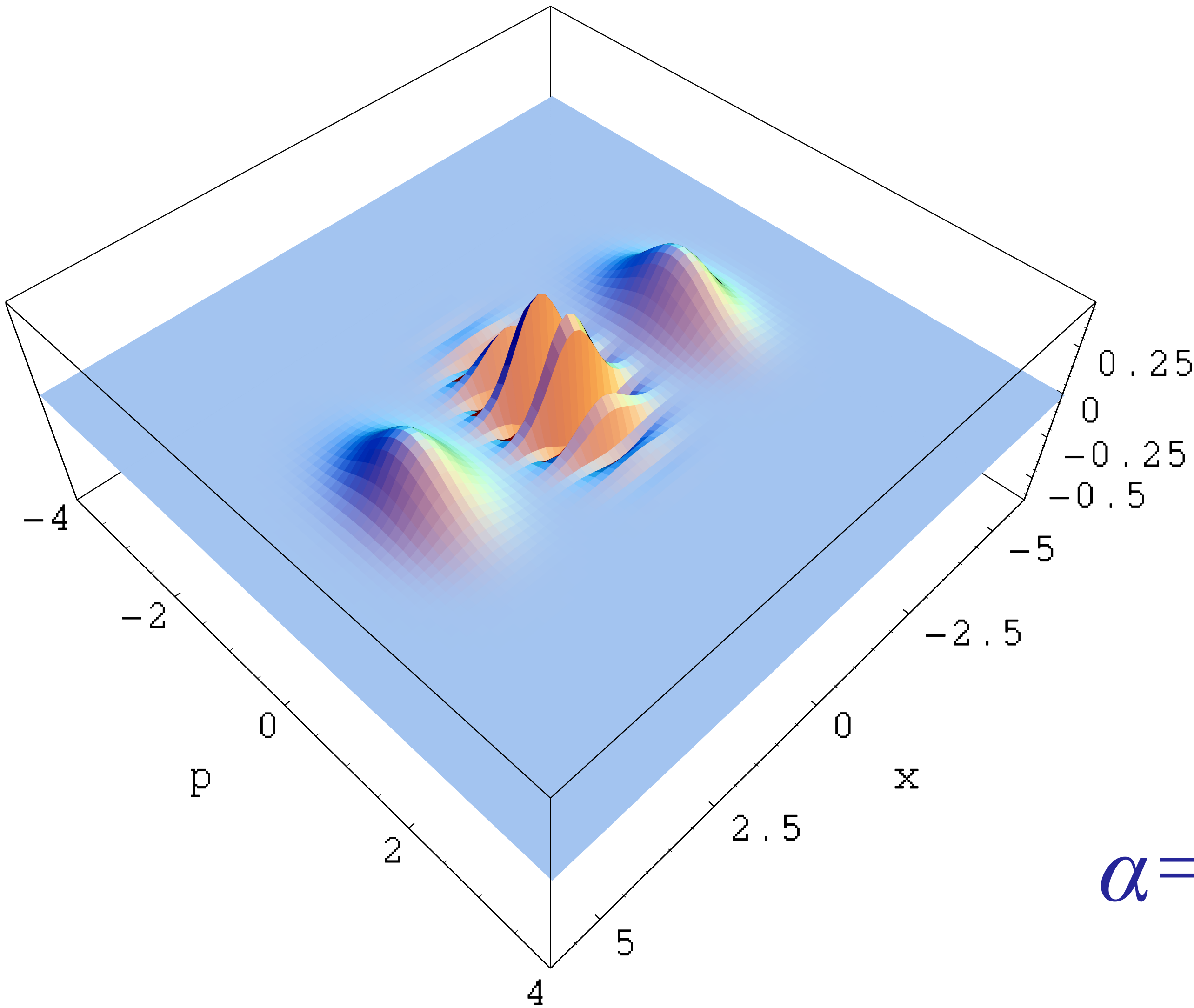
Feynman, *Lectures on Physics*, *Volume 3*



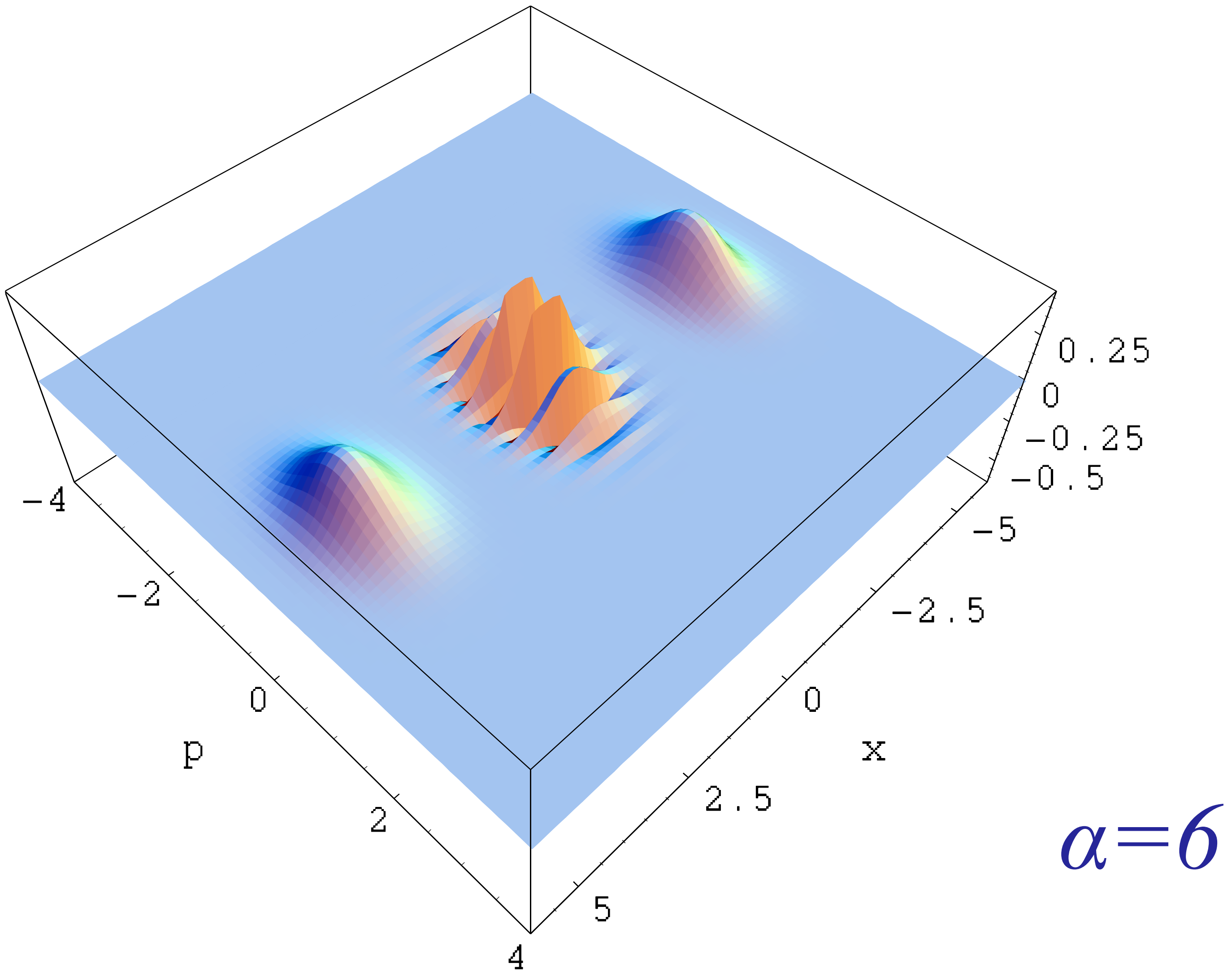
Wigner function of $|\alpha\rangle + |-\alpha\rangle$



$\alpha = 2$



α=4



0.25
0
-0.25
-0.5
-4
-2
0
2
4
p
-5
-2.5
0
2.5
5
x
α=6

Interference-based general measure for bosonic systems

C.-W. Lee and H. Jeong, *Phys. Rev. Lett.* 106, 220401 (2011)

$$\mathcal{I}(\rho) \sim \int d^2\xi \left(\xi_r^2 + \xi_i^2\right) \left|\chi\left(\xi\right)\right|^2$$

$$\chi(\xi) = \mathrm{Tr}[\hat{D}(\xi)\hat{\rho}]$$

$$\hat{D}(\xi) = e^{\xi\hat{a}^{+} - \xi^{*}\hat{a}}$$

$$W(\beta) = \frac{1}{\pi^2}\int \exp[\xi^{*}\beta - \xi\beta^{*}]\chi(\xi)d^2\xi$$

General measure for bosonic systems

C.-W. Lee and H. Jeong, *Phys. Rev. Lett.* 106, 220401 (2011)

$$\int d^2\xi \left(\xi_r^2 + \xi_i^2\right) \left|\chi\left(\xi\right)\right|^2$$

$$\chi(\xi) = \mathrm{Tr}[\hat{D}(\xi)\hat{\rho}]$$

- For an ***arbitrary*** state, it simultaneously quantifies (1) *how far-separate the component states of the superposition are* ***and*** (2) *the degree of genuine quantum coherence between the component states against their classical mixture*.
- It can be applied to *any* harmonic oscillator systems such as light fields.
- Independent of the decomposition of the component states: easy to calculate.

General measure for bosonic systems

C.-W. Lee and H. Jeong, *Phys. Rev. Lett.* 106, 220401 (2011)

$$\mathcal{I}(\rho) = \frac{1}{2\pi^M}\int d^2\boldsymbol{\xi} \sum_{m=1}^{M} \left[|\xi_m|^2 - 1\right] |\chi(\boldsymbol{\xi})|^2$$
$$= \frac{\pi^M}{2}\int d^2\boldsymbol{\alpha}\, W(\boldsymbol{\alpha}) \sum_{m=1}^{M} \left[-\frac{\partial^2}{\partial\alpha_m \partial\alpha_m^*} - 1\right] W(\boldsymbol{\alpha})$$

- $\mathcal{I}(\rho)$ is equivalent to the purity decay rate of the state as

$$\mathcal{I}(\rho) = -\frac{1}{2}\frac{d\mathcal{P}(\rho)}{d\tau}$$

where $\mathcal{P}(\rho) = \mathrm{Tr}[\rho^2]$

for a standard decoherence model (loss for a *photonic system)*.

✓ Invariant under passive linear optics operations such as the displacement operations, beam splitting, and phase shifts.

- Approximately measurable without full tomography (Jeong *et al*., J. Opt. Soc. Am. 2014)

General measure for bosonic systems

C.-W. Lee and H. Jeong, *Phys. Rev. Lett.* 106, 220401 (2011)

$$
\begin{aligned}
\mathcal{I}(\rho) &= \frac{1}{2\pi^M}\int d^2\boldsymbol{\xi} \sum_{m=1}^{M}\left[|\xi_m|^2 - 1\right]|\chi(\boldsymbol{\xi})|^2 \\
&= \frac{\pi^M}{2}\int d^2\boldsymbol{\alpha}\, W(\boldsymbol{\alpha}) \sum_{m=1}^{M}\left[-\frac{\partial^2}{\partial\alpha_m\partial\alpha_m^*} - 1\right] W(\boldsymbol{\alpha})
\end{aligned}
$$

- $\mathcal{I}(\rho)$ is equivalent to the purity decay rate of the state as

$$\mathcal{I}(\rho) = -\frac{1}{2}\frac{d\mathcal{P}(\rho)}{d\tau}$$

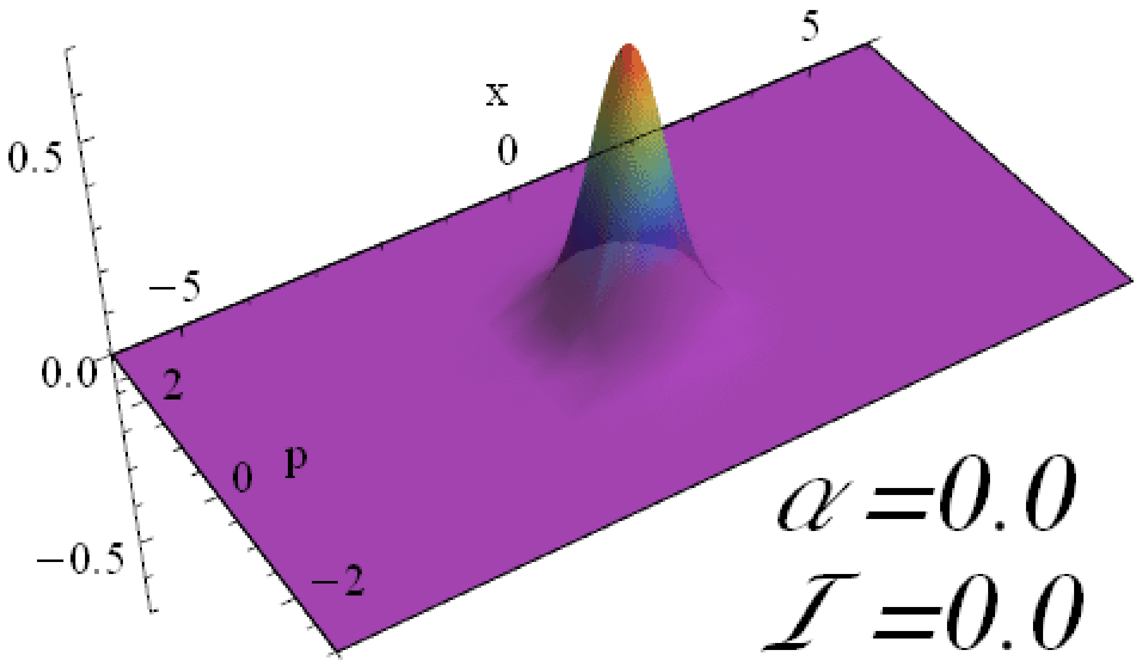
where $\mathcal{P}(\rho) = \mathrm{Tr}[\rho^2]$

for a standard decoherence model (loss for a photonic system).

✓ Invariant under passive linear optics operations such as the displacement operations, beam splitting, and phase shifts.

- $\mathcal{I}(\rho)$ can be applied to arbitrary spin systems with some modifications (C.-Y. Park *et al.*, Phys. Rev. A 94, 052105 (2016)).

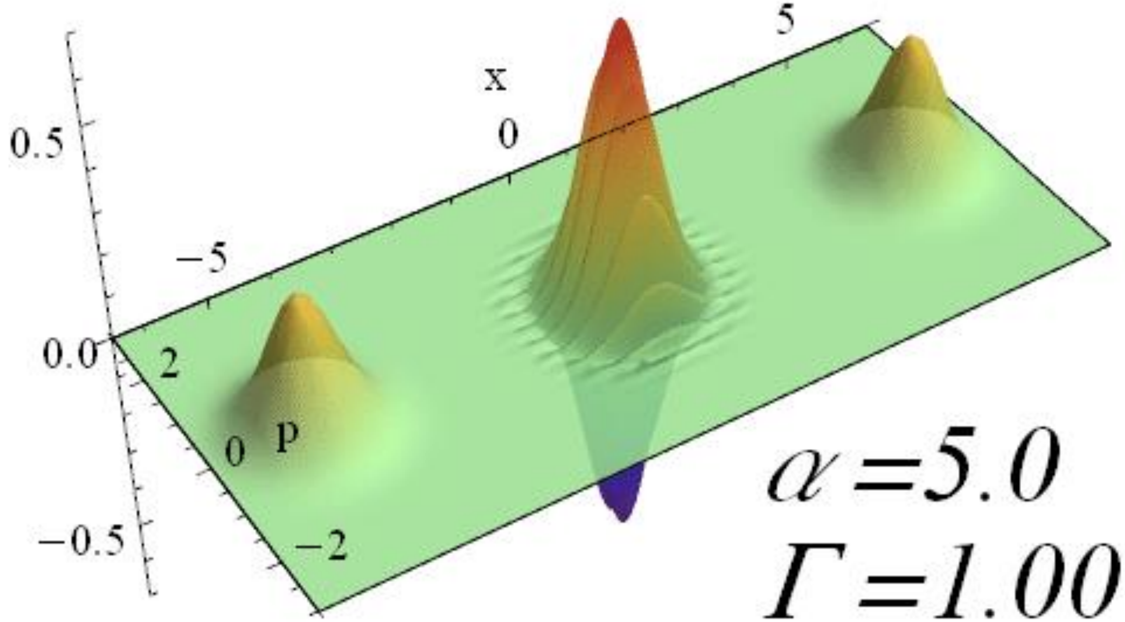
Macroscopic quantumness $\mathcal{I}$ of $|\alpha\rangle + |-\alpha\rangle$ with increasing α



Wigner function

Macroscopic quantumness $\mathcal{I}$ of

$$\rho_{\rm scs} = N_\Gamma[|\alpha\rangle\langle\alpha| + |-\alpha\rangle\langle-\alpha| + \Gamma(|\alpha\rangle\langle-\alpha| + |-\alpha\rangle\langle\alpha|)]$$



Wigner function

$\alpha = 5.0$

$\Gamma = 1.00$

$\mathcal{I} = 25.0$

Macroscopically quantum?

- Invariant under passive linear optics operations such as the displacement operations, beam splitting, and phase shifts.

- Coherent state: $I(|\alpha\rangle) = 0$ regardless of the value of α.
- Well known states in the "Schrödinger-cat family" with the maximum values of "quantum macroscopicity" I, i.e., the average photon number of the corresponding state:
  - ✓ Superposition of coherent states: $|\alpha\rangle + |-\alpha\rangle$
  - ✓ NOON state: $|n\rangle|0\rangle + |0\rangle|n\rangle$
  - ✓ GHZ state: $|H\rangle^N + |V\rangle^N$
  - ✓ Hybrid entanglement: $\frac{1}{\sqrt{2}}\left(|H\rangle|\alpha\rangle + |V\rangle|-\alpha\rangle\right)$

- **“Micro-macro” entanglement or “macro-macro” entanglement**

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[159] N. Bruno, A. Martin, P. Sekatski, N. Sangouard, R.T. Thew, N. Gisin, Nat. Phys. 9 (2013) 545.
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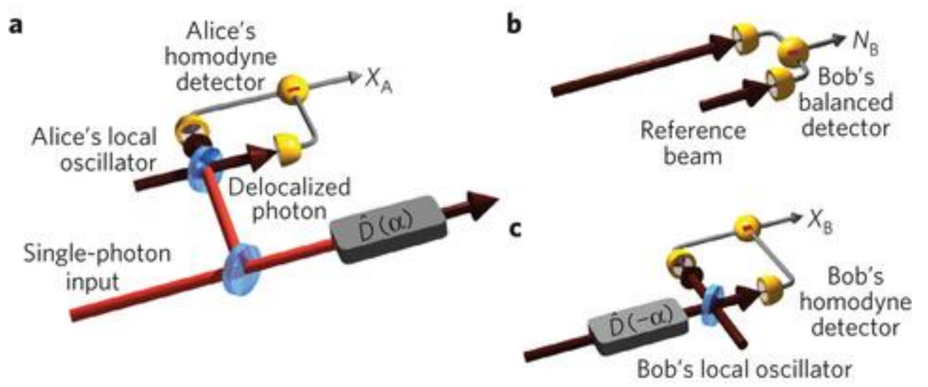
Micro-macro entanglement

N. Bruno *et al.*, Nature Physics 9, 545 (2013); A. I. Lvovsky *et al.* Nature Physics 9, 541 (2013)



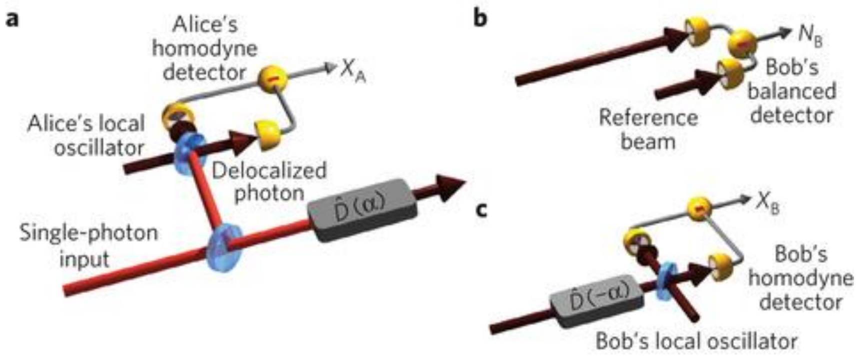
$$
\begin{aligned}
& D(\alpha)(|0\rangle|1\rangle + |1\rangle\,|0\rangle) \\
& \quad = D(\alpha)(|+\rangle|-\rangle - |-\rangle\,|+\rangle) \\
& \qquad = \underbrace{D(\alpha)|+\rangle}|-\rangle - \underbrace{D(\alpha)|-\rangle}\,|+\rangle
\end{aligned}
$$

$$|\pm\rangle = |0\rangle \pm |1\rangle$$

- $D(\alpha)|+\rangle$ and $D(\alpha)|-\rangle$ are distinguishable by a single-shot photon number measurement with a high probability [Sekatski *et al.*, PRA 012116 (2014)].

Micro-macro entanglement

N. Bruno *et al.*, Nature Physics 9, 545 (2013); A. I. Lvovsky *et al.* Nature Physics 9, 541 (2013)



$$D_1(\alpha)\left(|0\rangle_1|1\rangle_2+|1\rangle_1|0\rangle_2\right)$$
$$=D_1(\alpha)\left(|+\rangle_1|-\rangle_2-|-\rangle_1|+\rangle_2\right)$$

$$|\pm\rangle=|0\rangle\pm|1\rangle$$

- $D(\alpha)(|0\rangle+|1\rangle)=D(\alpha)|+\rangle$ and $D(\alpha)(|0\rangle-|1\rangle)=D(\alpha)|-\rangle$ are distinguishable by a single-shot photon number measurement with a high probability [Sekatski *et al*., PRA 012116 (2014)].
- However, if so, even a coherent state

$$|\alpha\rangle=D(\alpha)\left(|0\rangle+|1\rangle\right)+D(\alpha)\left(|0\rangle-|1\rangle\right)$$

should be interpreted as a macroscopic superposition state. Is it acceptable?

- The value of quantum macroscopicity is only I=1 regardless of the value of α.

H. Jeong, M. Kang and H. Kwon, Special Issue on Macroscopic Quantumness, Optics Communications 337, 12 (2015).

General measure for spin systems

F. Fröwis, W. Dür, New J. Phys. 14 (2012) 093039

- A quantum system is 'macroscopic' ***if*** there exists $\hat{A}$ such that

$$\max_{\hat{A} \in \mathcal{A}} F(\rho, \hat{A}) = O(N^2)$$

where F is quantum Fisher information and $\hat{A}$ is an additive operator $A = \sum_{i=1}^{N} A^{(i)}$.

$$F(\rho, \hat{A}) = 2 \sum_{i,j} \frac{(\lambda_i - \lambda_j)^2}{\lambda_i + \lambda_j} \left| \langle i | \hat{A} | j \rangle \right|^2, \quad \text{where} \quad \rho = \sum_i \lambda_i |i\rangle \langle i|$$

✓ $F=O(N)$ for a product state $|\psi^-\rangle^{\otimes N/2}$

✓ $F=O(N^2)$ for a GHZ state $|\mathrm{GHZ}_L\rangle = \frac{1}{\sqrt{2}} \left(|0_L\rangle^{\otimes N} + |1_L\rangle^{\otimes N} \right)$

- Applicable to arbitrary spin systems.
- Operational meaning in relation to quantum metrology.

General measures

- *It should be applicable to a wide range of states, not limited to a specific type of states.*
- *It should be able to quantify the degree of a genuine superposition against a classical mixture together with its effective size factor.*

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[13] C.-W. Lee, H. Jeong, Phys. Rev. Lett. 106 (2011) 220401.
[14] F. Fröwis, W. Dür, New J. Phys. 14 (2012) 093039.

General measures

- *It should be applicable to a wide range of states, not limited to a specific type of states.*
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[13] C.-W. Lee, H. Jeong, Phys. Rev. Lett. 106 (2011) 220401.
[14] F. Fröwis, W. Dür, New J. Phys. 14 (2012) 093039.

- [13] corresponds to sensitivity to decoherence while [14] is sensitivity to phase shifts.
- For pure states, these two measures become 'identical' – the maximum variance of an arbitrarily chosen observable $\boldsymbol{A}$.

General framework for quantum macroscopicity

B. Yadin and V. Vedral, Phys. Rev. A **93**, 022122 (2016)

- Assume ρ in terms of eigenbasis $|i\rangle$ of observable $A = \sum_i a_i |i\rangle\langle i|$
- δ-coherence: $\rho^{(\delta)} := \sum_{i,j:\, a_i - a_j = \delta} \rho_{i,j} |i\rangle\langle j|$
- Free-operation: $\mathcal{E}(\rho)^{(\delta)} = \mathcal{E}(\rho^{(\delta)})\ \forall \delta \in \Delta$
- Conditions for a macroscopic coherence measure $M(\rho)$

(M1): $M(\rho) \geq 0$ and $M(\rho) = 0 \iff \rho = \rho^{(0)}$.
(M2a): $M(\rho) \geq M(\mathcal{E}(\rho))$ for a trace-preserving free operation $\mathcal{E}$.
(M2b): $M(\rho) \geq \sum_\alpha p_\alpha M(\mathcal{E}_\alpha(\rho)/p_\alpha)$, where $\mathrm{Tr}\mathcal{E}_\alpha(\rho) = p_\alpha$.
(M3): $M(\sum_i p_i \rho_i) \leq \sum_i p_i M(\rho_i)$.
(M4): $M(|i\rangle + |j\rangle) > M(|k\rangle + |l\rangle)$, when $|a_i - a_j| > |a_k - a_l|$.

- ✓ (M1, M2a, M2b): requirements for a monotone
- ✓ (M3): convexity
- ✓ (M4): requirement for a ***macroscopicity*** measure

Disturbance-based measure of macroscopic coherence

H. Kwon, C.-Y. Park, K. C. Tan, H. Jeong, New J. Physics 19, 043024 (2017)

For given observable $\hat{A} = \sum_i a_i |i\rangle \langle i|$,

✓ Coarse-grained measurement: $\hat{Q}_x^\sigma = \sum_i \sqrt{q_i^\sigma(x)}\, |i\rangle \langle i|$

where $q_i^\sigma(x) := \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(a_i - x)^2}{2\sigma^2}}$

✓ Quantum state after the measurement: $\Phi_\sigma(\rho) = \int_{-\infty}^{\infty} \hat{Q}_x^\sigma \rho \hat{Q}_x^\sigma dx$

✓ Quantum state disturbance by coarse-grained measurement

$$M_\sigma(\rho) := D(\rho, \Phi_\sigma(\rho))$$

for the Bures distance $D_B(\rho, \tau) = 2 - 2\sqrt{\mathcal{F}(\rho, \tau)}$

and quantum relative entropy $D_R(\rho, \tau) = S(\rho || \tau)$

satisfies all the conditions (M1) – (M4) for every σ>0.

Disturbance-based measure of macroscopic coherence

H. Kwon, C.-Y. Park, K. C. Tan, H. Jeong, New J. Physics 19, 043024 (2017)

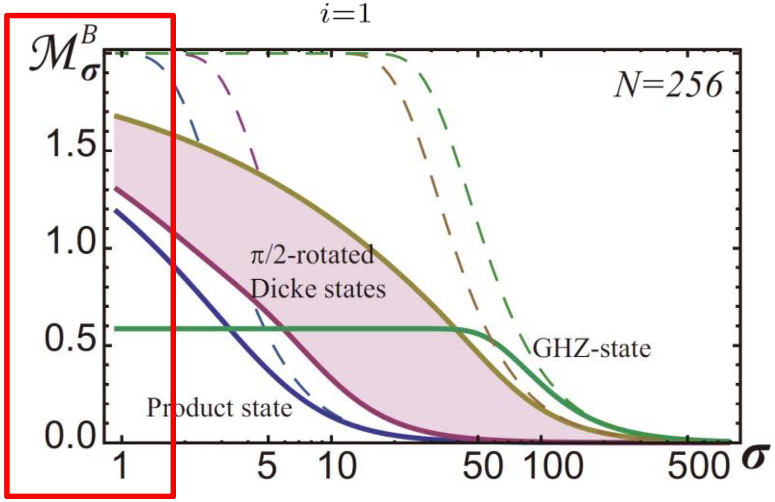
- Examples

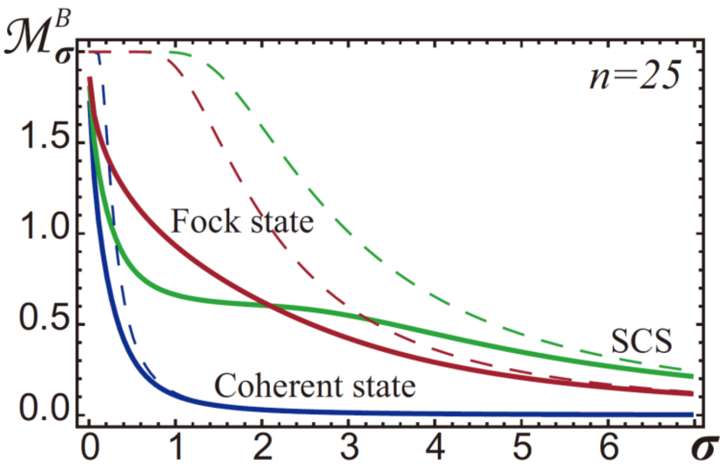
1) N-particle spin system with total spin measurement:

$$\hat{A} = \sum_{i=1}^{N} \vec{\alpha}^{(i)} \cdot \hat{\sigma}^{(i)}$$

2) Bosonic system with quadrature measurement:

$$\hat{X} = (\hat{a} + \hat{a}^\dagger)/2$$





※ For precise measurement ($\sigma \to 0$), product states have larger values of M than the GHZ-state

→ ***The criteria of macroscopic coherence by Yadin and Vedral, (M1)-(M4), are insufficient.***

✓ Our solution: Take the coarse-graining scale to the classical measurement regime, $\sigma \gg \sqrt{N}$

➔ Quantifying the size of a superposition between (classically) distinct states.

Quantum information processing using light

- ***All-optical quantum information processing***
  - ***How to overcome limitations of single-photon qubits?***
  - ***Non-deterministic…***

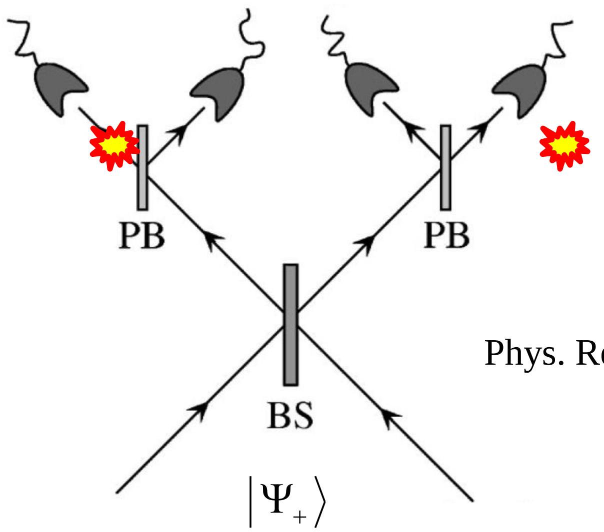
Bell-state measurement with single-photon qubits

Lütkenhaus *et al*., Phys. Rev. A (1999)

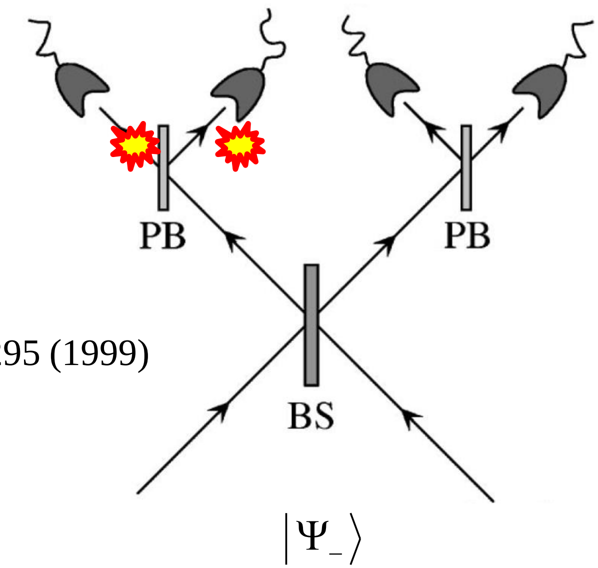
$$|\Phi_\pm\rangle = \frac{1}{\sqrt{2}}(|H\rangle|H\rangle \pm |V\rangle|V\rangle)$$

$$|\Psi_\pm\rangle = \frac{1}{\sqrt{2}}(|H\rangle|V\rangle \pm |V\rangle|H\rangle)$$

$$|qubit\rangle = a|H\rangle + b|V\rangle$$



Phys. Rev. A 59, 3295 (1999)

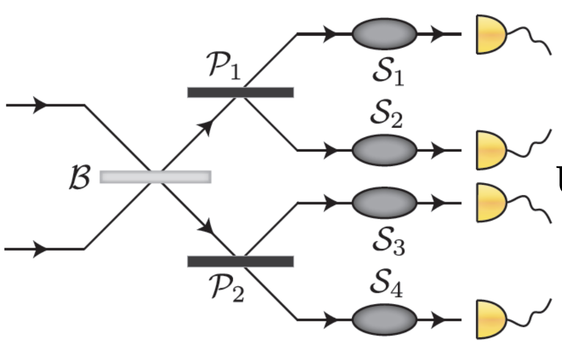


- **Only two among the four Bell states can be detected using linear optics and photodetectors.**
- **The success probability of teleportation for a single-photon qubit cannot exceed 50%.**

Schemes to enhance success probabilities of Bell-state measurements for single-photon qubits

- Ancillary entangled states
  - W. *P. Grice, Phys. Rev A 84, 042331 (2011)*
  - *F. Ewert and P. van Loock, Phys. Rev. Lett. 113, 140403 (2014)*

- In-line squeezing operations
  - H. A. Zaidi and P. van Loock,
    Phys. Rev. Lett. 110, 260501 (2013)



Up to 64.2%

- *Additional resources or operations are required.*

Quantum information processing using light

- ***All-optical quantum information processing***
  - ***How to overcome limitations of single-photon qubits?***

  - ***With more photons…***
    - ✓ ***Coherent-state (cat-state) qubits***
    - ✓ ***Hybrid scheme based on squeezed resource***
    - ✓ ***Optical hybrid qubits***
    - ✓ ***Multi-photon (GHZ-type) qubits***

Coherent-state qubits and Bell states

(HJ and M. S. Kim, *Phys. Rev.* A 65, 042305 (2002); *Phys. Rev. A*. 68, 042319 (2003) *etc*).

- Qubit encoding with coherent states

$$|\alpha\rangle \rightarrow |0_L\rangle, \quad |-\alpha\rangle \rightarrow |1_L\rangle,$$

$$|\psi_{qubit}\rangle = a|\alpha\rangle + b|-\alpha\rangle$$

c.f. single photon qubit for LOQC

$$a|H\rangle + b|V\rangle$$

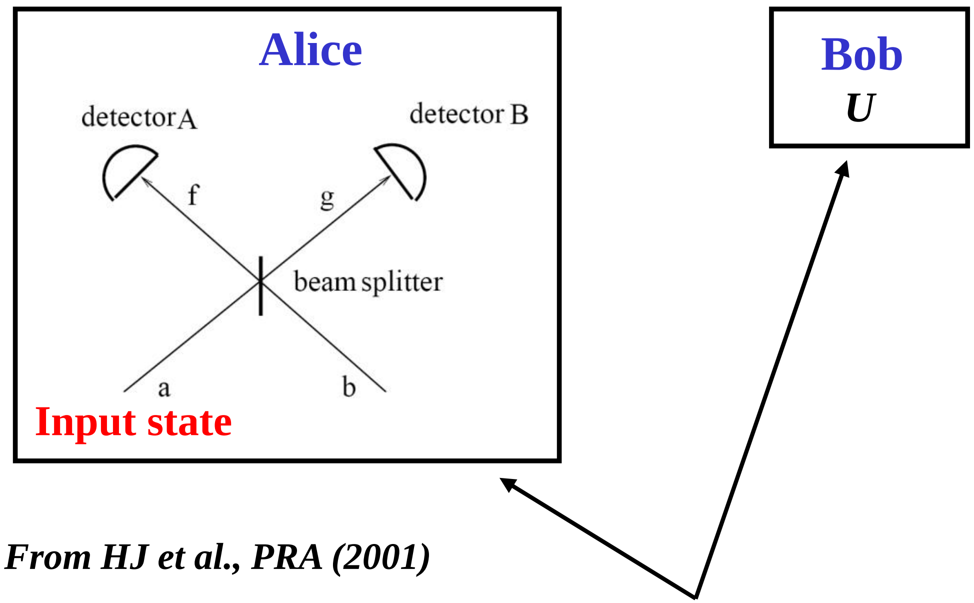
- Bell states with coherent states

$$|B_{1,2}\rangle = N_{\pm}(|\alpha\rangle|\alpha\rangle \pm |-\alpha\rangle|-\alpha\rangle),$$

$$|B_{3,4}\rangle = N_{\pm}(|\alpha\rangle|-\alpha\rangle \pm |-\alpha\rangle|\alpha\rangle)$$

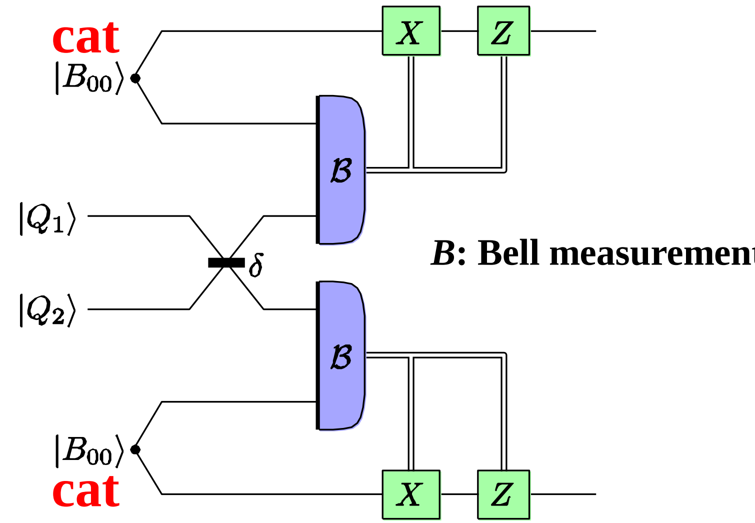
Teleportation and quantum gates using coherent state qubits

(van Enk and Hirota, PRA 2001, Jeong *et al*., PRA2001, Jeong and Kim, PRA 2002, Ralph *et al*., PRA 2003)



From HJ et al., PRA (2001)

$$N_{-}(|\alpha\rangle|-\alpha\rangle-|-\alpha\rangle|\alpha\rangle)$$

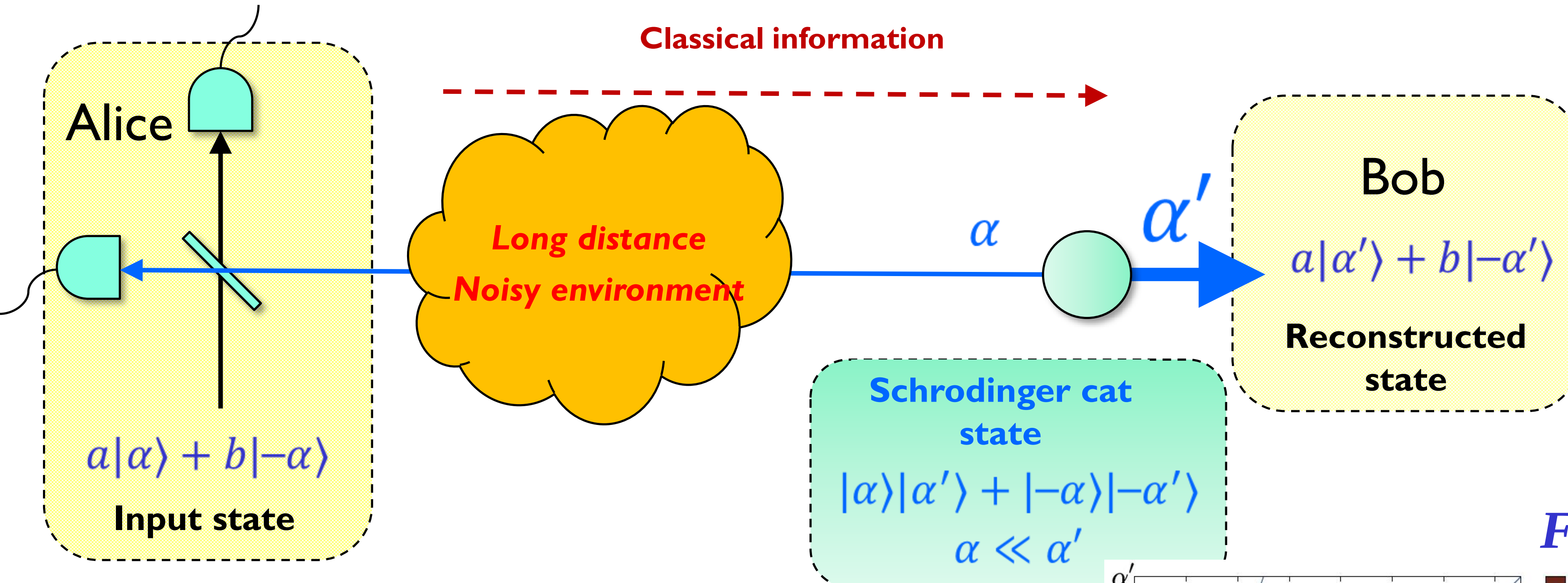


$$|B_{00}\rangle = N_{+}(|\alpha\rangle|\alpha\rangle+|-\alpha\rangle|-\alpha\rangle)$$

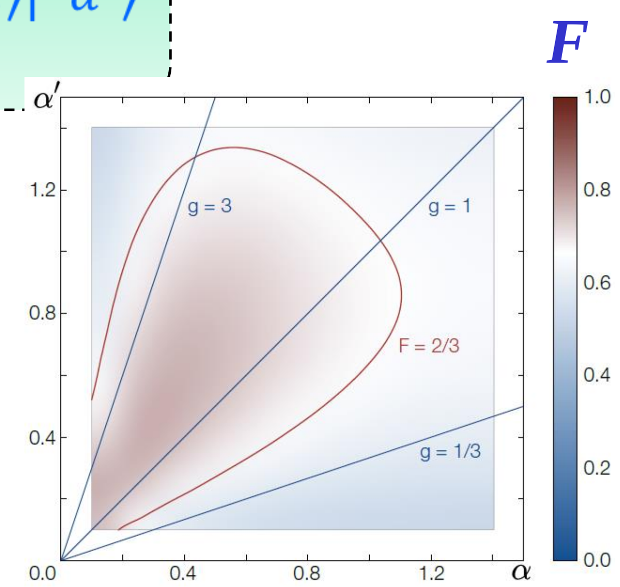
From Ralph et al., PRA (2003)

- **Deterministic Bell-state measurement and teleportation**
- **Sensitive to photon losses**
- **Optimized value α~1.6 (α>1.2 is required for fault-tolerant quantum computing) [Lund *et al.* PRL 2008]**

Tele-amplification of coherent-state qubits



J. S. Neergaard-Nielsen, Y. Eto, C.-W. Lee, HJ and M. Sasaki, *Nature Photonics* 7, 439-443 (2013).

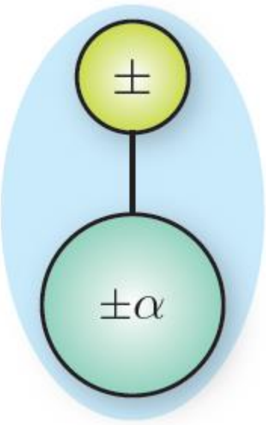


Hybrid approach:

S.-W. Lee and H. Jeong, "Near-deterministic quantum teleportation and resource-efficient quantum computation using linear optics and hybrid qubits," **Phys. Rev. A 87, 022326 (2013).**

$$a\underbrace{|H\rangle|\alpha\rangle}_{|0_L\rangle}+b\underbrace{|V\rangle|-\alpha\rangle}_{|1_L\rangle}$$

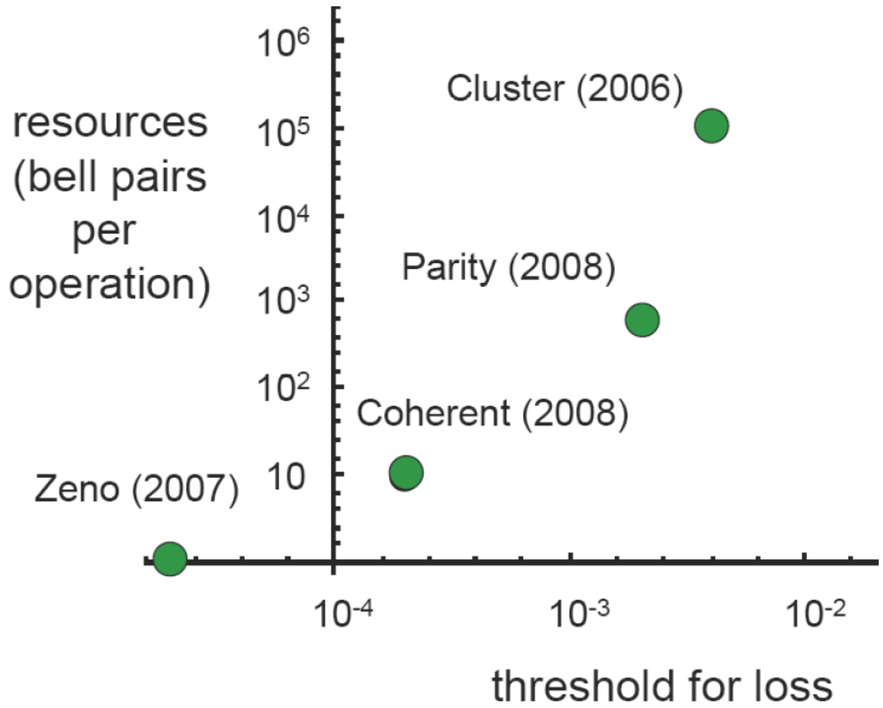
“Optical hybrid qubit”



“Single-photon” mode

“Coherent-state” mode

Comparison with previous schemes



T. C. Ralph and G. J. Pryde, Progress in Optics, Ed. E.Wolf, 54, 209 (2009)

Comparison with previous schemes

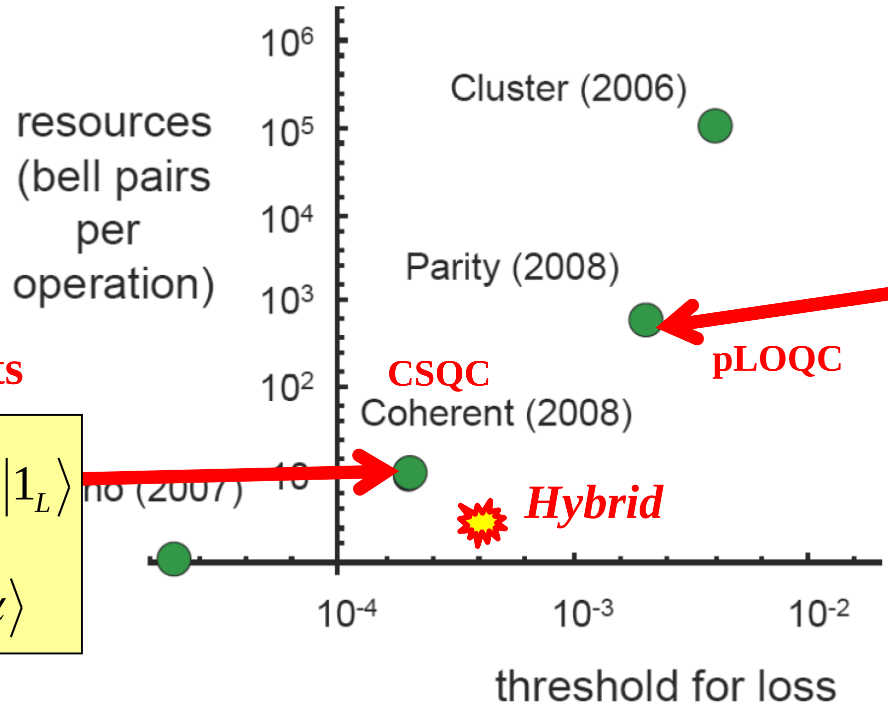
Resource

$$\frac{1}{\sqrt{N}}\left(|\alpha\rangle|\alpha\rangle + |-\alpha\rangle|-\alpha\rangle\right)$$

Coherent state qubits

$$|\alpha\rangle \to |0_L\rangle, \quad |-\alpha\rangle \to |1_L\rangle$$

$$|\psi_{qubit}\rangle = a|\alpha\rangle + b|-\alpha\rangle$$



Single photon qubits

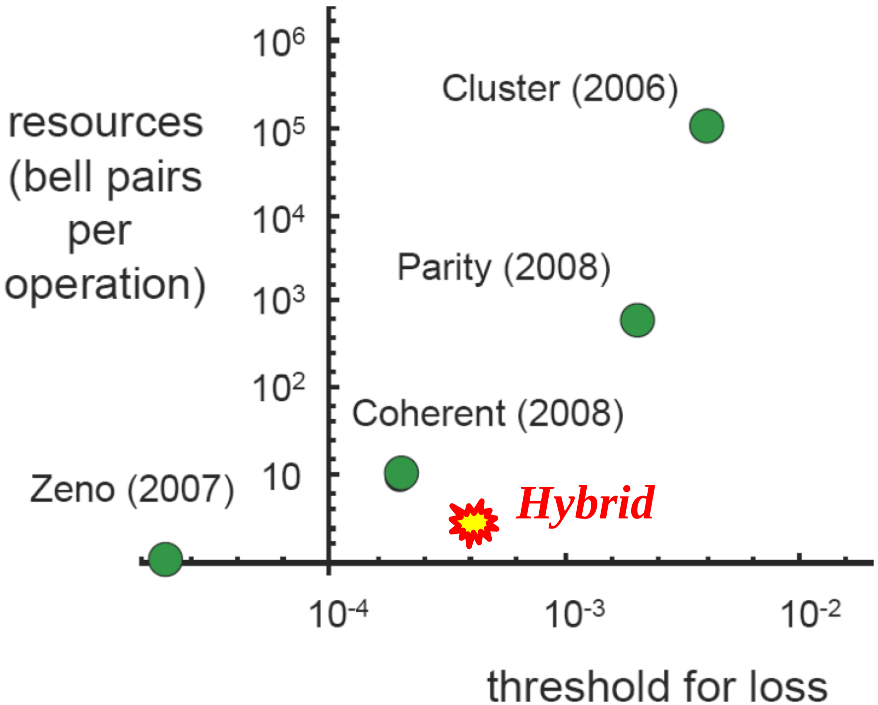
$$|H\rangle \to |0_L\rangle, \quad |V\rangle \to |1_L\rangle$$

$$|\psi_{qubit}\rangle = a|H\rangle + b|V\rangle$$

Resource

$$\frac{1}{\sqrt{2}}\left(|H\rangle|H\rangle + |V\rangle|V\rangle\right)$$

Comparison with previous schemes

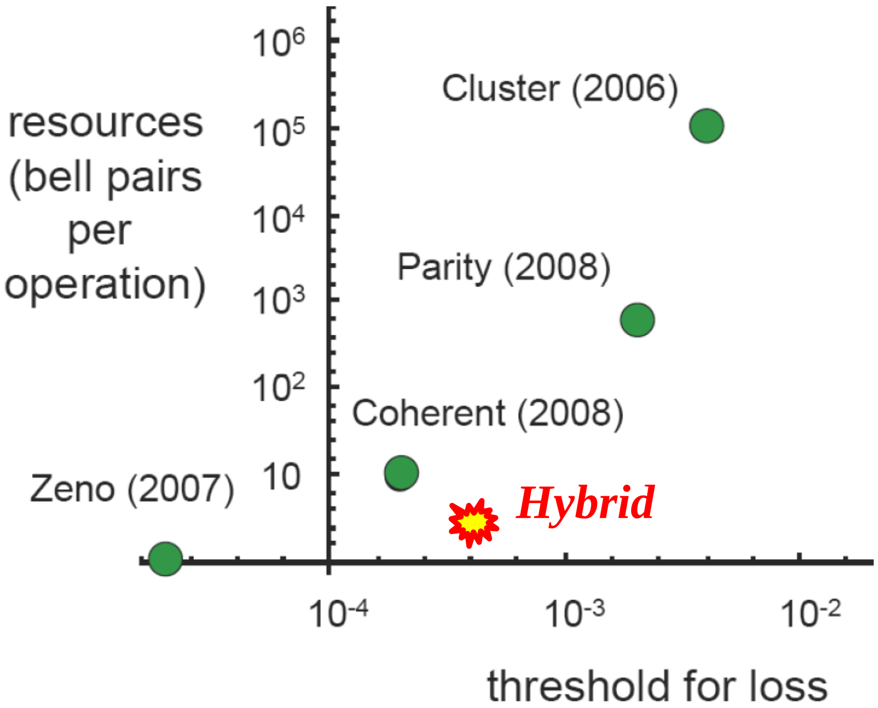


Resource

$$\frac{1}{\sqrt{2}}\left(|H\rangle|\alpha\rangle+|V\rangle|-\alpha\rangle\right)$$

The hybrid approach seems to outperform the previous ones with hybrid entangled states of $\alpha \approx 1.1$ as resources.

Comparison with previous schemes



Resource

$$\frac{1}{\sqrt{2}}\left(|0\rangle|\alpha\rangle+|1\rangle|-\alpha\rangle\right)$$

The hybrid approach seems to outperform the previous ones with hybrid entangled states of $\alpha \approx 1.1$ as resources.

Coherent states and single photons

- **Coherent state:** $|\alpha\rangle = e^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle$ ***"Classical"***

✓ Coherent states are most classical among all pure states

- semi-classical descriptions available

- most robust against decoherence ("pointer states")

✓ The two coherent states | α > and | -α > are "classically" (or macroscopically) distinguishable for α >>1, *i.e.*, they can be well discriminated by a homodyne measurement (HD) with limited efficiency.

(For 70% of HD efficiency: *D*≈99.7% for α=1.6 and *D*>99.9% for α=2.0.)

- **Single photon:** $|1\rangle$ ***"Non-classical"***

✓ Discrete light quantum containing the minimum quantized amount of energy available at a given frequency.

✓ Negative values in well-known quasi-probability distributions such as the Wigner function.

Hybrid entanglement

- ***Hybrid entanglement between a single photon and a coherent state:***

$$\frac{1}{\sqrt{2}}\left(|0\rangle|\alpha\rangle+|1\rangle|-\alpha\rangle\right)$$

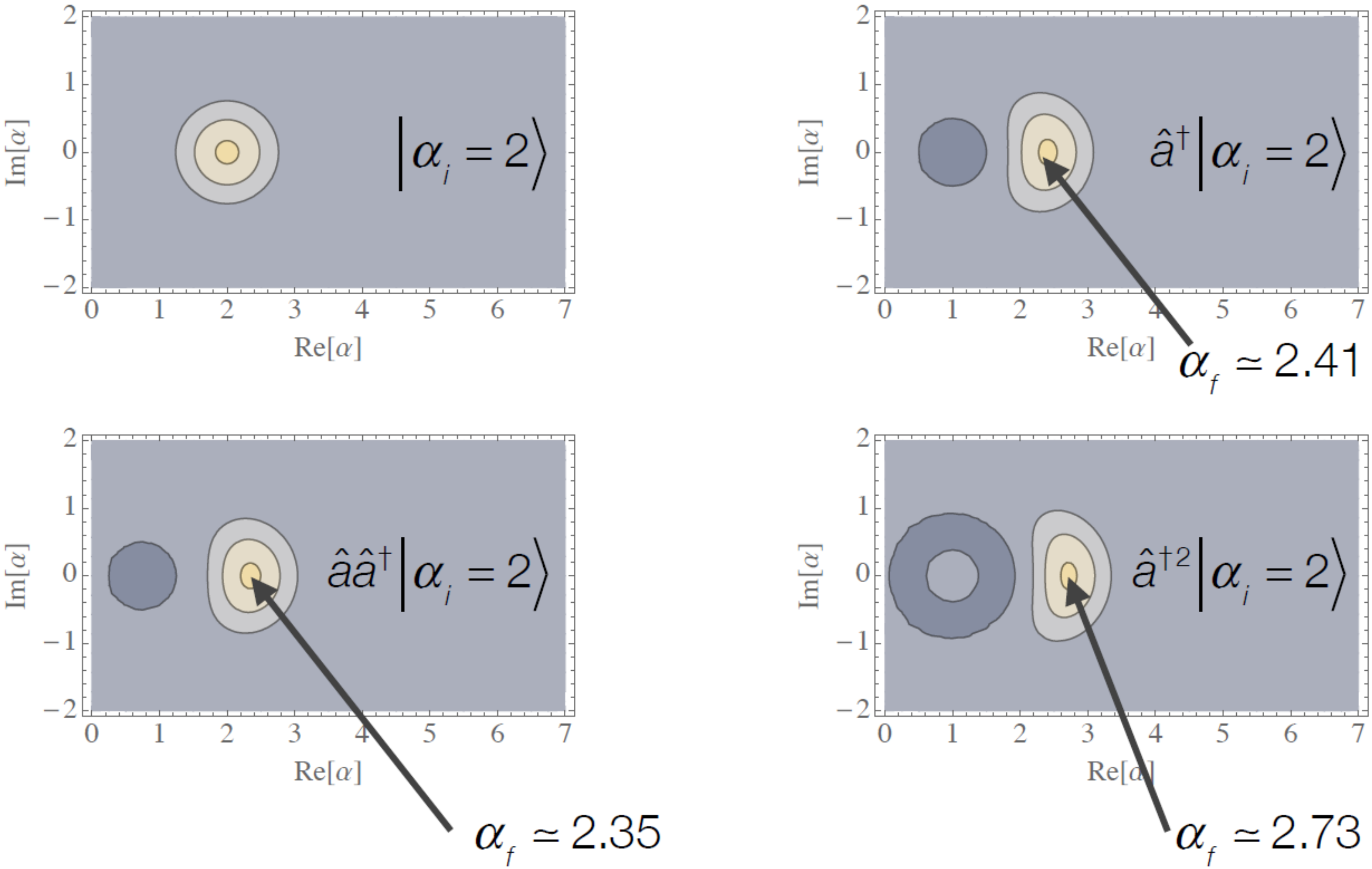
$$\frac{1}{\sqrt{2}}\left(|e\rangle|Alive\rangle+|g\rangle|Dead\rangle\right)$$

- The closest optical analogy to Schrödinger's *Gedankenexperiment* when α is reasonably large.

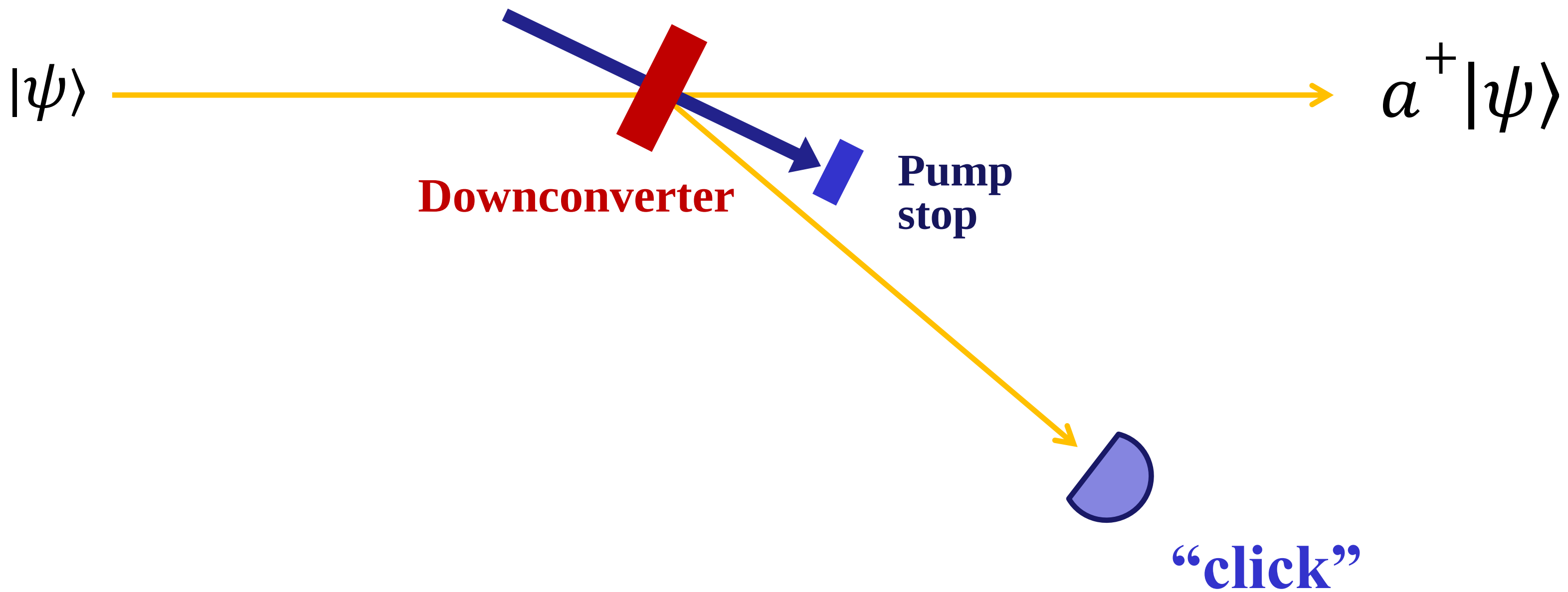
How to generate hybrid entanglement

- Cross-Kerr nonlinearity?
- Many fundamental problems lie in the way of realizing this type of interaction. [J. H. Shapiro, *Single-photon Kerr nonlinearities do not help quantum computation.* Phys. Rev. A 73, 062305 (2006); J. H. Shapiro & M. Razavi, *Continuous-time cross-phase modulation and quantum computation.* New J. Phys. 9, 16 (2007).]

Wigner Functions of Amplified Coherent States



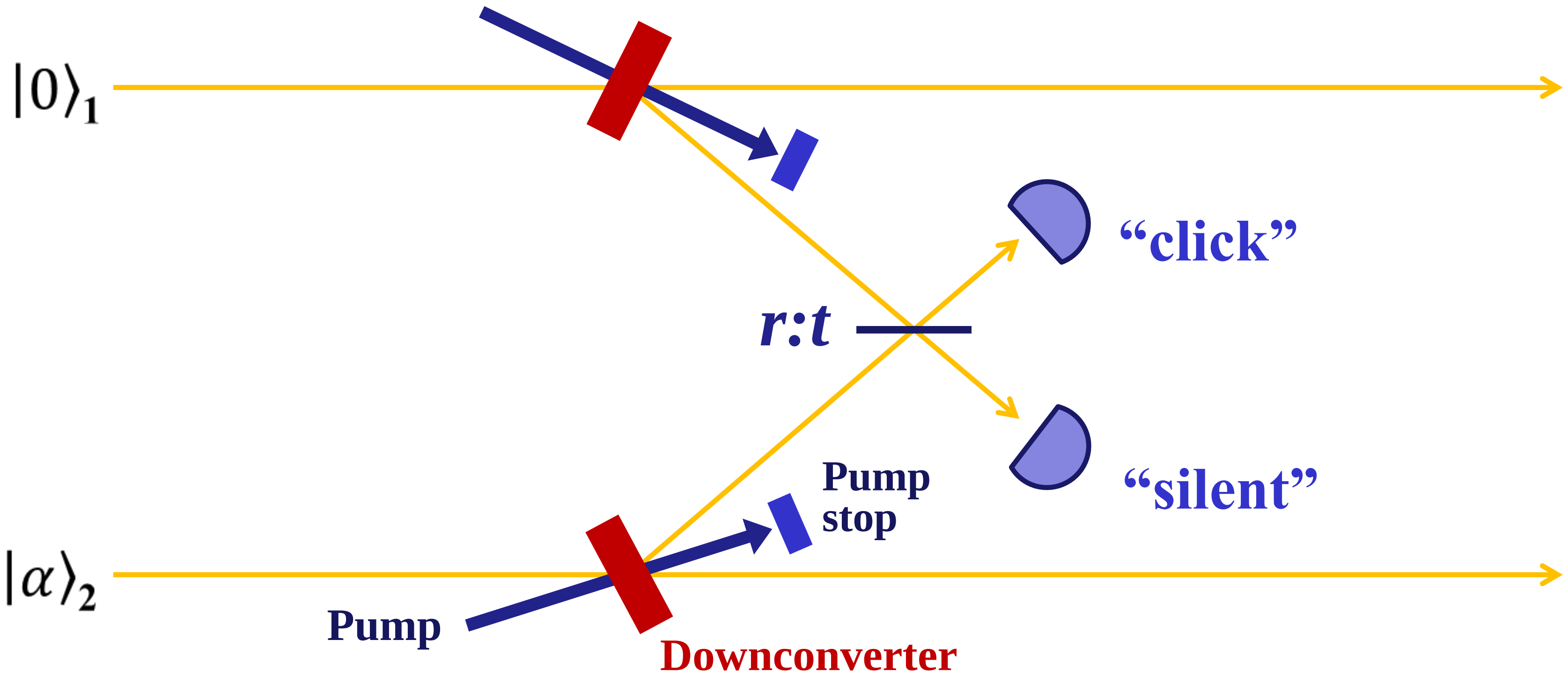
Single photon addition



[Zavatta *et al*., Science, 2004]

Generation of hybrid entanglement

H. Jeong, A. Zavatta, M. Kang, S.-W. Lee, L.S. Costanzo, S. Grandi, T.C. Ralph & M. Bellini, Nature Photonics 8, 564 (2014).



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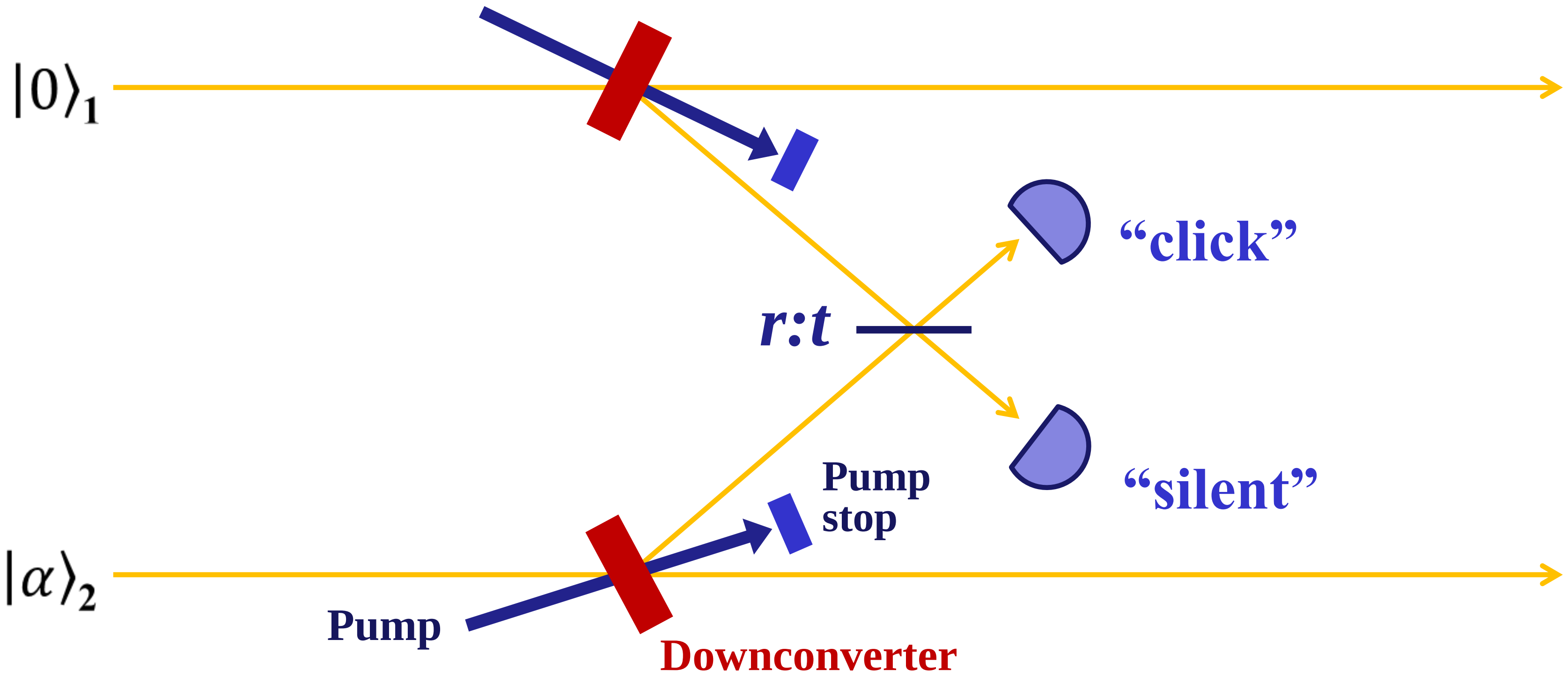

$$r a^{+} |0\rangle|\alpha\rangle + t|0\rangle a^{+} |\alpha\rangle$$

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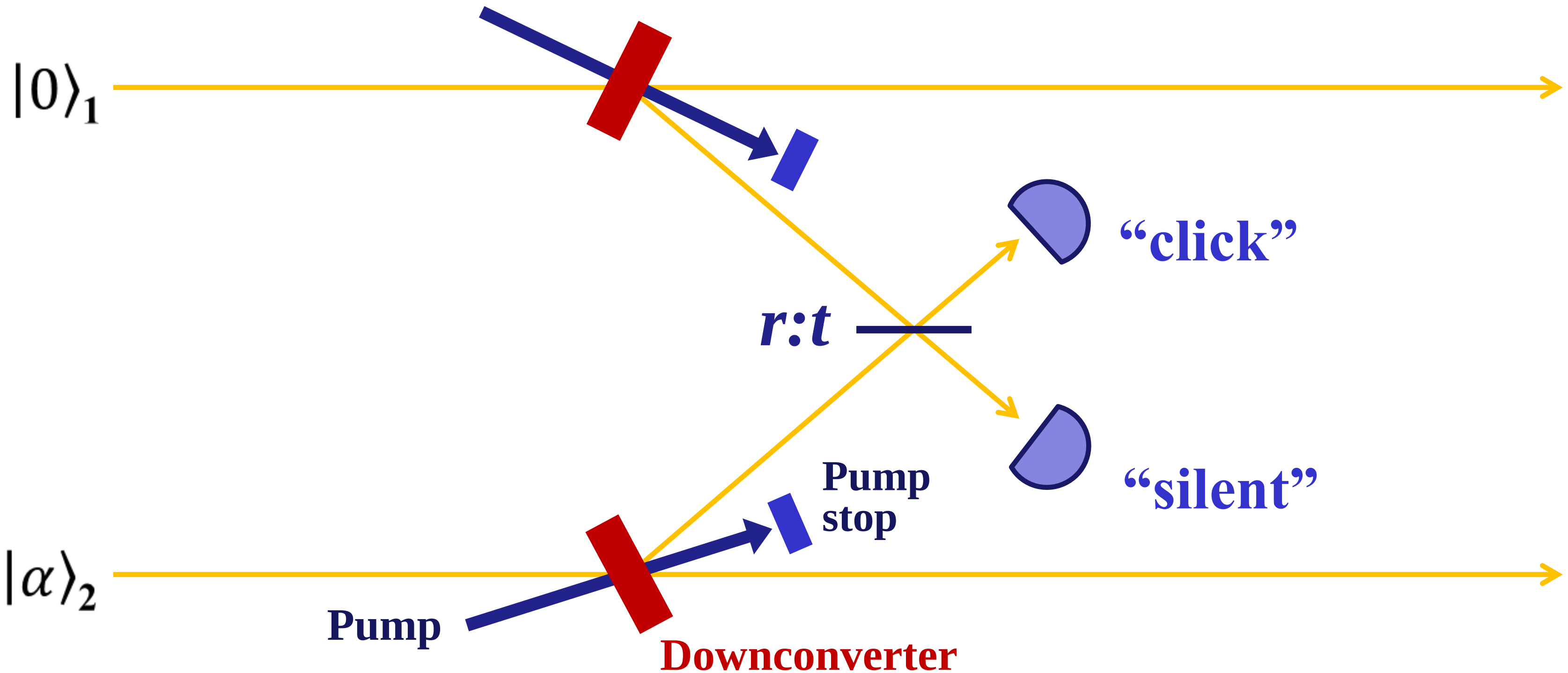
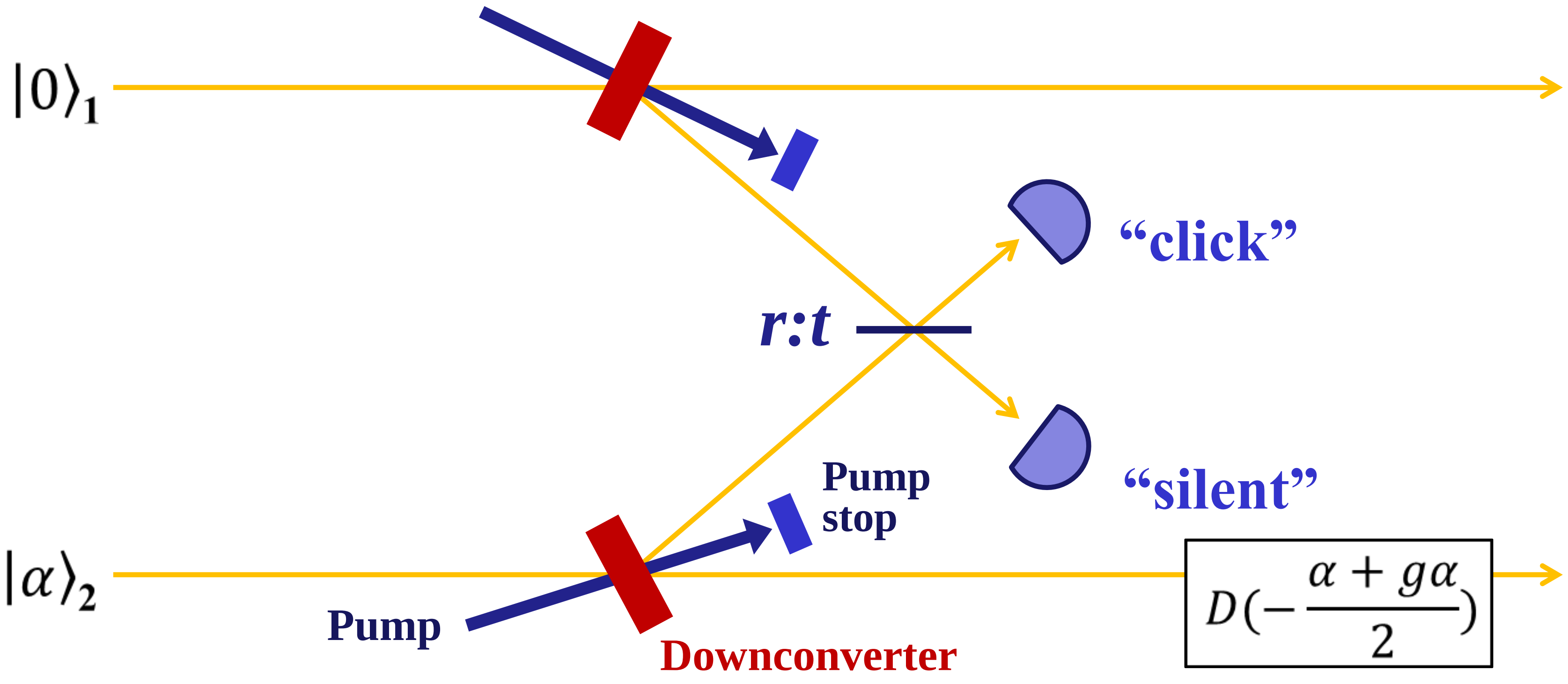


$$r a^{+}|0\rangle|\alpha\rangle + t|0\rangle a^{+}|\alpha\rangle \approx |1\rangle|\alpha\rangle + |0\rangle|g\alpha\rangle \quad \text{with } g>1$$

Generation of hybrid entanglement

H. Jeong, A. Zavatta, M. Kang, S.-W. Lee, L.S. Costanzo, S. Grandi, T.C. Ralph & M. Bellini, Nature Photonics 8, 564 (2014).



$$\approx |1\rangle|\alpha\rangle + |0\rangle|g\alpha\rangle$$

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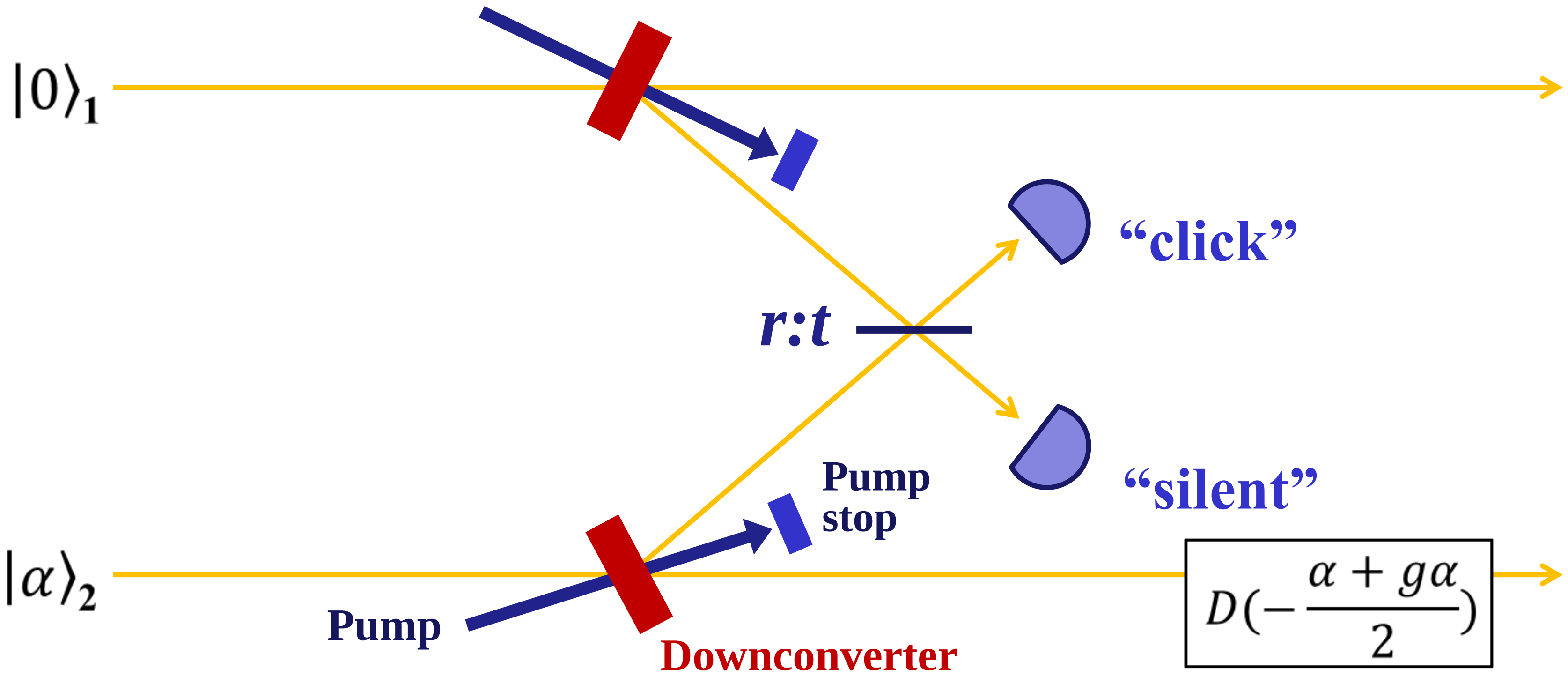

$$\approx |1\rangle|\alpha\rangle + |0\rangle|g\alpha\rangle \qquad \rightarrow \; |0\rangle|\alpha_f\rangle + |1\rangle|-\alpha_f\rangle$$

Generation of hybrid entanglement

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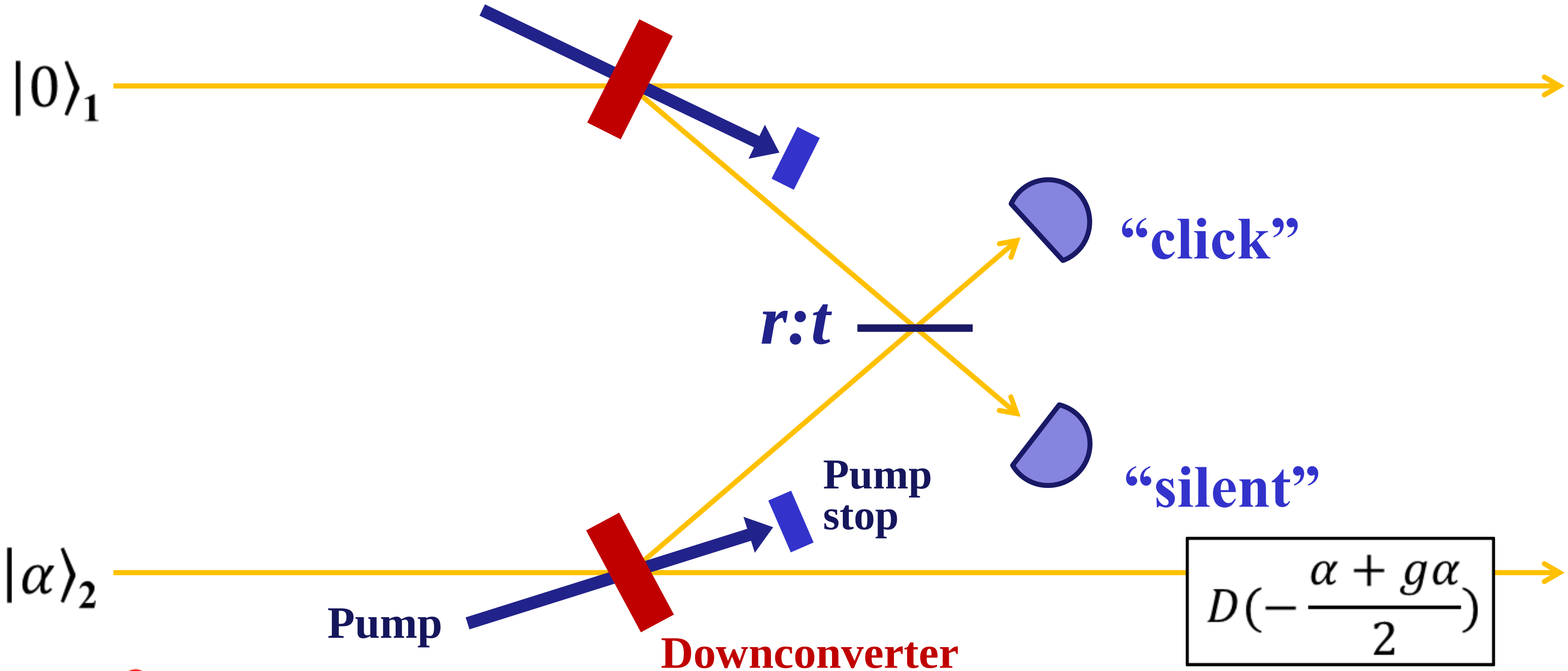
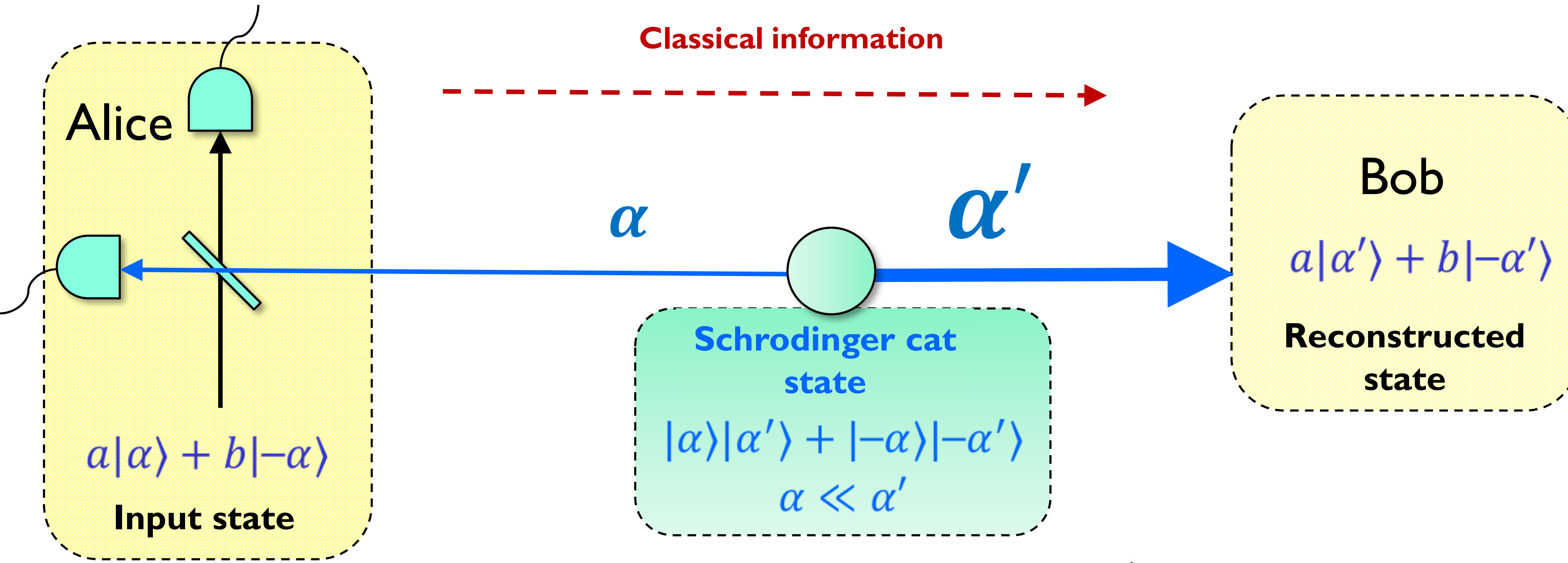


α=2

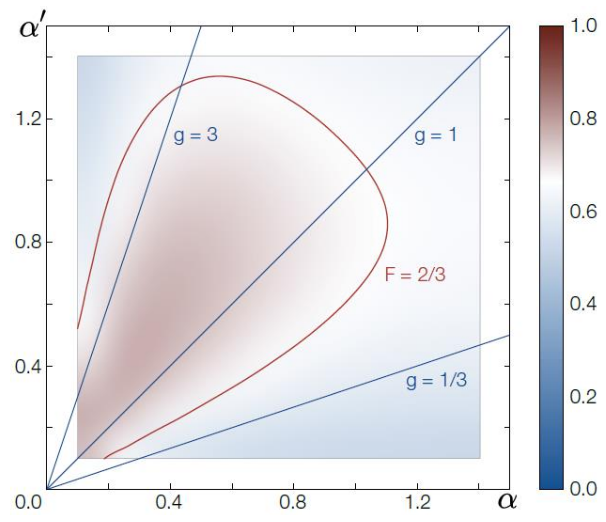
$$\approx |1\rangle|\alpha\rangle + |0\rangle|g\alpha\rangle \qquad \rightarrow \; |0\rangle|\alpha_f\rangle + |1\rangle|-\alpha_f\rangle$$

$F > 0.99$ **for** $\alpha_f = 0.21$

Tele-amplification of coherent-state qubits



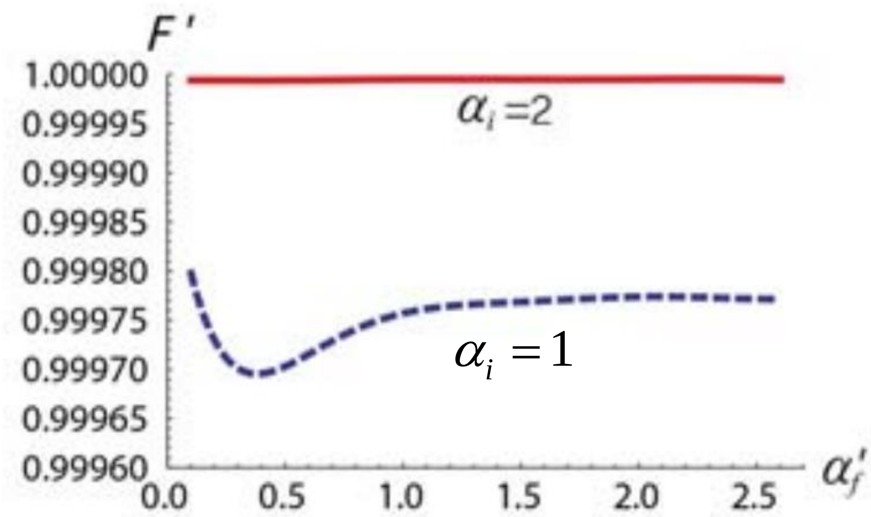
J. S. Neergaard-Nielsen, Y. Eto, C.-W. Lee, HJ and M. Sasaki, *Nature Photonics* (2013)



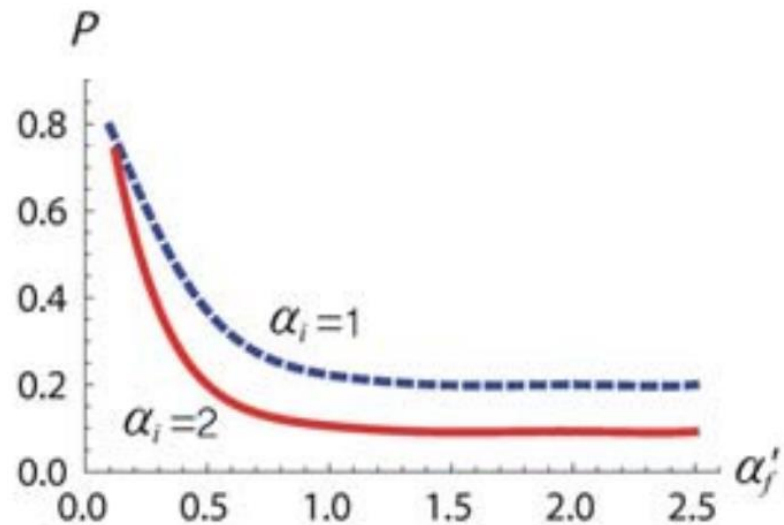
Tele-amplification of hybrid entanglement

- Both fidelities and amplitudes are increased using another ***entangled coherent state*** and two ***single photon*** detectors.

Improved Fidelity

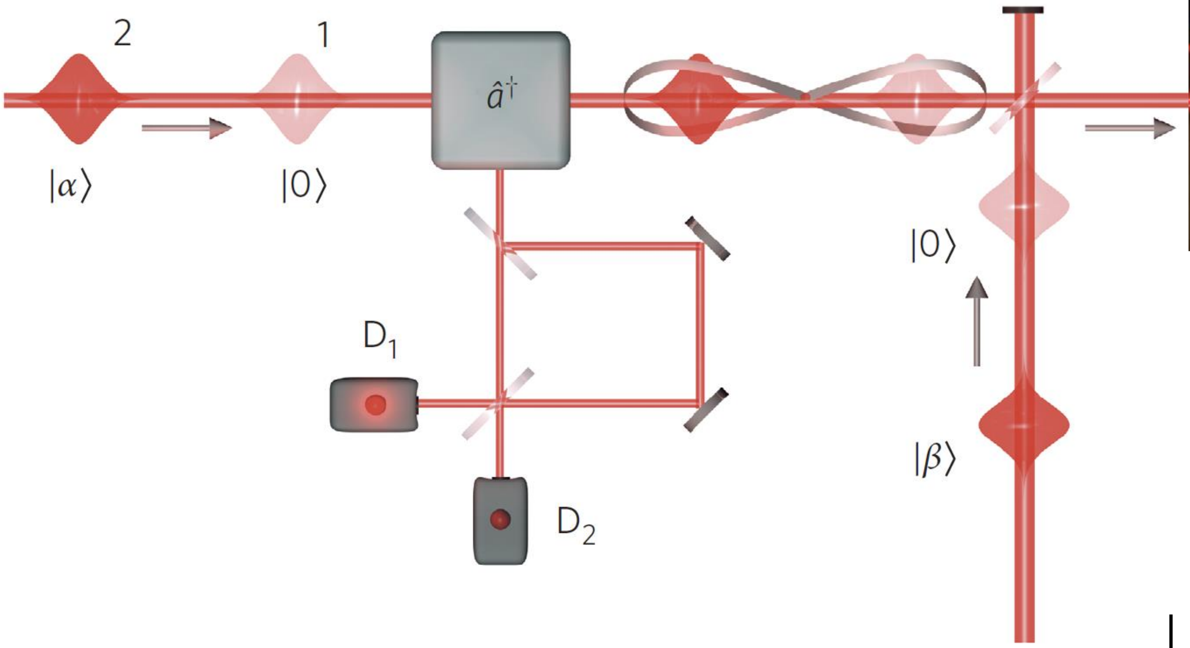


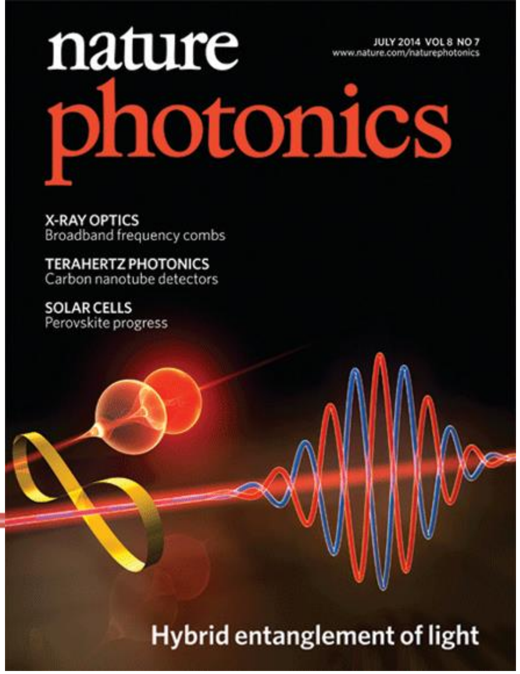
Success Probability



H. Jeong, A. Zavatta, M. Kang, S.-W. Lee, L.S. Costanzo, S. Grandi, T.C. Ralph & M. Bellini, Nature Photonics 8, 564 (2014).

Experimental generation of hybrid entanglement





$$\alpha_f \approx 0.31$$

$$F \approx 0.76$$

$$\mathrm{NPT} \approx 0.45$$

$$|0\rangle|\alpha\rangle + |1\rangle|-\alpha\rangle$$

- ✓ H. Jeong, A. Zavatta, M. Kang, S.-W. Lee, L.S. Costanzo, S. Grandi, T.C. Ralph & M. Bellini, Nature Photonics 8, 564 (2014).
- ✓ (c.f.) O. Morin, K. Huang, J. Liu, H. Le Jeannic, C. Fabre and J. Laurat, Nature Photonics 8, 570 (2014).

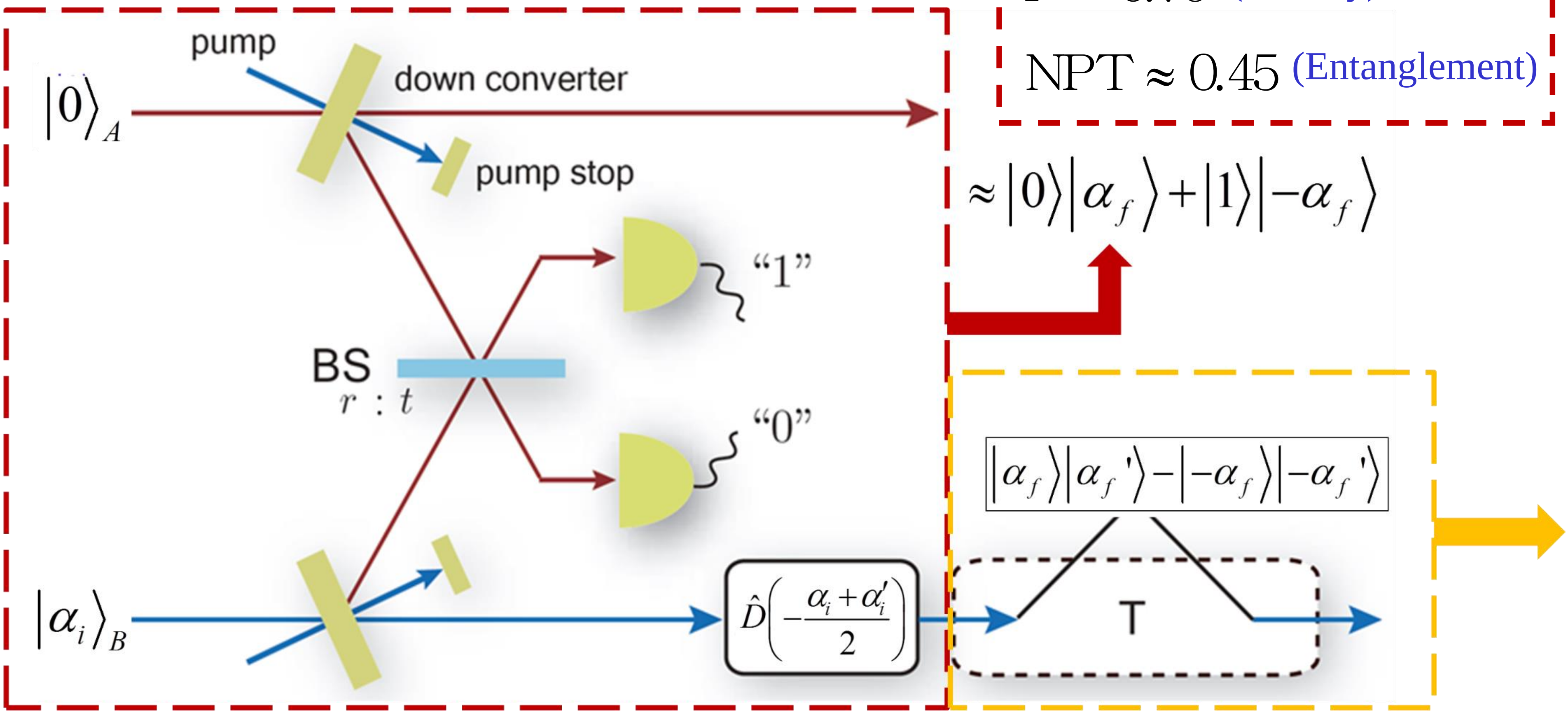
Entangling quantum and classical states of light:
Hyunseok Jeong, Alessandro Zavatta, Minsu Kang, Seung-Woo Lee, Luca S. Costanzo, Samuele Grandi, Timothy C. Ralph & Marco Bellini, Nature Photonics 8, 564 (2014).

$\alpha_f \approx 0.31$ (Final amplitude)

$F \approx 0.76$ (Fidelity)

$\mathrm{NPT} \approx 0.45$ (Entanglement)

Experimentally realized part for small-size hybrid entanglement



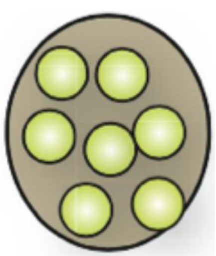
$$\approx |0\rangle|\alpha_i'\rangle + |1\rangle|\alpha_i\rangle$$

Quantum computing using multi-photon qubits

S.-W. Lee, K. Park, T. C. Ralph, and HJ, Phys. Rev. Lett. 114, 113603 (2015)

- Muliti-photon qubits

$$a|H\rangle^{\otimes N} + b|V\rangle^{\otimes N} = a\underbrace{|H\rangle|H\rangle|H\rangle\cdots}_{|0_L\rangle} + b\underbrace{|V\rangle|V\rangle|V\rangle\cdots}_{|1_L\rangle}$$

N photons  $a|0_L\rangle \pm b|1_L\rangle$

Quantum computing using multi-photon qubits

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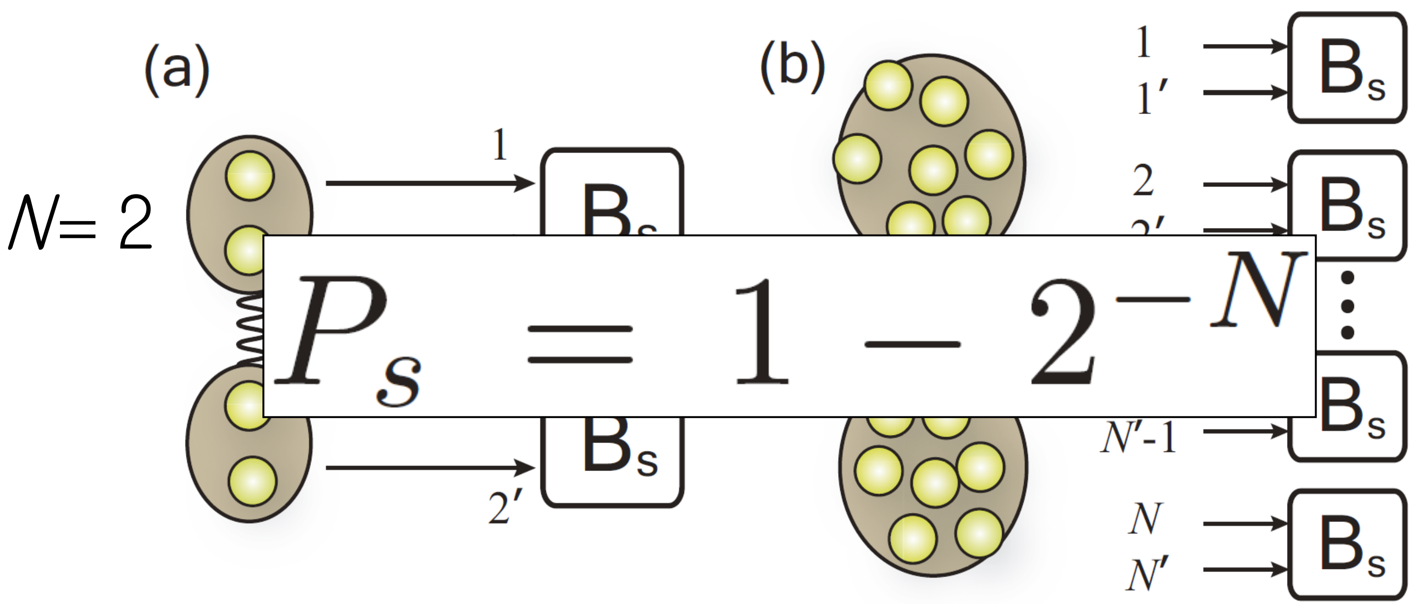
- Muliti-photon qubits

$$a|H\rangle^{\otimes N} + b|V\rangle^{\otimes N} = a|H\rangle|H\rangle|H\rangle\cdots + b|V\rangle|V\rangle|V\rangle\cdots$$



$$P_s = 1 - 2^{-N}$$

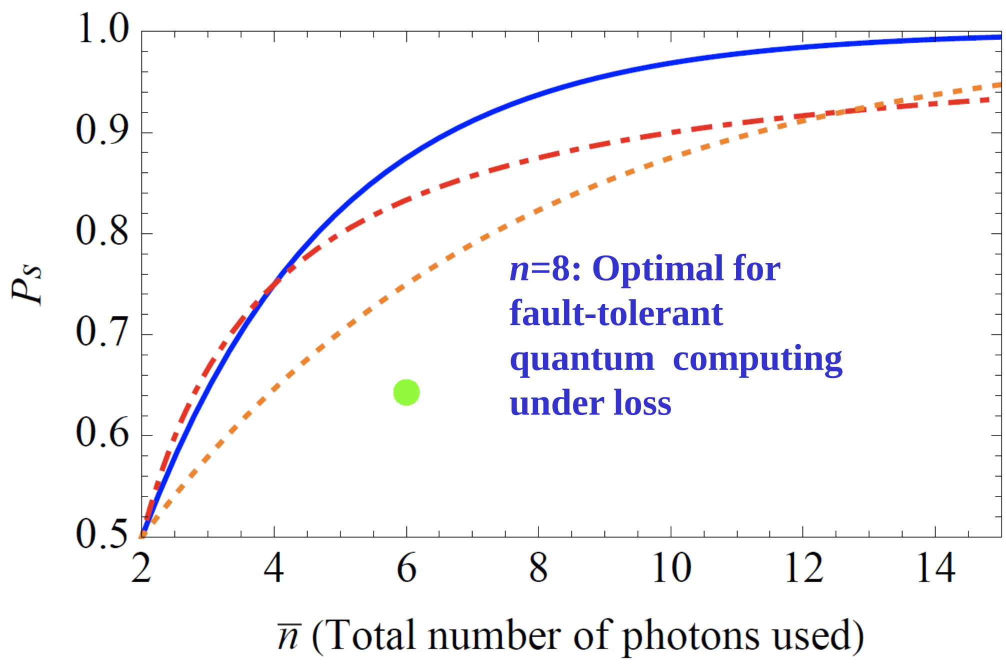
Quantum computing using multi-photon qubits

S.-W. Lee, K. Park, T. C. Ralph, and HJ, Phys. Rev. Lett. 114, 113603 (2015)

- Nearly deterministic teleportation using multi-photon qubits and on-off photodetectors.
- Multi-photon GHZ entanglements are required as resource.

$$P_s = 1 - 2^{-N}$$

- **Blue: Multi-photon qubits**
- Red: Ancillary states (Grice)
- Orange: Ancillary states (Ewerd and van Loock)
- Green: Squeezing-based (Zaidi and van Loock)

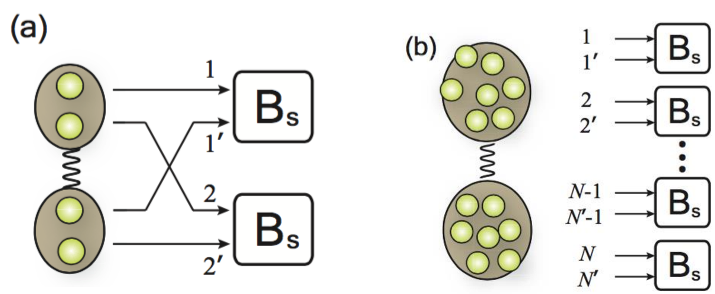


Fast & efficient all-photonic quantum repeater for long-distance quantum communication

based on loss-tolerant Bell-state measurement

S.-W. Lee, T. C. Ralph, and H. Jeong, in preparation

- ✓ **All-photonic** using passive linear optics and photon on-off measurement.
- ✓ **No quantum memory required**
- ✓ **Nearly deterministic** S.-W. Lee, *et al.* Phys. Rev. Lett. 114, 113603 (2015)



success prob. when using M blocks of N photons (total NM photons)

$$P_s = 1 - 2^{-NM}$$

rapidly increase to unit as the photons number increases.

- ✓ **Loss-tolerant**

Remarks

- We have suggested general and sensible measures of "quantum macroscopicity" based on (i) phase space structures and (ii) sensitivity to coarse-grained measurements, respectively. They are useful for quantifying macroscopic coherence for arbitrary continuous-variable states and spin systems.
- Alternative types of qubits such as coherent-state qubits, hybrid qubits and multi-photon qubits may be useful to overcome limitations of single-photon qubits for efficient quantum information processing.

THANK YOU