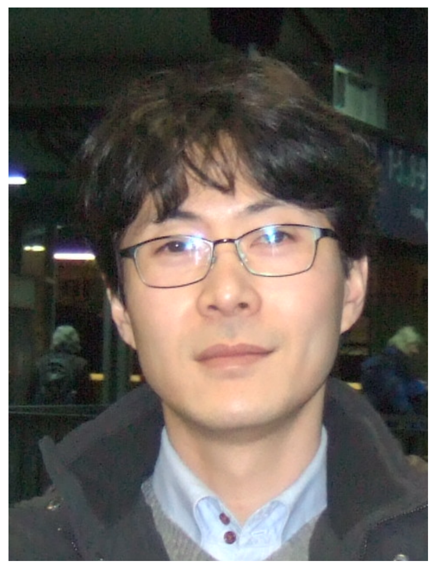
Measurement-driven Quantum Engines



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At a single temperature, the average work done by a system through a cyclic deformation of a parameter of the system **cannot be positive**

(Kelvin-Planck statement)

$$W \leq 0$$

(Jarzynski, Crooks, Tasaki)

A heat engine operated at a single temperature is possible with the help of feedback control (FC)

$$W \le k_B T \ln \gamma$$

for $\gamma > 1$ work done can be positive

(Sagawa & Ueda, Morikuni & Tasaki)

$\gamma = 1$ in the absence of FC

For quantum systems with the help of measurement, work can be positive

$$W > 0$$

at a single temperature in a cycle without FC

For quantum systems with help of measurement, work can be positive

$$W > 0$$

at single temperature without feedback control

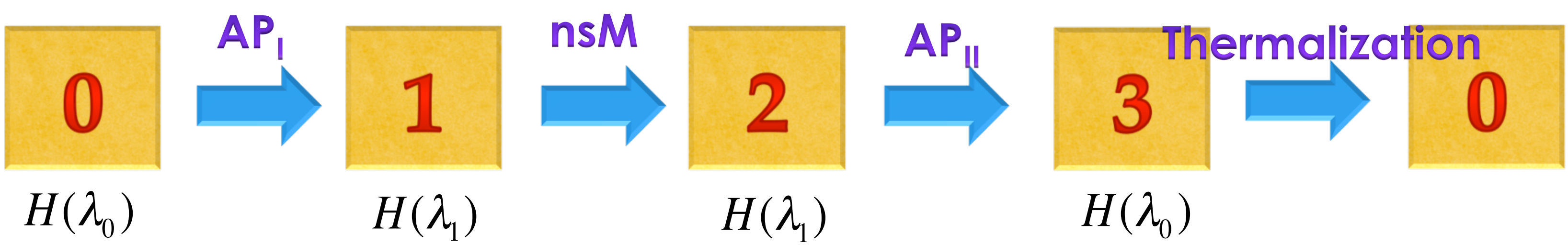
For any quantum mechanical system, work can be extractable:

$W > 0$ at single temperature in a cycle without FC, *if*

(1)the system initially in equilibrium state (**0**)

(2)work strokes (AP_I , AP_{II}) should be adiabatically slow

(3)nonselective and minimally disturbing measurement (**nsM**) is performed between the strokes



(Yi, Talkner & Kim)

Measurement Schemes:

An example of spin-1/2 system under the projective measurement

$|\uparrow\rangle$ spin-up state

$|\downarrow\rangle$ spin-down state

$$\sigma_z|\uparrow\rangle = +1|\uparrow\rangle \quad \sigma_z|\downarrow\rangle = (-1)|\downarrow\rangle$$

$$\langle\downarrow|\uparrow\rangle = \langle\uparrow|\downarrow\rangle = 0 \quad \langle\uparrow|\uparrow\rangle = \langle\downarrow|\downarrow\rangle = 1$$

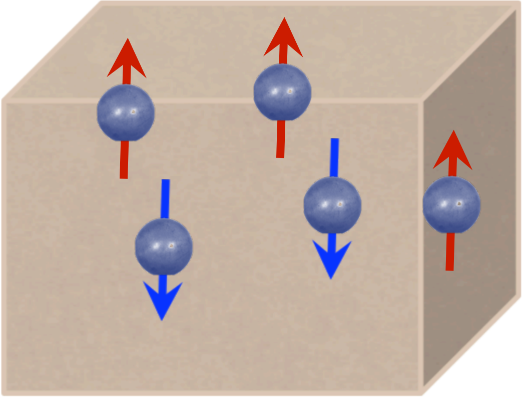
PROJECTION OPERATORS

$$\boxed{\begin{aligned} \Pi_\uparrow &= |\uparrow\rangle\langle\uparrow| \\ \Pi_\downarrow &= |\downarrow\rangle\langle\downarrow| \end{aligned}}$$

$$\begin{aligned} \sigma_z &= +1|\uparrow\rangle\langle\uparrow| + (-1)|\downarrow\rangle\langle\downarrow| \\ &= +1\Pi_\uparrow + (-1)\Pi_\downarrow \end{aligned}$$

Density operator ρ for a mixed state ($\rho^2 \neq \rho$)

$$\rho = p_\uparrow |\uparrow\rangle\langle\uparrow| + p_\downarrow |\downarrow\rangle\langle\downarrow| = \begin{pmatrix} p_\uparrow & 0 \\ 0 & p_\downarrow \end{pmatrix}$$

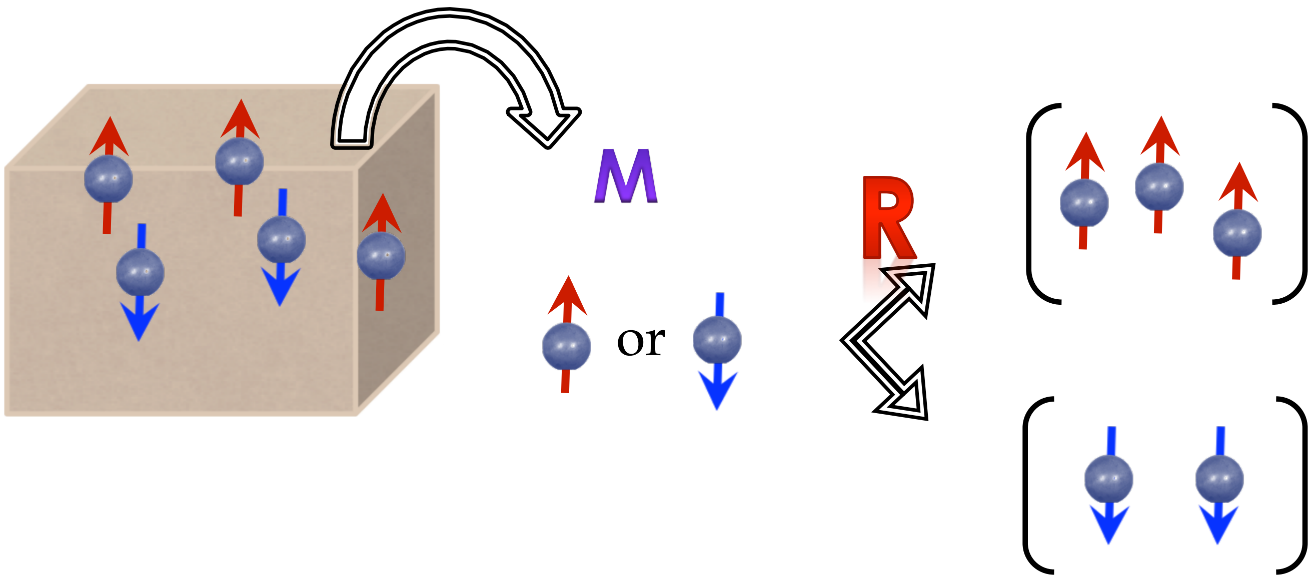


$$p_\uparrow = Tr\Pi_\uparrow \rho \Pi_\uparrow = \frac{3}{5}$$

$$p_\downarrow = Tr\Pi_\downarrow \rho \Pi_\downarrow = \frac{2}{5}$$

$$Tr\rho = p_\uparrow + p_\downarrow = 1$$

Measurement (M) with registration (R): selective measurement



$$\rho_{\uparrow} = \frac{\Pi_{\uparrow}\rho\Pi_{\uparrow}}{Tr\,\Pi_{\uparrow}\rho\Pi_{\uparrow}} = |\uparrow\rangle\langle\uparrow|$$

$$\rho_{\downarrow} = |\downarrow\rangle\langle\downarrow|$$

pure state $(\rho^2_{\uparrow,\downarrow} = \rho_{\uparrow,\downarrow})$

$$Tr\,\rho_{\uparrow,\downarrow} = 1$$

von Neumann entropy(degree of uncertainty)

before the measurement

$$S_{prior} = -Tr\rho \ln \rho = -p_{\uparrow} \ln p_{\uparrow} - p_{\downarrow} \ln p_{\downarrow} \approx 0.67$$

after the measurement with R

$$S_{post} = p_{\uparrow} S_{\uparrow} + p_{\downarrow} S_{\downarrow} = 0$$

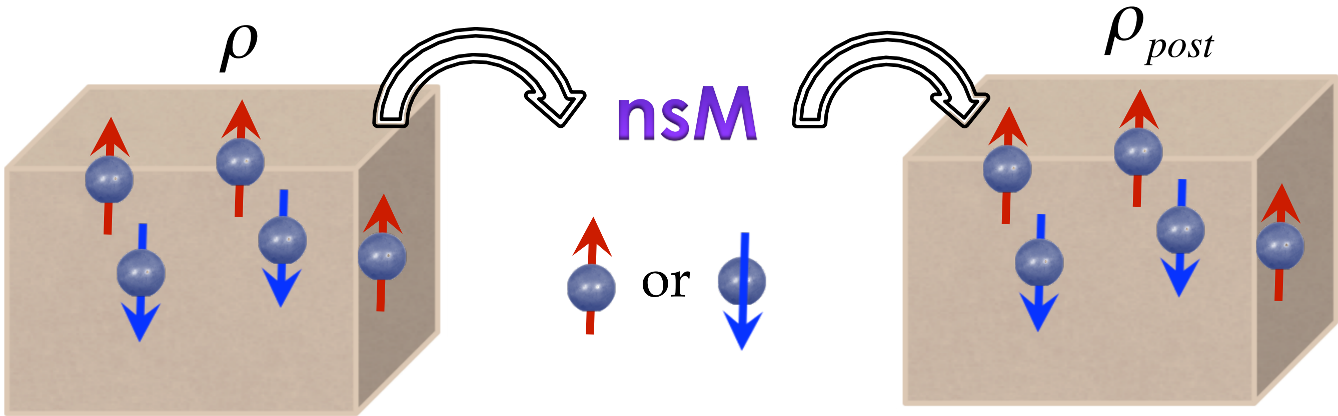
von Neumann entropy
of a pure state =0

$$S_{\uparrow,\downarrow} = -Tr\rho_{\uparrow,\downarrow} \ln \rho_{\uparrow,\downarrow} = 0$$

$$\boxed{S_{prior} - S_{post} = 0.67}$$

Non-selective measurement: (**nsM**)

Measurement **without** registration



$$\rho_{post} = \Pi_{\uparrow}\rho\Pi_{\uparrow} + \Pi_{\downarrow}\rho\Pi_{\downarrow} = \rho$$

$$\left[\Pi_{\uparrow},\rho\right] = \left[\Pi_{\downarrow},\rho\right] = 0$$

$$\Pi_{\uparrow,\downarrow}^{2} = \Pi_{\uparrow,\downarrow} \qquad \text{(idempotent)}$$

$$\Pi_{\uparrow} + \Pi_{\downarrow} = 1 \qquad \text{(complete)}$$

Non-selective Measurement (**nsM**) of an **incompatible** observable

density matrix before **nsM** $\quad \rho = p_{\uparrow}|\uparrow\rangle\langle\uparrow| + p_{\downarrow}|\downarrow\rangle\langle\downarrow|$

incompatible observable (*x*-component of spin)

$$\sigma_x = +1|\uparrow;x\rangle\langle\uparrow;x| + (-1)|\downarrow;x\rangle\langle\downarrow;x|$$
$$= +1\Pi_{\uparrow}^{x} + (-1)\Pi_{\downarrow}^{x}$$

$$\sigma_x|\uparrow;x\rangle = +1|\uparrow;x\rangle$$
$$\sigma_x|\downarrow;x\rangle = -1|\downarrow;x\rangle$$

$$\left[\Pi_{\uparrow}^{x},\rho\right] \neq 0 \quad \left[\Pi_{\downarrow}^{x},\rho\right] \neq 0$$

Non-selective Measurement (**nsM**) of an **incompatible** observable

density matrix before **nsM** $\rho = p_{\uparrow} |\uparrow\rangle\langle\uparrow| + p_{\downarrow} |\downarrow\rangle\langle\downarrow|$

density matrix after **nsM**

$$\rho_{post} = \Pi^x_{\uparrow}\rho\Pi^x_{\uparrow} + \Pi^x_{\downarrow}\rho\Pi^x_{\downarrow} = \frac{1}{2}|\uparrow\rangle\langle\uparrow| + \frac{1}{2}|\downarrow\rangle\langle\downarrow| \neq \rho$$

change in von Neumann entropy

$$S_{prior} - S_{post} = -p_{\uparrow}\ln p_{\uparrow} - p_{\downarrow}\ln p_{\downarrow} - \ln 2 \leq 0$$

energy change caused by nsM of an incompatible observable

Hamiltonian $H = \varepsilon_{\uparrow} |\uparrow\rangle\langle\uparrow| + \varepsilon_{\downarrow} |\downarrow\rangle\langle\downarrow|$ $\varepsilon_{\uparrow} < \varepsilon_{\downarrow}$

average energy before nsM

$$E_{prior} = TrH\rho = \varepsilon_{\uparrow} p_{\uparrow} + \varepsilon_{\downarrow} p_{\downarrow}$$

average energy after nsM

$$E_{post} = TrH\rho_{post} = (\varepsilon_{\uparrow} + \varepsilon_{\downarrow}) / 2$$

$$E_{prior} - E_{post} = \varepsilon_{\uparrow}(p_{\uparrow} - 1/2) + \varepsilon_{\downarrow}(p_{\downarrow} - 1/2)$$

$$= \underbrace{(\varepsilon_{\uparrow} - \varepsilon_{\downarrow})}_{<0}(p_{\uparrow} - p_{\downarrow})/2$$

$$p_{\uparrow} + p_{\downarrow} = 1$$
$$\varepsilon_{\uparrow} < \varepsilon_{\downarrow}$$

If the lower energy state $|\uparrow\rangle$ is more populated than the higher energy state $|\downarrow\rangle$ $(p_{\uparrow} > p_{\downarrow})$

$$E_{prior} < E_{post}$$

For any systems having no population inversion,

a minimally disturbing measurement performed on the system

increases the system energy.

Population inversion:
low lying energy states are more populated than higher energy states

Minimally disturbing measurement: $M_\alpha = M_\alpha^+$

$$\sum_\alpha M_\alpha M_\alpha^+ = 1$$

Is $\Delta E_M = E_{post} - E_{prior}$ work or heat?

ΔE_M does NOT result from change of any parameter of the considered system;
hence, it cannot be associated with work.

ΔE_M is akin to HEAT.

Suspose a system which is described by

Hamiltonian $H(t)$ and a density operator $\rho(t)$ at time t

average energy of the system $E(t) = TrH(t)\rho(t)$

rate of the average energy change

$$\frac{dE}{dt} = \underbrace{Tr\frac{dH}{dt}\rho}_{P_W} + \underbrace{TrH\frac{d\rho}{dt}}_{P_Q}$$

P_W work rate (power)

P_Q heat transfer rate

For an isolated system under unitary evolution (no heat flow)

(informationally)

density operator evolution $$\frac{d\rho}{dt} = \frac{1}{i\hbar}[H,\rho]$$

heat transfer rate $$P_Q = TrH\frac{d\rho}{dt} = \frac{1}{i\hbar}TrH[H,\rho]$$

$$= \frac{1}{i\hbar}\left(Tr\,HH\rho - TrH\rho H\right) = 0$$

work rate $$P_W = Tr\rho\frac{dH}{dt}$$

vanishes if
Hamiltonian is constant in time.

For an open system (coupled to an environment)

density matrix evolution

effect of environment

$$\frac{d\rho}{dt} = \frac{1}{i\hbar}[H,\rho] + \boxed{\sum_k \left(L_k \rho L_k^+ - \frac{1}{2}\left\{ L_k^+ L_k, \rho \right\} \right)}$$

L_k : Lindblad operators

heat transfer rate

$$P_Q = TrH\frac{d\rho}{dt}$$

$$= \frac{1}{2}Tr\rho\left(\sum_k L_k^+[H,L_k] + [L_k^+,H]L_k \right)$$

A system subject to measurements is an open system

density matrix evolution

continuous nonselective

effect of measurement

$$\frac{d\rho}{dt} = \frac{1}{i\hbar}[H,\rho] - \gamma\left(\rho - \sum_k M_k \rho M_k^+\right)$$

Measurement operators for spin-1/2 ($k = 1,2$)

$M_1 = \Pi_\uparrow, M_2 = \Pi_\downarrow$ (spin-z measurement)

$M_1 = \Pi_\uparrow^x, M_2 = \Pi_\downarrow^x$ (spin-x measurement)

heat transfer rate

$$P_Q \propto TrH \sum_k [\rho, M_k] M_k^+$$

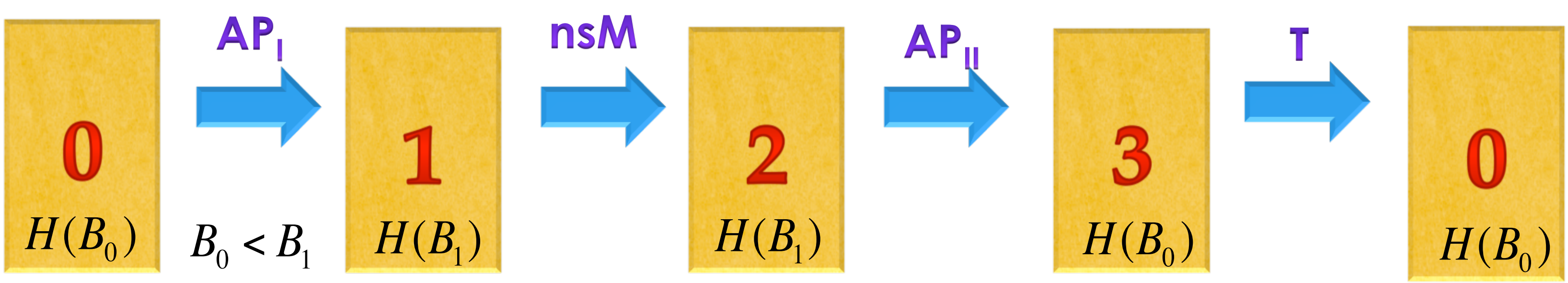
Non-zero even for a constant Hamiltonian, provided $\left[M_k, \rho\right] \neq 0$

Measurement-driven Spin-1/2 Engine

An engine cycle

working substance (WS): spin-1/2 in the presence of magnetic fields aligned along *z*-direction

Hamiltonian $H(B) = -\mu_B B \sigma_z$



The field strength should vary **ADIABATICALLY**

unitary evolution for $\mathrm{AP_I}$

$$U_{0\to1}^{ad} = |\uparrow;B_1\rangle\langle\uparrow;B_0| + |\downarrow;B_1\rangle\langle\downarrow;B_0|$$

$|\sigma=\uparrow,\downarrow;B\rangle$:the eigenstate of the Hamiltonian with field strength B

Density operator at the state 1

$$\rho(1) = U_{0\to1}^{ad}\rho(0)(U_{0\to1}^{ad})^+$$

Quantum adiabatic theorem

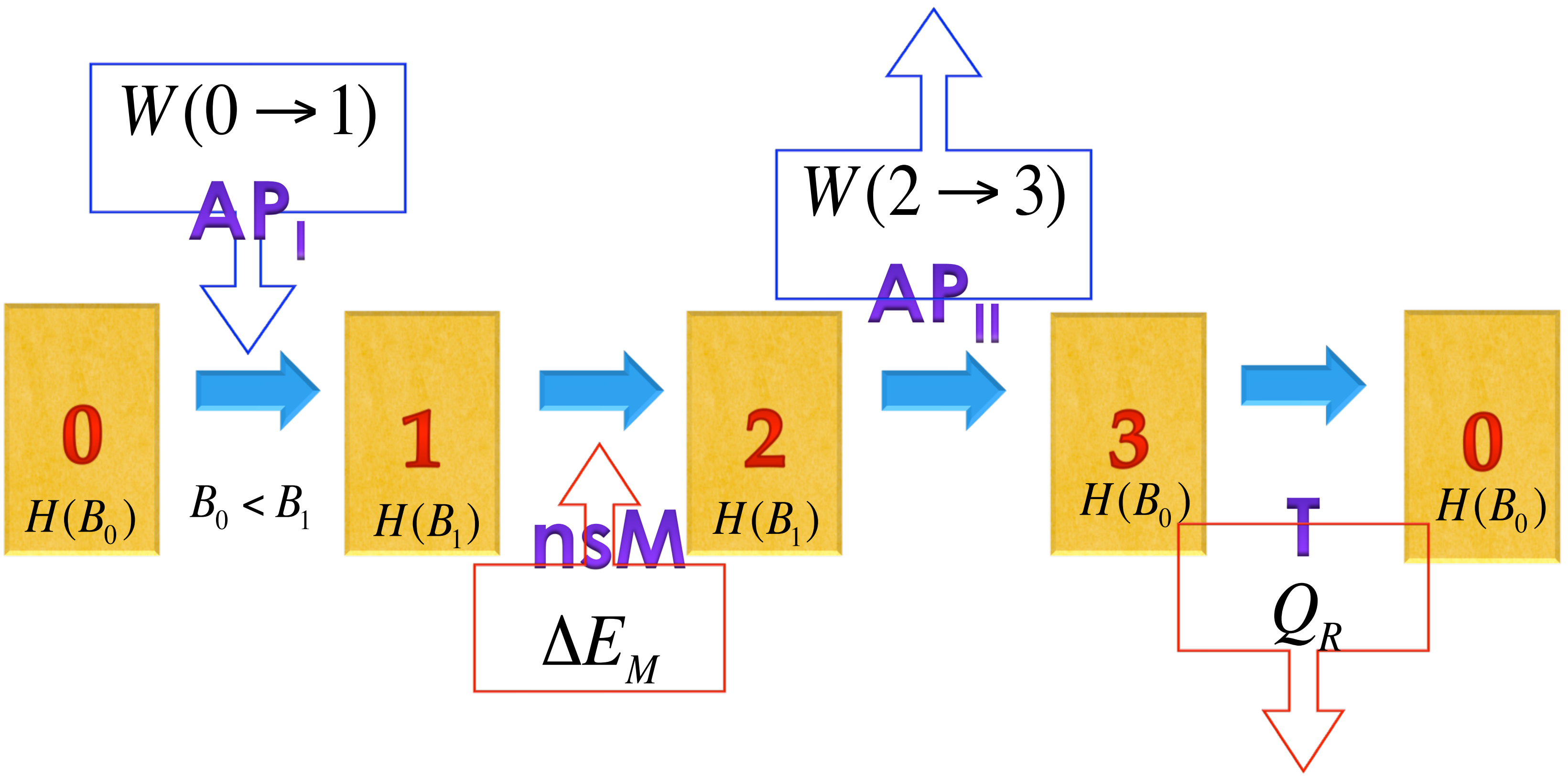
A quantum system remains in its instantaneous eigenstate for a sufficiently slow perturbation

Density operator at the state 2

$$\rho(2) = \Pi_{\uparrow}^{x}\rho(1)\Pi_{\uparrow}^{x} + \Pi_{\downarrow}^{x}\rho(1)\Pi_{\downarrow}^{x}$$

nonselective measurement of spin-x component

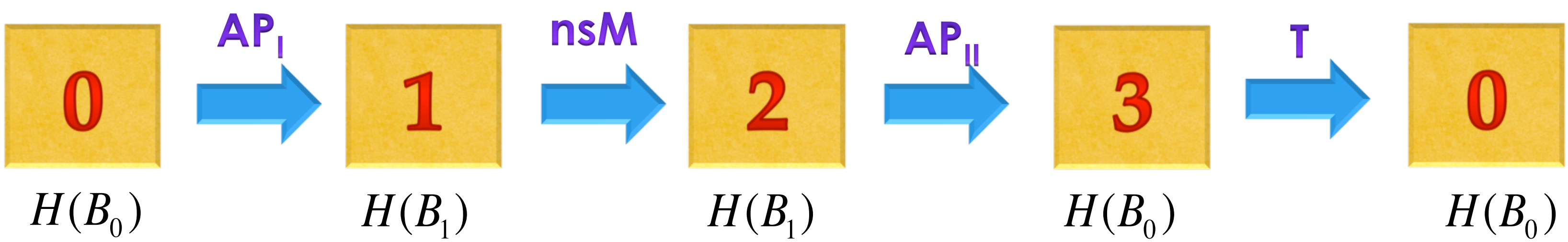
Work and Heat



Work

$$W(0 \to 1) = TrH(B_0)\rho(0) - TrH(B_1)\rho(1)$$

$$W(2 \to 3) = TrH(B_1)\rho(2) - TrH(B_0)\rho(3)$$



Heat input by nsM

$$\Delta E_M = TrH(B_1)\rho(2) - TrH(B_1)\rho(1)$$

Density operators

initial state $\rho(0) = \dfrac{e^{-\beta_0 H(B_0)}}{Tr e^{-\beta_0 H(B_0)}}$

state reached after $\mathrm{AP_I}$ $\rho(1) = U^{ad}_{0\to1}\rho(0)(U^{ad}_{0\to1})^{+}$

post-measurement state $\rho(2) = \Pi^x_{\uparrow}\rho(1)\Pi^x_{\uparrow} + \Pi^x_{\downarrow}\rho(1)\Pi^x_{\downarrow}$

state reached after $\mathrm{AP_{II}}$ $\rho(3) = U^{ad}_{2\to3}\rho(2)(U^{ad}_{2\to3})^{+}$

TOTAL Work done by WS during one engine cycle

$$W_{tot} = W(0 \to 1) + W(2 \to 3) = \mu_B (B_1 - B_0) \tanh(\beta_0 \mu_B B_0) > 0$$

W_{tot} >0 indicates that work can be extractable

Maximal work that can be extracted via unitary transformation, free of *inner friction*

Heat input by nsM

$$\Delta E_M = \mu_B B_1 \tanh(\beta_0 \mu_B B_0) > 0$$

Efficiency

$$\eta = \frac{W_{tot}}{\Delta E_M} = 1 - \frac{B_0}{B_1} = 1 - \frac{T_0}{T_1}$$

$$W_{tot} = \mu_B (B_1 - B_0) \tanh(\beta_0 \mu_B B_0)$$

T_0 initial temperature

$$\frac{B_0}{T_0} = \frac{B_1}{T_1}$$

This **MIGHT LOOK LIKE** the **Carnot efficiency** but…

the engine cycle is **IRREVERSIBLE**

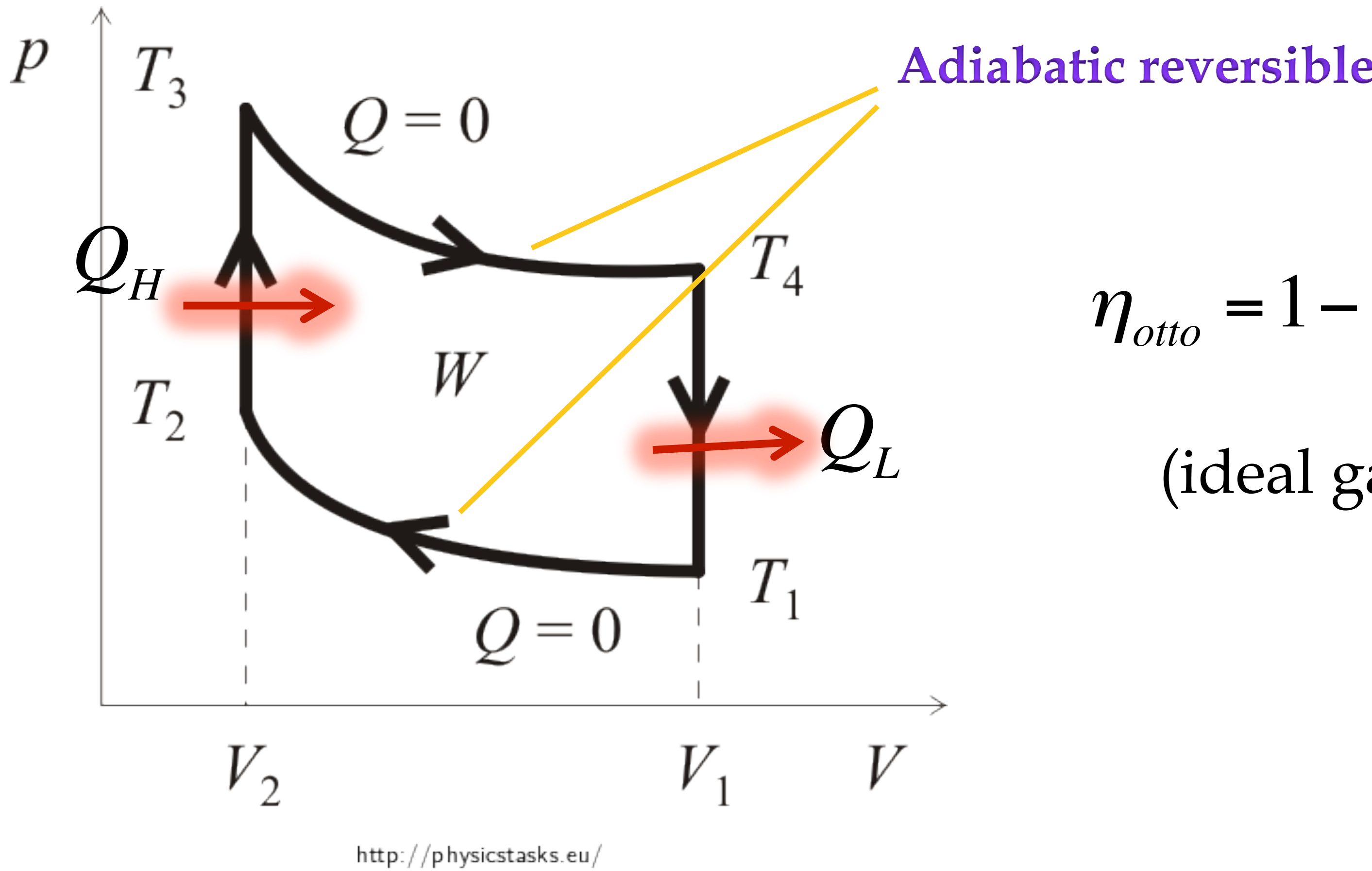
$$\begin{array}{ccccccccc} 0 & \xrightarrow{\mathrm{AP_I}} & 1 & \xrightarrow{\mathrm{nsM}} & 2 & \xrightarrow{\mathrm{AP_{II}}} & 3 & \xrightarrow{\mathrm{T}} & 0 \\ H(B_0) & & H(B_1) & & H(B_1) & & H(B_0) & & H(B_0) \end{array}$$

During **T** process,
the WS is brought into a thermal contact with a reservoir at $T_0 = 1/(k_B\beta_0)$.
After some time elapsed, the WS recovers the initial equilibrium state **0**.

Heat transferred from the WS to the reservoir

$$Q_R = E_3 - E_0 = \mu_B B_0 \tanh(\beta\mu_B B_0) > 0$$

Otto cycle

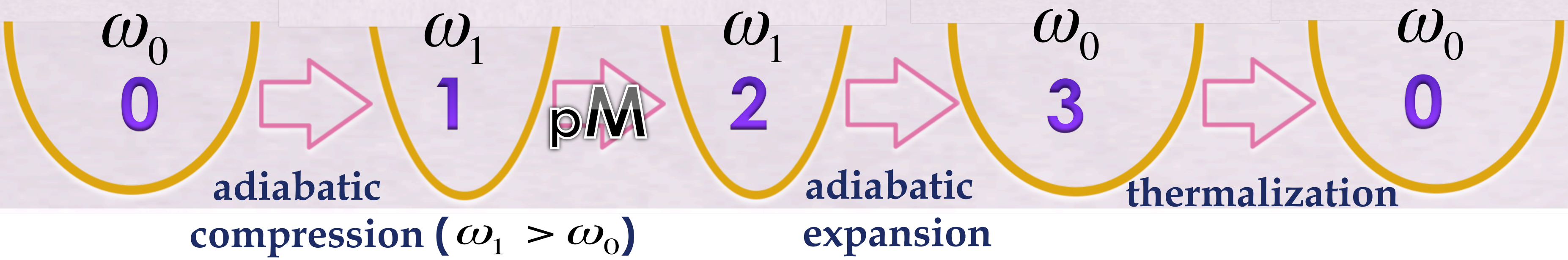


$$\eta_{otto} = 1 - \frac{T_2}{T_1}$$

(ideal gas)

a particle in a harmonic potential

ω_i oscillation frequency



pM : position-measurement with finite fuzziness σ

measurement operator $$M_x = \frac{1}{(2\pi\sigma^2)^{1/4}} \exp\left[-\frac{(\hat{x}-x)^2}{4\sigma^2}\right]$$

$\sigma \to 0, M_x$ in effects becomes projective measurement

$\hat{x}$ position operator

extracted work

$$W = (1-\gamma)\frac{\hbar^2}{8m\sigma^2}$$

$$\gamma = \left(\frac{\hbar\omega_0}{\hbar\omega_1}\right)$$

energy level spacing
before the compression

energy level spacing
after the compression

heat input by the measurement

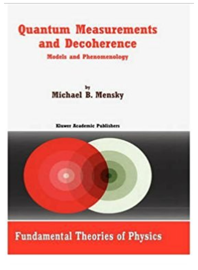
$$\Delta E_M = \frac{\hbar^2}{8m\sigma^2}$$

measurement-induced uncertainty

$$\left\langle \delta p^2 \right\rangle_{post} = \frac{\hbar^2}{4\sigma^2} + \left\langle \delta p^2 \right\rangle_{prior}$$

vanishes in the classical realm ($\hbar \to 0$)

diverges for precise measurement ($\sigma \to 0$)



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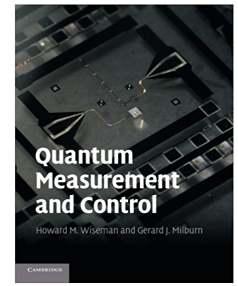

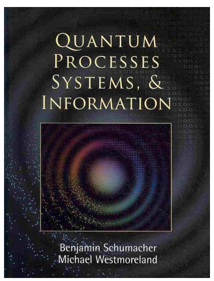

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