# Quantum coherence in thermodynamics: coherent second laws and passive memories

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3rd KIAS Workshop on Quantum Information and Thermodynamics, Korea Institute for Advanced Study
20 September 2017

# Outline

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- Every CPTP map has a Kraus operator representation

$$\mathcal{E}(\rho) = \sum_j K_j \rho K_j^\dagger,$$

with $\sum_j K_j^\dagger K_j = \mathbb{1}_{\mathcal{H}}$.

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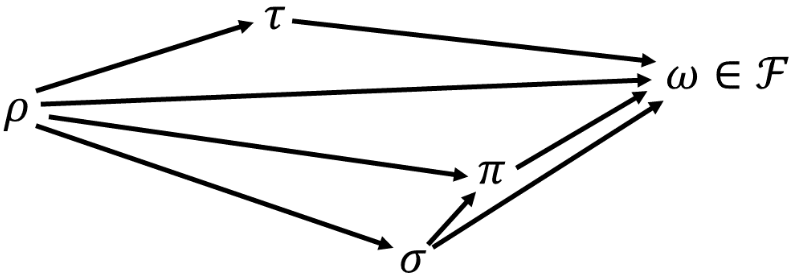
Partial order of resources under allowed operations



# Examples of quantum resource theories

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- Resource theory of thermal inequilibrium
  - Free operations: Energy-conserving interactions
  - Free states: Thermal states

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System $\mathcal{S}$ under Hamiltonian $H$

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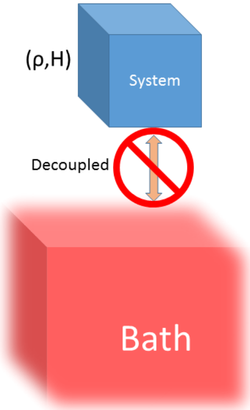

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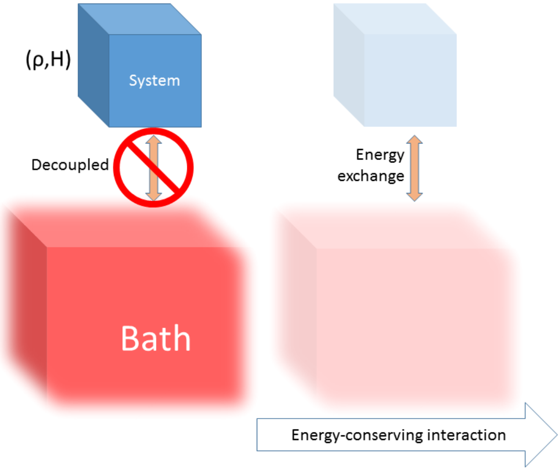

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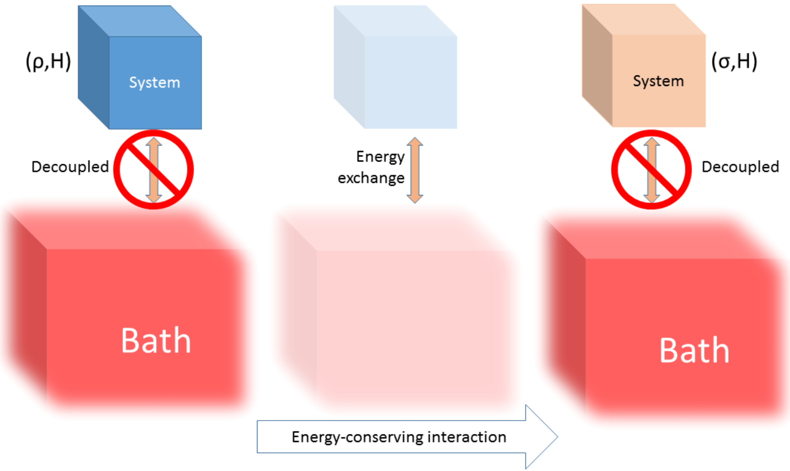

# Coherence under thermal operations

- Coherence is useful in quantum computing, work extraction, etc. [Åberg, *Phys. Rev. Lett.* **113** (2014)]; [Skrzypczyk *et al.*, *Nat. Commun.* **5** (2014)]; [Korzekwa *et al.*, *New J. Phys.* **5** (2016)]

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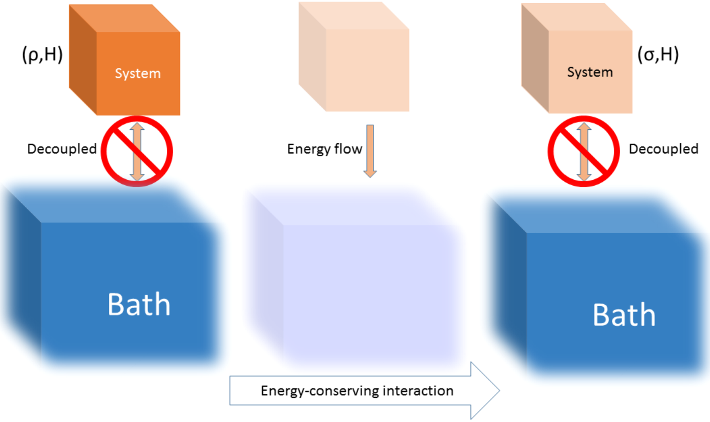
- Hard to get exact “second laws” because of complex interplay between entropies and time-translation symmetry
- Our work: Some simple laws in the low-temperature limit

Thermal operations in the low-temperature regime



# “Cooling maps” emerge

In the low-temperature regime, possible channels have elegant Kraus operator representation:

1. Some *diagonal* Kraus operators

$$K_i = \sum_j \lambda_j^{(i)} \left|E_j\right\rangle \left\langle E_j\right| ;$$

2. Other Kraus operators of the form

$$J_{jk} = \mu_{jk} \left|E_j\right\rangle \left\langle E_k\right| ,$$

one for each pair $(j, k)$ with $j < k$. (Only one nonzero matrix element in every such Kraus operator)

“Cooling maps” model

# Results: Upper-triangular majorization

$\rho \mapsto \sigma$ possible only if

$$\rho_{dd} \geq \sigma_{dd}$$

$$\rho_{d-1,d-1} + \rho_{dd} \geq \sigma_{d-1,d-1} + \sigma_{dd}$$

$$\vdots$$

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- These conditions emerge as the low-temperature limit of the conditions in the coherence-less case.
- But there are *more conditions*—on the off-diagonal terms.

# Full low-temperature condition

**Theorem**

*For states $\rho$, $\sigma$, define matrix Q:*

$$Q_{jk} = \begin{cases} \min\left(\frac{\sigma_{jj}}{\rho_{jj}}, 1\right), & \text{if } j = k; \\ \frac{\sigma_{jk}}{\rho_{jk}}, & \text{if } j \neq k. \end{cases}$$

*Then, $\rho \mapsto \sigma$ possible via cooling maps iff:*

1. $Q \geq 0$;
2. $\sum_{k \geq j} \rho_{kk} \geq \sum_{k \geq j} \sigma_{kk}$ $\forall j$ *("Upper-triangular majorization")*

[VN and Gour, *Nat. Commun.* **6** (2015)]

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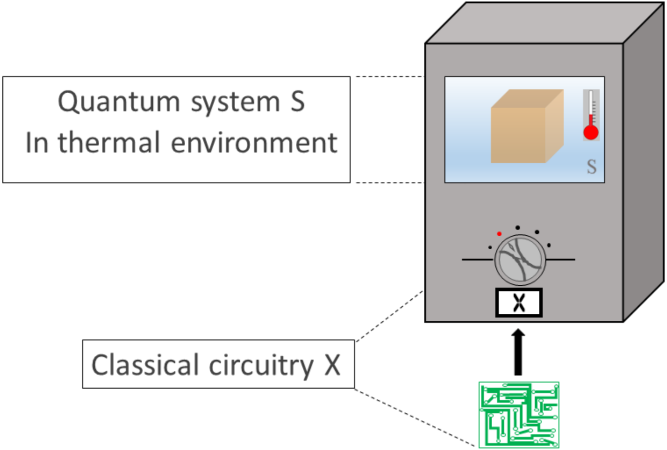
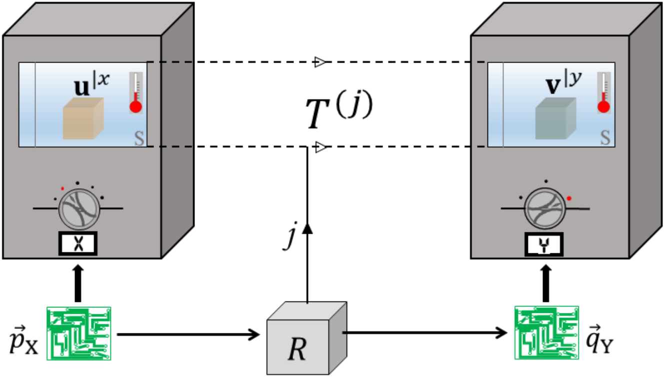
# Quantum thermal device controlled by classical components



# Classically-controlled thermal operations



# Classically-controlled thermal operations

Results:

- Characterization of free states and resources
- Quantification of resources
- Conditions for resource conversion:
  - Single-copy
  - Many-copy
  - State-to-ensemble
  - Ensemble-to-state

[VN and Gour, *Phy. Rev. A* **95** (2017)]

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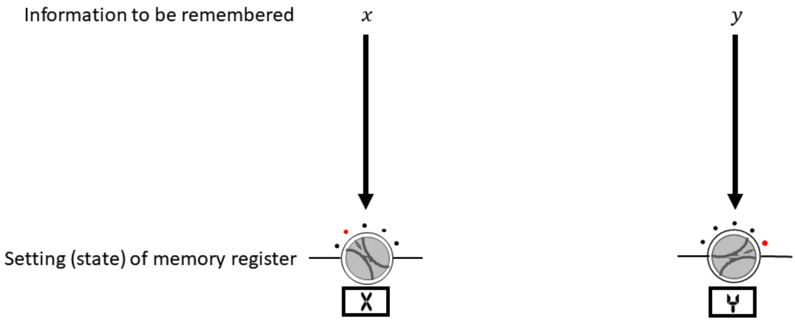

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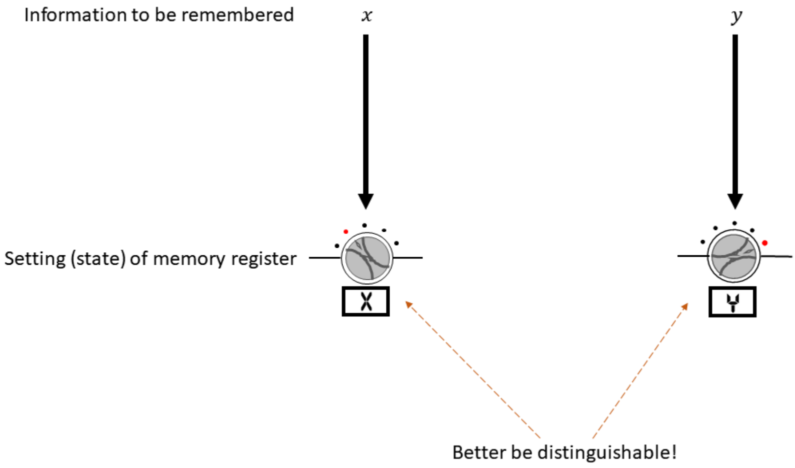

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- "Reading memory" $\equiv$ "distinguishing states $\mathcal{E}_x(\rho)$"
- Memory capacity limited by
  - Encoding operation repertoire $\mathcal{O}$;
  - Blank tape state $\rho$

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# Thermally passive qubit memory

Initial state of the memory determines accessible states under TO

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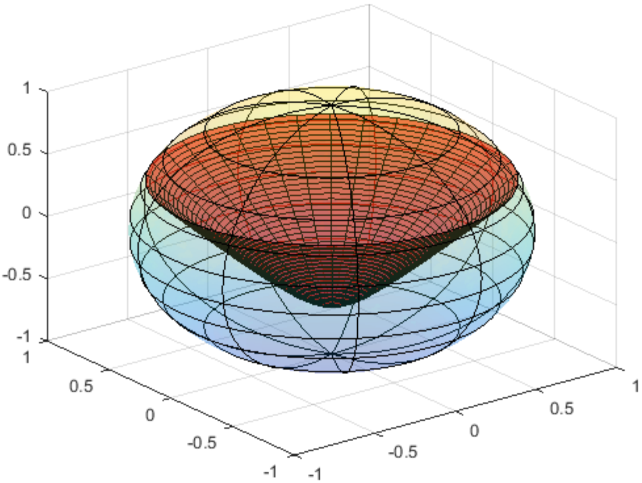

# Thermally passive qubit memory

Formal definition of memory capacity:

$$I(\rho) := \sup_{\{(p_j,\mathcal{E}_j)\}_j} \chi\left[\{p_j, \mathcal{E}_j(\rho)\}\right], \tag{1}$$

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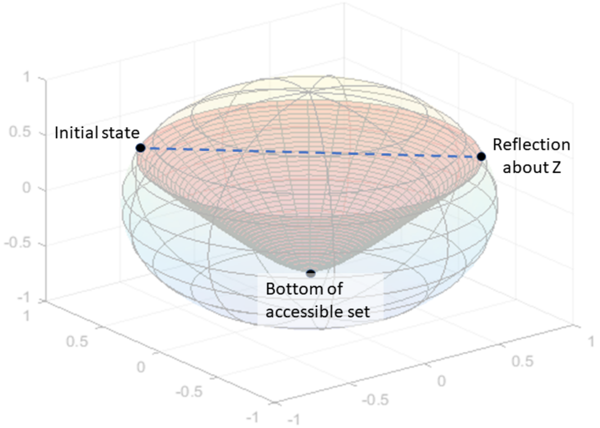
where

$$\chi\left[\{p_j, \sigma_j\}\right] = S\left(\sum_j p_j \sigma_j\right) - \sum_j p_j S\left(\sigma_j\right). \tag{2}$$

(Holevo bound on accessible information)

# Optimal encoding scheme

Three-state ensemble attains the optimum rate:



# Qubit memory capacity

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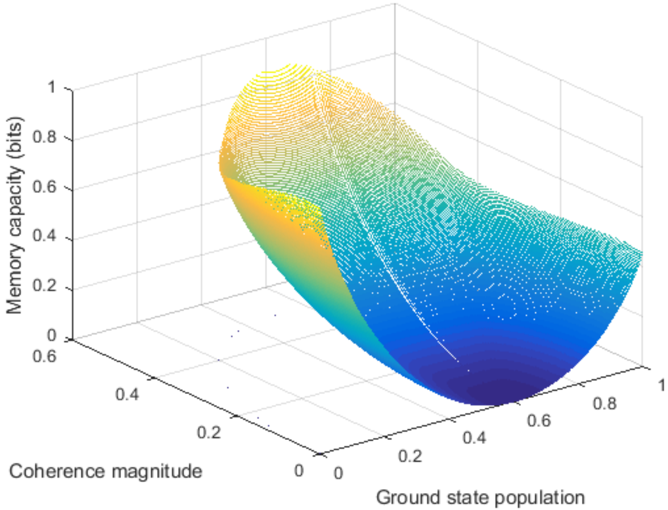

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- Passive memories under other resource theories

# Thanks!

- Participants, organizers, and sponsors, KIAS Workshop
- Collaborators, including supervisors G. Gour and M. Gu
- My institution NTU