From unitary dynamics to statistical mechanics in isolated quantum systems

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L. D'Alessio, Y. Kafri, A. Polkovnikov, and MR, *From Quantum Chaos and Eigenstate Thermalization to Statistical Mechanics and Thermodynamics*, Adv. Phys. **65**, 239-362 (2016).

Outline

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1. Motivation
   - Foundations of quantum statistical mechanics
   - Experiments with ultracold gases
2. Quantum chaos and random matrix theory
   - Classical and quantum chaos
   - Random matrix theory
3. Dynamics and thermalization (quantum chaotic systems)
   - Quantum mechanics vs statistical mechanics
   - Dynamics and equilibration
   - Thermalization
   - Eigenstate thermalization
4. Dynamics and generalized thermalization (integrable systems)
   - Generalized Gibbs ensemble (GGE)
5. Summary

Foundations of quantum statistical mechanics

Quantum ergodicity: John von Neumann ‘29 (Proof of the ergodic theorem and the H-theorem in quantum mechanics)

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Recent works (keywords)

Tasaki '98
(From Quantum Dynamics to the Canonical Distribution. . .)

Goldstein, Lebowitz, Tumulka, and Zanghi '06
(Canonical Typicality)

Popescu, Short, and A. Winter '06
(Entanglement and the foundation of statistical mechanics)

Goldstein, Lebowitz, Mastrodonato, Tumulka, and Zanghi '10
(Normal typicality and von Neumann's quantum ergodic theorem)

MR and Srednicki '12
(Alternatives to Eigenstate Thermalization)

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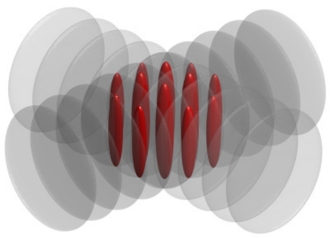
Experiments with ultracold bosons in one dimension



Effective 1D δ potential

M. Olshanii, PRL **81**, 938 (1998).

$$U_{1D}(x) = g_{1D}\delta(x)$$

Lieb-Liniger parameter

$$\gamma = \frac{m g_{1D}}{\hbar^2 \rho}$$

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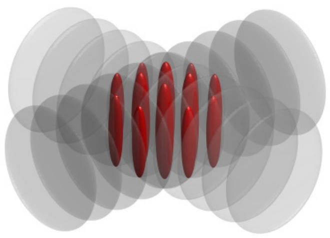

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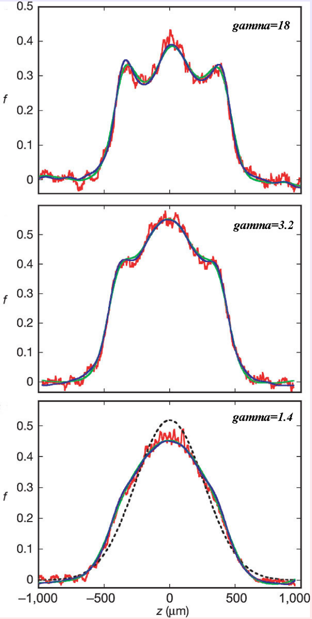
Observables (density and momentum distribution functions) equilibrated to nonthermal distributions

T. Kinoshita, T. Wenger, and D. S. Weiss, Nature **440**, 900 (2006).



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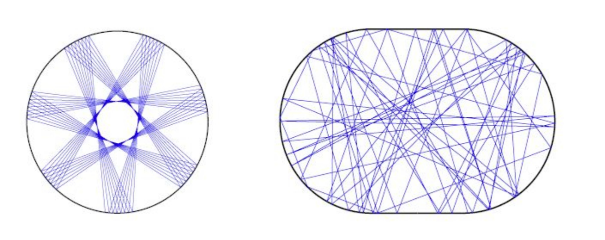
Classical chaos and integrability

Particle trajectories in a circular cavity and a Bunimovich stadium (scholarpedia)



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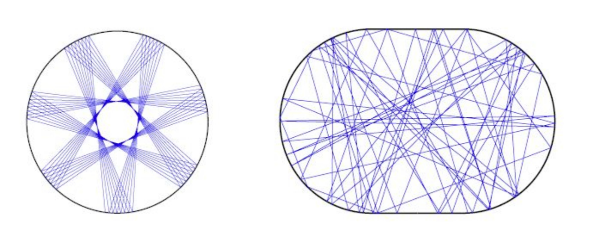

- A Hamiltonian $H(\mathbf{p},\mathbf{q})$, with $\mathbf{q}=(q_1,\cdots,q_N)$ and $\mathbf{p}=(p_1,\cdots,p_N)$, is said to be integrable if there are N functionally independent constants of the motion $\mathbf{I}=(I_1,\cdots,I_N)$ in involution:

$$\{I_\alpha,H\}=0,\quad \{I_\alpha,I_\beta\}=0,\quad \text{where}\quad \{f,g\}=\sum_{i=1,N}\frac{\partial f}{\partial q_i}\frac{\partial g}{\partial p_i}-\frac{\partial f}{\partial p_i}\frac{\partial g}{\partial q_i}.$$

Liouville's integrability theorem: $(\mathbf{p},\mathbf{q})\to(\mathbf{I},\mathbf{\Theta})$, so that $H(\mathbf{p},\mathbf{q})\to H(\mathbf{I})$.

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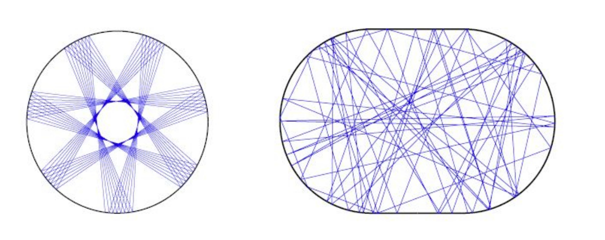

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- Chaos: exponential sensitivity of the trajectories to perturbations

Semi-classical limit: Statistics of energy levels

- Berry-Tabor conjecture (1977): The statistics of level spacings of quantum systems whose classical counterpart is integrable is described by a Poisson distribution. (Energy eigenvalues behave like a sequence of independent random variables.)

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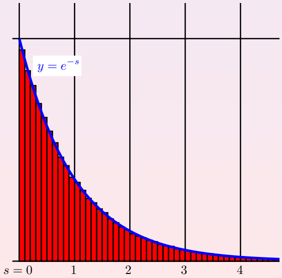
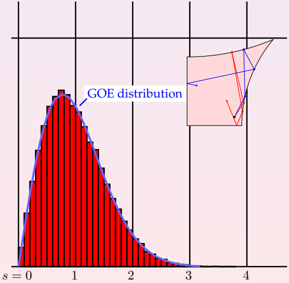
Distribution of level spacings: rectangular and chaotic cavities





Z. Rudnik, Notices AMS **55**, 32 (2008).

Integrability to quantum chaos transition

Spinless fermions (hard-core bosons) in one dimension

$$\hat{H} = \sum_{i=1}^{L} \left\{ -t \left(\hat{f}_i^\dagger \hat{f}_{i+1} + \text{H.c.} \right) + V \hat{n}_i \hat{n}_{i+1} - t' \left(\hat{f}_i^\dagger \hat{f}_{i+2} + \text{H.c.} \right) + V' \hat{n}_i \hat{n}_{i+2} \right\}$$

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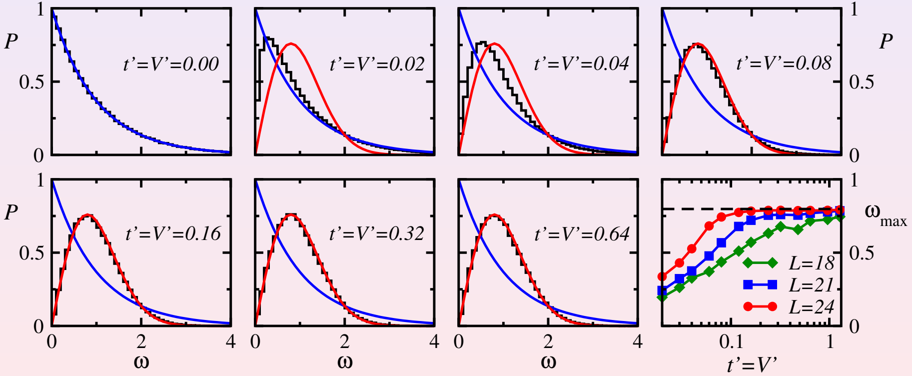
Level spacing distribution $(N_f = L/3)$



L. Santos and MR, PRE **81**, 036206 (2010); PRE **82**, 031130 (2010).

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Random matrix theory

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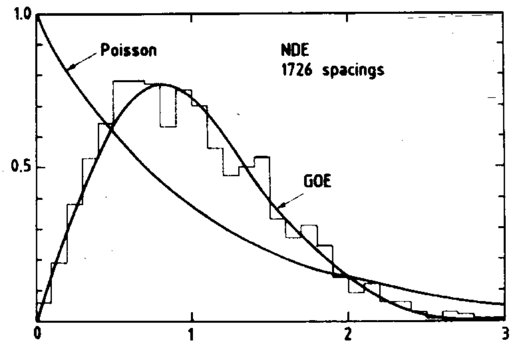
Distribution of level spacings for the "Nuclear Data Ensemble"



T. Guhr *et al.*, Physics Reports **299**, 189 (1998).

Distribution of level spacings $P(\omega)$

$P(\omega)$ can be understood using 2×2 matrices

$$\begin{bmatrix} \varepsilon_1 & \frac{V}{\sqrt{2}} \\ \frac{V^*}{\sqrt{2}} & \varepsilon_2 \end{bmatrix}, \qquad E_{1,2} = \frac{\varepsilon_1 + \varepsilon_2}{2} \pm \frac{1}{2}\sqrt{(\varepsilon_1 - \varepsilon_2)^2 + 2|V|^2}.$$

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For systems that are invariant under time reversal, $\hat{H}$ can be written as a real matrix. Draw ε_1, ε_2, and V from a Gaussian distribution with zero mean and variance σ.

$$P(\omega \equiv E_1 - E_2) = \frac{1}{(2\pi)^{3/2}\sigma^3} \int d\varepsilon_1 \int d\varepsilon_2 \int dV\, \delta\left(\sqrt{(\varepsilon_1 - \varepsilon_2)^2 + 2V^2} - \omega\right) \times \exp\left(-\frac{\varepsilon_1^2 + \varepsilon_2^2 + V^2}{2\sigma^2}\right).$$

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$$P(\omega) = \frac{\omega}{2\sigma^2} \exp\left[-\frac{\omega^2}{4\sigma^2}\right]$$

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Wigner Surmise (Wigner-Dyson distribution)

$P(\omega) = A_\beta\, \omega^\beta \exp[-B_\beta \omega^2]$, where $\beta = 1$ (GOE) and $\beta = 2$ (GUE)

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Exact results from quantum mechanics

If the initial state is not an eigenstate of $\widehat{H}$

$$|\psi_{\text{ini}}\rangle \neq |\alpha\rangle \quad \text{where} \quad \widehat{H}|\alpha\rangle = E_\alpha|\alpha\rangle \quad \text{and} \quad E = \langle\psi_{\text{ini}}|\widehat{H}|\psi_{\text{ini}}\rangle,$$

then observables $\hat{O}$ evolve in time:

$$O(\tau) \equiv \langle\psi(\tau)|\widehat{O}|\psi(\tau)\rangle \quad \text{where} \quad |\psi(\tau)\rangle = e^{-i\widehat{H}\tau}|\psi_{\text{ini}}\rangle.$$

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One can rewrite

$$O(\tau) = \sum_{\alpha,\beta} C^\star_\alpha C_\beta e^{i(E_\alpha - E_\beta)\tau} O_{\alpha\beta} \quad \text{using} \quad |\psi_{\text{ini}}\rangle = \sum_\alpha C_\alpha|\alpha\rangle.$$

Taking the infinite time average (diagonal ensemble $\hat{\rho}_{\text{DE}} \equiv \sum_\alpha |C_\alpha|^2|\alpha\rangle\langle\alpha|$)

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which depends on the initial conditions through $C_\alpha = \langle\alpha|\psi_{\text{ini}}\rangle$.

Energy fluctuations after a sudden quench

Initial state $|\psi_{\text{ini}}\rangle = \sum_\alpha C_\alpha |\alpha\rangle$ is an eigenstate of $\widehat{H}_{\text{ini}}$. At $t = 0$

$$\widehat{H}_{\text{ini}} \rightarrow \widehat{H} = \widehat{H}_{\text{ini}} + \widehat{W} \qquad \text{with} \quad \widehat{W} = \sum_j \hat{w}(j) \quad \text{and} \quad \widehat{H}|\alpha\rangle = E_\alpha |\alpha\rangle.$$

MR, V. Dunjko, and M. Olshanii, Nature **452**, 854 (2008).

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The energy fluctuations after a quench, ΔE, are:

$$\Delta E = \sqrt{\sum_\alpha E_\alpha^2 |C_\alpha|^2 - (\sum_\alpha E_\alpha |C_\alpha|^2)^2} = \sqrt{\langle \psi_{\text{ini}}|\widehat{W}^2|\psi_{\text{ini}}\rangle - \langle \psi_{\text{ini}}|\widehat{W}|\psi_{\text{ini}}\rangle^2},$$

or

$$\Delta E = \sqrt{\sum_{j_1, j_2 \in \sigma} [\langle \psi_{\text{ini}}|\hat{w}(j_1)\hat{w}(j_2)|\psi_{\text{ini}}\rangle - \langle \psi_{\text{ini}}|\hat{w}(j_1)|\psi_{\text{ini}}\rangle \langle \psi_{\text{ini}}|\hat{w}(j_2)|\psi_{\text{ini}}\rangle]} \overset{N\to\infty}{\propto} \sqrt{N},$$

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where N is the total number of lattice sites.
Since $E \propto N$, then the ratio

$$\frac{\Delta E}{E} \overset{N\to\infty}{\propto} \frac{1}{\sqrt{N}},$$

so, as in traditional ensembles, it vanishes in the thermodynamic limit.

MR, V. Dunjko, and M. Olshanii, Nature **452**, 854 (2008).

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Numerical experiments in one dimension

Hard-core boson Hamiltonian

$$\hat{H} = \sum_{i=1}^{L} \left\{ -t \left(\hat{b}_i^\dagger \hat{b}_{i+1} + \text{H.c.} \right) + V \hat{n}_i \hat{n}_{i+1} - t' \left(\hat{b}_i^\dagger \hat{b}_{i+2} + \text{H.c.} \right) + V' \hat{n}_i \hat{n}_{i+2} \right\}$$

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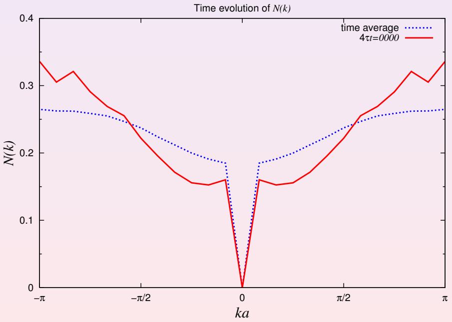
Nonequilibrium dynamics in 1D (density-density structure factor)



MR, PRL **103**, 100403 (2009).

$N_b = 8$ hard-core bosons

$N = 24$ lattice sites

Fix $t' = V'$ and "quench"
$t_{\text{ini}} = 0.5, V_{\text{ini}} = 2$
$\rightarrow t_{\text{fin}} = 1, V_{\text{fin}} = 1$

Integrated results for $L = 24$, $N_b = 8$

Relative difference

$$\delta N(\tau) = \frac{\sum_k |N(k,\tau) - N_{\rm DE}(k)|}{\sum_k N_{\rm DE}(k)}$$

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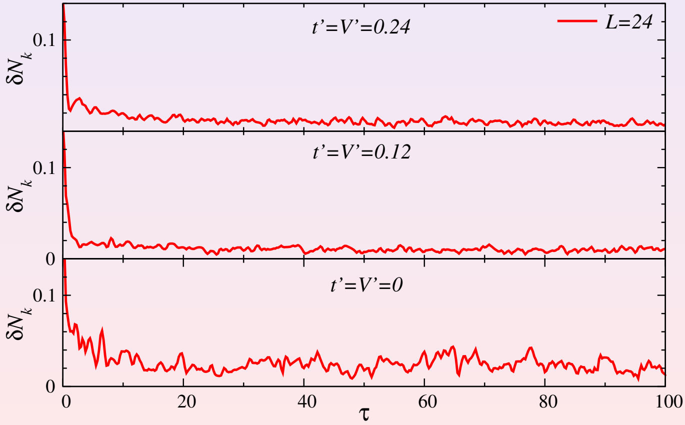

Scaling of the integrated results with system size

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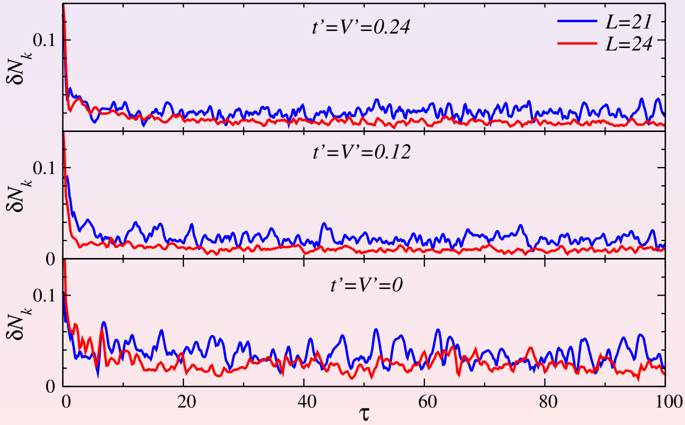

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Statistical description after relaxation (nonintegrable)

Canonical calculation

$$O_{\mathsf{CE}} = \mathrm{Tr}\left\{\hat{O}\hat{\rho}_{\mathsf{CE}}\right\}$$

$$\hat{\rho}_{\mathsf{CE}} = Z_{\mathsf{CE}}^{-1}\exp\left(-\hat{H}/k_B T\right)$$

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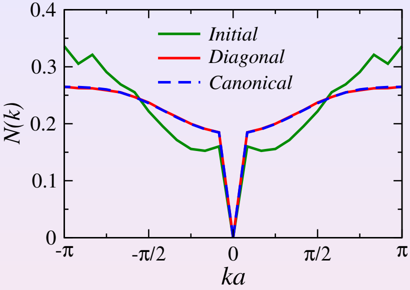
Structure factor



Statistical description after relaxation (nonintegrable)

Canonical calculation

$$O_{\mathsf{CE}} = \mathrm{Tr}\left\{\hat{O}\hat{\rho}_{\mathsf{CE}}\right\}$$

$$\hat{\rho}_{\mathsf{CE}} = Z_{\mathsf{CE}}^{-1} \exp\left(-\hat{H}/k_B T\right)$$

$$Z_{\mathsf{CE}} = \mathrm{Tr}\left\{\exp\left(-\hat{H}/k_B T\right)\right\}$$

$$E = \mathrm{Tr}\left\{\hat{H}\hat{\rho}_{\mathsf{CE}}\right\} \qquad T = 3.0$$

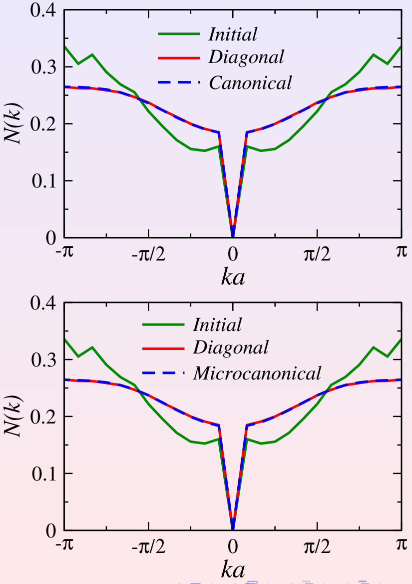
Microcanonical calculation

$$O_{\mathsf{ME}} = \frac{1}{N_{\text{states}}} \sum_{\alpha} \langle \Psi_\alpha | \hat{O} | \Psi_\alpha \rangle$$

with $E - \Delta E < E_\alpha < E + \Delta E$

N_{states} : # of states in the window

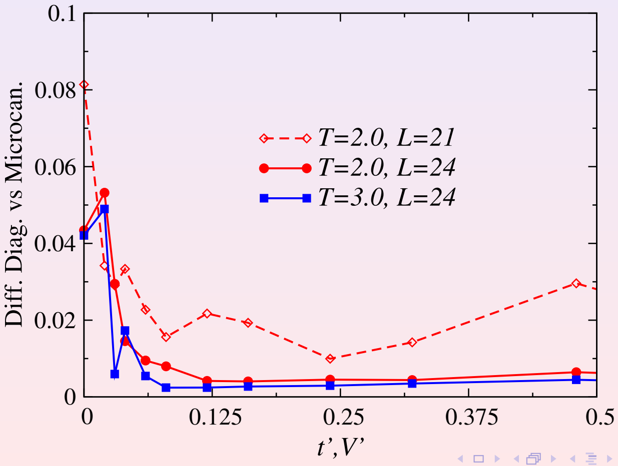
Structure factor



Thermalization and the lack thereof at integrability

Relative difference

$$\frac{\sum_k |N_{\mathsf{DE}}(k) - N_{\mathsf{ME}}(k)|}{\sum_k N_{\mathsf{DE}}(k)}$$



Outline

Eigenstate thermalization

Paradox?

$$\sum_{\alpha} |C_\alpha|^2 O_{\alpha\alpha} = \frac{1}{N_{E,\Delta E}} \sum_{|E-E_\alpha|<\Delta E} O_{\alpha\alpha}$$

Left hand side: Depends on the initial conditions through $C_\alpha = \langle \alpha | \psi_{\text{ini}} \rangle$
Right hand side: Depends only on the energy

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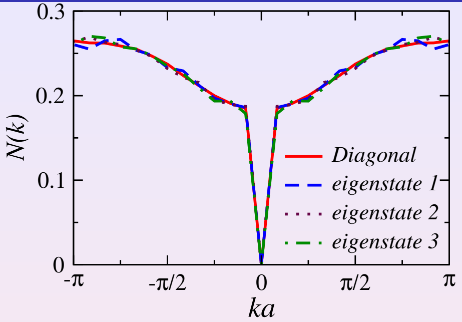
Eigenstate thermalization hypothesis (ETH): diagonal part
[Deutsch, PRA **43** 2046 (1991); Srednicki, PRE **50**, 888 (1994);
MR, Dunjko, and Olshanii, Nature **452**, 854 (2008).]

> The expectation value $\langle \alpha | \widehat{O} | \alpha \rangle$ of a few-body observable $\widehat{O}$ in an eigenstate of the Hamiltonian $|\alpha\rangle$, with energy E_α, of a large interacting many-body system equals the thermal average of $\widehat{O}$ at the mean energy E_α:
>
> $$\langle \alpha | \widehat{O} | \alpha \rangle = O_{\text{ME}}(E_\alpha)$$

ETH – away from integrability ($t' = V' = 0.24$)

Structure factor

Eigenstates with energies closest to E



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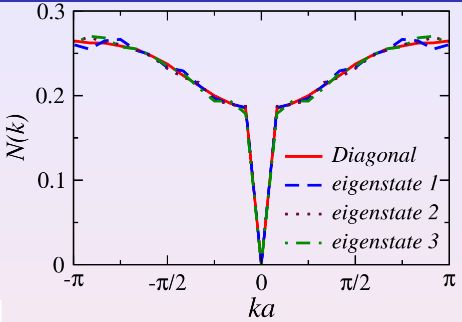

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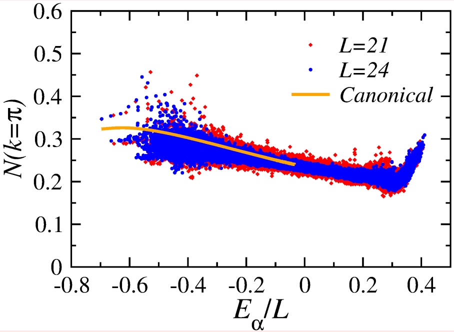


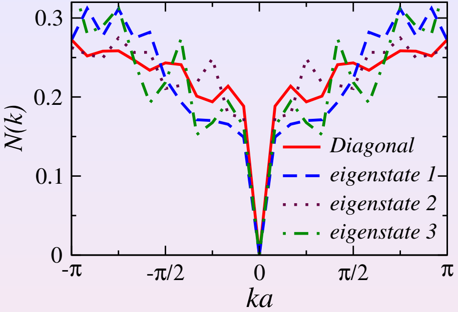
$N(k = \pi)$ vs eigenstate energy

There is no eigenstate thermalization at the edges of the spectrum (there is no quantum chaos either)

Breakdown of ETH at integrability ($t' = V' = 0$)

Structure factor

Eigenstates with energies closest to E



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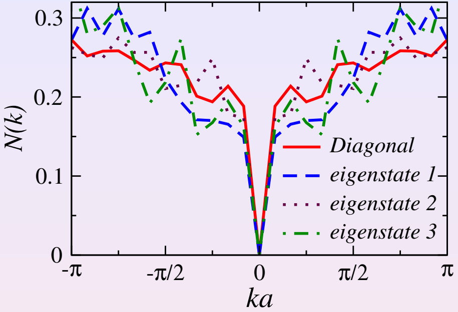
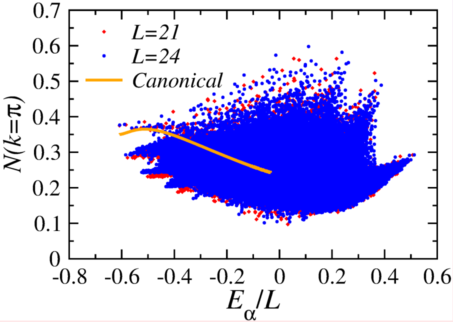


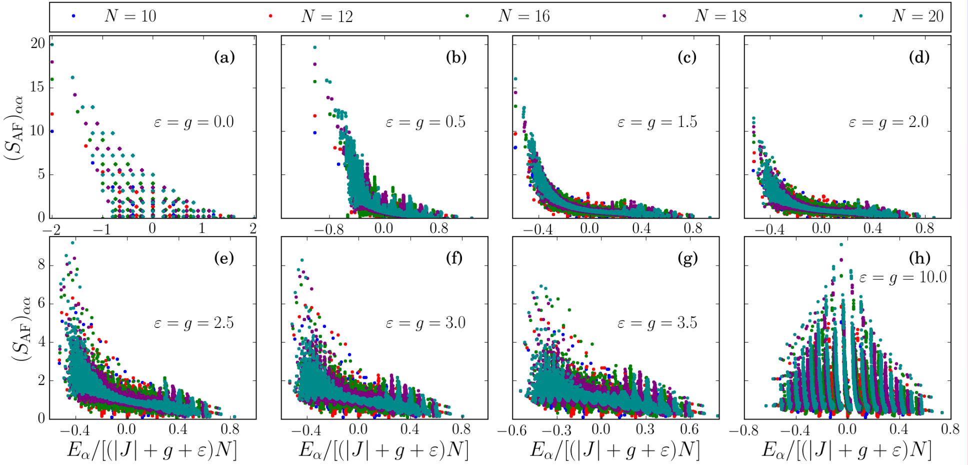


$N(k = \pi)$ vs eigenstate energy

In finite systems, eigenstate thermalization breaks down close to integrable points (there is no quantum chaos either). Quantum KAM?

Diagonal part of ETH (2D AF-TFIM)

Hamiltonian: $\hat{H} = J \sum_{\langle \mathbf{i},\mathbf{j} \rangle} \hat{\sigma}^z_{\mathbf{i}} \hat{\sigma}^z_{\mathbf{j}} + g \sum_{\mathbf{i}} \hat{\sigma}^x_{\mathbf{i}} + \varepsilon \sum_{\mathbf{i}} \hat{\sigma}^z_{\mathbf{i}},$

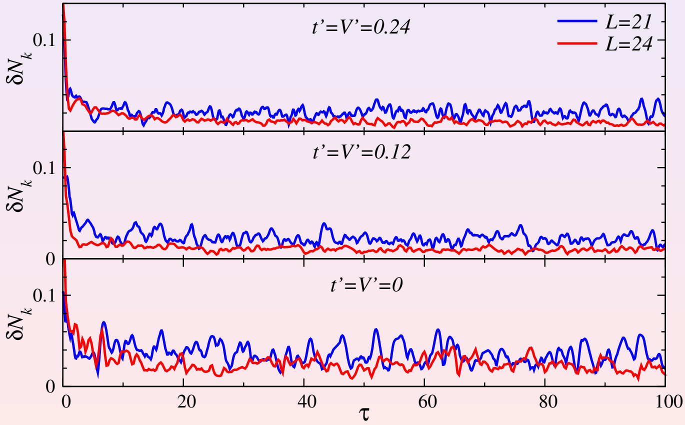


R. Mondaini, K. R. Fratus, M. Srednicki, and MR, PRE **93**, 032104 (2016).

Smallness of the time fluctuations

Relative difference

$$\delta N(\tau) = \frac{\sum_k |N(k,\tau) - N_{\mathrm{DE}}(k)|}{\sum_k N_{\mathrm{DE}}(k)}$$



Time fluctuations

Are they small because of dephasing?

$$O(t)-\overline{O(t)} = \sum_{\substack{\alpha,\beta\\ \alpha\neq\beta}} C_\alpha^\star C_\beta e^{i(E_\alpha-E_\beta)t} O_{\alpha\beta} \sim \sum_{\substack{\alpha,\beta\\ \alpha\neq\beta}} \frac{e^{i(E_\alpha-E_\beta)t}}{N_{\text{states}}} O_{\alpha\beta}$$

$$\sim \frac{\sqrt{N_{\text{states}}^2}}{N_{\text{states}}} O_{\alpha\beta}^{\text{typical}} \sim O_{\alpha\beta}^{\text{typical}}$$

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$$\sim \frac{\sqrt{N^2_{\text{states}}}}{N_{\text{states}}} O^{\text{typical}}_{\alpha\beta} \sim O^{\text{typical}}_{\alpha\beta}$$

Time average of $O(t)$

$$\overline{O(t)} = \sum_\alpha |C_\alpha|^2 O_{\alpha\alpha}$$

$$\sim \sum_\alpha \frac{1}{N_{\text{states}}} O_{\alpha\alpha} \sim O^{\text{typical}}_{\alpha\alpha}$$

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Dephasing is not enough.
One needs $O^{\text{typical}}_{\alpha\beta} \ll O^{\text{typical}}_{\alpha\alpha}$

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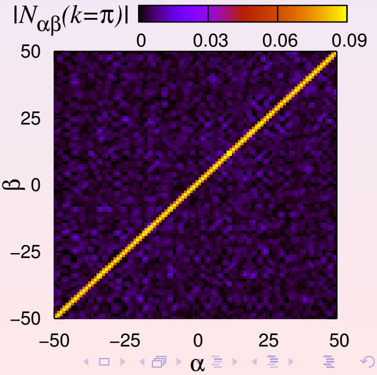

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MR, PRA **80**, 053607 (2009).

Eigenstate thermalization hypothesis

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M. Srednicki, J. Phys. A **32**, 1163 (1999).

$$O_{\alpha\beta} = O(E)\delta_{\alpha\beta} + e^{-S(E)/2} f_O(E,\omega) R_{\alpha\beta}$$

where $E \equiv (E_\alpha + E_\beta)/2$, $\omega \equiv E_\alpha - E_\beta$, $S(E)$ is the thermodynamic entropy at energy E, and $R_{\alpha\beta}$ is a random number with zero mean and unit variance.

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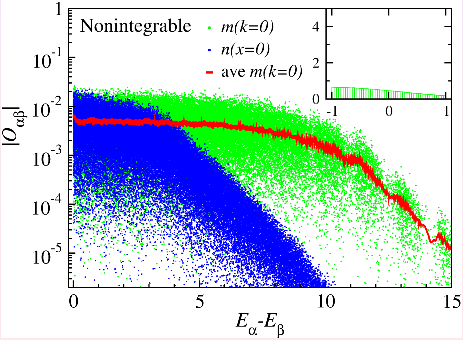
Off-diagonal matrix elements [histogram of $(|O_{\alpha\beta}| - |O_{\alpha\beta}|_{\text{ave}})/|O_{\alpha\beta}|_{\text{ave}}$]



E. Khatami, G. Pupillo, M. Srednicki, and MR, PRL **111**, 050403 (2013).

Matrix elements of Hermitian operators within RMT

Let $\hat{O} = \sum_i O_i |i\rangle\langle i|$, where $\hat{O}|i\rangle = O_i|i\rangle$,

$$O_{\alpha\beta} \equiv \langle\alpha|\hat{O}|\beta\rangle = \sum_i O_i \langle\alpha|i\rangle\langle i|\beta\rangle = \sum_i O_i (\psi_i^\alpha)^* \psi_i^\beta$$

$|\alpha\rangle$ and $|\beta\rangle$ are eigenstates of a random matrix. Averaging over $|\alpha\rangle$ and $|\beta\rangle$ (random orthogonal unit vectors in arbitrary bases): $\overline{(\psi_i^\alpha)^*(\psi_i^\beta)} = \frac{1}{\mathcal{D}}\delta_{\alpha\beta}$.

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This means that (to leading order):

$$\overline{O_{\alpha\alpha}} = \frac{1}{\mathcal{D}} \sum_i O_i \equiv \bar{O}, \quad \text{while} \quad \overline{O_{\alpha\beta}} = 0 \quad \text{for} \quad \alpha \neq \beta.$$

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One can further show that ($\eta = 2$ for GOE and $\eta = 1$ for GUE):

$$\overline{O_{\alpha\alpha}^2} - \overline{O_{\alpha\alpha}}^2 = \eta\overline{|O_{\alpha\beta}|^2} = \frac{\eta}{\mathcal{D}^2} \sum_i O_i^2 \equiv \frac{\eta}{\mathcal{D}}\overline{O^2}.$$

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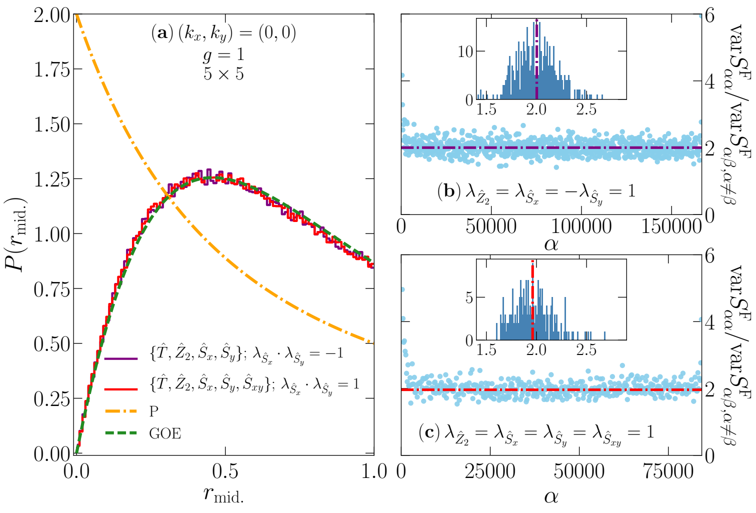
Combining these results one can write

$$O_{\alpha\beta} \approx \bar{O}\delta_{\alpha\beta} + \sqrt{\frac{\overline{O^2}}{\mathcal{D}}}R_{\alpha\beta},$$

where $R_{\alpha\beta}$ is a random variable (real for GOE and complex for GUE).

Ratio of variances in the 2D F-TFIM

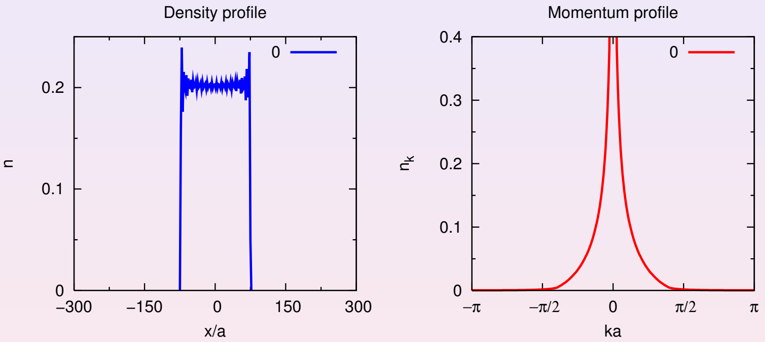
Hamiltonian: $\hat{H} = -J \sum_{\langle \mathbf{i},\mathbf{j} \rangle} \hat{\sigma}^z_{\mathbf{i}} \hat{\sigma}^z_{\mathbf{j}} + g \sum_{\mathbf{i}} \hat{\sigma}^x_{\mathbf{i}}.$



R. Mondaini and MR, PRE **96**, 012157 (2017).

Outline

Relaxation dynamics in an integrable HCB system



MR, V. Dunjko, V. Yurovsky, and M. Olshanii, PRL **98**, 050405 (2007).

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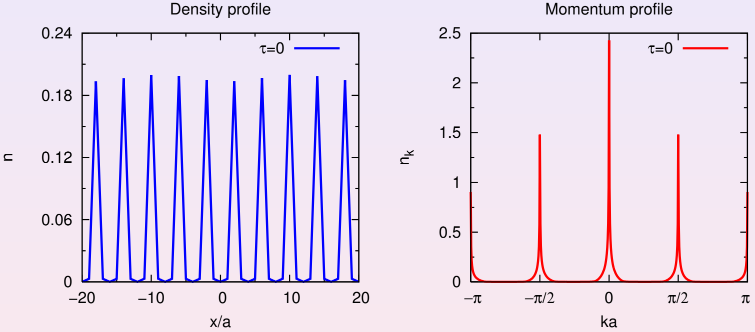

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Generalized Gibbs ensemble (GGE)

Thermal equilibrium

$$\hat{\rho} = Z^{-1} \exp\left[-\left(\hat{H} - \mu\hat{N}\right)/k_B T\right]$$

$$Z = \mathrm{Tr}\left\{\exp\left[-\left(\hat{H} - \mu\hat{N}\right)/k_B T\right]\right\}$$

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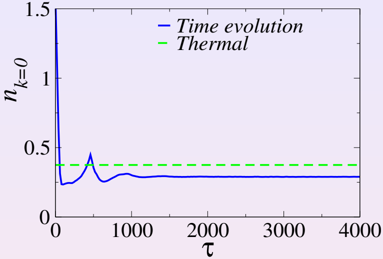
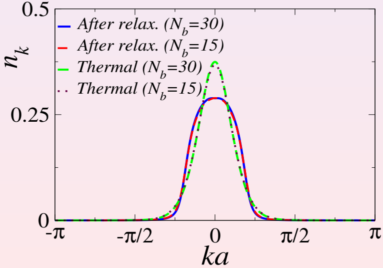
Evolution of $n_{k=0}$



n_k after relaxation



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MR, PRA **72**, 063607 (2005).

Integrals of motion

(underlying noninteracting fermions)

$$\hat{H}_F \hat{\gamma}^{f\dagger}_m |0\rangle = E_m \hat{\gamma}^{f\dagger}_m |0\rangle$$

$$\left\{\hat{I}_m\right\} = \left\{\hat{\gamma}^{f\dagger}_m \hat{\gamma}^f_m\right\}$$

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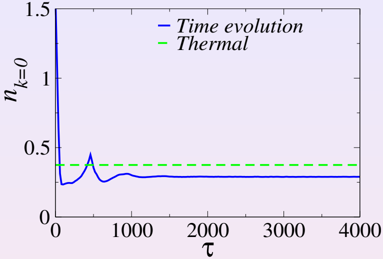

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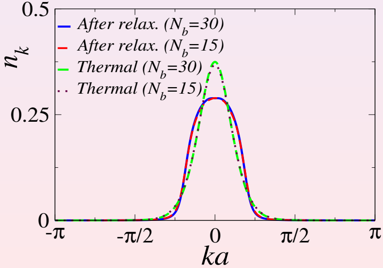

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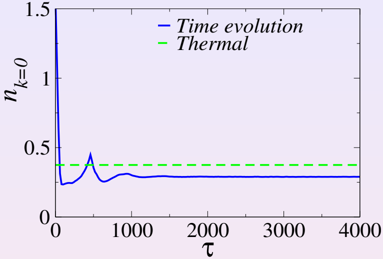


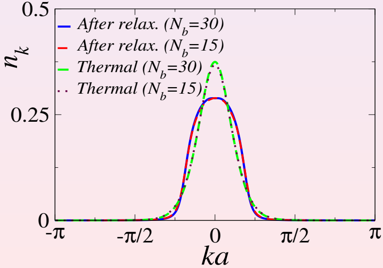
Generalized Gibbs ensemble

$$\hat{\rho}_{\text{GGE}} = Z_c^{-1} \exp\left[-\sum_m \lambda_m \hat{I}_m\right]$$

$$Z_c = \mathrm{Tr}\left\{\exp\left[-\sum_m \lambda_m \hat{I}_m\right]\right\}$$

$$\mathrm{Tr}\left\{\hat{I}_m \hat{\rho}_{\text{GGE}}\right\} = \langle \hat{I}_m \rangle_{\tau=0}$$

n_k after relaxation



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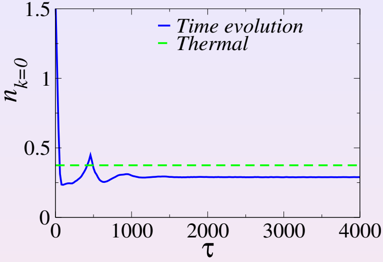
The constraint

$$\mathrm{Tr}\left\{\hat{I}_m \hat{\rho}_{\text{GGE}}\right\} = \langle \hat{I}_m \rangle_{\tau=0}$$

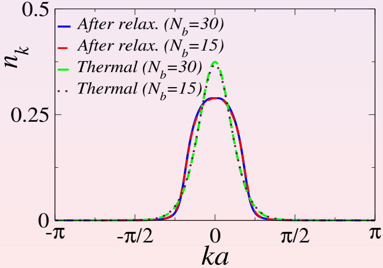
results in

$$\lambda_m = \ln\left[\frac{1 - \langle \hat{I}_m \rangle_{\tau=0}}{\langle \hat{I}_m \rangle_{\tau=0}}\right]$$

Evolution of $n_{k=0}$



n_k after relaxation



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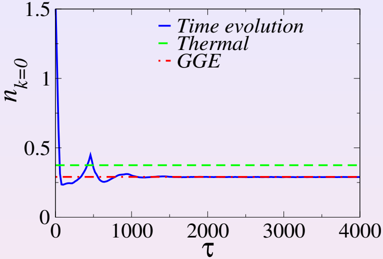

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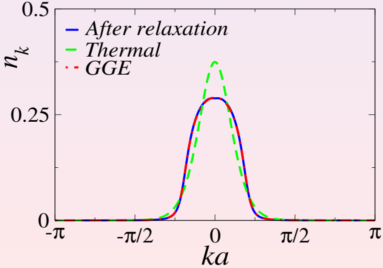

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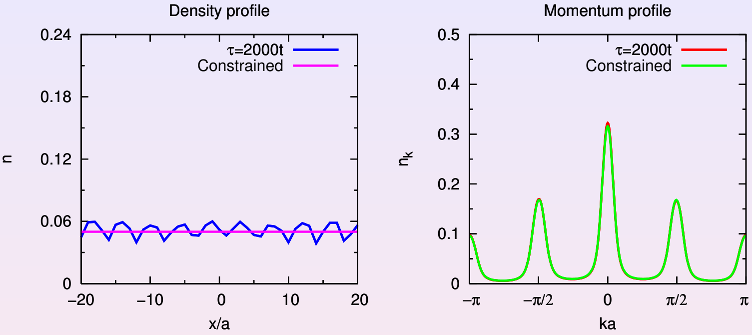
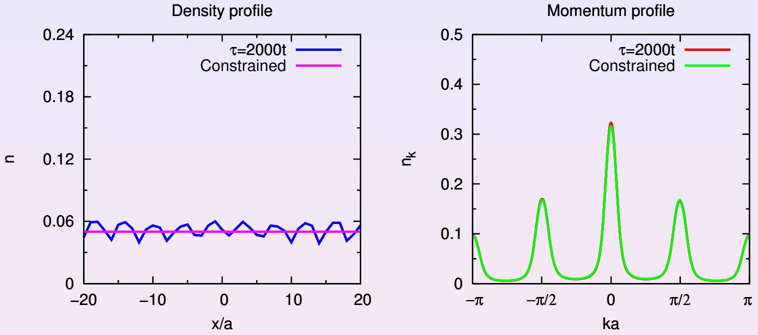


Generalized Gibbs ensemble (GGE)



Why does the GGE work?

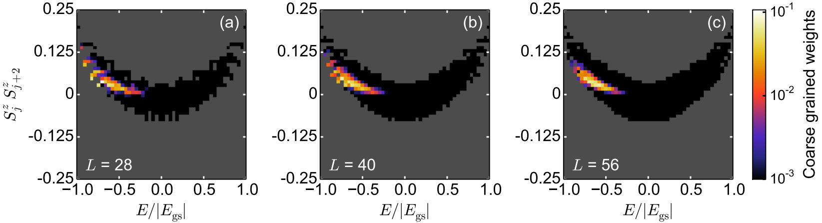
Generalized eigenstate thermalization:

A. C. Cassidy, C. W. Clark, and MR, Phys. Rev. Lett. **106**, 140405 (2011).
L. Vidmar and MR, J. Stat. Mech. 064007 (2016).

Relevant to generalized thermodynamic Bethe ansatz approaches:
J.-S. Caux and F. H. L. Essler, Phys. Rev. Lett. **110**, 257203 (2013).

Generalized eigenstate thermalization (1D-TFIM)

Weight of eigenstate expectation values in the diagonal ensemble



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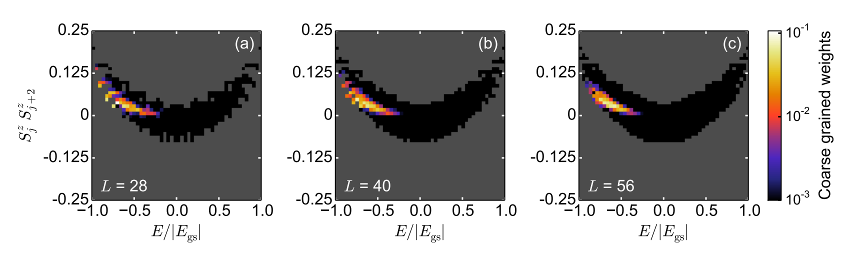

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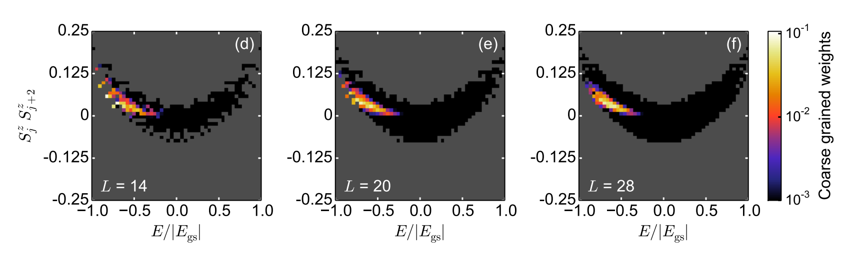

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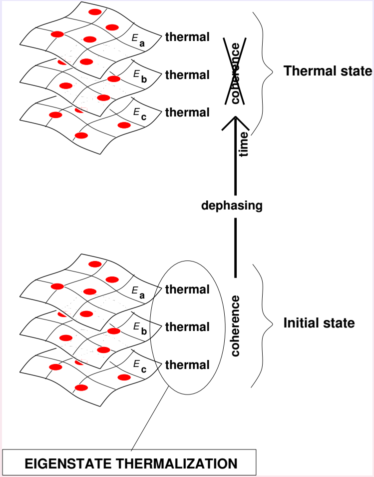

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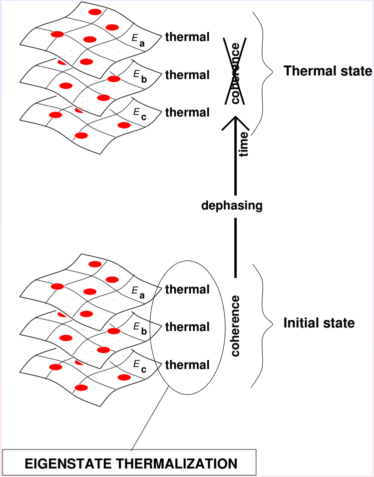

Collaborators

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- Yariv Kafri (Technion)
- Ehsan Khatami (San Jose State U)
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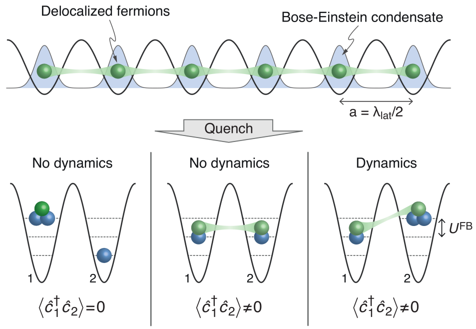
- Maxim Olshanii (U Mass Boston)
- Anatoli Polkovnikov (Boston U)
- Guido Pupillo (U Strasbourg)
- Lea F. Santos (Yeshiva U)
- Mark Srednicki (UC Santa Barbara)
- Lev Vidmar (Penn State)

Supported by:

Coherence after quenches in Bose-Fermi mixtures



S. Will, D. Iyer, and MR
Nat. Commun. **6**, 6009 (2015).

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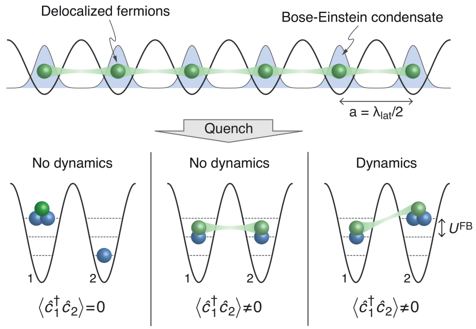

$$\langle \hat{c}^\dagger_{j\neq 0} \hat{c}_0 \rangle(t) = \frac{n_F \sin[\pi n_F j] e^{2n_B[\cos(U^{\rm FB} t/\hbar)-1]}}{j},$$

n_F and n_B are the fermion and boson fillings.

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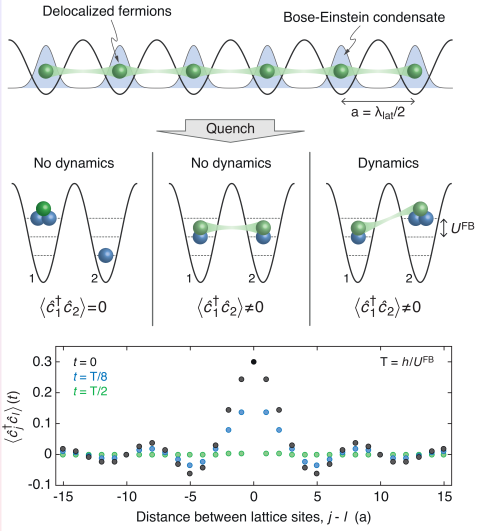

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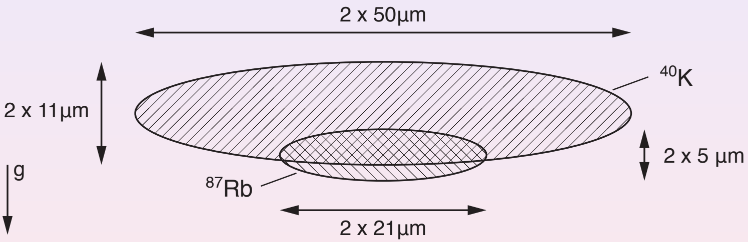

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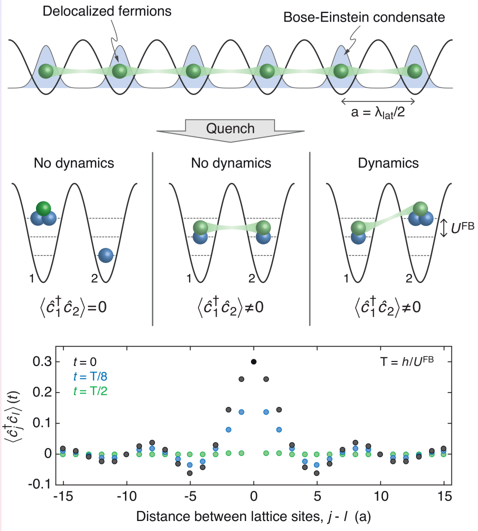

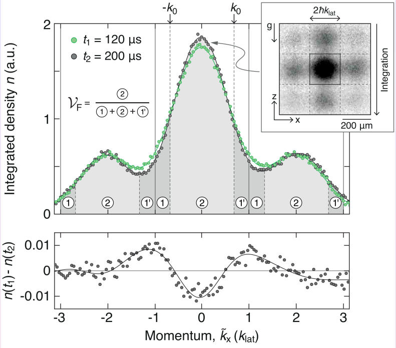

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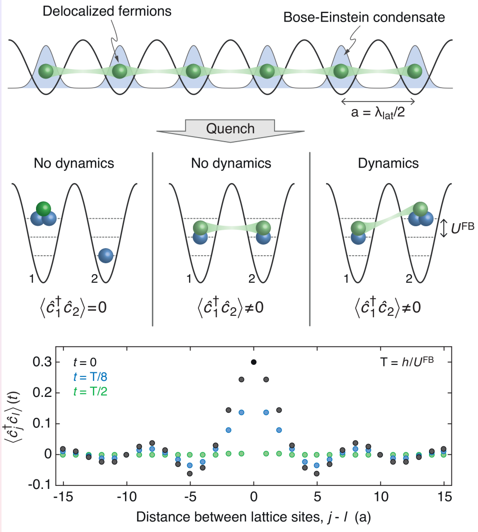

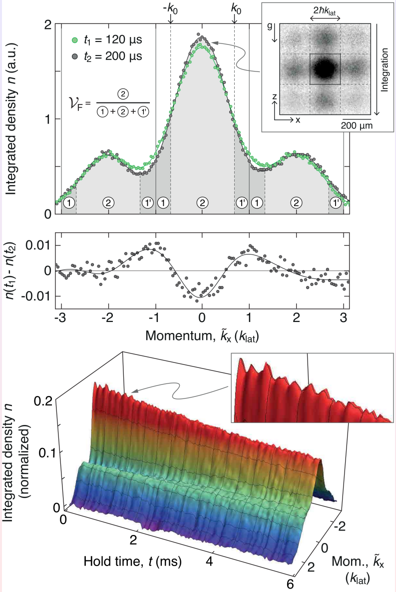

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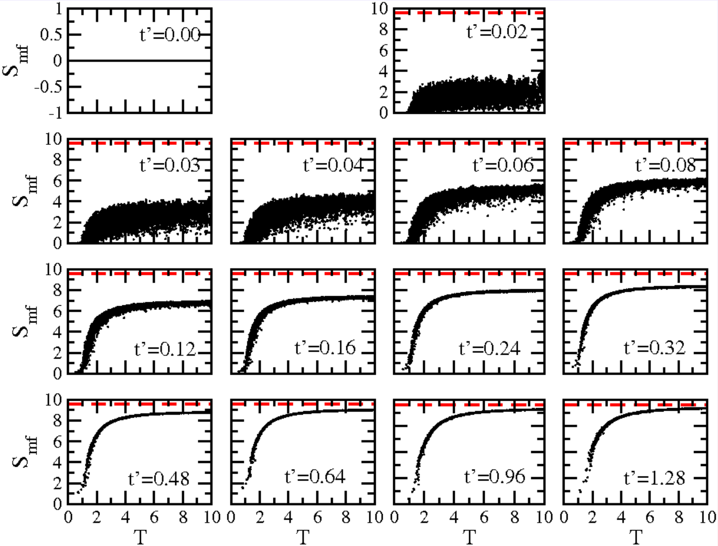
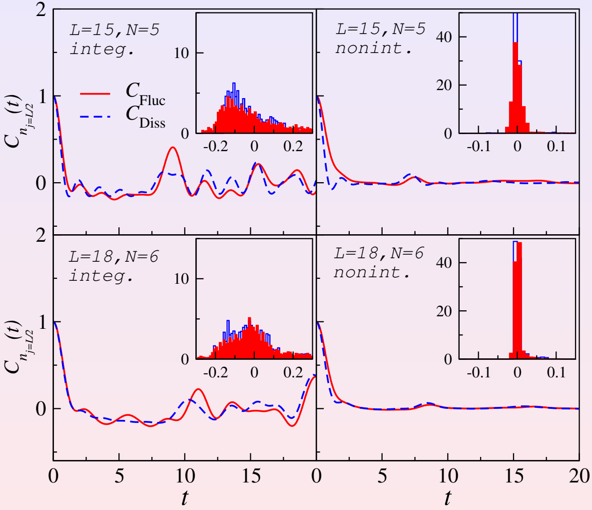
Information entropy ($S_j = -\sum_{k=1}^{D} |c_j^k|^2 \ln |c_j^k|^2$)



L.F. Santos and MR, PRE **81**, 036206 (2010); PRE **82**, 031130 (2010).

Fluctuation-dissipation theorem (dipolar bosons)

Occupation in the center of the trap ($n_{j=L/2}$)



Hamiltonian

$$\hat{H} = -J \sum_{j=1}^{L-1} \left(\hat{b}_j^\dagger \hat{b}_{j+1} + \text{H.c.} \right) + V \sum_{j<l} \frac{\hat{n}_j \hat{n}_l}{|j-l|^3} + g \sum_j x_j^2 \, \hat{n}_j$$

magnetic atoms, polar molecules

Relaxation dynamics

$$O(t) = C(t) O(t=0)$$

where

$$C(t) = \frac{\overline{O(t+t')O(t')}}{\overline{(O(t'))^2}}$$

Srednicki, JPA **32**, 1163 (1999).

E. Khatami, G. Pupillo, M. Srednicki, and MR, PRL **111**, 050403 (2013).